Matching between TMD and Collinear Factorizations

Nobuo Sato
ODU/JLab

POETIC 19
LBL

A historical note on DY p_{T} spectrum

VOLUME 40, NUMBER 17 PHYSICAL REVIEW LETTERS 24 APRIL 1978

Testing Quantum Chromodynamics in the Hadroproduction of Real and Virtual Photons

F. Halzen and D. M. Scott
University of Wisconsin, Department of Physics, Madison, Wisconsin 53706
(Received 12 January 1978)

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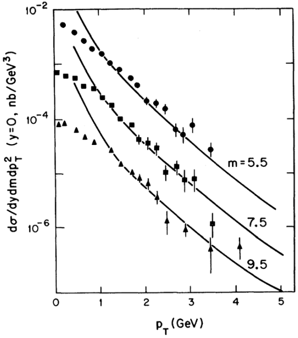
$pp \quad \sqrt{s} = 27.4 \text{ GeV} \quad y = 0$



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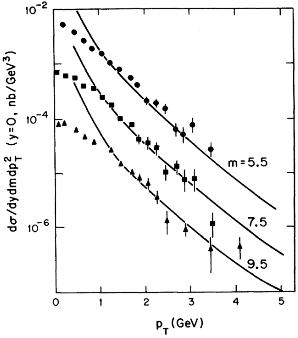

of lepton pairs shown in Figs. 2 and 3. Our procedure[7] predicts normalization as well as the shape of the p_T distribution, and therefore the agreement with the data for large values of p_T is a definite success of the theory and a detailed experimental study of the large-p_T behavior for different values of m constitutes the most direct and meaningful test of the theory. Notice the

Fact check...

PDFs from 1978 (PhysRevD.18.3378)

$$u_v(x) = 2.99(1-x)^4(1+5.99x-2.63x^{1/2})x^{-1/2},$$

$$d_v(x) = 1.02(1-x)^5(1+5.75x)x^{-1/2},$$

$$s(x) = 0.44(1-x)^9x^{-1}(0.8760+3.087x-4.408x^2$$

$$-74.07x^3+209.9x^4-102.3x^5).$$

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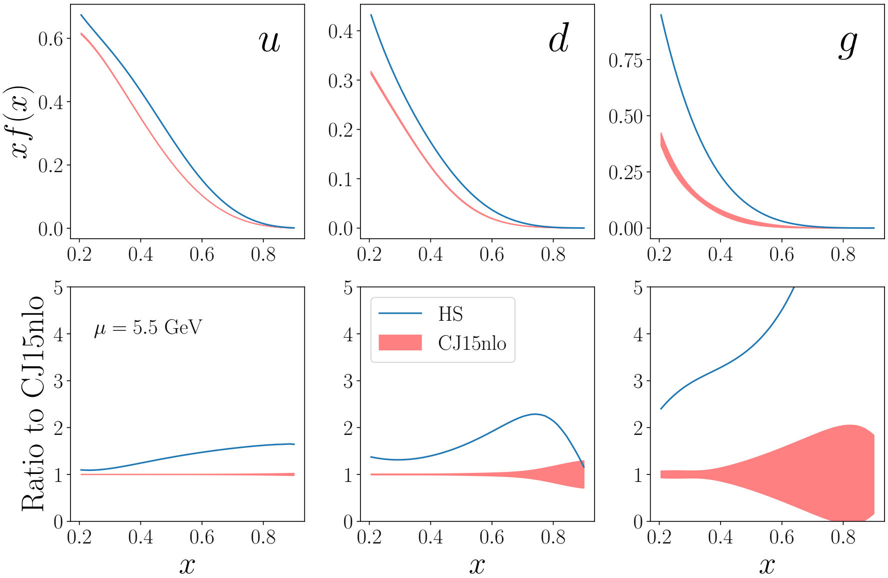
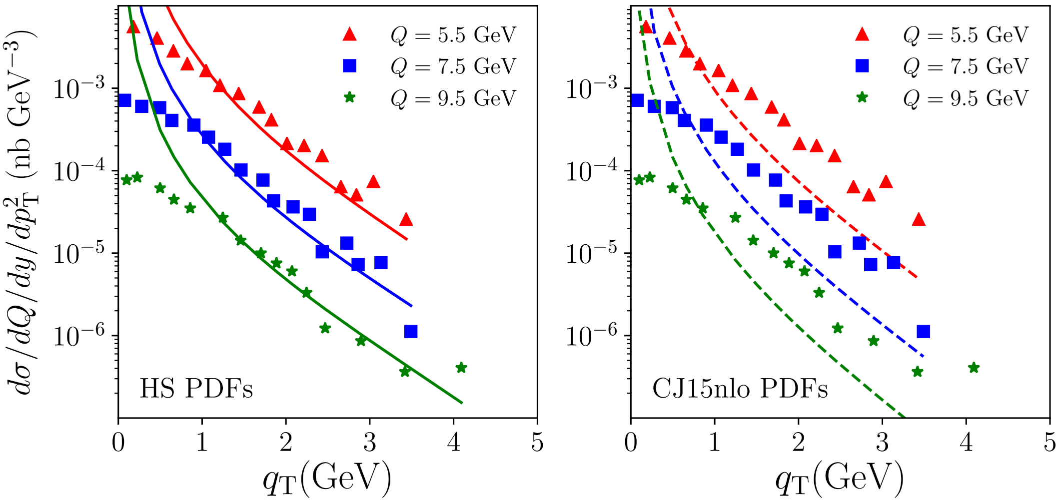
$$G^P(x) = 3(1-x)^5/x,$$

so that indeed

$$\int_0^1 xG^P(x)dx = 0.5$$

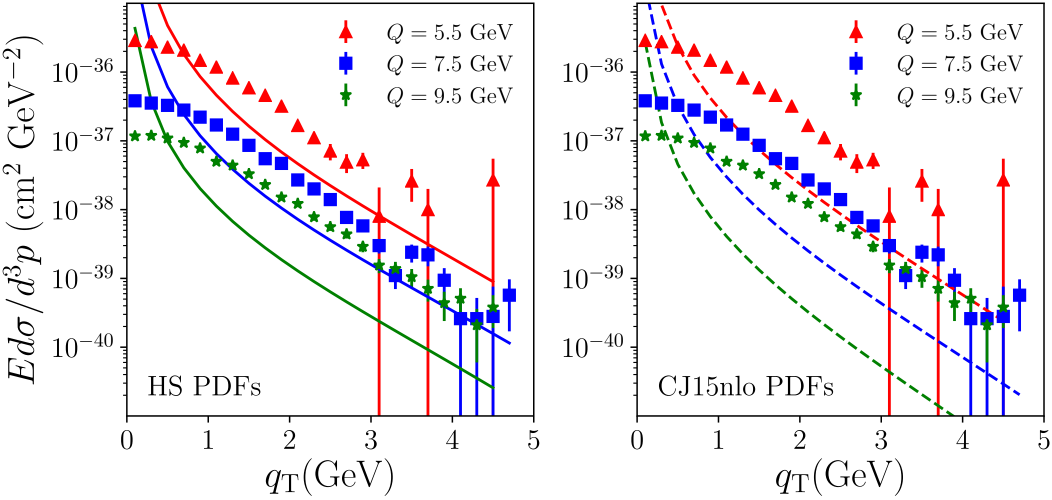

u
d
g
x f(x)
Ratio to CJ15nlo
μ = 5.5 GeV
HS
CJ15nlo
x
x
x

1978   2015



Data from Phys.Rev.Lett. 40 (1978) 1117

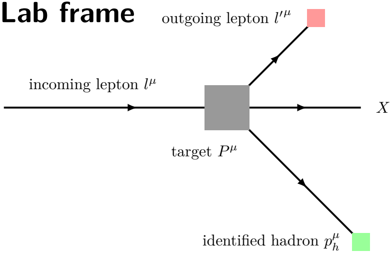
1978   2015

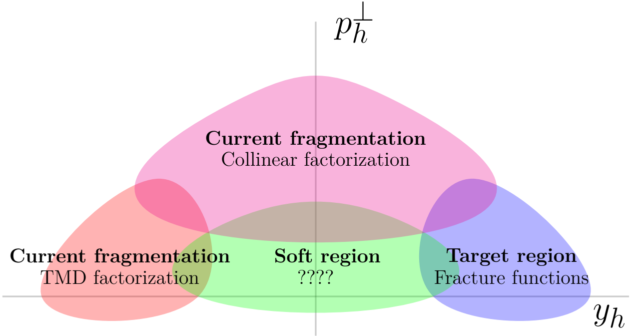


Data from Phys.Rev. D23 (1981) 604-633

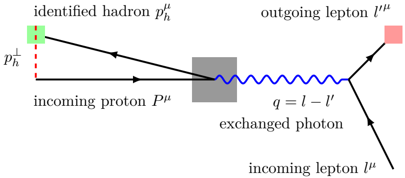
SIDIS

Lab frame





Breit frame



- **Key question** : How is $p_h^\perp$ generated at short distances?
- **Different regions** are sensitive to distinct physical mechanisms

Nucleon structures accessible in SIDIS

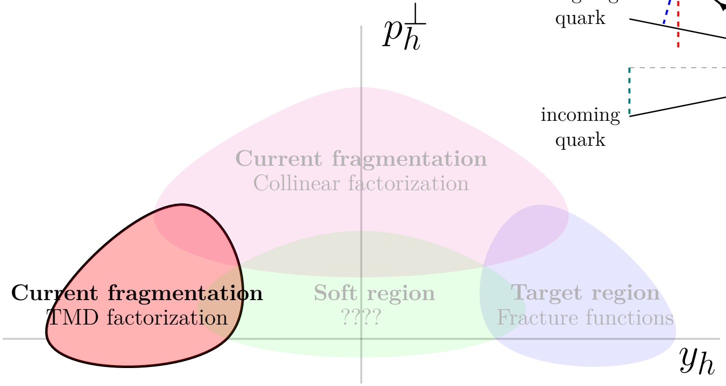
F_i	Standard label	β_i
F_1	$F_{UU,T}$	1
F_2	$F_{UU,L}$	ε
F_3	F_{LL}	$S_{\|\|}\lambda_e\sqrt{1-\varepsilon^2}$
F_4	$F_{UT}^{\sin(\phi_h+\phi_S)}$	$\|\vec{S}_\perp\|\varepsilon\sin(\phi_h+\phi_S)$
F_5	$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\sin(\phi_h-\phi_S)$
F_6	$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\varepsilon\sin(\phi_h-\phi_S)$
F_7	$F_{UU}^{\cos 2\phi_h}$	$\varepsilon\ \cos(2\phi_h)$
F_8	$F_{UT}^{\sin(3\phi_h-\psi_S)}$	$\|\vec{S}_\perp\|\varepsilon\sin(3\phi_h-\phi_S)$
F_9	$F_{LT}^{\cos(\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\lambda_e\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)$
F_{10}	$F_{UL}^{\sin\ 2\phi_h}$	$S_{\|\|}\varepsilon\sin(2\phi_h)$
F_{11}	$F_{LT}^{\cos\phi_S}$	$\|\vec{S}_\perp\|\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S$
F_{12}	$F_{LL}^{\cos\phi_h}$	$S_{\|\|}\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h$
F_{13}	$F_{LT}^{\cos(2\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)$
F_{14}	$F_{UL}^{\sin\phi_h}$	$S_{\|\|}\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h$
F_{15}	$F_{LU}^{\sin\phi_h}$	$\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h$
F_{16}	$F_{UU}^{\cos\phi_h}$	$\sqrt{2\varepsilon(1+\varepsilon)}\ \cos\phi_h$
F_{17}	$F_{UT}^{\sin\phi_S}$	$\|\vec{S}_\perp\|\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S$
F_{18}	$F_{UT}^{\sin(2\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)$

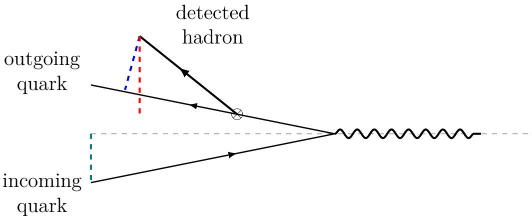
$$\frac{d\sigma}{dx\ dy\ d\Psi\ dz\ d\phi_h\ dP_{hT}^2} \sim \sum_{i=1}^{18} F_i(x,z,Q^2,P_{hT}^2)\beta_i$$

Name	Symbol	meaning
upol. PDF	f_1^q	U. pol. quarks in U. pol. nucleon
pol. PDF	g_1^q	L. pol. quarks in L. pol. nucleon
Transversity	h_1^q	T. pol. quarks in T. pol. nucleon
Sivers	$f_{1T}^{\perp(1)q}$	U. pol. quarks in T. pol. nucleon
Boer-Mulders	$h_1^{\perp(1)q}$	T. pol. quarks in U. pol. nucleon
Boer-Mulders	$h_1^{\perp(1)q}$	T. pol. quarks in U. pol. nucleon
⋮	⋮	⋮
FF	D_1^q	U. pol. quarks to U. pol. hadron
Collins	$H_1^{\perp(1)q}$	T. pol. quarks to U. pol. hadron
⋮	⋮	⋮

Regions in SIDIS

small transverse momentum

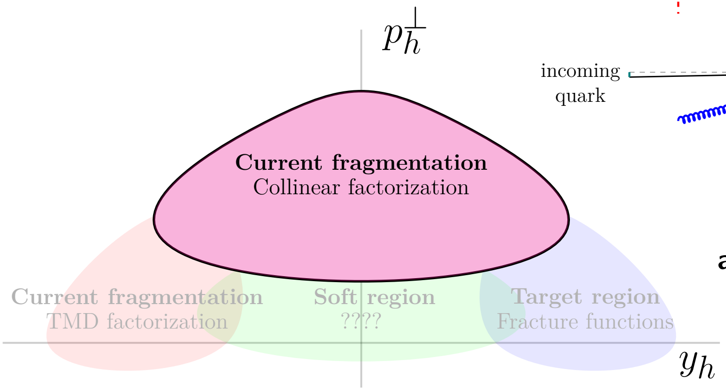


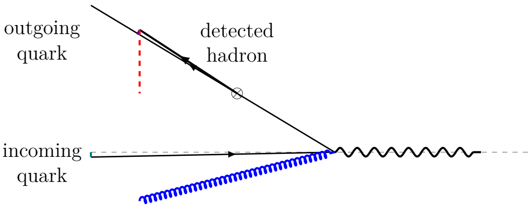


aka W

Regions in SIDIS

large transverse momentum

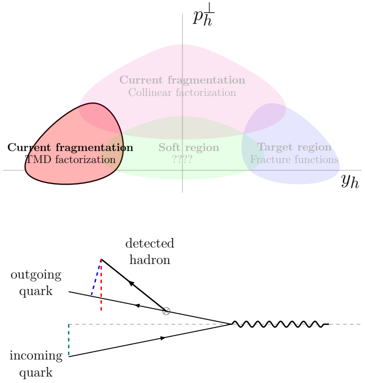




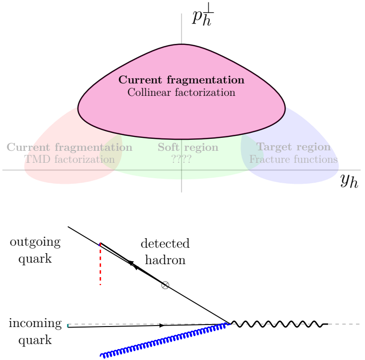
aka FO **(=fixed order)**

Regions in SIDIS

small transverse momentum W

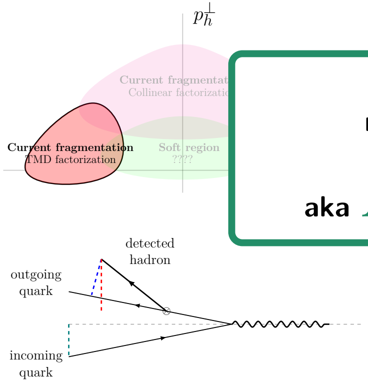


large transverse momentum FO

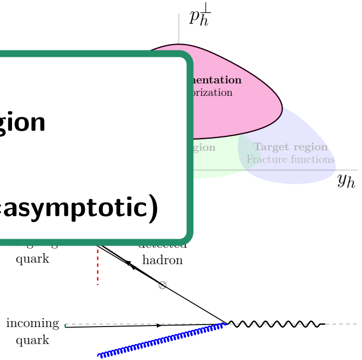


Regions in SIDIS

small transverse momentum W



large transverse momentum FO



matching region

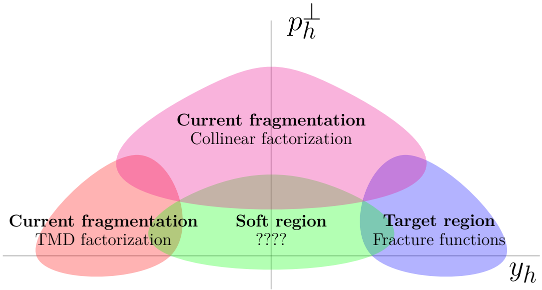
aka ASY **(=asymptotic)**

What is large or small transverse momentum?

- Scale separation

$$q_{\mathrm{T}}/Q$$

$$z = \frac{P \cdot p_h}{P \cdot q}, \qquad q_{\mathrm{T}} = p_h^{\perp}/z$$



- Merging factorization theorems

$$\frac{d\sigma}{dx dQ^2 dz dp_h^{\perp}} = \mathrm{W} + \mathrm{FO} - \mathrm{ASY} + \mathcal{O}(m^2/Q^2)$$

$$\sim \mathrm{W} \quad \text{for } q_{\mathrm{T}} \ll Q$$

$$\sim \mathrm{FO} \quad \text{for } q_{\mathrm{T}} \sim Q$$

Small transverse momentum Collins, Rogers PRD91 (2015)

$$\mathrm{W} = \sum_f H_f(Q,\mu) \int \frac{d^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2} e^{-i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}} F_{f/N}(x,\boldsymbol{b}_\mathrm{T},\mu,\zeta_F) D_{h/f}(z,\boldsymbol{b}_\mathrm{T},\mu,\zeta_D) + O(q_\mathrm{T}^2/Q^2)$$

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- **CSS evolution equation**

$$\frac{\partial \ln F_{f/N}(x,\boldsymbol{b}_\mathrm{T},\zeta_F,\mu)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(\boldsymbol{b}_\mathrm{T},\mu)$$

+ Related to vacuum matrix elements of products of Wilson Lines
+ Independent of flavor, target and spin
+ Independent of x
+ Universal across TMDs and processes

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- **RG equations**

$$\frac{d\tilde{K}(\boldsymbol{b}_\mathrm{T},\mu)}{d\ln\mu} = -\gamma_K(\alpha_S(\mu))$$

$$\frac{d\ln F_{f/N}(\boldsymbol{b}_\mathrm{T},\mu)}{d\ln\mu} = \gamma_f(\alpha_S(\mu),1) - \frac{1}{2}\gamma_K(\alpha_S(\mu))\ln\frac{\zeta_F}{\mu^2}$$

$$\frac{d}{d\ln\mu}\ln H(Q,\mu) = -2\gamma_f(\alpha_S(\mu),1) + \gamma_K(\alpha_S(\mu))\ln\frac{Q^2}{\mu^2}$$

Small transverse momentum Collins, Rogers PRD91 (2015)

Valid for $0 \le q_{\rm T} \ll Q$

$$
\begin{aligned}
\mathrm{W} = &\sum_f H_f(Q,\mu) \int \frac{d^2 \boldsymbol{b}_{\rm T}}{(2\pi)^2} e^{-i \boldsymbol{q}_{\rm T}\cdot \boldsymbol{b}_{\rm T}} \\
&\times e^{-g_{f/N}(x,b_{\rm T},b_{\max})} \int_x^1 \frac{d\hat{x}}{\hat{x}} \boldsymbol{f}_{f/N}(\hat{x},\mu_{b_*}) \tilde{C}_{f/p}(x/\hat{x}, b_*, \mu_{b_*}^2, \alpha_S(\mu_{b_*})) \\
&\times e^{-g_{h/f}(z,b_{\rm T},b_{\max})} \int_z^1 \frac{d\hat{z}}{\hat{z}^3} \boldsymbol{d}_{h/f}(\hat{z},\mu_{b_*}) \tilde{C}_{h/f}(z/\hat{z}, b_*, \mu_{b_*}^2, \alpha_S(\mu_{b_*})) \\
&\times \left(\frac{Q^2}{Q_0^2}\right)^{-g_K(b_{\rm T},b_{\max})} \left(\frac{Q^2}{\mu_{b_*}^2}\right)^{\tilde{K}(b_*,\mu_{b_*})} \\
&\times \exp\left[\int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[2\gamma(\alpha_S(\mu'),1) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(\alpha_S(\mu'))\right]\right]
\end{aligned}
$$

Quantities in pink are non-perturbative

Large transverse momentum

Valid for $q_{\mathrm{T}} \sim Q$

$$\mathrm{FO} = \sum_q e_q^2 \int_{\frac{q_{\mathrm{T}}^2}{Q^2}\frac{xz}{1-z}+x}^{1} \frac{d\xi}{\xi - x}\; H(\xi)\; \boldsymbol{f}_q(\xi,\mu)\; \boldsymbol{d}_q(\zeta(\xi),\mu) + O(\alpha_S^2) + O(m^2/q^2)$$

- **Attention**: $\left(\frac{q_{\mathrm{T}}^2}{Q^2}\frac{xz}{1-z} + x\right) < \xi < 1$
- **large q_{T} probes large ξ in PDFs**
- Can be useful in collinear global fits

Matching region

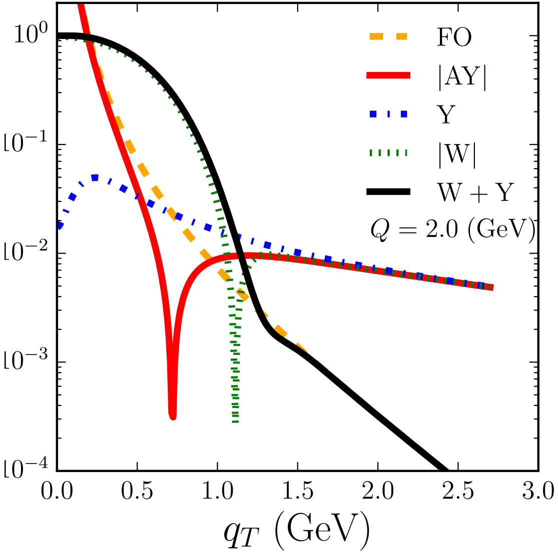
Valid for $0 \ll q_\mathrm{T} \ll Q$

$$\mathrm{ASY} \sim \boldsymbol{d}(z;\mu) \int_x^1 \frac{d\xi}{\xi} \boldsymbol{f}(\xi;\mu) P(x/\xi) + \boldsymbol{f}(x;\mu) \int_z^1 \frac{d\zeta}{\zeta} \boldsymbol{d}(\zeta;\mu) P(z/\zeta)$$
$$+ 2C_F \boldsymbol{f}(x;\mu) \boldsymbol{d}(z;\mu) \left(\ln\left(\frac{Q^2}{q_\mathrm{T}}\right) - \frac{3}{2} \right)$$

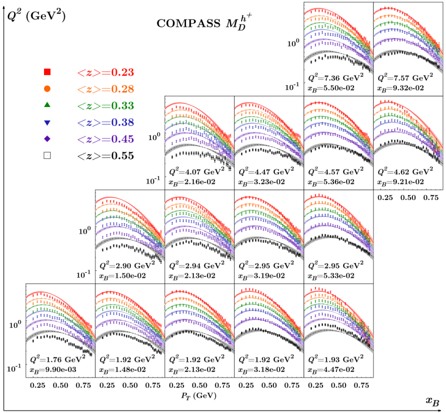
+ Interpolates between W and FO
+ $\mathrm{FO} - \mathrm{ASY} \equiv \mathrm{Y}$

Toy example: how we expect the W+FO-ASY to work

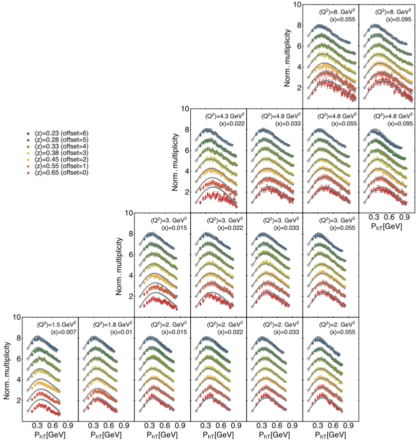
$$\frac{d\sigma}{dx dQ^2 dz dp_h^\perp}$$



Existing phenomenology



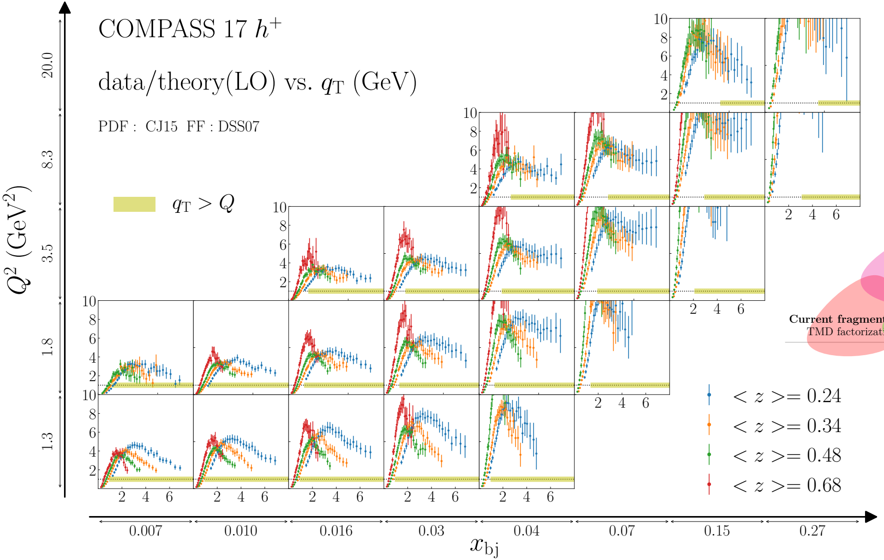
Anselmino et al

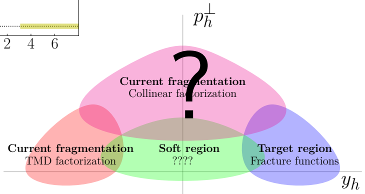


Bacchetta et al

- These analyses used only W (Gaussian, CSS) $\rightarrow$ no FO nor ASY
- Samples with $q_T/Q \sim 1.63$ have been included
- **BUT TMDs are only valid for $q_T/Q \ll 1$!**

FO @ LO predictions (DSS07) Gonzalez, Rogers, NS, Wang PRD98 (2018)



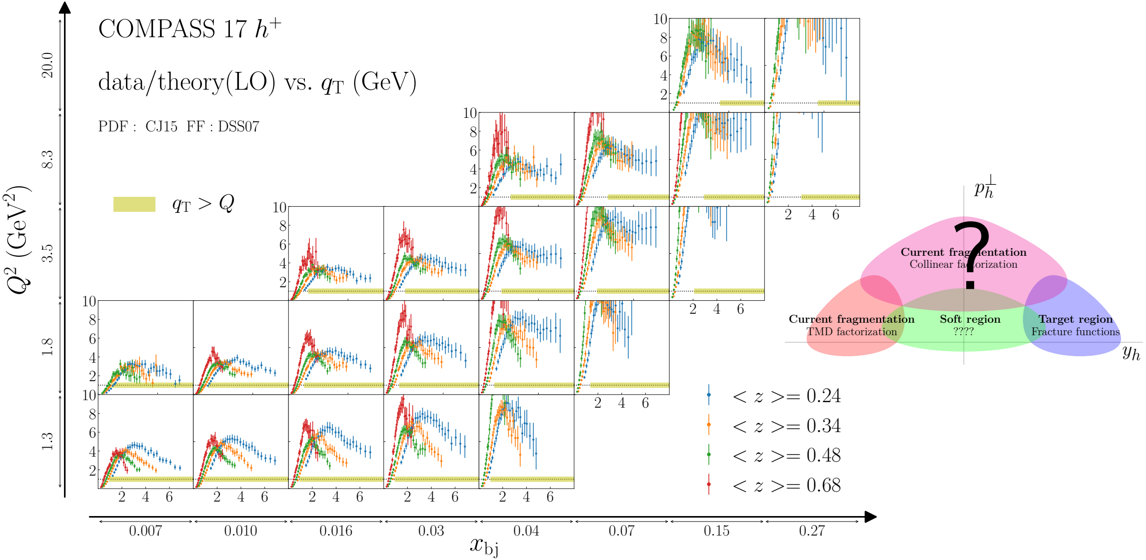


Trouble with large transverse momentum

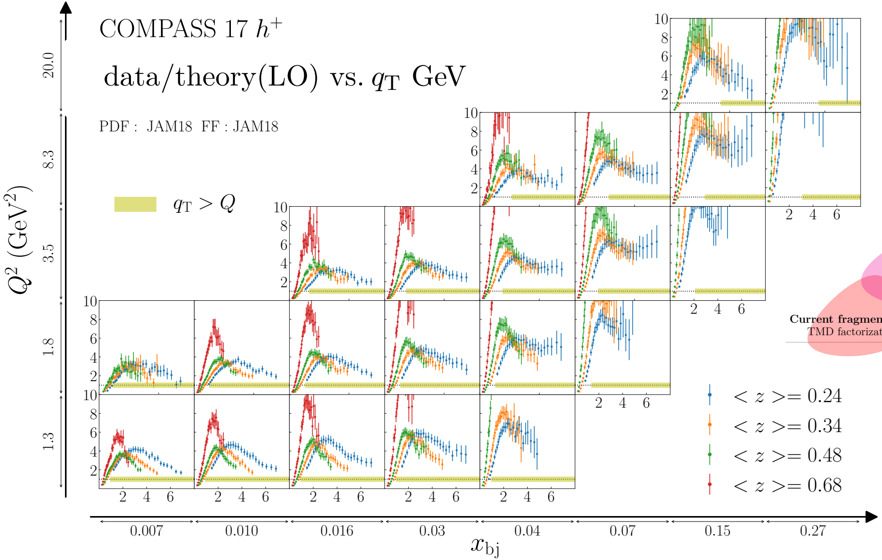
$$\text{FO} = \sum_q e_q^2 \int_{\frac{q_T^2}{Q^2}\frac{xz}{1-z}+x}^{1} \frac{d\xi}{\xi - x}\ H(\xi)\ \boldsymbol{f}_q(\xi,\mu)\ \boldsymbol{d}_q(\zeta(\xi),\mu) + O(\alpha_S^2) + O(m^2/q^2)$$

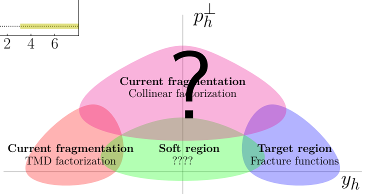
+ FFs needs to be updated?

FO @ LO predictions (DSS07) Gonzalez, Rogers, NS, Wang PRD98 (2018)



FO @ LO predictions (JAM18) Gonzalez, Rogers, NS, Wang PRD98 (2018)





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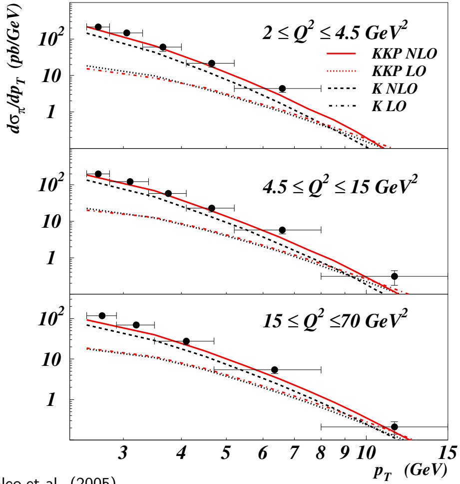
+ $O(\alpha_S^2)$ corrections might be important

order α_S^2 corrections to FO



Daleo,et al. (2005)
PRD.71.034013

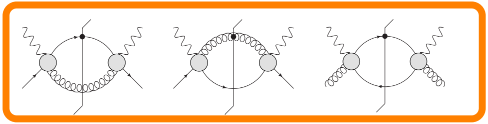
- There are strong indications that order α_S^2 corrections are very important
- An order of magnitude correction at small p_T.
- As a sanity check, we need to have an independent calculation

$O(\alpha_S^2)$ **calculation** (J. Gonzalez-Hernandes, T.C Rogers, NS, B. Wang - PRD99 (2019))

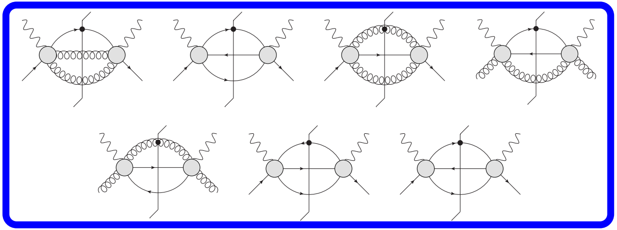
$$W^{\mu\nu}(P, q, P_H) = \int_{x-}^{1+} \frac{d\xi}{\xi} \int_{z-}^{1+} \frac{d\zeta}{\zeta^2} \hat{W}_{ij}^{\mu\nu}(q, x/\xi, z/\zeta) f_{i/P}(\xi) d_{H/j}(\zeta)$$

$$\{\mathrm{P}_g^{\mu\nu}\hat{W}_{\mu\nu}^{(N)}; \mathrm{P}_{PP}^{\mu\nu}\hat{W}_{\mu\nu}^{(N)}\} \equiv \frac{1}{(2\pi)^4}\int \{|M_g^{2\to N}|^2; |M_{pp}^{2\to N}|^2\}\, \mathrm{d}\Pi^{(N)} - \mathrm{Subtractions}$$

Born/Virtual

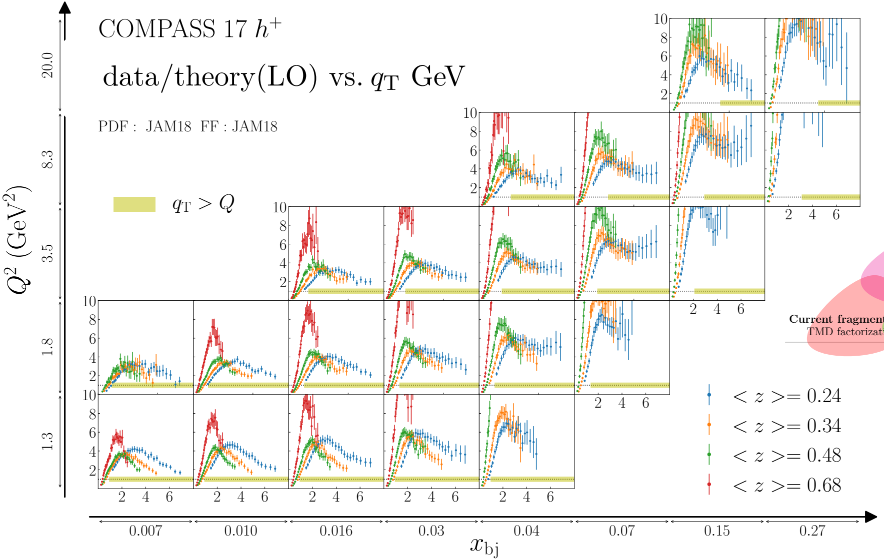


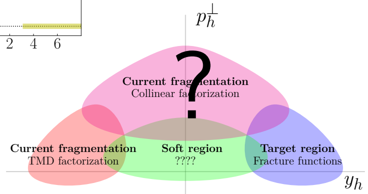
Real



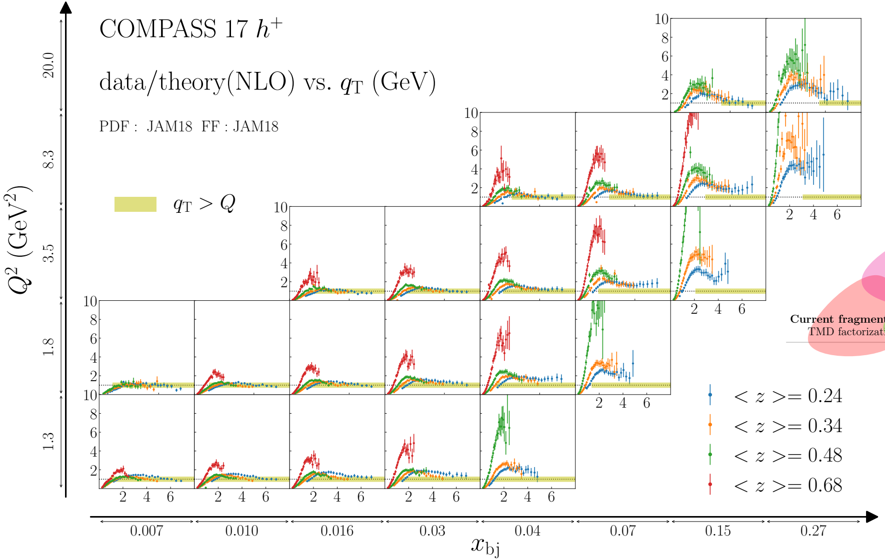
- ✓ Generate all $2 \to 2$ and $2 \to 3$ squared amplitudes
- ✓ Evaluate $2 \to 2$ virtual graphs (Passarino-Veltman)
- ✓ Integrate 3-body PS analytically
- ✓ Check cancellation of IR poles

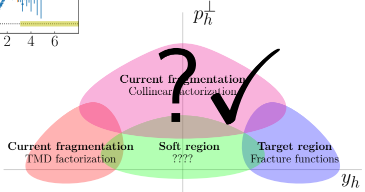
FO @ LO predictions (JAM18)





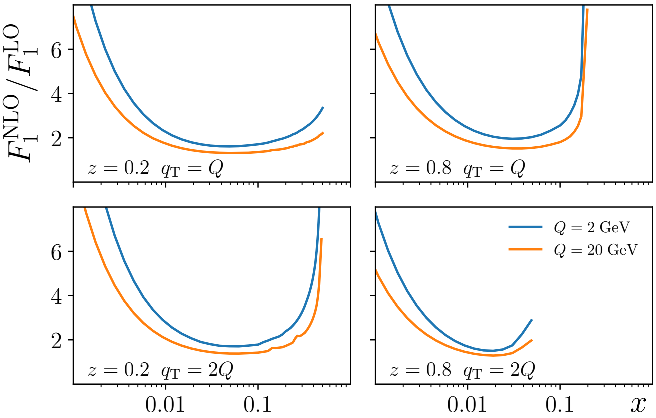
FO @ NLO (JAM18)


COMPASS 17 h^+
data/theory(NLO) vs. q_T (GeV)
PDF : JAM18 FF : JAM18
$q_T > Q$
Q^2 (GeV^2)
20.0
8.3
3.5
1.8
1.3
x_{bj}
0.007
0.010
0.016
0.03
0.04
0.07
0.15
0.27
$< z >= 0.24$
$< z >= 0.34$
$< z >= 0.48$
$< z >= 0.68$


$p_h^\perp$
Current fragmentation
Collinear factorization
Current fragmentation
TMD factorization
Soft region
????
Target region
Fracture functions
y_h

Understanding the large x

(J. Gonzalez-Hernandes, T.C Rogers, NS, B. Wang - PRD99 (2019))

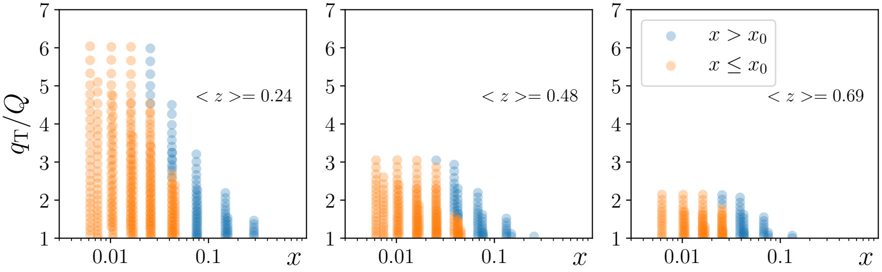


- Large corrections threshold corrections are observed
- The x at the minimum can be used as an indicator of where such corrections are expected to be large

Understanding the large x

(J. Gonzalez-Hernandes, T.C Rogers, NS, B. Wang - PRD99 (2019))

COMPASS kinematics



- The blue region might receive large threshold corrections
- This can potential explain why the $O(\alpha_S^2)$ fail to describe the data at large x

Summary and outlook (the next steps)

- A new combined PDF/FF (charged hadrons) global analysis with the inclusion of COMPASS q_{T} data is on the way (JAM19+)

- Need to identify regions of kinematics where FO and data are compatible

- Explore/combined alternative approaches based on power corrections $\rightarrow$ Liu, Qiu (arXiv:1907.06136)

- SIDIS region pheno analysis based on tools developed at Boglione, et al. (arXiv:1904.12882)