Overview of the Large-Momentum Effective Theory Approach

Yong Zhao
Brookhaven National Laboratory
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Review articles

- Lin et al., “Parton distributions and lattice QCD calculations: a community white paper”, Prog.Part.Nucl.Phys. 100 (2018)
- C. Monahan, “Recent Developments in x-dependent Structure Calculations”, PoS LATTICE2018 (2018) 018
- K. Cichy and M. Constantinou, “A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results”, Adv.High Energy Phys. 2019 (2019)
- Y. Zhao, “Unravelling High-Energy Hadron Structures with Lattice QCD”, Int.J.Mod.Phys. A33 (2019)

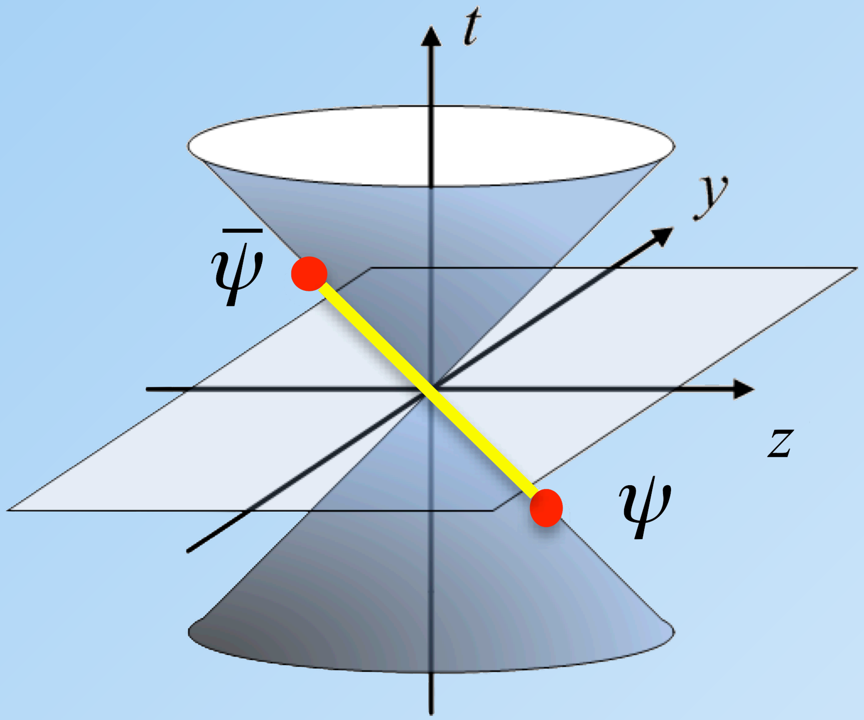
Outline

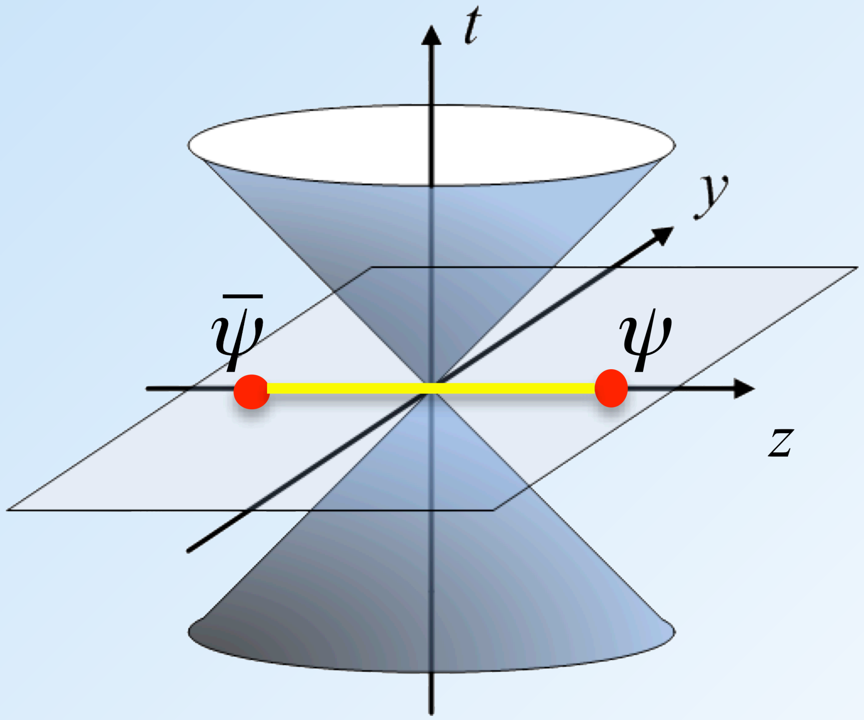
- Large-momentum effective theory
    - Formalism
    - Factorization formulas
- Lattice calculation of collinear distributions
    - Renormalization
    - Power corrections
    - Perturbative matching
    - Lattice calculations
    - Systematics
    - Other approaches
- TMDs from lattice QCD
    - Quasi-TMDs and relation to TMDs
    - Collins-Soper kernel from lattice
- Summary and outlook

A novel approach to calculate light-cone PDFs

- Large-Momentum Effective Theory:

- Ji, PRL110 (2013);
- Ji, SCPMA57 (2014).





PDF $q(x)$:
Cannot be calculated on the lattice

Quasi-PDF $\tilde{q}(x, P^z)$:
Directly calculable on the lattice

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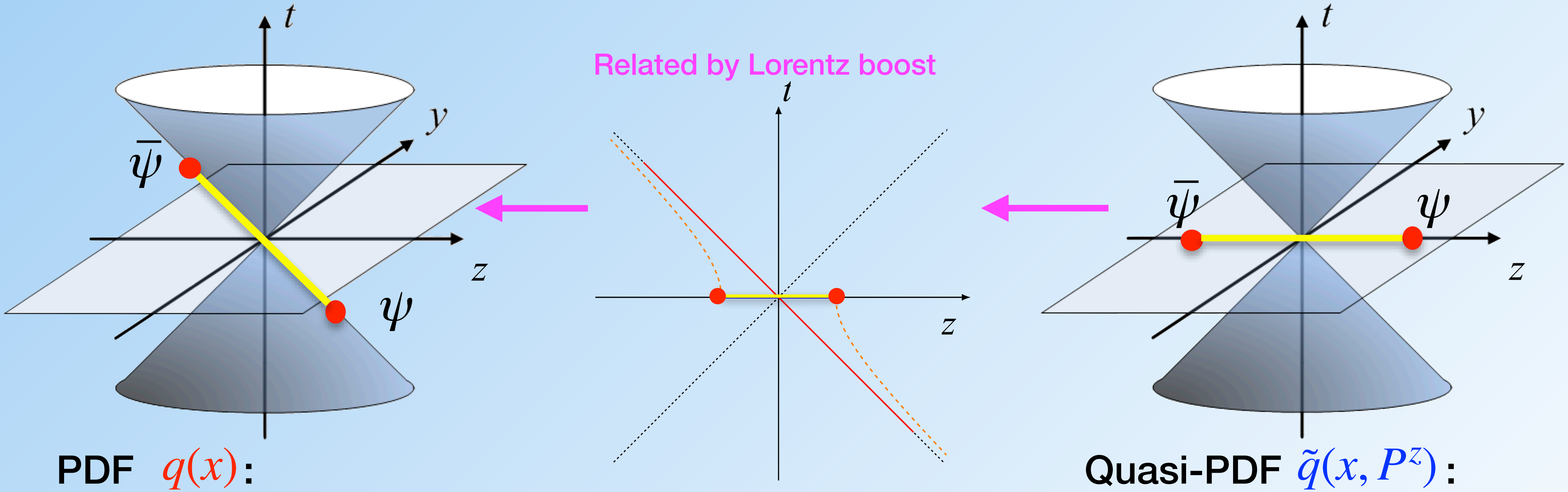

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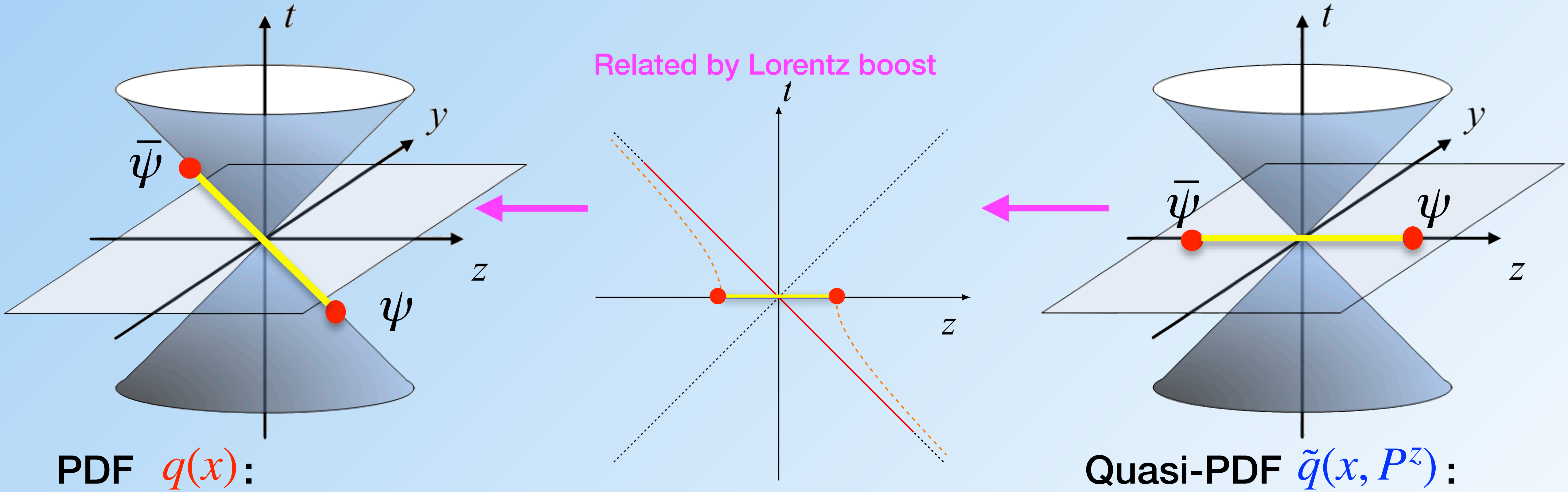


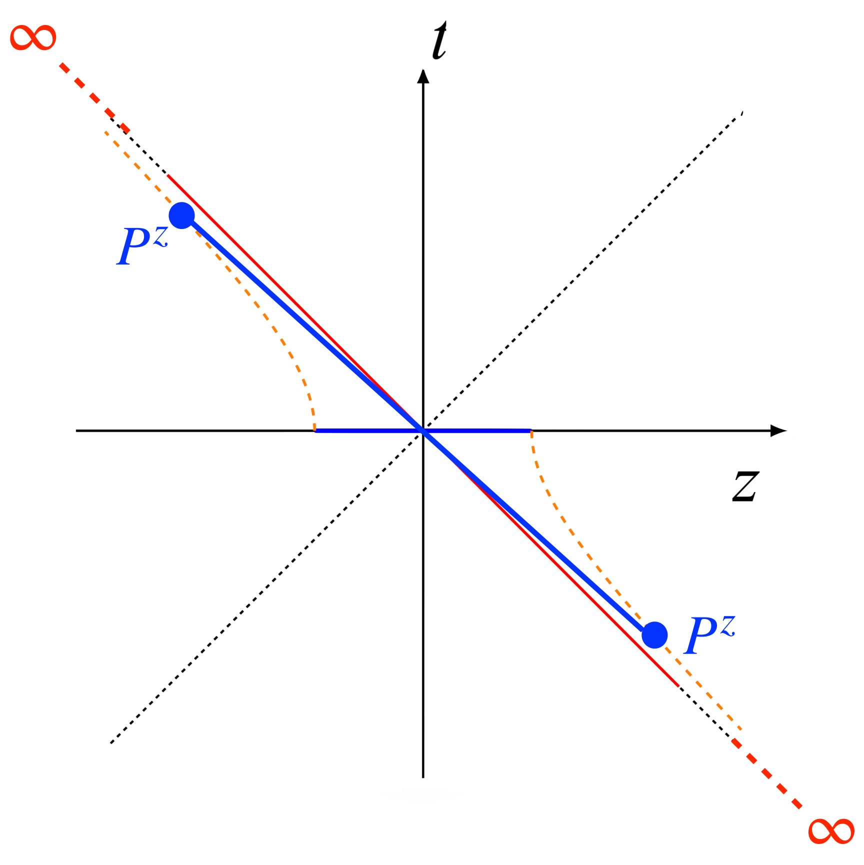
PDF $q(x)$:
Cannot be calculated on the lattice

Calculating the quasi-PDF at hadron momentum P^z is equivalent to boosting it.

Quasi-PDF $\tilde{q}(x, P^z)$:
Directly calculable on the lattice

A novel approach to calculate light-cone PDFs

$$\lim_{P^z\to\infty}\tilde{q}(x,P^z)=?$$ ✗



Instead of taking $P^z\to\infty$ limit, one can perform an expansion for large but finite P^z:

$$\tilde{q}(x,P^z,\mu)=\int\frac{dy}{|y|}C\left(\frac{x}{y},\frac{\mu}{yP^z}\right)q(y,\mu)+O\left(\frac{M^2}{P_z^2},\frac{\Lambda_{\rm QCD}^2}{x^2P_z^2}\right)$$

- X. Xiong, X. Ji, J.-H. Zhang and Y.Z., PRD90 (2014);
- Y.-Q. Ma and J. Qiu, PRD98 (2018), PRL 120 (2018);
- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., PRD98 (2018).

- $\tilde{q}(x,P^z)$ and $q(x)$ have the same infrared physics (nonperturbative), but different ultraviolet (UV) physics (perturbative);
- Therefore, the matching coefficient C is perturbative, which controls the logarithmic dependences on P^z.

Factorization formulas

- Non-singlet quark PDFs:

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) q(y, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

- Y.-Q. Ma and J. Qiu, PRD98 (2018), PRL 120 (2018);
- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., PRD98 (2018).

- Gluon PDF and singlet quark PDF:

$$\tilde{q}_i(x, P_z, \mu) = \int_{-1}^{1} \frac{dy}{|y|} \left[\sum_j C_{q_i q_j}\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) q_j(y, \mu) + C_{qg}\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) g(y, \mu) \right] + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

$$\tilde{g}(x, P_z, \mu) = \int_{-1}^{1} \frac{dy}{|y|} \left[\sum_j C_{gq}\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) q_j(y, \mu) + C_{gg}\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) g(y, \mu) \right] + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

- Wang et al., EPJC78 (2018), JHEP1805 (2018);
- Zhang et al., arXiv: 1904.00978.

- Non-singlet quark GPD:

$$\tilde{F}_{\tilde{\gamma}^z}(x, \xi, t, \mu) = \int_{-1}^{1} \frac{dy}{|\xi|} C\left(\frac{x}{\xi}, \frac{y}{\xi}, \frac{\mu}{\xi P^z}\right) F_{\gamma^+}(y, \xi, t, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{t}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

$$= \int_{-1}^{1} \frac{dy}{|y|} \bar{C}\left(\frac{x}{y}, \frac{\xi}{y}, \frac{\mu}{yP^z}\right) F_{\gamma^+}(y, \xi, t, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{t}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

- Y.-S. Liu, Y.Z. et al., PRD100 (2019)

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Renormalization

- Multiplicative renormalizability (in the continuum theory)

$$\bar{\psi}_0(z)\frac{\Gamma}{2}W_0[z,0]\psi_0(0) = e^{\delta m|z|}\ Z_{j_1}Z_{j_2}\left[\bar{\psi}(z)\frac{\Gamma}{2}W[z,0]\psi(0)\right]_R$$

- X. Ji, J.-H. Zhang, and **Y.Z.**, PRL120 (2018);
- J. Green et al., PRL121 (2018);
- T. Ishikawa, Y.-Q. Ma, J. Qiu, S. Yoshida, PRD96 (2017).

$$\tilde{O}^g(z) = g_{\perp,\mu\nu}F^{n_1\mu}(z)W[z,0]F^{n_2\nu}(0)$$

$$n_1, n_2 \in \{\hat{z}, \hat{v}\},$$

$$\tilde{O}^g_0(z) = e^{\delta m|z|}Z_{j_1}Z_{j_2}Z_AZ_Q\left[\tilde{O}^g(z)\right]_R$$

$$\hat{z}^\mu = (0,0,0,1),\ \ \hat{v}^\mu = (1,0,0,0)$$

$$+e^{\delta m|z|}Z_{\text{mix}}\left[g_\perp^{\mu\nu}A_\mu(z)W[z,0]A_\nu(0)\right]_R\delta(z)\delta_{n_1\hat{z}}\delta_{n_2\hat{z}}$$

No mixing with quarks under renormalization!

- Zhang, Ji, Schaefer et al., PRL122 (2019);
- Li, Ma, and Qiu, PRL122 (2019).

Renormalization

- Operator mixing on the lattice (with broken chiral symmetry)
    - γ^z , finite mixing with 1 at O(a^0);
    - γ^t , no mixing at O(a^0).

- Constantinou and Panagopoulos, PRD96 (2017);
- J. Green et al., PRL121 (2018);
- Chen et al, arXiv:1710.01089.

    - $\gamma^z \gamma^5$, no mixing at O(a^0);
    - $\gamma^t \gamma^5$, finite mixing with 1 at O(a^0);

    - $i \gamma^x \gamma^z \gamma^5$, finite mixing with 1 at O(a^0);
    - $i \gamma^x \gamma^t \gamma^5$, no mixing at O(a^0).

Renormalization

- Perturbative renormalization (lattice perturbation theory)
  - Constantinou and Panagopoulos, PRD96 (2017);
  - Ishikawa et al., arXiv:1609.02018
  - Xiong, Luu and Meißner, arXiv: 1705.00246
- Nonperturbative renormalization:
  - Ishikawa et al., arXiv:1609.02018
  - Zhang et al. (LP3), PRD95 (2017)
  - Static quark-antiquark potential
  - RI/MOM
    - Constantinou and Panagopoulos, PRD96 (2017);
    - Stewart and Y.Z., PRD97 (2018)
    - Alexandrou et al. (ETMC), NPB923 (2017)
    - Chen et al. (LP3), PRD97 (2018)
    - Liu et al. (LP3), arXiv:1807.06566
  - Mixed schemes
    - J. Green et al., PRL121 (2018);
  - Smeared quasi-PDF in the gradient flow method
    - Monahan and Orginos, JHEP 1703 (2017)
    - Monahan, PRD97 (2018)

Nonperturbative renormalization

- RIMOM scheme:

Green's function:

$$G(b,p)=\sum_x \left\langle \gamma_5 S^\dagger(p,b+x)\gamma_5 U(b+x,x)\frac{\Gamma}{2}S(p,x)\right\rangle$$

Amputated Green's function (or vertex function):

$$\Lambda(b,p)=\left(\gamma_5\left[S^{-1}(p)\right]^\dagger\right)G(b,p)S^{-1}(p)$$

RI'/MOM scheme:

$$Z_{\mathcal{O}}^{-1}(b,p_R^\mu)Z_q(p_R^2)\ \mathrm{Tr}\left[\Lambda(b,p)\mathscr{P}\right]\Big|_{p=p_R}=\mathrm{Tr}\left[\Lambda^{\mathrm{tree}}(b,p_R)\mathscr{P}\right]\ ,$$

- I. Stewart and Y.Z., PRD97 (2018);
- Constantinou and Panagopoulos, PRD96 (2017).

$$Z_q(p_R^2)=\frac{1}{12}\mathrm{Tr}\left[S^{-1}(p)S^{\mathrm{tree}}(p)\right]\Big|_{p=p_R}$$

Parametrization of the amputated Green's function:

$$\Lambda_{\gamma^t}(z,p)=\tilde{F}_t\gamma^t\ +\ \tilde{F}_z\gamma^z\ +\ \tilde{F}_{\not{p}}\frac{p^t\not{p}}{p^2}$$

Choice of $\mathscr{P}$ must include γ^t.

- Y.-S. Liu, Y.Z. et al. (LP3), arXiv:1807.06566.

Perturbative matching

- Continuum limit of the renormalized matrix element:

$$\lim_{a\to 0} \frac{\langle P | \tilde{O}(z, a) | P \rangle}{Z_{\mathcal{O}}(z, p_\mu^R, \mu_R, a)} = \frac{\langle P | \tilde{O}(z, \epsilon) | P \rangle}{Z_{\mathcal{O}}(z, p_\mu^R, \mu_R, \epsilon)}$$

$$D = 4 - 2\epsilon$$

- Regularization-independence allows the matching to be done in continuum perturbation theory (with dimensional regularization.)

Two matching strategies

- Constantinou and Panagopoulos, PRD96 (2017);
- C. Alexandrou et al., ETM Collaboration, NPB923 (2017).

$$\begin{tikzcd}
\frac{\langle P|\tilde{O}(z,\epsilon)|P\rangle}{Z_{\mathcal{O}}(z,p^R_\mu,\mu_R,\epsilon)} \arrow[r] \arrow[dd, "\text{F.T.}"'] & \times\frac{Z_{\mathcal{O}}(z,p^R_z,\mu_R,\epsilon)}{Z^{\overline{\mathrm{MS}}}_{\mathcal{O}}(\epsilon,\mu)} = \frac{\langle P|\tilde{O}(z,\epsilon)|P\rangle}{Z^{\overline{\mathrm{MS}}}_{\mathcal{O}}(\epsilon,\mu)} \arrow[d, "\text{F.T.}"] \\
 & \tilde{q}(x,P^z,\mu) \arrow[d, "\text{Matching}"] \\
\tilde{q}(x,P^z,\mu_R,p^R_z) \arrow[r, "\text{Matching}"] & q(x,\mu)
\end{tikzcd}$$

Or

$$Z^{\mathrm{ratio}}_{\mathcal{O}}(\epsilon,\mu)/Z^{\mathrm{MMS}}_{\mathcal{O}}(\epsilon,\mu)$$

- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., PRD98 (2018);
- Alexandrou et al. (ETMC), PRD99 (2019)

- I. Stewart and Y.Z., PRD97 (2018);
- J.-W. Chen, Y.Z. et al., LP3 Collaboration, PRD97 (2018).

One-step matching

- Matching formula:

- I. Stewart and Y.Z., PRD97 (2018);

$$\tilde{q}(x, P^z, \mu_R, p_z^R) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, r, \frac{yP^z}{\mu}, \frac{yP^z}{p_z^R}\right) q(y, \mu) + O(1/P_z^2)$$

$$r = \frac{\mu_R^2}{(p_z^R)^2}$$

- Matching kernel:

$$C\left(\xi, r, \frac{p^z}{\mu}, \frac{p^z}{p_z^R}\right) = \delta(1-\xi) + \frac{\alpha_s C_F}{2\pi}\left[C_B\left(\xi, \frac{p^z}{\mu}\right) - \left|\frac{p^z}{p_z^R}\right| h\left(1 + \frac{p^z}{p_z^R}(\xi - 1), r\right)\right]_+^{(-\infty,\infty)}$$

$\xi = \frac{x}{y}$, $p^z = yP^z$

$$\left[f(x)\right]_+^{(-\infty,\infty)} = f(x) - \delta(x-1)\int_{-\infty}^{\infty} dy\, f(y)$$

😄Formally satisfying vector current (or particle number conservation): $\int_{-\infty}^{\infty} d\xi\ C(\xi) = 1$

😒Renormalization scale dependence to be cancelled after matching, making systematics analysis of discretization effects more complicated.

Two-step matching

- Scheme conversion:

- Constantinou and Panagopoulos, PRD96 (2017)

$$C(z, p_z^R, \mu_R, \mu) = \frac{Z_{\mathcal{O}}(z, p_z^R, \mu_R, \epsilon)}{Z_{\mathcal{O}}^{\overline{\mathrm{MS}}}(\epsilon, \mu)}$$

Renormalization scale dependence to be cancelled in the first step, useful for the systematics analysis.

- Matching in the MSbar scheme:

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) q(y, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{x^2 P_z^2}\right)$$

- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., PRD98 (2018)

😄 Systematics analysis of discretization effects is easier.

😒 Does not satisfy vector current (or particle number) conservation, thus lacking a cross check in the intermediate steps.

Modified schemes

- Ratio scheme:

$$C^{\rm ratio}(z,p_z^R,\mu_R,\mu)=\frac{C(z,p_z^R,\mu_R,\mu)}{1+\frac{\alpha_sC_F}{2\pi}\left[\frac{3}{2}\ln\frac{\mu^2z^2e^{2\gamma_E}}{4}+\frac{5}{2}\right]}$$

$$\lim_{z\to0}C(z,p_z^R,\mu_R,\mu)=1$$

- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., PRD98 (2018);
- A. Radyushkin, PLB781 (2018).

- Modified MSbar scheme:

$$C^{\rm MMS}(z,p_z^R,\mu_R,\mu)=\frac{C(z,p_z^R,\mu_R,\mu)}{Z^{\rm M\overline{MS}}_{\Gamma_{\gamma^0}}(z\bar\mu)}$$

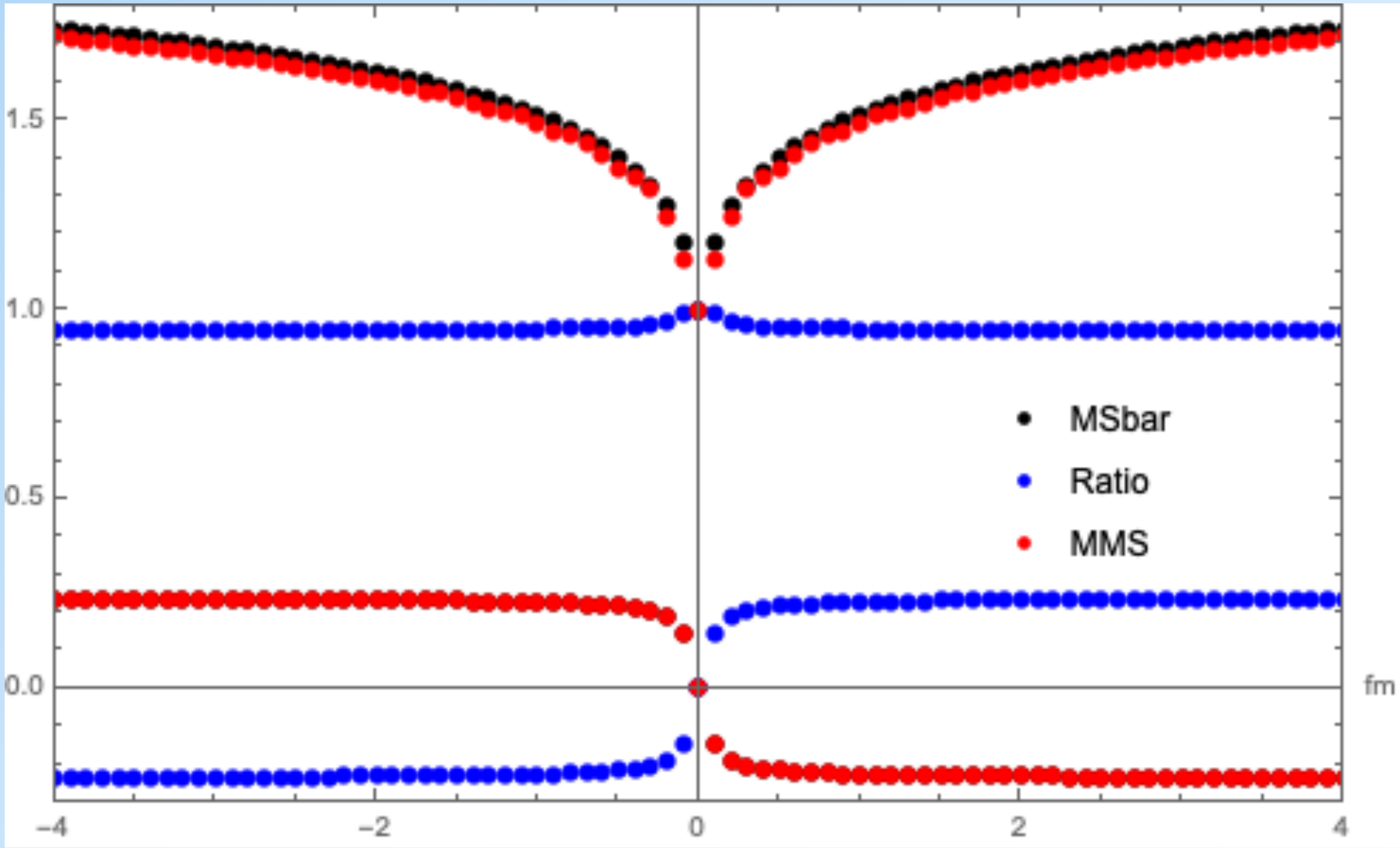
$$\begin{aligned}Z^{\rm M\overline{MS}}_{\Gamma_{\gamma^0}}(z\bar\mu)\; &=\; 1-\frac{\alpha_s}{2\pi}C_F\left(\frac{3}{2}\ln\left(\frac{1}{4}\right)+\frac{5}{2}\right)\\ &+\;\frac{3}{2}\frac{\alpha_s}{2\pi}C_F\left(i\pi\frac{|z\bar\mu|}{2z\bar\mu}-\mathrm{Ci}(z\bar\mu)+\ln(z\bar\mu)-\ln(|z\bar\mu|)-i\mathrm{Si}(z\bar\mu)\right)\\ &-\;\frac{3}{2}\frac{\alpha_s}{2\pi}C_Fe^{iz\bar\mu}\left(\frac{2\mathrm{Ei}(-iz\bar\mu)-\ln(-iz\bar\mu)+\ln(iz\bar\mu)+i\pi\mathrm{sgn}(z\bar\mu)}{2}\right)\end{aligned}$$

- C. Alexandrou et al. (ETMC), PRD99 (2019)

Comparison between ratio and MMS schemes

- Conversion factors:

$$P^z = 3.0\ \mathrm{GeV};\mu = 3.0\ \mathrm{GeV};p_z^R = 2.2\ \mathrm{GeV};\mu_R = 3.7\ \mathrm{GeV};\alpha_s = 0.258.$$



In collaboration with Yu-Sheng Liu

$$\int_{-z_{\max}}^{z_{\max}} \frac{d(zp^z)}{2\pi} e^{i\xi zp^z}\ Z_{\Gamma\gamma^0}^{\mathrm{MMS/Ratio}}(z\mu) \neq C^{\overline{\mathrm{MS}}}(\xi, \frac{\mu}{p^z}) - C^{\mathrm{MMS/Ratio}}(\xi, \frac{\mu}{p^z})$$

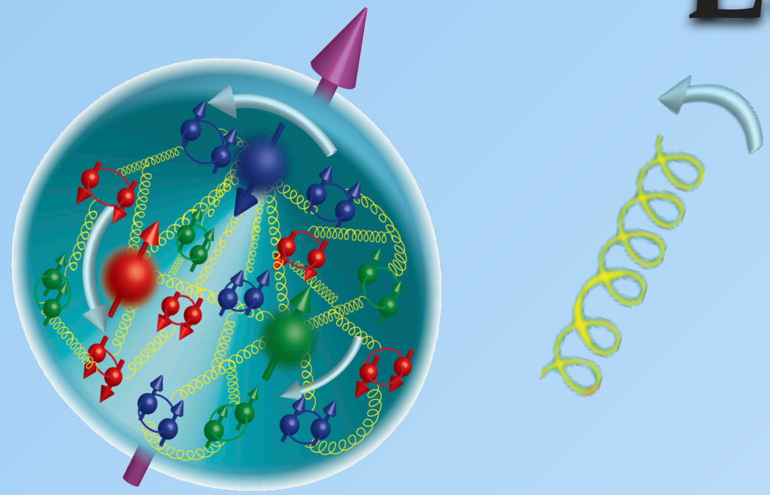
Power corrections

- Mass correction (M^2/P_z^2)
  - Derived to all power orders
    - Chen et al. (LP3), NPB911 (2016)
- Higher-twist correction ($\Lambda_{\rm QCD}^2/(x^2P_z^2)$)
  - Unconstraint in lattice calculations so far;
  - Uncontrolled at small x ($x \to 0$) and large x ($x \to 1$)? $q(x)\cdot O\left(\frac{\Lambda_{\rm QCD}^2}{x^2(1-x)P_z^2}\right)$
    - Braun, Vladimirov and Zhang, PRD99 (2019)
  - Extrapolating final results to $P^z \to \infty$ with $A(x)+B(x)/P_z^2$?

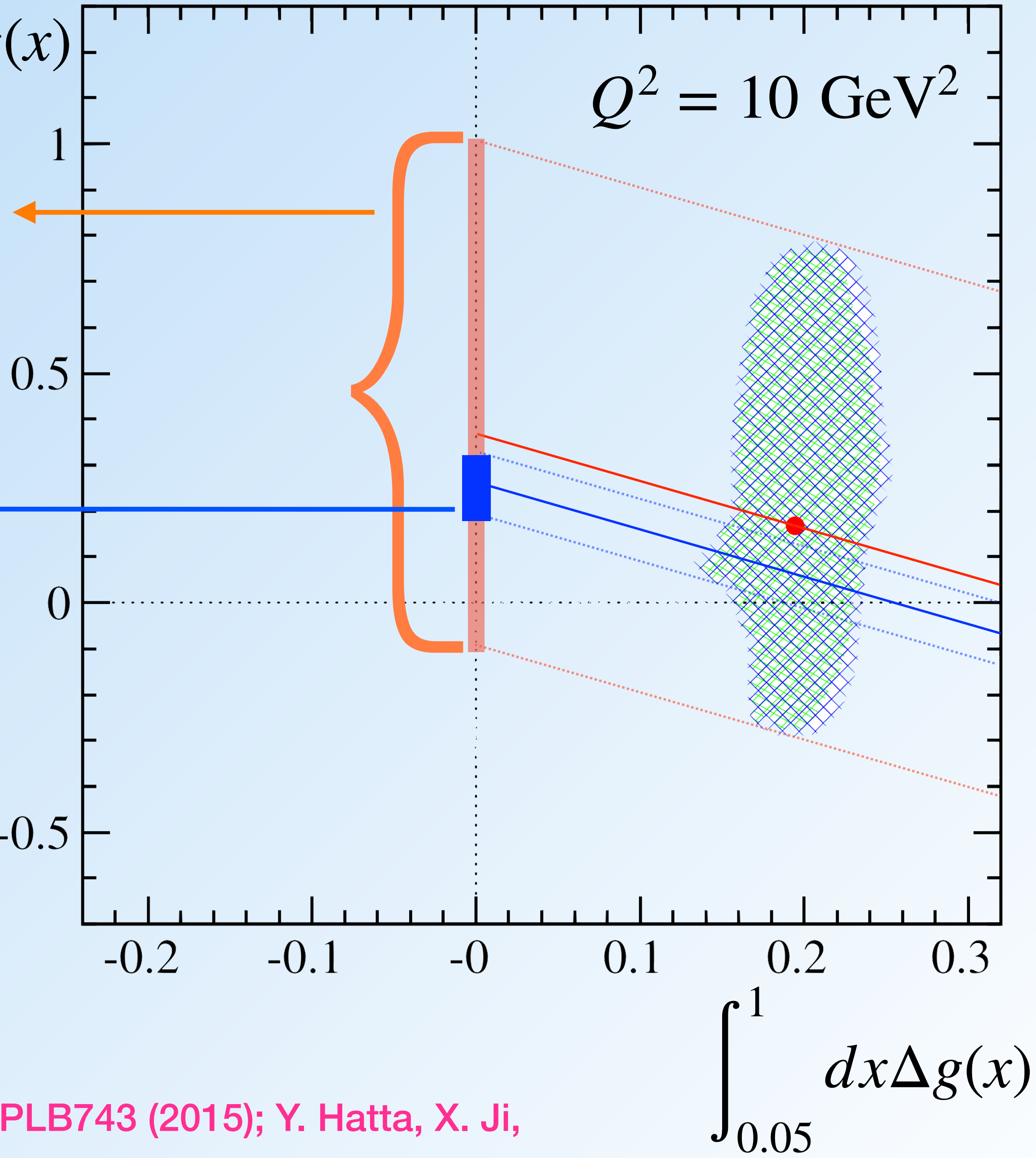
Lattice calculations

- Lattice Parton Physics Project (LP3) Collaboration
- European Twisted Mass Collaboration
- SBU-BNL Group
- χ-QCD Collaboration

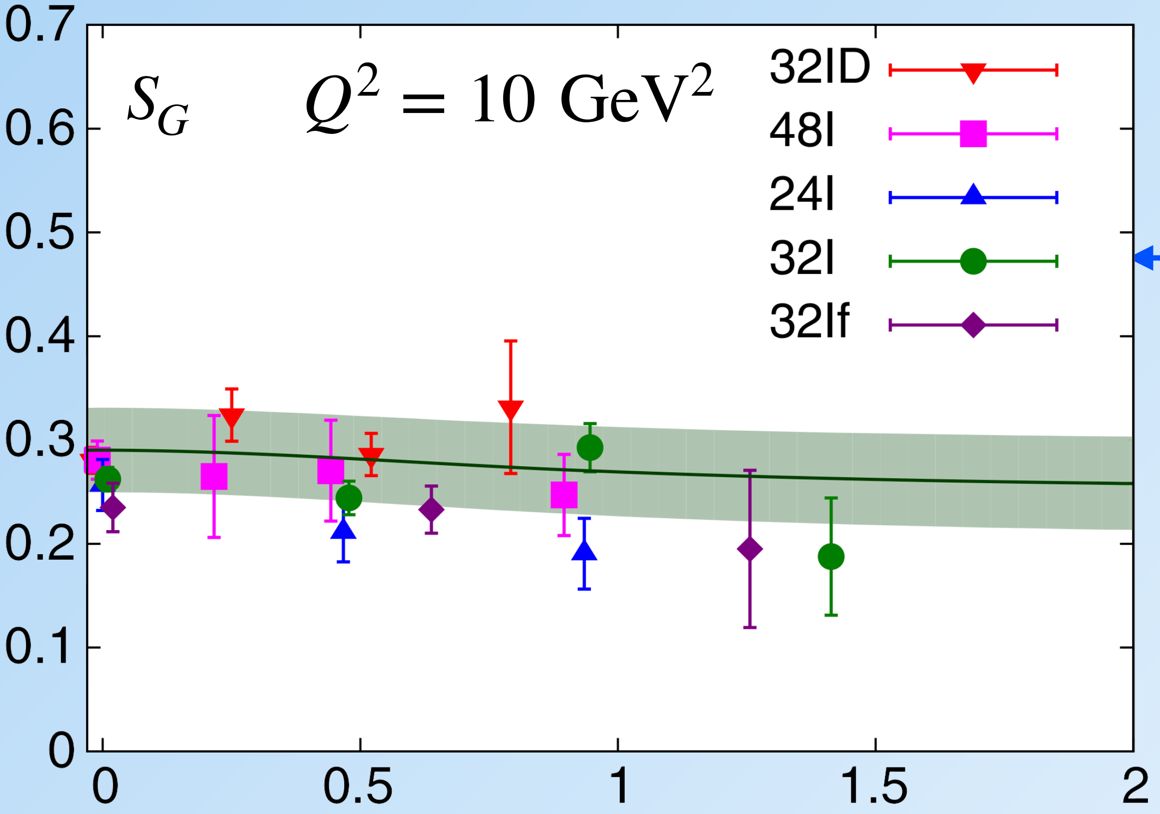
Lattice calculations



First lattice calculation of gluon spin (2017)



Up-to-date experimental fit:
de Florian, et al., PRL113 (2014).

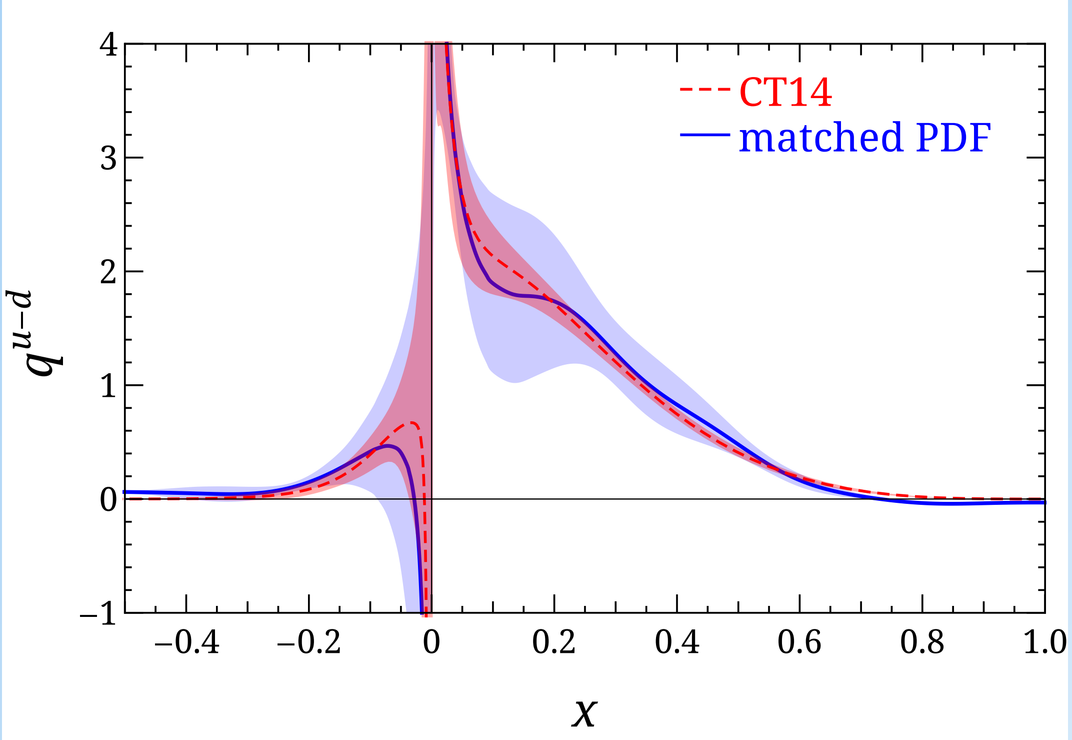


$\Delta G \approx S_G(\infty) = 0.251(47)(16)$

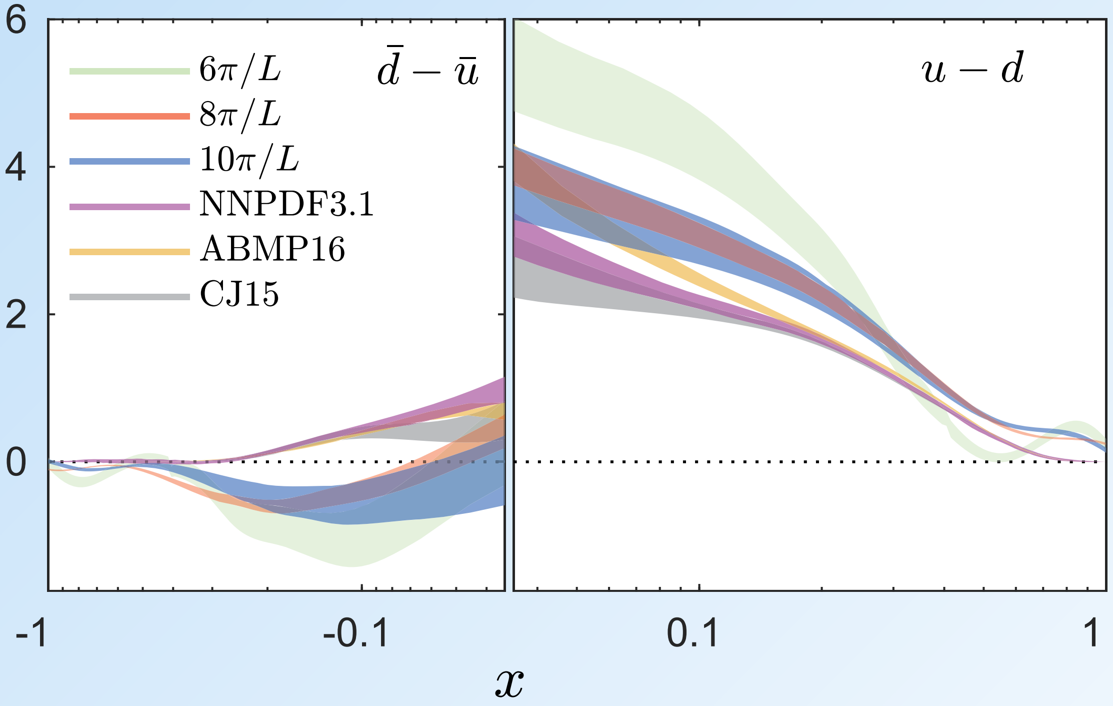
- X. Ji, J.-H. Zhang, and **Y.Z.**, PRL111 (2013); X. Ji, J.-H. Zhang, and **Y.Z.**, PLB743 (2015); Y. Hatta, X. Ji, and **Y.Z.**, PRD89 (2014);
- Y.-B. Yang, R. S. Suffian, **Y.Z.**, et al. (χQCD), PRL118 (2017).

Lattice calculations

- Iso-vector quark PDFs of the proton: $u(x) - d(x)$



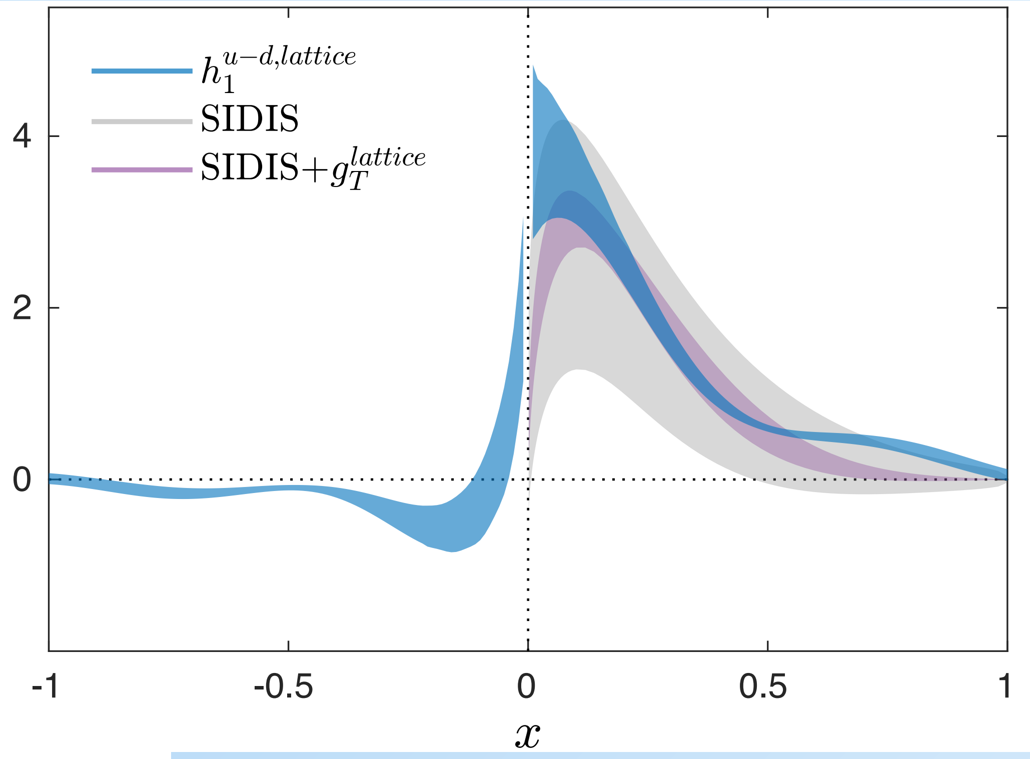
J.W. Chen, **Y.Z.** et al. (LP3), arXiv:1803.04393.



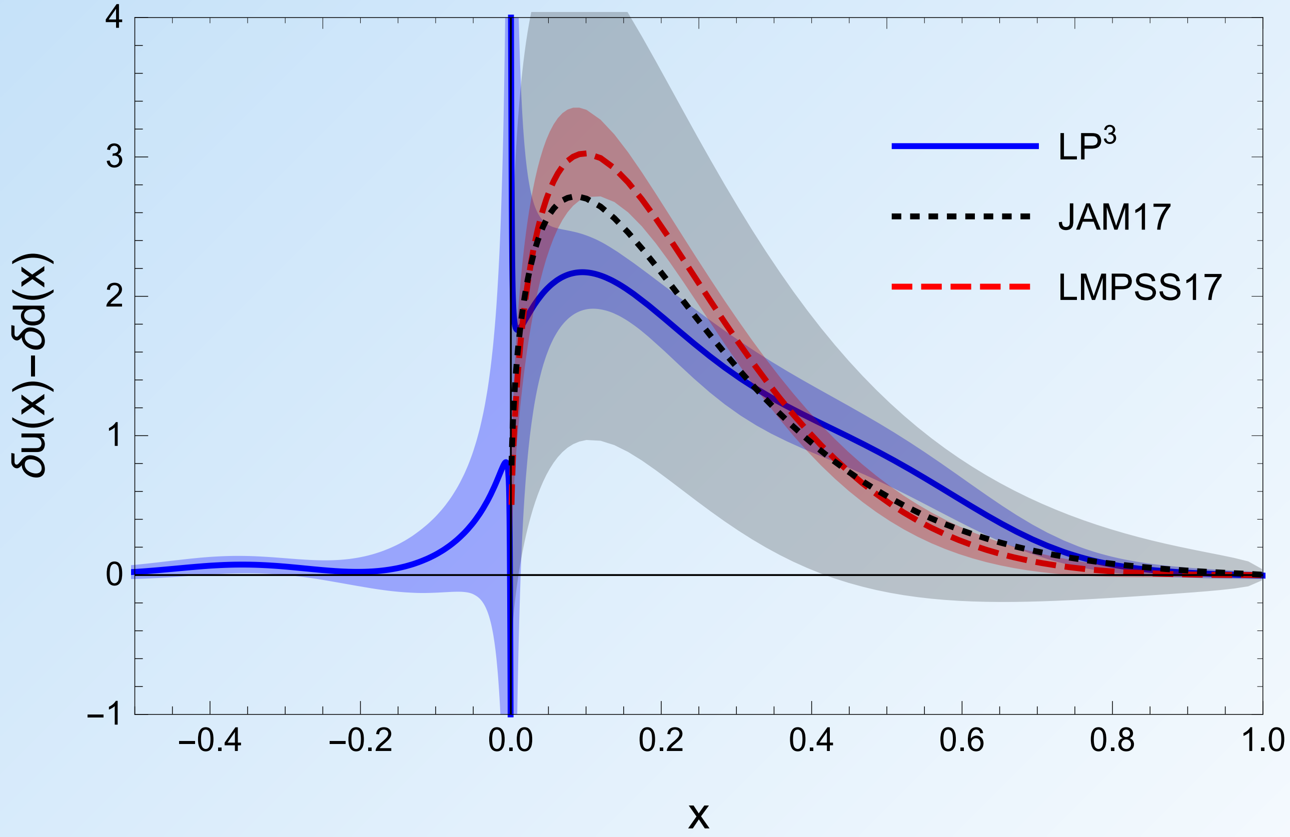
Alexandrou et al. (ETMC), PRL 121 (2018)

Lattice calculations

- Iso-vector quark PDFs of the proton: $\delta u(x) - \delta d(x)$



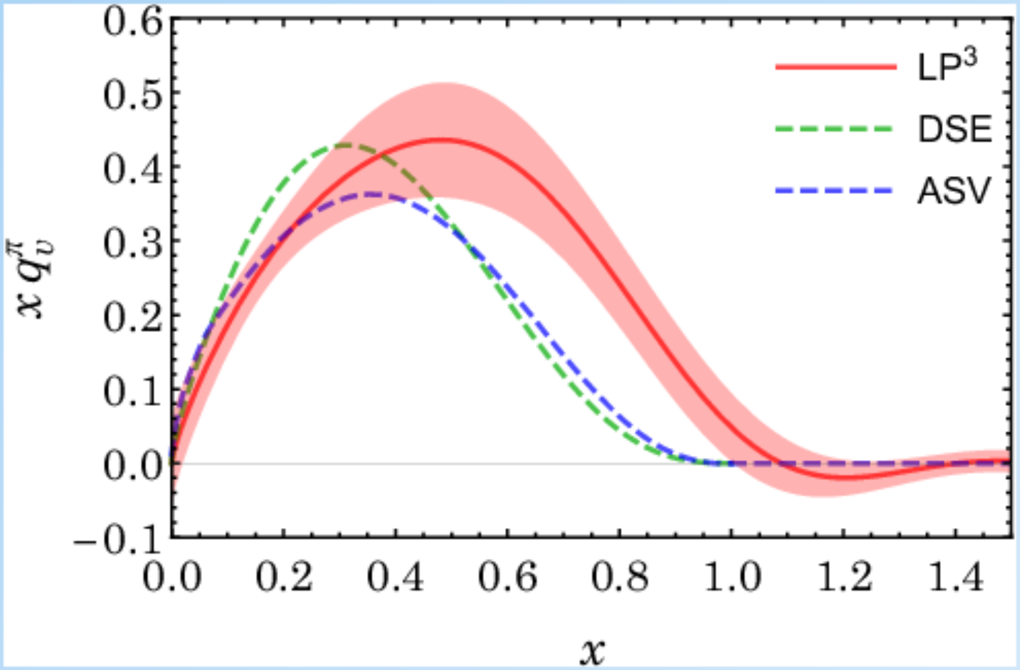
ETMC collaboration, PRD98 (2018).



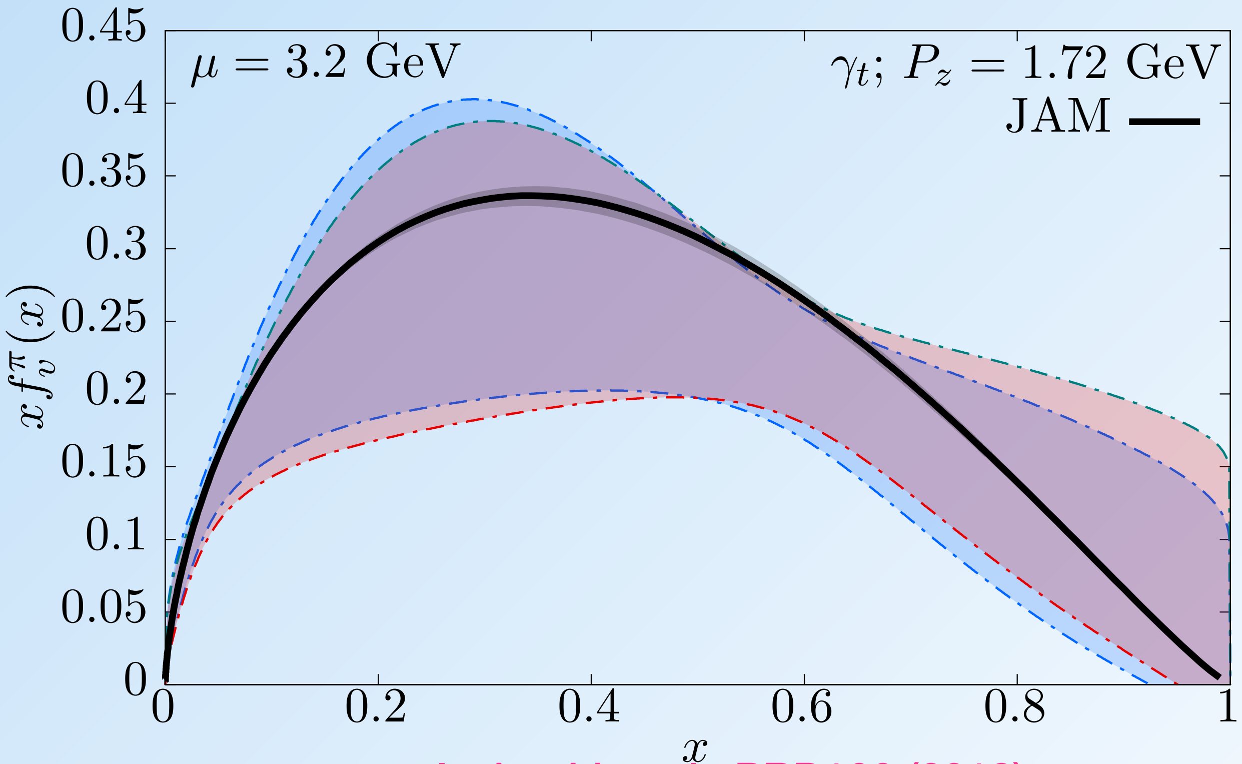
Y.-S. Liu, Y.Z., et al. (LP3), arXiv:1810.05043.

Lattice calculations

- Valence quark distribution of the pion:



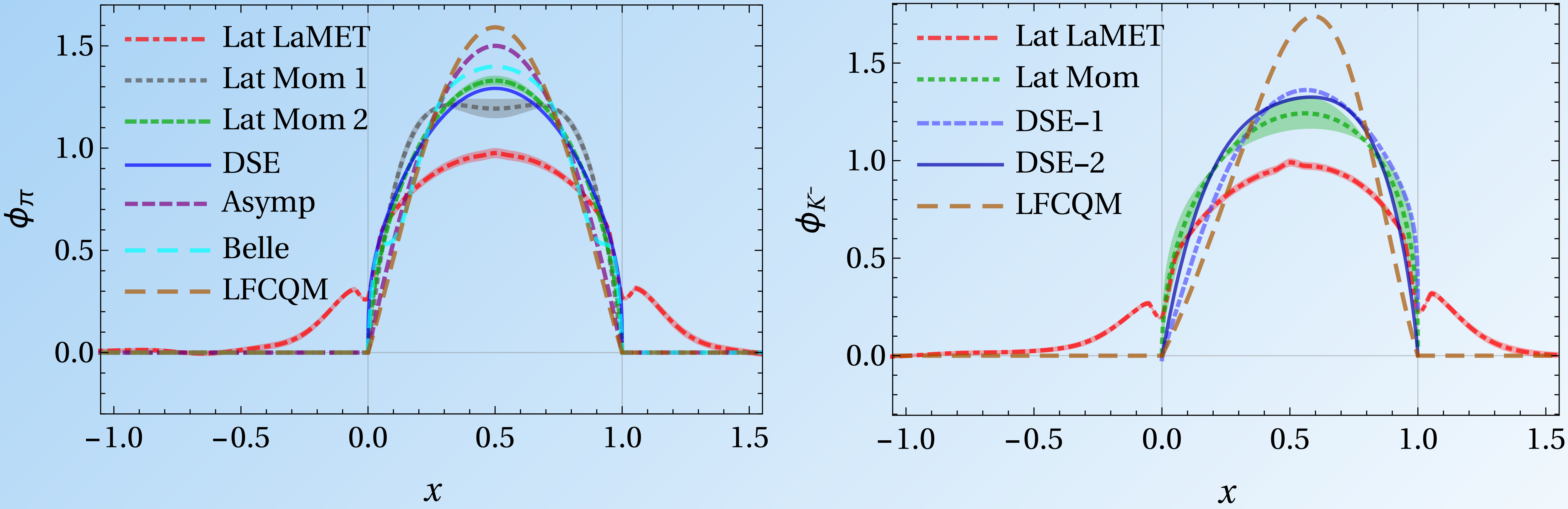
Zhang et al. (LP3), PRD100 (2019).



Izubuchi et al., PRD100 (2019).

Lattice calculations

- Light-cone distribution amplitudes:



Chen et al (LP3)., NPB939 (2019)

Systematics

- C. Alexandrou et al. (ETMC), PRD99 (2019)
- Y. S. Liu, Y.Z., et al. (LP3), arXiv:1807.06566

- Excited contamination at large hadron momentum

- Discretization effects, unknown nonperturbative contributions in the RI/MOM renormalization constant

  - Izubuchi et al., PRD100 (2019)

- Finite volume effects?

  - Briceno, Guerrero, Hansen and Monahan PRD99 (2018)

- Fourier transform

  - Lin et al. (LP3), PRD 98 (2018)
  - C. Alexandrou et al. (ETMC), PRD99 (2019)

- Power corrections

- Perturbative matching
  - Inversion of matching coefficient
  - Direct matching or fitting the PDF?
  - Higher-order perturbative matching

$$C^{-1}(x/y) = \left[\delta\left(1-\frac{x}{y}\right) + \alpha_s C^{(1)}(x/y) + \mathcal{O}(\alpha_s^2)\right]^{-1}$$

$$\approx \delta\left(1-\frac{x}{y}\right) - \alpha_s C^{(1)}(x/y) + \mathcal{O}(\alpha_s^2)$$

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) q(y, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{x^2 P_z^2}\right)$$

Other approaches

- Ioffe-time or pseudo distribution (position-space representation of the quasi-PDF)

A. Radyushkin, PRD96 (2017); K. Orginos et al., PRD96 (2017).

  - Nonperturbative renormalization

$$\frac{\langle P \neq 0 | \bar{\psi}_0(z)\frac{\Gamma}{2}W_0[z,0]\psi_0(0) | P \neq 0\rangle}{\langle P = 0 | \bar{\psi}_0(z)\frac{\Gamma}{2}W_0[z,0]\psi_0(0) | P = 0\rangle}$$

  - Factorization or OPE formula at short distance in position space

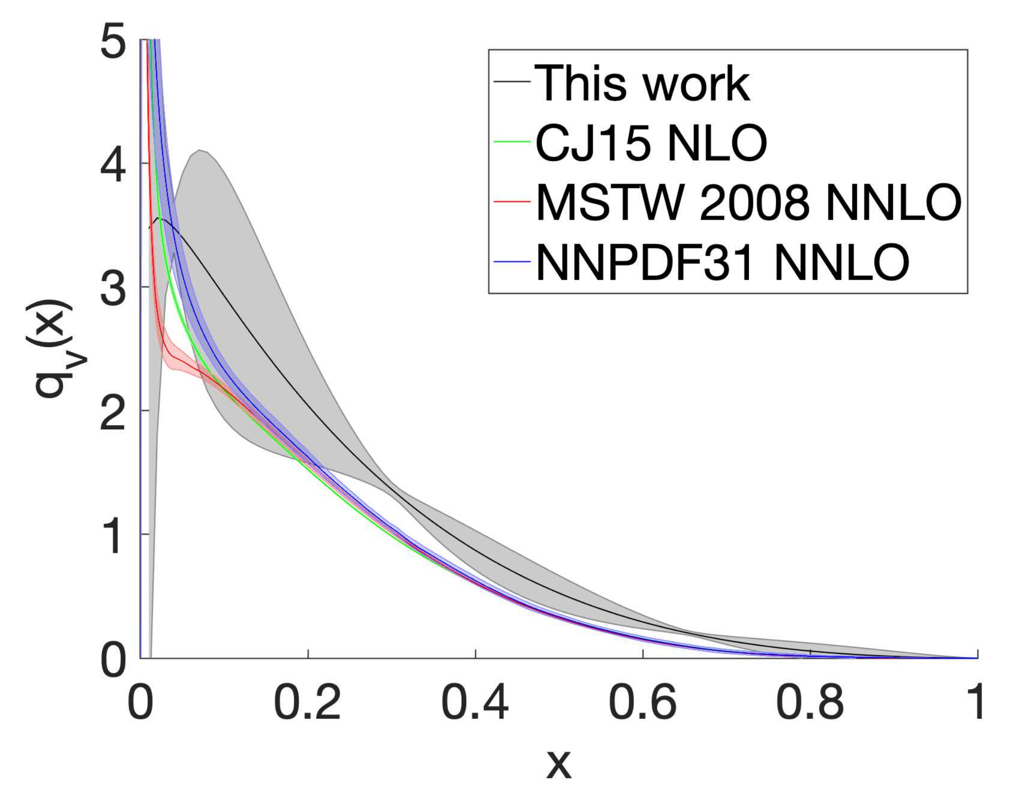
$$\frac{\langle P | \tilde{O}(z,\mu) | P\rangle}{2P^z} = \sum_{n=0} C_n(\mu^2 z^2)\frac{(-izP^z)^n}{n!}a_{n+1}(\mu)\left[1 - O\Big(\frac{M^2}{P_z^2}\Big)\right] + O(z^2\Lambda_{\rm QCD}^2)$$

$$= \int_{-1}^{1} d\alpha\ \mathscr{C}(\alpha, z^2\mu^2) \int_{-1}^{1} dy\ e^{-i\alpha y P^z z} q(y,\mu)$$

- A. Radyushkin, PLB781 (2018);
- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., PRD98 (2018).

Other approaches

- Ioffe-time or pseudo distribution (position-space representation of the quasi-PDF)



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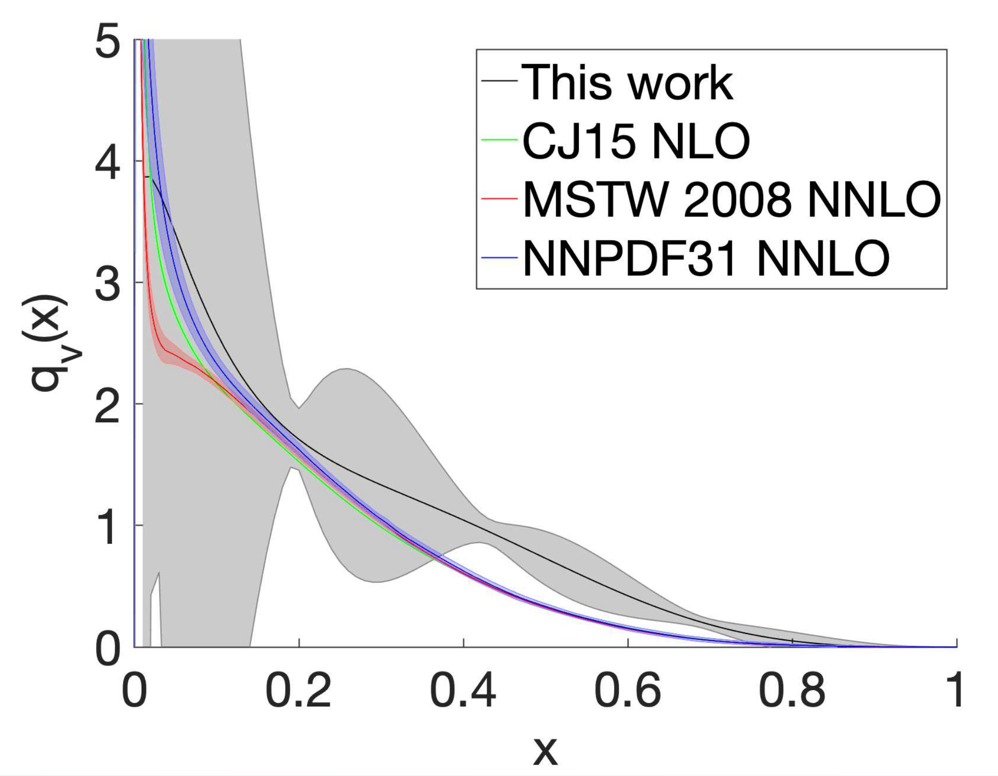

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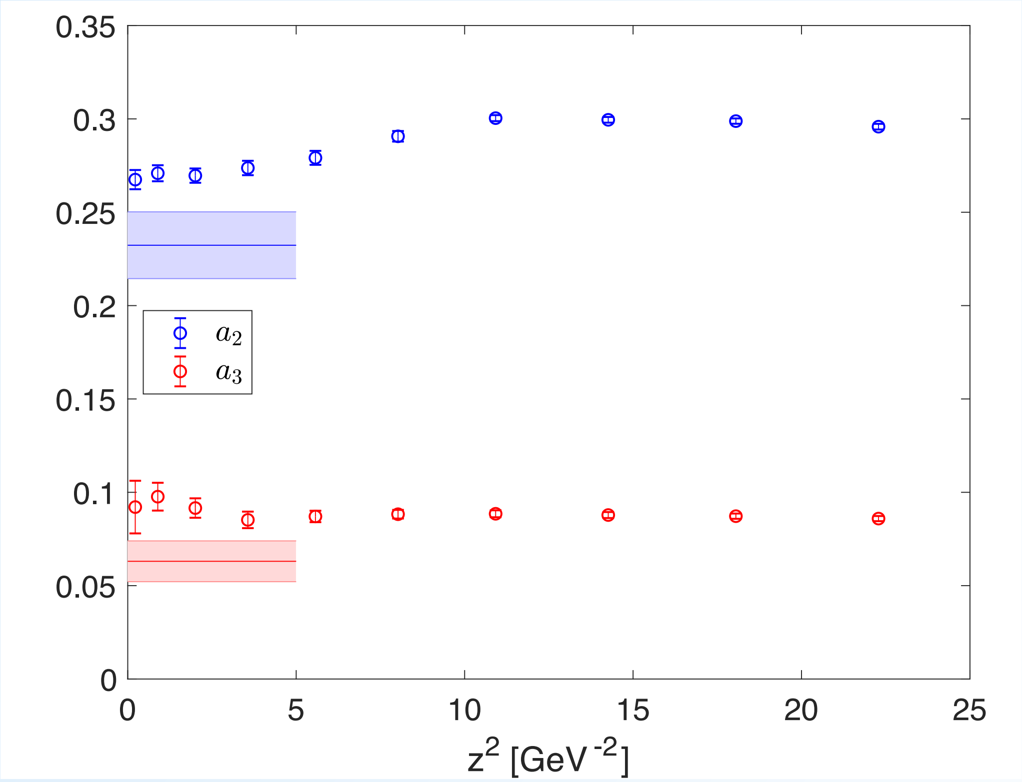
- Joo, Karpie et al., 1908.09771

Other approaches

- Extracting higher moments

$$\frac{\langle P|\tilde{O}(z,\mu)|P\rangle}{2P^z}=\sum_{n=0}C_n(\mu^2z^2)\frac{(-izP^z)^n}{n!}a_{n+1}(\mu)\left[1-O\left(\frac{M^2}{P_z^2}\right)\right]+O(z^2\Lambda_{\mathrm{QCD}}^2)$$

  - Small |z| needed to suppress higher twist corrections;
  - With suppression factor $(zP^z)^n/n!$, higher moment contributions need larger (zP^z) to beat the statistical errors in the correlator;
  - Large momentum P^z is the key.

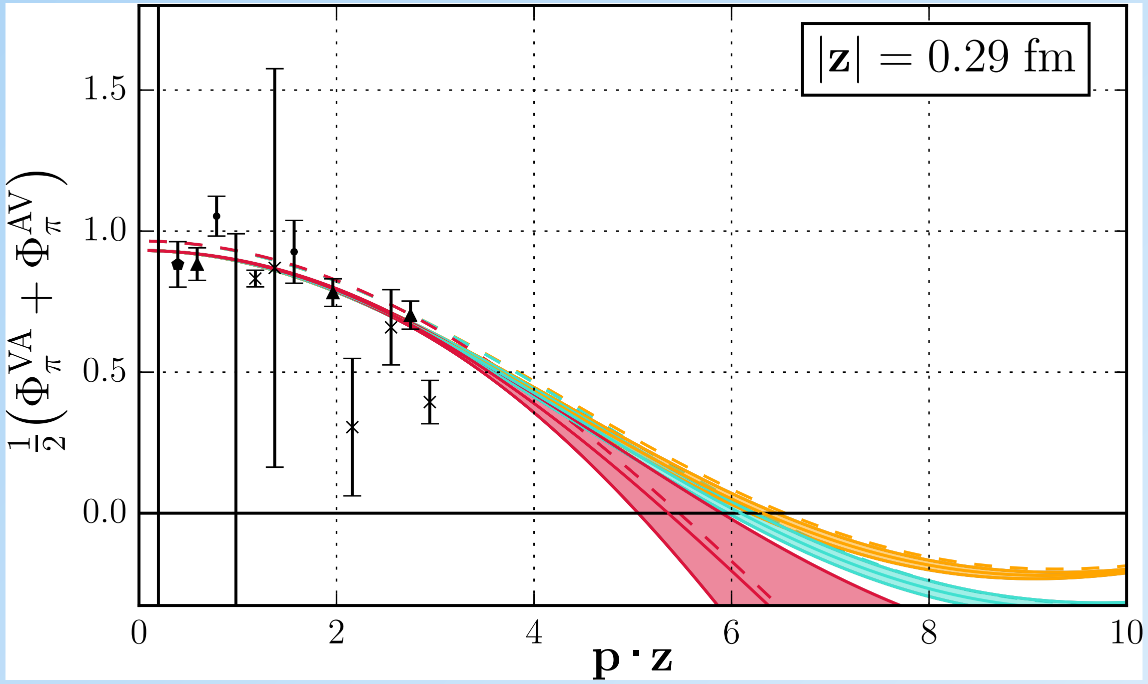


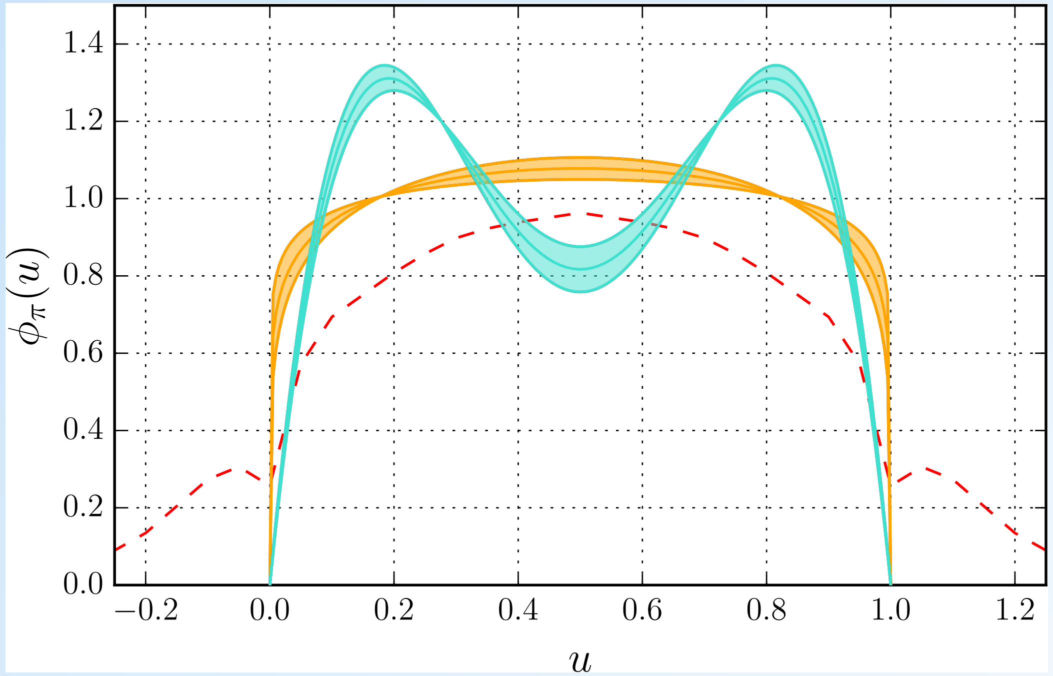
- Joo, Karpie et al., JHEP 1811 (2018) 178

Other approaches

- Pion DA from Euclidean current-current correlators:

$$\langle 0\,|\,J_X^{\dagger}(\frac{z}{2}J_Y(-\frac{z}{2}))\,|\,\pi^0(p)\rangle = f_\pi \frac{z\cdot p}{2\pi^2 z^4}\sum_{n=0}^{\infty} H_n^{XY}(z\cdot p,\mu)a_n^{\pi}(\mu)\,.$$





- Bali et al., PRD98 (2018)

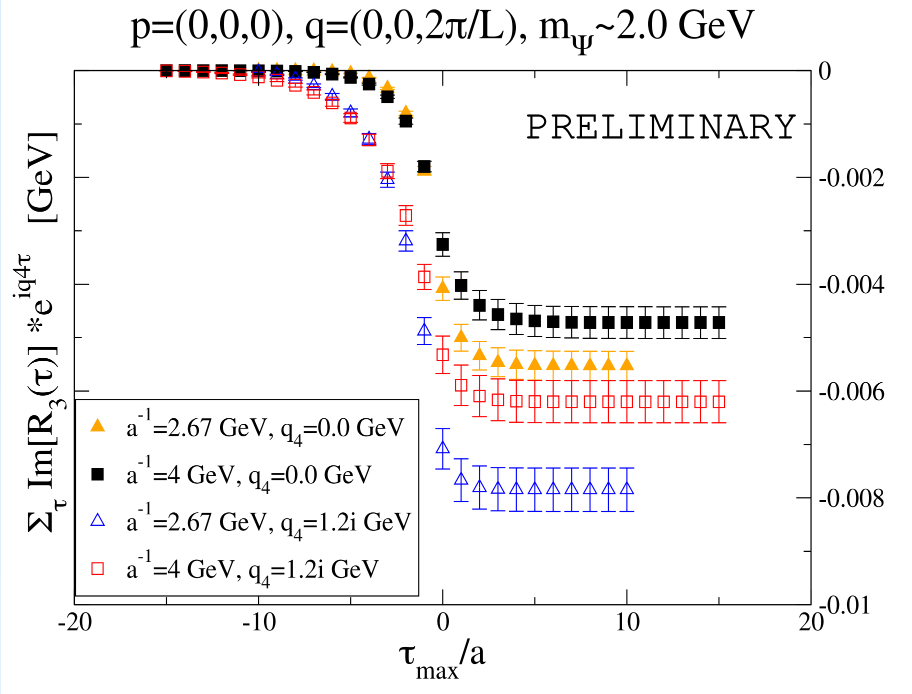
Other approaches

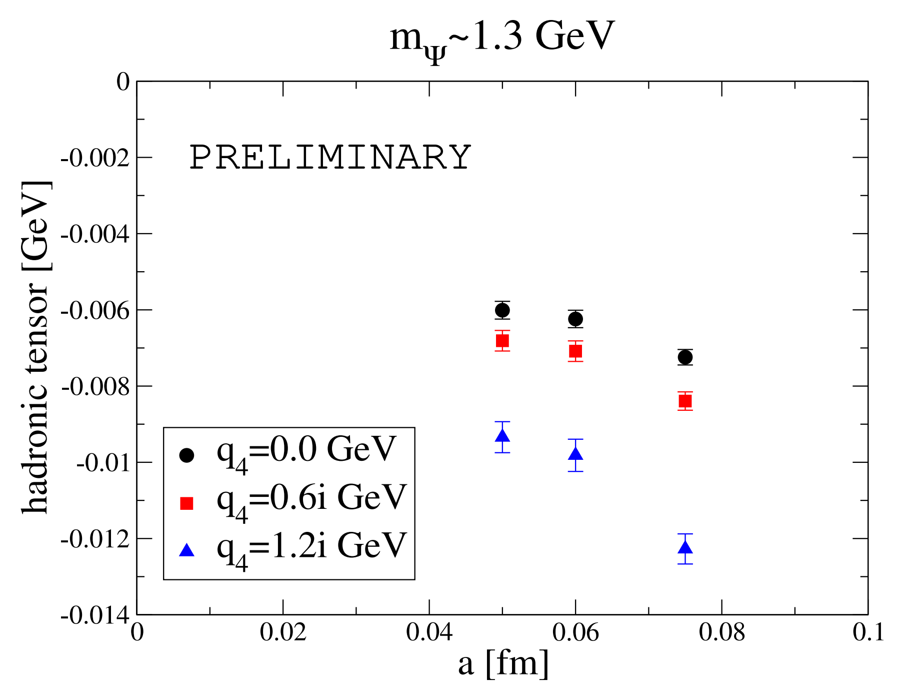
- Detmold and Lin, PRD73 (2006)

- OPE of current-current correlator involving a fictitious heavy quark

$$U_A^{[\mu\nu]}(q,p) = \int d^4x\, e^{iqx}\, \langle 0|T[A^{[\mu}_{\Psi,\psi}(x)\, A^{\nu]}_{\Psi,\psi}(0)]|\pi^+(p)\rangle$$

$$= 2if_\pi\varepsilon_{\mu\nu\rho\lambda}q^\rho p^\lambda \times \sum_{n=0,2,4\cdots}^{\infty}\left[\frac{\zeta^n C_n^2(\eta)}{(n+1)\tilde{Q}^2}\right]\mathscr{C}_W^{(n)}\left(m_\Psi,\tilde{Q},\mu\right)a_n(\mu)$$





- Detmold, Kanamori, Lin, Mondal and Y.Z., PoS LATTICE2018 (2018) 106

Other approaches

- “Lattice Cross Section”

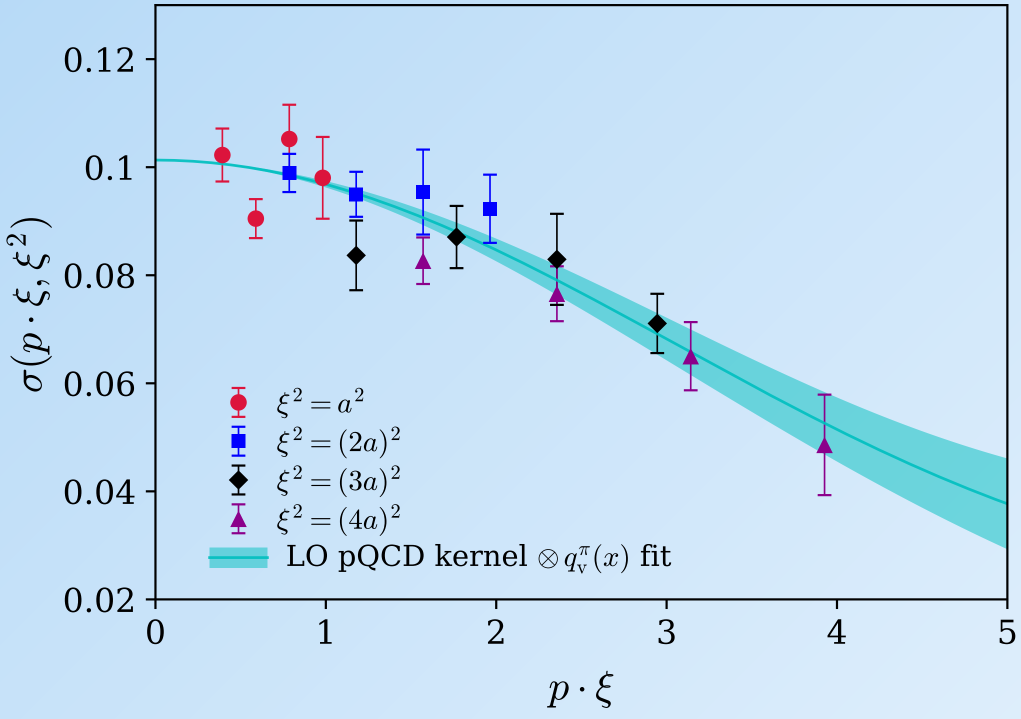
- Y.-Q. Ma and J. Qiu, PRD98 (2018), PRL 120 (2018);
- Suffian et al., PRD99 (2019)

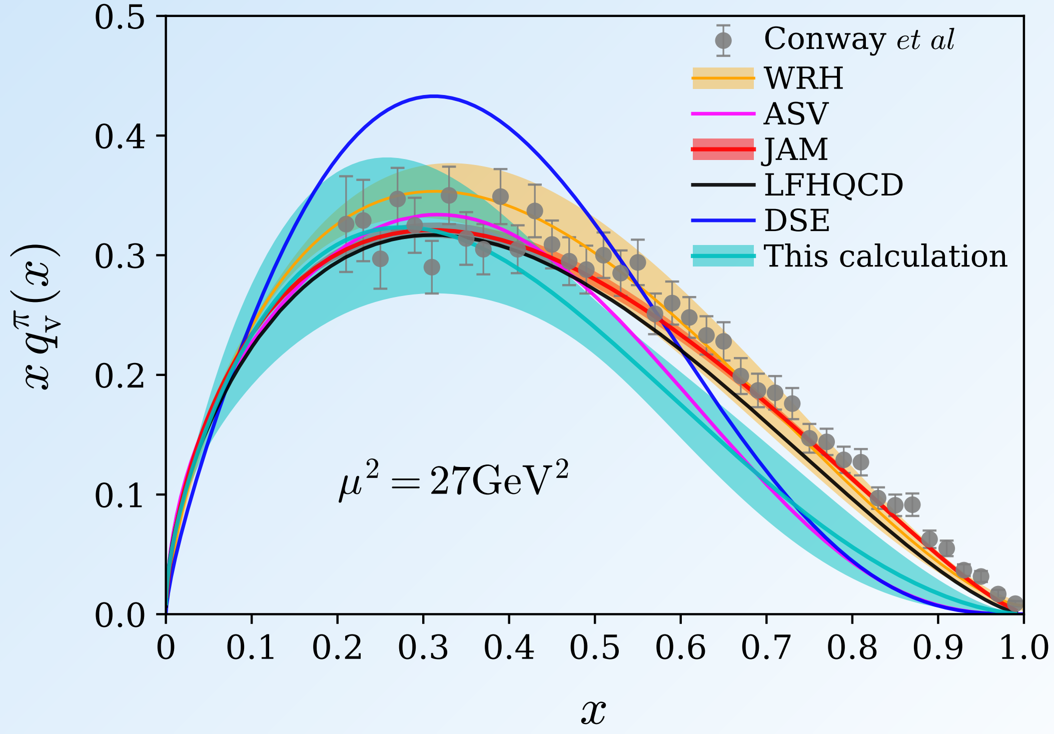
$$\sigma_{ij}^{\mu\nu}(\xi,p)= \langle\pi(p)|\,\mathcal{O}_{ij}^{\mu\nu}(\xi)\,|\pi(p)\rangle$$
$$= \xi^4\langle\pi(p)|\,\mathcal{J}_i^{\mu}(\xi/2)\,\mathcal{J}_j^{\nu}(-\xi/2)\,|\pi(p)\rangle$$

$$\frac{1}{2}\left[\sigma_{VA}^{\mu\nu}(\xi,p)+\sigma_{AV}^{\mu\nu}(\xi,p)\right]$$
$$\equiv \epsilon^{\mu\nu\alpha\beta}\xi_\alpha p_\beta T_1\left(\omega,\xi^2\right)+\left(p^\mu\xi^\nu-\xi^\mu p^\nu\right)T_2\left(\omega,\xi^2\right)$$

$$\mathcal{J}_\Gamma = \bar{q}\Gamma Q$$

$$T_i\left(\omega,\xi^2\right)=\sum_{a=q,\bar{q},g}\int_0^1\frac{dx}{x}f_a\left(x,\mu^2\right)C_i^a\left(x\omega,\xi^2,\mu^2\right)$$
$$+\mathcal{O}\left(\xi^2\Lambda_{\mathrm{QCD}}^2\right).$$





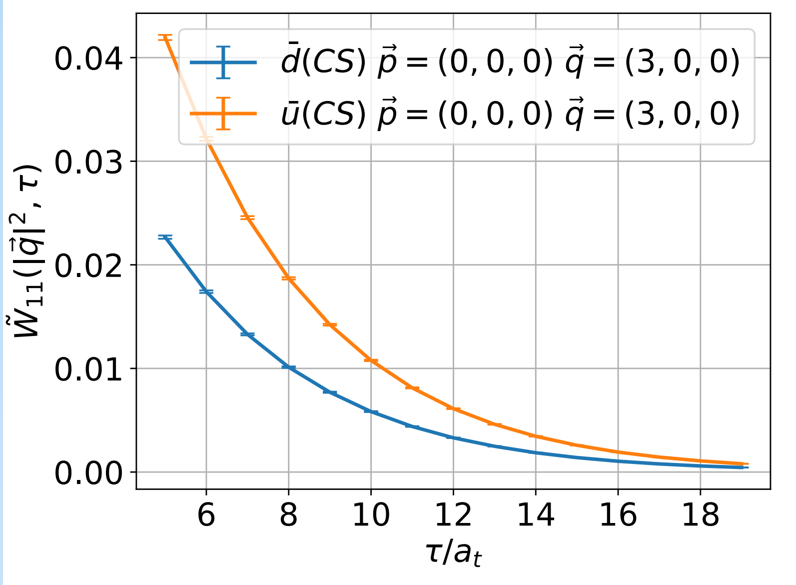
Other approaches

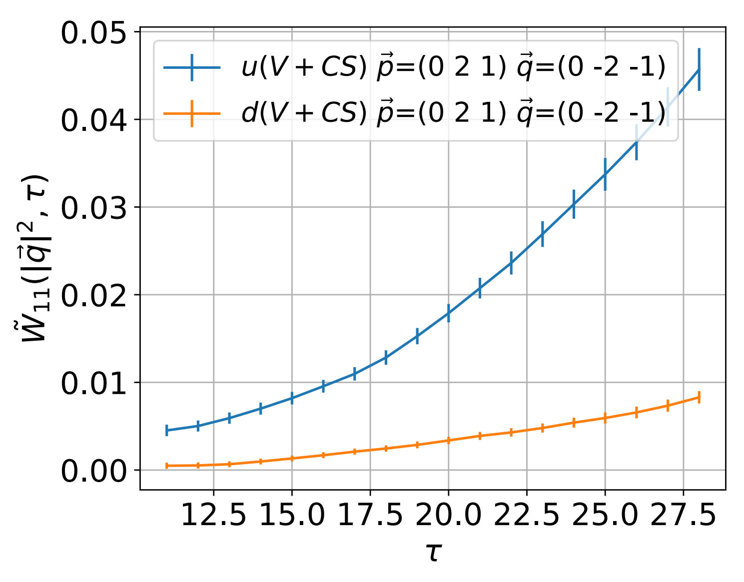
- Hadronic tensor from lattice QCD

$$C_4(\vec{p},\vec{q},\tau) = \sum_{\vec{x}_f} e^{-i\vec{p}\cdot\vec{x}_f} \sum_{\vec{x}_2\vec{x}_1} e^{-i\vec{q}\cdot(\vec{x}_2-\vec{x}_1)} \langle \chi_N(\vec{x}_f,t_f) J_\mu(\vec{x}_2,t_2) J_\nu(\vec{x}_1,t_1) \bar{\chi}_N(\vec{0},t_0)\rangle$$

$$C_2(\vec{p},\tau) = \sum_{\vec{x}_f} e^{-i\vec{p}\cdot\vec{x}_f} \langle \chi_N(\vec{x}_f,t_f) \bar{\chi}_N(\vec{0},t_0)\rangle,$$

$$\tilde{W}(\vec{p},\vec{q},\tau) \overset{t_f \gg t_2,t_1 \gg t_0}{=} \frac{E_N \mathrm{Tr}[\Gamma_e C_4(\vec{p},\vec{q},\tau)]}{m_N \mathrm{Tr}[\Gamma_e C_2(\vec{p},\tau)]} \qquad W_{\mu\nu}(q^2,\nu) = \frac{1}{2m_N i}\int_{c-i\infty}^{c+i\infty} d\tau\, e^{\nu\tau}\, \widetilde{W}_{\mu\nu}(\vec{q}^{\,2},\tau)$$





- Liang, Liu and Yang, EPJ Web Conf. 175 (2018)

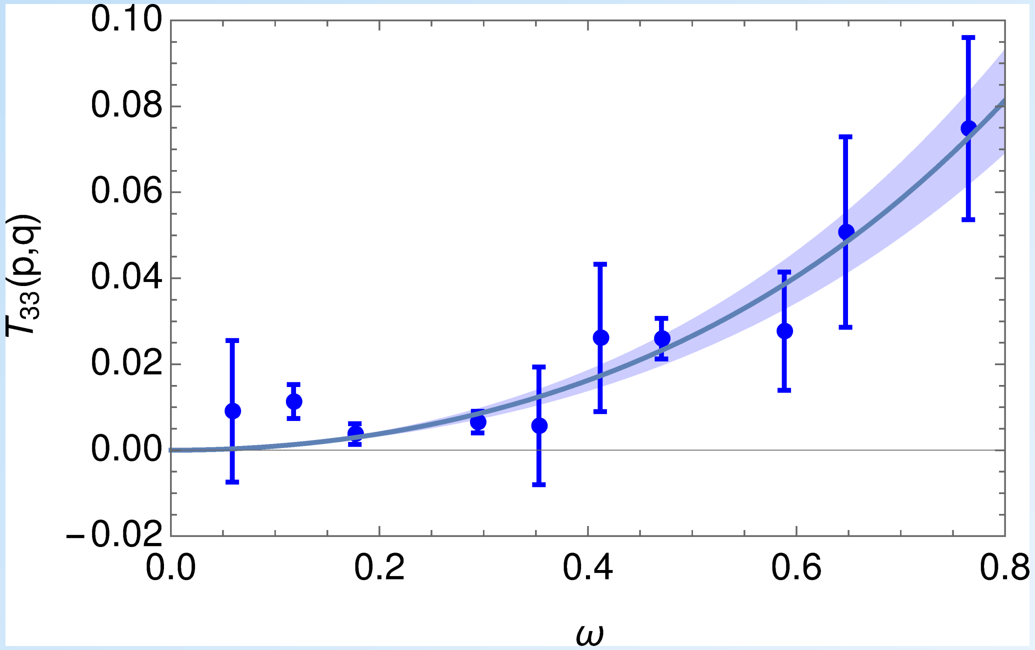
Other approaches

- OPE without OPE:

- A. J. Chambers et al. (QCDSF), PRL 118 (2017)

$$T_{\mu\nu}(p,q) = \rho_{\lambda\lambda'} \int \mathrm{d}^4x\, \mathrm{e}^{iq\cdot x} \langle p, \lambda' | T J_\mu(x) J_\nu(0) | p, \lambda \rangle$$

$$T_{33}(p,q) = \sum_{n=2,4,\cdots}^{\infty} 4\omega^n \int_0^1 dx\, x^{n-1} F_1(x, q^2)$$



Other approaches

- Restoration of Rotational Symmetry in the Continuum Limit of Lattice Field Theories
  - Davoudi and Savage PRD 86 (2012)

$$\hat{\theta}_{L,M}\left(\mathbf{x}; a, N\right) = \frac{3}{4\pi N^3}\sum_{\mathbf{n}}^{|\mathbf{n}|\leq N} \overline{\psi}\left(\mathbf{x}\right) U\left(\mathbf{x}, \mathbf{x}+\mathbf{n}a\right) \psi\left(\mathbf{x}+\mathbf{n}a\right)\ \ Y_{L,M}\left(\hat{\mathbf{n}}\right)$$

$$\begin{aligned}\hat{\theta}_{3,0}\left(\mathbf{x}; a, N\right) = {} & \frac{C^{(1)}_{30;10}\left(N\right)}{\Lambda}\mathcal{O}^{(1)}_{z}\left(\mathbf{x}; a\right) + \frac{C^{(3)}_{30;10}\left(N\right)}{\Lambda^3}\mathcal{O}^{(3)}_{z}\left(\mathbf{x}; a\right) + \frac{C^{(5)}_{30;10}\left(N\right)}{\Lambda^5}\mathcal{O}^{(5)}_{z}\left(\mathbf{x}; a\right) + \\ & \frac{C^{(5;RV)}_{30;10}\left(N\right)}{\Lambda^5}\mathcal{O}^{(5;RV)}_{z}\left(\mathbf{x}; a\right) + \frac{C^{(3)}_{30;30}\left(N\right)}{\Lambda^3}\underset{\substack{\uparrow \\ \boxed{\text{DESIRED } L = 3 \text{ OPERATOR}}}}{\mathcal{O}^{(3)}_{zzz}\left(\mathbf{x}; a\right)} + \frac{C^{(5)}_{30;30}\left(N\right)}{\Lambda^5}\mathcal{O}^{(5)}_{zzz}\left(\mathbf{x}; a\right) + \\ & \frac{C^{(5)}_{30;50}\left(N\right)}{\Lambda^5}\mathcal{O}^{(5)}_{zzzzz}\left(\mathbf{x}; a\right) + \mathcal{O}\left(\frac{\nabla^7_z}{\Lambda^7}\right)\end{aligned}$$

Outline

- Large-momentum effective theory
    - Formalism
    - Factorization formulas
- Lattice calculation of collinear distributions
    - Renormalization
    - Power corrections
    - Perturbative matching
    - Lattice calculations
    - Systematics
    - Other approaches
- TMDs from lattice QCD
    - Quasi-TMDs and relation to TMDs
    - Collins-Soper kernel from lattice
- Summary and outlook

Quasi-TMDPDF

For more details see Yong Zhao's talks on Monday and Thursday.

- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- Ji, Jin, Yuan, Zhang and Y.Z., PRD99 (2019);
- M. Ebert, I. Stewart, Y.Z., PRD99 (2019);
- M. Ebert, I. Stewart, Y.Z., JHEP09(2019)037.

- Definition:

$$\tilde f_q^{\rm TMD}(x, \vec b_T, \mu, P^z) = \int \frac{db^z}{2\pi} e^{ib^z(xP^z)} \tilde Z'(b^z, \mu, \tilde\mu) \tilde Z_{\rm UV}(b^z, \mu, a) \frac{\tilde B_q(b^z, \vec b_T, a, L, P^z)}{\sqrt{\tilde S_q(b_T, a, L)}}$$

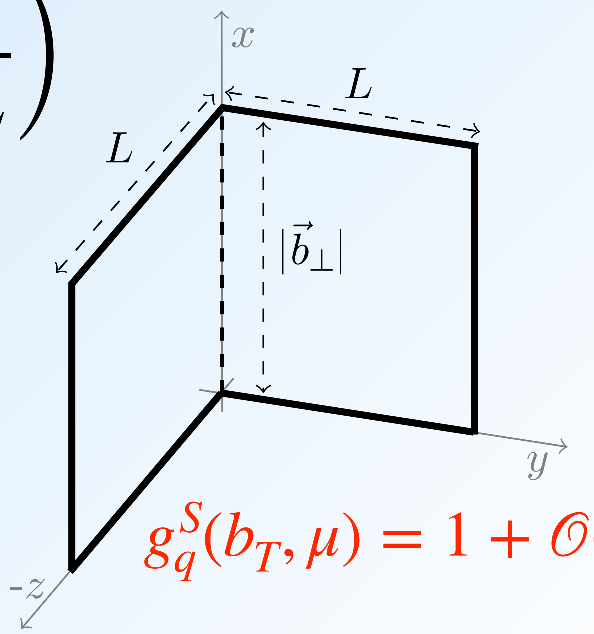
- Relationship to the physical TMDPDF:

$$\tilde f_{\rm ns}^{\rm TMD}(x, \vec b_T, \mu, P^z) = C_{\rm ns}^{\rm TMD}\left(\mu, xP^z\right)\, g_q^S(b_T, \mu)\, \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T)\ln\frac{(2xP^z)^2}{\zeta}\right]$$

$$\times f_{\rm ns}^{\rm TMD}(x, \vec b_T, \mu, \zeta) + \mathcal{O}\left(\frac{b_T}{L}, \frac{1}{b_T P^z}, \frac{1}{P^z L}\right)$$

$$b^z \sim \frac{1}{P^z} \ll b_T \ll L$$

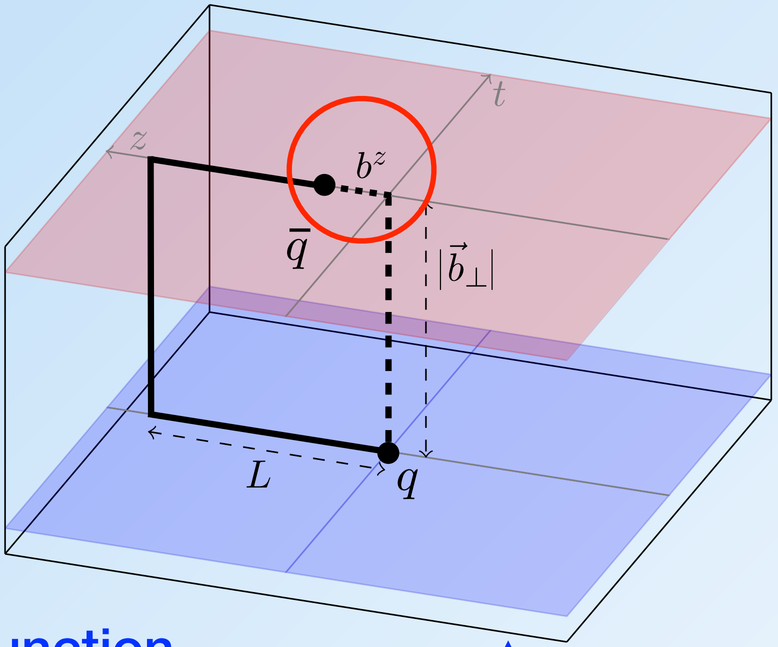
$C_{\rm ns}^{\rm TMD}\left(\mu, xP^z\right)$	Perturbative matching coefficient
$g_q^S(b_T, \mu)$	Nonperturbative function for large b_T, depending on the choice of the quasi-soft factor



$$g_q^S(b_T, \mu) = 1 + \mathcal{O}(\alpha_s^2)$$

Collins-Soper kernel of TMDPDF from lattice QCD

- M. Ebert, I. Stewart, Y.Z., PRD99 (2019);
- M. Ebert, I. Stewart, Y.Z., JHEP09(2019)037;
- M. Ebert, I. Stewart, Y.Z., in progress.



Quasi beam function
(or unsubtracted quasi-TMD)

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)}$$

$$\times \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z) \int db^z \; e^{ib^z xP_1^z} \; \tilde{Z}'(b^z, \mu, \tilde{\mu}) \tilde{Z}_{\rm UV}(b^z, \tilde{\mu}, a) \tilde{B}_{\rm ns}(b^z, \overrightarrow{b}_T, a, L, P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z) \int db^z \; e^{ib^z xP_2^z} \; \tilde{Z}'(b^z, \mu, \tilde{\mu}) \tilde{Z}_{\rm UV}(b^z, \tilde{\mu}, a) \tilde{B}_{\rm ns}(b^z, \overrightarrow{b}_T, a, L, P_2^z)}$$

Collins-Soper (CS) kernel does not depend on the external hadron state, which means that one can calculate it with a pion state including heavier than physical valence quarks.

A first look at the CS kernel

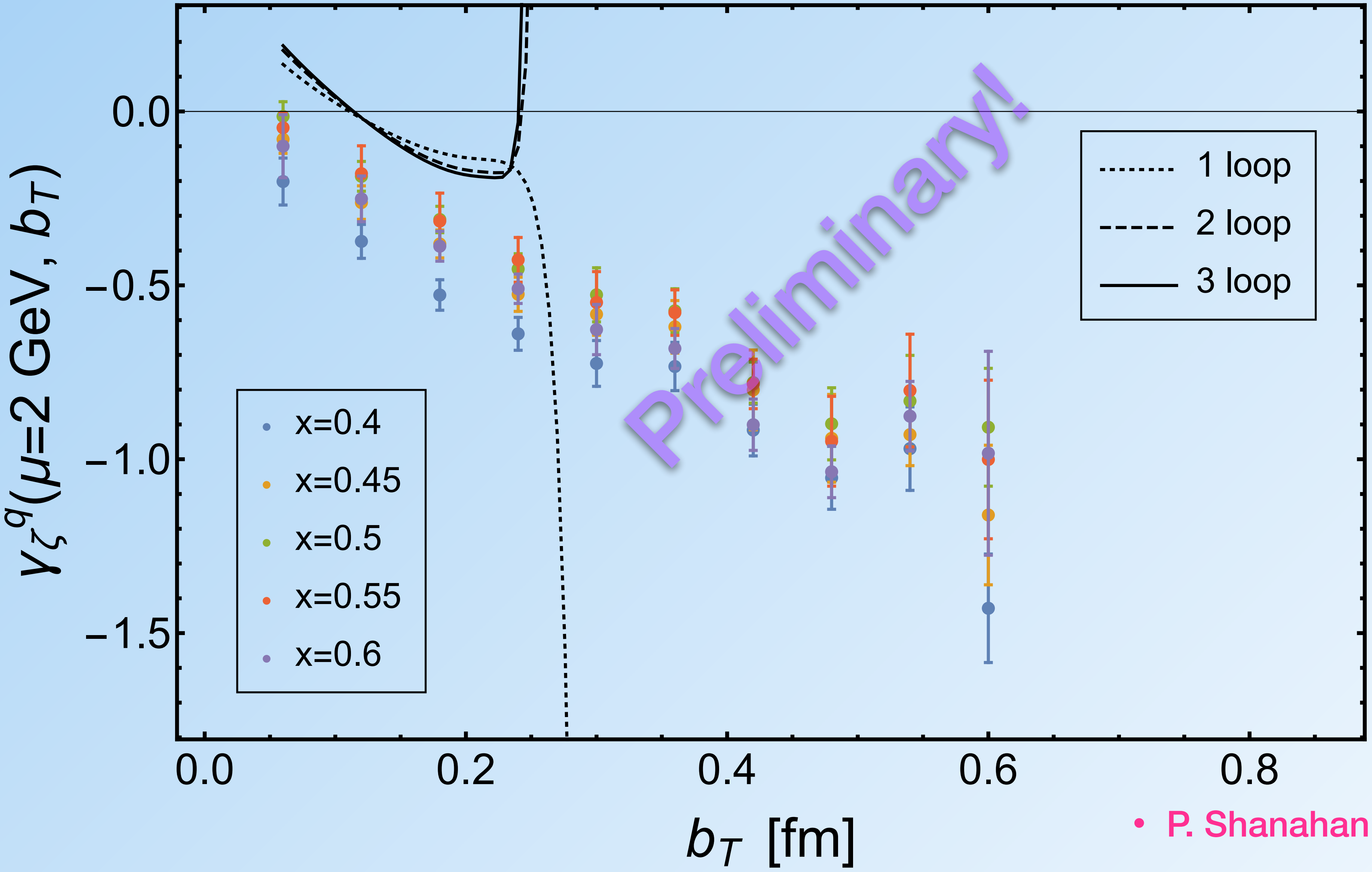
Caveat: low stats and operator mixings not considered.

$$\gamma_\zeta^i(\mu, b_T) = -2\int_{1/b_T}^{\mu} \frac{d\mu'}{\mu'}\Gamma^i_{\rm cusp}[\alpha_s(\mu')] + \gamma_\zeta^i[\alpha_s(1/b_T)]$$

α_s running with Nf=0.

$a = 0.06$ fm

$P_1^z = 1.3$ GeV, $P_2^z = 1.9$ GeV



- P. Shanahan, M. Wagman, Y.Z., in progress.

Conclusion

- A systematic procedure to calculate the collinear PDFs have already been established with the LaMET approach;
- Current lattice results have shown promising signs for the extraction fo the x-dependence of PDFs;
- There are still systematic uncertainties that need to be improved or constraint;
- Progress has also been made with other approaches;
- Extension of LaMET to TMDPDF have been under study, and progress is also being made in lattice calculations.