Proton pressure and shear distributions from lattice QCD

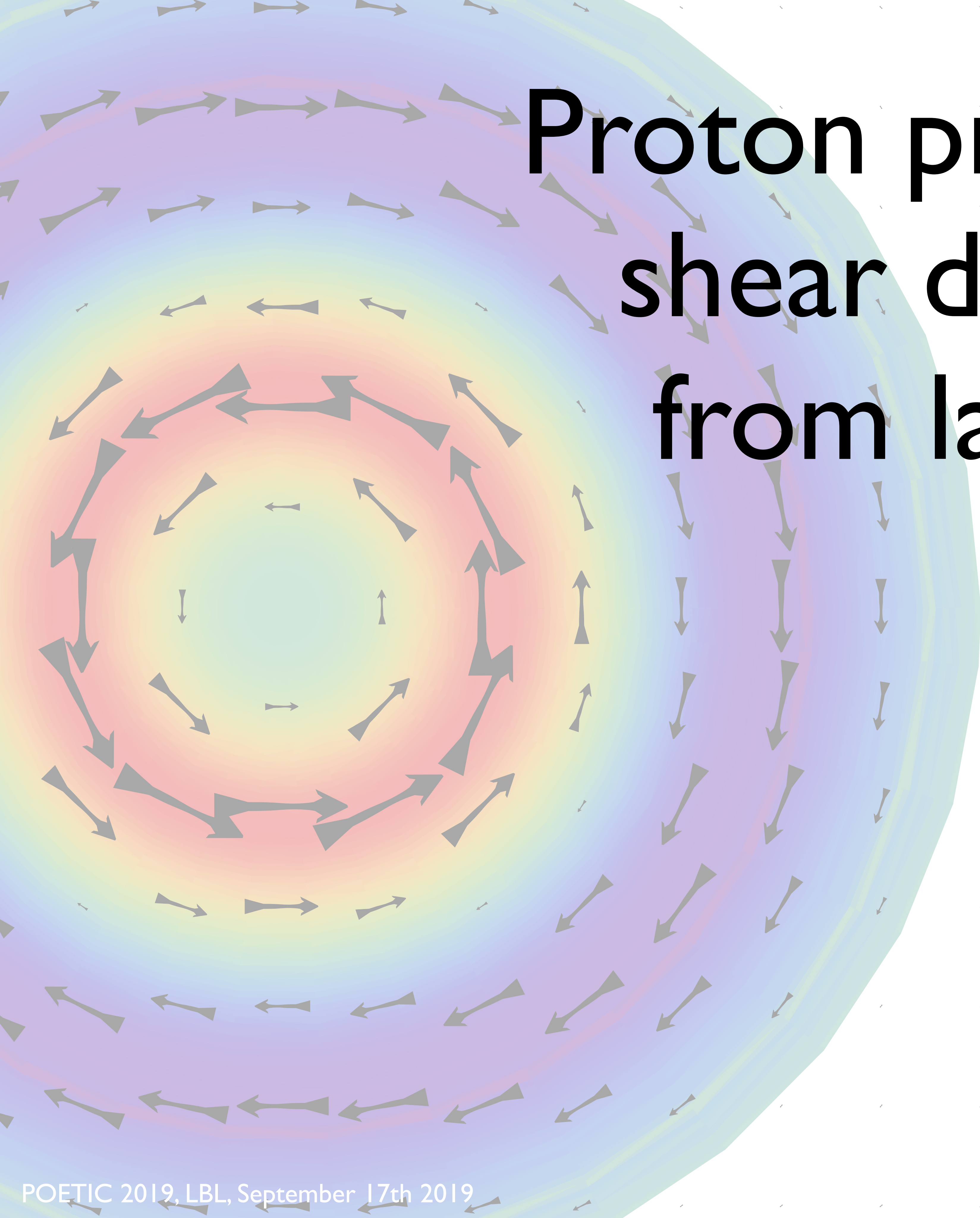
Will Detmold

work wth Phiala Shanahan

PRL (2019) arXiv:1810.07589

PRD (2019) arXiv:1810.04626

Massachusetts Institute of Technology



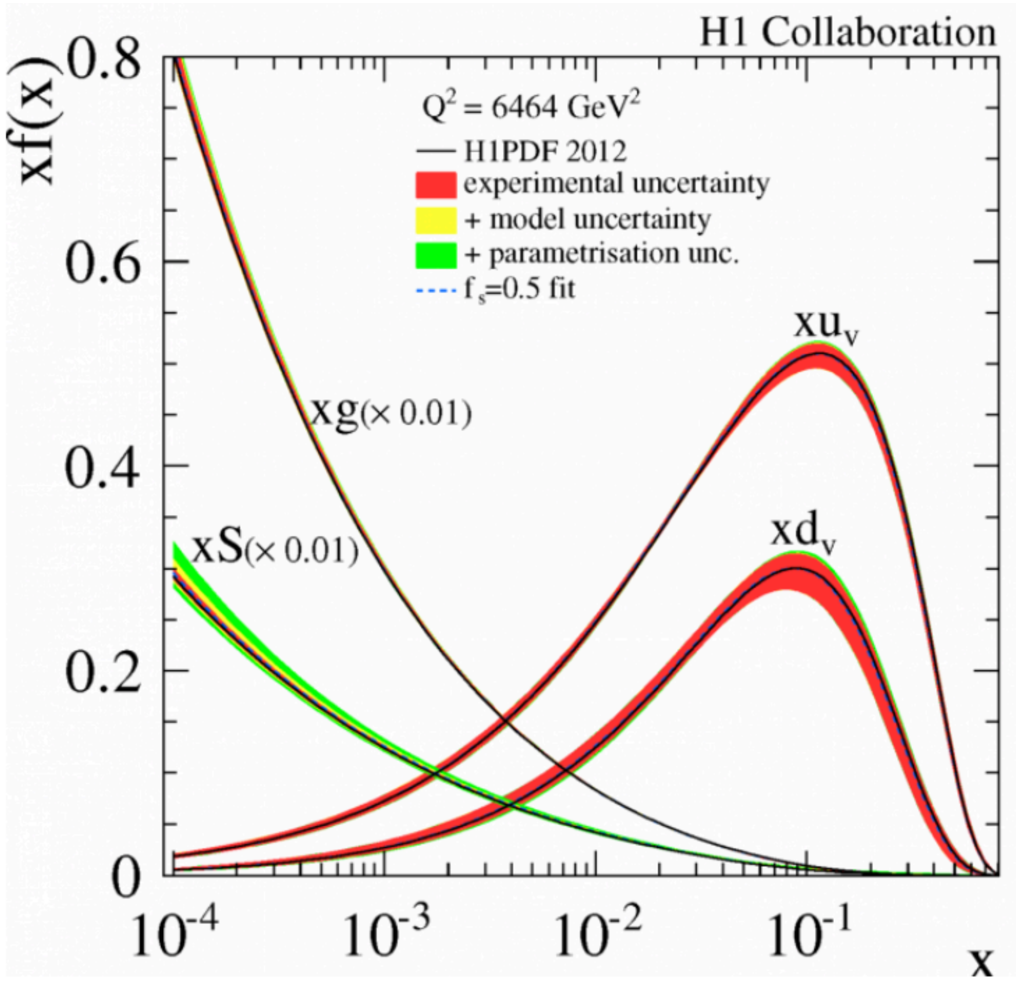
POETIC 2019, LBL, September 17th 2019

Gluon structure

Gluons offer a new window on nuclear structure

- Past 60+ years: detailed view of quark structure of nucleons
- Gluon structure also important
  - Unpolarised gluon PDF dominant at small longitudinal momentum fraction
- Other aspects of gluon structure relatively unexplored

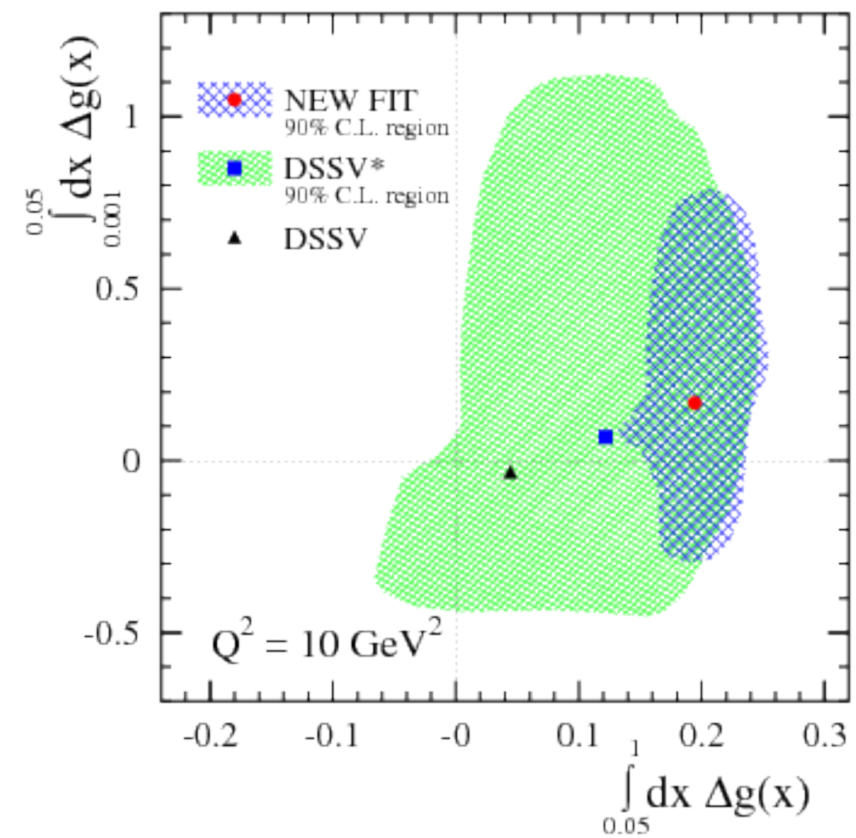
Parton distributions in the proton

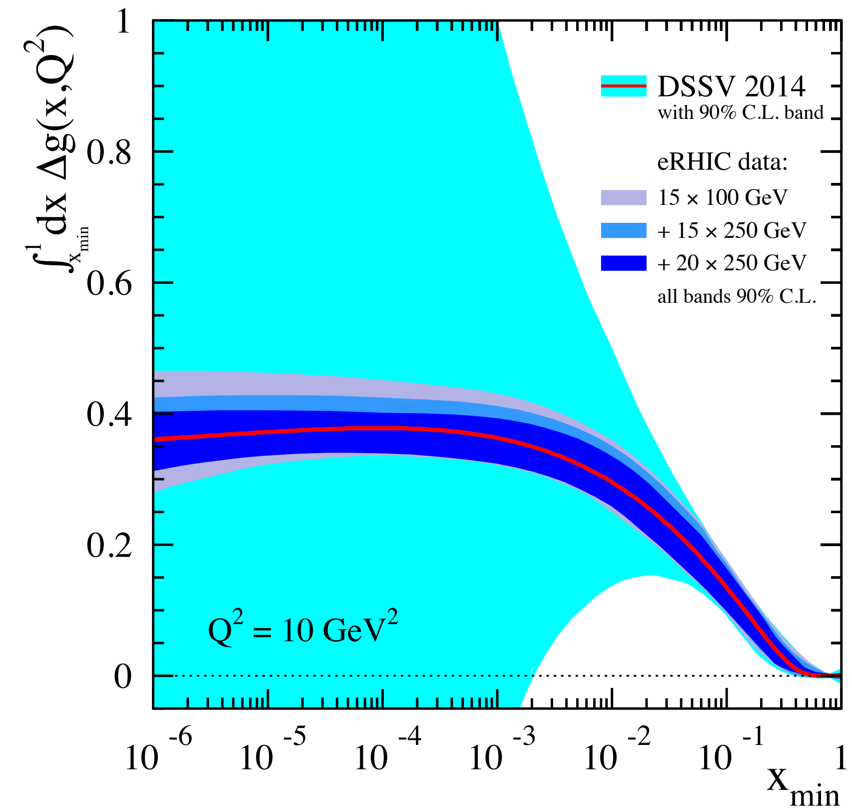


Longitudinal momentum fraction carried by parton

Gluon angular momentum

- Gluon helicity much less well constrained
  - Major focus of RHIC-spin program
  - Asymmetries in polarised $pp \rightarrow \pi X$, DX, BX, *jets*
- Orbital angular momentum of gluons even less understood
  - Gluon TMDs
- Further major motivation for EIC





de Florian et. al, Phys.Rev.Lett. 113, 012001 (2014)

Gluon structure

How much do gluons contribute to the proton's

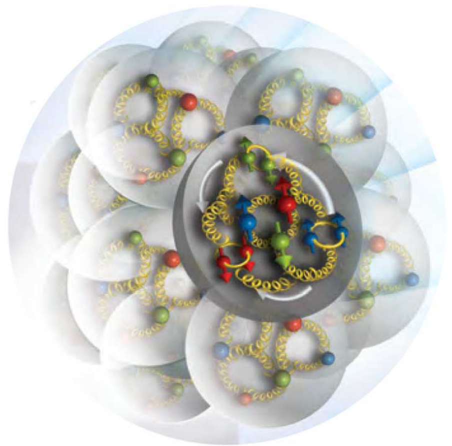
- Momentum
- Spin
- Mass
- D-term

What are the gluon distributions in a proton

- PDFs, GPDs, TMDs
- Pressure, Shear
- Gluon 'radius/radii'

How is the gluon structure of a proton modified in a nucleus

- Gluon 'EMC' effect
- Exotic glue



Gluon structure

First-principles QCD calculations

→ QCD benchmarks and predictions ahead of experiment

Cover image from EIC whitepaper arXiv::1212.1701

Energy-momentum tensor

Many of these properties derived from **Energy-Momentum Tensor** (conserved Noether current associated with Lorentz translations)

Matrix elements of traceless gluon EMT for spin-half nucleon:

$$\langle p', s' | G^{a}_{\{\mu\alpha} G^{a\alpha}{}_{\nu\}} | p, s \rangle = \bar{U}(p', s') \left(A_g(t)\, \gamma_{\{\mu} P_{\nu\}} + B_g(t)\, \frac{i\, P_{\{\mu} \sigma_{\nu\}\rho} \Delta^{\rho}}{2M_N} + D_g(t)\, \frac{\Delta_{\{\mu} \Delta_{\nu\}}}{4M_N} \right) U(p, s)$$

Generalised gluon form factors $\Delta_\mu = p'_\mu - p_\mu \quad P_\mu = (p_\mu + p'_\mu)/2 \quad t = \Delta^2$

- Three generalised gluon form factors $A_g(t),\ B_g(t),\ D_g(t)$
- Sum rules with quark pieces in forward limit
  - Momentum fraction $A_a(0) = \langle x \rangle_a$ ➡ $\sum_{a=q,g} A_a(0) = 1$
  - Spin $J_a(t) = \frac{1}{2}(A_a(t) + B_a(t))$ ➡ $\sum_{a=q,g} J_a(0) = \frac{1}{2}$
  - D-terms $D_a(0)$ unknown but equally fundamental!

D-term

D-term GFF encodes the pressure **and** shear **distributions in the nucleon** (Breit frame)

$$s(r) = -\frac{r}{2}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\widetilde{D}(r), \quad p(r) = \frac{1}{3}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\widetilde{D}(r),$$

$$\widetilde{D}(r) = \int \frac{d^3\vec{p}}{2E(2\pi)^3}\, e^{-i\vec{p}\cdot\vec{r}}\, D(-\vec{p}^{\,2})$$

- Quark and gluon shear forces individually well-defined (i.e., scale-dependent partial contributions $s_{q,g}(r)$
- Pressure defined only for the total system (pieces depend also on GFFs related to the trace terms of the EMT that cancel in the sum)

Generalised parton distributions

GFFs correspond to lowest moments of GPDs:

$$\int_0^1 \mathrm{d}x\; H_g(x,\xi,t) = A_g(t) + \xi^2 D_g(t)\,, \qquad \int_0^1 \mathrm{d}x\; E_g(x,\xi,t) = B_g(t) - \xi^2 D_g(t)$$

$$\int_{-1}^1 \mathrm{d}x\; x\,H_q(x,\xi,t) = A_q(t) + \xi^2 D_q(t)\,, \qquad \int_{-1}^1 \mathrm{d}x\; x\,E_q(x,\xi,t) = B_q(t) - \xi^2 D_q(t)$$

- **Quark GPDs:** constraints from JLab, HERA, COMPASS, by DVCS, DVMP, future improvements from JLab 12GeV
- **Gluon GPDs:** almost unknown from experiment, future constraints are a central goal of EIC

Leading twist nucleon gluon GPDs:

Gluon field-strength tensor

$$\Delta_\mu = p'_\mu - p_\mu \quad P_\mu = (p_\mu + p'_\mu)/2$$

$$t = \Delta^2 \quad n^2 = 0 \quad \xi = -\tfrac{1}{2} n\cdot\Delta / n\cdot P$$

$$\int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p', s' | G_a^{\{\mu\alpha}(-\frac{\lambda}{2}n) \left[\mathcal{U}^{(A)}_{[-\frac{\lambda}{2}n, \frac{\lambda}{2}n]}\right]_{ab} G_{b\alpha}^{\;\;\nu\}}(\frac{\lambda}{2}n) | p, s\rangle$$

$$= \frac{1}{2}\left(H_g(x,\xi,t) \bar{U}(p',s') P^{\{\mu}\gamma^{\nu\}} U(p,s) + E_g(x,\xi,t) \bar{U}(p',s') \frac{P^{\{\mu} i\sigma^{\nu\}\alpha}\Delta_\alpha}{2M} U(p,s)\right) + \ldots\,,$$

GPDs(Bjorken x, skewness, mom transfer)

D-term from JLab DVCS

Recent experimental determination of DVCS D-term and extraction of proton pressure distribution

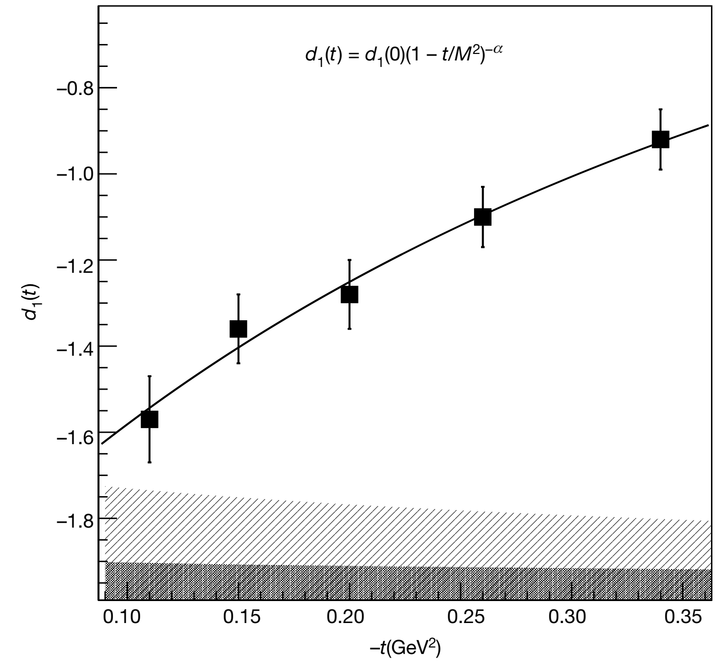
V. D. Burkert, L. Elouadrhiri, and F. X. Girod, Nature 557, 396 (2018)

$$s(r) = -\frac{r}{2}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\widetilde{D}(r), \quad p(r) = \frac{1}{3}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\widetilde{D}(r)$$

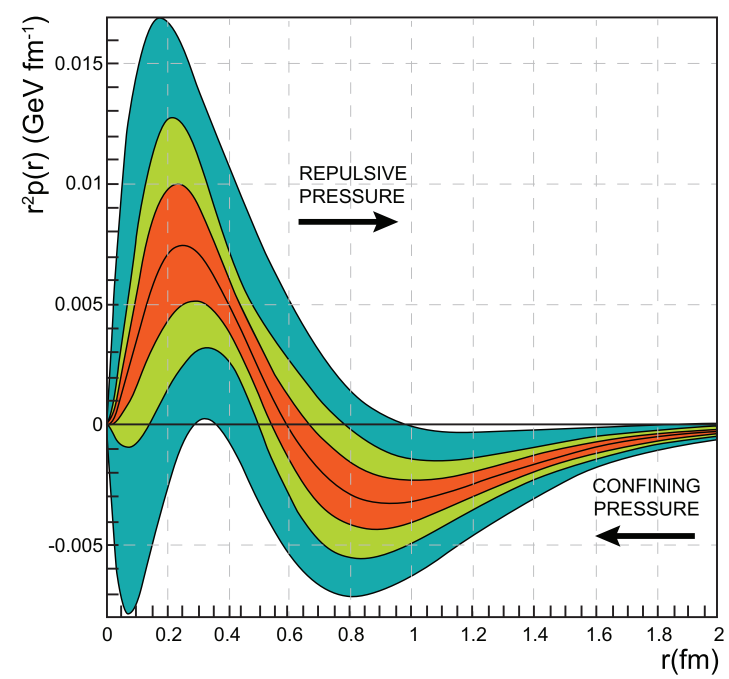
- Strong repulsive pressure near the centre of the proton
- Binding pressure at greater distances.
- Peak pressure near the centre ~10^{35} Pascal, greater than pressure estimated for neutron stars
- **Key assumptions:** gluon D-term same as quark term, tripole form factor model, $D_u(t,\mu) = D_d(t,\mu)$

Use lattice QCD to test assumptions in pressure extraction

DVCS (quark) D-term



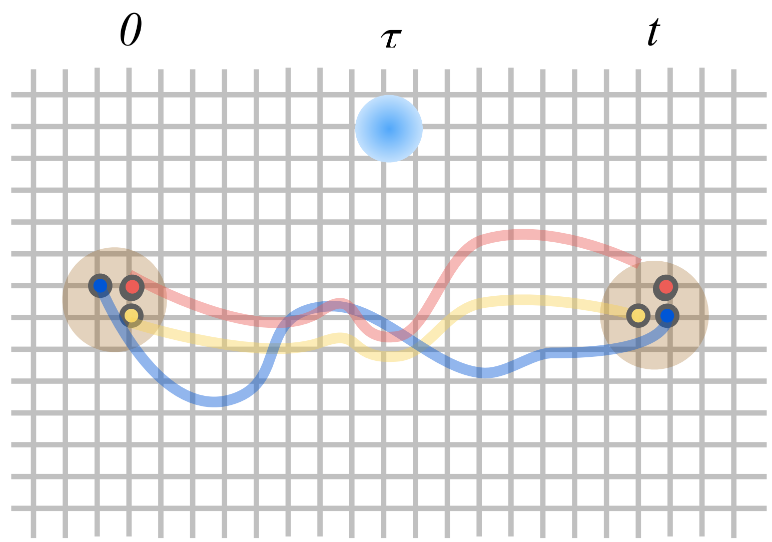
Radial pressure distribution



Lattice correlations

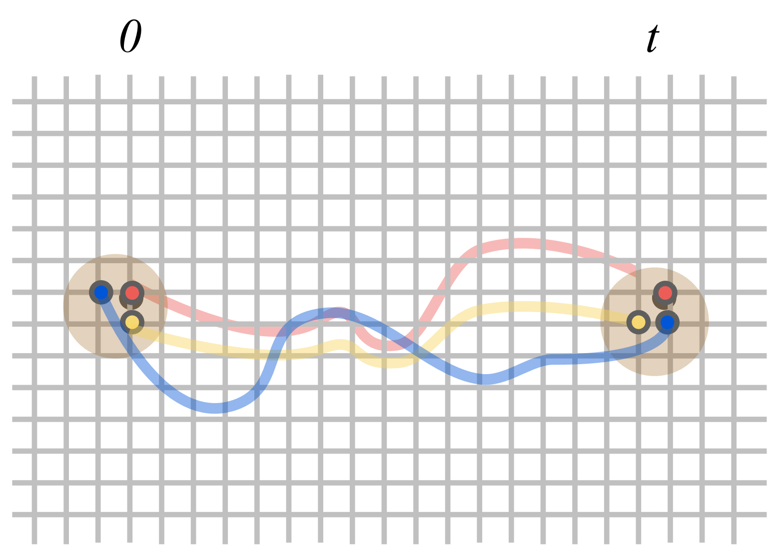
LQCD: extract matrix element from ratio of correlators

$C_3(t,\tau) =$



$$\frac{C_3(t,\tau)}{C_2(t)} \propto A_2, \qquad 0 \ll \tau \ll t$$

$C_2(t) =$



Gluon GFFs from LQCD

Construct system of equations for **generalised gluon form factors**

Ratios of 3pt and 2pt correlation functions:

$$R_{s;\mathfrak{R},i}(\vec{p},\vec{p}\,',t_f,\tau) = \frac{C^{3\mathrm{pt}}_{s;\mathfrak{R},i}(\vec{p},\vec{p}\,',t_f,\tau)}{C^{2\mathrm{pt}}_{s}(\vec{p}\,',t_f)}\sqrt{\frac{C^{2\mathrm{pt}}_{s}(\vec{p},t_f-\tau)C^{2\mathrm{pt}}_{s}(\vec{p}\,',t_f)C^{2\mathrm{pt}}_{s}(\vec{p}\,',\tau)}{C^{2\mathrm{pt}}_{s}(\vec{p}\,',t_f-\tau)C^{2\mathrm{pt}}_{s}(\vec{p},t_f)C^{2\mathrm{pt}}_{s}(\vec{p},\tau)}} \xrightarrow{t_f\gg\tau\gg 0} \frac{\mathrm{Tr}\left[\Gamma_s(\not{p}'+M_N)\mathcal{F}_i[A_g,B_g,D_g](\not{p}+M_N)\right]}{8\sqrt{E^{(N)}_{\vec{p}}E^{(N)}_{\vec{p}\,'}(E^{(N)}_{\vec{p}}+M_N)(E^{(N)}_{\vec{p}\,'}+M_N)}}$$

$$\mathcal{F}_{\mu\nu}[A_g,B_g,D_g] = A_g(t)\,\gamma_{\{\mu}P_{\nu\}} + B_g(t)\,\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2M_N} + D_g(t)\,\frac{\Delta_{\{\mu}\Delta_{\nu\}}}{4M_N}$$

Generalised gluon form factors

$\Delta_\mu = p'_\mu - p_\mu \quad P_\mu = (p_\mu + p'_\mu)/2 \quad t = \Delta^2$

- Nucleon spin up/down: $\Gamma_{s=\pm 1}$
- Sink and operator momenta:

$$|\vec{p'}\,|^2 \leq 5(2\pi/L)^2$$

$$|\vec{\Delta}|^2 \leq 18(2\pi/L)^2$$

- Operator index choices: two different irreducible representations of H(4)

$$\mathcal{O}^{\tau_3^{(6)}}_{i=\{1,\ldots,6\}} = \left\{\frac{(-i)^{\delta_{\nu 0}}}{\sqrt{2}}(\mathcal{O}_{\mu\nu}+\mathcal{O}_{\nu\mu}),\ \ 0\leq\mu<\nu\leq 3\right\}$$

$$\mathcal{O}^{\tau_1^{(3)}}_{1} = \frac{1}{2}(\mathcal{O}_{11}+\mathcal{O}_{22}-\mathcal{O}_{33}+\mathcal{O}_{00}),\ \ \ldots,$$

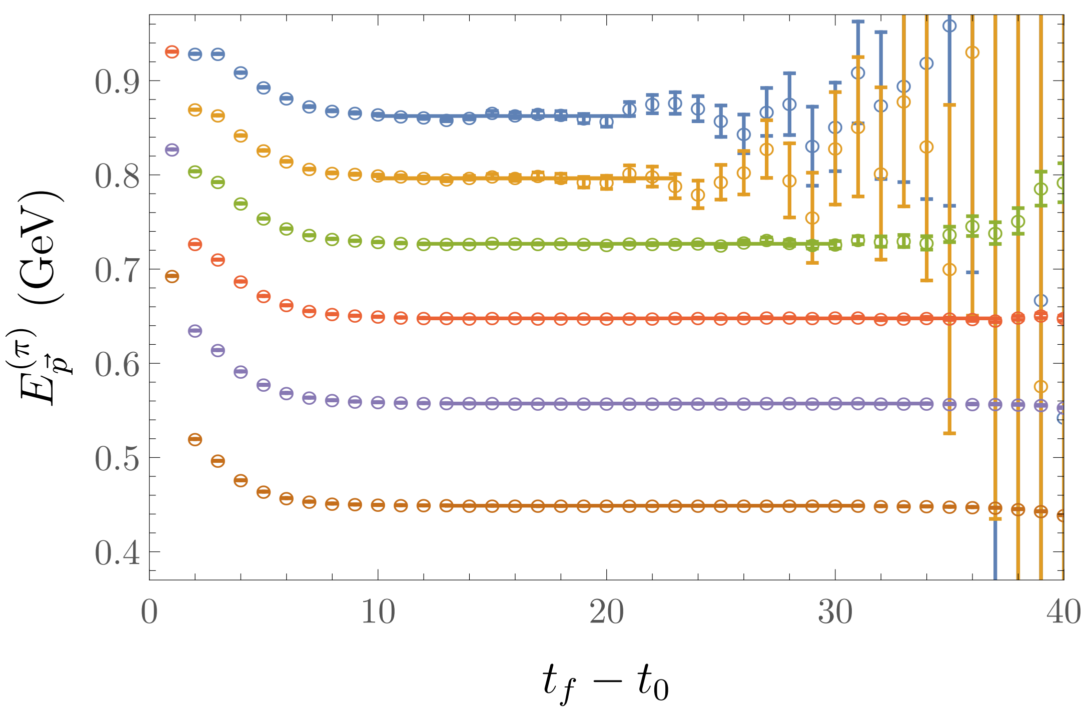
Gluon GFFs from LQCD

One ensemble, **m_π ~450 MeV**

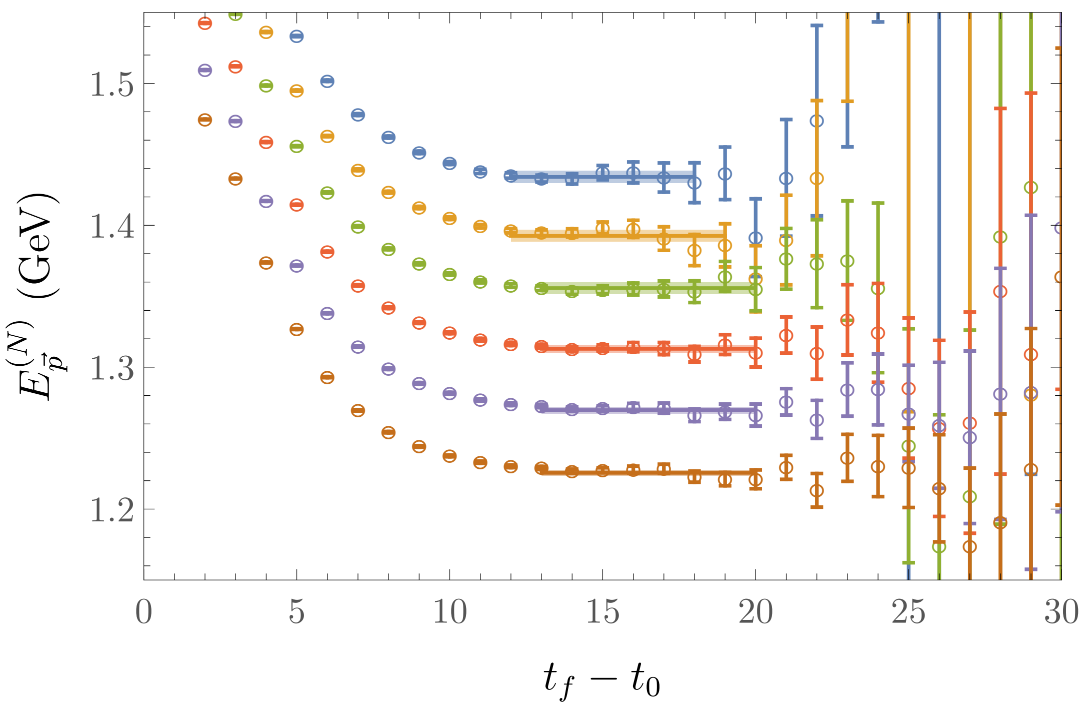
L/a	T/a	β	am_l	am_s	a (fm)	L (fm)	T (fm)	m_π (MeV)	m_K (MeV)	$m_\pi L$	$m_\pi T$	N_{cfg}	N_{meas}
32	96	6.1	-0.2800	-0.2450	0.1167(16)	3.7	11.2	450(5)	596(6)	8.5	25.6	2821	203

Clean plateaus in effective masses for $|\vec{p'}|^2 \leq 5(2\pi/L)^2$

Pion

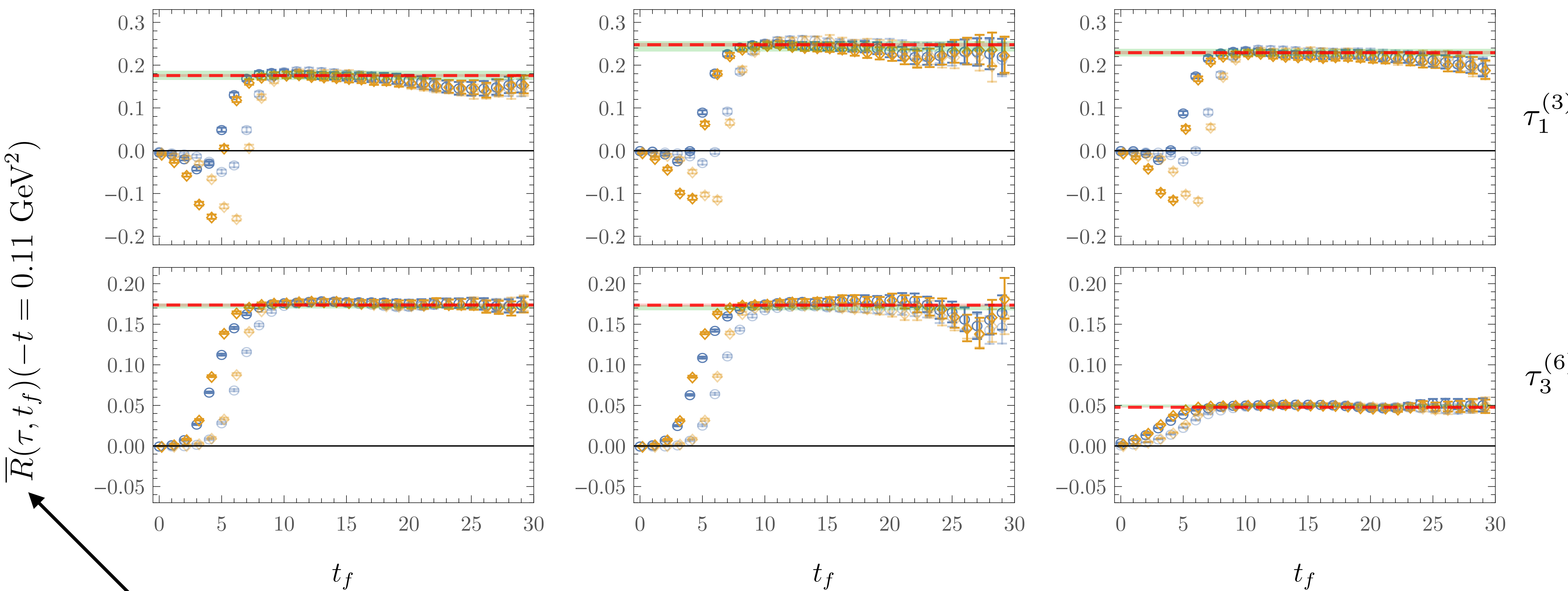


Nucleon



Gluon GFFs from LQCD

PION: Clean signals in 3pt/2pt ratios (examples)



τ = 5 (dark points), τ=7 (pale points)

Ratios that give same linear combination of GFFs are averaged

Plateau fits

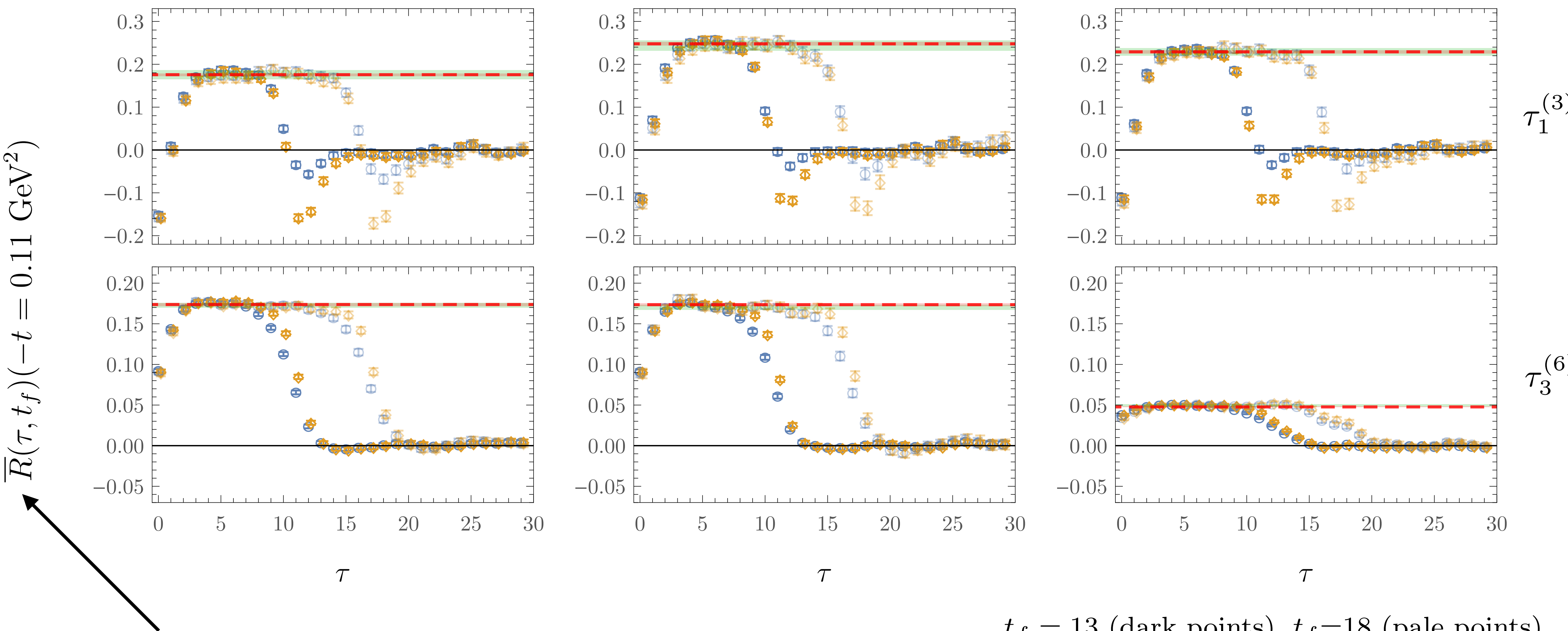
Solution of system of equations projected back

smeared-point

smeared-smeared

Gluon GFFs from LQCD

PION: Clean signals in 3pt/2pt ratios (examples)



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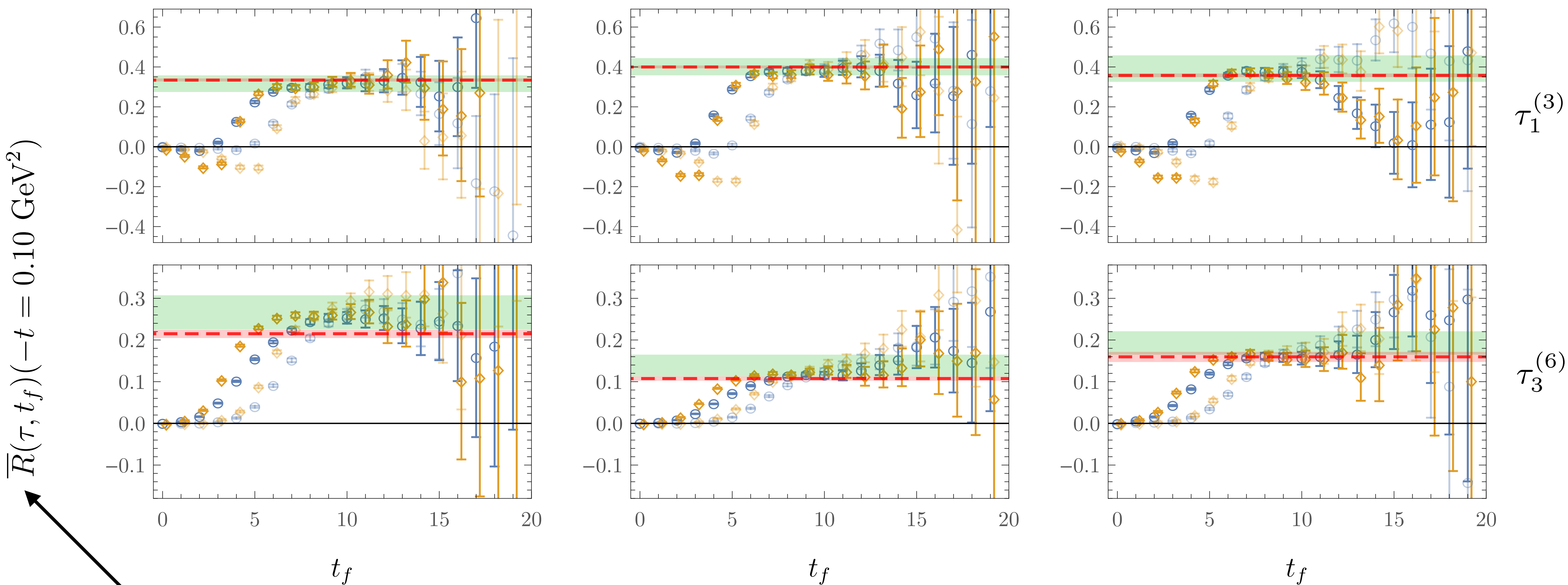
Gluon GFFs from LQCD

NUCLEON: Clean signals in 3pt/2pt ratios (examples)



Ratios that give same linear combination of GFFs are averaged

Plateau fits

Solution of system of equations projected back

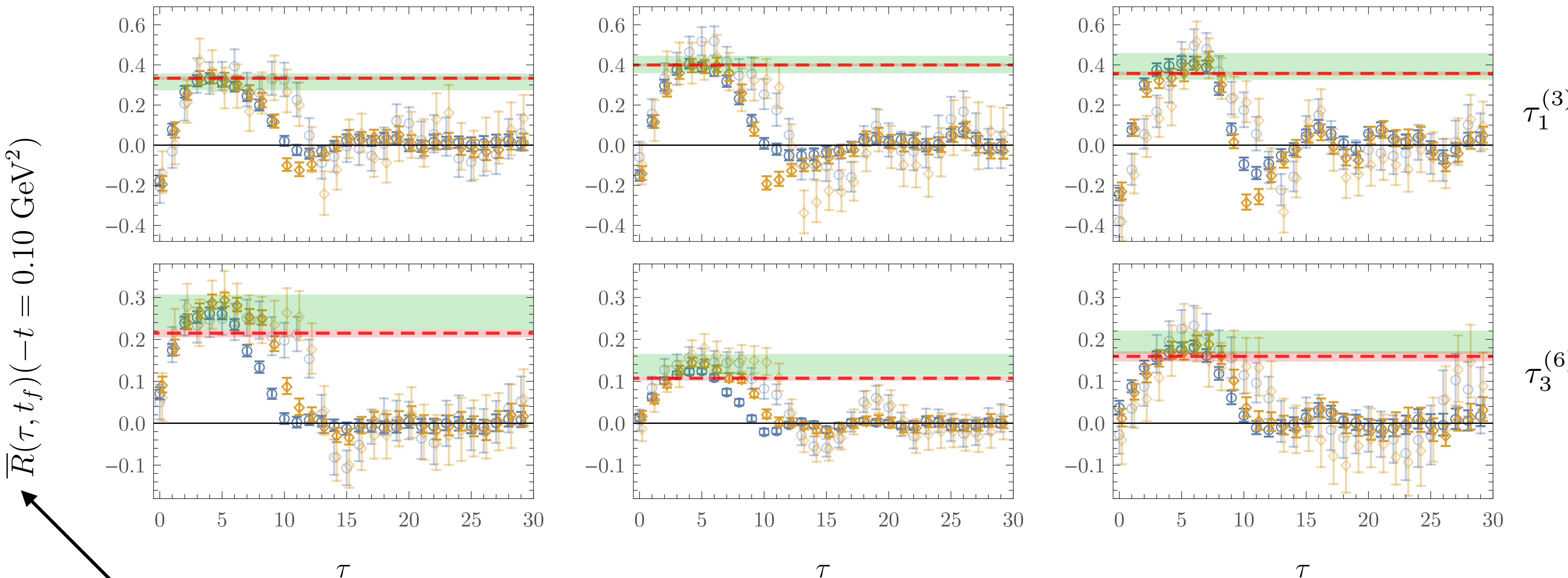
τ = 4 (dark points), τ=6 (pale points)

smeared-point

smeared-smeared

Gluon GFFs from LQCD

NUCLEON: Clean signals in 3pt/2pt ratios (examples)



$t_f = 12$ (dark points), t_f=14 (pale points)

Ratios that give same linear combination of GFFs are averaged

Plateau fits

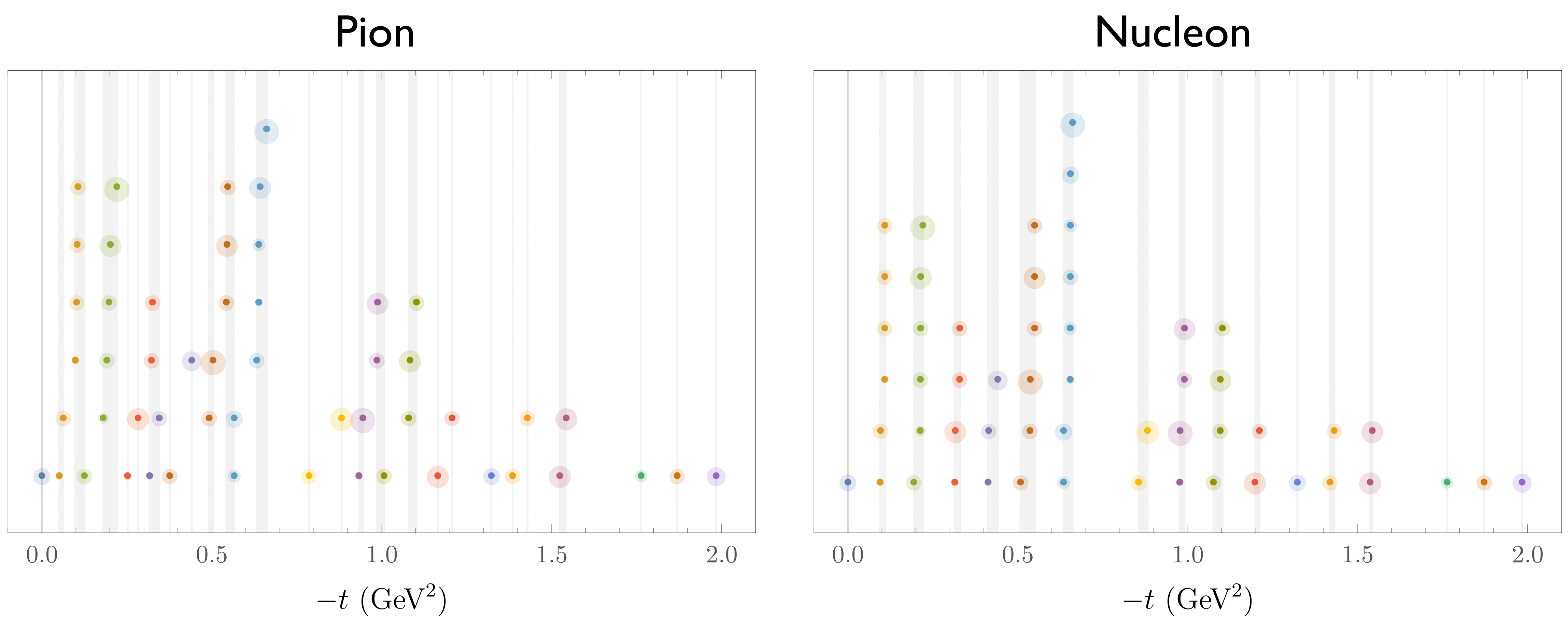
Solution of system of equations projected back

smeared-point

smeared-smeared

Gluon GFFs from LQCD

Solve system of equations for GFFs in bins in $t = (p' - p)^2$



- Colour coding: three momentum transfer $\vec{\Delta}^2 = (\vec{p}\,' - \vec{p})^2$
- Point size $\propto$ number of three-momenta at that $\vec{\Delta}^2$
- Grey bands: bins in t

Renormalisation

Non-perturbative RI-MOM renormalisation of gluon operator

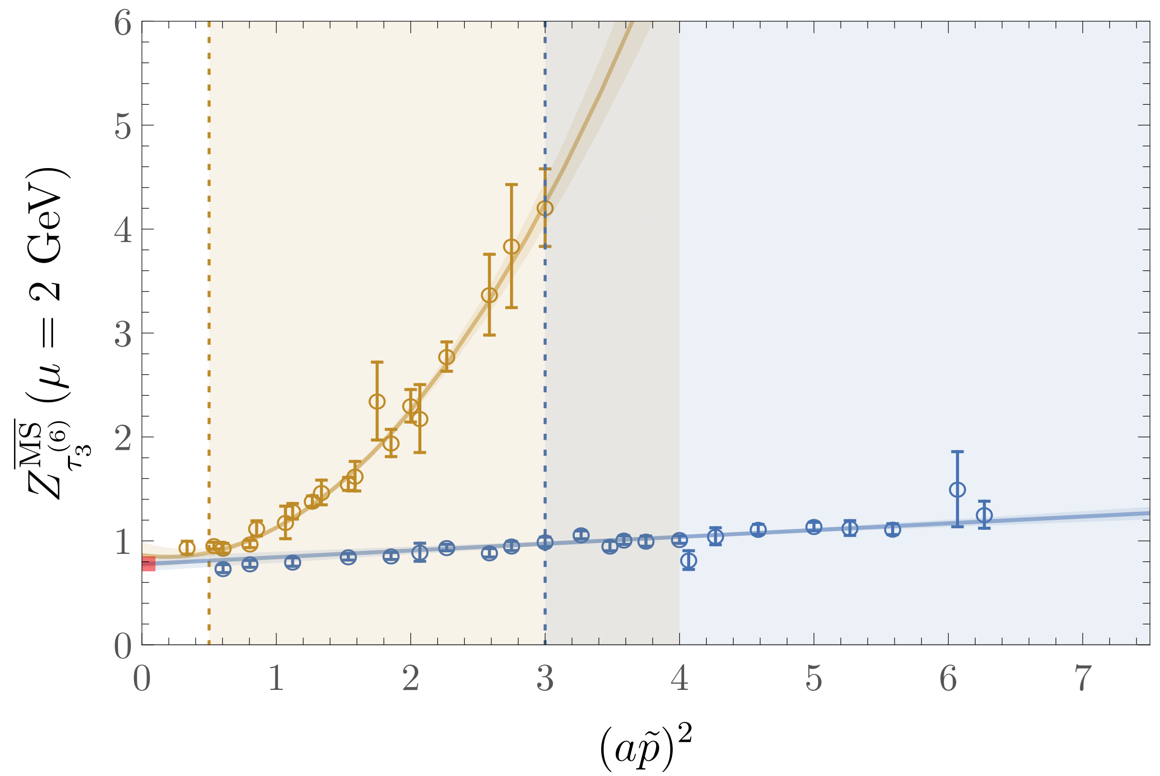
- Mixing with quark operator neglected
  Found to be small in lattice PT e.g., Alexandrou et al., 1611.06901
- One-loop perturbative matching to $\overline{\mathrm{MS}}$ scheme: Yang et al., 1612.02855

$$\mathcal{O}^{\overline{\mathrm{MS}}}(\mu^2) = Z_{\mathcal{O}}^{\overline{\mathrm{MS}}}(\mu^2)\mathcal{O}^{\mathrm{latt}} = \mathcal{R}^{\overline{\mathrm{MS}}}(\mu^2,\mu_R^2) Z_{\mathcal{O}}^{\mathrm{RI\text{-}MOM}}(\mu_R^2)\mathcal{O}^{\mathrm{latt}}$$

Calculate RI-MOM coefficient using Landau-gauge fixed gluon 2pt function

$$\left(Z_{\hat{\mathcal{O}}}^{\mathrm{RI\text{-}MOM}}(\mu_R^2)\right)^{-1} = \left.\frac{4p^2\langle \hat{\mathcal{O}}_{\alpha\beta}\mathrm{Tr}[A_\tau(p)A_\tau(-p)]\rangle}{\Lambda_{\hat{\mathcal{O}}}^{\mathrm{tree}}(p)\langle \mathrm{Tr}[A_\tau(p)A_\tau(-p)]\rangle}\right|_{\substack{p^2=\mu_R^2\\ \tau\neq\alpha\neq\beta\\ ,p_\tau=0}}$$

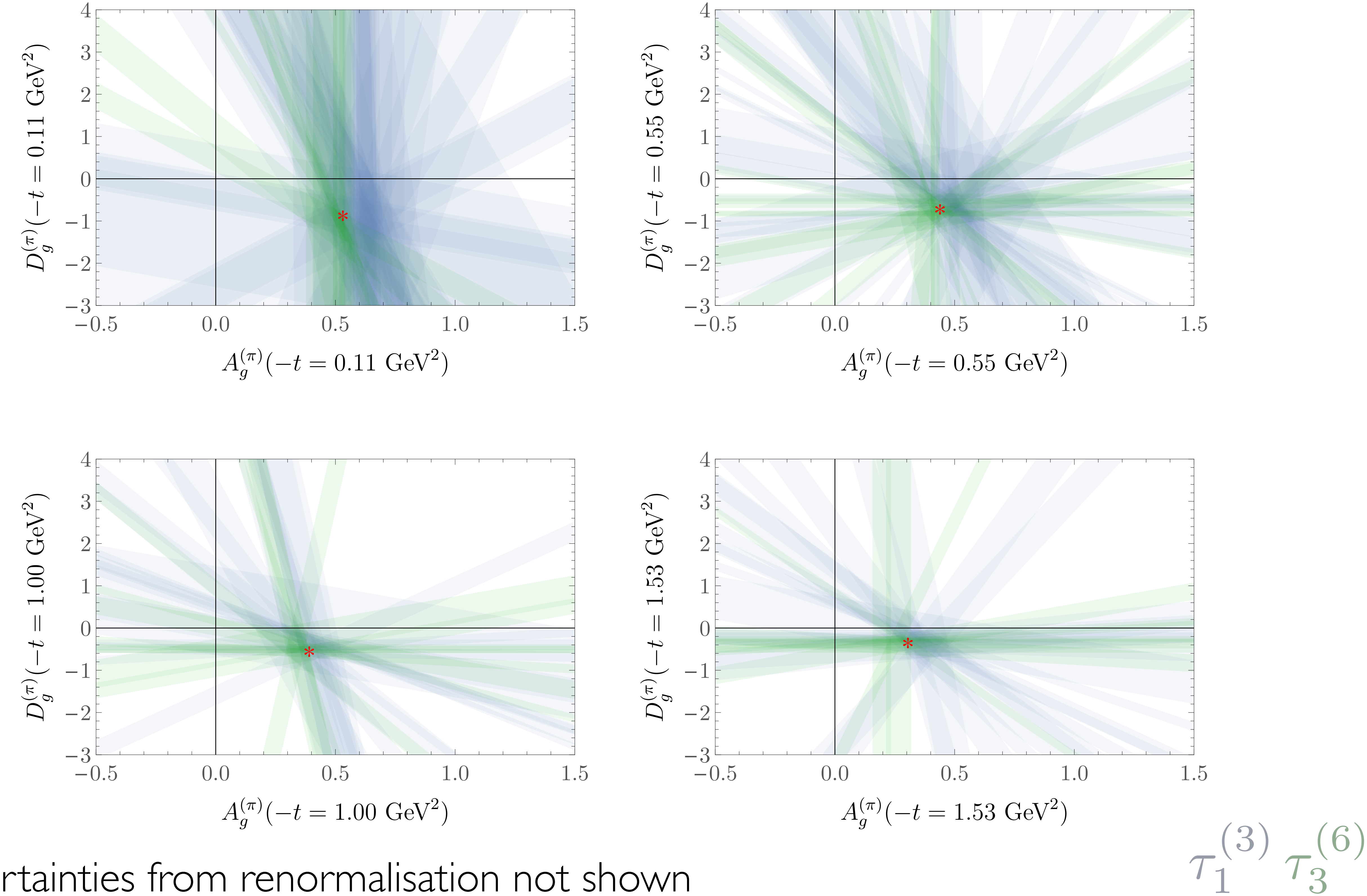
$$\Lambda_{\hat{\mathcal{O}}}^{\mathrm{tree}}(p) = \langle \hat{\mathcal{O}}_{\alpha\beta}^{\mathfrak{R}}\mathrm{Tr}[A_\tau(p)A_\tau(-p)]\rangle_{\mathrm{amp.}}^{\mathrm{tree}}$$



Wilson-flowed gluon 2pts

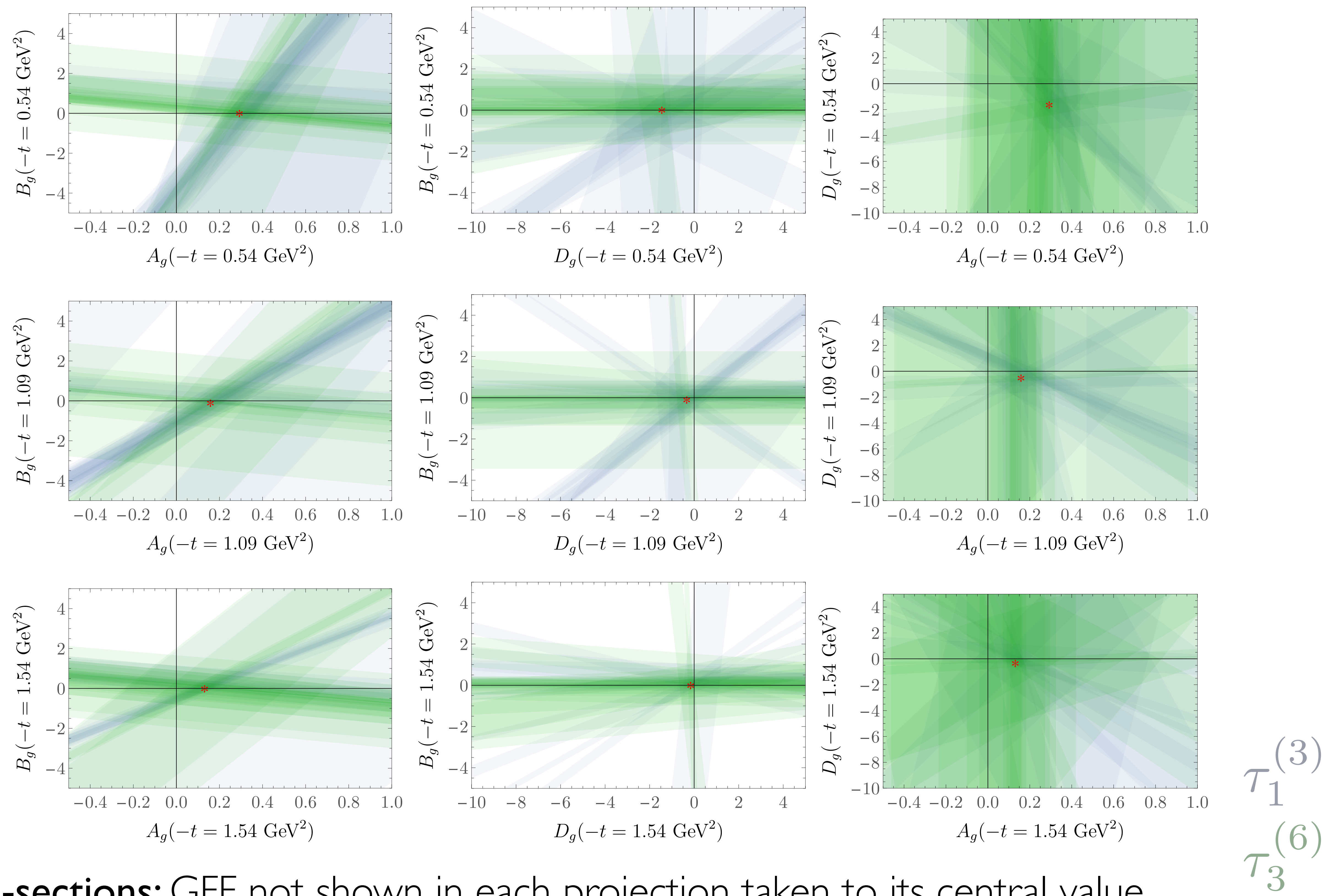
No flow in 2pts

Gluon GFFs from LQCD



Uncertainties from renormalisation not shown

Gluon GFFs from LQCD



Cross-sections: GFF not shown in each projection taken to its central value

LQCD Pion GFFs

Pion gluon GFFs m_π ~450 MeV

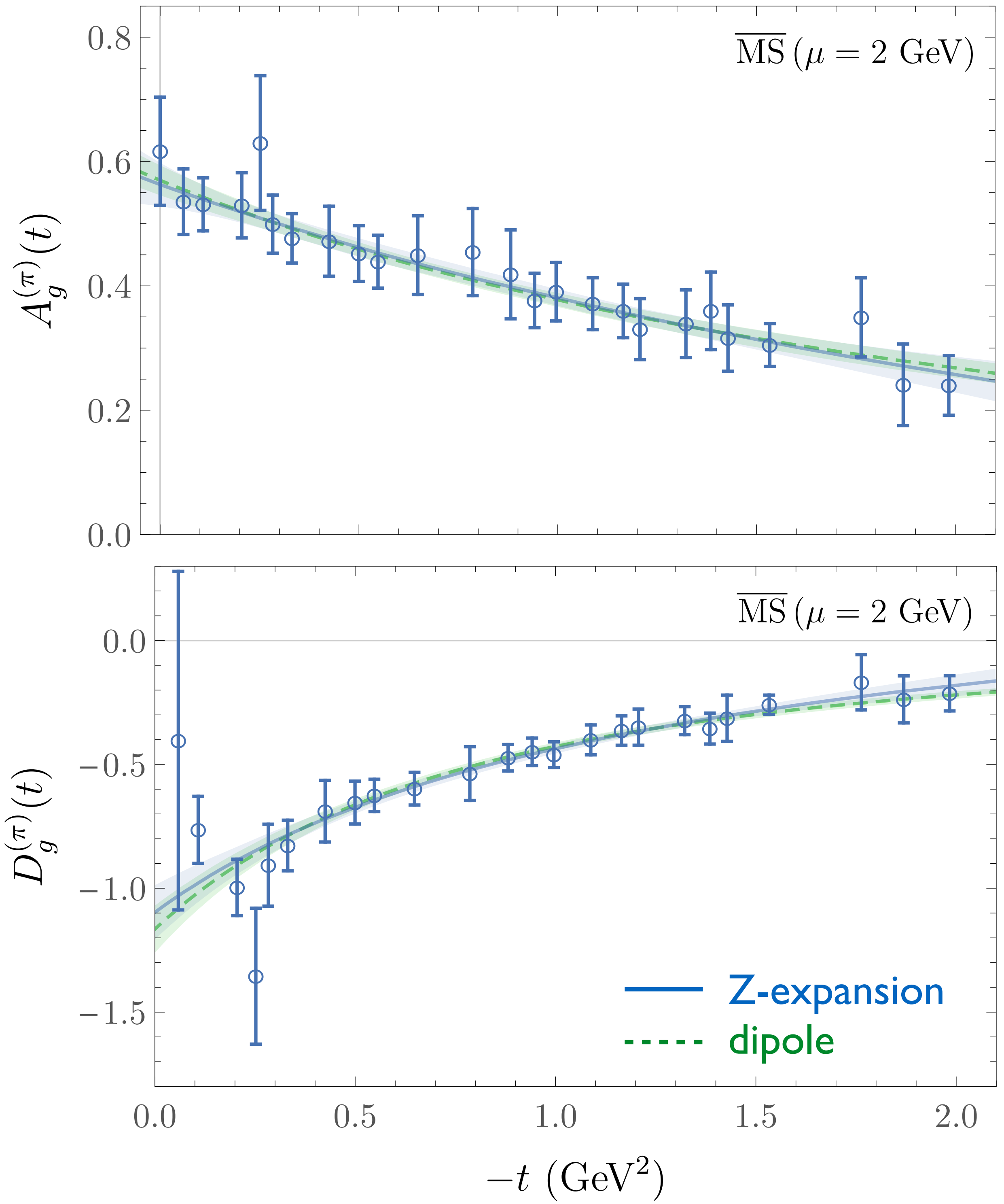
Solve system of equations simultaneously for both hypercubic irreps for each binned four-momentum transfer

Dipole-like fall-off with momentum transfer

- Momentum fraction $A_a(0) = \langle x \rangle_a$

$$\sum_{a=q,g} A_a(0) = 1$$

- D-terms $D_a(0)$ related to pressure and shear distributions



Shanahan & Detmold PRL (2019) arXiv:1810.07589, PRD (2019) arXiv:1810.04626

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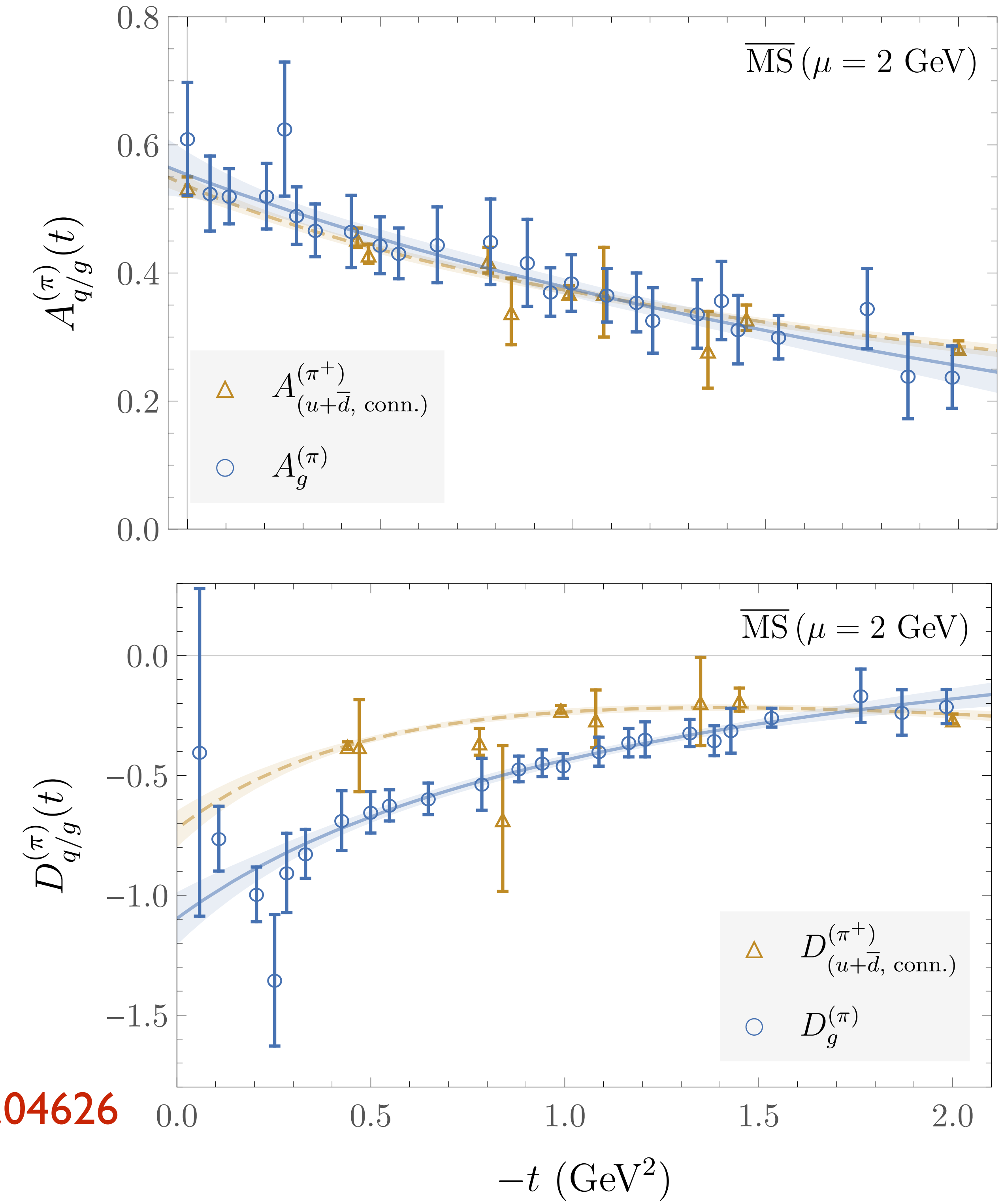
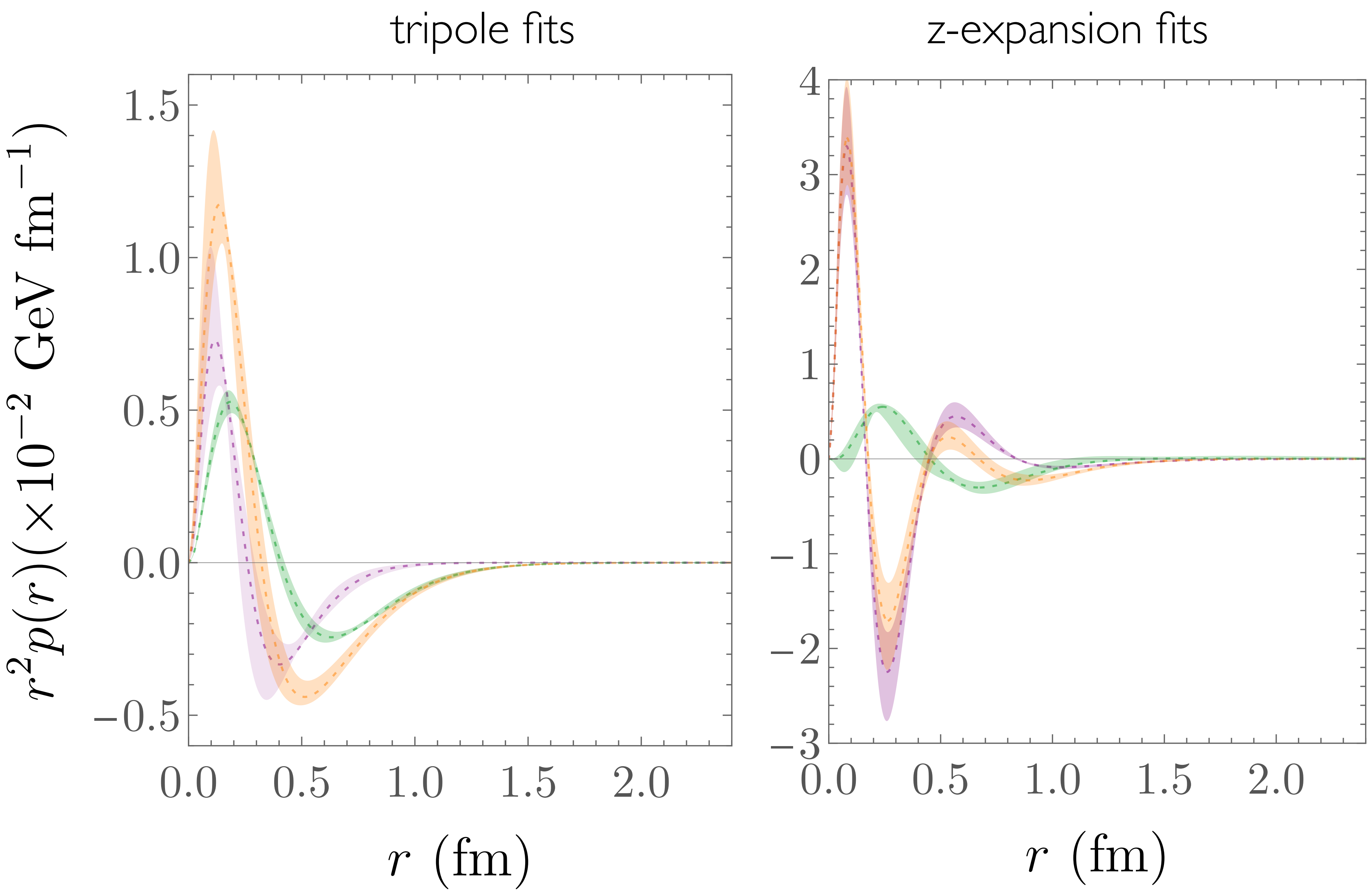


gluon: Shanahan, Detmold, PRD (2019) arXiv:1810.04626

quark: Brommel Ph.D. thesis (2007) m_π ~840 MeV

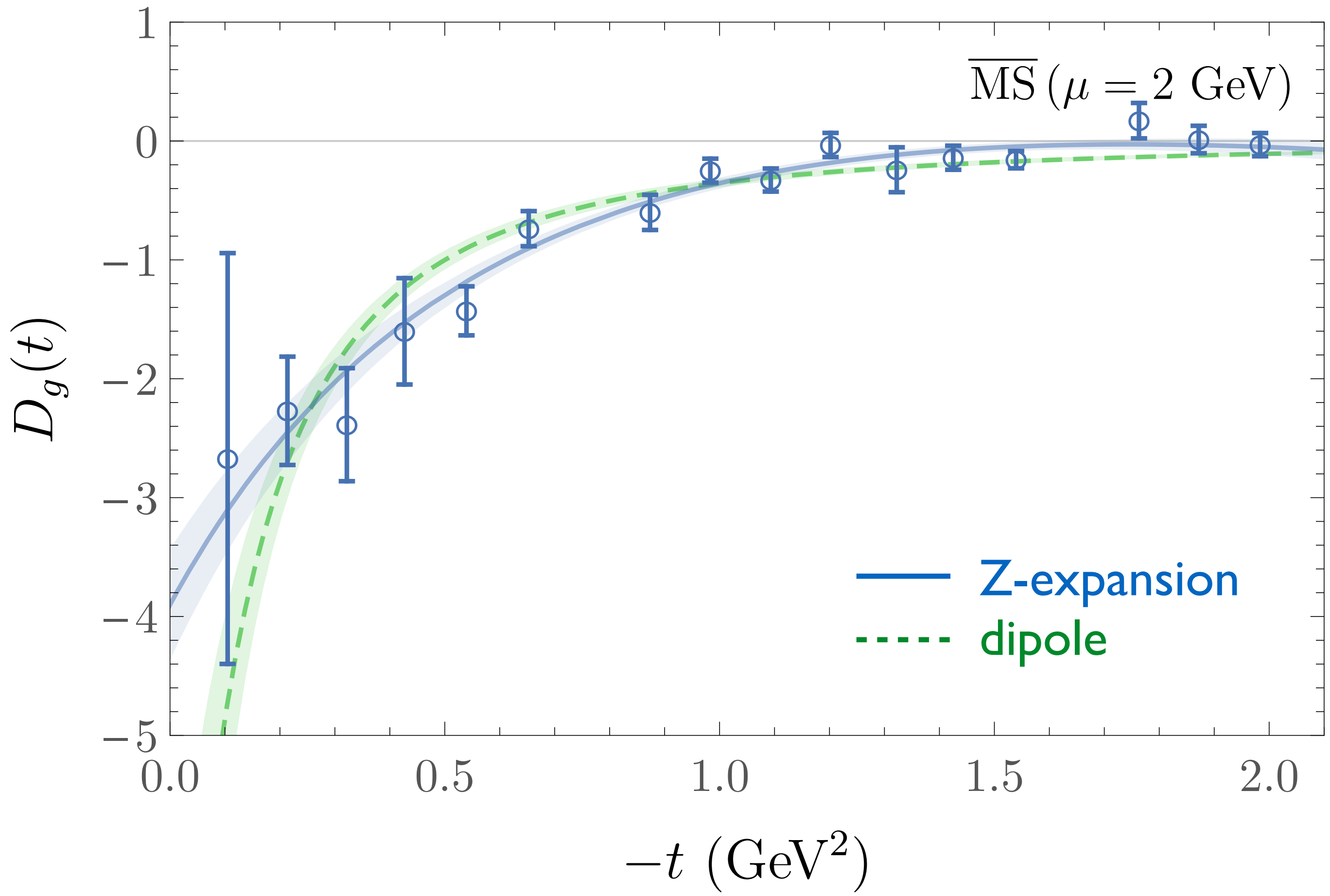
LQCD pion pressure

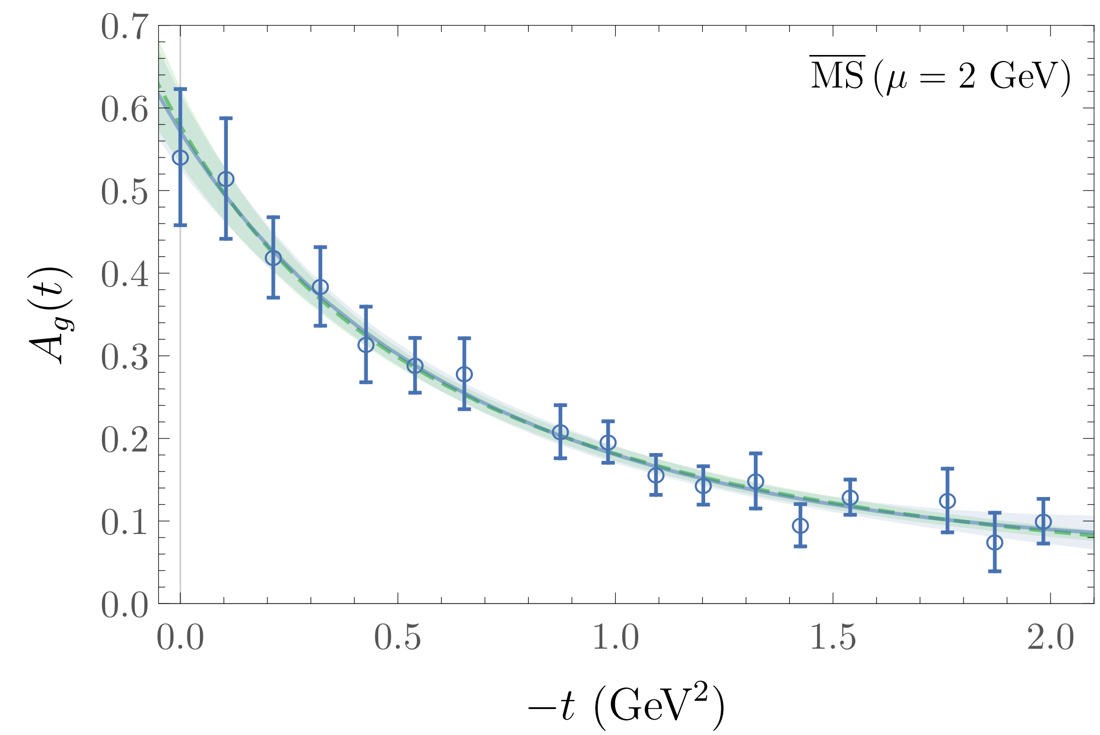


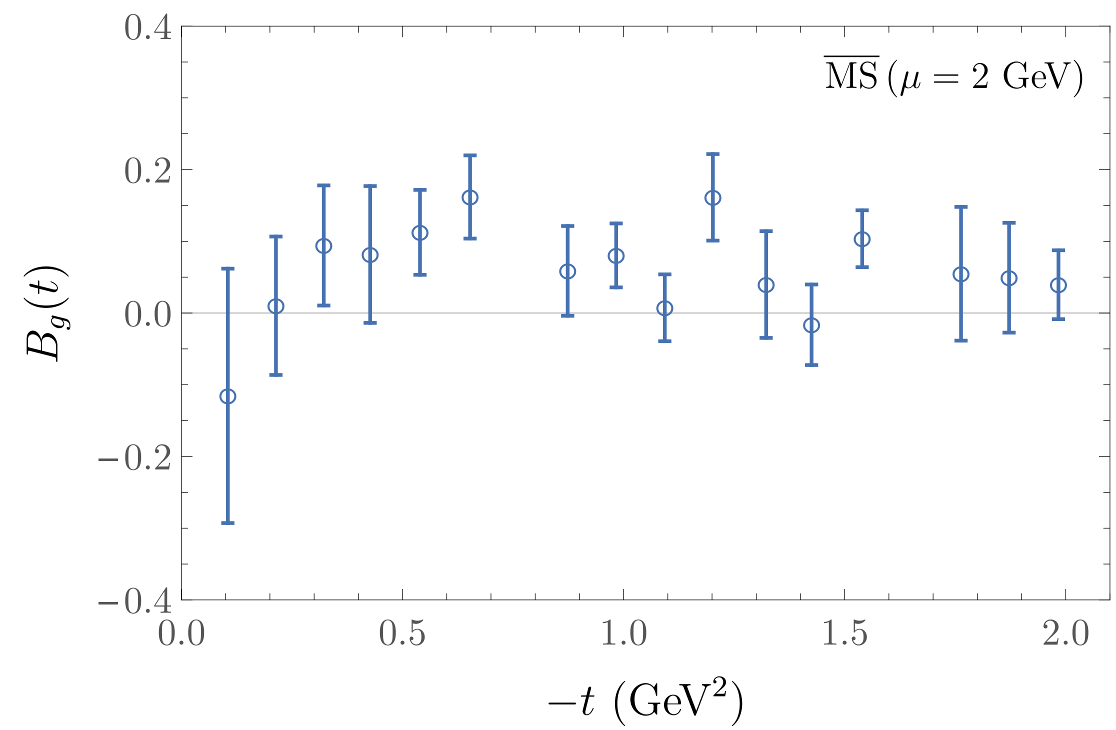
LQCD Nucleon GFFs

Nucleon gluon GFFs, m_π ~450 MeV

Dipole-like fall-off with momentum transfer



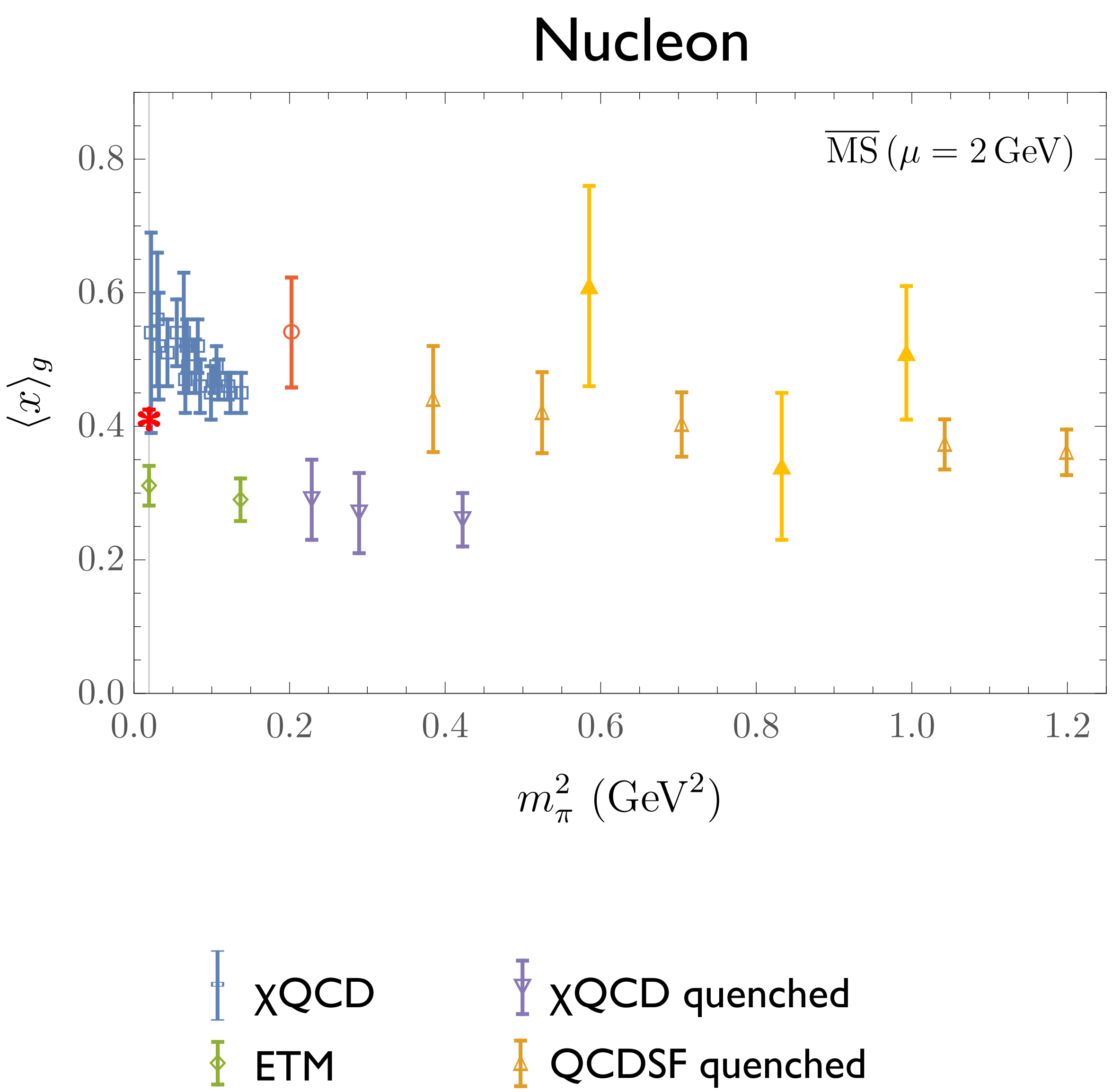


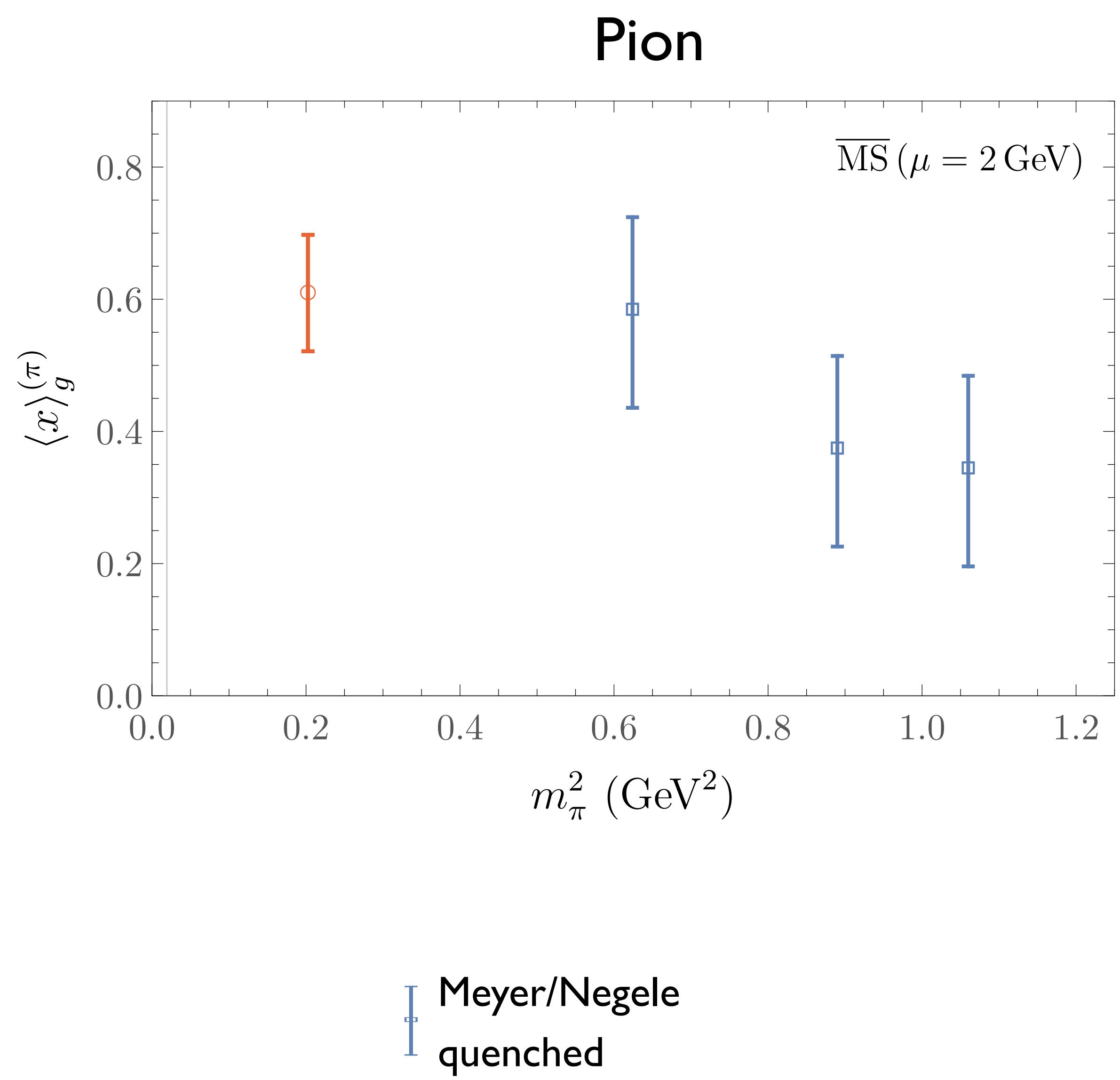


Shanahan, Detmold, arXiv: PRD (2019) arXiv:1810.04626

Gluon momentum fraction

Gluon momentum fraction $A_a(0) = \langle x \rangle_a$

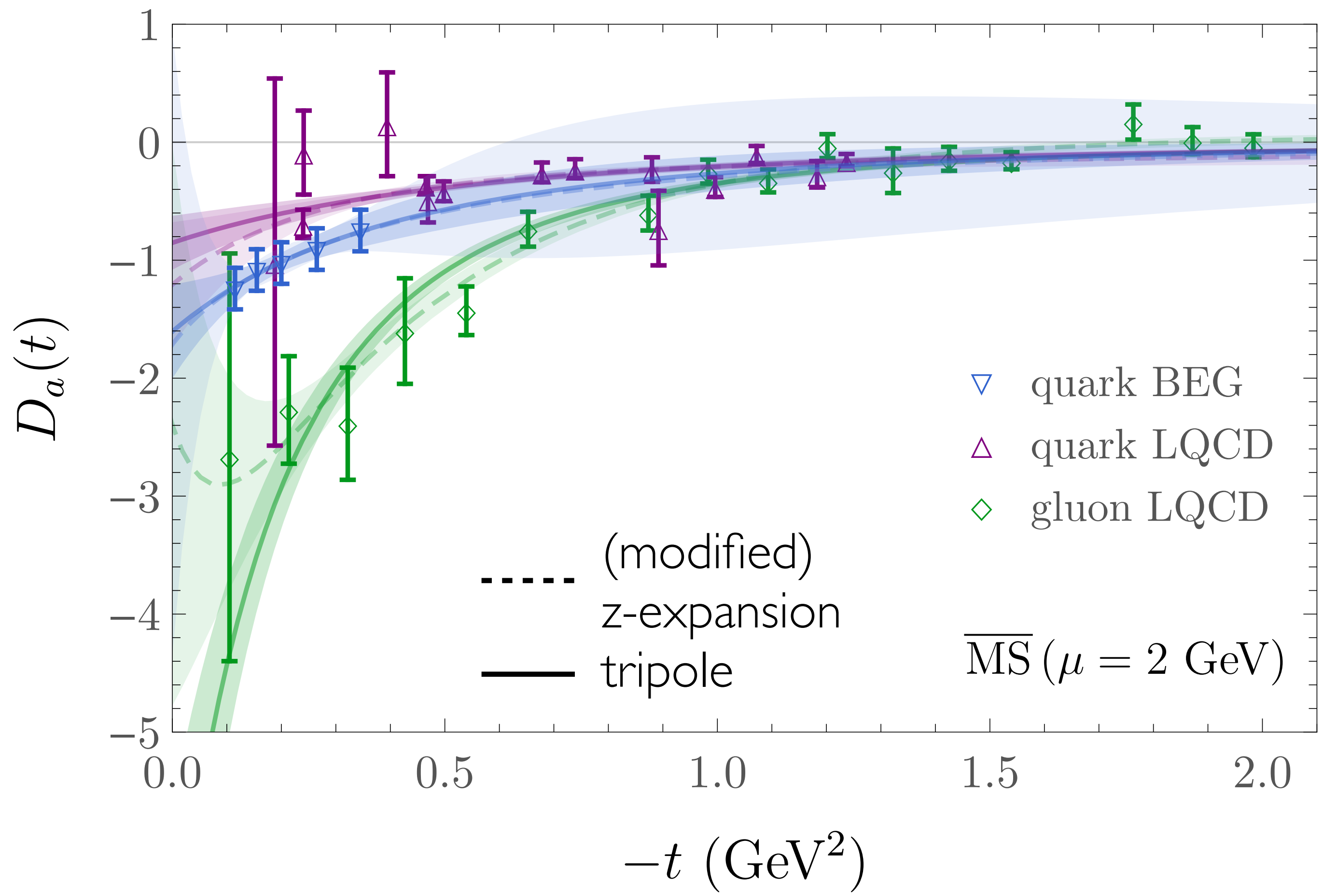


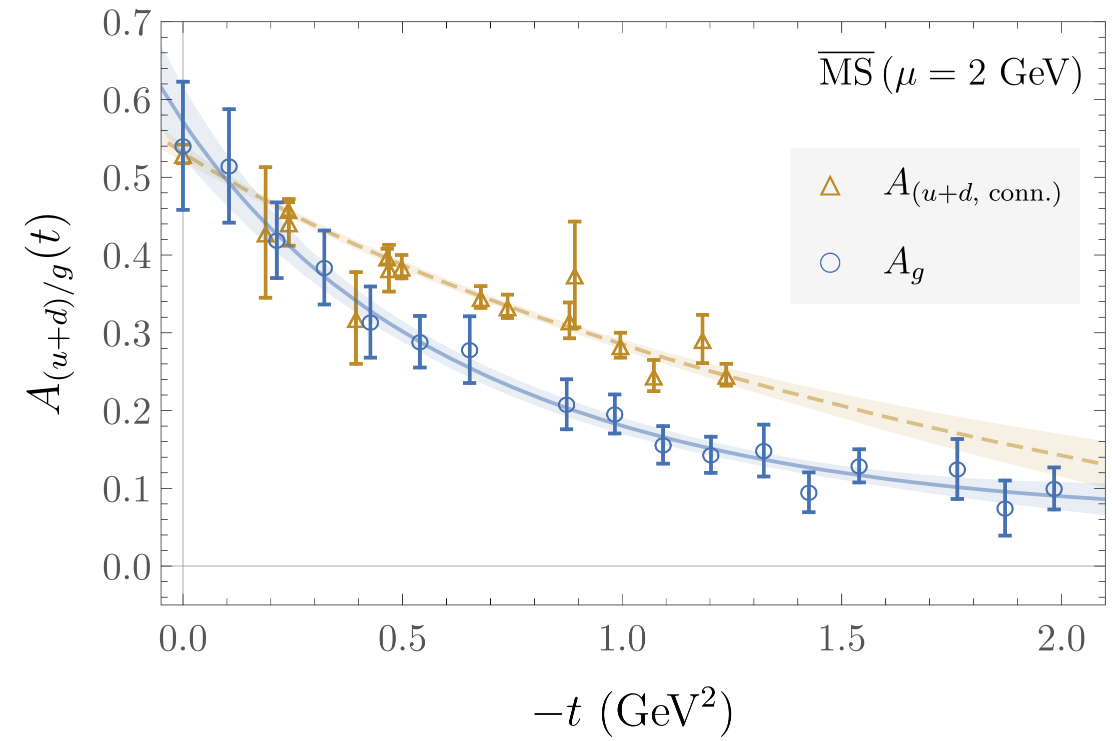


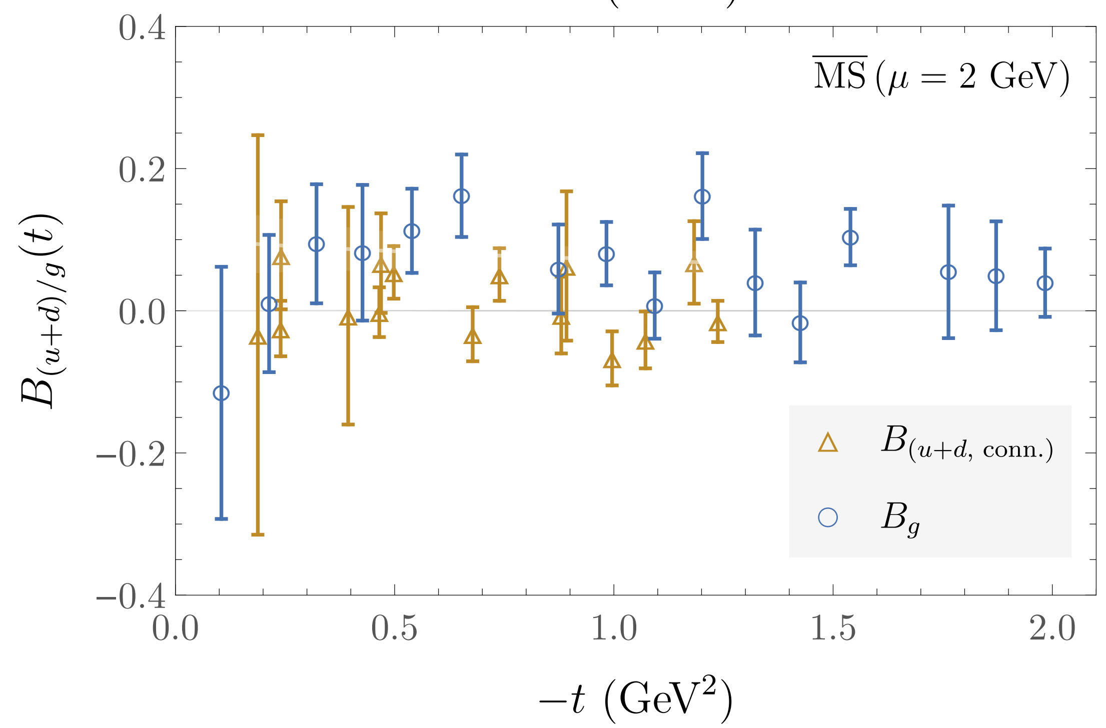
LQCD Nucleon GFFs

Nucleon gluon GFFs, m_π ~450 MeV

Tripole-like fall-off with momentum transfer







Gluon GFFs: Shanahan, Detmold, PRD (2019) arXiv:1810.04626

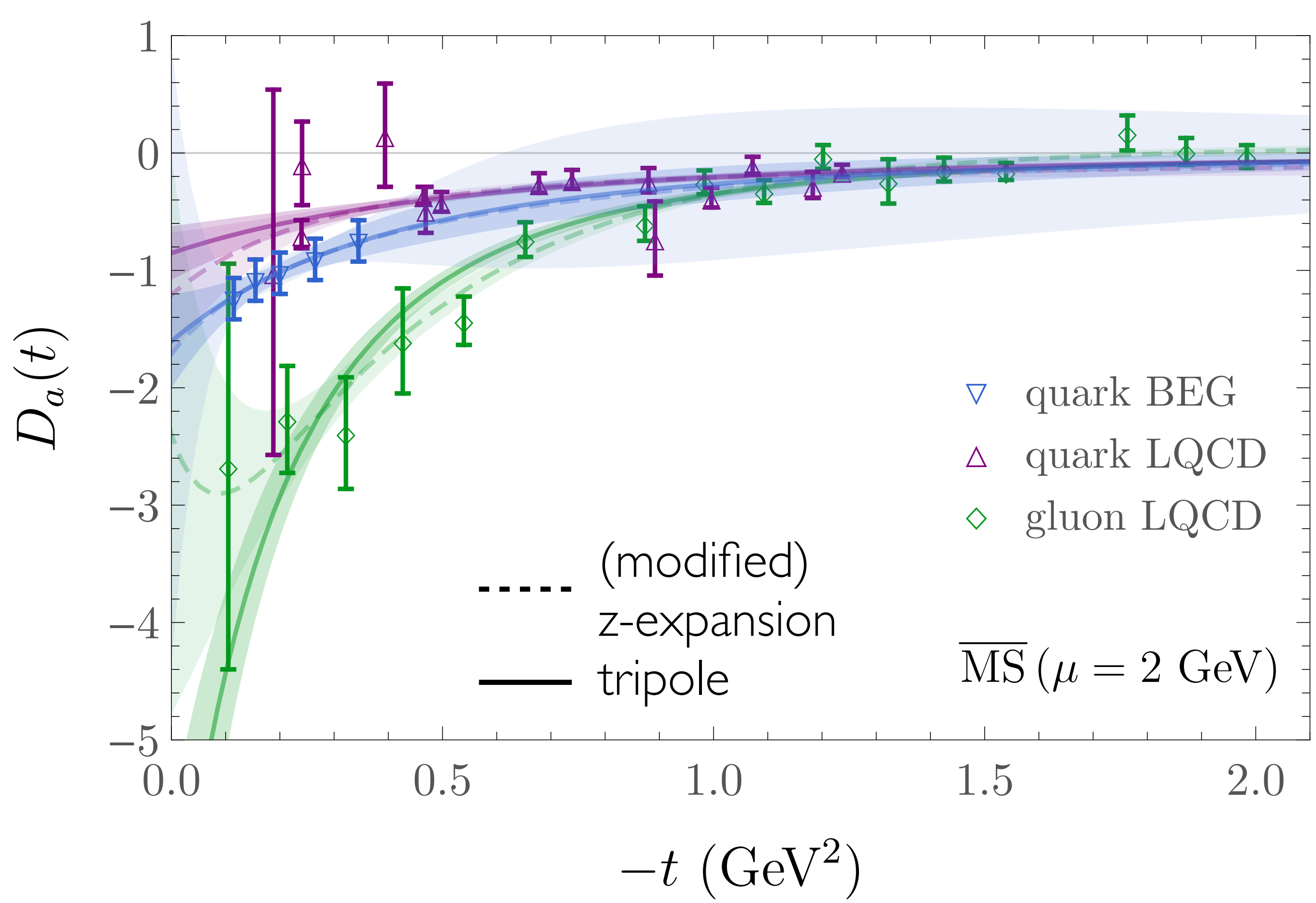
Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)

Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

Nucleon D-term GFFs

Nucleon gluon GFFs, m_π ~450 MeV

Tripole-like fall-off with momentum transfer



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Key assumptions in pressure extraction from DVCS

- **Gluon D-term same as quark term in magnitude and shape**

  Factor of ~2 difference in magnitude, somewhat different t-dependence

- **Tripole form factor model**

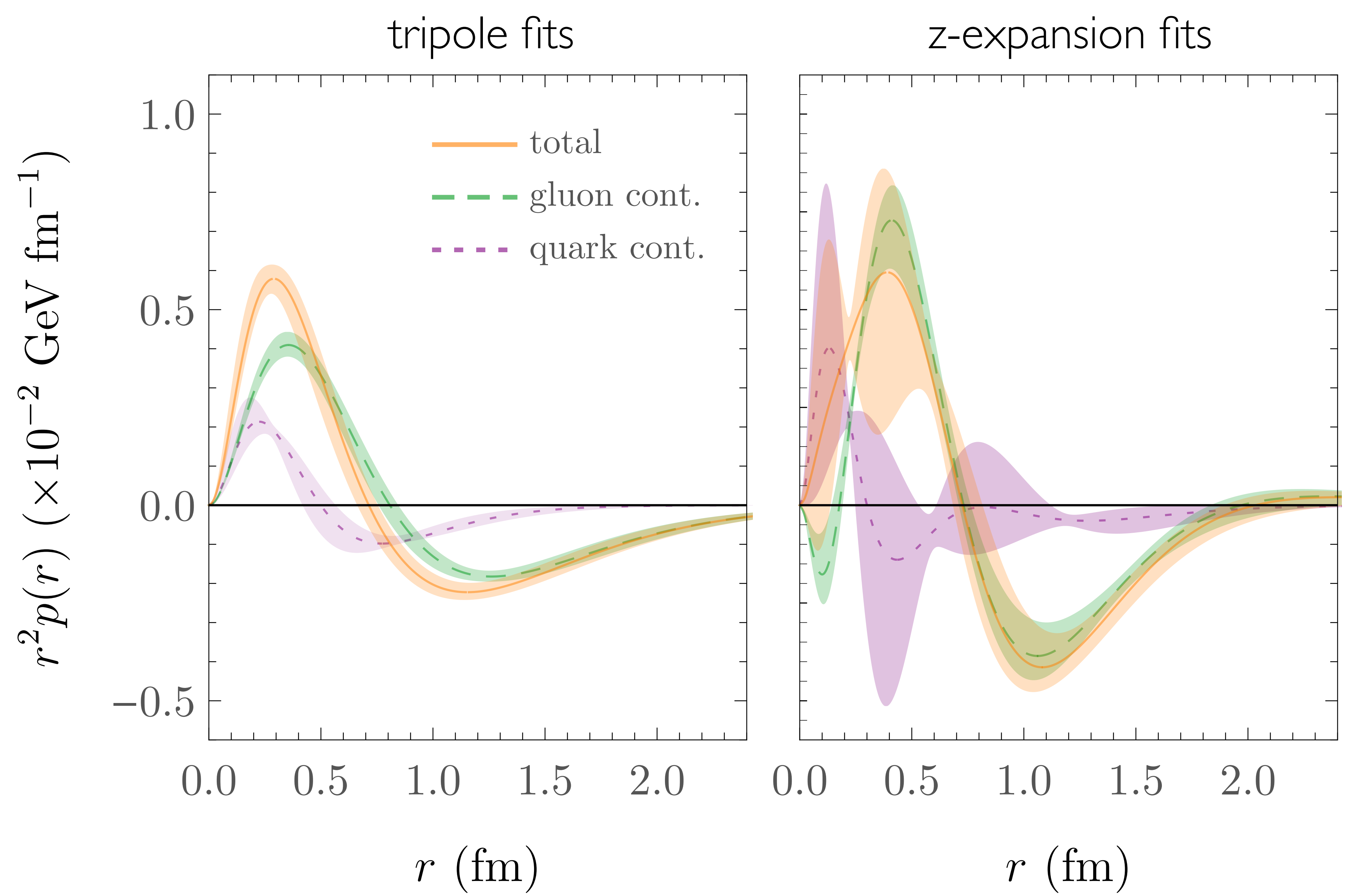
  LQCD results consistent with ansatz, but more general form is less well constrained

- **Isovector quark D-term vanishes**

  $D_{u-d}(t)$ ~0 from other LQCD studies

LQCD proton pressure

Nucleon pressure using LQCD results for quark and gluon GFFs, m_π ~450 MeV



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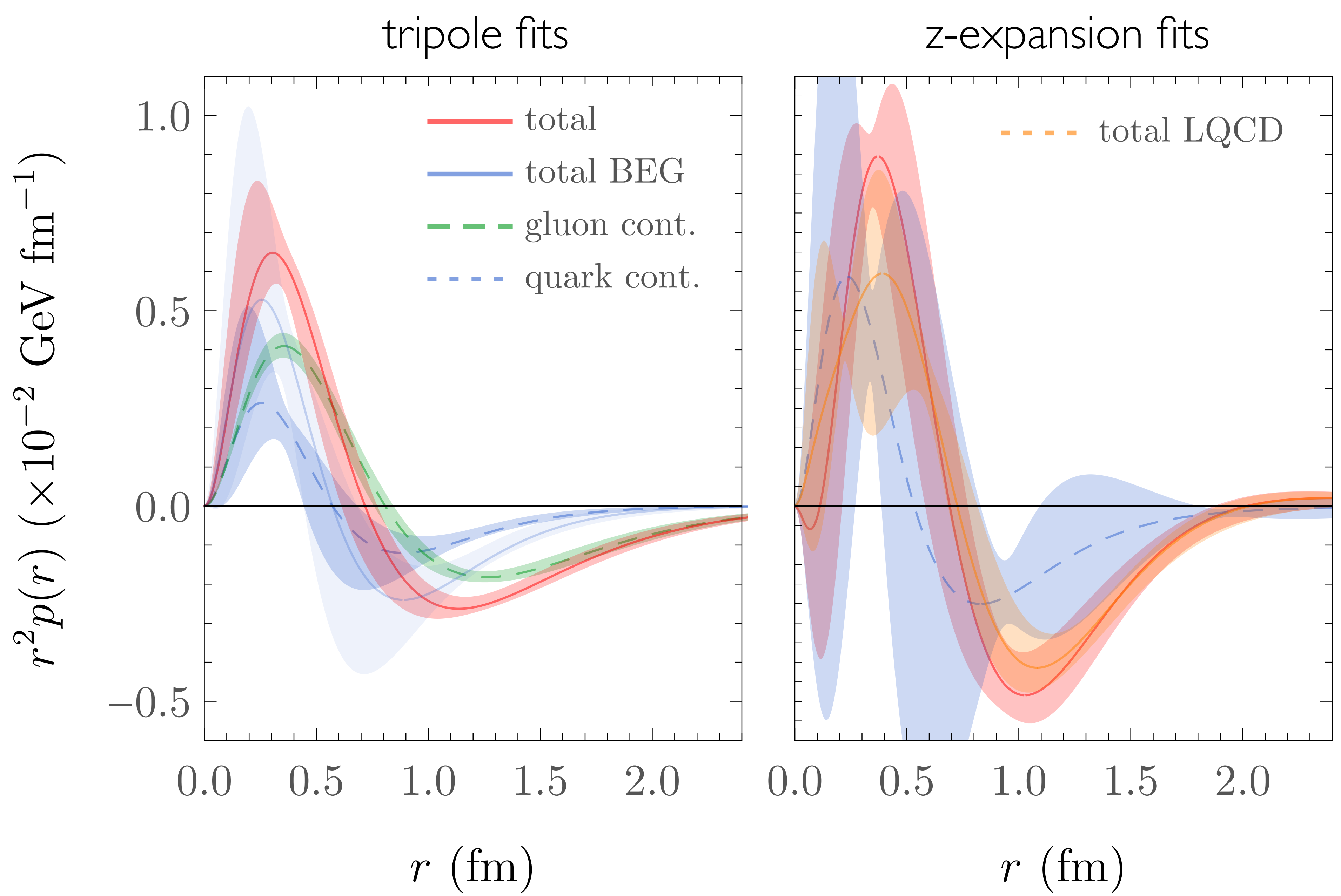
LQCD + Expt proton pressure

Nucleon pressure using LQCD results for gluon GFF, JLab results for quark GFF



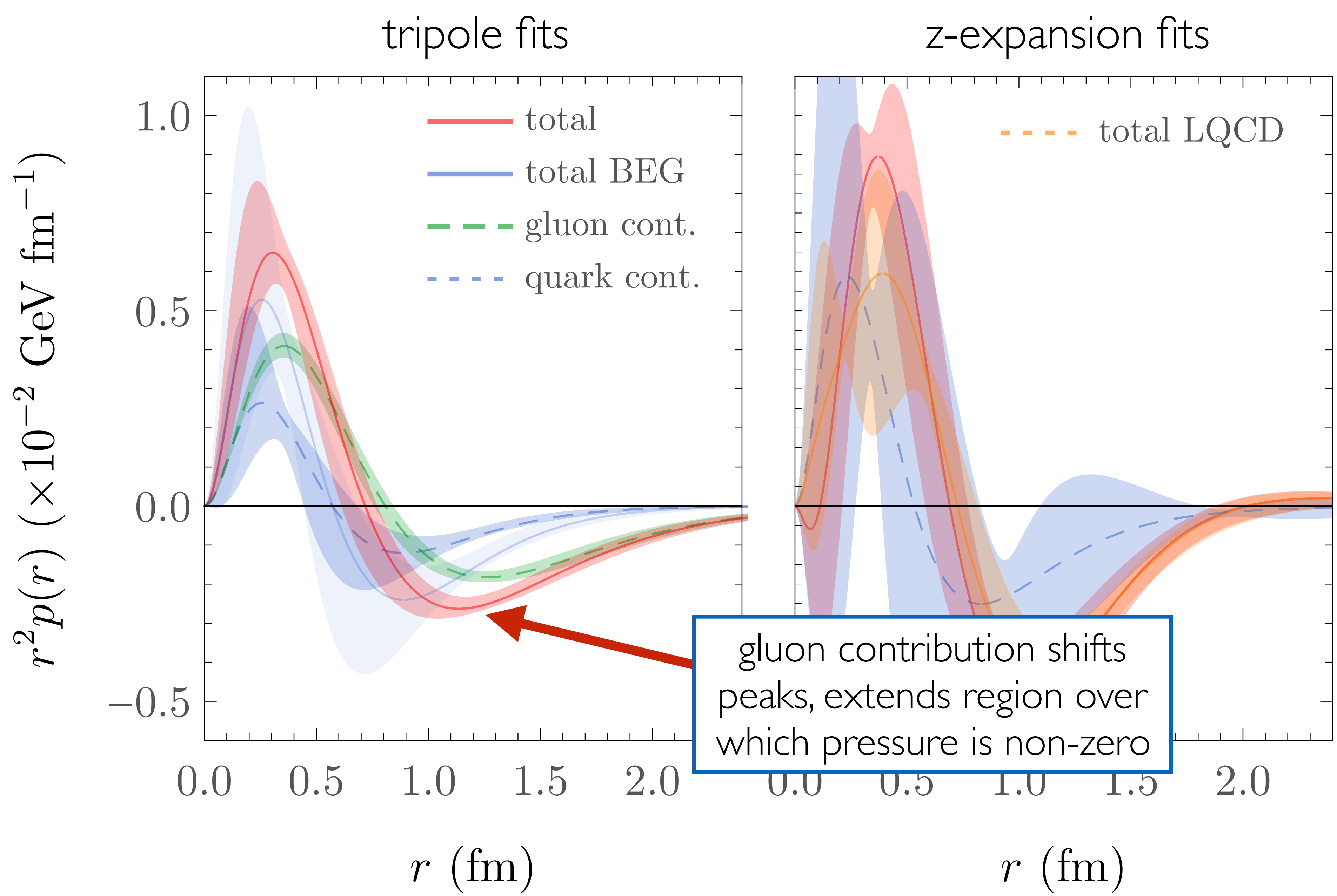
Gluon GFFs: Shanahan, Detmold, PRD (2019) arXiv:1810.04626, PRL (2019) arXiv:1810.07589

Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)

Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

LQCD + Expt proton pressure

Nucleon pressure using LQCD results for gluon GFF, JLab results for quark GFF

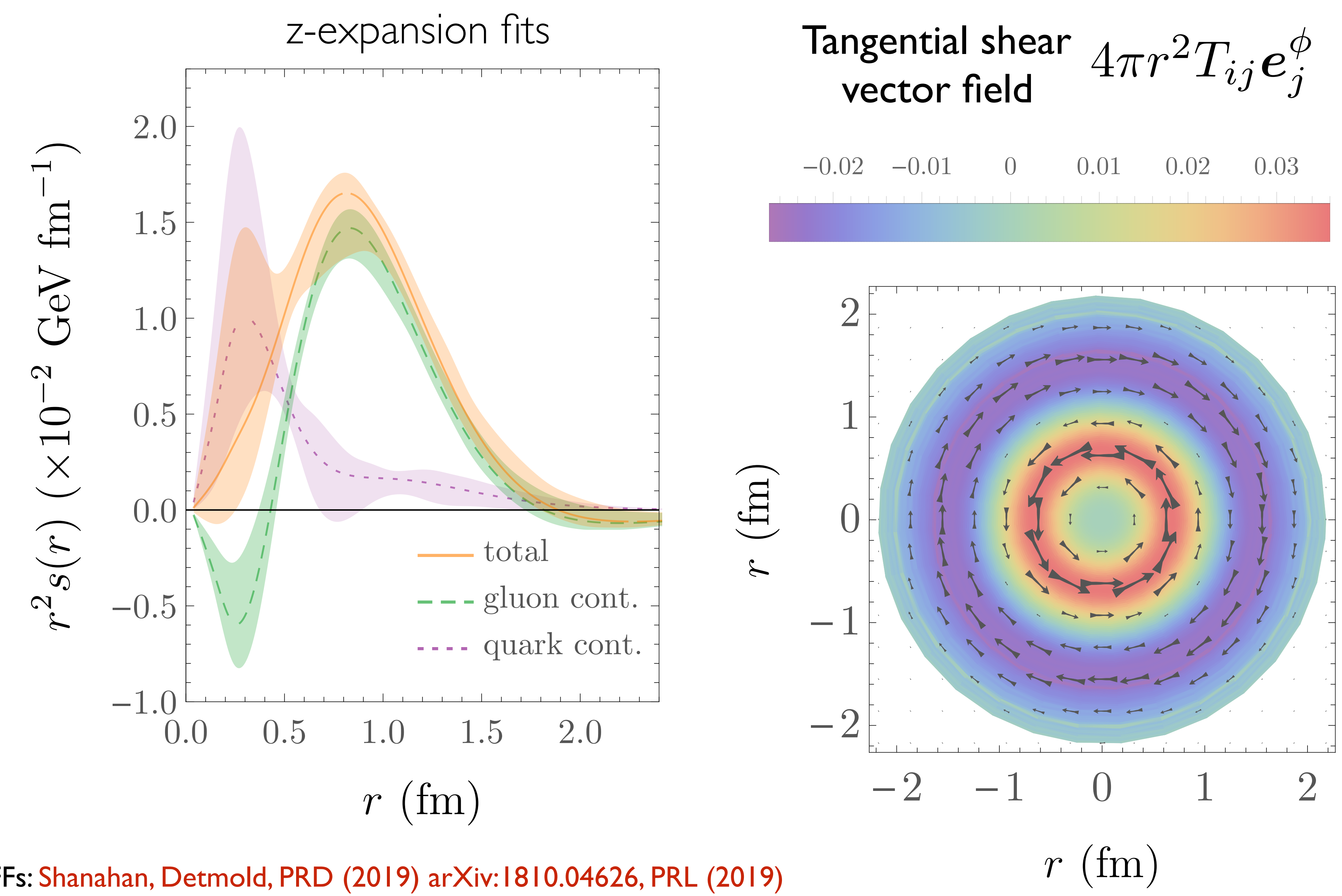


Gluon GFFs: Shanahan, Detmold, PRD (2019) arXiv:1810.04626, PRL (2019) arXiv:1810.07589

Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)

Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

LQCD proton shear



Gluon GFFs: Shanahan, Detmold, PRD (2019) arXiv:1810.04626, PRL (2019) arXiv:1810.07589

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Gluon structure from LQCD

LQCD calculations of proton and pion energy momentum tensor

- Gluon and quark gravitational form factors
- Shear and pressure distributions
- Complements recent experimental studies
  - Support analysis assumptions
  - Suggest target kinematics for future model independent extractions at JLab12 and EIC