B-meson light-cone distribution amplitude from lattice QCD

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OUTLINE

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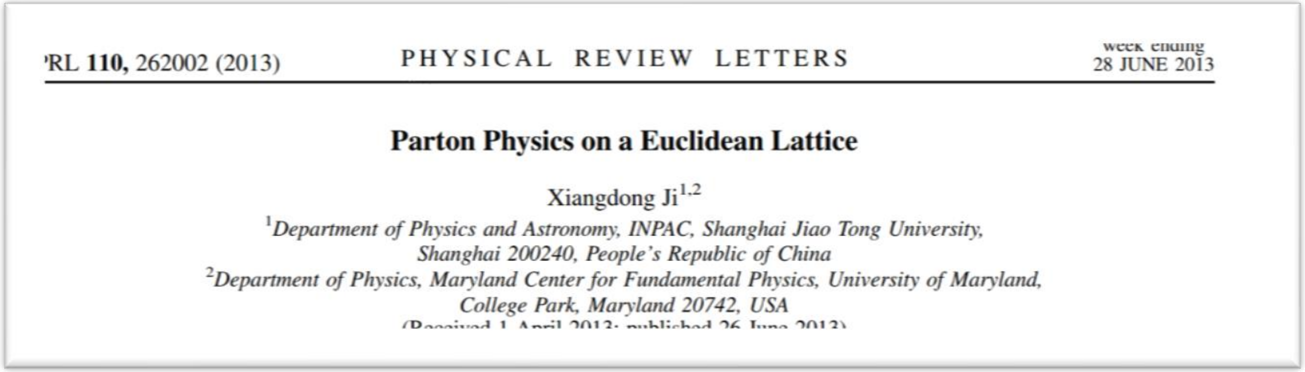
Introduction

B-meson light-cone distribution amplitudes (LCDAs) serve as indispensable ingredients for

1. Establishing QCD factorization theorems of exclusive B-meson decay amplitudes.
2. Precision calculations of the B-meson decay observables.
3. Constructing the light cone sum rules of numerous hadronic matrix elements.

But our knowledge of these distribution amplitudes is very limited. They encode information of the non-perturbative strong interaction dynamics.

LaMET



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Parton Physics on a Euclidean Lattice

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Large-momentum effective field theory: LaMET

LaMET is a theory allowing ab initio computation of light-cone physics on a Euclidean lattice!

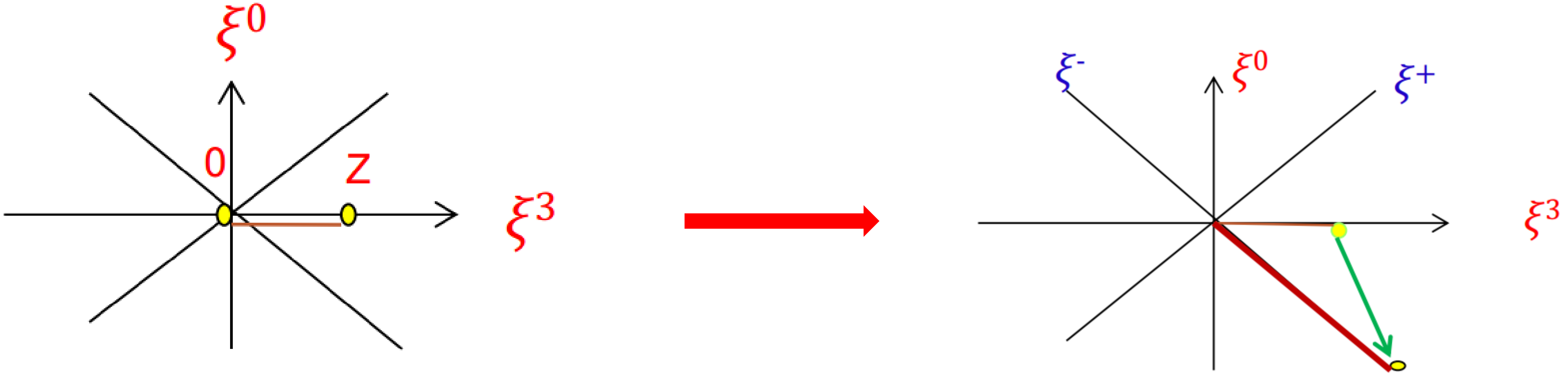
Step 1: Constructing lattice operators and evaluate the ME

Step 2: Lattice calculations

Step 3: Extracting the light-cone physics from the lattice ME

LaMET

- Calculate the equal-time correlators (quasi quantities) instead of the light-cone ones.
- The matrix elements defined by these equal-time correlators can be simulated on the lattice.
- The quasi observables can be factorized as the convolution of a matching coefficient and the corresponding light-cone



LaMET

1. Quark PDFs.

X.Xiong, X. Ji, J.-H. Zhang, Y. Zhao Phys.Rev.D 2014
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2. Transverse momentum dependent (TMD) PDFs.

X. Ji, P.Sun, X. Xiong and F. Yuan, Pys. Rev. D 2015
X.Ji,L.-C. Jin, F.Yuan, J.-H. Zhang, Y. Zhao Phys.Rev.D 2015
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3. Generalized parton distributions (GPDs).

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4. Light-cone distribution amplitudes (LCDAs).

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Hiroyuki Kawamura and Kazuhiro Tanaka, Proceedings of Science 2017
J. Xu, Q.-A. Zhang and S. Zhao, Phys. Rev. D 2018
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5. Gluon PDFs.

W. Wang, S. Zhao and R.Zhu, Eur.Phys.J. C 2018
W. Wang and S. Zhao, JHEP 2018
……

LCDA

Lattice calculation → Quasi-DA → Normal-LCDA

LQCD   Matching

Matching the Quasi Meson Distribution Amplitude in RI/MOM scheme

Yu-Sheng Liu, Wei Wang, Ji Xu, Qi-An Zhang, Shuai Zhao and Yong Zhao

Phys.Rev. D99 (2019) no.9, 094036

Derive the one-loop matching coefficient that matches RI/MOM quasi-DA in the Landau gauge to $\overline{MS}$ LCDA within the framework of LaMET.

$$\widetilde{\phi}(\Gamma, x, P^z, \mu_R, p_R^z) = \int_0^1 dy\, C_\Gamma\left(x, y, r, \frac{P^z}{\mu}, \frac{P^z}{p_R^z}\right) \phi(\Gamma, y, \mu) + \mathcal{O}\left(\frac{M^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(P^z)^2}\right)$$

It is natural to extend this logic to B-meson LCDA.

B-meson LCDA

B-meson light-cone distribution amplitudes: The leading-twist LCDA $\tilde{\phi}_B^+(\eta,\mu)$ in coordinate space is defined by the renormalized HQET matrix element of a light-ray operator,

$$\langle 0 \left| (\overline{q}_s Y_s)(\eta\overline{n}) \not{n} \gamma_5 \left(Y_s^\dagger h_v\right)(0) \right| \overline{B}(v) \rangle = i\tilde{f}_B(\mu) m_B \tilde{\phi}_B^+(\eta,\mu)$$

$$Y_s(\eta\,\bar{n}) = \mathrm{P}\left\{\mathrm{Exp}\left[i\,g_s \int_{-\infty}^{\eta} dx\,\bar{n}\cdot A_s(x\,\bar{n})\right]\right\},$$

Applying the Fourier transformation for $\tilde{\phi}_B^+(\eta,\mu)$ leads to the momentum-space distribution function

$$\phi_B^+(\omega,\mu) = \frac{1}{2\,\pi}\int_{-\infty}^{+\infty} d\eta\, e^{i\,\bar{n}\cdot v\,\omega\,\eta}\,\tilde{\phi}_B^+(\eta - i\,\epsilon,\mu)\,.$$

B-meson Quasi-DA

Following the construction presented before, we can define the B-meson quasi distribution amplitude as

$$i\tilde{f}_B(\mu)\, m_B\, \varphi_B^+(\xi,\mu) = \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi}\, e^{i\, n_z\cdot v\, \xi\, \tau}$$

$$\langle 0|\, (\bar{q}_s\, Y_s)\, (\tau\, n_z)\;\; \not{n}_z\, \gamma_5\, \left(Y_s^\dagger\, h_v\right)(0)|\bar{B}(v)\rangle\,,$$

The correlation direction has switched to z axis, $n_z = (0,0,0,1)$

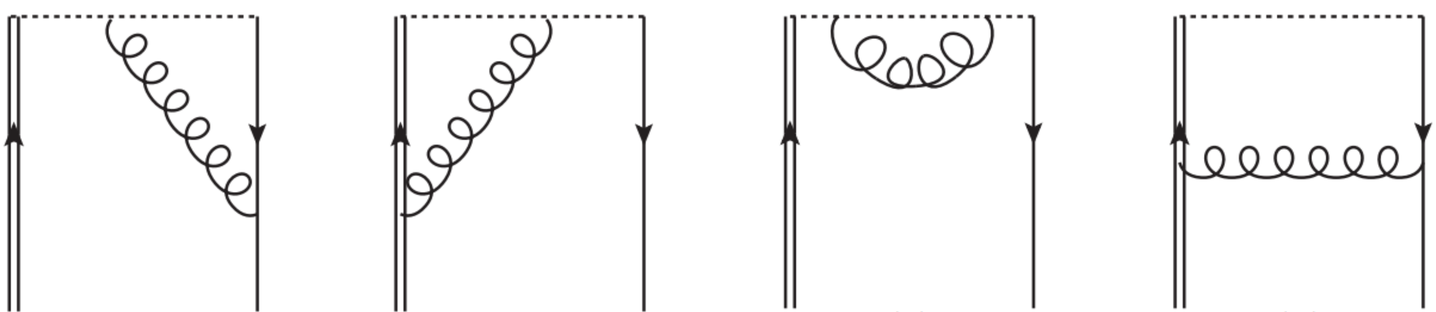
Factorization Formula

Factorization formula:

Based on the hard-collinear factorization, it is straightforward to have the factorization of quasi-DA as:

$$\varphi_B^+(\xi,\mu) = \int_0^\infty d\omega\, \underline{H(\xi,\,\omega,\,n_z\cdot v,\,\mu)}\,\phi_B^+(\omega,\mu) + O\left(\frac{\Lambda_{\rm QCD}}{n_z\cdot v\,\xi}\right).$$

We extract the coefficients by calculating these diagrams below from both LCDA and qausi-DA sides,



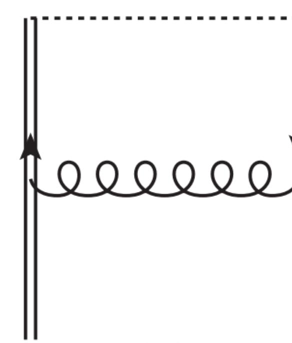
(a) (b) (c) (d)

One loop matching coefficient

The plus distribution is defined by (with $a > 1$)

$$\{\mathcal{F}(\xi,\omega)\}_{\oplus} = \mathcal{F}(\xi,\omega) - \delta(\xi-\omega)\int_0^{a\,\xi} dt\,\mathcal{F}(\xi,t)$$

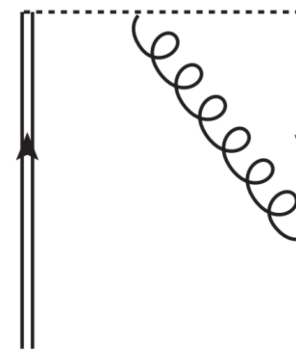
One can readily identify that the hard correction from the 1-loop box diagram is power suppressed



(d)

One loop matching coefficient

1. For LCDA side，



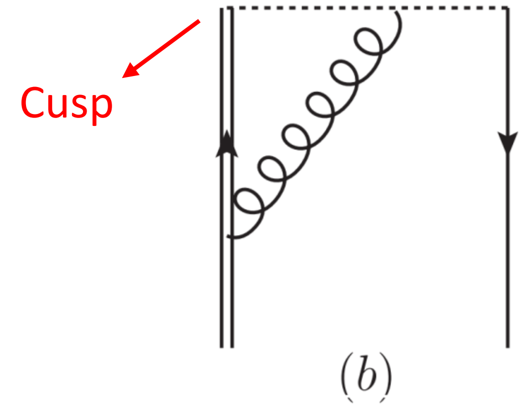
$$(a) = \frac{\alpha_s}{2\pi}C_F\left[\left(-\frac{1}{\varepsilon}\frac{\omega}{(\omega-k_0)k_0} - \frac{\omega}{(\omega-k_0)k_0}\ln\frac{\mu^2 k_0^2}{-k^2\omega(k_0-\omega)}\right)\theta(0<\omega<k_0)\right]_{\oplus}$$
$$-\frac{\alpha_s}{2\pi}C_F\delta(\omega-k_0)\left(\frac{1}{\varepsilon}\left(\ln\frac{a}{a-1}-1\right)+\ln\frac{\mu^2}{-k^2}(\ln a-1)-\ln(a-1)\ln\frac{a\mu^2}{-k^2}\right.$$
$$\left.+\mathrm{Li}_2(2,1-a)+\frac{1}{2}\ln^2(a-1)+\ln^2 a-2+\frac{\pi^2}{6}\right),$$

2. For quasi-DA side，

$$(a)_1 = \frac{\alpha_s}{4\pi}C_F\left[\frac{1}{k_0(\omega-k_0)}\left(-k_0+2\omega\ln\frac{-\omega}{k_0-\omega}\right)\theta(\omega<0)\right]+\frac{\alpha_s}{4\pi}C_F\delta(\omega-k_0)\left(\frac{1}{2\varepsilon}+2-\frac{\pi^2}{3}-\ln 2+\ln\frac{\tilde{\mu}}{v_z\omega}\right),$$
$$(a)_2 = \frac{\alpha_s}{4\pi}C_F\left[\frac{1}{k_0(\omega-k_0)}\left(2\omega-k_0-2\omega\ln\frac{4k_0^2v_z^2}{-k^2}\right)\theta(0<\omega<k_0)\right]_{\oplus}$$
$$+\frac{\alpha_s}{4\pi}C_F\delta(\omega-k_0)\left(\ln\frac{16a^2}{a-1}-2\left(1+\ln\frac{a}{a-1}\ln\frac{4k_0^2v_z^2}{-k^2}+\ln\frac{-k^2}{k_0^2v_z^2}\right)\right),$$
$$(a)_3 = \frac{\alpha_s}{4\pi}C_F\left[\frac{1}{k_0(\omega-k_0)}\left(k_0-2\omega\ln\frac{\omega}{\omega-k_0}\right)\theta(\omega>k_0)\right]_{\oplus}+\frac{\alpha_s}{4\pi}C_F\delta(\omega-k_0)\left(\frac{1}{2\varepsilon}+2-\ln 2+\ln\frac{\tilde{\mu}}{v_z\omega}\right).$$

One loop matching coefficient

1. For LCDA side，



$$(b) = \frac{\alpha_s}{2\pi}C_F\left[\left(\frac{1}{\varepsilon}\frac{1}{\omega-k_0}+\frac{1}{\omega-k_0}\ln\frac{\mu^2}{(\omega-k_0)^2}\right)\theta(\omega-k_0)\right]_{\oplus}$$
$$-\frac{\alpha_s}{2\pi}C_F\delta(\omega-k_0)\left(\frac{1}{2\varepsilon^2}+\frac{1}{2\varepsilon}\ln\frac{\mu^2}{\omega^2}+\frac{\pi^2}{24}+\ln^2\frac{\omega}{\mu}\right).$$

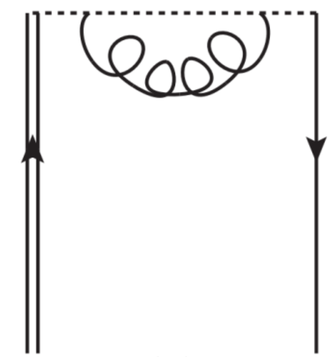
2. For quasi-DA side，

$$(b) = \frac{\alpha_s}{4\pi}C_F\frac{v_z}{v_0}\left[\frac{\theta(\omega-k_0)}{\omega-k_0}\left(\frac{1}{\varepsilon}+\ln v_0^2-2\ln(v_0-v_z)+\ln\frac{\tilde{\mu}^2}{v_z^2(\omega-k_0)^2}\right)\right]_{\oplus}$$
$$+\left[\frac{\theta(k_0-\omega)}{\omega-k_0}\left(\frac{1}{\varepsilon}+\ln v_0^2-2\ln(v_0+v_z)+\ln\frac{\tilde{\mu}^2}{v_z^2(k_0-\omega)^2}\right)\right]_{\oplus}$$
$$-\frac{\alpha_s}{4\pi}C_F\frac{v_z}{v_0}\delta(\omega-k_0)\left(\frac{1}{\varepsilon}\ln\frac{(v_0+v_z)(a-1)}{v_0-v_z}+2\ln(a-1)\ln\mu+\ln\frac{v_0+v_z}{v_0-v_z}\ln\frac{\mu^2}{4}+2\ln\frac{v_0-v_z}{v_0}\ln(v_z\omega)\right.$$
$$\left.-2\ln\frac{v_0+v_z}{v_0}\ln(\omega v_z(a-1))-\ln(a-1)\ln(\omega^2v_z^2(a-1))-\mathrm{HPL}((-,+),-\frac{v_z}{v_0})\right).$$

One loop matching coefficient

1. **For LCDA side,**

$$\bar{n}^2 = 0 \quad \Rightarrow \quad (c) = 0\,.$$



(c)

2. **For quasi-DA side,**

$$(c) \; = \; \frac{\alpha_s}{2\pi} C_F \left[\frac{\theta(k_0 - \omega)}{k_0 - \omega} + \frac{\theta(\omega - k_0)}{\omega - k_0} \right]_{\oplus} - \frac{\alpha_s}{2\pi} C_F \delta\left(\omega - k_0\right) \left(\frac{1}{\varepsilon} + 2 - \ln 4 + \ln \frac{\tilde{\mu}^2}{v_z^2 \omega^2} \right) \,.$$

One loop matching coefficient

$$\varphi_B^+(\xi,\mu) \; = \; \int_0^\infty d\omega \, H(\xi,\,\omega,\,n_z\cdot v,\,\mu)\,\phi_B^+(\omega,\mu) + O\left(\frac{\Lambda_{\rm QCD}}{n_z\cdot v\,\xi}\right) .$$

The obtained hard function reads,

$$\begin{aligned}
H(\xi,\,\omega,\,n_z\cdot v,\,\mu) \; = \; & \delta(\xi-\omega) + \frac{\alpha_s(\mu)\,C_F}{4\,\pi}\left\{\frac{1}{\omega-\xi}\left[3-2\,\ln\left(\frac{\mu}{2\,n_z\cdot v\,(\omega-\xi)}\right)-\frac{2\,\xi}{\omega}\,\ln\left(\frac{\xi}{\xi-\omega}\right)\right]\theta(-\xi)\,\theta(\omega)\right.\\
& + \left\{\frac{1}{\omega-\xi}\left[3-2\left(1+\frac{2\,\xi}{\omega}\right)\ln\left(\frac{\mu}{2\,n_z\cdot v\,(\omega-\xi)}\right)-\frac{2\,\xi}{\omega}\left(\ln\left(\frac{\omega-\xi}{\xi}\right)+1\right)\right]\right\}_\oplus \theta(\xi)\,\theta(\omega-\xi)\\
& + \left\{\frac{1}{\xi-\omega}\left[3-2\,\ln\left(\frac{\mu}{2\,n_z\cdot v\,(\xi-\omega)}\right)-\frac{2\,\xi}{\omega}\,\ln\left(\frac{\xi}{\xi-\omega}\right)\right]\right\}_\oplus \theta(\omega)\,\theta(\xi-\omega)\\
& \left. + 2\left[\ln^2\frac{\mu}{n_z\cdot v\,\xi}-3\,\ln\frac{\mu}{n_z\cdot v\,\xi}+f(a)\right]\delta(\xi-\omega)\right\},
\end{aligned}$$

$$\begin{aligned}
f(a) \; = \; & \ln\frac{a^2}{4(a-1)^3}\,\ln\frac{\mu}{n_z\cdot v\;\xi}+\ln(a-1)\,\ln\frac{8\,(a-1)}{a}\\
& + {\rm Li}_2(1-a)+\ln a\,\ln\left(\frac{a}{4}\right)-\frac{1}{2}\ln(a-1)\\
& + \ln(8\,a)+\ln^2 2+\frac{\pi^2}{8}-3
\end{aligned}$$

Perspectives for lattice calculations

It will be instructive to understand the characteristic feature of $\varphi_B^+(\xi,\mu)$ with distinct non-perturbative models of $\phi_B^+(\omega,\mu)$

Taking advantage of the two phenomenological models

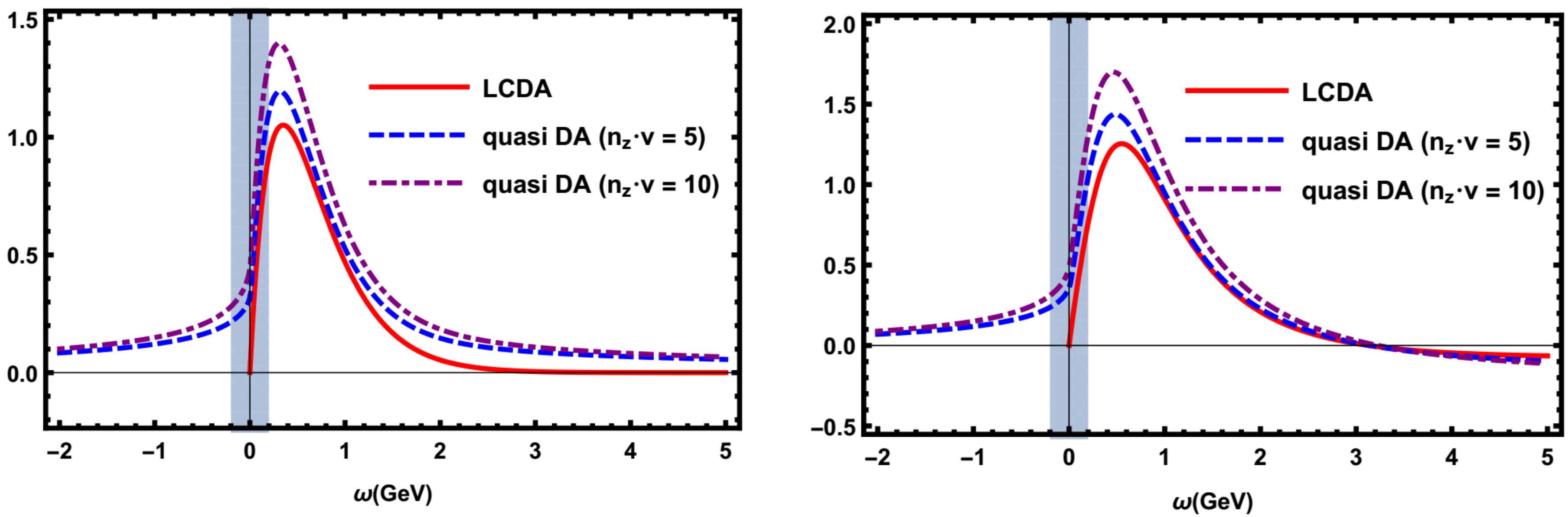
$$\phi^+_{B,\,\mathrm{I}}(\omega,\,\mu = 1.5\,\mathrm{GeV}) \;=\; \frac{\omega}{\omega_0^2}e^{-\omega/\omega_0},$$

$$\phi^+_{B,\,\mathrm{II}}(\omega,\,\mu = 1.5\,\mathrm{GeV}) \;=\; \left[\frac{1}{k^2+1} - \frac{2\,(\sigma_B^{(1)}-1)}{\pi^2}\,\ln k\right] \times \frac{4}{\pi\,\omega_0}\,\frac{k}{k^2+1},\;\; k = \frac{\omega}{1.5\,\mathrm{GeV}},$$

We can imply the shapes of $\varphi_B^+(\xi,\mu)$

Perspectives for lattice calculations

The resulting ω-shapes of the B-meson quasi-distribution amplitude $\varphi_B^+(\xi = \omega, \mu = 1.5\,\mathrm{GeV})$



Summary

- We have proposed new method for the model-independent determination of the light-cone distribution amplitude (LCDA) of the B-meson in heavy quark effective theory (HQET).

- Derive the one-loop matching coefficient that matches quasi-DA to LCDA within the framework of LaMET.

- These results are of importance for exploring the delicate flavor structure of the SM and beyond at the LHCb and Belle II experiments.

Thank you !

Back Up

ues of $n_z \cdot v$, where the reference values of the logarithmic inverse moments $\omega_0 = 350\,\text{MeV}$ and $\sigma_B^{(1)} = 1.4$ are taken for the illustration purpose. It is evident that

FIG. 2: The resulting ω-shapes of the B-meson quasi-distribution amplitude $\varphi_B^+(\xi = \omega, \mu = 1.5\,\text{GeV})$ in bHQET from the hard-collinear factorization theorem (10) and from the two non-perturbative models of $\phi_B^+(\omega, \mu = 1.5\,\text{GeV})$ presented in (14), with two different values of $n_z \cdot v$. The shadow region of $|\omega| \leq 200\,\text{MeV}$ is excluded due to inapplicability of the hard-collinear factorization formula (10) for $|n_z \cdot v\ \omega| \leq 1.0\,\text{GeV}$.

B-meson light-cone distribution amplitude from lattice QCD

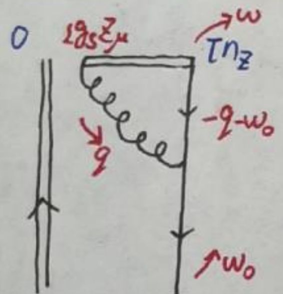
We use projection to handle the spinor part,

$$v(k)\gamma^z(-\not{q}-\not{k})\gamma^z\gamma_5 u_v(P_b) \to \mathrm{Tr}\left[\frac{1+\not{v}}{2}\not{v}\gamma_5\not{k}\gamma^z(-\not{q}-\not{k})\gamma^z\gamma_5\right].$$

$$n_{+\mu} = \frac{1}{\sqrt{2}}(1,0,0,1), \quad n_{-\mu} = \frac{1}{\sqrt{2}}(1,0,0,-1), \quad v = v_+ n_- + v_- n_+ \,,$$

第一个图 (Normal term)

此图其实不受 v 的影响.



$$\langle\tilde{\Phi}\rangle^{(a)} = \bar{v}(\omega_0)\int\frac{d^dq}{(2\pi)^d}(i g_s\gamma^\mu\tilde{\mu}^\varepsilon)\frac{i(-\not{q}-\not{\omega}_0)}{(q+\omega_0)^2}(i g_s z_\mu\tilde{\mu}^\varepsilon)\gamma^z\gamma_5\frac{-i}{q^2}\frac{1}{i q_z}u_v(P_b)\,\delta(\omega-\omega_0 z)$$

$$= i g_s^2 C_F(\tilde{\mu})^{2\varepsilon}\int\frac{d^dq}{(2\pi)^d}\frac{1}{q_z}\underbrace{\frac{1}{q^2}}_{D_1}\underbrace{\frac{1}{(q+\omega_0)^2}}_{D_2}\underbrace{\bar{v}(\omega_0)\gamma^z(-\not{q}-\not{\omega}_0)\gamma^z\gamma_5 u_v(P_b)}_{\text{trace}}$$

计算就是标准的 quasi 算法，最后结果分三个区域，$y<-1$，$-1<y<0$，$0<y$.

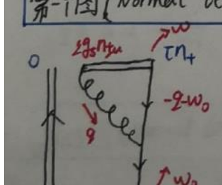
$$\begin{cases} n_{+\mu} = \frac{1}{\sqrt{2}}(1,0,0,1) \\ n_{-\mu} = \frac{1}{\sqrt{2}}(1,0,0,-1) \end{cases} \qquad v_\mu = (1,0,0,0) = \frac{1}{\sqrt{2}}(n_{+\mu}+n_{-\mu}) \qquad n_+\cdot v = \frac{1}{\sqrt{2}}$$

About off-shellness,

$\omega_0^2 = 2\omega_0^+\omega_0^-$, $\omega_0^- = \frac{\omega_0^2}{2\omega_0^+}$, 设 $q^+ = y\omega_0^+$, $x = \frac{\omega}{\omega_0^+}$.

$$i g_s^2 C_F\frac{1}{2\pi}\frac{1}{2\pi}(2\pi i)\frac{1}{4\pi} = \frac{-g_s^2 C_F}{8\pi^2} = -\frac{\alpha_s}{2\pi}C_F.$$

第一个图 (Normal term)



$$\langle\Phi\rangle^{(a)} = \bar{v}(\omega_0)\int\frac{d^dq}{(2\pi)^d}(i g_s\gamma^\mu\tilde{\mu}^\varepsilon)\frac{i(-\not{q}-\not{\omega}_0)}{(q+\omega_0)^2}(i g_s n_{+\mu}\tilde{\mu}^\varepsilon)\not{n}_+\gamma_5\frac{-i}{q^2}\frac{1}{i q^+}u_v(P_b)\,\delta(\omega-\omega_0^+)$$

$$= i g_s^2 C_F(\tilde{\mu}^{2\varepsilon})\int\frac{d^dq}{(2\pi)^d}\frac{1}{q^+}\underbrace{\frac{1}{q^2}}_{D_1}\underbrace{\frac{1}{(q+\omega_0)^2}}_{D_2}\underbrace{\bar{v}(\omega_0)\not{n}_+(-\not{q}-\not{\omega}_0)\not{n}_+\gamma_5 u_v(P_b)}_{\text{求 trace}}$$

B-meson light-cone distribution amplitude from lattice QCD

Multiplicative renormalization: To facilitate the lattice QCD evaluation of the quasi-distribution amplitude $\varphi_B^+(\xi,\mu)$, it is of vita importance to show that such quasi-quantity will renormalize multiplicatively to all orders in perturbation theory applying the lattice regularization scheme. For this purpose, it has proven to be most convenient employing the one-dimensional auxiliary field formalism for the contour integrals introduced in [31].

The resulting Lagrangian for the ultra-collinear gluon interactions with both the effective bottom-quark field h_v and the auxiliary field $\mathcal{Q}$ can be written as

$$\mathcal{L} = \mathcal{L}_{\rm bHQET} + \bar{\mathcal{Q}}(x)\,(i n_z \cdot D_n - \delta m)\,\mathcal{Q}(x), \qquad (5)$$

where the "dynamical" mass term originates from the self-energy correction to the $\mathcal{Q}$ field in the dimensionful cut-off scheme [32], in analogy to the scheme-dependent residual mass term in the HQET formalism [33–35], and the ultra-collinear covariant derivative $D_n^\mu = \partial^\mu - i\,g_s\,T^a\,A_n^{a,\,\mu}$. Alternatively, the ultraviolet (UV) linear divergences from the Wilson-line corrections in (4) can be removed by introducing the proper subtraction term defined by a simpler matrix element but with the same power divergences [36–38]. It is straightforward to rewrite the non-local operator defining the B-meson quasi-distribution amplitude as follows [39, 40]

$$\begin{aligned}\mathcal{O}(\tau\,n_z,0) &= [\bar{\chi}_n(\tau\,n_z)\;\not{n}_z\,\gamma_5\,\mathcal{Q}(\tau\,n_z)]\;[\bar{\mathcal{Q}}(0)\,h_v(0)] \\ &\equiv \mathcal{J}_{\chi\mathcal{Q}}(\tau\,n_z)\;\mathcal{J}_{\mathcal{Q}h_v}(0)\,, \end{aligned}\qquad (6)$$

Thanks to the heavy-quark spin symmetry and the light-quark chiral symmetry for the effective Lagrangian $\mathcal{L}$, both of the two currents $\mathcal{J}_{\chi\mathcal{Q}}$ and $\mathcal{J}_{\mathcal{Q}h_v}$ renormalize multiplicatively under radiative corrections [41]

$$\begin{aligned}\mathcal{J}_{\chi\mathcal{Q}}(\tau\,n_z) &= Z_{\chi\mathcal{Q}}^{(R)}\,\mathcal{J}_{\chi\mathcal{Q}}^{(R)}(\tau\,n_z,\mu)\,, \\ \mathcal{J}_{\mathcal{Q}h_v}(0) &= Z_{\mathcal{Q}h_v}^{(R)}\,\mathcal{J}_{\mathcal{Q}h_v}^{(R)}(0,\mu)\,. \end{aligned}\qquad (7)$$

at all orders in α_s. We are therefore led to conclude the autonomous renormalization of the composite non-local operator $\mathcal{O}(\tau\,n_z,0)$, namely

$$\mathcal{O}(\tau\,n_z,0) = Z_{\chi\mathcal{Q}}^{(R)}\,Z_{\mathcal{Q}h_v}^{(R)}\,\mathcal{O}^{(R)}(\tau\,n_z,0,\mu). \qquad (8)$$

B-meson light-cone distribution amplitude from lattice QCD

Box diagram

Non-physical region ,

$$\frac{\alpha s\ cf}{4\pi}\ \frac{1}{v0\ \omega0z}\ \frac{1}{\omega-\omega0z}\left(2\ v0\ \omega\ \mathrm{Log}\left[\frac{\omega}{\omega-\omega0z}\right]-(vz\ (\omega-\omega0z)+v0\ (\omega+\omega0z))\ \mathrm{Log}\left[1+\frac{2\ v0\ \omega0z}{(v0+vz)\ (\omega-\omega0z)}\right]\right)$$

Vz -> infinity

0

Physical region,

$$\frac{\alpha s\ cf}{4\pi}\ \frac{1}{\sqrt{1+vz^2}}\ \frac{1}{\omega0z}\ \frac{1}{\omega-\omega0z}\left(-\left(\omega0z\left(\sqrt{1+vz^2}-vz\right)+\left(\sqrt{1+vz^2}+vz\right)\omega\right)\left(\mathrm{Log}\left[\frac{\sqrt{1+vz^2}+vz}{\sqrt{1+vz^2}-vz}\right]-i\pi\right)-2\sqrt{1+vz^2}\ \omega\ \mathrm{Log}\left[\frac{\rho}{4}\right]\right)$$

Vz -> infinity

$$\frac{cf\ \alpha s\ \omega\ (-\mathrm{Log}[\rho]+2\ \mathrm{Log}[\tau])}{2\pi\ (\omega-\omega0z)\ \omega0z}$$

Same as result of LCDA box diagram.

B-meson light-cone distribution amplitude from lattice QCD

LCDA: $\omega' = \frac{v^+}{\sqrt{2}}\omega'_L$; quasi: $\omega = v^z\,\omega_L$

另外，定义 $k_0 = \frac{k^+}{v^+} = \frac{k^z}{v^z}$.

Compare with plus function for light-meson,

$$\{\mathcal{F}(x,y)\}_\oplus = \mathcal{F}(x,y) - \delta(x-y)\int_0^1 dt\mathcal{F}(x,t)$$

$$v^\mu = \left(\frac{1}{\sqrt{1-\beta^2}}, 0, 0, \frac{\beta}{\sqrt{1-\beta^2}}\right)$$

2. *Evolution equations.*—The LCDA is given by the Fourier transform

$$\phi_+^B(\omega,\mu) = \frac{1}{2\pi}\int d\tau\, e^{i\omega\tau}\,\tilde{\phi}_+^B(\tau,\mu) \qquad (2)$$

of a function $\tilde{\phi}_+^B(\tau,\mu)$ defined in terms of a B-meson matrix element in HQET. Denoting by h the effective heavy-quark field and by q_s the soft spectator quark, and using a mass-independent normalization of meson states, we write [2]

$$\langle 0\,|\,\bar{q}_s(z)\,S_n(z,0)\not{n}\,\Gamma\,h(0)\,|\bar{B}(v)\rangle = -\frac{iF(\mu)}{2}\,\tilde{\phi}_+^B(\tau,\mu) \times \mathrm{tr}\left(\not{n}\,\Gamma\,\frac{1+\not{v}}{2}\,\gamma_5\right). \qquad (3)$$

B-meson light-cone distribution amplitude from lattice QCD

VERIFICATION OF THE RESULT.

We want to make comparison of $\ln\mu$ dependence between our result and Lange-Neubert kernel, in their paper 'Renormalization-Group Evolution of the B-Meson Light-Cone Distribution Amplitude', it follows that the LCDA obeys the evolution equation,

$$\frac{d}{d\ln\mu}\phi_+^B(\omega,\mu) = -\int_0^\infty d\omega'\gamma_+(\omega,\omega',\mu)\phi_+^B(\omega',\mu)\,, \tag{11}$$

here,

$$\gamma_+(\omega,\omega',\mu) \;=\; \left[4\ln\frac{\mu}{\omega}-2\right]\delta(\omega-\omega')+\omega\Gamma(\omega,\omega',\alpha_s)\,. \tag{12}$$

In our paper, the plus function definition of our result is different from their paper, we first convert our plus function to their plus function, this will induce a δ term,

$$F_+(\text{our paper}) = F_+(\text{L-N paper}) + \delta(\omega-\omega')\int_{2\omega}^\infty dt F. \tag{13}$$

So the second line in Eq.(10) would contribute a δ term (we only focus on $\ln\mu$ term),

$$\delta(\omega-\omega')\int_{2\omega}^\infty d\omega'\frac{-2\omega}{\omega'(\omega'-\omega)}\ln\mu^2 = -\ln 4\ln\mu^2\,. \tag{14}$$

Add this term to the last line in Eq.(10), we have (we only focus on $\ln\mu$ term),

$$D \equiv \left[(2\ln(2v^z)+1)\ln\mu^2 - 2\ln\mu^2 + 2\ln\left(\frac{\omega}{\mu}\right)^2\right]\delta(\omega-\omega') - \ln 4\ln\mu^2\,. \tag{15}$$

So,

$$\frac{dD}{d\ln\mu} = 4\ln\frac{\mu v^z}{\omega} - 2\,. \tag{16}$$

The definition in LCDA, in our paper, of ω (our paper) is $v^+\omega$ (ω defined in L-N paper), and when we doing matching, we make a re-scale, $\omega \to \frac{\omega}{\sqrt{2}}$, so the ω' appears in our final matching coefficient is

$$\omega'\ (\text{our paper}) = v^z\omega'\ (\text{L-N paper})\,, \tag{17}$$

in large v^z limit. And the Eq.(16) written in Lange-Neubert's notation is,

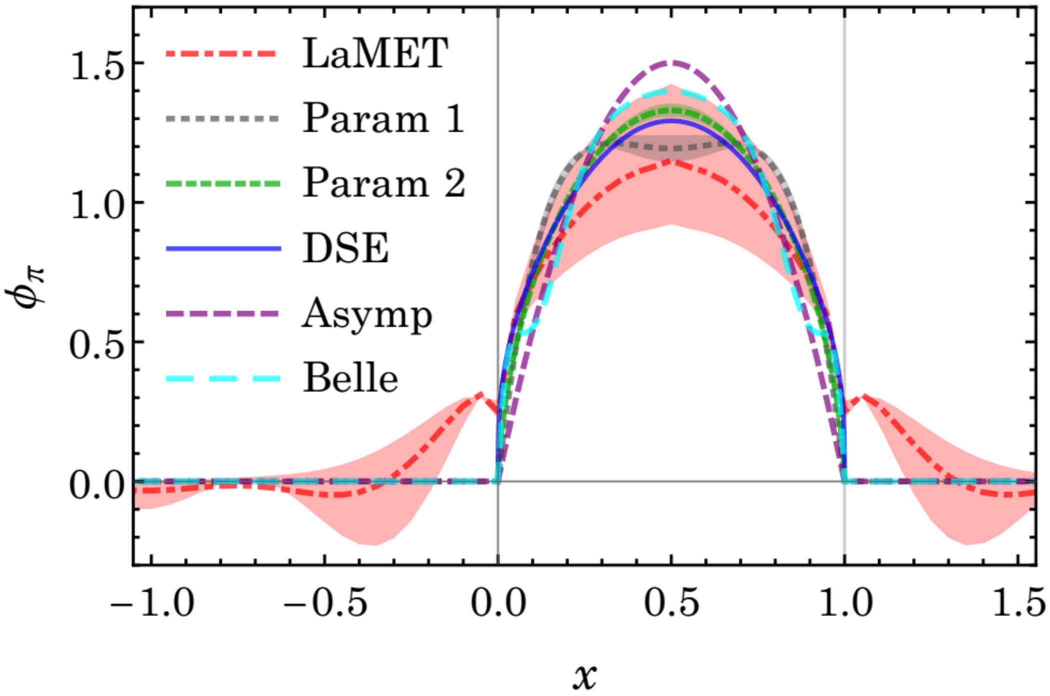
$$\frac{dD}{d\ln\mu} = 4\ln\frac{\mu}{\omega} - 2\,. \tag{18}$$

This is as same as the δ term in Eq.(12).

Sketch of LCDA in PRD

The matching coefficients have been calculated in dimensional regularization and transverse momentum cutoff schemes.

J.-H. Zhang, J.-W. C, X. Ji, Lu. J, H.-W. L Phys.Rev. D 2017



Light-cone distribution amplitudes of vector meson in large momentum effective theory

$$\langle P|O(\gamma^\mu\gamma_5,0)|0\rangle = if_P P^\mu\,,$$
$$\langle P,\epsilon_{||}|O(\gamma^\mu,0)|0\rangle = f_V^{||} M_V \epsilon_{||}^\mu\,,$$
$$\langle P,\epsilon_\perp|O(\sigma^{\mu\nu},0)|0\rangle = if_V^\perp(\epsilon_\perp^\mu P^\nu - \epsilon_\perp^\nu P^\mu)$$

$$\mathcal{O}_V^\Gamma(\xi^-) = \bar{\psi}(\xi^-)\Gamma W(\xi^-,0)\psi(0), \tag{1}$$

where $\Gamma = \gamma^+\gamma_\perp^\alpha$ for transversely polarized vector meson, and $\Gamma = \gamma^+$ for longitudinally polarized vector meson. $W(\xi^-,0)$ is the Wilson line with the end points $(0,\xi^-,0_\perp)$ and $(0,0,0_\perp)$. In LCDAs the Wilson line is light-like

$$W(\xi^-,0) = P\exp\Big[-ig_s\int_0^{\xi^-} n\cdot A(\lambda n)d\lambda\Big], \tag{2}$$

where P denotes that the exponential is path ordered. We also need the Fourier transformation of these operators, which are denoted by $O_V^\Gamma(x)$

$$O_V^\Gamma(x) = \int\frac{d\xi^-}{2\pi}e^{-ix\xi^-P^+}\mathcal{O}_V^\Gamma(\xi^-), \tag{3}$$

Matching the Quasi Meson Distribution Amplitude in RI/MOM scheme

The LCDAs are defined by the matrix elements of non-local gauge invariant quark bilinear operators, in which the two fermion fields are separated in the n direction.

$$\mathcal{O}_V^\Gamma(\xi^-) = \bar{\psi}(\xi^-)\Gamma W(\xi^-, 0)\psi(0),$$

Fourier transformation

$$O_V^\Gamma(x) = \int \frac{d\xi^-}{2\pi} e^{-ix\xi^- P^+} \mathcal{O}_V^\Gamma(\xi^-),$$

Take longitudinally polarized vector for instance,

$$f_V \frac{m_V}{P^+} \epsilon^{*+} \phi_V^{\parallel}(x,\mu) = \langle V, P, \epsilon^* | O_V^{\parallel}(x) | 0 \rangle,$$

$$f_V \frac{m_V}{P^+} \epsilon^{*+} = \langle V, P, \epsilon^* | \mathcal{O}_V^{\parallel}(0) | 0 \rangle$$

So,

$$\phi_V^\Gamma(x,\mu) = \frac{\langle V, P, \epsilon^* | O_V^\Gamma(x) | 0 \rangle}{\langle V, P, \epsilon^* | \mathcal{O}_V^\Gamma(0) | 0 \rangle}.$$

Matching the Quasi Meson Distribution Amplitude in RI/MOM scheme

Similarly, for Quasi-DAs

$$\widetilde{\mathcal{O}}^{\Gamma}(z) = \bar{\psi}(z)\Gamma W(z,0)\psi(0),$$

$$f_V \epsilon_z^* \frac{m_V}{P_z} \widetilde{\phi}_V^{\parallel}(x, P_z) = \langle V, P, \epsilon^* | \widetilde{\mathcal{O}}_V^{\parallel}(x) | 0 \rangle,$$

$$f_V \epsilon_z^* \frac{m_V}{P_z} = \langle V, P, \epsilon^* | \widetilde{\mathcal{O}}_V^{\parallel}(0) | 0 \rangle$$

So, we have

$$\widetilde{\phi}_V^{\Gamma}(x, \mu) = \frac{\langle V, P, \epsilon^* | \widetilde{\mathcal{O}}_V^{\Gamma}(x) | 0 \rangle}{\langle V, P, \epsilon^* | \widetilde{\mathcal{O}}_V^{\Gamma}(0) | 0 \rangle}.$$

The factorization formula,

$$\begin{aligned} &\widetilde{\phi}_R(\Gamma, x, P^z, \mu_R, p_R^z) \\ &= \int_0^1 dy\, C_\Gamma\left(x, y, r, \frac{P^z}{\mu}, \frac{P^z}{p_R^z}\right) \phi(\Gamma, y, \mu) \\ &\quad + \mathcal{O}\left(\frac{M^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(P^z)^2}\right), \end{aligned}$$

Matching the Quasi Meson Distribution Amplitude in RI/MOM scheme

The renormalized quasi-DA in the RI/MOM scheme can be matched to LCDA through the factorization formula,

$$\begin{aligned}
&\widetilde{\phi}_R(\Gamma, x, P^z, \mu_R, p^z_R) \\
&= \int_0^1 dy\, C_\Gamma\left(x, y, r, \frac{P^z}{\mu}, \frac{P^z}{p^z_R}\right) \phi(\Gamma, y, \mu) \\
&\quad + \mathcal{O}\left(\frac{M^2}{(P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(P^z)^2}\right),
\end{aligned}$$

$C_\Gamma\,(x, y)\,|_{tree} = \delta(x-y)$

where $r = \mu_R^2/(p^z_R)^2$.

The bare matching coefficient

$$C_B^{(1)}\left(\Gamma, x, y, \frac{P^z}{\mu}\right) = \widetilde{\phi}_B^{(1)}(\Gamma, x, y, P^z) - \phi^{(1)}(\Gamma, x, y, \mu)$$

Matching the Quasi Meson Distribution Amplitude in RI/MOM scheme

We have calculated $\Gamma = \gamma^+\gamma_5,\ \gamma^+,\ \gamma^+\gamma_\perp$ for pseudoscalar, longitudinally polarized vector and transversely polarized vector meson LCDAs;
$\Gamma = \gamma^z\gamma_5,\ \gamma^t,\ \gamma^z\gamma_\perp$ for pseudoscalar, longitudinally polarized vector and transversely polarized vector meson quasi-DAs, respectively.

Since we take the on-shell limit to obtain the bare matching coefficient, $C_B^{(1)}$.

$$C_B^{(1)}\left(\Gamma, x, y, \frac{P_z}{\mu}\right) = \frac{\alpha_s C_F}{2\pi}\begin{cases} [H_1(\Gamma, x, y)]_{+(y)} & x<0<y \\ [H_2(\Gamma, x, y, P^z/\mu)]_{+(y)} & 0<x<y \\ [H_2(\Gamma, 1-x, 1-y, P^z/\mu)]_{+(y)} & y<x<1 \\ [H_1(\Gamma, 1-x, 1-y)]_{+(y)} & y<1<x \end{cases}$$

where

$$H_1(\Gamma, x, y) = \begin{cases} \frac{1+x-y}{y-x}\frac{1-x}{1-y}\ln\frac{y-x}{1-x} + \frac{1+y-x}{y-x}\frac{x}{y}\ln\frac{y-x}{-x} & \Gamma = \gamma^z\gamma_5 \text{ and } \gamma^t \\ \frac{1}{y-x}\frac{1-x}{1-y}\ln\frac{y-x}{1-x} + \frac{1}{y-x}\frac{x}{y}\ln\frac{y-x}{-x} & \Gamma = \gamma^z\gamma_\perp \end{cases},$$

$$H_2\left(\Gamma, x, y, \frac{P_z}{\mu}\right) = \begin{cases} \frac{1+y-x}{y-x}\frac{x}{y}\ln\frac{4x(y-x)(P^z)^2}{\mu^2} + \frac{1+x-y}{y-x}\left(\frac{1-x}{1-y}\ln\frac{y-x}{1-x} - \frac{x}{y}\right) & \Gamma = \gamma^z\gamma_5 \\ \frac{1+y-x}{y-x}\frac{x}{y}\left(\ln\frac{4x(y-x)(P^z)^2}{\mu^2} - 1\right) + \frac{1+x-y}{y-x}\frac{1-x}{1-y}\ln\frac{y-x}{1-x} & \Gamma = \gamma^t \\ \frac{1}{y-x}\frac{x}{y}\ln\frac{4x(y-x)(P^z)^2}{\mu^2} + \frac{1}{y-x}\left(\frac{1-x}{1-y}\ln\frac{y-x}{1-x} - \frac{x}{y}\right) & \Gamma = \gamma^z\gamma_\perp \end{cases}.$$

Matching the Quasi Meson Distribution Amplitude in RI/MOM scheme

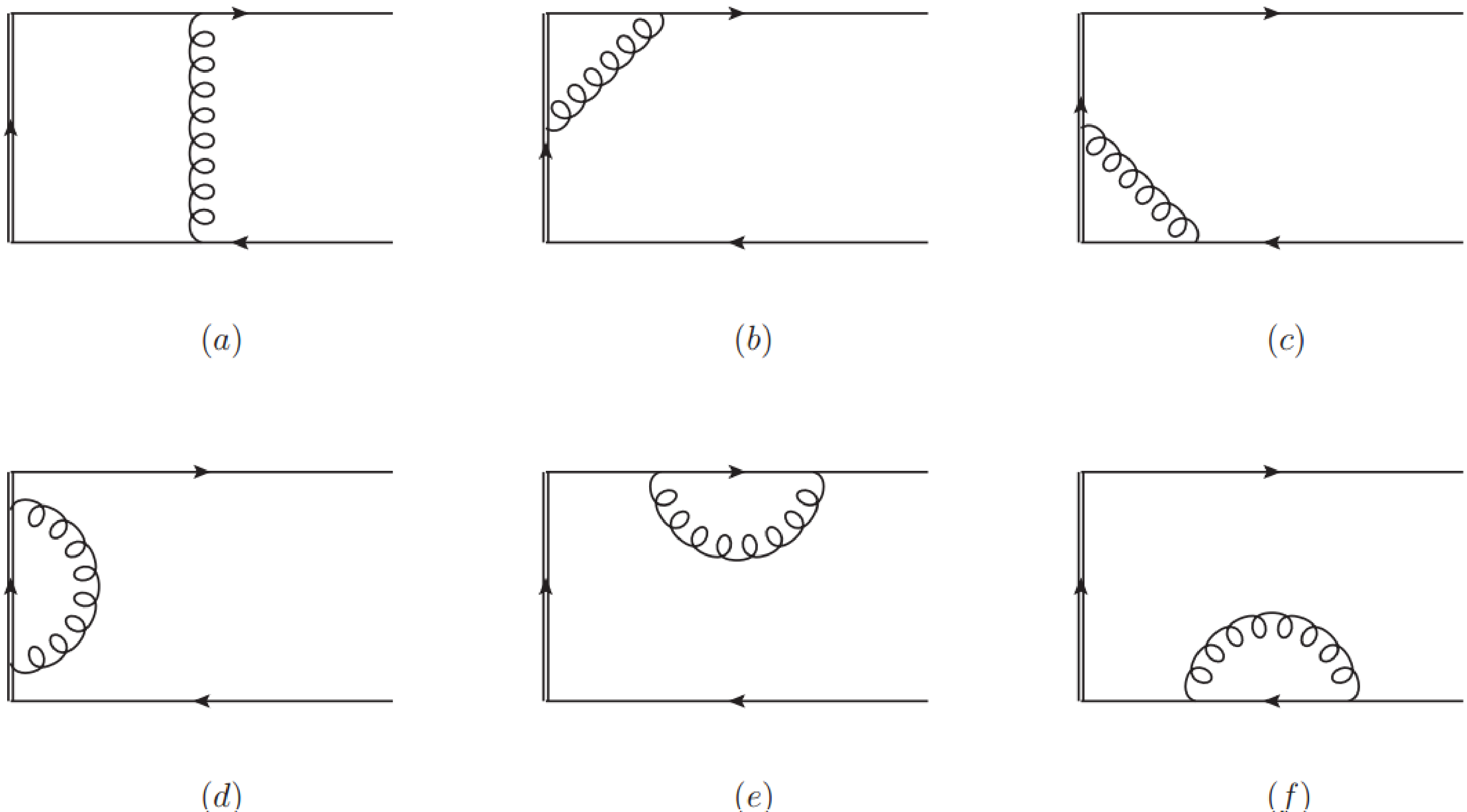


FIG. 1: Feynman diagrams for LCDAs and quasi-DAs at one loop level. The double line denotes the Wilson line.

Matching the Quasi Meson Distribution Amplitude in RI/MOM scheme

In order to combine the "real" and "virtual" contributions (defined in Ref. [?]) in a compact form at one-loop level, we introduce a plus function $[h(x,y)]_{+(y)}$ which is defined as

$$\int dx[h(x,y)]_{+(y)}g(x) = \int dx\ h(x,y)[g(x) - g(y)] \quad (22)$$

Quark Generalized Parton Distributions

The momentum fraction $x \in [-1, 1]$, which falls into the following three regions:

- $x \in [-1, -\xi]$, both momentum fractions $x + \xi$ and $x - \xi$ are negative: emission and reabsorption of antiquarks with respective momentum fractions $\xi - x$ and $-\xi - x$.
- $x \in [-\xi, \xi]$, one has $x + \xi > 0$ but $x - \xi < 0$: a quark with momentum fraction $x + \xi$ and an antiquark with $\xi - x$ emitted from the initial proton.
- $x \in [\xi, 1]$ both $x + \xi$ and $x - \xi$ are positive: emission and reabsorption of a quark.

The first and third case are commonly referred to as **DGLAP regions** and the second as **ERBL region**.

Large Momentum Effective Theory (LaMET)

Relating *parton physics observables* to *equal-time correlators in a large momentum nucleon states* (quasi-observables).

- Light-cone observables: $p_z \to \infty$, then $\Lambda \to \infty$.
  Quasi observables: $\Lambda \to \infty$, then $p_z \to \infty$.
  These two limits **do not commute**!
- They have **same IR** but different UV behaviours, while the UV difference is **controllable** and **calculable**.

Factorization formula between light-cone and quasi GPDs:

$$\mathcal{H}(x,\xi,t,p_z) = \int_{-1}^{1} \frac{dy}{|y|} Z_H\Big(\frac{x}{y},\frac{\xi}{y},\frac{\mu}{p_z}\Big) H(y,\xi,t,\mu) + \mathcal{O}\Big(\frac{M^2}{p_z^2},\frac{\Lambda_{QCD}^2}{p_z^2}\Big).$$

Order of limits

- Thus the difference between the matrix elements o and O is the order of limits:

  o: $P \to \infty$, followed by UV cut-off

  O: UV cut-off imposed first, followed by $P \to \infty$

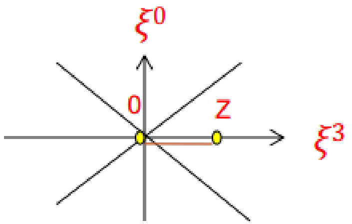
- This is the starndard set-up for effective field theory, such as HQET. The generic argument for factorization follow through. Hence we have large-momentum effective field theory: LaMET.
- Perturbative proof case by case.

PPT

A Euclidean quasi-distribution

- Consider space correlation in a large momentum P in the z-direction.

$$\tilde{q}(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(z)\gamma^z \times \exp\left(-ig\int_0^z dz' A^z(z')\right)\psi(0)|P\rangle$$



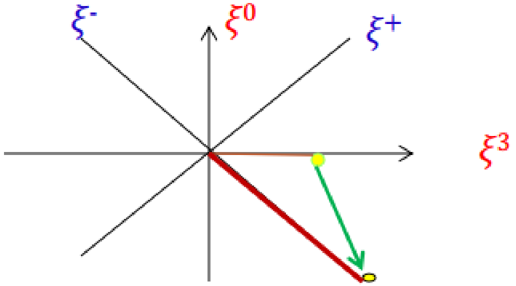
- Quark fields separated along the z-direction
- The gauge-link along the z-direction
- The matrix element depends on the momentum P.

Taking the limit P-> ∞ first

- After renormalizing all the UV divergences, one has the standard quark distribution!
  - One can prove this using the standard OPE
  - One can also see this by writing

    |P> = U(Λ(p)) |p=0>

  and applying the boost operator on the gauge link.



Step 1: Constructing lattice operators and evaluate the ME

- Construct a *frame-dependent, Euclidean* quasi-operator "O".
- In the IMF limit, O becomes a light-cone (light-front, parton) operator *o*.

$$O_1 = A^0 \;\rightarrow\; \mathrm{o} = \Lambda\, A^+$$

There are many operators leading to the same light-cone operator.

$$O_2 = A^3 \;\rightarrow\; \mathrm{o} = \Lambda$$
$$O_3 = \alpha A^0 + (1-\alpha)A^3 \;\rightarrow\; \mathrm{o} = \Lambda\, A^+$$

Step 2: lattice calculations

- Compute the matrix element of O on a lattice
- It will depend on the momentum of the hadron P, O(P,a).
- It also depends on the details of the lattice actions (UV specifics).

Step 3: Extracting the light-cone physics from the lattice ME

- Extract light-front physics o(μ) from O(P,a) at large P through a EFT matching condition or factorization theorem,

$$O(P,a) = Z(\frac{\mu}{P})o(\mu) + \frac{c_2}{P^2} + \frac{c_4}{P^4} + \cdots$$

  Where Z is perturbatively calculable.

- Infrared physics of O(P,a) is entirely captured by the parton physics o(μ). In particular, it contains all the collinear divergence when P gets large.

Renormalization in Large Momentum Effective Theory of Parton Physics

Xiangdong Ji,[1,2] Jian-Hui Zhang,[3] and Yong Zhao[4]

II. EFFECTIVE QCD WITH AUXILIARY "HEAVY-QUARK" FIELD

To be concrete, consider the following non-local operator

$$O(x,y) = \overline{\psi}(x)\Gamma L(x,y)\psi(y), \quad (1)$$

where $\psi, \bar{\psi}$ denote the bare quark fields and Γ a Dirac structure. $L(x,y)$ is a path-ordered gauge link from y to x,

$$L(x,y) = \mathcal{P}\exp\left(-ig\int_0^1 d\lambda \frac{dz^\mu}{d\lambda}A_\mu(z(\lambda))\right) \quad (2)$$

IV. RENORMALIZATION OF NON-LOCAL OPERATORS IN DIMENSIONAL REGULARIZATION

Now let us consider the renormalization of the non-local operator, such as in Eq. (1), which appears in the LaMET approach to parton physics. In the effective theory, the non-local operator becomes a product of local composite operators,

$$O(z_2,z_1) = \bar{j}(z_2)j(z_1), \quad (10)$$

after we replace the non-local Wilson line by the product of two auxiliary "heavy quark" fields. This operator can be multiplicatively renormalized by

$$O(z_2,z_1) = Z_{\bar{j}}Z_j O_R(z_2,z_1) \quad (11)$$

to all orders in perturbation theory with $Z_j = Z_q^{1/2}Z_Q^{1/2}Z_{V_j}$, where Z_q, Z_Q, Z_{V_j} are the renormalization constant for the light-quark, heavy-quark and vertex, respectively.

To study the renormalization property of the above operator, we introduce an "heavy quark" auxiliary field (color source without spin degrees of freedom) Q with color 3, and its conjugate $\overline{Q}$, with color $\bar{3}$. We extend QCD to include this "heavy quark" interaction with the gluon field, and introduce the following Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm QCD} + \overline{Q}(x)in\cdot DQ(x), \quad (3)$$

where $D_\mu = \partial_\mu + igt^aA^a_\mu$ is the covariant derivative in fundamental representation. For a real heavy quark or time-like Wilson line, n^μ is a timelike vector $n^\mu = (1,0,0,0)$, whereas for a spacelike Wilson line n^μ can be chosen as $n^\mu = (0,0,0,1)$. We will focus on the latter case in the following, although the discussion may go through for any n. So long as there is a non-zero time component n^0, the "heavy quark" is dynamical, but follows a designated world line.

In the above theory, we can replace the bilocal operator $O(x,y)$ by a new operator,

$$O(x,y) = \overline{\psi}(x)\Gamma Q(x)\overline{Q}(y)\psi(y). \quad (4)$$