GPDs from LaMET

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SEPTEMBER 17TH, 2019

POETIC 2019

BERKELEY LAB

GPDs from LaMET

Matching Quasi-GPDs in the RI/MOM scheme

Phys. Rev. **D**100 (2019) no.3, 034006
arXiv:1902.00307 [hep-ph]

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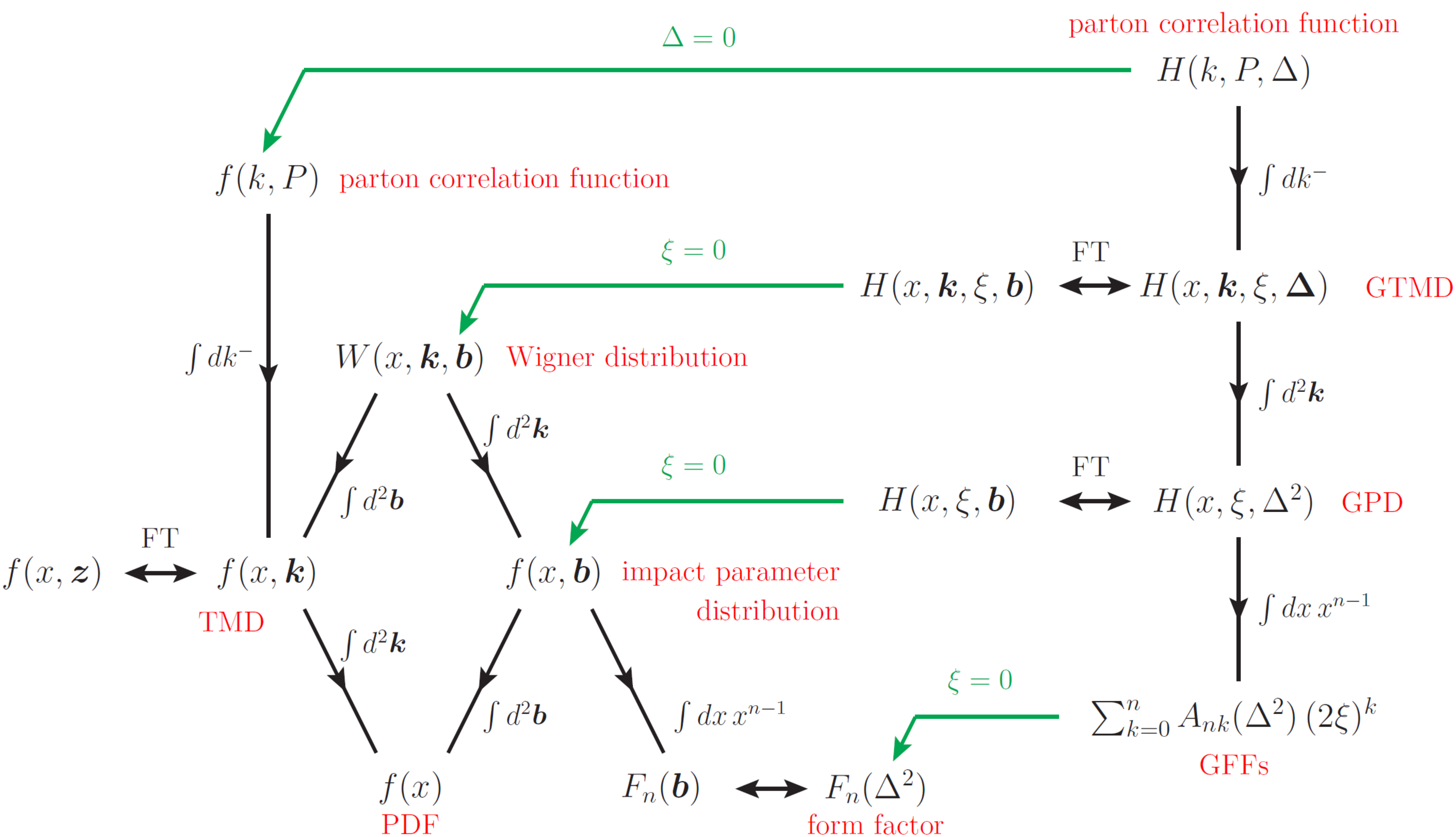
Generalized Parton Distribution

- Rich theoretical implications
  - DGLAP (ξ<x<1), ERBL (|x|<ξ) evolutions
  - The spin structure of the nucleon
  - Angular momentum of parton
  - Sum rules
  - Form factors
  - three-dimensional image of partons inside hadrons
  - And more…
- GPD Global fit
  - Not as constrained as PDF
  - Difficult to extract from experiment
  - More kinematic parameters dependence than PDF

Eur. Phys. J. **A**52 (2016) no.6, 149
arXiv:1512.01328 [hep-ph]

Markus Diehl: Introduction to GPDs and TMDs



PDF and Parent-GPD

- Both are defined on the light-cone coordinate $\xi^{\pm} = \frac{t \pm z}{\sqrt{2}}$

$$q(\bar{\Gamma}, x, \mu) = \int \frac{d\zeta^-}{4\pi} e^{-ix\zeta^- P^+} \langle P, S | \bar{\psi}(\zeta^-) \bar{\Gamma} \lambda^a W_+(\zeta^-, 0) \psi(0) | P, S \rangle$$

$$F(\bar{\Gamma}, x, \xi, t, \mu) = \int \frac{d\zeta^-}{4\pi} e^{-ix\zeta^- P^+} \langle P'', S'' | \bar{\psi}\left(\frac{\zeta^-}{2}\right) \bar{\Gamma} \lambda^a W_+\left(\frac{\zeta^-}{2}, -\frac{\zeta^-}{2}\right) \psi\left(-\frac{\zeta^-}{2}\right) | P', S' \rangle$$

where $x \in [-1,1]$ is the momentum fraction, $\bar{\Gamma}$ is gamma matrices, λ is a matrix in flavor space, and the gauge link is

$$W_+(\zeta_2^-, \zeta_1^-) = P \exp\left[-ig_s \int_{\zeta_1^-}^{\zeta_2^-} A^+(\eta^-) d\eta^-\right]$$

- $\bar{\Gamma} = \gamma^+, \gamma^+\gamma_5$, and $i\sigma^{+\perp}$ correspond to unpolarized, helicity, and transversity PDF/parent GPD.

GPD

- GPDs: Lorentz decomposition of parent-GPDs

$$F(\bar{\Gamma}, x, \xi, t, \mu) = \frac{1}{2P^+}\bar{u}(P'', S'')\Big\{H(\bar{\Gamma}, x, \xi, t, \mu)\bar{\Gamma} + E(\bar{\Gamma}, x, \xi, t, \mu)\frac{[\not{\Delta}, \bar{\Gamma}]}{4M} + H'(\bar{\Gamma}, x, \xi, t, \mu)\frac{P^{[+}\Delta^{\perp]}}{M^2} + E'(\bar{\Gamma}, x, \xi, t, \mu)\frac{\gamma^{[+}P^{\perp]}}{M}\Big\}u(P', S')$$

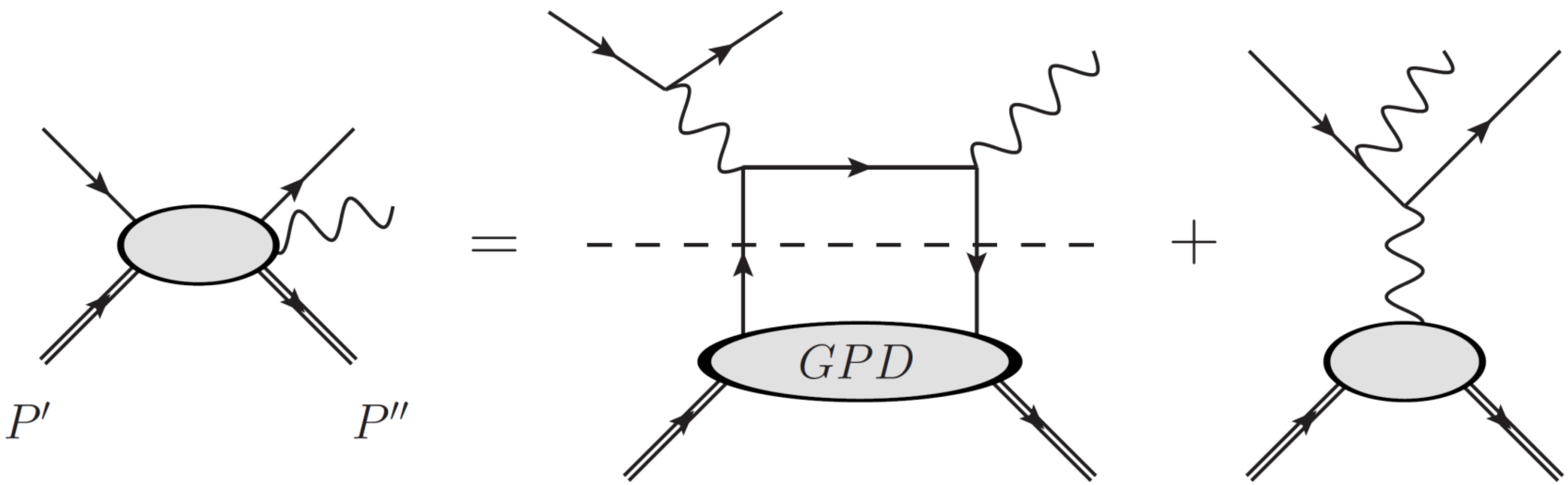
- H' and E' are only non-zero for transversity GPD.
- Kinematics: $P^\mu \equiv \dfrac{P''^\mu + P'^\mu}{2} = (P^0, 0, 0, P^z)$

$$\Delta \equiv P'' - P', \quad t \equiv \Delta^2$$

skewness: $\xi \equiv -\dfrac{P''^+ - P'^+}{P''^+ + P'^+} = -\dfrac{\Delta^+}{2P^+}$

GPD in experiments

- For example, $ep \to ep\gamma$ process
  - Deeply Virtual Compton Scattering (DVCS)
  - Bethe-Heitler



- GPDs can also be studied other processes, such as $ep \to ep\pi^0, \gamma p \to \mu^+\mu^- p, ep \to ep\mu^+\mu^-$, etc.

Large Momentum Effective Theory

- LaMET proposed by Xiangdong Ji [1]
- Light-cone observables:
  - defined in infinite momentum frame
- quasi-observables:
  - equal time correlation functions
  - frame dependent (momentum of the external hadron state)
- LaMET relates light-cone observable and quasi-observables with large momentum.
  - LC and quasi-observables have the same IR but different UV.
  - Lattice calculation of quasi-observables using LaMET can be improved systematically.

[1] X. Ji, PRL 110, 262002 (2013); Sci. China Phys. Mech. Astron. 57, 1407 (2014).

Quasi-PDF and Quasi-Parent-GPD

- Both are defined by equal-time correlators

$$\widetilde{q}(\Gamma, x, P^z, \widetilde{\mu}) = \frac{1}{N}\int \frac{dz}{4\pi} e^{ixzP^z} \langle P, S|\bar{\psi}(z)\Gamma\lambda^a W_z(z,0)\psi(0)|P, S\rangle$$

$$\widetilde{F}(\Gamma, x, \widetilde{\xi}, t, P^z, \widetilde{\mu}) = \frac{1}{N}\int \frac{dz}{4\pi} e^{ixzP^z} \langle P'', S''|\bar{\psi}\left(\frac{z}{2}\right)\Gamma\lambda^a W_z\left(\frac{z}{2}, -\frac{z}{2}\right)\psi\left(-\frac{z}{2}\right)|P', S'\rangle$$

where $x \in (-\infty, \infty)$ is the momentum fraction, Γ is gamma matrices, λ is a matrix in flavor space, and the gauge link is

$$W_z(z_2, z_1) = P\exp\left[ig_s\int_{z_1}^{z_2} A^z(z')dz'\right]$$

- $\Gamma = \{\gamma^z, \gamma^t\}, \{\gamma^z\gamma_5, \gamma^t\gamma_5\}$, and $\{i\sigma^{z\perp}, i\sigma^{t\perp}\}$ correspond to unpolarized, helicity, and transversity quasi-PDF/quasi-parent-GPD.

Operator Mixing on Lattice

- The quasi-operator of PDF (GPD) might mix with the scalar operator ($\Gamma = 1$) for some choice of Γ on lattice [1].
  - Calculation of lattice perturbation theory
  - Examining symmetry of operator on lattice
- To avoid operator mixing at $\mathcal{O}(a^0)$, we choose

$$\Gamma = \gamma^t, \gamma^z\gamma_5, \text{ and } i\sigma^{z\perp}$$

for unpolarized, helicity, and transversity parent quasi-GPD.

- The nonlocal quark operator mixing pattern has been classified [2].

[1] M. Constantinou and H. Panagopoulos, PRD96, 054506 (2017); J. Green, K. Jansen, and F. Steens, PRL 121, 022004 (2018); C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, H. Panagopoulos, and F. Steffens, NPB923, 394 (2017); J.-W. Chen, T. Ishikawa, L. Jin, H.-W. Lin, Y.-B. Yang, J.-H. Zhang, and Y. Zhao, PRD97, 014505 (2018).
[2] J.-W. Chen, T. Ishikawa, L. Jin, H.-W. Lin, Y.-B. Yang, J.-H. Zhang, and Y. Zhao, (2017), arXiv:1710.01089 [hep-lat].

Quasi-GPD

- Quasi-GPDs: Lorentz decomposition of quasi-parent-GPDs

$$\widetilde{F}(\Gamma, x, \xi, t, P^z, \widetilde{\mu}) = \frac{1}{2P^t}\bar{u}(P'', S'')\Big\{\widetilde{H}(\Gamma, x, \xi, t, P^z, \widetilde{\mu})\Gamma + \widetilde{E}(\Gamma, x, \xi, t, P^z, \widetilde{\mu})\frac{[\not{\Delta}, \Gamma]}{4M}$$
$$+ \widetilde{H}'(\bar{\Gamma}, x, \xi, t, \mu)\frac{P^{[z}\Delta^{\perp]}}{M^2} + \widetilde{E}'(\bar{\Gamma}, x, \xi, t, \mu)\frac{\gamma^{[z}P^{\perp]}}{M}\Big\}u(P', S')$$

- $\widetilde{H}'$ and $\tilde{E}'$ are only non-zero for transversity quasi-GPD.
- Kinematics: $P^\mu \equiv \dfrac{P''^\mu + P'^\mu}{2} = (P^0, 0, 0, P^z)$

$$\Delta \equiv P'' - P', \;\; t \equiv \Delta^2$$

quasi-skewness: $\widetilde{\xi} = -\dfrac{P''^z - P'^z}{P''^z + P'^z} = -\dfrac{\Delta^z}{2P^z} = \xi + \mathcal{O}\left(\dfrac{M^2}{P_z^2}\right)$

Factorization

- Using operator product expansion, we can show that

$$\widetilde{F}(\Gamma, x, \xi, t, P^z, \mu) = \int_{-1}^{1} \frac{dy}{|\xi|} \bar{C}_\Gamma \left(\frac{x}{\xi}, \frac{y}{\xi}, \frac{\mu}{\xi P^z} \right) F(\bar{\Gamma}, y, \xi, t, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{t}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2} \right)$$
$$= \int_{-1}^{1} \frac{dy}{|y|} C_\Gamma \left(\frac{x}{y}, \frac{\xi}{y}, \frac{\mu}{y P^z} \right) F(\bar{\Gamma}, y, \xi, t, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{t}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2} \right)$$

where

$$C_\Gamma \left(\frac{x}{y}, \frac{\xi}{y}, \frac{\mu}{y P^z} \right) = \left| \frac{y}{\xi} \right| \bar{C}_\Gamma \left(\frac{x}{\xi}, \frac{y}{\xi}, \frac{\mu}{\xi P^z} \right)$$

- $\xi \to 0$ and $t \to 0$: recover quasi-PDF factorization
- $\xi \to 1$ and $t \to 0$: recover quasi-DA factorization

Renormalization

UV divergence of the quasi-observable

only depends on the operator,

not the external state.

RI/MOM Scheme

$$\widetilde{O}(\Gamma, z) = \bar{\psi}(z)\,\Gamma W_z\,(z, 0)\,\psi\,(0)$$

- The quantum corrections of quasi-PDF matrix element in an off-shell quark state vanish at a given momentum

$$Z(\Gamma, z, a, \mu_R, p_R^z) = \left. \frac{\langle p, s|\widetilde{O}(\Gamma, z, a)|p, s\rangle}{\langle p, s|\widetilde{O}(\Gamma, z, a)|p, s\rangle_{\mathrm{tree}}} \right|_{\{\widetilde{\mu}\}}$$

The subtraction point is specified by scales $\tilde{\mu} = \{\mu_R, p_R^z\}$. $\langle p, s|\tilde{O}(\Gamma, z, a)|p, s\rangle$ is obtained from the amputated Green's function $\Lambda_\Gamma(z, p, a)$ of $\tilde{O}(\Gamma, z, a)$ which is calculated on lattice with a projection operator $\mathcal{P}$ for the Dirac matrix

$$\langle p, s|\widetilde{O}(\Gamma, z, a)|p, s\rangle = \mathrm{Tr}\left[\Lambda(\Gamma, z, a, p)\mathcal{P}\right]$$

- UV divergence of quasi-GPD is the same as quasi-PDF!

Factorization in RI/MOM scheme

- Factorization formula

$$\widetilde{F}(\Gamma, x, \xi, t, P^z, \mu_R, p_R^z) = \int_{-1}^{1} \frac{dy}{|y|} C_\Gamma \left(\frac{x}{y}, \frac{\xi}{y}, r, \frac{yP^z}{\mu}, \frac{yP^z}{p_R^z}\right) F(\bar{\Gamma}, y, \xi, t, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{t}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

- Suggest the matching coefficient for all GPDs are the same!
- GPDs with on-shell massless quark state at tree level

$$H^{(0)}(\bar{\Gamma}, x, \xi, t) = \widetilde{H}^{(0)}(\Gamma, x, \xi, t, p^z) = \delta(1-x)$$
$$H'^{(0)} = \widetilde{H}'^{(0)} = E^{(0)} = \widetilde{E}^{(0)} = E'^{(0)} = \widetilde{E}'^{(0)} = 0$$

- At one-loop level, one only need to match $\widetilde{H}$ and H

$$\widetilde{H}(\Gamma, x, \xi, t, P^z, \mu_R, p_R^z) = \int_{-1}^{1} \frac{dy}{|y|} C_\Gamma \left(\frac{x}{y}, \frac{\xi}{y}, r, \frac{yP^z}{\mu}, \frac{yP^z}{p_R^z}\right) H(\bar{\Gamma}, y, \xi, t, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{t}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

Matching Coefficients

- Matching coefficient up to NLO

$$C_\Gamma\left(x,\xi,r,\frac{p^z}{\mu},\frac{p^z}{p^z_R}\right) = \delta(1-x) + \left[f_1\left(\Gamma,x,\xi,\frac{p^z}{\mu}\right) - \left|\frac{p^z}{p^z_R}\right| f_2\left(\Gamma,\frac{p^z}{p^z_R}(x-1)+1,r\right)\right]_+$$
$$+\,\delta_{\Gamma,i\sigma^{z\perp}}\delta(1-x)\frac{\alpha_s C_F}{4\pi}\ln\left(\frac{\mu^2}{\mu_R^2}\right) + \mathcal{O}(\alpha_s^2)$$

- The generalized plus function

$$\int dx[h(x)]_+ g(x) = \int dx\, h(x)\left[g(x) - g(1)\right]$$

- f_1 is the bare matching coefficient:
  - difference of bare quasi-GPD and renormalize LCGPD
  - gauge, scheme, IR cutoff independent
- f_2: quasi-PDF counterterm depending on gauge and scheme

Matching Coefficients cont'd

- Bare matching coefficient

$$f_1\left(\Gamma, x, \xi, \frac{p^z}{\mu}\right) = \frac{\alpha_s C_F}{2\pi}\begin{cases} G_1(\Gamma, x, \xi) & x < -\xi \\ G_2(\Gamma, x, \xi, p^z/\mu) & |x| < \xi \\ G_3(\Gamma, x, \xi, p^z/\mu) & \xi < x < 1 \\ -G_1(\Gamma, x, \xi) & x > 1 \end{cases}$$

- Counterterms in Landau gauge with minimal projection

$$f_2(\gamma^t, x, r) = \frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{-3r^2+13rx-8x^2-10rx^2+8x^3}{2(r-1)(x-1)(r-4x+4x^2)} + \frac{-3r+8x-rx-4x^2}{2(r-1)^{3/2}(x-1)}\tan^{-1}\frac{\sqrt{r-1}}{2x-1} & x > 1 \\ \frac{-3r+7x-4x^2}{2(r-1)(1-x)} + \frac{3r-8x+rx+4x^2}{2(r-1)^{3/2}(1-x)}\tan^{-1}\sqrt{r-1} & 0 < x < 1 \\ -\frac{-3r^2+13rx-8x^2-10rx^2+8x^3}{2(r-1)(x-1)(r-4x+4x^2)} - \frac{-3r+8x-rx-4x^2}{2(r-1)^{3/2}(x-1)}\tan^{-1}\frac{\sqrt{r-1}}{2x-1} & x < 0 \end{cases}$$

$$f_2(\gamma^z\gamma_5, x, r) = \frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{3r-(1-2x)^2}{2(r-1)(1-x)} - \frac{4x^2(2-3r+2x+4rx-12x^2+8x^3)}{(r-1)(r-4x+4x^2)^2} + \frac{2-3r+2x^2}{(r-1)^{3/2}(x-1)}\tan^{-1}\frac{\sqrt{r-1}}{2x-1} & x > 1 \\ \frac{1-3r+4x^2}{2(r-1)(1-x)} + \frac{-2+3r-2x^2}{(r-1)^{3/2}(1-x)}\tan^{-1}\sqrt{r-1} & 0 < x < 1 \\ -\frac{3r-(1-2x)^2}{2(r-1)(1-x)} + \frac{4x^2(2-3r+2x+4rx-12x^2+8x^3)}{(r-1)(r-4x+4x^2)^2} - \frac{2-3r+2x^2}{(r-1)^{3/2}(x-1)}\tan^{-1}\frac{\sqrt{r-1}}{2x-1} & x < 0 \end{cases}$$

$$f_2(i\sigma^{z\perp}, x, r) = \frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{3}{2(1-x)} + \frac{r-2x}{(r-1)(r-4x+4x^2)} + \frac{-r+2x-rx}{(r-1)^{3/2}(x-1)}\tan^{-1}\frac{\sqrt{r-1}}{2x-1} & x > 1 \\ \frac{1-3r+2x}{2(r-1)(1-x)} + \frac{r-2x+rx}{(r-1)^{3/2}(1-x)}\tan^{-1}\sqrt{r-1} & 0 < x < 1 \\ -\frac{3}{2(1-x)} - \frac{r-2x}{(r-1)(r-4x+4x^2)} - \frac{-r+2x-rx}{(r-1)^{3/2}(x-1)}\tan^{-1}\frac{\sqrt{r-1}}{2x-1} & x < 0 \end{cases}$$

Matching Coefficients cont'd cont'd

$$G_1(\gamma^t, x, \xi) = G_1(\gamma^z\gamma_5, x, \xi) = -\left[\frac{1}{x-1} - \frac{x}{2\xi} + \frac{1+x}{2(1+\xi)}\right] \ln\frac{x-1}{x+\xi} + (\xi \to -\xi)$$

$$G_1(i\sigma^{z\perp}, x, \xi) = -\frac{x+\xi}{(x-1)(1+\xi)} \ln\frac{x-1}{x+\xi} + (\xi \to -\xi)$$

$$G_2\left(\gamma^t, x, \xi, p^z/\mu\right) = \frac{(x+\xi)(1-x+2\xi)}{2(1-x)\xi(1+\xi)}\left[\ln\frac{4(1-x)^2(x+\xi)(p^z)^2}{(\xi-x)\mu^2} - 1\right] + \frac{x+\xi^2}{\xi(1-\xi^2)}\ln\frac{\xi-x}{1-x}$$

$$G_2\left(\gamma^z\gamma_5, x, \xi, p^z/\mu\right) = G_2\left(\gamma^t, x, \xi, p^z/\mu\right) + \frac{x+\xi}{\xi(1+\xi)}$$

$$G_2\left(i\sigma^{z\perp}, x, \xi, p^z/\mu\right) = \frac{x+\xi}{(1-x)(1+\xi)}\left[\ln\frac{4(1-x)^2(x+\xi)(p^z)^2}{(\xi-x)\mu^2} - 1\right] + \frac{2\xi}{1-\xi^2}\ln\frac{\xi-x}{1-x}$$

$$G_3\left(\gamma^t, x, \xi, p^z/\mu\right) = \frac{1+x^2-2\xi^2}{(1-x)(1-\xi^2)}\left[\ln\frac{4\sqrt{x^2-\xi^2}(1-x)(p^z)^2}{\mu^2} - 1\right] + \frac{x+\xi^2}{2\xi(1-\xi^2)}\ln\frac{x+\xi}{x-\xi}$$

$$G_3\left(\gamma^z\gamma_5, x, \xi, p^z/\mu\right) = G_3\left(\gamma^t, x, \xi, p^z/\mu\right) + 2\frac{1-x}{1-\xi^2}$$

$$G_3\left(i\sigma^{z\perp}, x, \xi, p^z/\mu\right) = \frac{2(x-\xi^2)}{(1-x)(1-\xi^2)}\left[\ln\frac{4\sqrt{x^2-\xi^2}(1-x)(p^z)^2}{\mu^2} - 1\right] + \frac{\xi}{1-\xi^2}\ln\frac{x+\xi}{x-\xi}$$

Limits of Matching Coefficients

$$C_\Gamma\left(x,\xi,r,\frac{p^z}{\mu},\frac{p^z}{p^z_R}\right)=\delta(1-x)+\left[f_1\left(\Gamma,x,\xi,\frac{p^z}{\mu}\right)-\left|\frac{p^z}{p^z_R}\right|f_2\left(\Gamma,\frac{p^z}{p^z_R}(x-1)+1,r\right)\right]_+$$
$$+\,\delta_{\Gamma,i\sigma^{z\perp}}\delta(1-x)\frac{\alpha_s C_F}{4\pi}\ln\left(\frac{\mu^2}{\mu_R^2}\right)+\mathcal{O}(\alpha_s^2)$$

- Recovering quasi-PDFs matching coefficients
  - $\xi\to 0$ and $t\to 0$
  - For zero skewness $\xi=0$, the matching coefficient of GPD is the same as the one of PDF.

- Recovering quasi-DAs matching coefficients
  - $\xi\to\frac{1}{2y-1}$, $\frac{x}{\xi}\to 2x-1$, and $p^z\to\frac{p^z}{2}$
  - No extra δ-function for transversely polarized vector meson DA ($\Gamma=i\sigma^{z\perp}$) due to an extra local operator in the denominator in the definition of DA.

Summary

- Several proposed experiments: EIC, EICC, LHeC, etc, are going to further explore the structure of hadron.
- Quasi-GPDs on lattice are marginally more difficult than quasi-PDFs.
- With the help of LaMET
  - Improve global fit in parameter space which is difficult to measure.
  - Produce prediction on various distribution functions in parton physics before the experiments.
- For GPD, it is a great opportunity!
  One can obtain predictions ahead of experiments!!!

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