Matching for gluon and singlet quark quasi distributions in LaMET

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Outline

Recent PDFs results at Lattice

LP3 Collaboration, 1803.04393,1807.07431; Alexandrou et al,1803.02685

for unpolarized and polarized nonsinglet quark distributions:

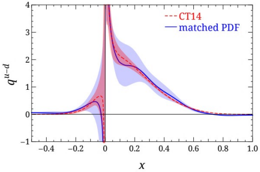


FIG. 4. Our final PDF renormalized at 3 GeV and compared with CT14 [63] at $(\mu_R, p_z^R) = (3.7, 2.2)$ GeV. It is consistent with NNPDF3.1 distribution [64] and CJ15 [65]. Our results agree nicely with the global-analysis PDF.

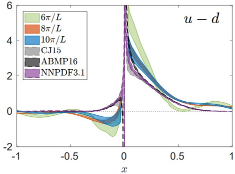
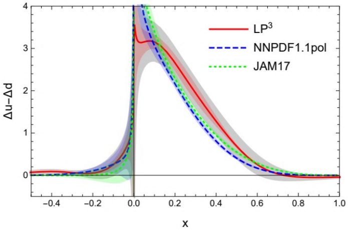
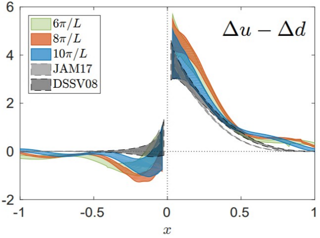


FIG. 4: Comparison of unpolarized PDF at momenta $\frac{6\pi}{L}$ (green band), $\frac{8\pi}{L}$ (orange band), $\frac{10\pi}{L}$ (blue band), and ABMP16 [39] (NNLO), NNPDF [40] (NNLO) and CJ15 [38] (NLO) phenomenological curves.





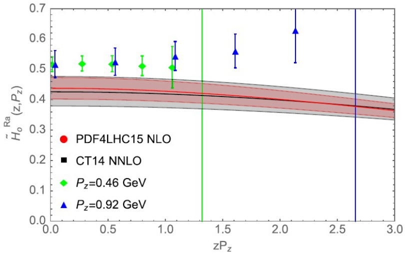
First results on gluon quasi distribution
Fan, Yang, et al., PRL121, 242001(2018)

for unpolarized gluon quasi distributions:

$$\tilde{g}\left(x, P_z^2, \mu\right) = \int \frac{\mathrm{d}z}{\pi x} e^{-ixzP_z} \tilde{H}_0^R\left(z, P_z, \mu\right)$$

where $\tilde{H}_0\left(z, P_z\right) = \langle P \left|\mathcal{O}_0(z)\right| P\rangle$ and $\mathcal{O}_0$ is defined by

$$\mathcal{O}_0 \equiv \frac{P_0\left(\mathcal{O}\left(F^t_\mu, F^{\mu t}; z\right) - \frac{1}{4} g^{tt} \mathcal{O}\left(F^\mu_\nu, F^\nu_\mu; z\right)\right)}{\frac{3}{4} P_0^2 + \frac{1}{4} P_z^2}$$

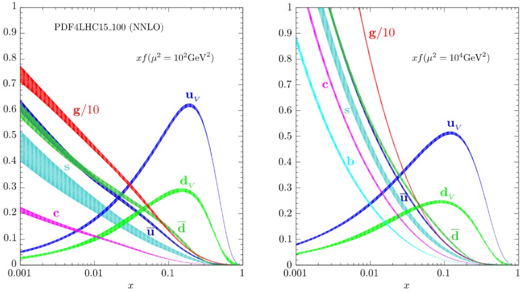


Gluon distribution functions

- play more important roles in higher energy hadron
- mixing with the singlet quark distribution functions.

$$f_{g/H}(x,\mu) = \int \frac{d\xi^-}{2\pi x P^+} e^{-ixP^+\xi^-} \langle P|F_a^{+i}(\xi^-)\mathcal{W}(\xi^-,0)F_a^{+i}(0)|P\rangle,$$

where $F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - g f_{abc} A_b^\mu A_c^\nu$.



Gluon quasi distribution functions

- first proposed by X.D. Ji, PRL110,262002(2013).

$$
\begin{aligned}
& x\tilde{f}_{g/H}(x,\mu^2,P^z) \\
&= \int \frac{dz}{2\pi P^z} e^{izxP^z} \langle P| G^z_{\ \mu}(z) \mathcal{P} \exp\left(-ig\int_0^z d\eta A^z(\eta)\right) G^{\mu z}(0)|P\rangle,
\end{aligned}
$$

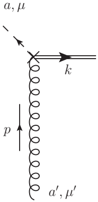
- Matching for light cone PDFs:

$$
\begin{aligned}
\tilde{f}_{i/H}(x,P^z) &= \int_0^1 \frac{dy}{y} C_{ij}\left(\frac{x}{y},\frac{\mu}{P^z}\right) f_{j/H}(y,\mu) \\
&= C_{ij}\left(\xi,\frac{\mu}{P^z}\right) \otimes f_{j/H}(y),
\end{aligned}
$$

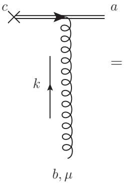
where $\xi \equiv x/y$, and i,j denotes quark q and gluon g.

Gluon quasi distribution functions

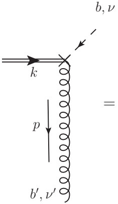
- Feynman Rules:
  $n^\mu = (0, 0, 0, -1)$ with $n^2 = -1$ for quasi distributions;
  $n^\mu = (1, -1, 0, 0)$ with $n^2 = 0$ for light cone distributions.



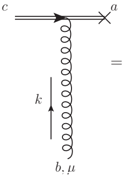
$$= i\left[n\cdot(p-k)\, g_{\perp\mu}^{\mu'} - p_{\perp\mu} n^{\mu'}\right]\delta^{a}_{a'}$$



$$= -ig f^{abc}\frac{n^\mu}{n\cdot k}$$



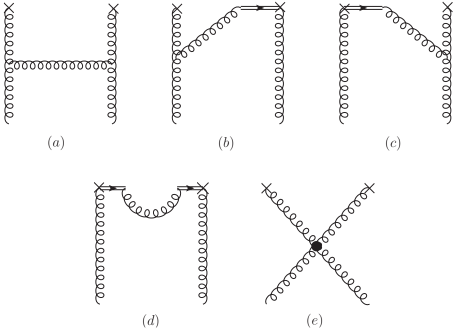
$$= -i\left[p_\perp^{\nu} n^{\nu'} - n\cdot(p-k)\, g_\perp^{\nu\nu'}\right]\delta^{b}_{b'}$$

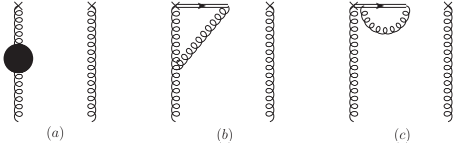


$$= ig f^{abc}\frac{n^\mu}{n\cdot k}$$

One Loop Diagrams

real and virtual corrections





Linear divergences at cut-off scheme

- Calculation scheme
  UV cut-off on transverse momemtum, IR regulator with small gluon mass, Feynman gauge
- A straightforward calculation yields the quasi PDF
- Extra linear divergences for both the self energy of gauge link and real diagrams

$$\tilde{f}_{g/g}(x,P^z)\Big|_{\mathrm{Fig.}(a)}$$
$$= -\frac{4i\pi\alpha_s C_A}{xP^z}\int\frac{dk^0}{(2\pi)^3}\int_0^\Lambda dk_\perp \frac{1}{\left(-(k^0)^2+k_\perp^2+m_g^2+x^2(P^z)^2\right)^2}$$
$$\times\frac{1}{\left(-(k^0)^2+2k^0\sqrt{m_g^2+(P^z)^2}+k_\perp^2+(P^z)^2x(x-2)\right)}$$
$$\times\left[x^2(P^z)^2\left(-(k^0)^2-2k^0\sqrt{m_g^2+(P^z)^2}-m_g^2+(P^z)^2x(x+2)\right)+2(k_\perp^2)^2+k_\perp^2(P^z)^2(5x^2+4)\right].$$

Linear divergences in the self energy of gauge link

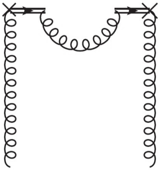
similar to the quark quasi distribution

Fig. (d) gives

$$\tilde{f}^{(1)}_{g/g}(x, P^z, \Lambda)\Big|_{\mathrm{Fig.}(d)} = \frac{\alpha_s C_A}{2\pi}\begin{cases} \frac{x}{1-x} + \frac{x}{(x-1)^2}\frac{\Lambda}{P^z}, & x > 1 \\ \frac{x}{x-1} + \frac{x}{(x-1)^2}\frac{\Lambda}{P^z}, & 0 < x < 1 \\ \frac{x}{x-1} + \frac{x}{(x-1)^2}\frac{\Lambda}{P^z}, & x < 0 \end{cases}$$

Due to

$$\int d^3k\, n^2 \frac{k^4}{k^6} \sim n^2 \frac{\Lambda}{P^z}.$$



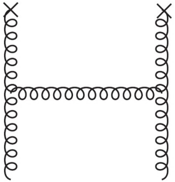
(d)

Linear divergences in the real diagrams

New, absent in quark case

For example, Fig. (a) gives

$$\tilde{f}^{(1)}_{g/g}(x, P^z, \Lambda)\Big|_{\mathrm{Fig.}\,real.(a)} = \frac{\alpha_s C_A}{2\pi x} \begin{cases} \left(2x^3 - 3x^2 + 2x - 2\right) \ln \frac{x-1}{x} & \\ +2x^2 - \frac{5x}{2} + \frac{8}{3} + \frac{3}{4}\frac{\Lambda}{P^z}\,, & x > 1 \\ \left(2x^3 - 3x^2 + 2x - 2\right) \ln \frac{(x^2-x+1)m_g^2}{4x(1-x)(P^z)^2} & \\ +\frac{1}{6}x\left(4x(8x-9) - \frac{9}{(x-1)x+1} + 42\right) & \\ -\frac{8}{3} + \frac{3}{4}\frac{\Lambda}{P^z}\,, & 0 < x < 1 \\ -\left(2x^3 - 3x^2 + 2x - 2\right) \ln \frac{x-1}{x} & \\ -2x^2 + \frac{5x}{2} - \frac{8}{3} + \frac{3}{4}\frac{\Lambda}{P^z}\,, & x < 0 \end{cases}$$



(a)

Cut Off scheme VS DR

- Cut off scheme breaks the gauge symmetry
  Linear divergences mixes with other operators
- Dimensional regularization scheme preserves gauge invariance
  Linear divergences (1/(D-3)) only exist in self energies of gauge link

Auxiliary field approach

Polyakov, NPB164(1980),171;Dorn, Fortschr.Phys.34(1986),11

- consider the renormalization of gauge link
- expand gauge link

$$W(x_i, x_f, C) = 1 + \sum_{k=1}^{\infty} \frac{(-ig)^k}{k!} P \int_C A_{\mu_1}(x_1) \cdots A_{\mu_k}(x_k)\, \mathrm{d}x_1^{\mu_1} \cdots dx_k^{\mu_k}$$

and

$$\begin{aligned} &\frac{(-ig)^k}{k!} \int_C P\left(A_{\mu_1}(x_1) \cdots A_{\mu_k}(x_k)\right) dx_1^{\mu_1} \cdots dx_k^{\mu_k} \\ =&(ig)^k \int_0^{\sigma_e} \theta(\sigma_k - \sigma_{k-1}) \cdots \theta(\sigma_2 - \sigma_1)\, T_{a_k} \cdots T_{a_1} A_{\mu_k}^{a_k}(x(\sigma_k)) \\ &\cdots A_{\mu_1}^{a_1}(x(\sigma_1))\, \dot{x}^{\mu_k}(\sigma_k) \cdots \dot{x}^{\mu_1}(\sigma_1)\, d\sigma_1 \cdots d\sigma_k \end{aligned}$$

- in auxiliary field approach language:
  A field $\mathcal{Q}(\sigma)$ is defined in one dimensional parameter space interacting with $A_\mu(x)$.

Auxiliary field approach

Polyakov, NPB164(1980),171;Dorn, Fortschr.Phys.34(1986),11

- in auxiliary field approach language

$$\begin{array}{lll} Q\text{ -propagator:} & \delta_{mn}\theta\left(\sigma_2-\sigma_1\right), & \delta_{a_0}\theta\left(\sigma_2-\sigma_1\right) \\ Q-A\text{ vertex:} & ig\dot{x}_\mu(\sigma)\left(T_a\right)_{mn}, & gf_{abc}\dot{x}_\mu(\sigma) \end{array} \quad (1)$$

The effective Lagrangian with an auxiliary adjoint "heavy quark" field (denoted as $\mathcal{Q}$) can be written as

$$\mathcal{L}=\mathcal{L}_{\mathrm{QCD}}+\overline{\mathcal{Q}}(x)(in\cdot D)\mathcal{Q}(x),$$

where $D_\mu=\partial_\mu+igt_aA_{a,\mu}$ is the covariant derivative in the adjoint representation.

Multiplicatively renormallization in auxiliary field approach

- Ji, Zhang, Zhao, PRL120,112001(2018)
- Ishikawa, Ma, Qiu, Yoshida, PRD96,094019(2017)
- Green, Jansen, Steffens, PRL121,022004(2018)
- Wang, Shuai, JHEP1805,142(2018)
- Li, Ma, Qiu, PRL122,062002(2019)
- Zhang, Ji, Schäfer, Wang, Zhao, PRL122,142001(2019)

The renormalization of gluon quasi operator is a little complicated

$$O_g^{\mu\nu}(z,0) = F^{\mu\alpha}(z)\mathcal{W}(z,0)F_{\alpha}^{\ \nu}(0) = J_1^{\mu\alpha}(z)\overline{J}_{1,\alpha}^{\ \nu}(0),$$

where

$$J_1^{\mu\alpha}(z_2) = F_a^{\mu\alpha}(z_2)\mathcal{Q}_a(z_2),\ \overline{J}_{1,\alpha}^{\ \nu}(z_1) = \overline{\mathcal{Q}}_b(z_1)F_{b,\alpha}^{\ \nu}(z_1).$$

Multiplicatively renormallization in auxiliary field approach

Zhang, Ji, Schäfer, Wang, Zhao, PRL122,142001(2019).

- $J_1^{\mu\nu}$ will mix with other operators

$$
\begin{aligned}
J_1^{\mu\nu} &= F_a^{\mu\nu}\mathcal{Q}_a, \\
J_2^{\mu\nu} &= n_\rho(F_a^{\mu\rho}n^\nu - F_a^{\nu\rho}n^\mu)\mathcal{Q}_a/n^2, \\
J_3^{\mu\nu} &= (-in^\mu A_a^\nu + in^\nu A_a^\mu)((in\cdot D - m)\mathcal{Q})_a/n^2,
\end{aligned}
$$

- Their renormalizations are

$$
\begin{pmatrix} J_{1,R}^{\mu\nu} \\ J_{2,R}^{\mu\nu} \\ J_{3,R}^{\mu\nu} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} & Z_{13} \\ 0 & Z_{22} & Z_{23} \\ 0 & 0 & Z_{33} \end{pmatrix} \begin{pmatrix} J_1^{\mu\nu} \\ J_2^{\mu\nu} \\ J_3^{\mu\nu} \end{pmatrix},
$$

Appropriate gluon quasi distribution operators

Zhang, Ji, Schäfer, Wang, Zhao, PRL122,142001(2019).

Only four kinds of multiplicatively renormalizable operators

$$
\begin{aligned}
O_g^{(1)}(z,0) &\equiv F^{ti}(z)\mathcal{W}(z,0)F_i{}^{t}(0),\\
O_g^{(2)}(z,0) &\equiv F^{zi}(z)\mathcal{W}(z,0)F_i{}^{z}(0),\\
O_g^{(3)}(z,0) &\equiv F^{ti}(z)\mathcal{W}(z,0)F_i{}^{z}(0),\\
O_g^{(4)}(z,0) &\equiv F^{z\mu}(z)\mathcal{W}(z,0)F_\mu{}^{z}(0),
\end{aligned}
$$

The renoramalized operators are:

$$
\begin{aligned}
O_{g,R}^{(1)}(z_2,z_1) &= Z_{11}^2 e^{\overline{\delta m}|z_2-z_1|}F^{ti}(z_2)\mathcal{W}(z_2,z_1)F_i{}^{t}(z_1)\\
O_{g,R}^{(2)}(z_2,z_1) &= Z_{22}^2 e^{\overline{\delta m}|z_2-z_1|}F^{zi}(z_2)\mathcal{W}(z_2,z_1)F_i{}^{z}(z_1)\\
O_{g,R}^{(3)}(z_2,z_1) &= Z_{11}Z_{22} e^{\overline{\delta m}|z_2-z_1|}F^{ti}(z_2)\mathcal{W}(z_2,z_1)F_i{}^{z}(z_1)\\
O_{g,R}^{(4)}(z_2,z_1) &= Z_{22}^2 e^{\overline{\delta m}|z_2-z_1|}F^{z\mu}(z_2)\mathcal{W}(z_2,z_1)F_\mu{}^{z}(z_1).
\end{aligned}
$$

Consider the gluon and quark mixing

$$\begin{pmatrix} O_g^{(n)}(z,0) \\ O_q^s(z,0) \end{pmatrix} = \begin{pmatrix} Z_{11}(z) & Z_{12}(z)/z \\ zZ_{21}(z) & Z_{22}(z) \end{pmatrix} \begin{pmatrix} O_{g,R}^{(n)}(z,0) \\ O_{q,R}^s(z,0) \end{pmatrix},$$

The renormalization factors in the above mixing matrix can be determined using the following renormalization conditions

$$\left.\frac{\mathrm{Tr}[\Lambda_{22}(p,z)\mathcal{P}]_R}{\mathrm{Tr}[\Lambda_{22}(p,z)\mathcal{P}]_{\mathrm{tree}}}\right|_{\substack{p^2=-\mu_R^2\\ p_z=p_z^R}} = 1, \quad \left.\frac{[P_{ij}^{ab}\Lambda_{11}^{ab,ij}(p,z)]_R}{[P_{ij}^{ab}\Lambda_{11}^{ab,ij}(p,z)]_{\mathrm{tree}}}\right|_{\substack{p^2=-\mu_R^2\\ p_z=p_z^R}} = 1,$$

$$\left.\mathrm{Tr}[\Lambda_{12}(p,z)\mathcal{P}]_R\right|_{\substack{p^2=-\mu_R^2\\ p_z=p_z^R}} = 0, \quad \left.[P_{ij}^{ab}\Lambda_{21}^{ab,ij}(p,z)]_R\right|_{\substack{p^2=-\mu_R^2\\ p_z=p_z^R}} = 0.$$

where $\Lambda_{\{11,12\}}$ ($\Lambda_{\{21,22\}}$) denote the amputated Green's functions of $O_g^{(n)}$ (O_q^s) in an offshell gluon and quark state, respectively. $\mathcal{P}$ and P_{ij}^{ab} are projection operators.

Factorization for gluon and singlet quark quasi PDFs

The formulae to extract the light cone gluon and quark distributions

$$\tilde{f}^{(n)}_{g/H}(x, P^z, p^R_z, \mu_R) = \int_{-1}^{1} \frac{dy}{|y|} \Big[C_{gg}\Big(\frac{x}{y}, \frac{\mu_R}{p^R_z}, \frac{yP^z}{\mu}, \frac{yP^z}{p^R_z}\Big) f_{g/H}(y, \mu)$$

$$+ C_{gq}\Big(\frac{x}{y}, \frac{\mu_R}{p^R_z}, \frac{yP^z}{\mu}, \frac{yP^z}{p^R_z}\Big) f_{q_j/H}(y, \mu)\Big] + \mathcal{O}\Big(\frac{M^2}{(P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(P^z)^2}\Big),$$

$$\tilde{f}_{q_i/H}(x, P^z, p^R_z, \mu_R) = \int_{-1}^{1} \frac{dy}{|y|} \Big[C_{q_i q_j}\Big(\frac{x}{y}, \frac{\mu_R}{p^R_z}, \frac{yP^z}{\mu}, \frac{yP^z}{p^R_z}\Big) f_{q_j/H}(y, \mu)$$

$$+ C_{qg}\Big(\frac{x}{y}, \frac{\mu_R}{p^R_z}, \frac{yP^z}{\mu}, \frac{yP^z}{p^R_z}\Big) f_{g/H}(y, \mu)\Big] + \mathcal{O}\Big(\frac{M^2}{(P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(P^z)^2}\Big),$$

where the scale p^R_z, μ_R are the renormalization scales in RI/MOM scheme.

But only one appropriate sort of gluon renormalizable operators in RI/MOM

Partonic quasi-PDF can be written as follows:

$$x\tilde{f}^{(n)}_{g/g}(x,\rho) = \left[x\tilde{f}^{(n)}_{g/g}(x,\rho)\right]_{+} + \tilde{c}^{(n)}\delta(x-1),$$

which corresponds to the matrix element of local operators

$$\tilde{c}^{(n)} = \frac{1}{p_z^2} N^{(n)} \langle g(p)|O_g^{(n)}(0,0)|g(p)\rangle.$$

$$\tilde{c}^{(1,g)} = \frac{\alpha_s C_A}{12\pi\epsilon}\frac{p^2}{p^2+p_z^2} + \mathcal{O}(\epsilon^0),$$

$$\tilde{c}^{(2,g)} = -\frac{\alpha_s C_A}{12\pi\epsilon}\frac{p^2}{p_z^2} + \mathcal{O}(\epsilon^0),$$

$$\tilde{c}^{(3,g)} = \mathcal{O}(\epsilon^0),$$

$$\tilde{c}^{(4,g)} = \frac{\alpha_s C_A}{3\pi\epsilon}\frac{p^2}{p_z^2} + \mathcal{O}(\epsilon^0),$$

One loop matching coefficient for gluon to gluon

The one-loop matching coefficient is given by the difference in the renormalized quasi-PDF and lightcone PDF

$$xC_{gg}^{(3,1)}(x, r, \frac{p_z}{\mu}, \frac{p_z}{p_z^R}) = \left[x\tilde{f}_{g/g}^{(3,1)}(x, \rho \to 0) - xf_{g/g}^{(1)}\left(x, \frac{\mu^2}{-p^2}\right) - (x\tilde{f}_{g/g}^{(3,1)})_{C.T.}\right]_+ + \left(\tilde{c}^{(3,g)}_{\mathrm{RI/MOM}} - c^{3,g}_{\overline{\mathrm{MS}}}\right)\delta(x-1),$$

where the $\ln(-p^2)$ dependence in each individual term cancels out in the combination on the r.h.s., and the counterterm in the RI/MOM scheme can be determined from the renormalization condition above as

$$(x\tilde{f}_{g/g}^{(3,1)})_{C.T.} = \left|\frac{p_z}{p_z^R}\right| x\tilde{f}_{g/g}^{(3,1)}\left(\frac{p_z}{p_z^R}(x-1)+1, \frac{\mu_R^2}{(p_z^R)^2}\right)$$

local terms:

$$\tilde{c}^{(3,g)}_{\mathrm{RI/MOM}} = 1 - \frac{\alpha_s T_f}{3\pi}\left(-\ln\frac{-p^2}{\mu_R^2}\right), \quad c^{3,g}_{\overline{\mathrm{MS}}} = 1 - \frac{\alpha_s T_f}{3\pi}\left(-\ln\frac{-p^2}{\mu^2}+\frac{5}{3}\right).$$

distributions in $p^2 \to 0$ limit

$$[x\tilde{f}^{(3,1)}_{g/g}(x,\rho\to 0)]_+ = \frac{\alpha_s C_A}{2\pi}\begin{cases}\left[\frac{2(1-x+x^2)^2}{x-1}\ln\frac{x-1}{x}+\frac{4x^3-6x^2+8x-5}{2(x-1)}\right]_+, & x>1\\ \left[\frac{2(1-x+x^2)^2}{x-1}\ln\frac{\rho}{4}+\frac{12x^4-24x^3+30x^2-17x+5}{2(x-1)}\right]_+, & 0<x<1\\ \left[-\frac{2(1-x+x^2)^2}{x-1}\ln\frac{x-1}{x}-\frac{4x^3-6x^2+8x-5}{2(x-1)}\right]_+, & x<0.\end{cases}$$

$$\left[xf^{(1)}_{g/g}\left(x,\frac{\mu^2}{-p^2}\right)\right]_+ = \theta(x)\theta(1-x)\left\{\frac{\alpha_s C_A}{2\pi}\left[\frac{2(1-x+x^2)^2}{x-1}\ln\frac{-p^2x(1-x)}{\mu^2}+2x^3-2x^2+3x-2\right]_+ - \frac{\alpha_s C_A}{4\pi}\left[\frac{x}{1-x}\right]_+\right\},$$

Result2

Counter term in RI/MOM scheme

$$\left[x\tilde{f}^{(3,1)}_{g/g}(x,\rho)\right]_+ \frac{2\pi}{\alpha_s C_A}\Big|_{\substack{p^2 = -\mu_R^2 \\ p_z = p_z^R}} =$$

$$\begin{cases} \left[\frac{-(\rho-4)^2(\rho-1)+8(\rho+2)x^4-16(\rho+2)x^3-2\left(\rho^2+8\rho-24\right)x^2+\left(6\rho^2+20\rho-32\right)x}{8(\rho-1)^2(x-1)}\frac{1}{\sqrt{1-\rho}}\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}} \right. \\ \left. +\frac{4x^3}{(2x-1)\left(\rho+4x^2-4x\right)}+\frac{8x^4-16x^3-22x^2+34x-9}{4(\rho-1)(x-1)(2x-1)}-\frac{8x^3(x-1)}{\left(\rho+4x^2-4x\right)^2}+\frac{3(2x-1)x}{2(\rho-1)^2}-\frac{4x+1}{4(x-1)}\right]_+, \\ \left[\frac{-(\rho-4)^2(\rho-1)+8(\rho+2)x^4-16(\rho+2)x^3-2\left(\rho^2+8\rho-24\right)x^2+\left(6\rho^2+20\rho-32\right)x}{8(\rho-1)^2(x-1)}\frac{1}{\sqrt{1-\rho}}\ln\frac{1-\sqrt{1-\rho}}{1+\sqrt{1-\rho}} \right. \\ \left. +\frac{-30x^2+34x-9}{4(\rho-1)(x-1)}+\frac{3\left(4x^3-4x^2+x\right)}{2(\rho-1)^2}+\frac{6x+1}{4(x-1)}\right]_+, \\ \left[-\frac{-(\rho-4)^2(\rho-1)+8(\rho+2)x^4-16(\rho+2)x^3-2\left(\rho^2+8\rho-24\right)x^2+\left(6\rho^2+20\rho-32\right)x}{8(\rho-1)^2(x-1)}\frac{1}{\sqrt{1-\rho}}\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}} \right. \\ \left. -\frac{4x^3}{(2x-1)\left(\rho+4x^2-4x\right)}+\frac{-8x^4+16x^3+22x^2-34x+9}{4(\rho-1)(x-1)(2x-1)}+\frac{8x^3(x-1)}{\left(\rho+4x^2-4x\right)^2}-\frac{3(2x-1)x}{2(\rho-1)^2}+\frac{4x+1}{4(x-1)}\right]_+, \end{cases}$$

where $\rho = -p^2/pz^2$.

One loop matching coefficient for gluon in quark

The one-loop matching coefficient is

$$xC^{(3,1)}_{g/q}\left(x, r, \frac{p_z}{\mu}, \frac{p_z}{p_z^R}\right) = \left[x\tilde{f}^{(3,1)}_{g/q}(x, \rho \to 0) - xf^{(1)}_{g/q}\left(x, \frac{\mu^2}{-p^2}\right) - (x\tilde{f}^{(3,1)}_{g/q})_{C.T.}\right],$$

The renormalized mixing contribution in lightcone

$$xf^{(1)}_{g/q}\left(x, \frac{\mu^2}{-p^2}\right) = \frac{\alpha_s C_F}{2\pi}\left[(1+(1-x)^2)\ln\frac{\mu^2}{-p^2 x(1-x)} + x(1-x) - 2\right].$$

For the quasi-PDF,

$$x\tilde{f}^{(3,1)}_{g/q}(x, \mu, P^z) = \frac{\alpha_s C_F}{2\pi}\begin{cases} -\frac{5\rho^2-10\rho+(8\rho+4)x^2-4(\rho+2)x+8}{4(\rho-1)^2}\frac{1}{\sqrt{1-\rho}}\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}} \\ -\frac{(\rho-4)\rho+8(2\rho+1)x^3-4\left(\rho^2+2\rho+6\right)x^2+2\left(3\rho^2-2\rho+8\right)x}{2(1-\rho)^2\left(\rho+4x^2-4x\right)}, & x>1 \\ -\frac{5\rho^2-10\rho+(8\rho+4)x^2-4(\rho+2)x+8}{4(\rho-1)^2}\frac{1}{\sqrt{1-\rho}}\ln\frac{1-\sqrt{1-\rho}}{1+\sqrt{1-\rho}} \\ -\frac{(2x-1)(\rho+2(\rho+2)x-4)}{2(1-\rho)^2}, & 0<x<1 \\ \frac{5\rho^2-10\rho+(8\rho+4)x^2-4(\rho+2)x+8}{4(\rho-1)^2}\frac{1}{\sqrt{1-\rho}}\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}} \\ +\frac{(\rho-4)\rho+8(2\rho+1)x^3-4\left(\rho^2+2\rho+6\right)x^2+2\left(3\rho^2-2\rho+8\right)x}{2(1-\rho)^2\left(\rho+4x^2-4x\right)}, & x<0. \end{cases}$$

$$(x\tilde{f}^{(3,1)}_{g/q})_{C.T.} = \left|\frac{p_z}{p_z^R}\right| x\tilde{f}^{(3,1)}_{g/q}\left(\frac{p_z}{p_z^R}(x-1)+1, \frac{\mu_R^2}{(p_z^R)^2}\right).$$

One loop matching coefficient for quark in gluon

The one-loop matching coefficient is

$$C_{qg}^{(1)}\left(x, r, \frac{p_z}{\mu}, \frac{p_z}{p_z^R}\right) = \left[\tilde{f}_{q/g}^{(1)}(x, \rho \to 0) - f_{q/g}^{(1)}\left(x, \frac{\mu^2}{-p^2}\right) - (\tilde{f}_{q/g}^{(1)})_{C.T.}\right],$$

where

$$f_{q/g}^{(1)}\left(x, \frac{\mu^2}{-p^2}\right) = \frac{\alpha_s T_f}{2\pi}\left[(x^2 + (1-x)^2)\ln\frac{\mu^2}{-p^2 x(1-x)} - 1\right],$$

$$\tilde{f}_{q/g}^{(1)}(x, \rho)$$

$$= \frac{\alpha_s T_f}{2\pi}\begin{cases} -\frac{\rho^2-2\rho+4(\rho+2)x^2-4(\rho+2)x+4}{4(1-\rho)^{3/2}}\frac{1}{\sqrt{1-\rho}}\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}} - \frac{(2x-1)\left(-(\rho-4)\rho+4(\rho+2)x^2-4(\rho+2)x\right)}{2(1-\rho)^{3/2}\left(\rho+4x^2-4x\right)}; \\ -\frac{\rho^2-2\rho+4(\rho+2)x^2-4(\rho+2)x+4}{4(1-\rho)^{3/2}}\frac{1}{\sqrt{1-\rho}}\ln\frac{1-\sqrt{1-\rho}}{1+\sqrt{1-\rho}} - \frac{-\rho+12x^2-12x+4}{2(1-\rho)^{3/2}}, \\ \frac{\rho^2-2\rho+4(\rho+2)x^2-4(\rho+2)x+4}{4(1-\rho)^{3/2}}\frac{1}{\sqrt{1-\rho}}\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}} - \frac{(2x-1)\left((\rho-4)\rho-4(\rho+2)x^2+4(\rho+2)x\right)}{2(1-\rho)^{3/2}\left(\rho+4x^2-4x\right)}, \end{cases}$$

$$(\tilde{f}_{q/g}^{(1)})_{C.T.} = \left|\frac{p_z}{p_z^R}\right|\tilde{f}_{q/g}^{(1)}\left(\frac{p_z}{p_z^R}(x-1)+1, \frac{\mu_R^2}{(p_z^R)^2}\right).$$

One-loop matching for polarized quasi-PDFs in RI/MOM scheme

To study the polarized gluon PDF

$$\Delta f_{g/H}(x,\mu) = i\epsilon_{\perp ij}\int \frac{d\xi^-}{2\pi x P^+} e^{-i\xi^- xP^+} \langle P|F^{+i}(\xi^- n_+)\mathcal{W}(\xi^- n_+, 0; L_{n_+})F^{j+}(0)|P\rangle,$$

We may use the following three operators to define the corresponding quasi-PDF

$$\Delta O_g^1(z,0) = i\epsilon_{\perp,ij}F^{ti}(z_2)\mathcal{W}(z_2,z_1)F^{tj}(z_1),$$
$$\Delta O_g^2(z,0) = i\epsilon_{\perp,ij}F^{zi}(z_2)\mathcal{W}(z_2,z_1)F^{zj}(z_1),$$
$$\Delta O_g^3(z,0) = i\epsilon_{\perp,ij}F^{ti}(z_2)\mathcal{W}(z_2,z_1)F^{zj}(z_1),$$

In RI/MOM, only $\Delta O_g^3(z,0)$ is appropriate.

One loop matching coefficient
the virtual contribution is same as unpolarized case

The one-loop matching coefficients are

$$x\Delta C_{gg}^{(3,1)}\left(x, r, \frac{p_z}{\mu}, \frac{p_z}{p_z^R}\right) = xC_{gg}^{(3,1)}\left(x, r, \frac{p_z}{\mu}, \frac{p_z}{p_z^R}\right) + \Big[\Big(x\Delta\tilde{f}_{g/g}^{(3,1)}(x, \rho \to 0) - x\tilde{f}_{g/g}^{(3,1)}(x, \rho \to 0)\Big) - \left(x\Delta f_{g/g}^{(3,1)}\left(x, \frac{\mu^2}{-p^2}\right) - xf_{g/g}^{(3,1)}\left(x, \frac{\mu^2}{-p^2}\right)\right) - (x\Delta\tilde{f}_{g/g}^{(3,1)})_{C.T.}\Big],$$

$$\Delta C_{qq}^{(1)}(x, r, \frac{p_z}{\mu}, \frac{p_z}{p_z^R}) = \left[\Delta\tilde{f}_{q/q,z}^{(1)}(x, \rho \to 0) - \Delta f_{q/q}^{(1)}\left(x, \frac{\mu^2}{-p^2}\right) - (\Delta\tilde{f}_{q/q}^{(1)})_{C.T.}\right]_+,$$

$$x\Delta C_{g/q}^{(3,1)}(x, r, \frac{p_z}{\mu}, \frac{p_z}{p_z^R}) = \left[x\Delta\tilde{f}_{g/q}^{(3,1)}(x, \rho \to 0) - x\Delta f_{g/q}^{(1)}\left(x, \frac{\mu^2}{-p^2}\right) - (x\Delta\tilde{f}_{g/q}^{(3,1)})_{C.T.}\right],$$

$$\Delta C_{qg}^{(1)}(x, r, \frac{p_z}{\mu}, \frac{p_z}{p_z^R}) = \left[\Delta\tilde{f}_{q/g}^{(1)}(x, \rho \to 0) - \Delta f_{q/g}^{(1)}\left(x, \frac{\mu^2}{-p^2}\right) - (\Delta\tilde{f}_{q/g}^{(1)})_{C.T.}\right],$$

Summary

- Multiplicatively renormalizable formulaes exist in gluon and singlet quark quasi distributions functions
- Only one sort of multiplicatively renormalizable formulae exists in RI/MONM
- One loop matching coefficients of gluon and singlet quark quasi distributions are obtained.

- Outlook
  - How to improve the accuracy of gluon quasi distributions at lattice?
  - How to improve the theoretical accuracy? NNLO?

Thank You!

Renormalization detail

The degeneracy of $J_2^{z\mu}$ and $J_1^{z\mu}$, which leads to

$$Z_{11} + Z_{12} = Z_{22}, \quad Z_{13} = Z_{23}.$$

As a result, the renormalization pattern can be simplified to

$$\begin{pmatrix} J_{1,R}^{z\mu} \\ J_{3,R}^{z\mu} \end{pmatrix} = \begin{pmatrix} Z_{22} & Z_{13} \\ 0 & Z_{33} \end{pmatrix} \begin{pmatrix} J_1^{z\mu} \\ J_3^{z\mu} \end{pmatrix}, J_{1,R}^{ti} = Z_{11} J_1^{ti}, J_{1,R}^{ij} = Z_{11} J_1^{ij}.$$

The reason that (J_1^{ti}, J_1^{ij}) and $J_1^{z\mu}$ have different renormalizations.

Divergences at D=4 and D=3

The one-loop diagrams that give rise to linearly divergent contributions to the operator $J_1^{\rho\nu}$, where the linear divergences cancel.

$$I_1^{\rho\nu} = \frac{\alpha_s C_A}{\pi}\left\{\frac{1}{d-4}(A_a^\nu n^\rho - A_a^\rho n^\nu) n\cdot\partial \mathcal{Q}_a/n^2 + \frac{\pi\mu}{d-3}(n^\rho A_a^\nu - n^\nu A_a^\rho)\mathcal{Q}_a + reg.\right\},$$

$$I_2^{\rho\nu} = \frac{\alpha_s C_A}{\pi}\left\{\frac{1}{d-4}\left[\frac{1}{4}F_a^{\rho\nu}\mathcal{Q}_a + \frac{1}{2}(F_a^{\rho\sigma} n_\nu n_\sigma - F_a^{\nu\sigma} n_\rho n_\sigma)\mathcal{Q}_a/n^2 + \frac{1}{2}(A_a^\rho n^\nu - A_a^\nu n^\rho) n\cdot\partial\mathcal{Q}_a/n^2\right] - \frac{\pi\mu}{d-3}(n^\rho A_a^\nu - n^\nu A_a^\rho)\mathcal{Q}_a + reg.\right\},$$