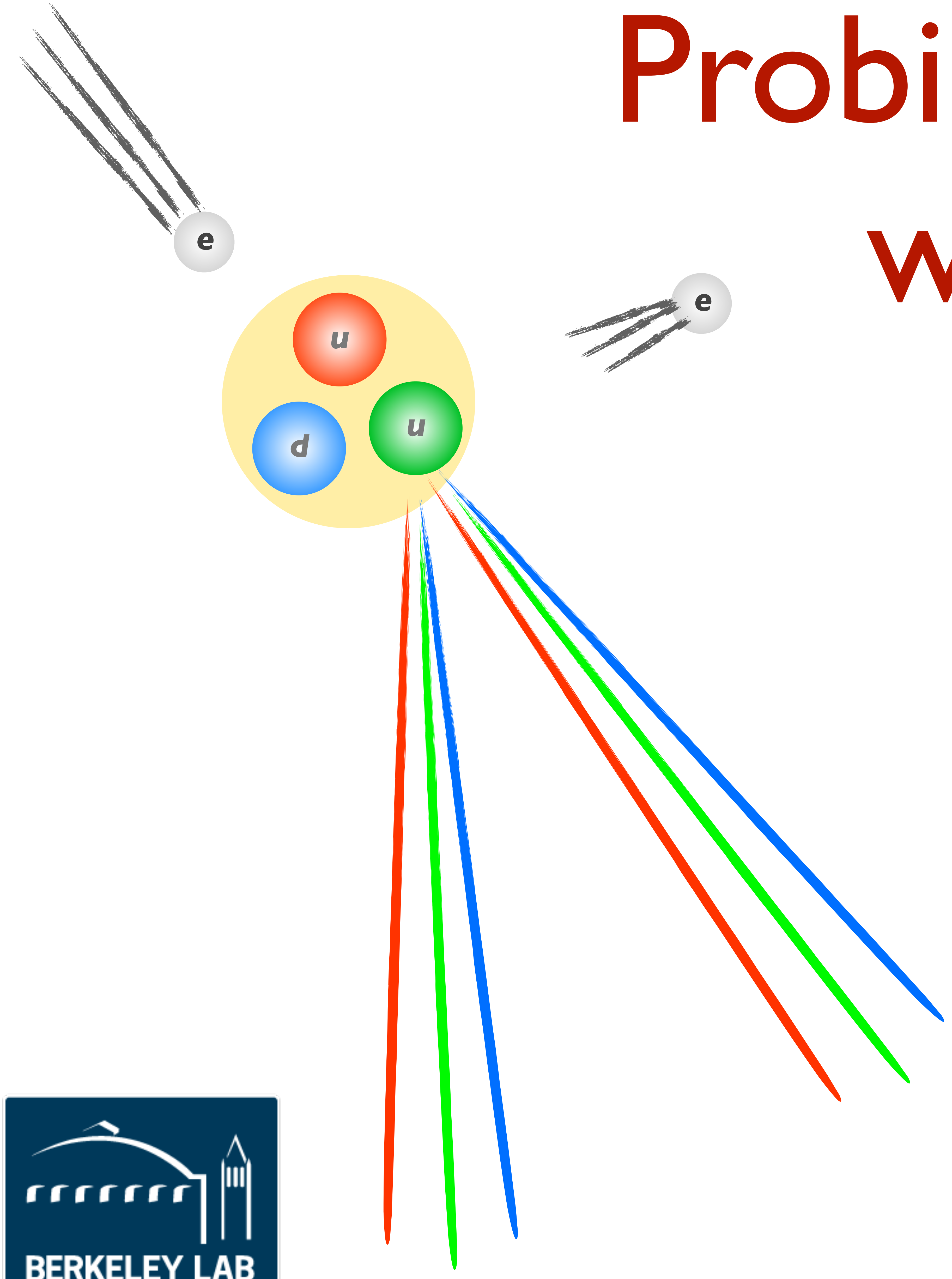

Probing the strong interaction with DIS event shapes

Christopher Lee, LANL

in collaboration with
D. Kang, I. Stewart and Y. Zhao

Jet Physics @ EIC workshop
at POETIC 2019, LBL
September 16, 2019

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Formation of Jets in QCD

Perturbative soft and collinear splittings happen at intermediate time

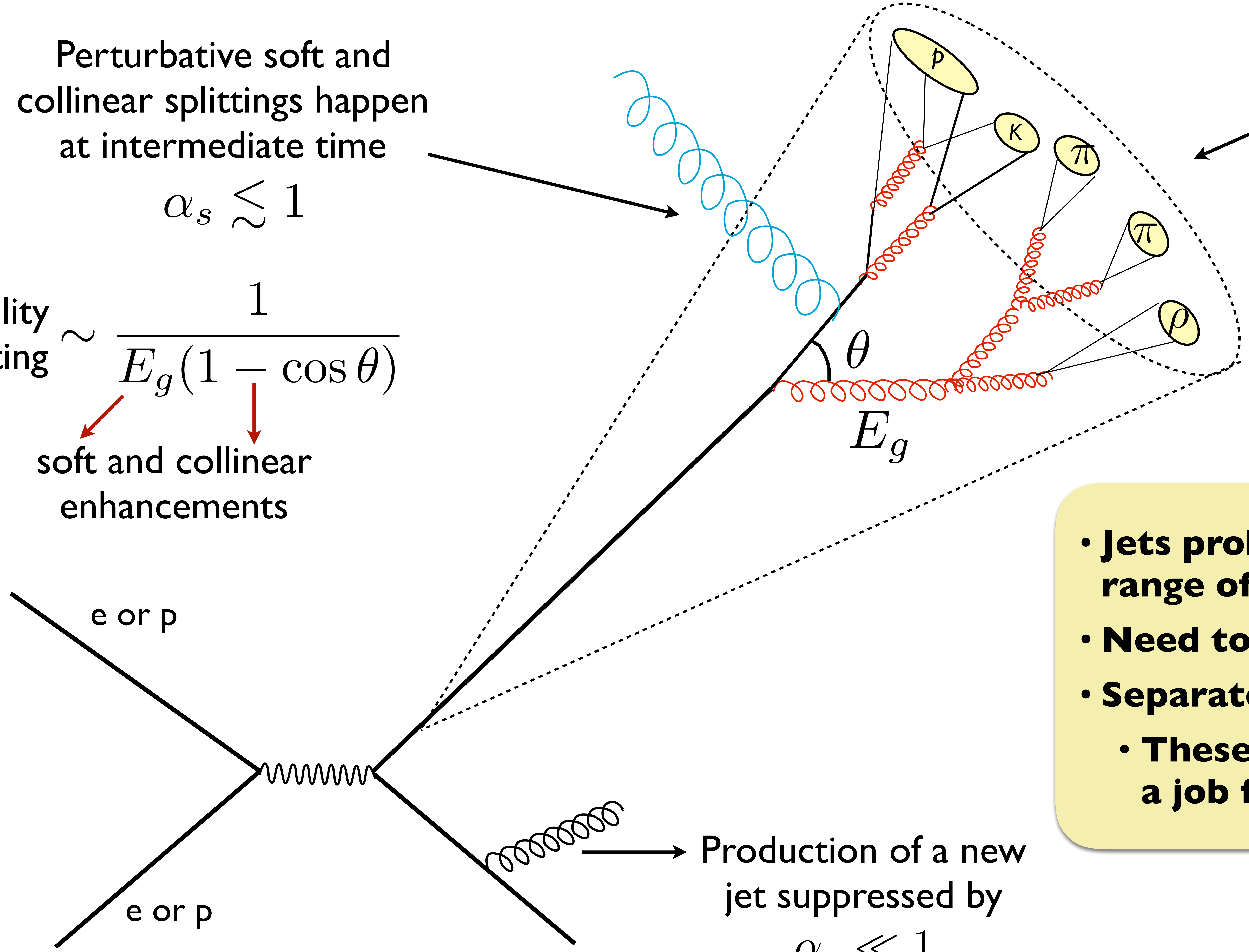
$\alpha_s \lesssim 1$

probability of splitting $\sim \dfrac{1}{E_g(1-\cos\theta)}$

soft and collinear enhancements

Hadronization at late time at low energy scale

$\alpha_s \gg 1$



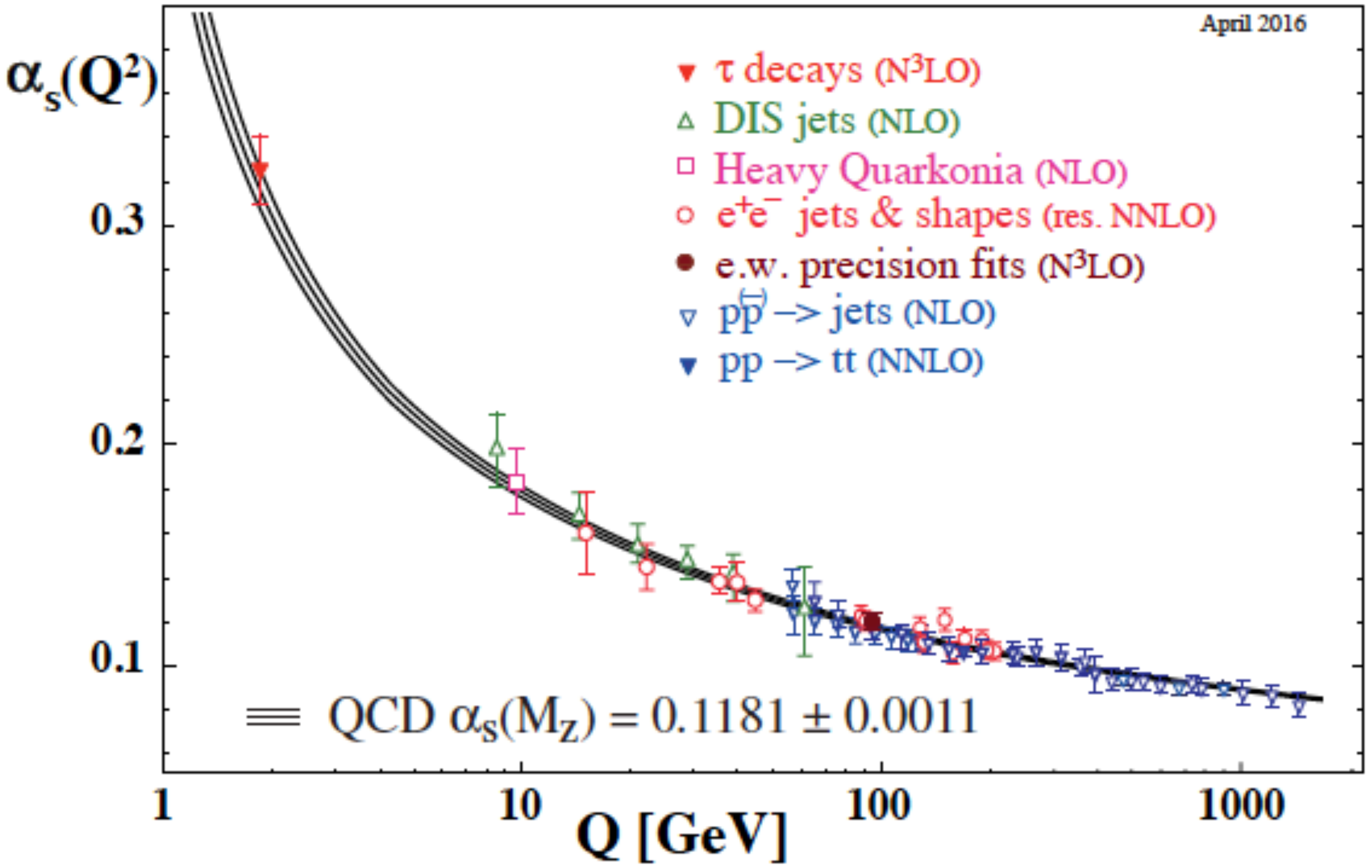
Production of a new jet suppressed by

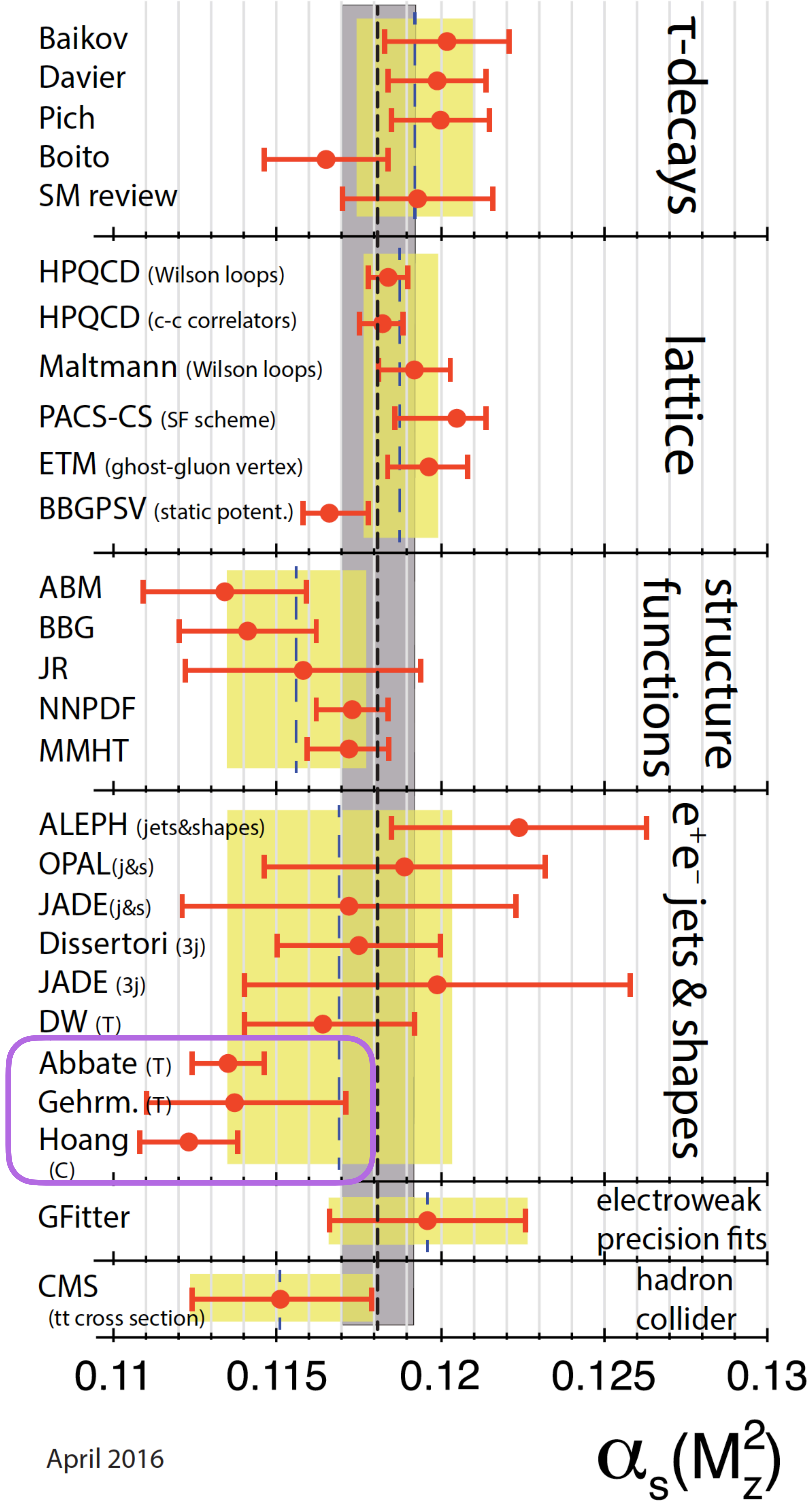
$\alpha_s \ll 1$

- **Jets probe strong interaction over wide range of scales**
- **Need to resum large perturbative logs**
- **Separate pert. and non-pert. physics**
  - **These are problems of *scale separation:* a job for EFT**

Event shapes and the strong coupling

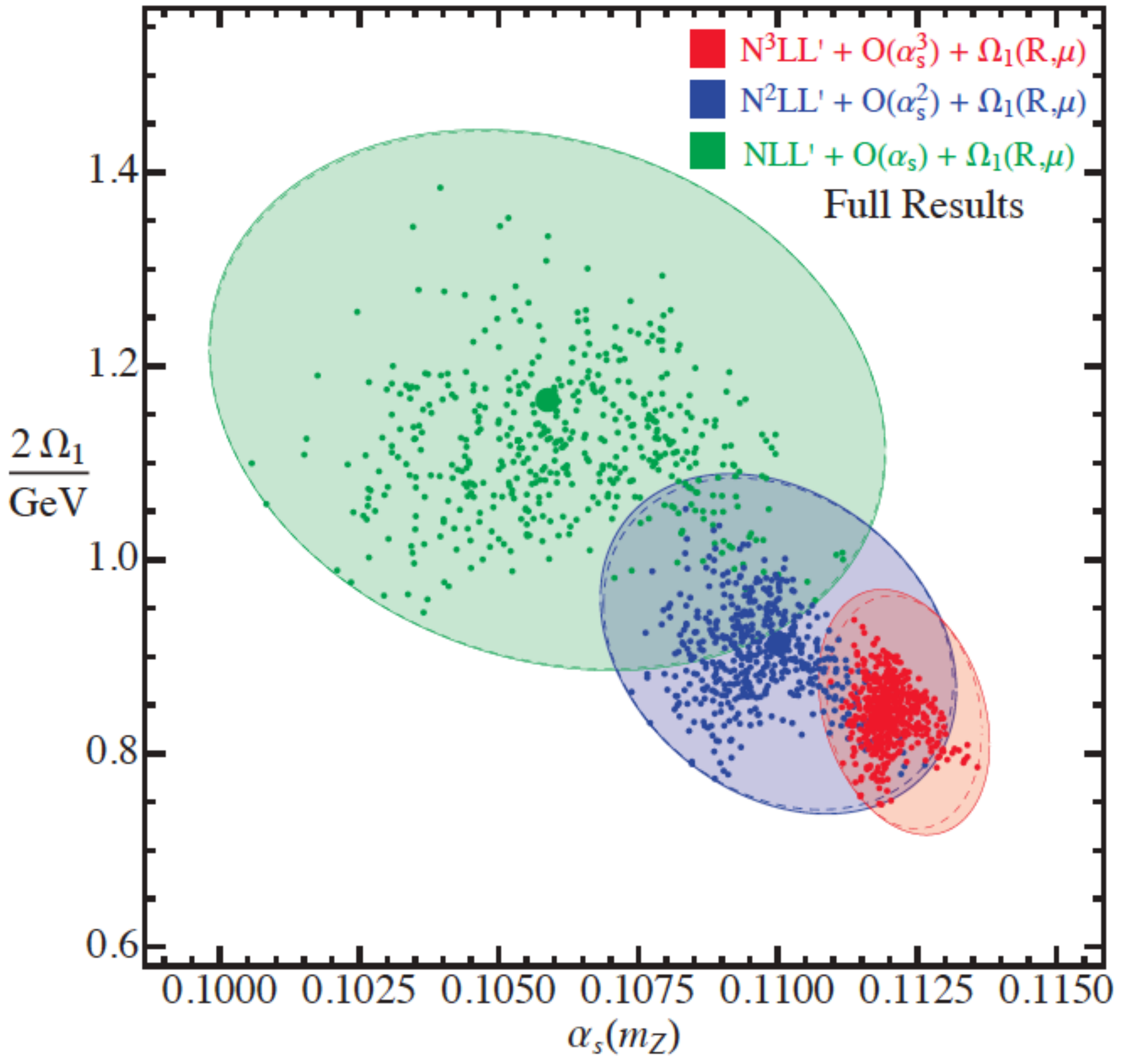
PDG 2016:

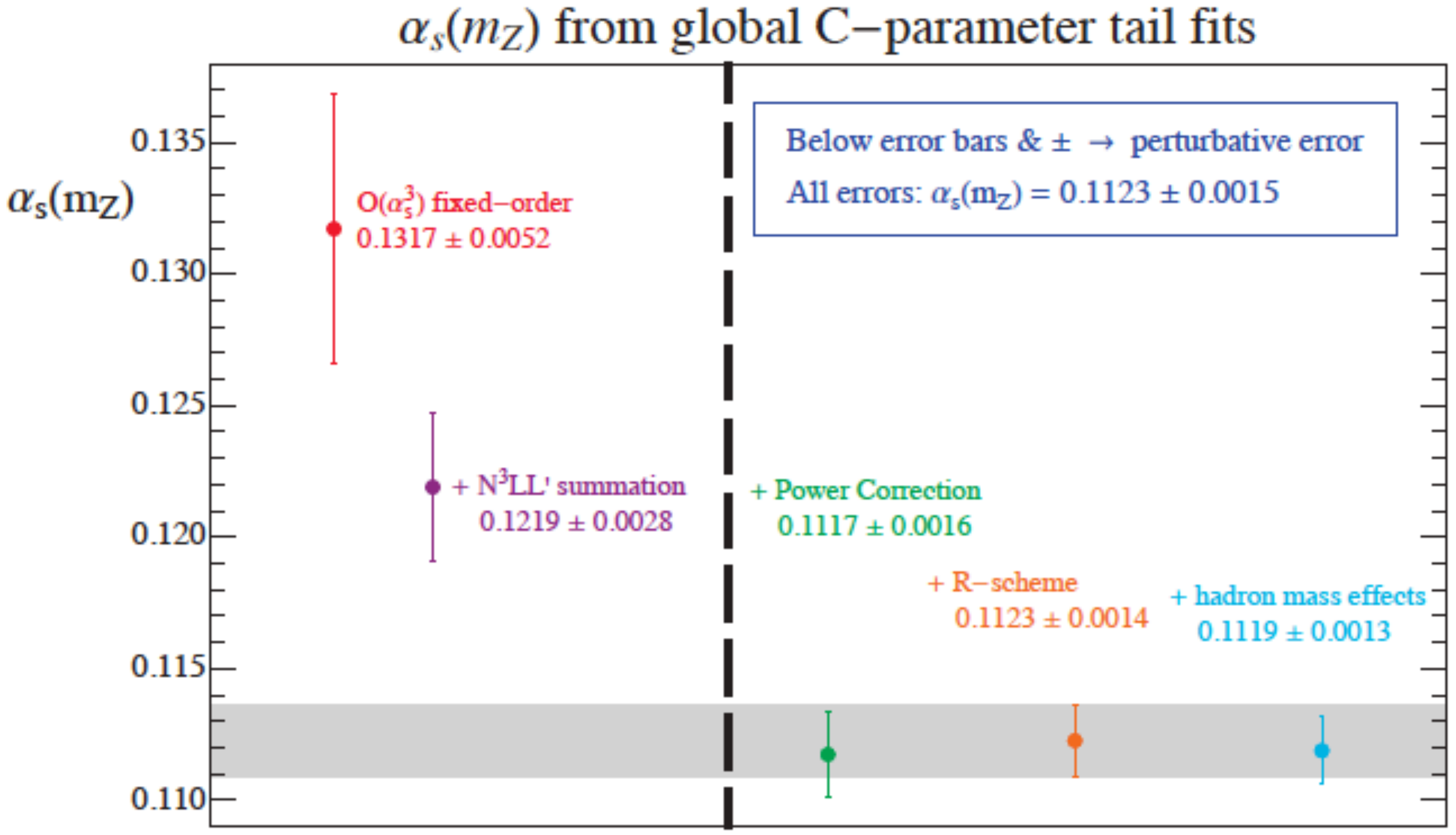




Event shapes

Hoang et al. , PRD 91 (2015) 094018:





Jets in DIS and the strong coupling

Process	Collab.	Value	Exp.	Th.	Total	(%)
(1) Inc. jets at low Q^2	H1	0.1180	0.0018	$^{+0.0124}_{-0.0093}$	$^{+0.0125}_{-0.0095}$	$^{+10.6}_{-8.1}$
(2) Dijets at low Q^2	H1	0.1155	0.0018	$^{+0.0124}_{-0.0093}$	$^{+0.0125}_{-0.0095}$	$^{+10.8}_{-8.2}$
(3) Trijets at low Q^2	H1	0.1170	0.0017	$^{+0.0091}_{-0.0073}$	$^{+0.0093}_{-0.0075}$	$^{+7.9}_{-6.4}$
(4) Combined low Q^2	H1	0.1160	0.0014	$^{+0.0094}_{-0.0079}$	$^{+0.0095}_{-0.0080}$	$^{+8.2}_{-6.9}$
(5) Trijet/dijet at low Q^2	H1	0.1215	0.0032	$^{+0.0067}_{-0.0059}$	$^{+0.0074}_{-0.0067}$	$^{+6.1}_{-5.5}$
(6) Inc. jets at medium Q^2	H1	0.1195	0.0010	$^{+0.0052}_{-0.0040}$	$^{+0.0053}_{-0.0041}$	$^{+4.4}_{-3.4}$
(7) Dijets at medium Q^2	H1	0.1155	0.0009	$^{+0.0045}_{-0.0035}$	$^{+0.0046}_{-0.0036}$	$^{+4.0}_{-3.1}$
(8) Trijets at medium Q^2	H1	0.1172	0.0013	$^{+0.0053}_{-0.0032}$	$^{+0.0055}_{-0.0035}$	$^{+4.7}_{-3.0}$
(9) Combined medium Q^2	H1	0.1168	0.0007	$^{+0.0049}_{-0.0034}$	$^{+0.0049}_{-0.0035}$	$^{+4.2}_{-3.0}$
(10) Inc. jets at high Q^2 (anti-k_T)	ZEUS	0.1188	$^{+0.0036}_{-0.0035}$	$^{+0.0022}_{-0.0022}$	$^{+0.0042}_{-0.0041}$	$^{+3.5}_{-3.5}$
(11) Inc. jets at high Q^2 (SIScone)	ZEUS	0.1186	$^{+0.0036}_{-0.0035}$	$^{+0.0025}_{-0.0025}$	$^{+0.0044}_{-0.0043}$	$^{+3.7}_{-3.6}$
(12) Inc. jets at high Q^2 (k_T; HERA I)	ZEUS	0.1207	$^{+0.0038}_{-0.0036}$	$^{+0.0022}_{-0.0023}$	$^{+0.0044}_{-0.0043}$	$^{+3.6}_{-3.6}$
(13) Inc. jets at high Q^2 (k_T; HERA II)	ZEUS	0.1208	$^{+0.0037}_{-0.0032}$	$^{+0.0022}_{-0.0022}$	$^{+0.0043}_{-0.0039}$	$^{+3.6}_{-3.2}$
(14) Inc. jets in γp (anti-k_T)	ZEUS	0.1200	$^{+0.0024}_{-0.0023}$	$^{+0.0043}_{-0.0032}$	$^{+0.0049}_{-0.0039}$	$^{+4.1}_{-3.3}$
(15) Inc. jets in γp (SIScone)	ZEUS	0.1199	$^{+0.0022}_{-0.0022}$	$^{+0.0047}_{-0.0042}$	$^{+0.0052}_{-0.0047}$	$^{+4.3}_{-3.9}$
(16) Inc. jets in γp (k_T)	ZEUS	0.1208	$^{+0.0024}_{-0.0023}$	$^{+0.0044}_{-0.0033}$	$^{+0.0050}_{-0.0040}$	$^{+4.1}_{-3.3}$
(17) Jet shape	ZEUS	0.1176	$^{+0.0013}_{-0.0028}$	$^{+0.0091}_{-0.0072}$	$^{+0.0092}_{-0.0077}$	$^{+7.8}_{-6.5}$
(18) Subjet multiplicity	ZEUS	0.1187	$^{+0.0029}_{-0.0019}$	$^{+0.0093}_{-0.0076}$	$^{+0.0097}_{-0.0078}$	$^{+8.2}_{-6.6}$
HERA average 2004		0.1186	± 0.0011	± 0.0050	± 0.0051	± 4.3
HERA average 2007		0.1198	± 0.0019	± 0.0026	± 0.0032	± 2.7

Table 1: Values of $\alpha_s(M_Z)$ extracted from jet observables at HERA together with their uncertainties (rows 1 to 18). The 2004 [10] and 2007 [11] HERA averages are shown in the last two rows.

Extractions from exclusive jet cross sections have order 10% uncertainty, dominated by theory

Improve to level of e^+e^-?

C. Glasman, in the Proceedings of the Workshop on Precision Measurements of α_s [1110.0016]

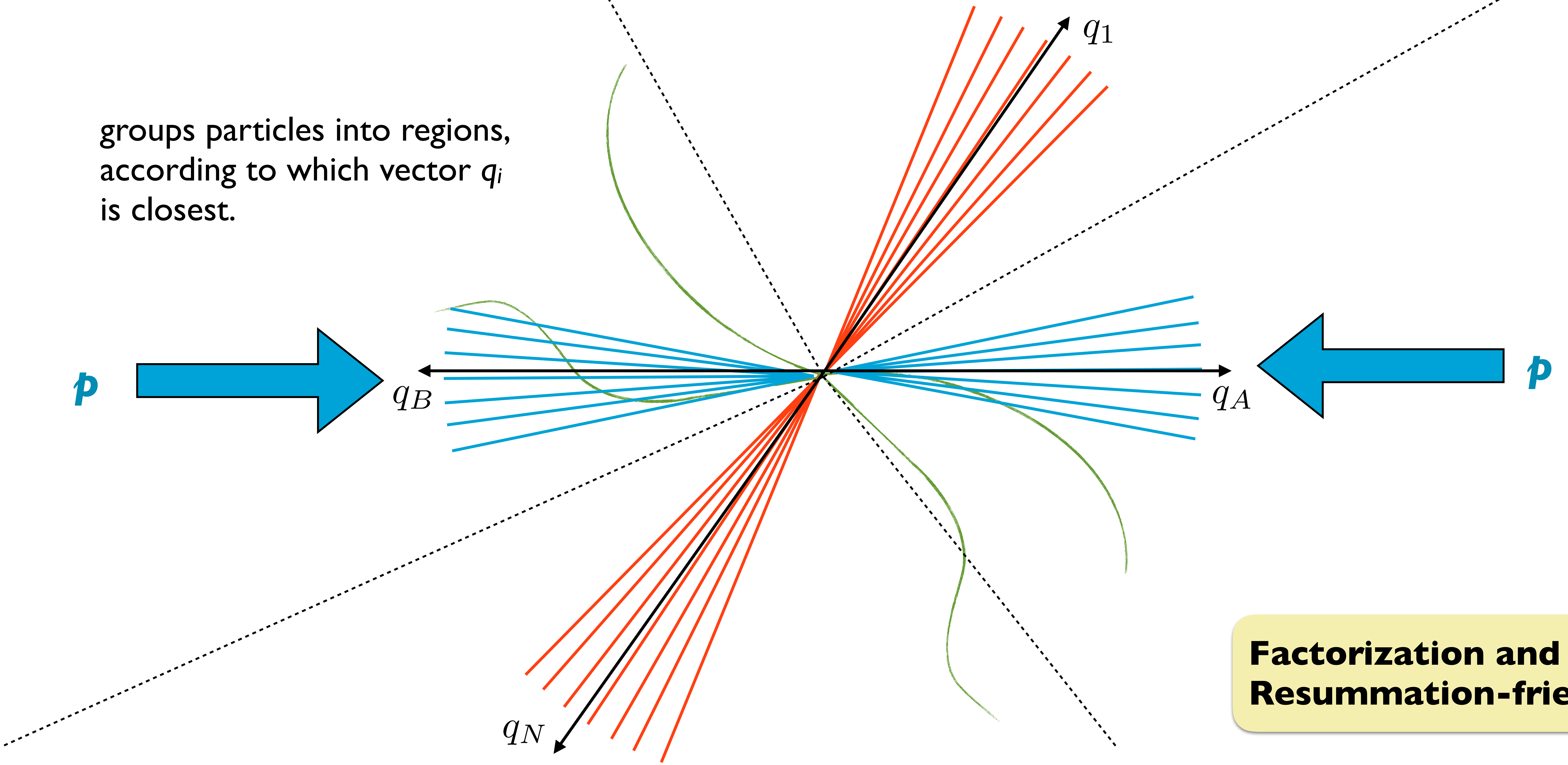
N-jettiness

Stewart, Tackmann, Waalewijn (2010)

- A global event shape measuring degree to which final state is N-jet-like.

$$\tau_N = \frac{2}{Q^2}\sum_k \min\{\underbrace{q_A\cdot p_k, q_B\cdot p_k}_{\#\text{ beams}}, \underbrace{q_1\cdot p_k, \ldots, q_N\cdot p_k}_{\#\text{ jets}}\}$$

groups particles into regions, according to which vector q_i is closest.

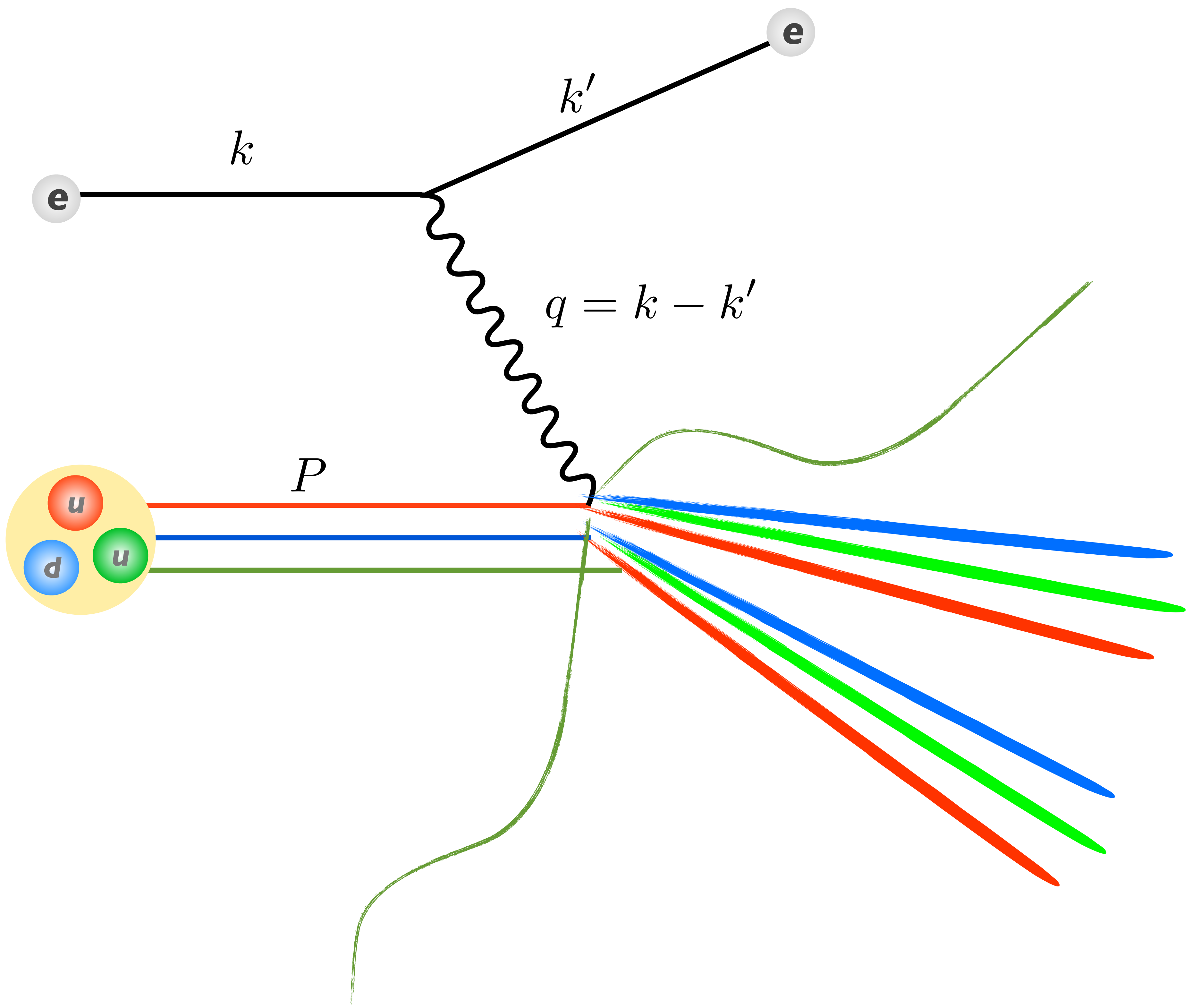


Factorization and Resummation-friendly

1-Jettiness in DIS

D. Kang, CL, I. Stewart (2013)

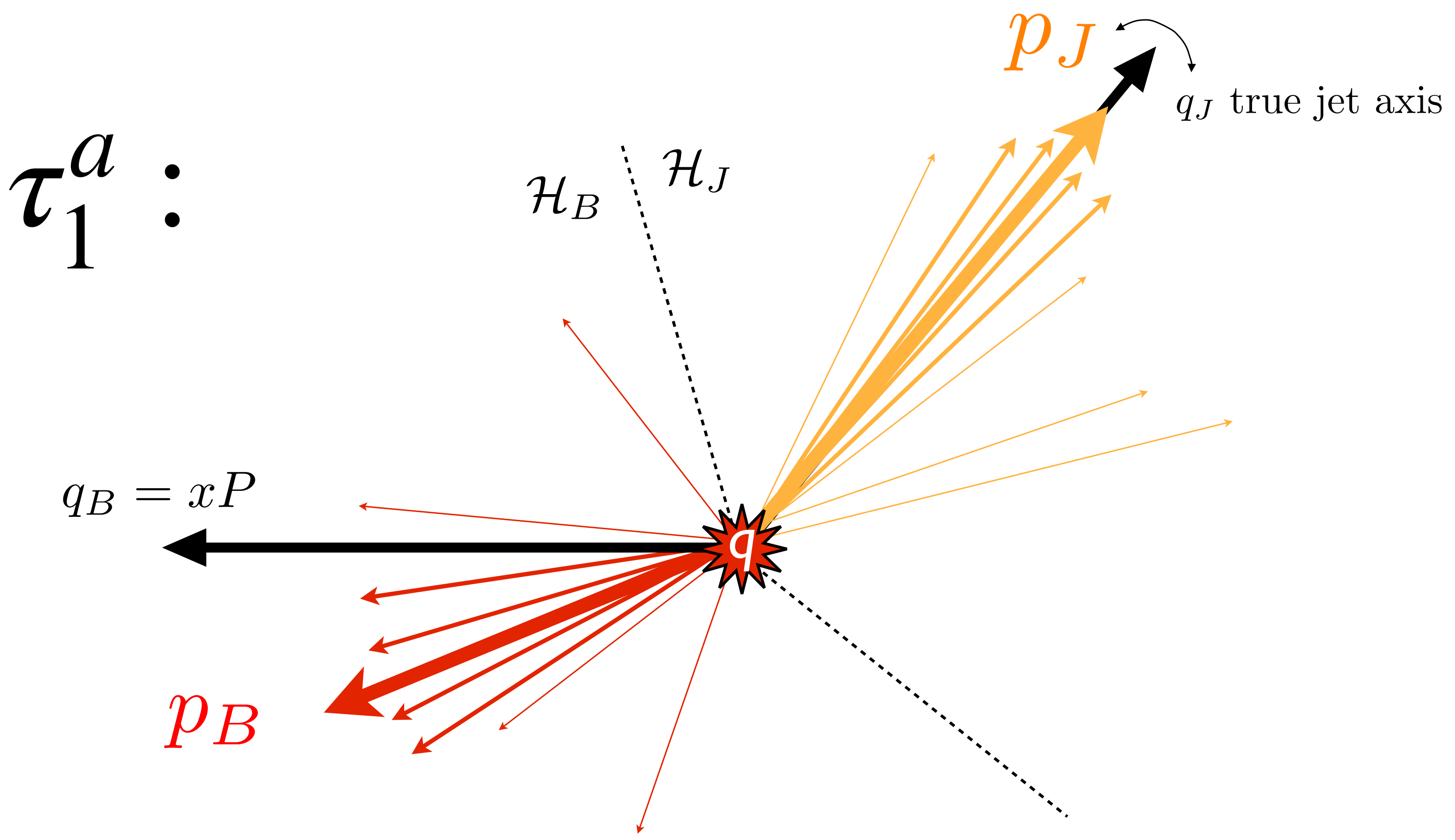
also Z. Kang, Liu, Mantry, Qiu (2012, 2013)



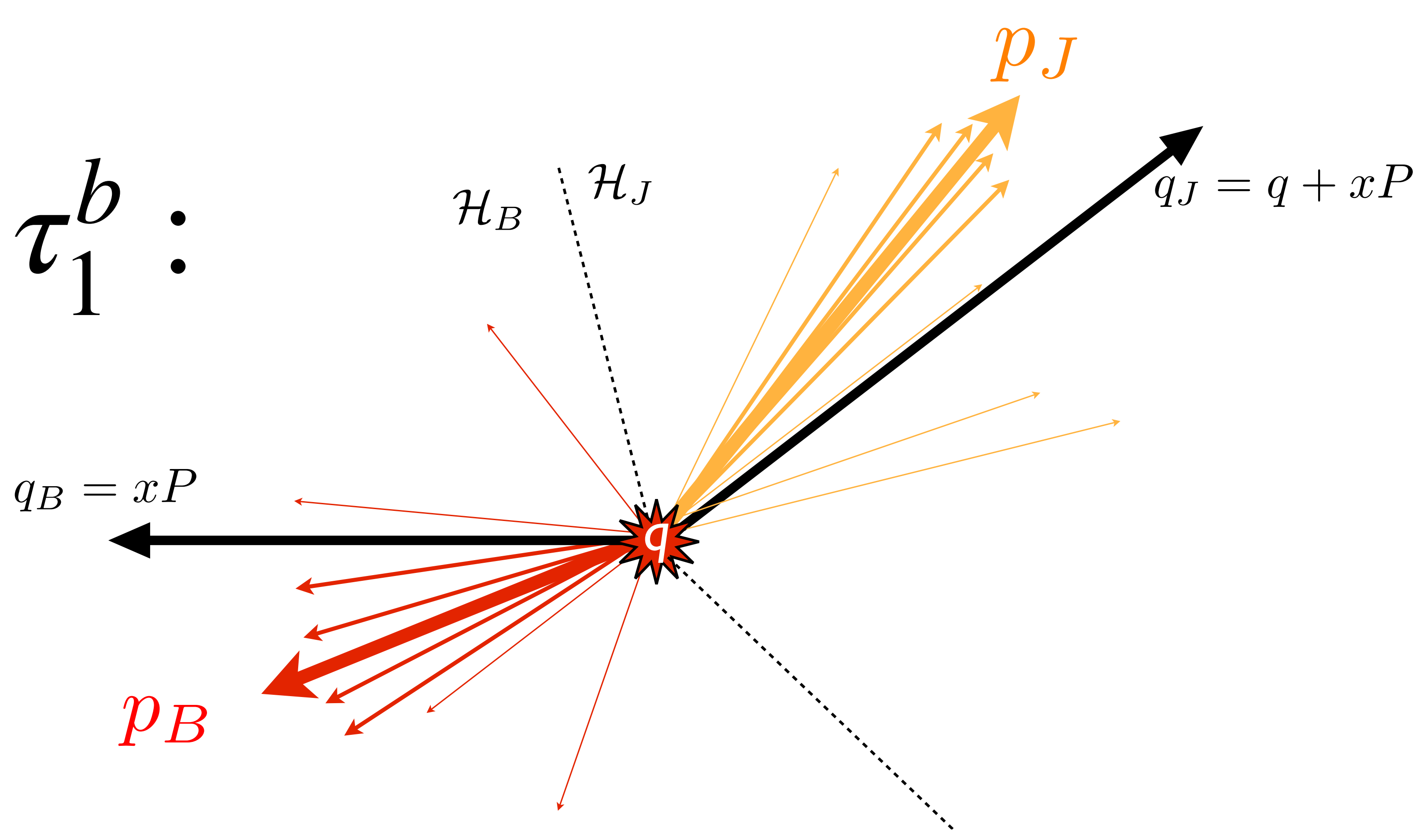
- "1-jettiness" in DIS measures final states with beam radiation + one additional jet

$$\tau_1 = \frac{2}{Q^2} \sum_{i \in X} \min\{q_B \cdot p_i, q_J \cdot p_i\}$$

τ_1^a :



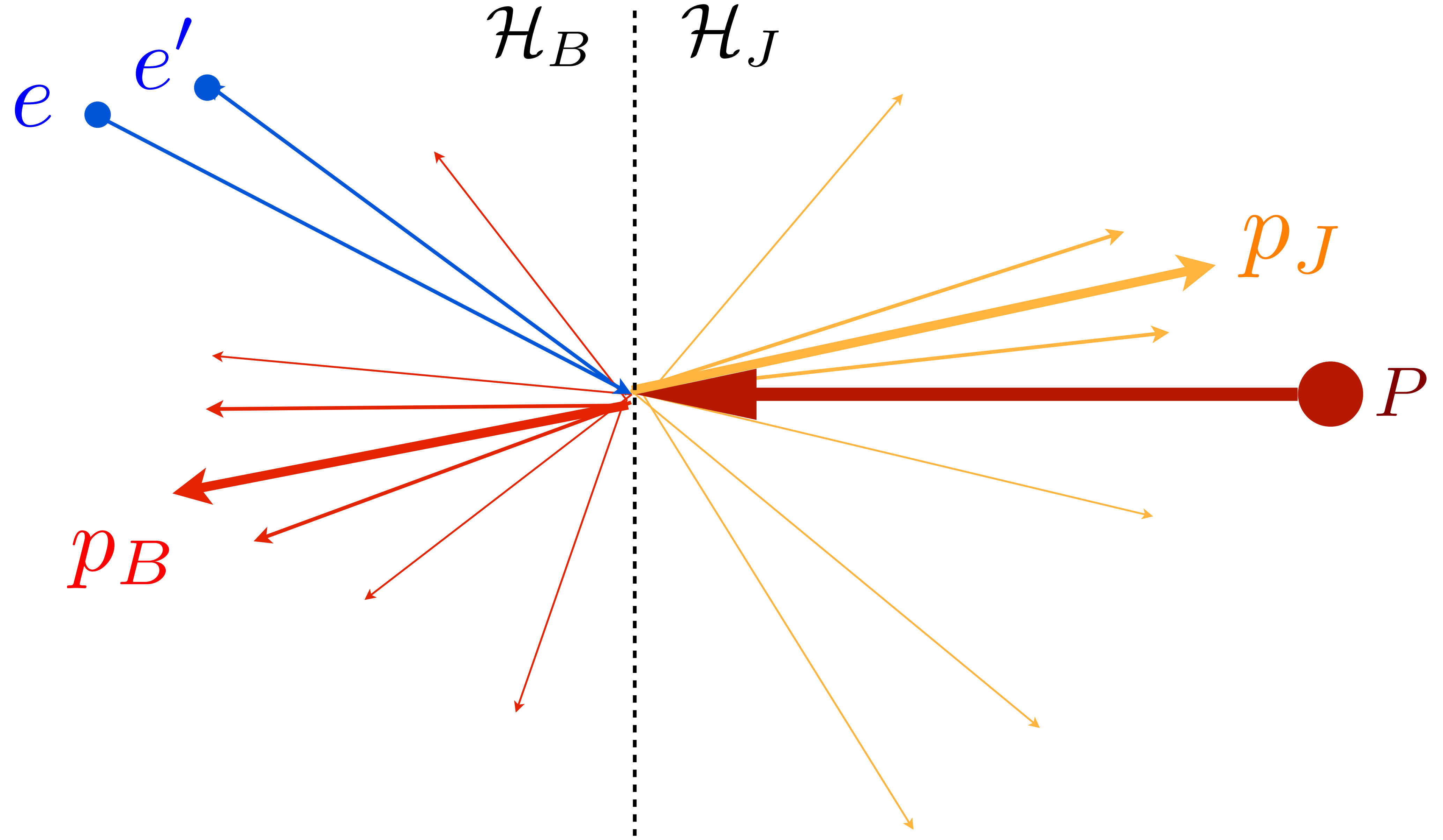
τ_1^b :



- Different choices of axes are possible: different sensitivity to ISR transverse momentum

DIS thrust

τ_1^b :



In the Breit frame:

$$\tau_1 = \frac{2}{Q^2}\sum_i \min\{q_B \cdot p_i, q_J \cdot p_i\}$$

$$\overset{\text{Breit}}{=} 1 - \frac{2}{Q}\sum_{i\in\mathcal{H}_J^b} p_z^i$$

$$q_B = xP$$
$$q_J = q + xP$$

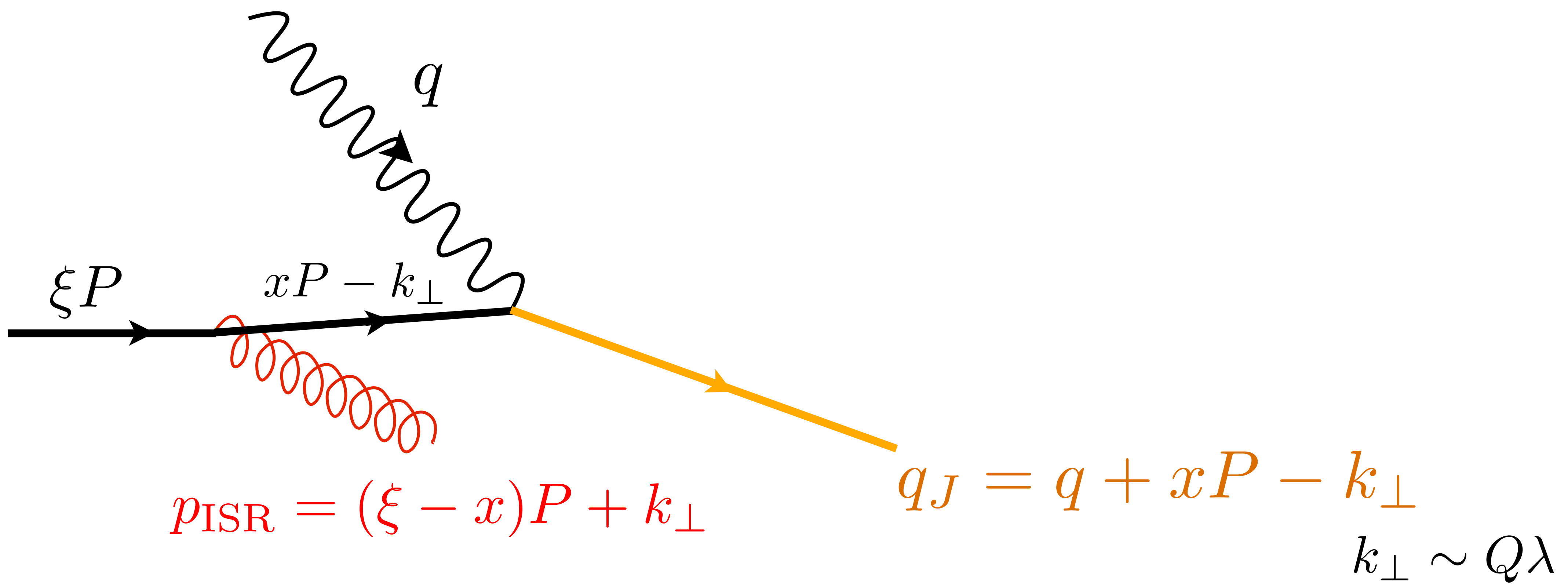
same as DIS thrust of Antonelli, Dasgupta, Salam (1999)

ultimately depends only on momentum in jet or "current" hemisphere

(thanks to momentum conservation)

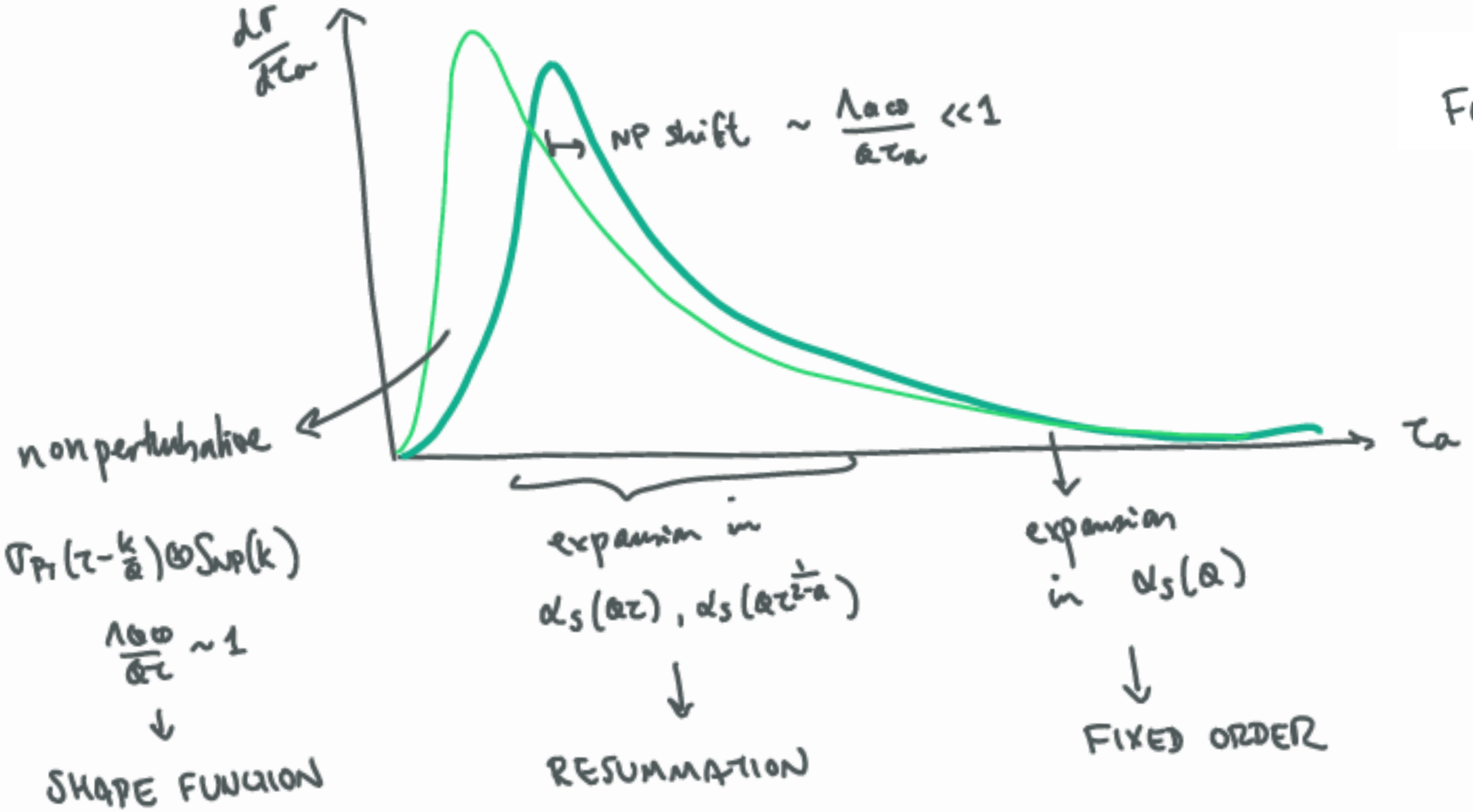
(not true of τ_1^a)

sensitive to ISR transverse momentum:



Sensitivity to strong coupling

τ_a's and similar event shapes probe QCD effects over wide range of scales, perturbative and nonperturbative



Full prediction:

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau_a} = \int dk\, \underbrace{\sigma_{PT}\left(\tau_a - \frac{k}{Q}\right)}\, S_{NP}(k)$$

$$= \sigma_{sing}^{resum}(\tau_a;\mu_{H,J,S}) + \sigma_{non\text{-}sing}^{F.O.}(\tau_a;\mu_{ns})$$

Fixed-order computation

D. Kang, CL, Stewart (2014)

Cross section:

$$\frac{d\sigma}{dx\, dQ^2\, d\tau} = L_{\mu\nu}(x, Q^2) W^{\mu\nu}(x, Q^2, \tau)$$

Hadronic tensor:

$$W^{\mu\nu}(x, Q^2, \tau) = \int d^4x\, e^{iq\cdot x} \langle P | J^{\mu\dagger}(x) \delta(\tau - \hat{\tau}) J^\nu(0) | P \rangle$$

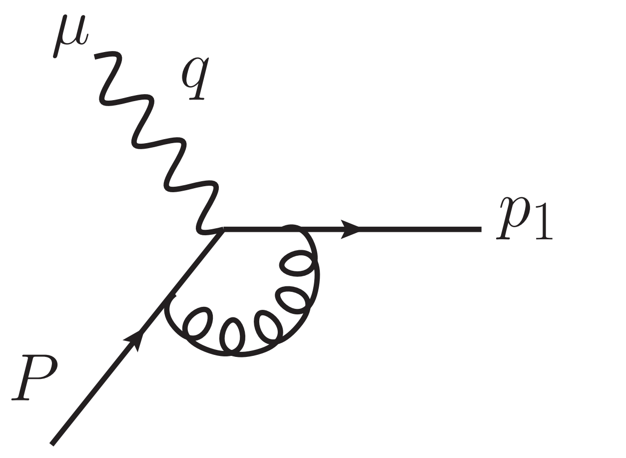
Measure thrust of final state:

$$W^j_{\mu\nu}(x, Q^2, \tau) = \frac{1}{s_j} \sum_n \int d\Phi_n \mathcal{M}^*_\mu(j(P) \to p_1 \ldots p_n) \mathcal{M}_\nu(j(P) \to p_1 \ldots p_n)$$
$$\times (2\pi)^D \delta^D(P + q - \sum_i p_i) \delta\Big(\tau - \tau(\{p_1 \ldots p_n\})\Big) \equiv \sum_n W^{j[n]}_{\mu\nu}$$

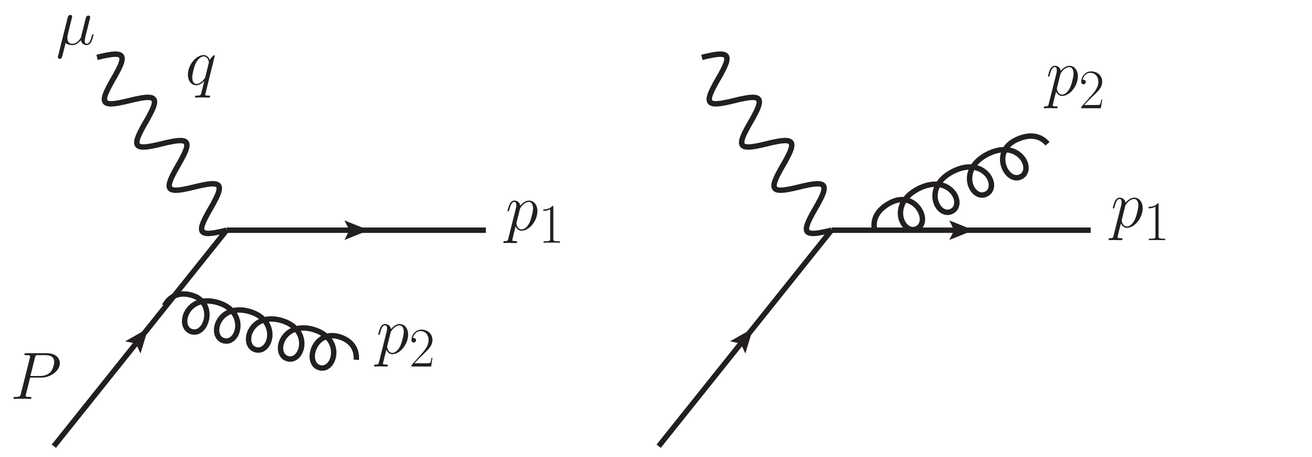
2-particle phase space:

$$W^{j[2]}_{\mu\nu} = \frac{1}{s_j} \frac{1}{8\pi} \left(\frac{4\pi}{Q^2}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \left(\frac{x}{1-x}\right)^\epsilon \int_0^1 \frac{dv}{v^\epsilon (1-v)^\epsilon} \mathcal{M}^*_\mu \mathcal{M}_\nu \delta\Big(\tau - \tau(x, v)\Big)$$

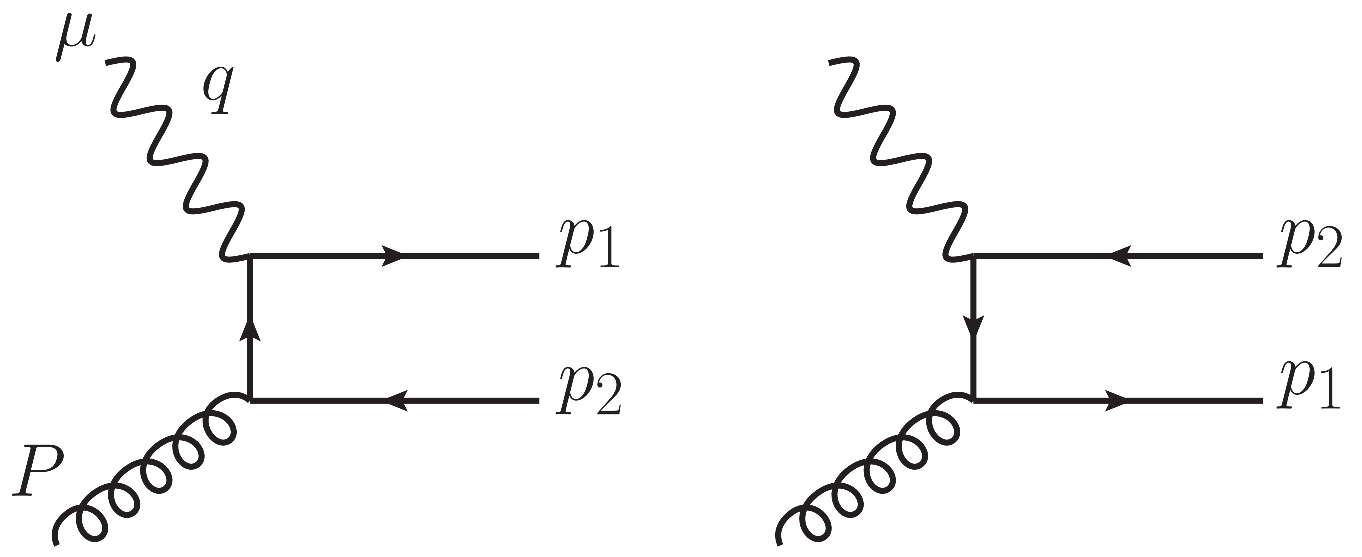
Diagrams to $\mathcal{O}(\alpha_s)$:



$\mathcal{M}^{\rm vir}_\mu$



$\mathcal{M}^{\rm real}_\mu$



$\mathcal{M}^{\rm real}_\mu$

$$p_1^\mu = Qv\frac{n_z^\mu}{2} + Q(1-v)\frac{1-x}{x}\frac{\bar{n}_z^\mu}{2} + p_\perp^\mu$$

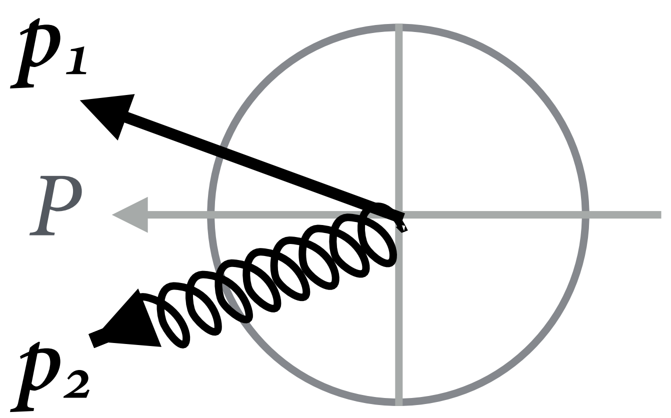
$$p_2^\mu = Q(1-v)\frac{n_z^\mu}{2} + Qv\frac{1-x}{x}\frac{\bar{n}_z^\mu}{2} - p_\perp^\mu$$

Fixed-order computation

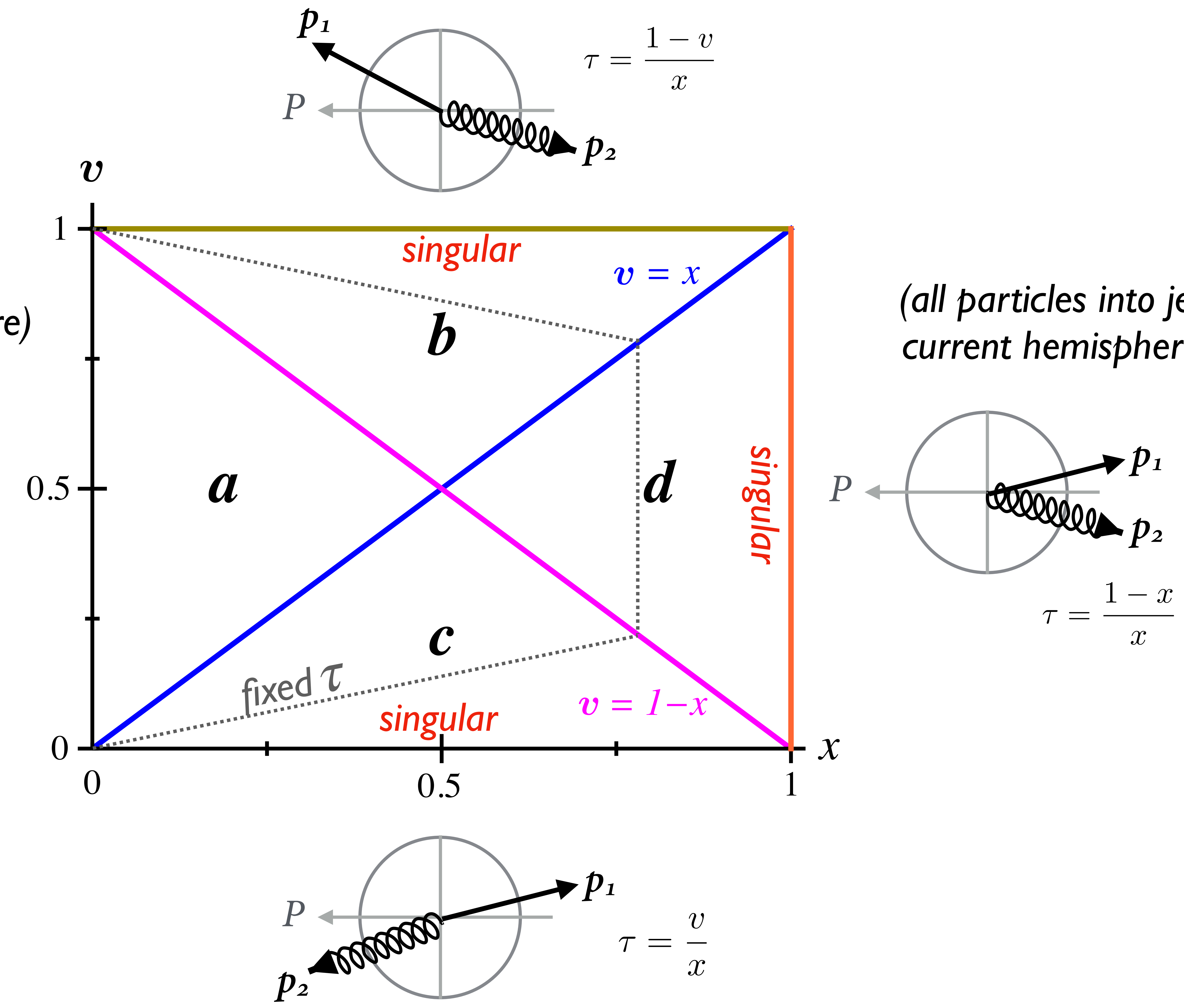
D. Kang, CL, Stewart (2014)

2-particle phase space:

(all particles into beam hemisphere)



$\tau = 1$



$$p_1^\mu = Qv\frac{n_z^\mu}{2} + Q(1-v)\frac{1-x}{x}\frac{\bar{n}_z^\mu}{2} + p_\perp^\mu$$

$$p_2^\mu = Q(1-v)\frac{n_z^\mu}{2} + Qv\frac{1-x}{x}\frac{\bar{n}_z^\mu}{2} - p_\perp^\mu$$

Fixed-order results

D. Kang, CL, Stewart (2014)

Structure functions:

$$W^{\mu\nu}(x,Q^2,\tau) = 4\pi\Big[T_1^{\mu\nu}\mathcal{F}_1(x,Q^2,\tau) + T_2^{\mu\nu}\frac{\mathcal{F}_2(x,Q^2,\tau)}{P\cdot q}\Big]$$

$$T_1^{\mu\nu} = -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}, \quad T_2^{\mu\nu} = \Big(P^\mu - q^\mu\frac{P\cdot q}{q^2}\Big)\Big(P^\nu - q^\nu\frac{P\cdot q}{q^2}\Big)$$

$$\mathcal{F}_L \equiv \mathcal{F}_2 - 2x\mathcal{F}_1$$

Group into singular and non-singular parts:

$$F_1(x,Q^2,\tau) = \sum_{i\in\{q,\bar q,g\}} (A_i + B_i)$$

$$F_L(x,Q^2,\tau) = \sum_{i\in\{q,\bar q,g\}} 4x\,A_i\,.$$

(integrated:)

$$F(x,Q^2,\tau) = \int_0^\tau d\tau' \mathscr{F}(x,Q^2,\tau)$$

$$\boxed{F_i = F_i^{\rm sing} + F_i^{\rm ns}}$$

Singular terms:

$$A_{q,g}^{\rm sing} = 0\,,$$

$$B_q^{\rm sing} = \sum_f Q_f^2\Big\{f_q(x)\Big[\frac{1}{2} - \frac{\alpha_s C_F}{4\pi}\Big(\frac{9}{2} + \frac{\pi^2}{3} + 3\ln\tau + 2\ln^2\tau\Big)\Big]$$
$$+ \frac{\alpha_s C_F}{4\pi}\int_x^1 \frac{dz}{z} f_q(x/z)\Big[\mathcal{L}_1(1-z)\,(1+z^2) + (1-z) + P_{qq}(z)\ln\frac{Q^2\tau}{\mu^2}\Big]\Big\}$$

$$B_g^{\rm sing} = \sum_f Q_f^2\frac{\alpha_s T_F}{2\pi}\int_x^1 \frac{dz}{z} f_g(x/z)\Big[1 - P_{qg}(z) + P_{qg}(z)\ln\frac{Q^2\tau(1-z)}{\mu^2}\Big]\,.$$

Need
$\ln\tau$
resummation

Fixed-order results

D. Kang, CL, Stewart (2014)

Non-singular terms:

$$A_q^{\rm ns} = \sum_f Q_f^2 \frac{\alpha_s C_F}{4\pi}\Big\{\Theta_0 \int_x^{\frac{1}{1+\tau}} dz\, f_q(\tfrac{x}{z})(2z\tau - 1) + \int_x^1 dz\, f_q(\tfrac{x}{z})\Big\}\,,$$

$$A_g^{\rm ns} = \sum_f Q_f^2 \frac{\alpha_s T_F}{\pi}\Big\{\Theta_0 \int_x^{\frac{1}{1+\tau}} dz\, f_g(\tfrac{x}{z})(2z\tau - 1)(1-z) + \int_x^1 dz\, f_g(\tfrac{x}{z})(1-z)\Big\}\,,$$

$$\begin{aligned} B_q^{\rm ns} = \sum_f Q_f^2 \frac{\alpha_s C_F}{4\pi}\Big\{&\Theta_0 \int_x^{\frac{1}{1+\tau}} \frac{dz}{z} f_q(\tfrac{x}{z})\Big[\frac{1-4z}{2(1-z)}(2z\tau - 1) + P_{qq}(z)\ln\frac{z\tau}{1-z\tau}\Big] \\ &+ f_q(x)(3\ln\tau + 2\ln^2\tau) + \int_x^1 \frac{dz}{z} f_q(\tfrac{x}{z})\Big[\mathcal{L}_0(1-z)\frac{1-4z}{2} - P_{qq}(z)\ln z\tau\Big]\Big\}\,, \end{aligned}$$

$$\begin{aligned} B_g^{\rm ns} = \sum_f Q_f^2 \frac{\alpha_s T_F}{2\pi}\Big\{&\Theta_0 \int_x^{\frac{1}{1+\tau}} \frac{dz}{z} f_g(\tfrac{x}{z})\Big[-(2z\tau - 1) + P_{qg}(z)\ln\frac{z\tau}{1-z\tau}\Big] \\ &- \int_x^1 \frac{dz}{z} f_g(\tfrac{x}{z})[1 + P_{qg}(z)\ln z\tau]\Big\}\,, \end{aligned}$$

$$\Theta_0 \equiv \Theta_0(\tau, x) \equiv \theta(\tau)\theta(1-\tau)\theta\Big(\frac{1-x}{x} - \tau\Big)$$

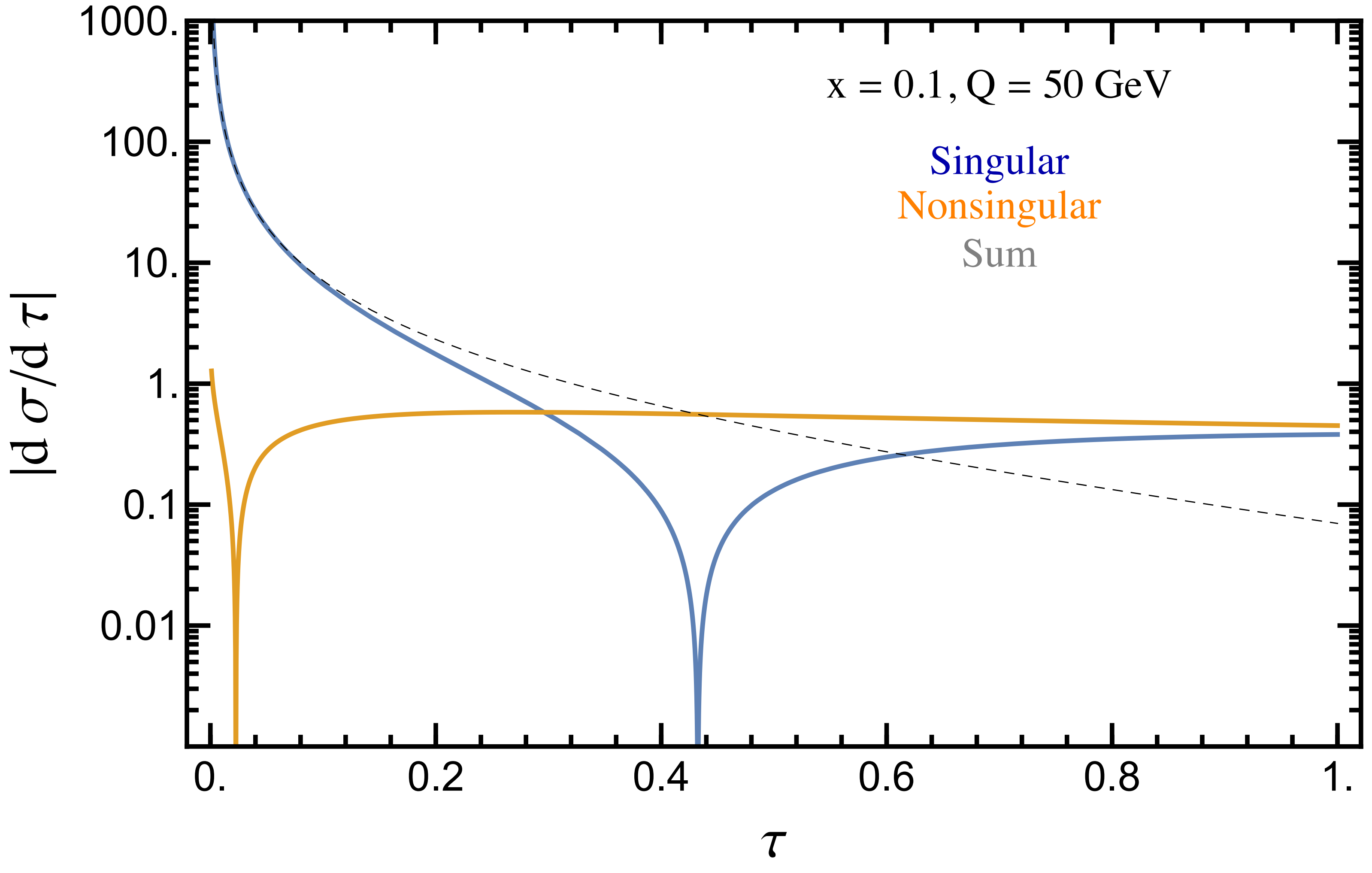
$$P_{qq}(z) \equiv \Big[\theta(1-z)\frac{1+z^2}{1-z}\Big]_+ = (1+z^2)\mathcal{L}_0(1-z) + \frac{3}{2}\delta(1-z)$$

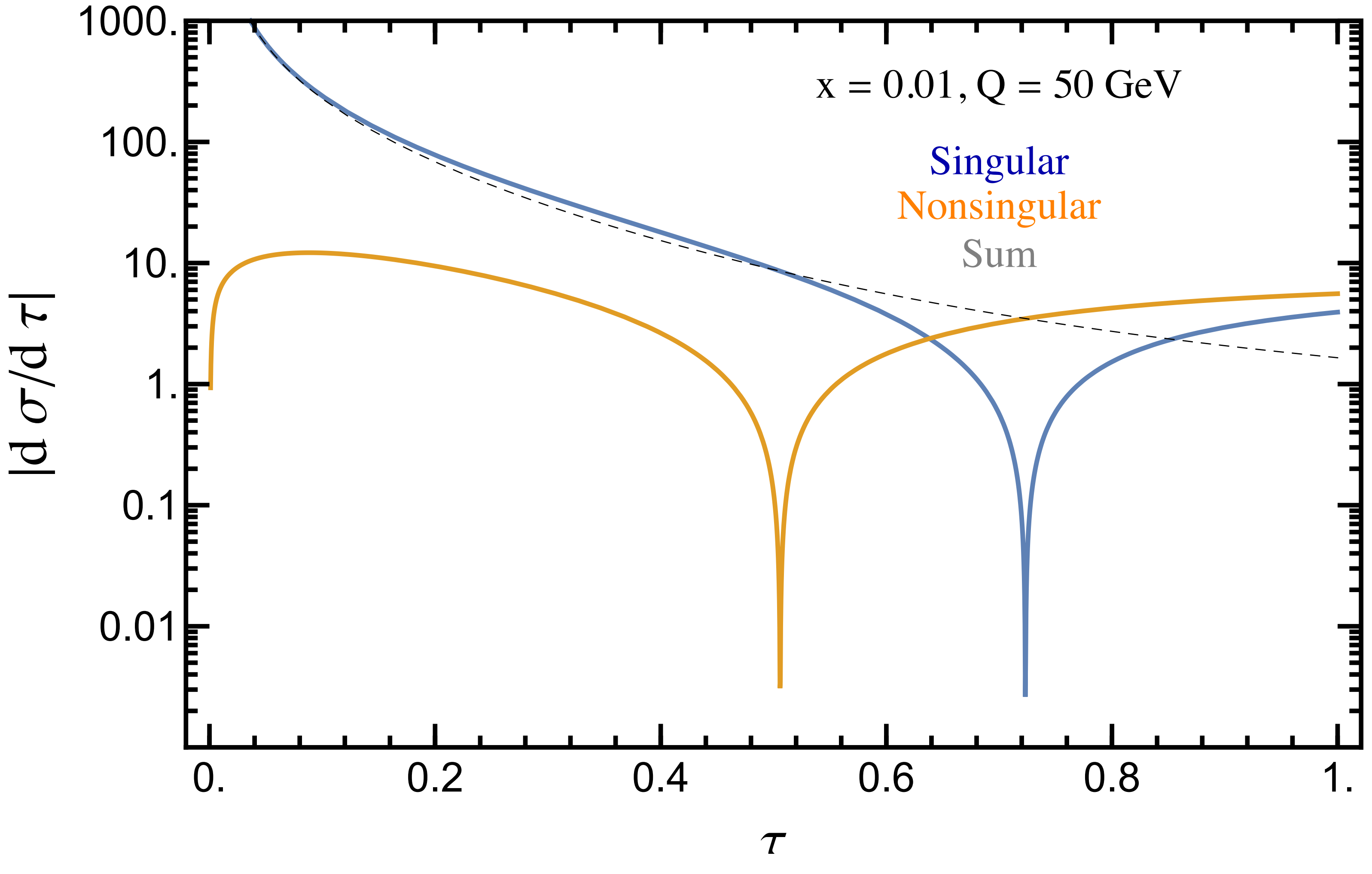
$$P_{qg}(z) \equiv \theta(1-z)[(1-z)^2 + z^2].$$

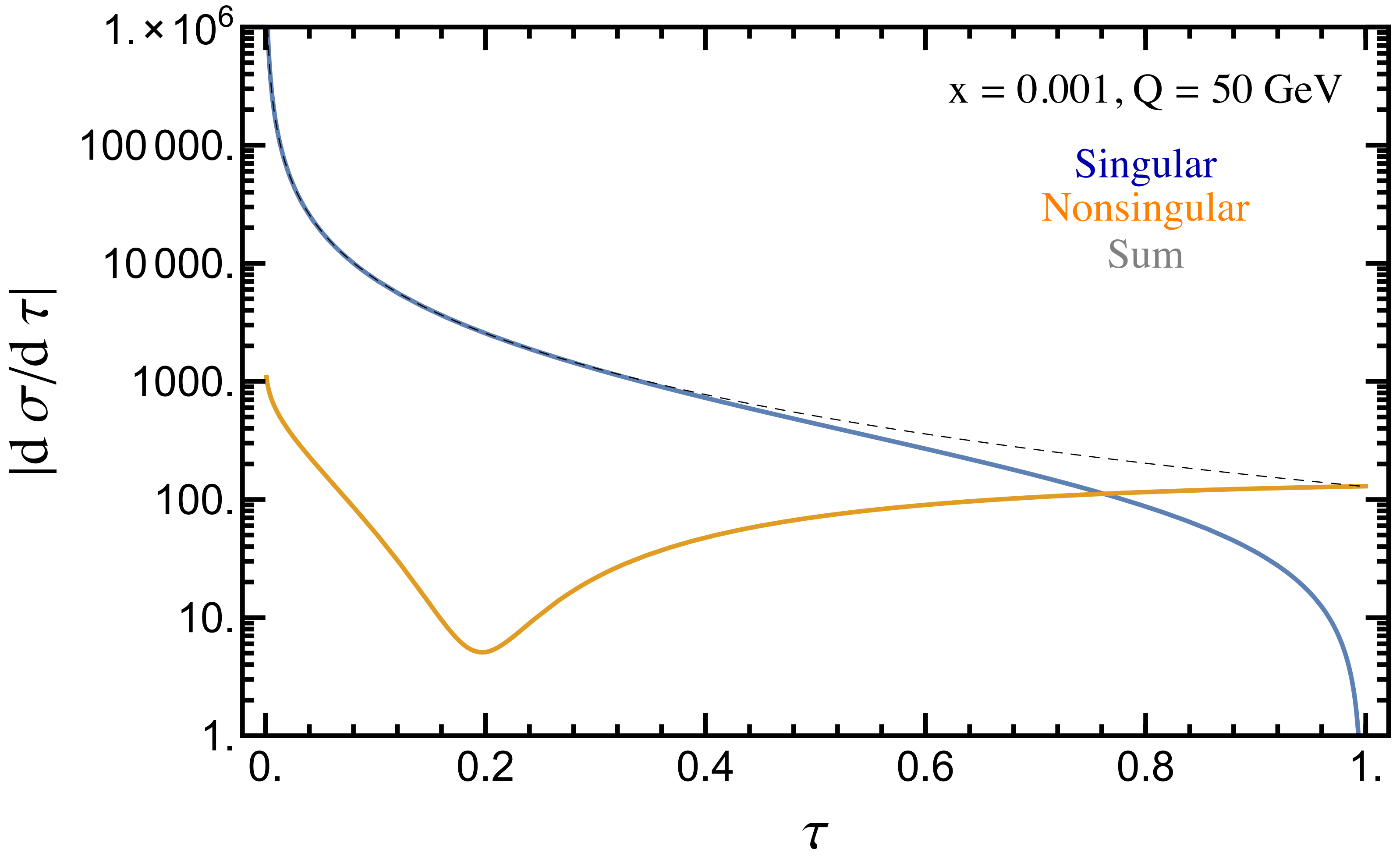
Singular vs. non-singular

D. Kang, CL, Stewart (2014)

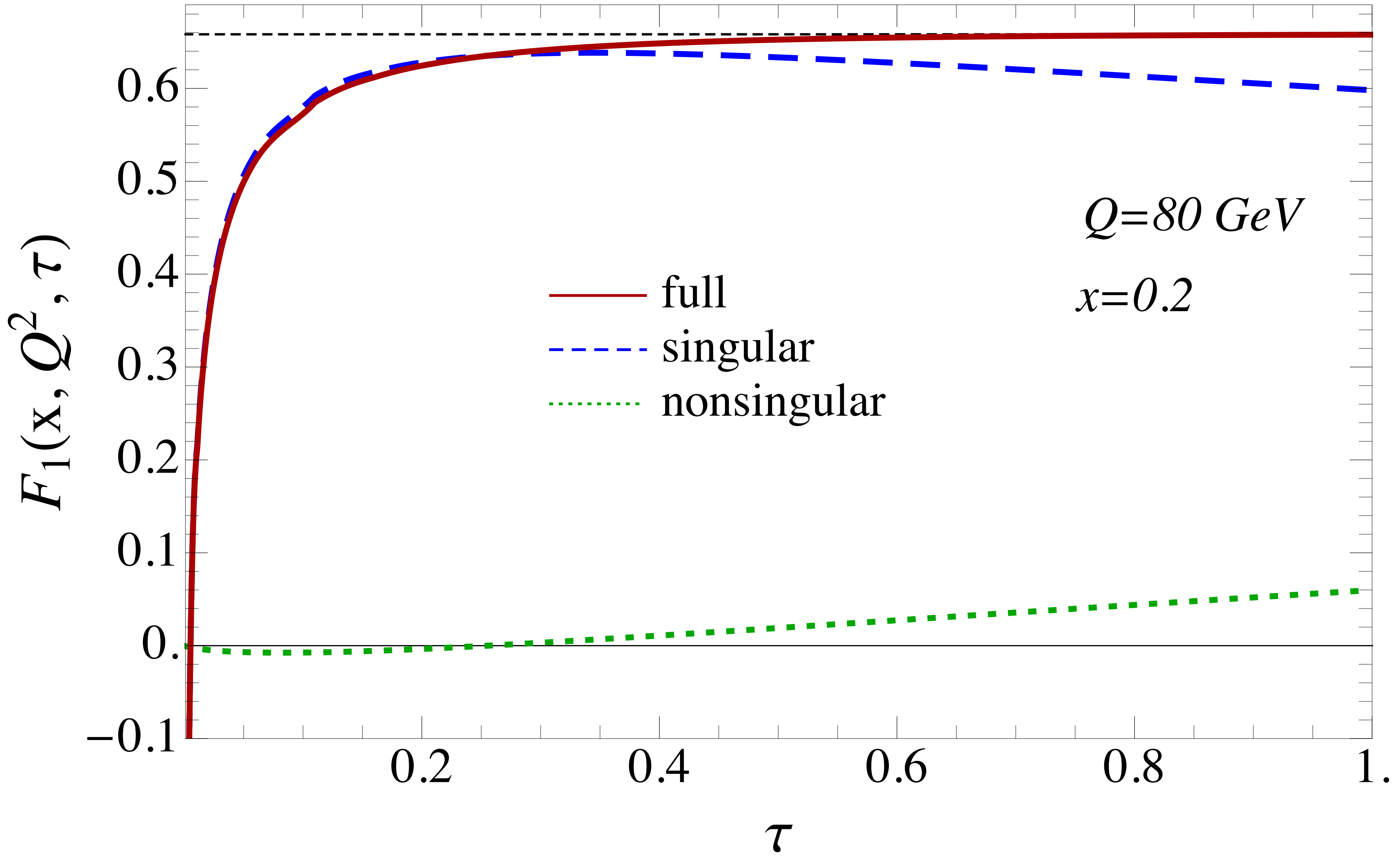
Contributions to differential thrust spectrum:





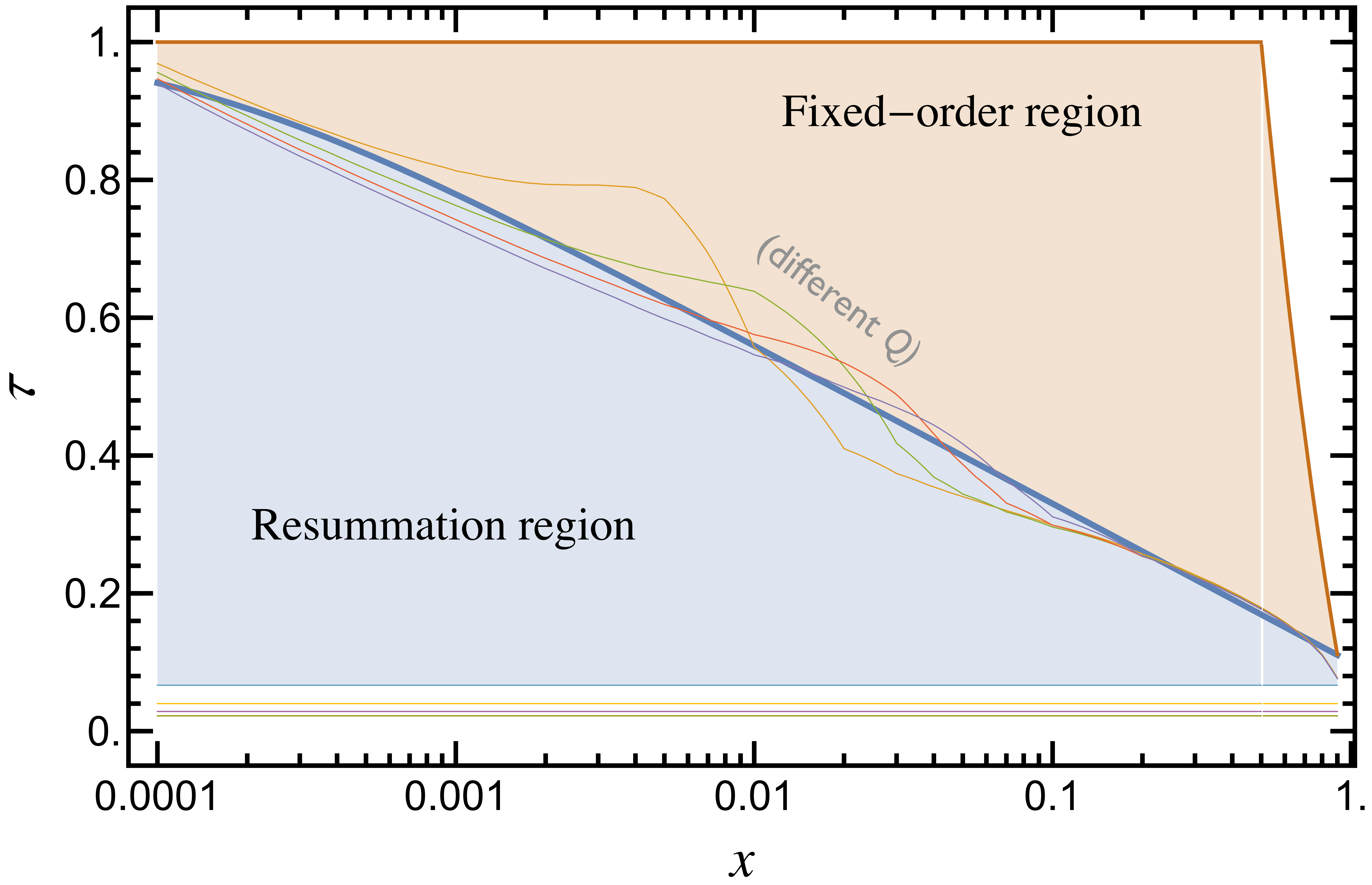


Add up to total integrated cross section:



Singular vs. non-singular

Region where resummation is important is thus a function of *x*:



Large Logs

- If we calculate event shape τ cross section in QCD perturbation theory, we will find:

$$\int_0^\tau d\tau \frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau} \sim \left[1 + \frac{\alpha_s}{4\pi}\Big(F_{12}\ln^2\tau - F_{11}\ln\tau + F_{10}\Big) + \left(\frac{\alpha_s}{4\pi}\right)^2\Big(F_{24}\ln^4\tau + F_{23}\ln^3\tau + F_{22}\ln^2\tau + F_{21}\ln\tau + F_{20}\Big) + \dots\right]$$

- In the narrow-jet limit $\tau \to 0$ the logs grow large and spoil the perturbative expansion. Reorganize the expansion:

$$\begin{aligned}
\ln\sigma(\tau) \sim \quad & \alpha_s(\ln^2\tau + \ln\tau) \\
+ & \alpha_s^2(\ln^3\tau + \ln^2\tau + \ln\tau) \\
+ & \alpha_s^3(\ln^4\tau + \ln^3\tau + \ln^2\tau + \ln\tau) \\
+ & \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots
\end{aligned}$$

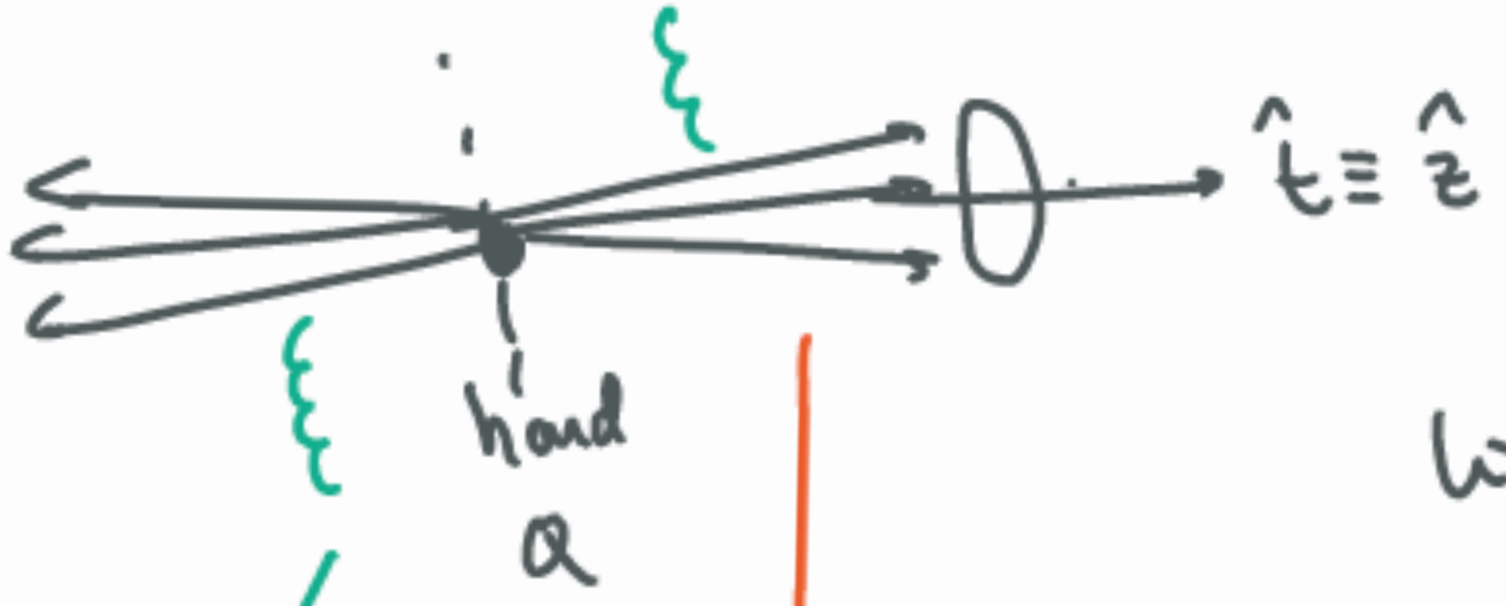
Leading Log (LL)	Next-to-Leading Log (NLL)	NNLL
$\sim \alpha^{-1}$	~ 1	$\sim \alpha$

New power counting when $\ln\tau \sim \frac{1}{\alpha}$: $\quad \sim \alpha^{-1} \quad \sim 1 \quad \sim \alpha$

- **These logs are of large ratios of disparate physical scales**
- **Need to identify and factor these scales**
- **Use RG evolution to resum the logs**

Momentum scales

Thrust: $M^2 = M_A^2 + M_B^2 = Q^2 \tau \quad \left(\text{if } \ll Q^2\right)$



$n = (1, +\hat{z})$
$\bar{n} = (1, -\hat{z})$

light-cone coordinates:

$$p^\mu = (\bar{n}\cdot p,\ n\cdot p,\ \vec{p}_\perp)$$

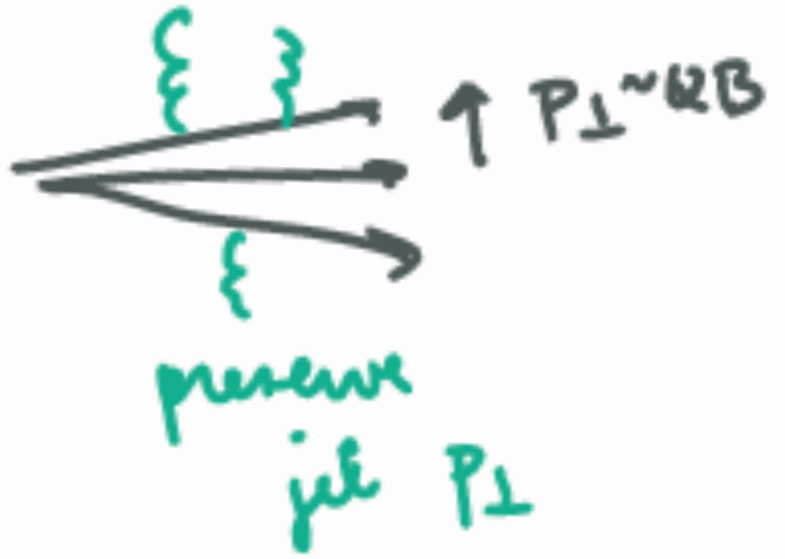
preserve jet mass

Collinear $p_c \sim \left(Q, \frac{M^2}{Q}, M\right) \sim Q(1, \tau, \sqrt{\tau})$

same

soft $k_s \sim \left(\frac{M^2}{Q}, \frac{M^2}{Q}, \frac{M^2}{Q}\right) \sim Q(\tau, \tau, \tau)$

Broadening:



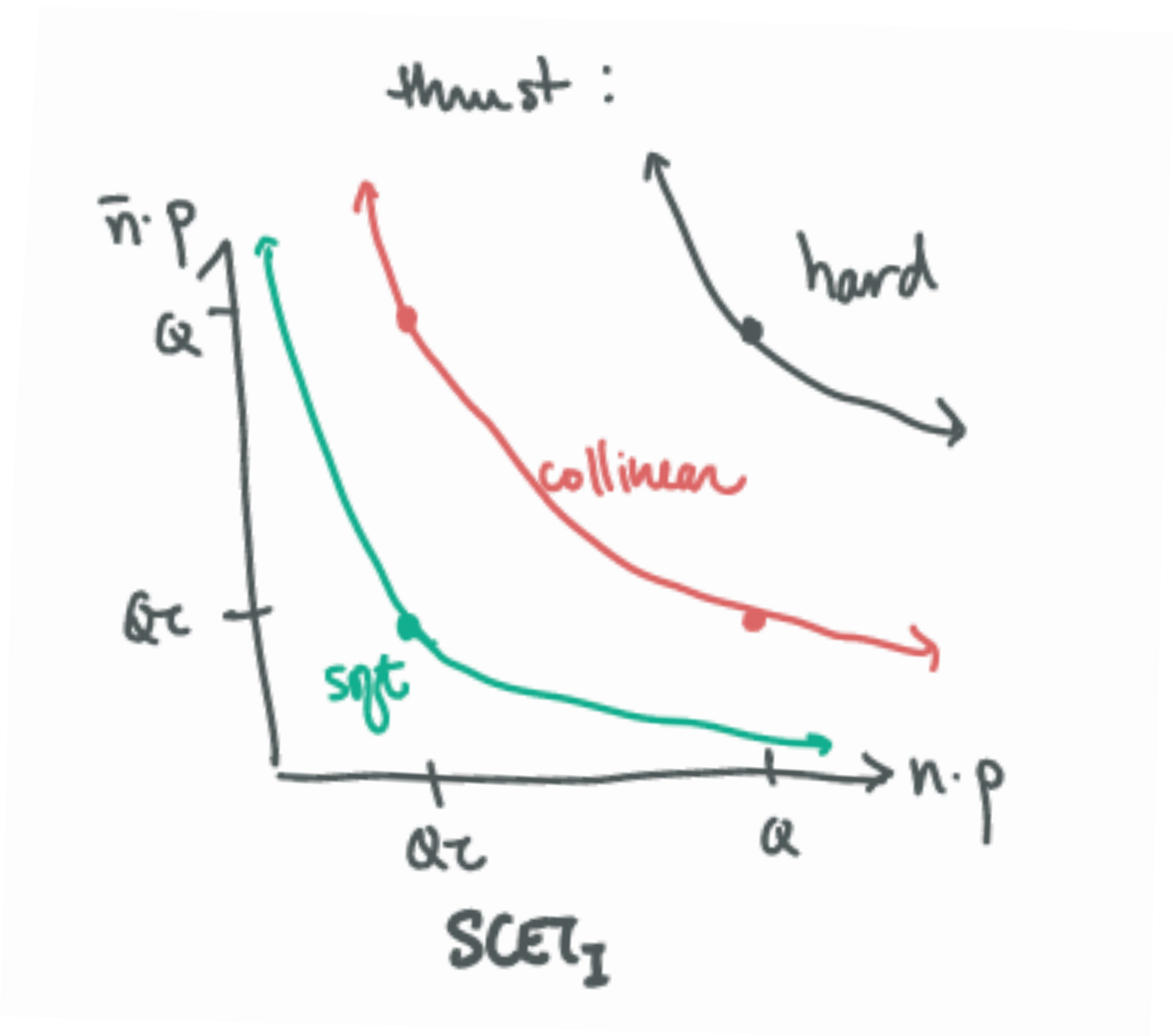
coll $p_c \sim (Q, QB^2, QB)$

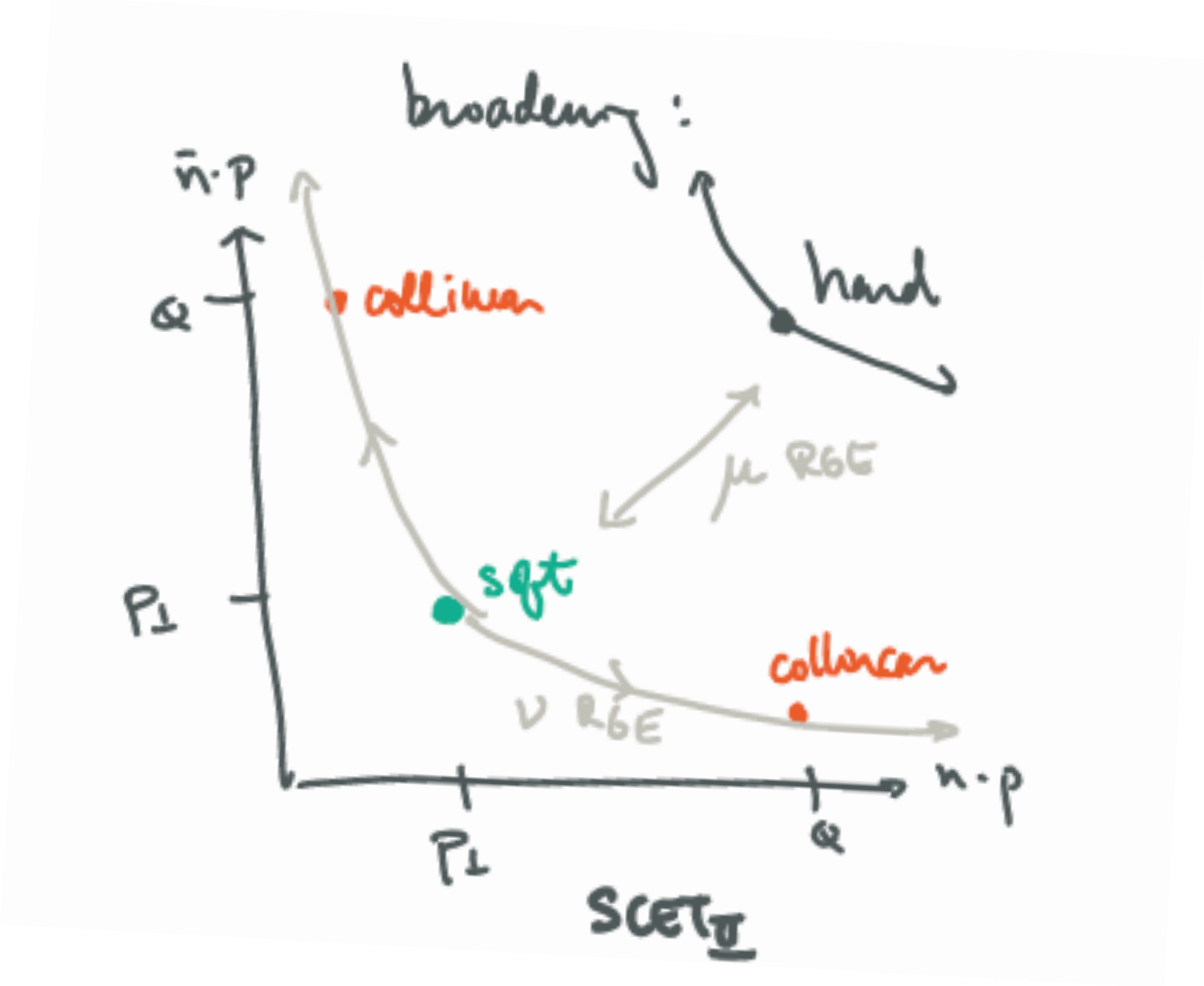
same

soft $k_s \sim Q(B, B, B)$

SCET modes

Bauer, Fleming, Luke, Pirjol, Stewart (2000-02)





Chiu, Jain, Neill, Rothstein (2011-12)

Factorization Theorem for DIS thrust

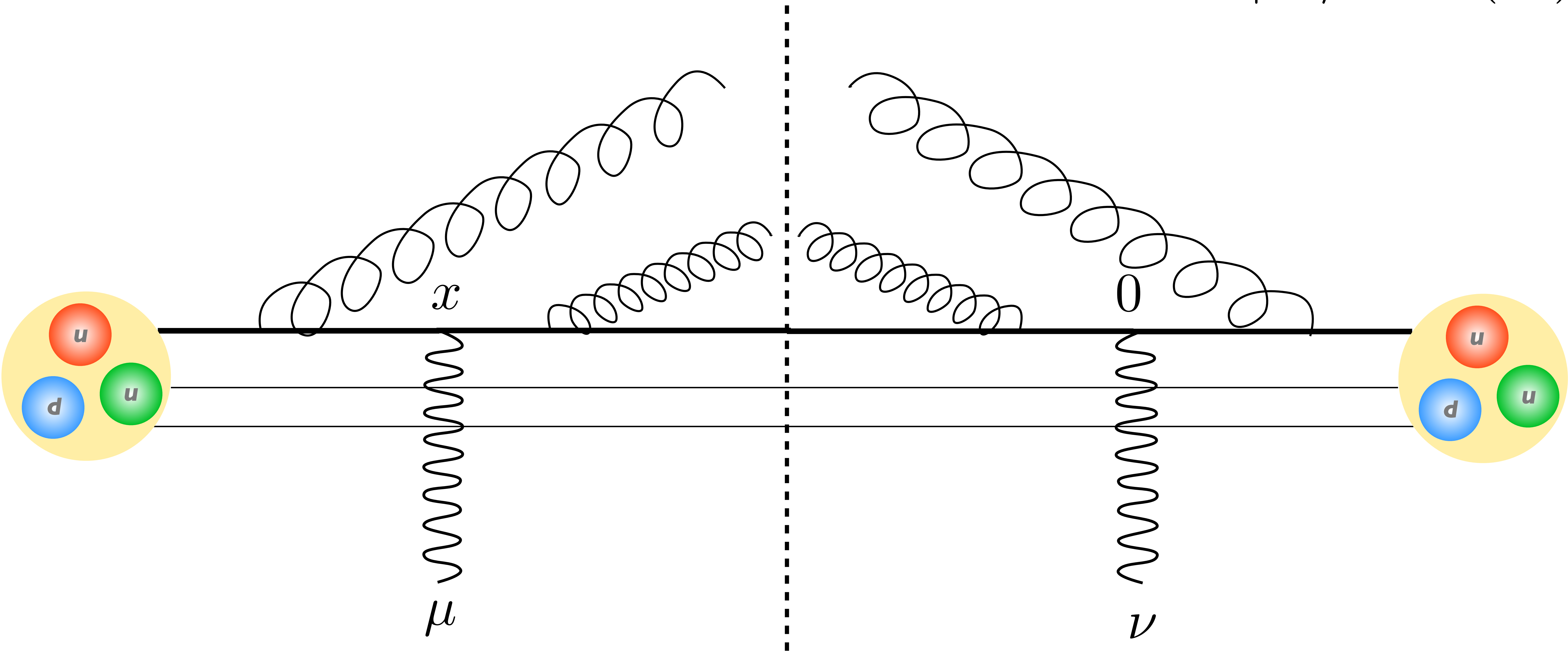
Start in QCD: $$\frac{d\sigma(x,Q^2)}{d\tau_1} = L_{\mu\nu}(x,Q^2) W^{\mu\nu}(x,Q^2,\tau_1)$$

leptonic tensor

hadronic tensor

$$W^{\mu\nu}(x,Q^2,\tau_1) = \int d^4x\, e^{iq\cdot x} \langle P|\bar{q}\gamma^\mu q(x)\delta(\tau_1 - \hat{\tau}_1)\bar{q}\gamma^\nu q(0)|P\rangle$$

$$\hat{\tau}_1|X\rangle = \tau_1(X)|X\rangle$$



Measure τ_1 of particles crossing the cut

Factorization Theorem for DIS thrust

Match onto 2-jet operators in SCET:

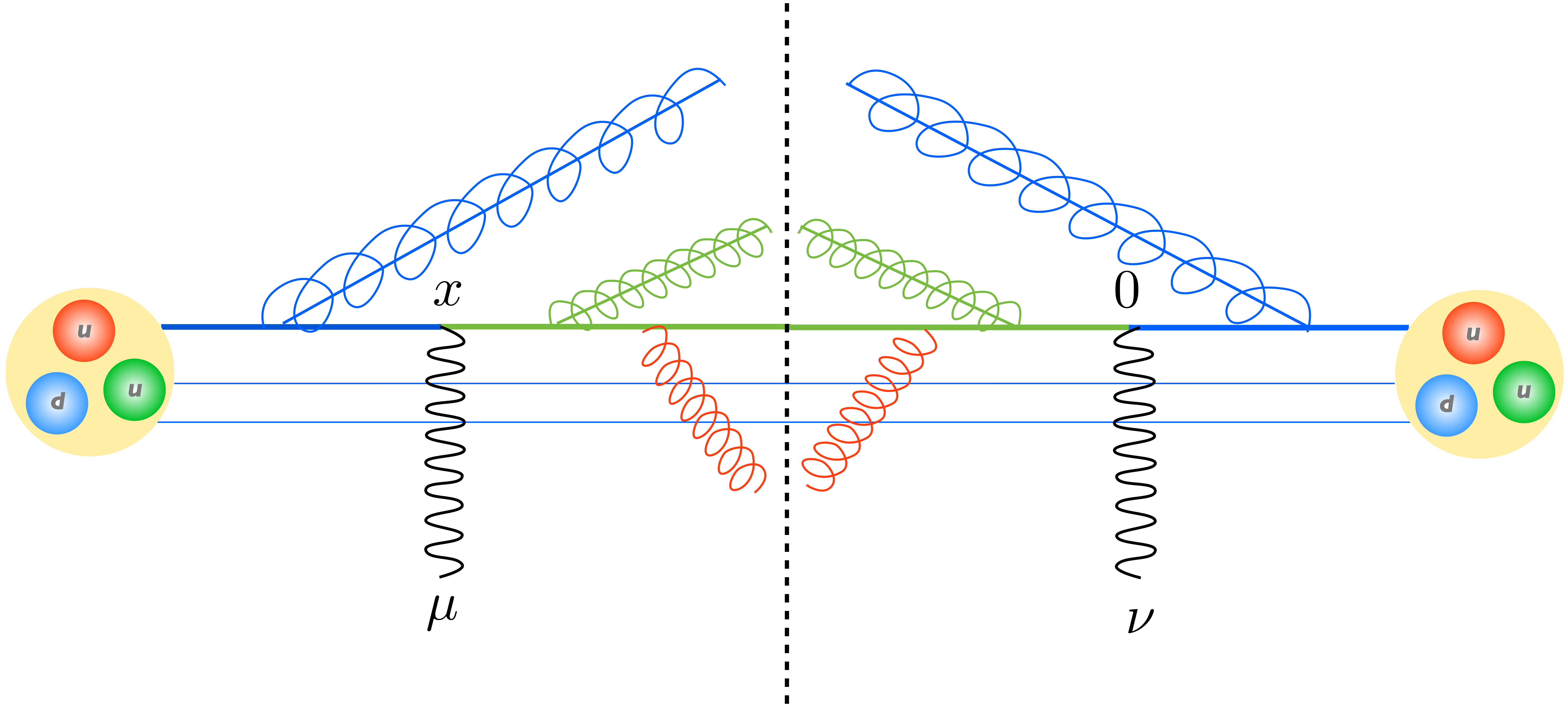
$$W_{\mu\nu}(x,Q^2,\tau_1) = \int d^4x\, e^{iq\cdot x} \sum_{n_1,n_2} \int d^3\tilde{p}_1 d^3\tilde{p}_2 e^{i(\tilde{p}_2-\tilde{p}_1)\cdot x} C_\mu^*(\tilde{p}_1,\tilde{p}_2) C_\mu(\tilde{p}_1,\tilde{p}_2)$$
$$\times \langle P_{n_B}|\bar{\chi}_{n_2,\tilde{p}_2}(x)\overline{T}[Y^\dagger_{n_2}(x)Y_{n_1}(x)]\chi_{n_1,\tilde{p}_1}(x)$$
$$\times \delta(\tau_1 - \hat{\tau}_1^{n_1} - \hat{\tau}_1^{n_2} - \tau_1^s)$$
$$\times \bar{\chi}_{n_1,\tilde{p}_1}(0) T[Y^\dagger_{n_1}(0)Y_{n_2}(0)]\chi_{n_2,\tilde{p}_2}(0)|P_{n_B}\rangle$$

collinear jet operators in SCET

$$\chi_n = [W_n \xi_n]$$

collinear Wilson line   collinear quark field

$Y_{n_{1,2}}$ soft gluon Wilson lines



Factorization Theorem for DIS thrust

Match onto 2-jet operators in SCET:

$$W_{\mu\nu}(x,Q^2,\tau_1) = \int d^4x\, e^{iq\cdot x} \sum_{n_1,n_2} \int d^3\tilde{p}_1 d^3\tilde{p}_2 e^{i(\tilde{p}_2-\tilde{p}_1)\cdot x} C_\mu^*(\tilde{p}_1,\tilde{p}_2) C_\mu(\tilde{p}_1,\tilde{p}_2)$$
$$\times \langle P_{n_B} | \bar{\chi}_{n_2,\tilde{p}_2}(x) \overline{T}[Y_{n_2}^\dagger(x) Y_{n_1}(x)] \chi_{n_1,\tilde{p}_1}(x)$$
$$\times \delta(\tau_1 - \hat{\tau}_1^{n_1} - \hat{\tau}_1^{n_2} - \tau_1^s)$$
$$\times \bar{\chi}_{n_1,\tilde{p}_1}(0) T[Y_{n_1}^\dagger(0) Y_{n_2}(0)] \chi_{n_2,\tilde{p}_2}(0) | P_{n_B} \rangle$$

$Y_{n_{1,2}}$ soft gluon Wilson lines

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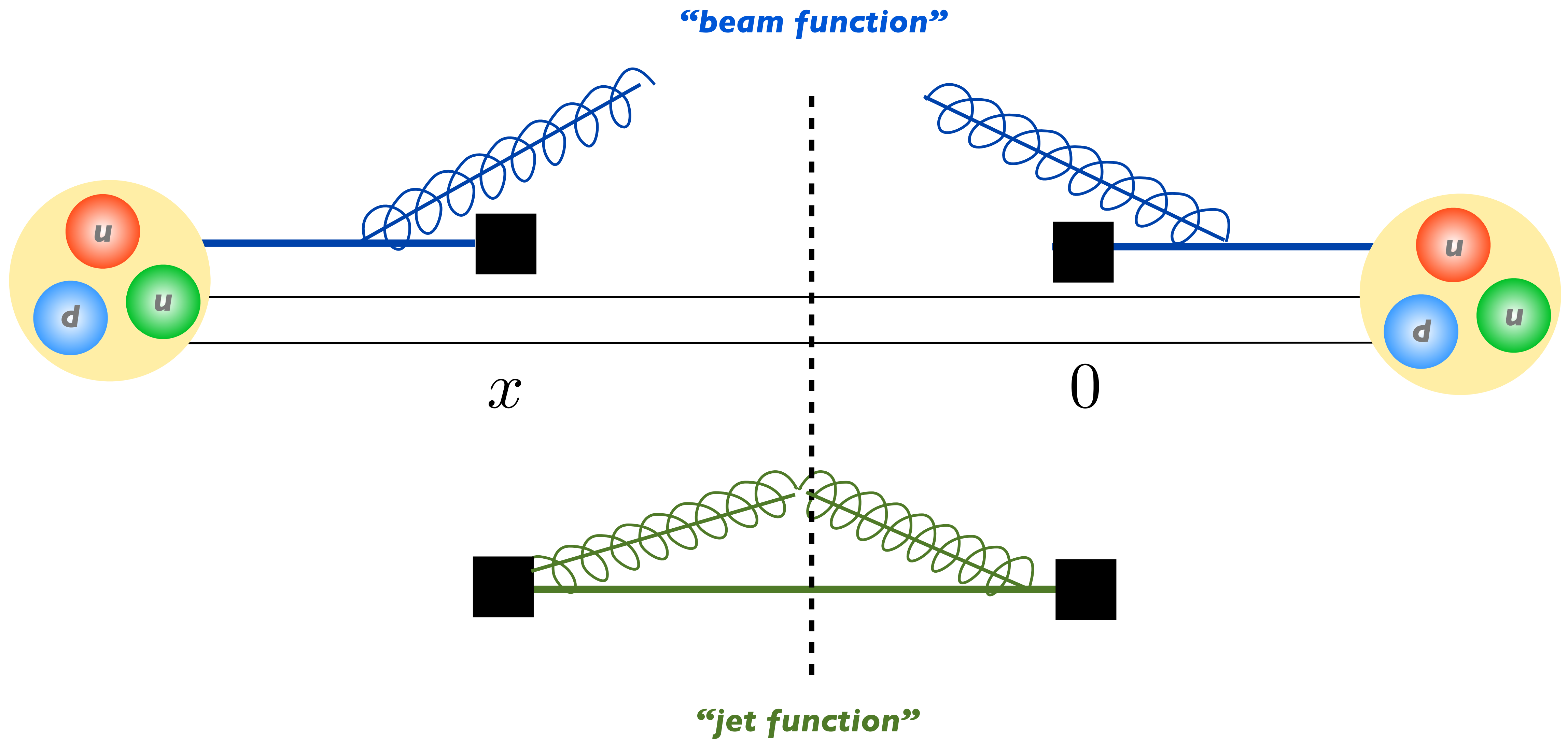
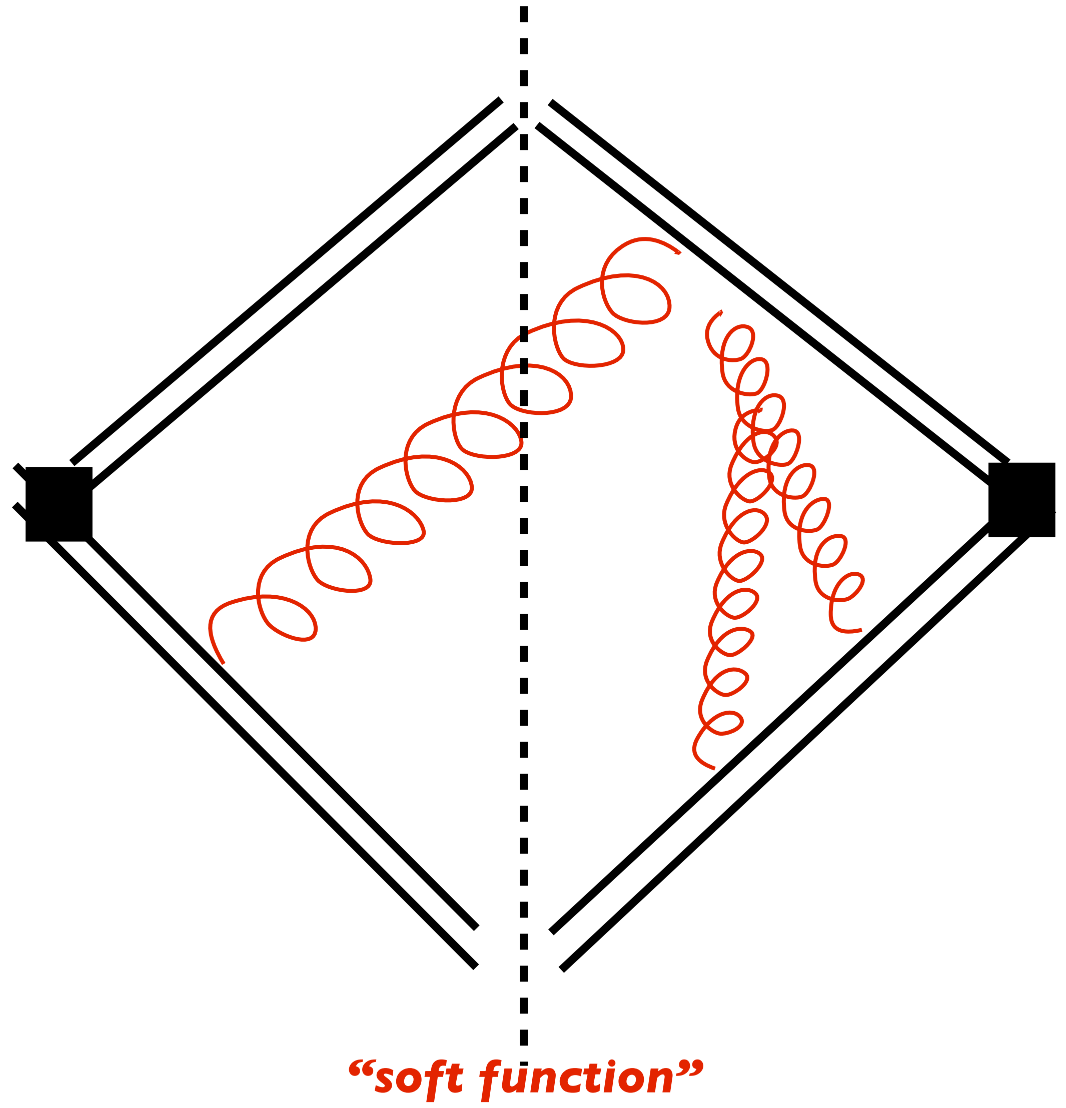
collinear quark field



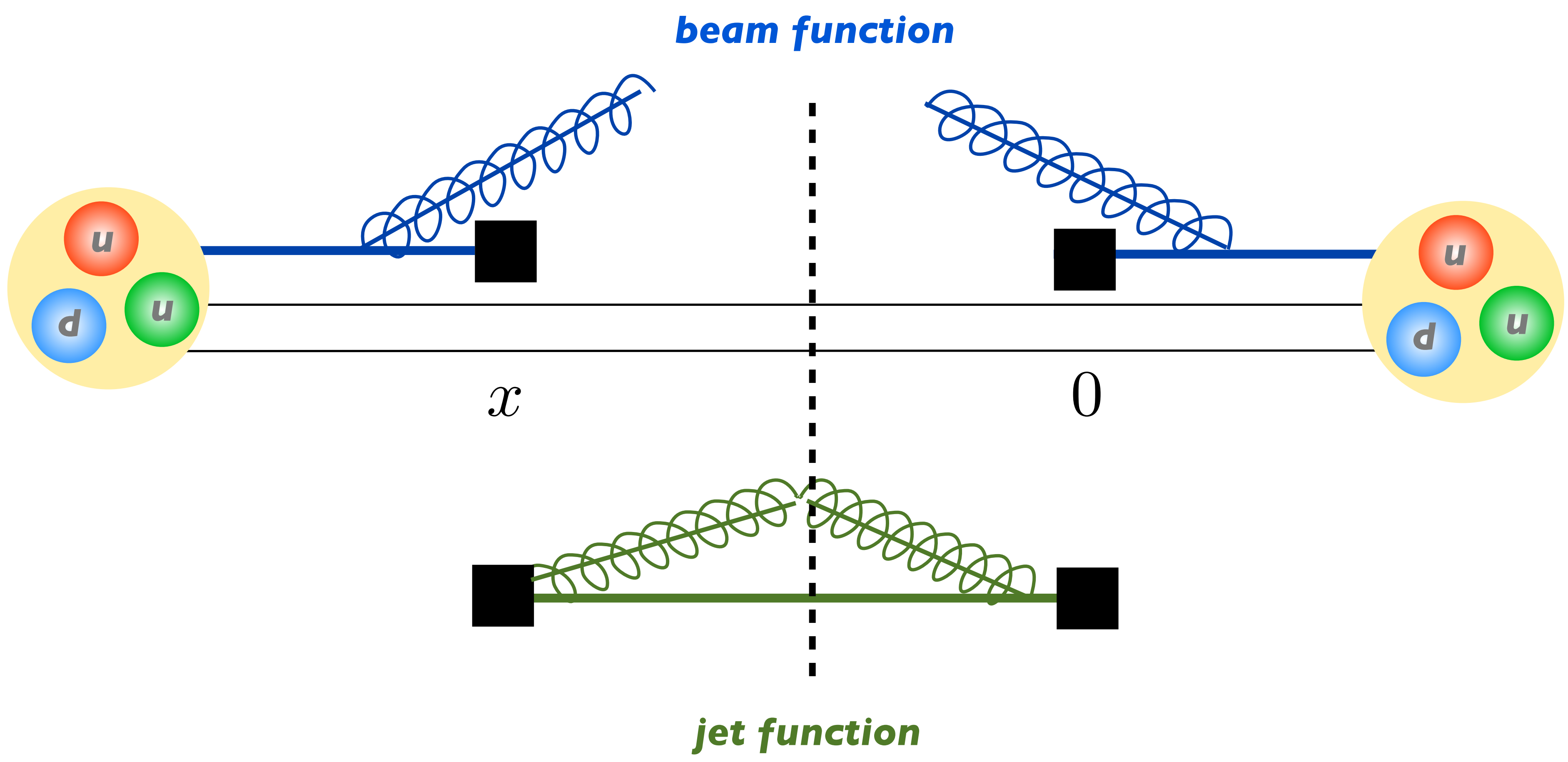


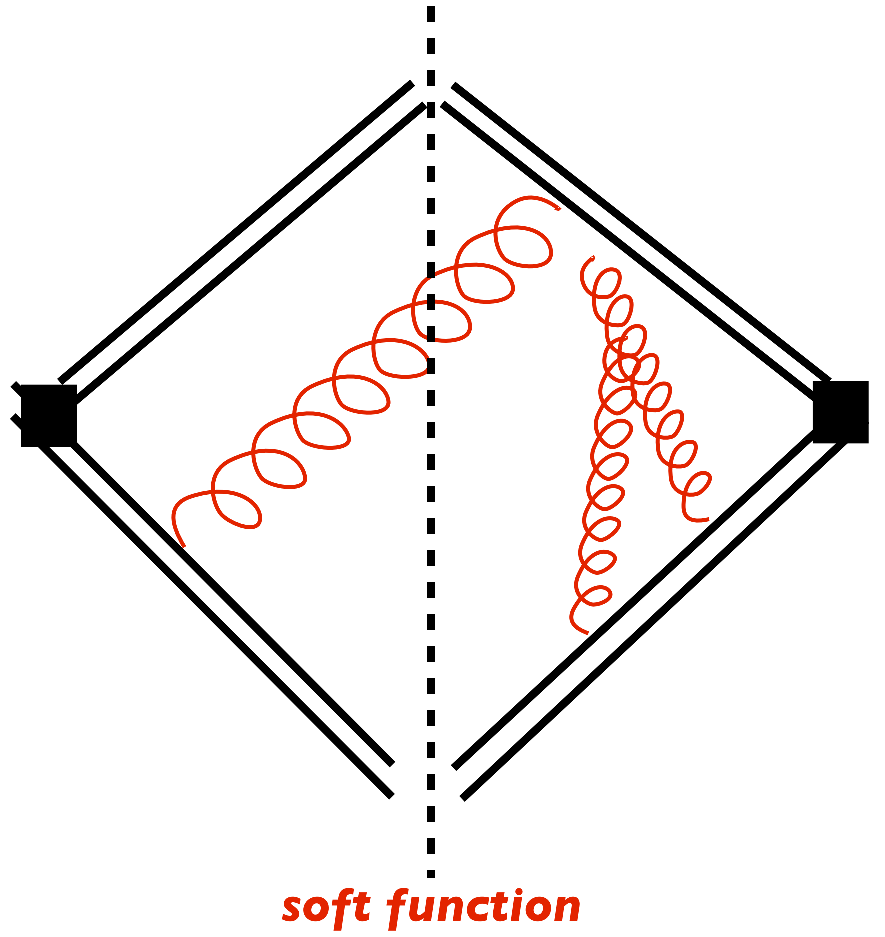
Factorization Theorem for DIS thrust

Factor collinear and soft matrix elements:

$$
\begin{aligned}
W_{\mu\nu}(x,Q^2,\tau_1) = &\int d^2\tilde{p}_\perp \int d\tau_J d\tau_B d\tau_S\ C^*(Q^2,\mu)C(Q^2,\mu)\ \delta\Big(\tau_1 - \frac{t_J}{s_J} - \frac{t_B}{s_B} - \frac{k_S}{Q_R}\Big) \\
&\times \langle 0|[Y^\dagger_{n'_J} Y^\dagger_{n'_B}](0)\delta(k_S - n'_J\cdot\hat{p}_{J'} - n'_B\cdot\hat{p}_{B'})[Y_{n'_B}Y_{n'_J}](0)|0\rangle \\
&\times \langle P_{n_B}|\bar{\chi}_{n_B}(0)\delta(Q_B\tau_B - n_B\cdot\hat{p}^{n_B})[\delta(\bar{n}_B\cdot q + \bar{n}_B\cdot\mathcal{P})\delta^2(\tilde{p}_\perp - \mathcal{P}_\perp)\chi_{n_B}](0)|P_{n_B}\rangle \\
&\times \langle 0|\chi_{n_J}(0)\delta(Q_J\tau_J - n_J\cdot\hat{p}^{n_J})\delta(\bar{n}_J\cdot q + \bar{n}_J\cdot\mathcal{P})\delta^2(q_\perp + \tilde{p}_\perp + \mathcal{P}_\perp)\bar{\chi}_{n_J}(0)|0\rangle
\end{aligned}
$$

(+ permutations)





$$
\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^b} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S \delta\Big(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big)
$$

$$
\times J_q(t_J - \mathbf{p}_\perp^2,\mu)\mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu)S(k_S,\mu)
$$

Factorization Theorem for DIS thrust

Factor collinear and soft matrix elements:

hard function

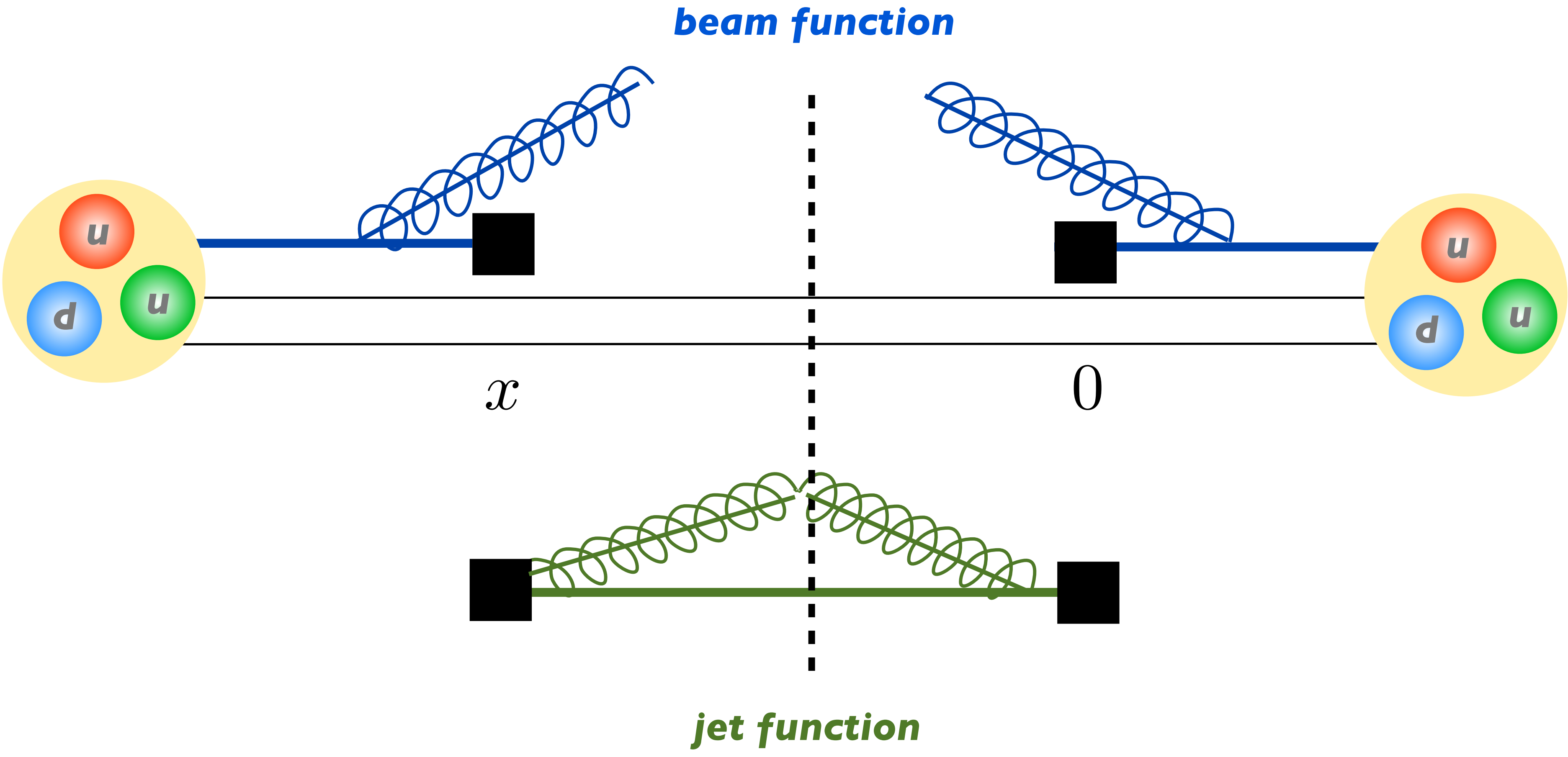
$$W_{\mu\nu}(x,Q^2,\tau_1) = \int d^2\tilde{p}_\perp \int d\tau_J d\tau_B d\tau_S \, \boxed{C^*(Q^2,\mu)C(Q^2,\mu)} \, \delta\Big(\tau_1 - \frac{t_J}{s_J} - \frac{t_B}{s_B} - \frac{k_S}{Q_R}\Big)$$

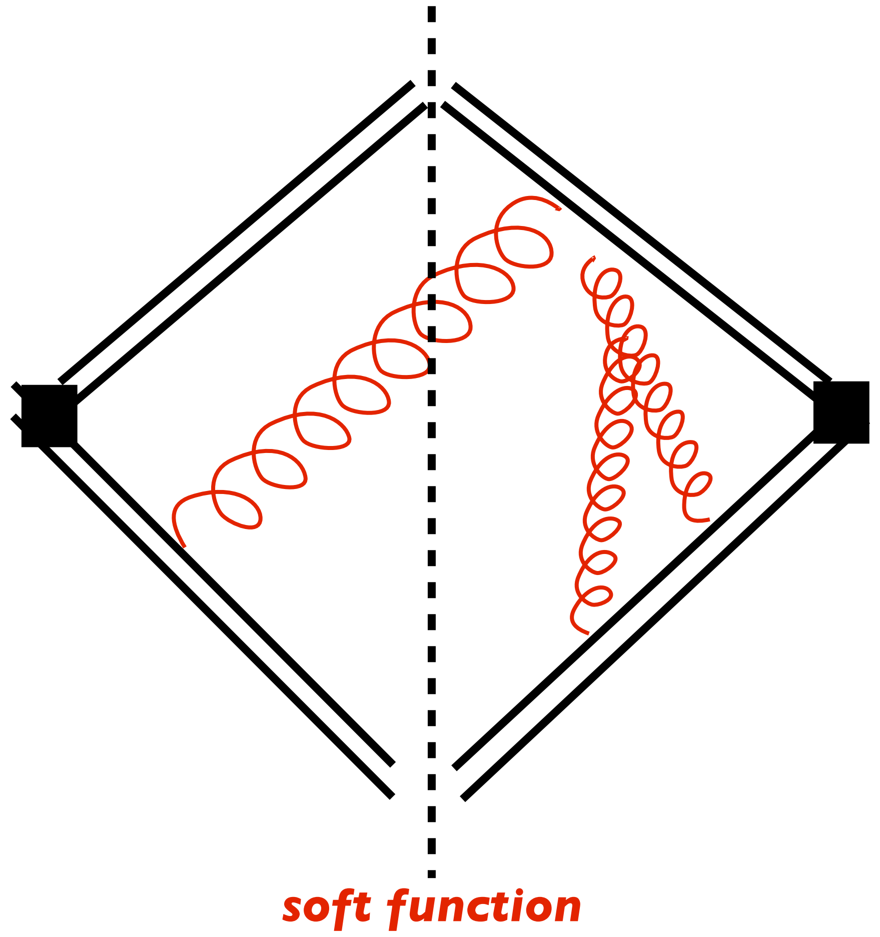
soft function $$\times \boxed{\langle 0|[Y^\dagger_{n'_J} Y^\dagger_{n'_B}](0)\delta(k_S - n'_J\cdot\hat{p}_{J'} - n'_B\cdot\hat{p}_{B'})[Y_{n'_B} Y_{n'_J}](0)|0\rangle}$$

beam function $$\times \boxed{\langle P_{n_B}|\bar{\chi}_{n_B}(0)\delta(Q_B\tau_B - n_B\cdot\hat{p}^{n_B})[\delta(\bar{n}_B\cdot q + \bar{n}_B\cdot\mathcal{P})\delta^2(\tilde{p}_\perp - \mathcal{P}_\perp)\chi_{n_B}](0)|P_{n_B}\rangle}$$

jet function $$\times \boxed{\langle 0|\chi_{n_J}(0)\delta(Q_J\tau_J - n_J\cdot\hat{p}^{n_J})\delta(\bar{n}_J\cdot q + \bar{n}_J\cdot\mathcal{P})\delta^2(q_\perp + \tilde{p}_\perp + \mathcal{P}_\perp)\bar{\chi}_{n_J}(0)|0\rangle}$$

(+ permutations)

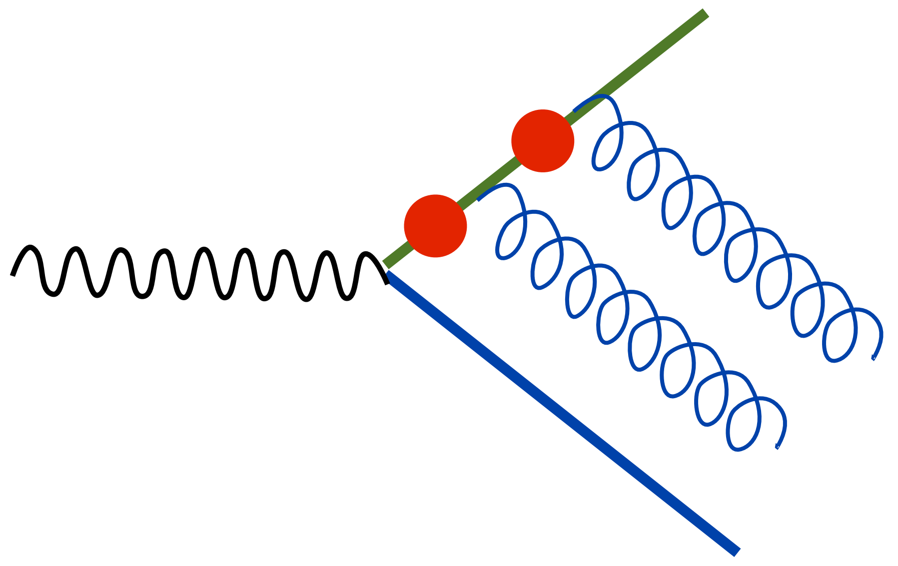


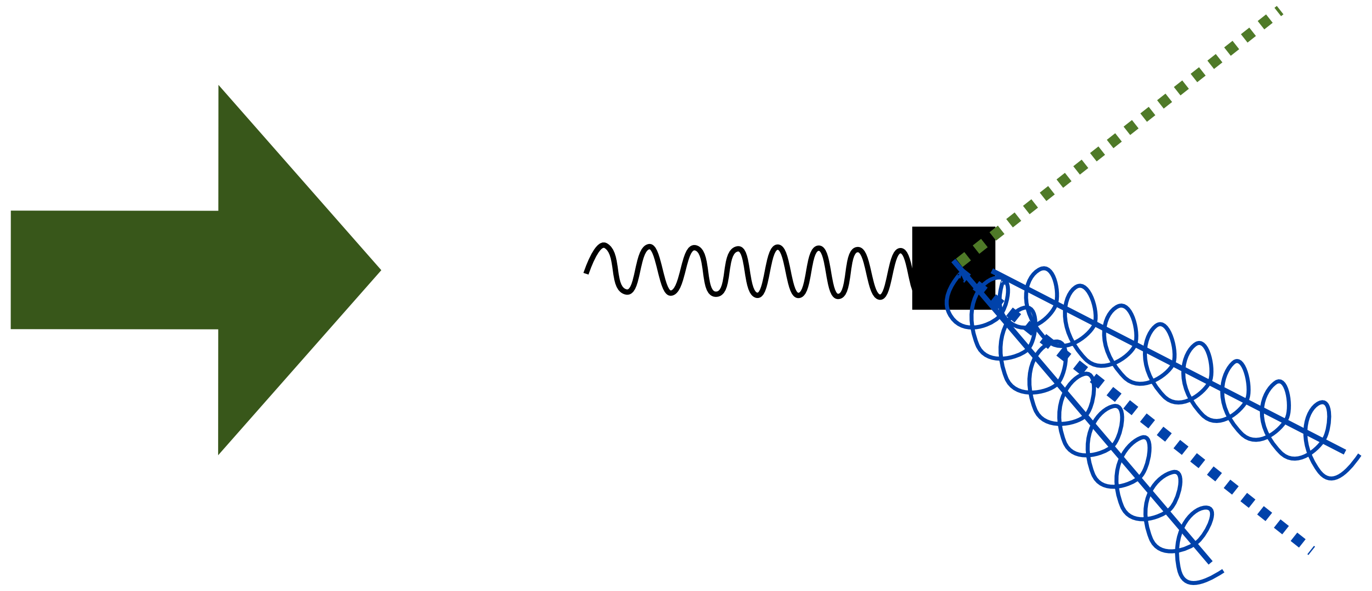


$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^b} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S \delta\Big(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big)$$
$$\times J_q(t_J - \mathbf{p}_\perp^2,\mu)\mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu)S(k_S,\mu)$$

Hard and Jet Functions

Hard function:

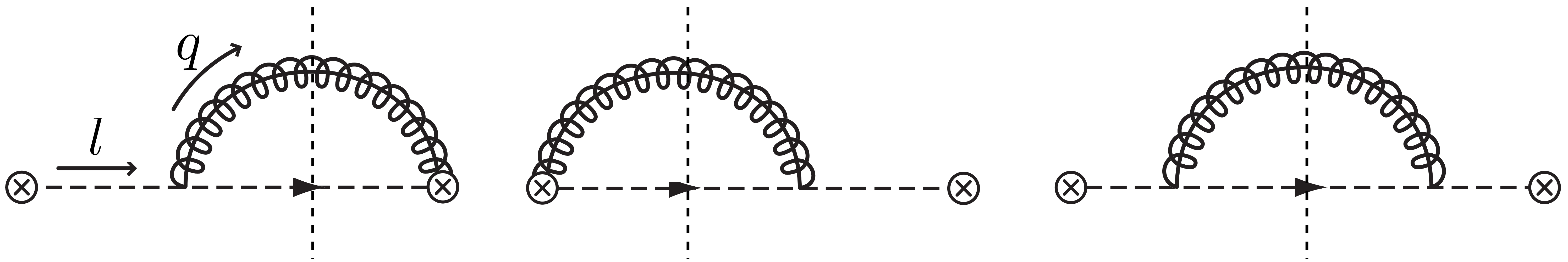




$$H(Q^2,\mu) = 1 + \frac{\alpha_s(\mu)C_F}{2\pi}\left(-\ln^2\frac{\mu^2}{Q^2} - 3\ln\frac{\mu^2}{Q^2} - 8 + \frac{\pi^2}{6}\right) + \ldots$$

known to 3 loops

Jet function:



$$J(t,\mu) = \delta(t) + \frac{\alpha_s(\mu)C_F}{4\pi}\left\{(7-\pi^2)\delta(t) - \frac{3}{\mu^2}\left[\frac{\mu^2\theta(t)}{t}\right]_+ + \frac{4}{\mu^2}\left[\frac{\theta(t)\ln(t/\mu^2)}{t/\mu^2}\right]_+\right\} + \ldots$$

known to 3 loops

Beam Function and PDFs

transverse momentum dependent beam function:

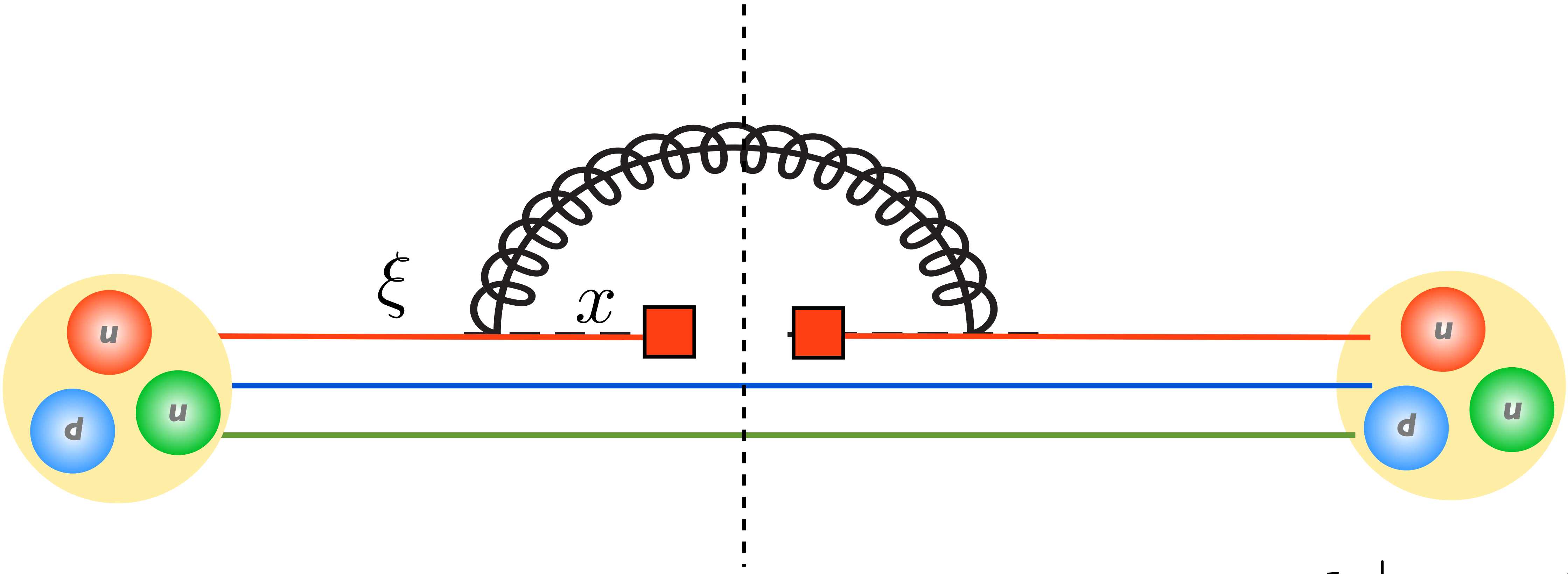
$$B(\omega k^+, x, k_\perp^2, \mu) = \frac{\theta(\omega)}{\omega} \int \frac{dy^-}{4\pi} e^{ik^+ y^-/2} \langle P_n(P^-)| \bar{\chi}_n\Big(y^- \frac{n}{2}\Big) \delta(xP^- - \bar{n}\cdot\mathcal{P}) \delta(k_\perp^2 - \mathcal{P}_\perp^2) \chi_n(0) |P_n(P^-)\rangle$$

match onto PDF

$$f(x,\mu) = \theta(\omega) \langle P_n(P^-)| \bar{\chi}_n(0) \delta(xP^- - \bar{n}\cdot\mathcal{P}) \chi_n(0) |P_n(P^-)\rangle$$

$$\mathcal{B}_q(t, x, \mathbf{k}_\perp^2, \mu) = \sum_j \int_x^1 \frac{d\xi}{\xi} \mathcal{I}_{ij}\Big(t, \frac{x}{\xi}, \mathbf{k}_\perp^2, \mu\Big) f_j(\xi, \mu)$$

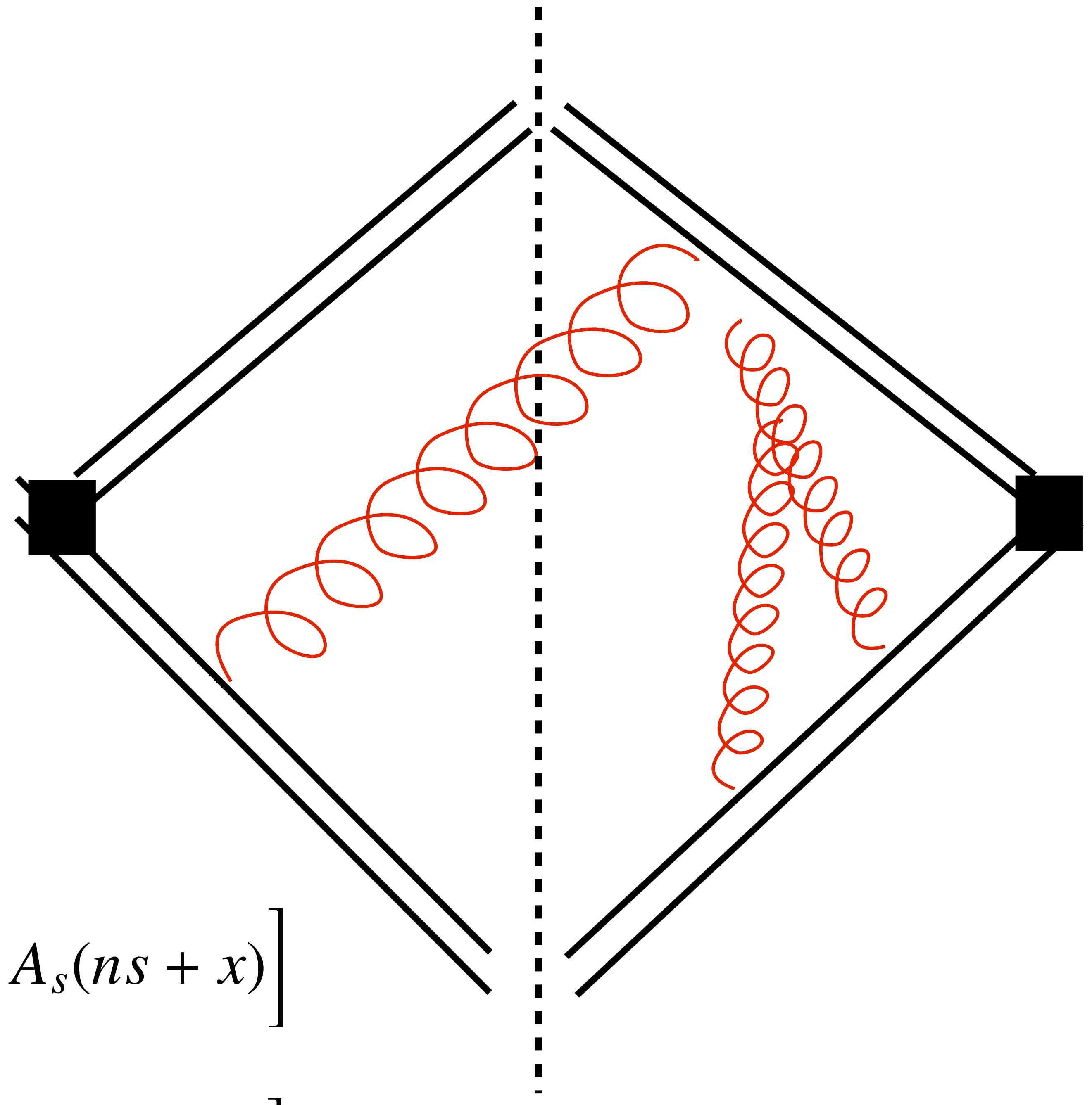
known to 2 loops;
anomalous dimension
known to 3 loops



Measure small light-cone momentum $k^+ = t/P^-$
and transverse momentum $\mathbf{k}_\perp$
of initial state radiation

Soft function

- Soft functions for e+e- dijets, DIS 1-jettiness, and *pp* beam thrust:



$$S_2(\ell_1,\ell_2,\mu) = \frac{1}{N_C}\,\mathrm{Tr}\sum_{i\in X_s}\left|\langle X_s|\,T[Y_n^{\pm\dagger}(0)Y_{\bar n}^{\pm}(0)]\,|0\rangle\right|^2$$

$$\times\,\delta\Big(\ell_1-\sum_{i\in X_s}\theta(\bar n\cdot k_i-n\cdot k_i)n\cdot k_i\Big)\delta\Big(\ell_2-\sum_{i\in X_s}\theta(n\cdot k_i-\bar n\cdot k_i)\bar n\cdot k_i\Big),$$

e⁺e⁻:	$++$
DIS:	$--$
***pp*:**	$+-$

$$Y_n^{+\dagger}(x) = P\exp\Big[ig\int_0^{\infty} ds\, n\cdot A_s(ns+x)\Big]$$

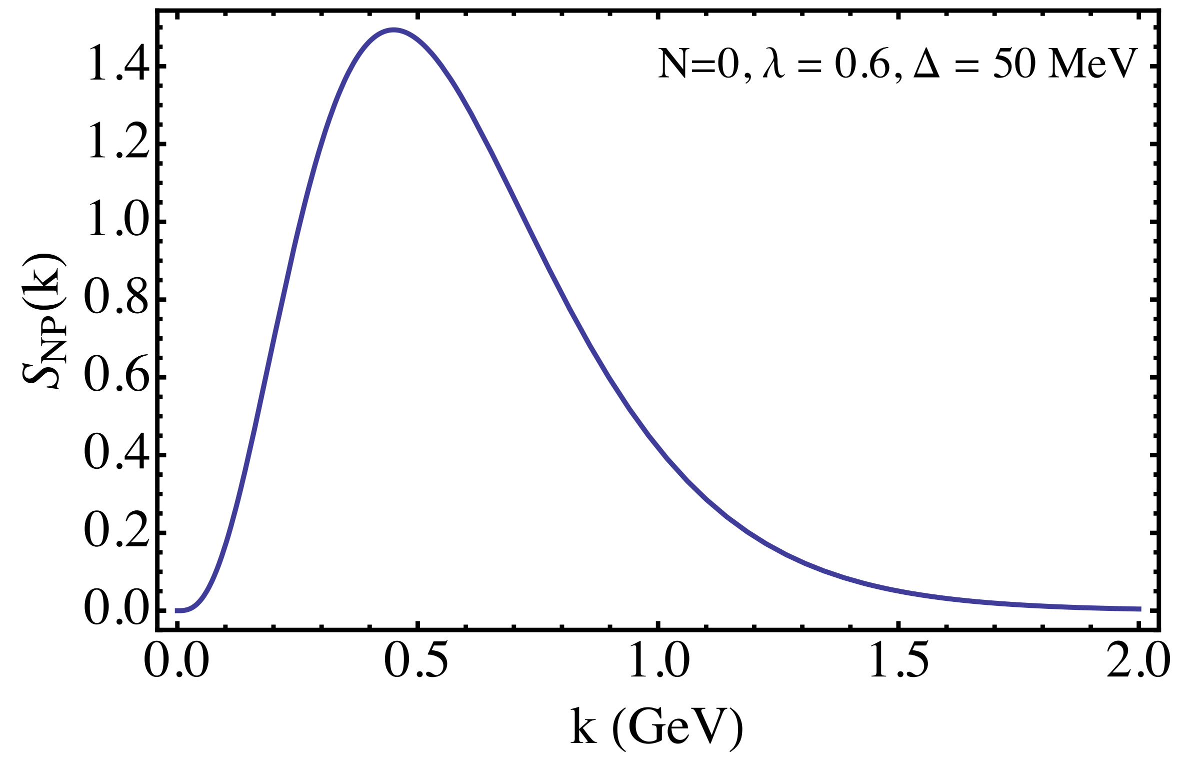
$$Y_n^{-}(x) = P\exp\Big[ig\int_{-\infty}^{0} ds\, n\cdot A_s(ns+x)\Big],$$

- Perturbatively, it is known that $\boxed{S_2^{ee}=S_2^{ep}=S_2^{pp}}$ to at least $\mathcal{O}(\alpha_s^2)$

D. Kang, Labun, CL (2015);
Boughezal, Liu, Petriello (2015)

$$\int S(k_s,\mu) = 1 + \frac{\alpha_s(\mu)}{4\pi}\left\{-\frac{8C_F}{1-a}\ln^2\frac{k}{\mu} + C_s^1(a)\right\}$$

Nonperturbative corrections



- In general, soft function expressed as convolution of perturbative part and nonperturbative shape function:

$$S(k_S,\mu) = \int dl\, S_{\rm PT}(k_S - l,\mu) S_{\rm NP}(l)$$

- For large enough $\tau\ (k_S)$, leading effect is a shift:

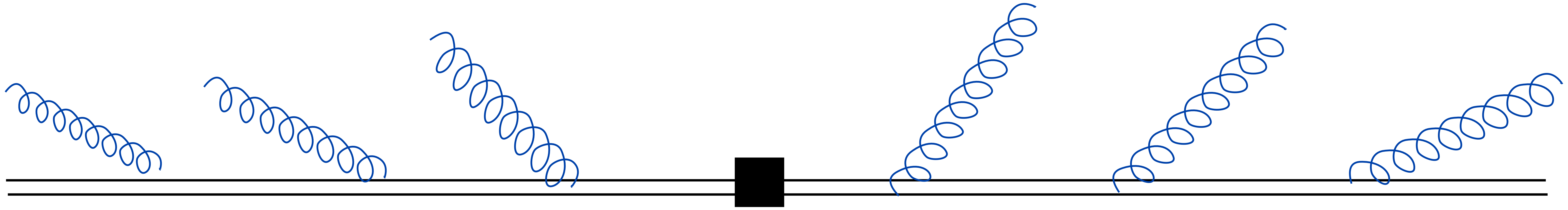
$$\langle e\rangle = \langle e\rangle_{\rm PT} + c_e \frac{\Omega_1}{Q}$$

c_e observable dependent, calculable coefficient

Ω_1 universal nonperturbative parameter

- Rigorous proof (and **field theory** definition of Ω_1) from factorization theorem and boost invariance of soft radiation:

$$\Omega_1 = \frac{1}{N_C}{\rm Tr}\langle 0|\overline{Y}_{\bar n}^\dagger Y_n^\dagger \underset{\text{“energy flow” operator}}{\underset{\downarrow}{\mathcal{E}_T(\eta)}} Y_n \overline{Y}_{\bar n}|0\rangle$$



soft radiation sees only direction, not energy, of original collinear partons, invariant to boosts along z

Evolution and resummation

- Easier to discuss in terms of Laplace transforms (or Fourier transforms to position space)

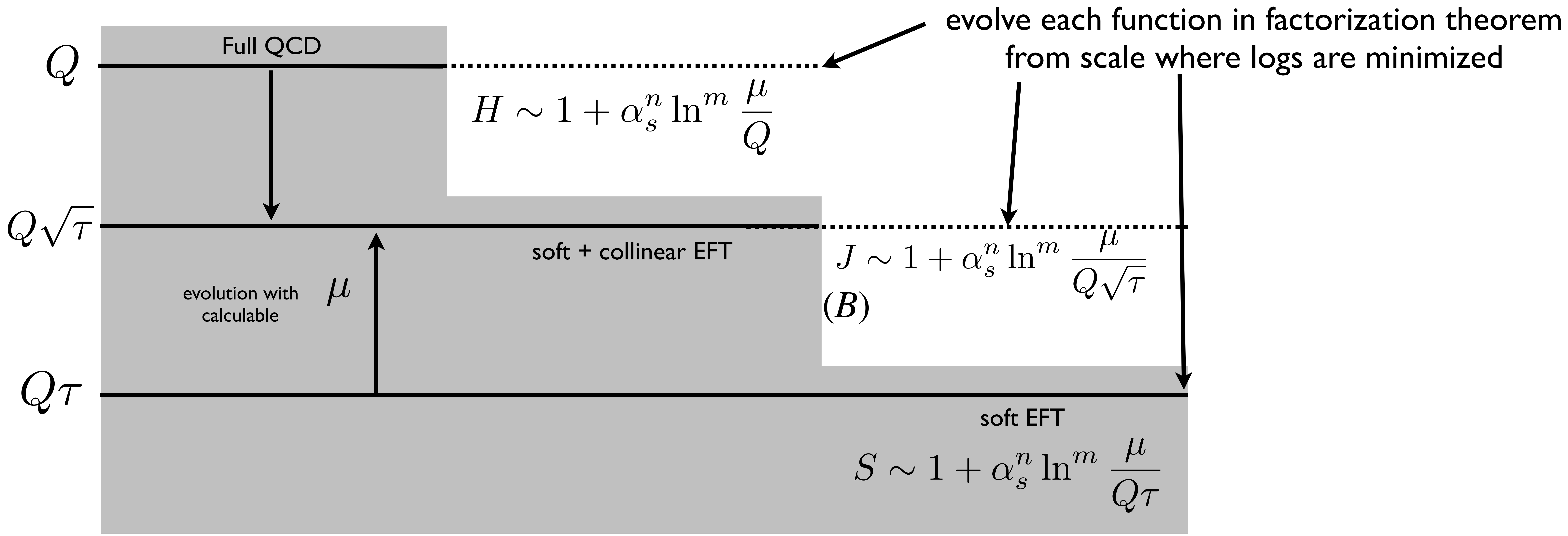
$$\tilde{\sigma}(\nu) = \int_0^\infty d\tau\, e^{-\nu\tau}\sigma(\tau)$$

- Turns factorization theorem into a simple product:

$$\tilde{\sigma}(\nu) = H(Q^2,\mu)\tilde{J}(Q^2/\nu,\mu)\tilde{B}(Q^2/\nu,x,\mu)\tilde{S}(Q/\nu,\mu)$$

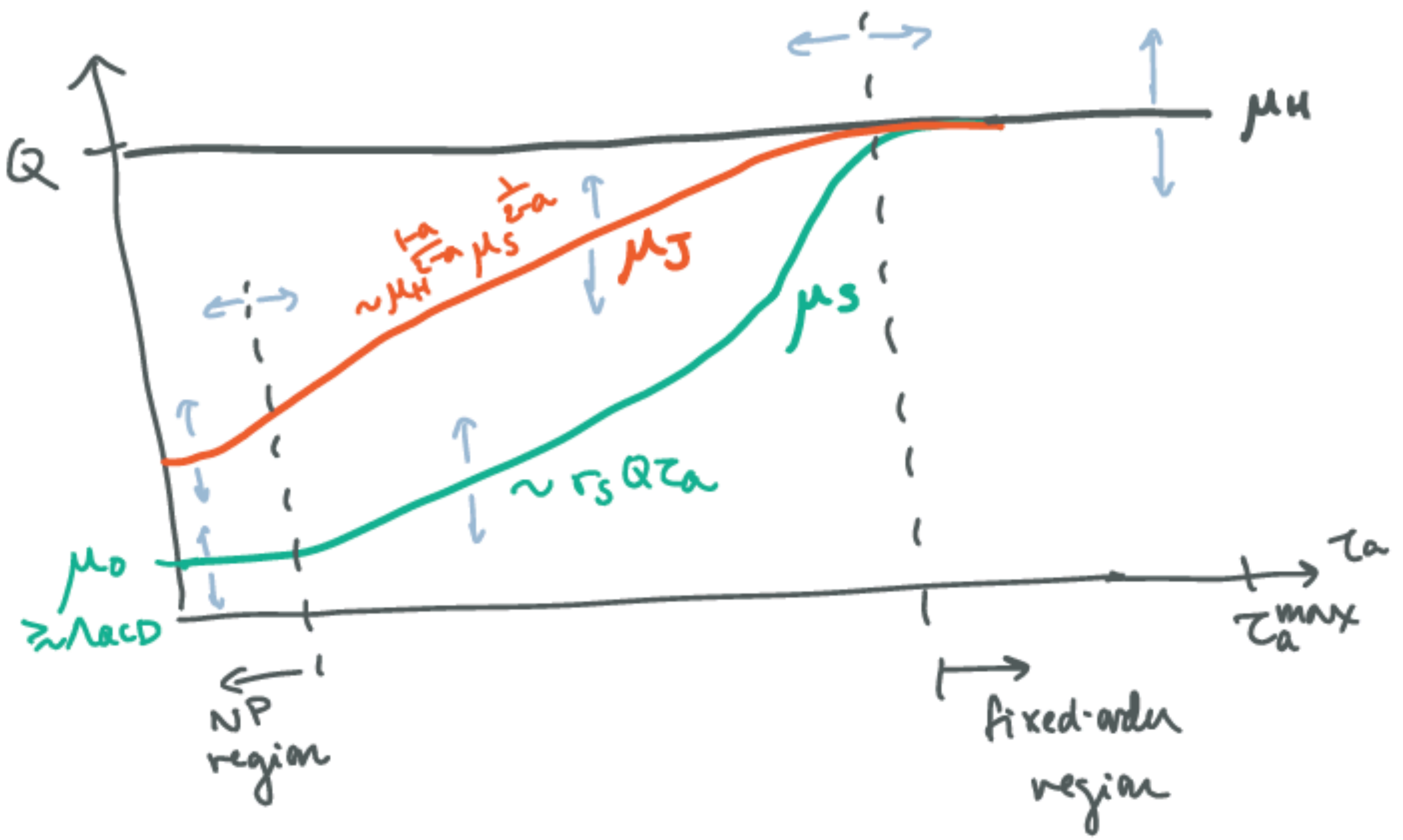
- RGE obeyed by Laplace-space jet and soft functions:

$$\mu\frac{d}{d\mu}\tilde{F}(Q^j/\nu,\mu) = \gamma_F(Q^j/\nu,\mu)\tilde{F}(Q^j/\nu,\mu)$$



Scale profiles

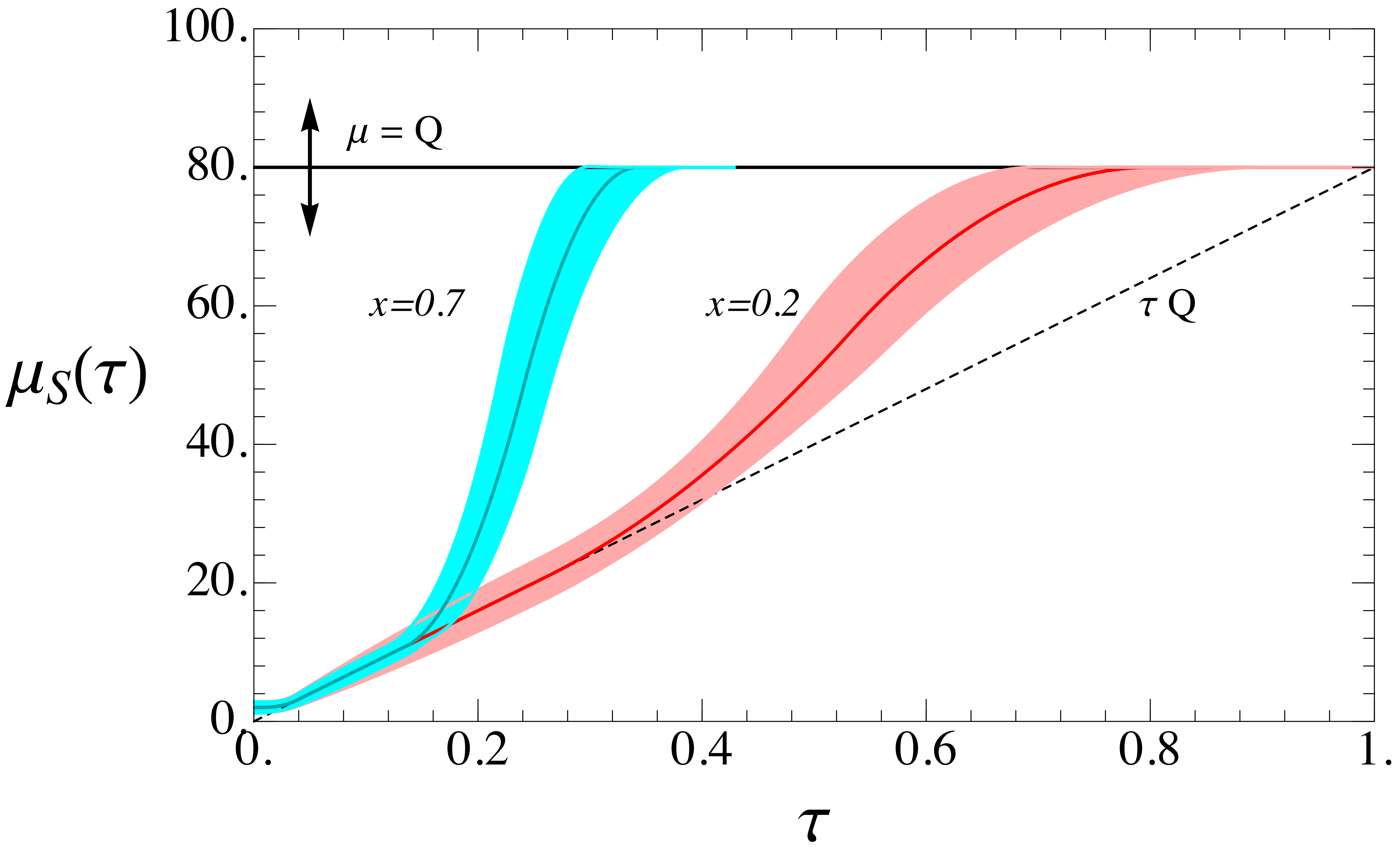
Freedom to choose μ_H, μ_J, μ_S (and μ_{ns}, R) allows not only log resummation but robust estimate of perturbative theory uncertainties in each region:



- allow scales to merge in fixed order region before $\tau_a = \tau_a^{max}$ (< 1)
- stable NP region to convolve shape function
- variation of all parameters to fully probe theory uncertainty

Scale profiles

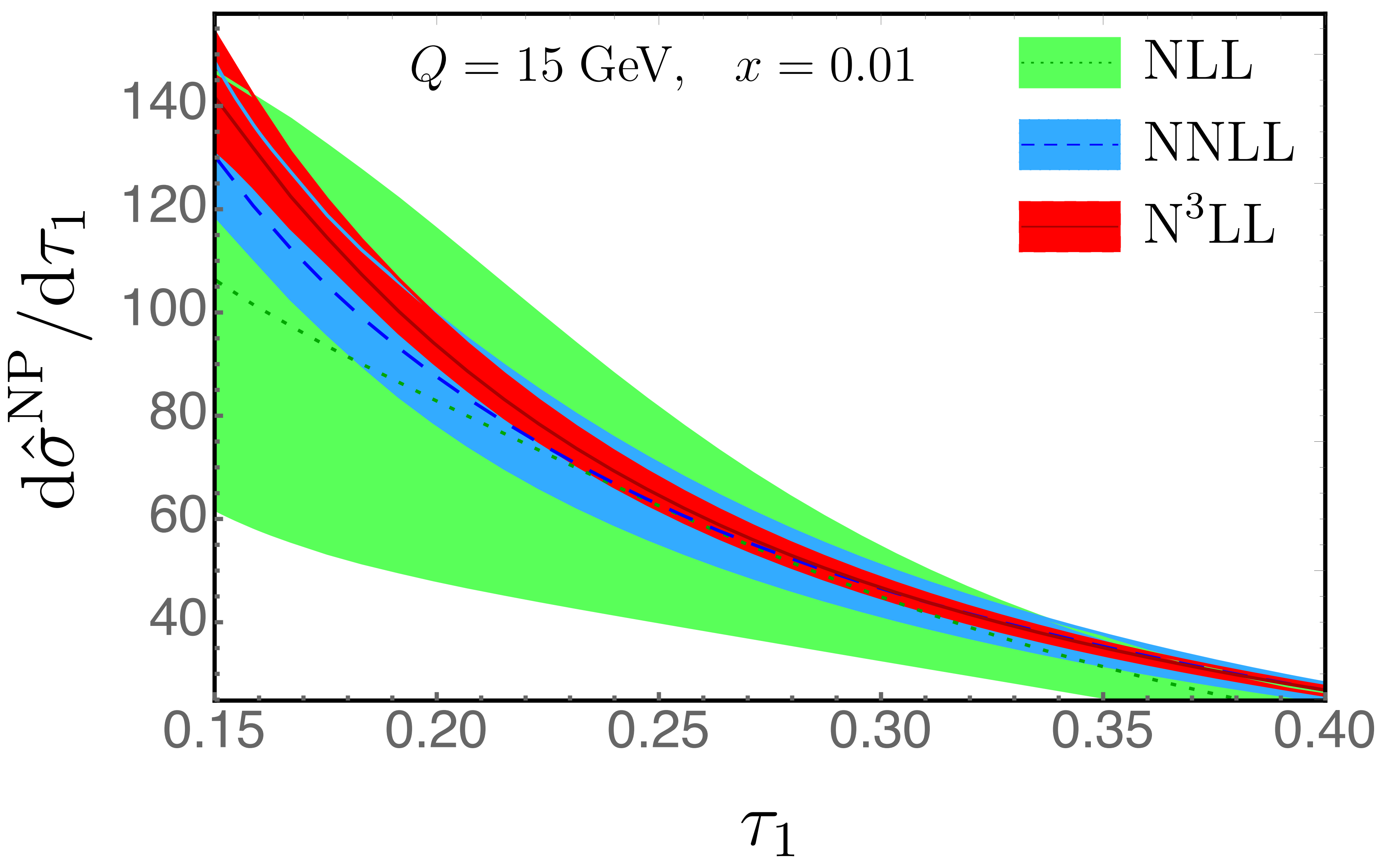
For DIS, these regions depend on *x*, e.g.:

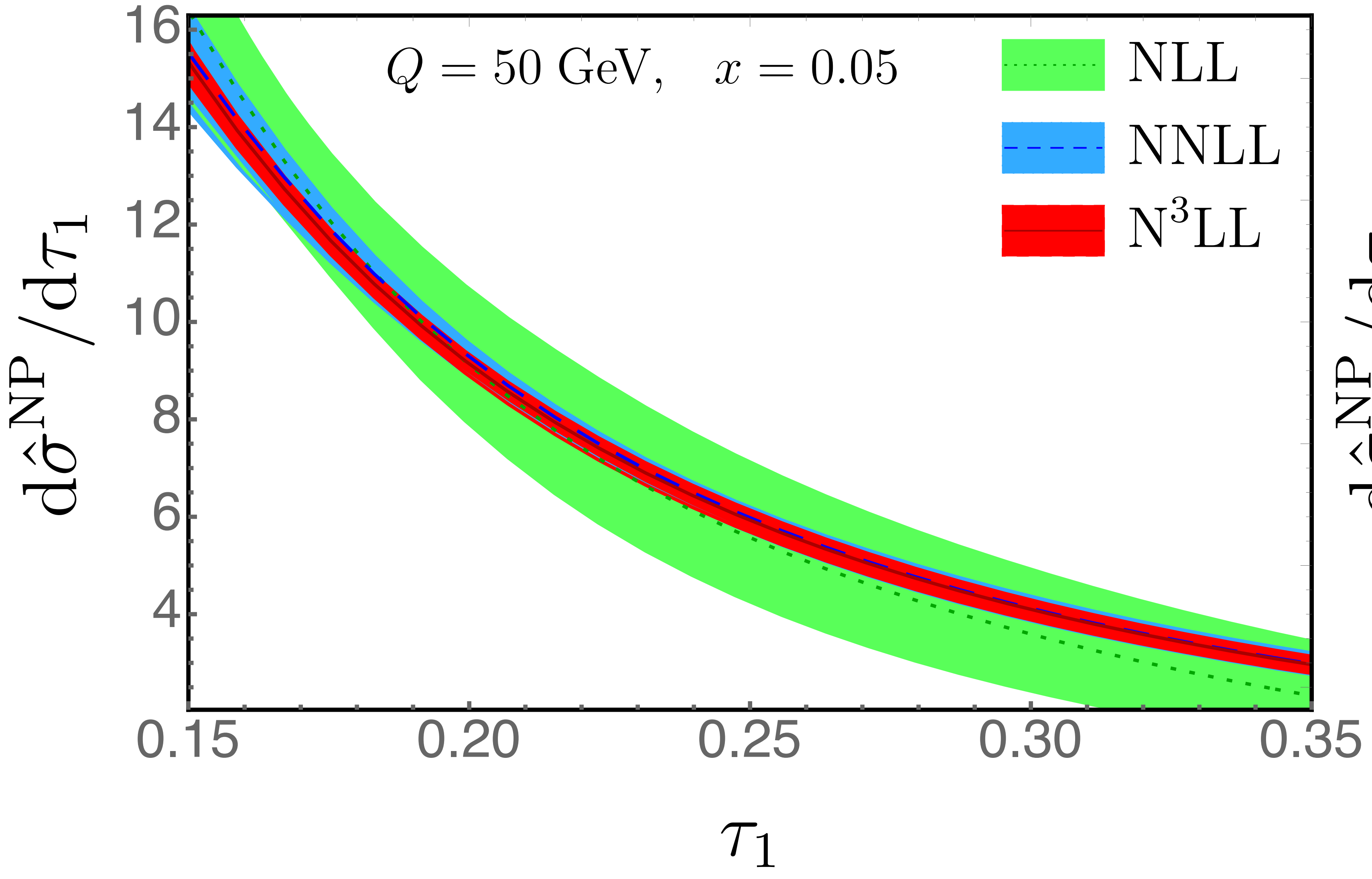


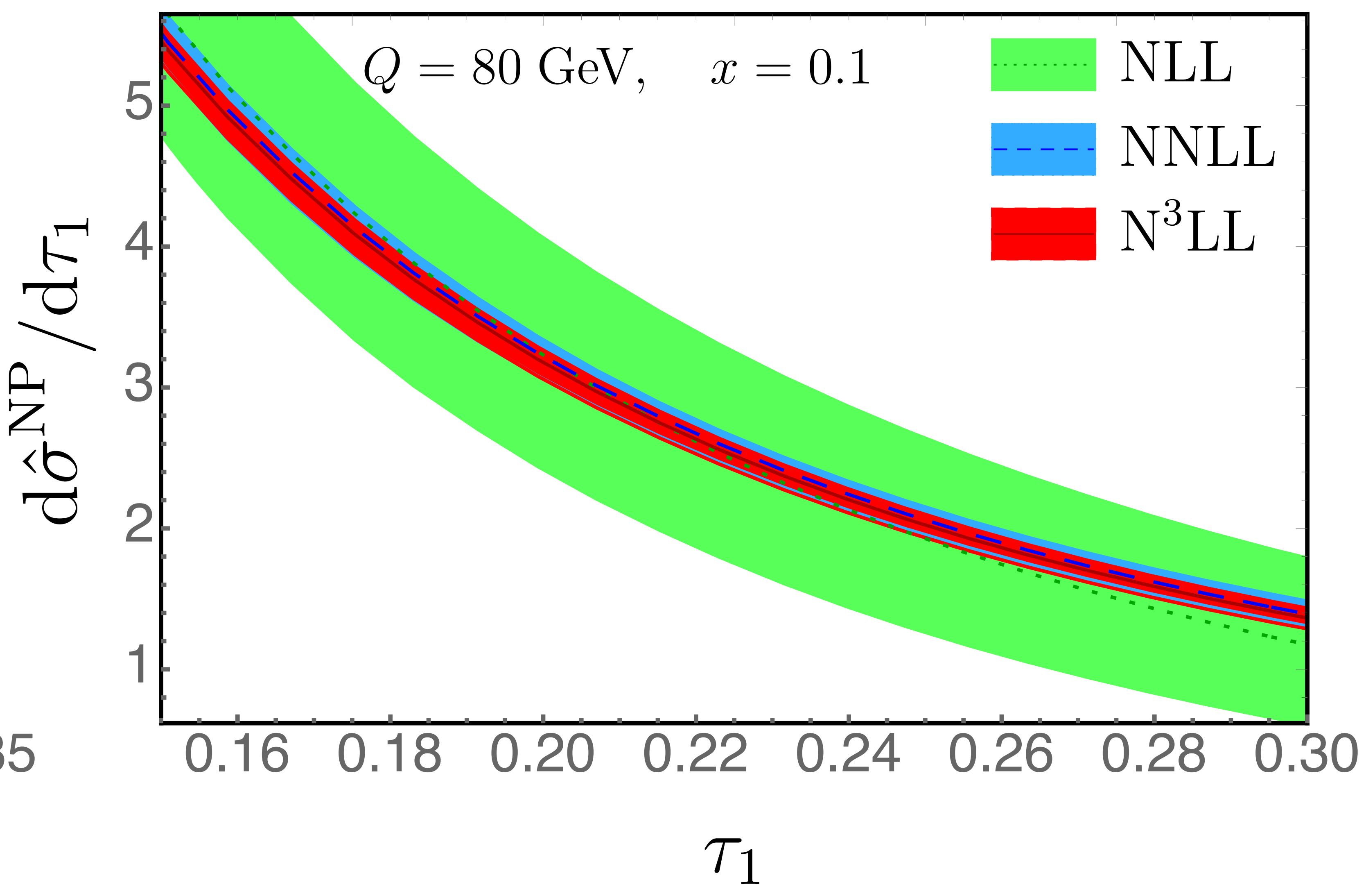
DIS thrust cross sections

Resummed to N³LL accuracy:

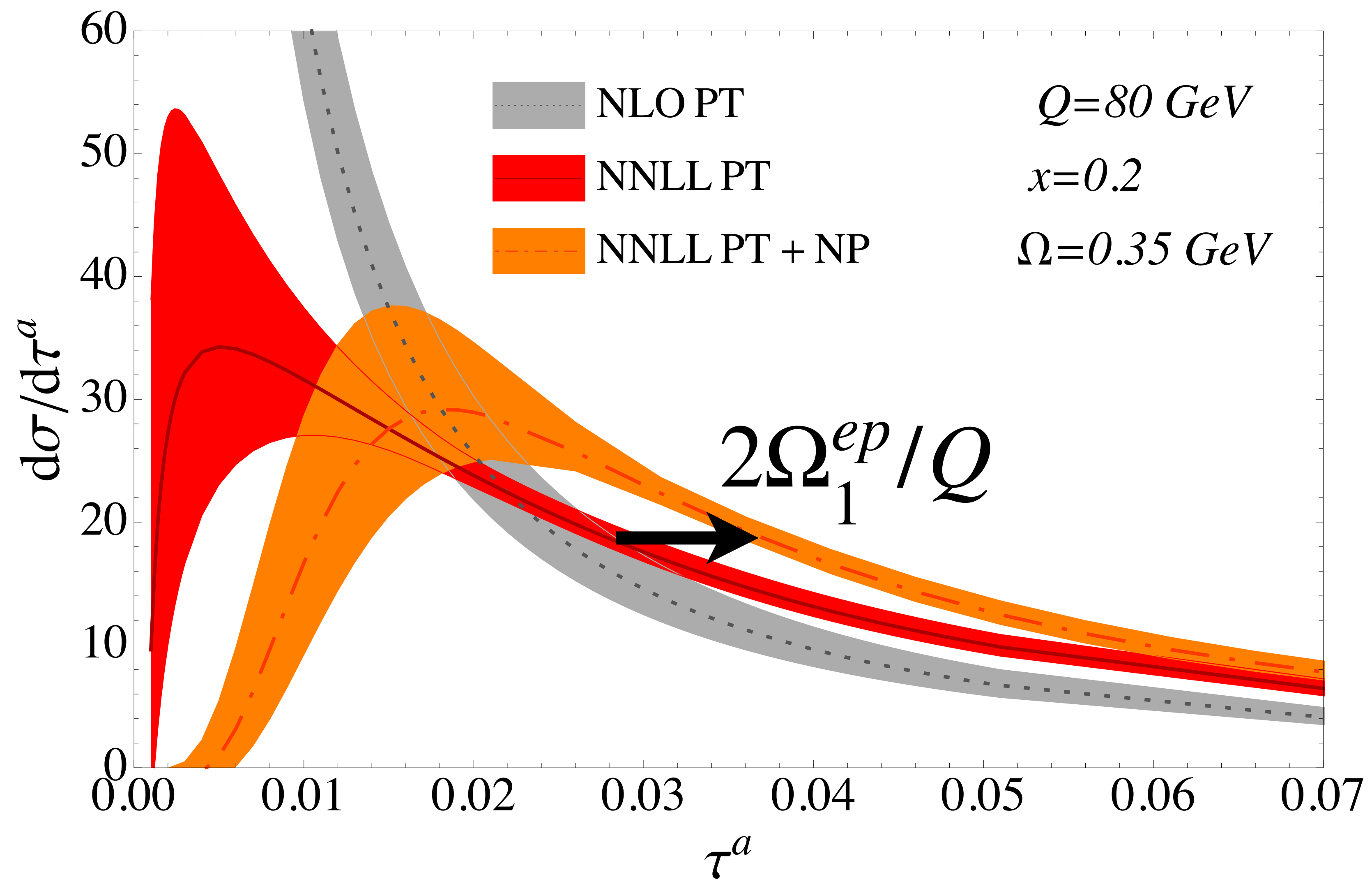
D. Kang, CL, Stewart (2015 and in progress)







include a nonperturbative shape function:



can be related to other DIS event shapes

D. Kang, CL, Stewart (2013)

or even *pp* to 1 jet observables

Stewart, Tackmann, Waalewijn (2014)

DIS thrust cross sections

Tail region, fixed *x*, low to high *Q*:

DIS thrust cross sections

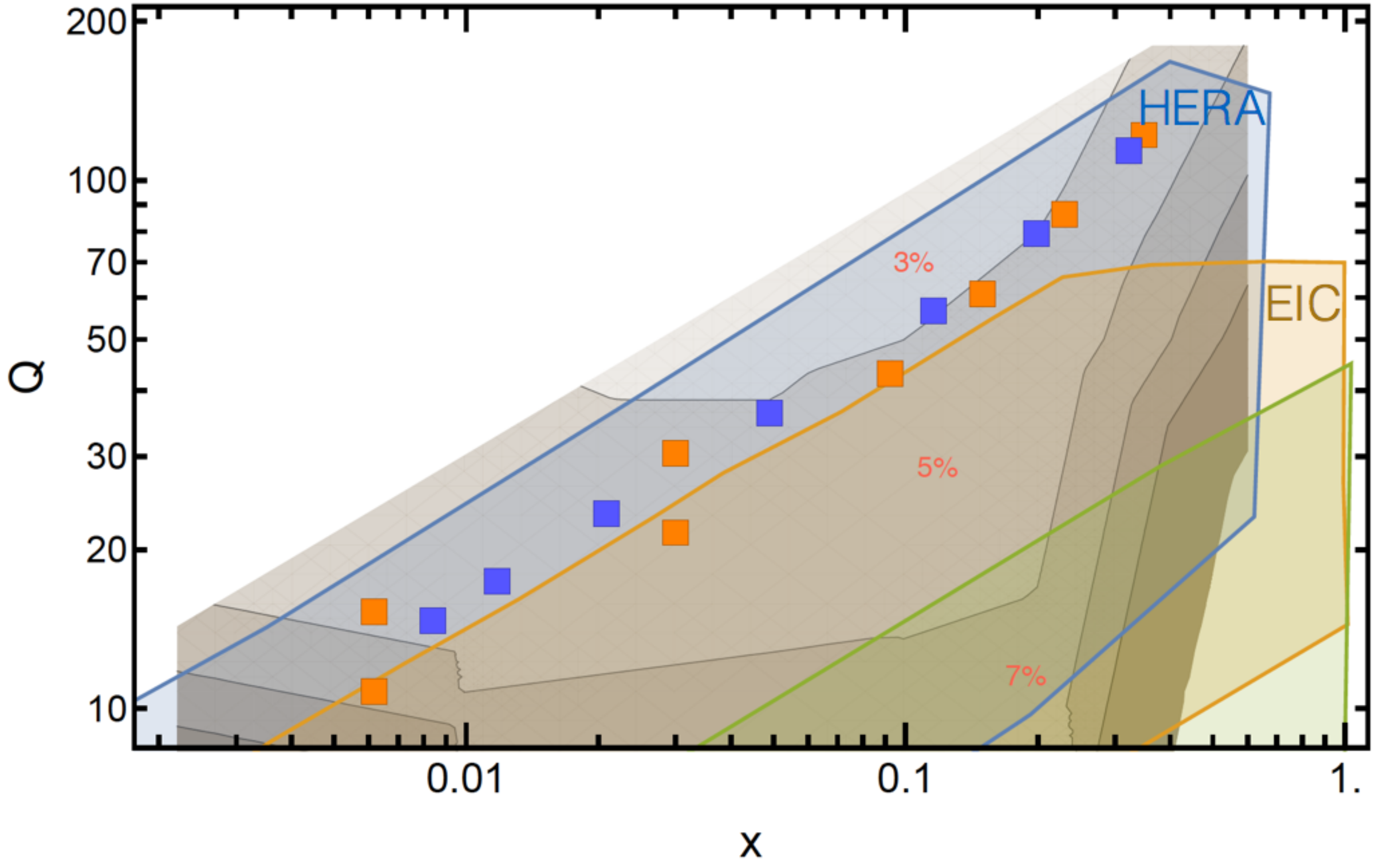
Tail region, fixed *Q*, low to high *x*:

DIS thrust cross sections

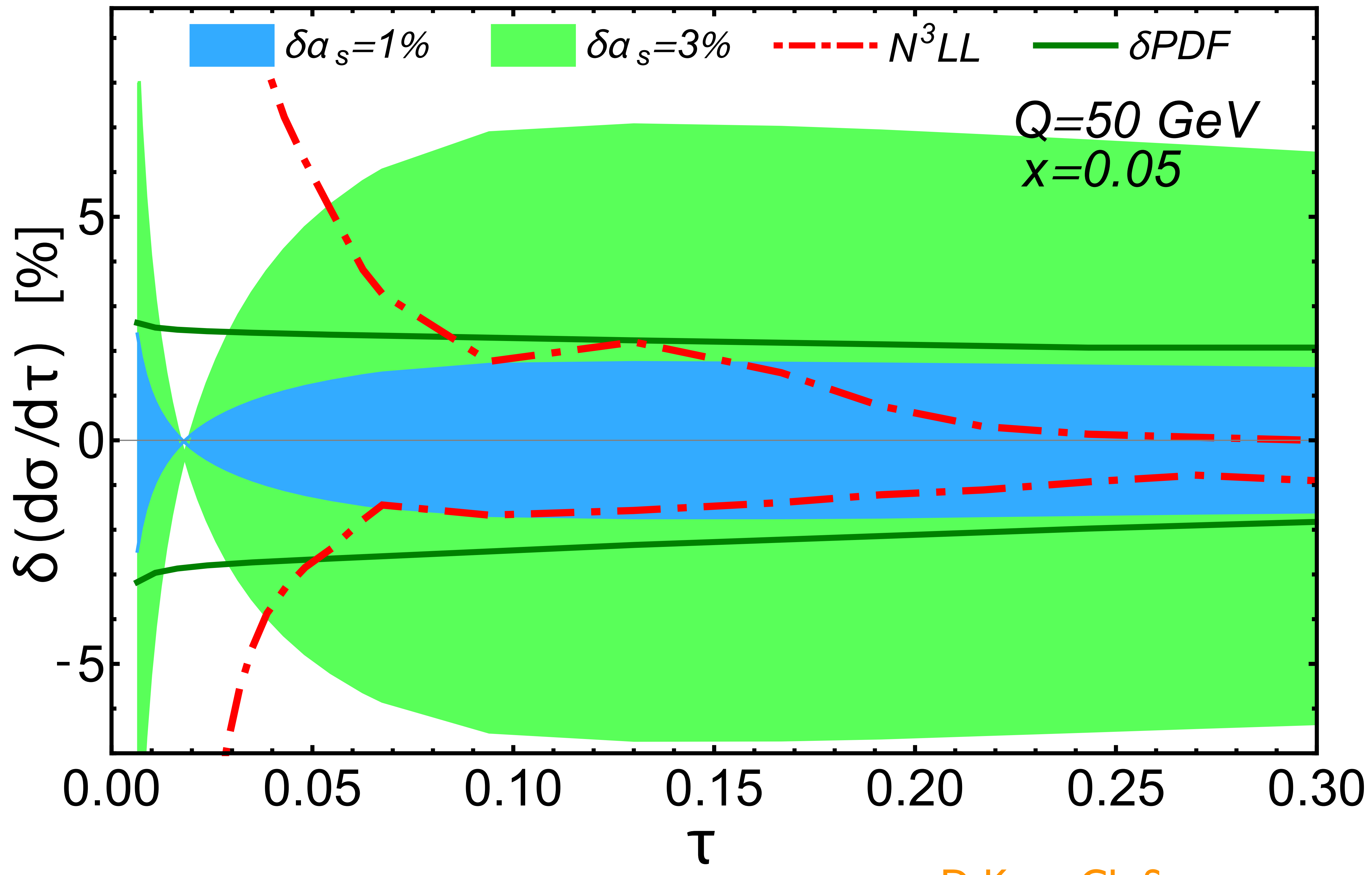
Full distribution:

Experimental sensitivity and strong coupling

Current theoretical uncertainty
vs. HERA or EIC coverage:



Current theoretical uncertainty on the order of 1% sensitivity to α_s and PDF uncertainties:



D. Kang, CL, Stewart
(2015 and in progress)

Outlook

- N^{3}LL resummed predictions for DIS thrust to be published soon.
- Event shapes in DIS promising candidates for precision determination of strong coupling, PDFs, and hadronization corrections
- Results from HERA or an EIC may shed light on "low" value of LEP event shape determinations of strong coupling