Collinear matching for TMD distributions in the background field method

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based on JHEP05 (2019) 125

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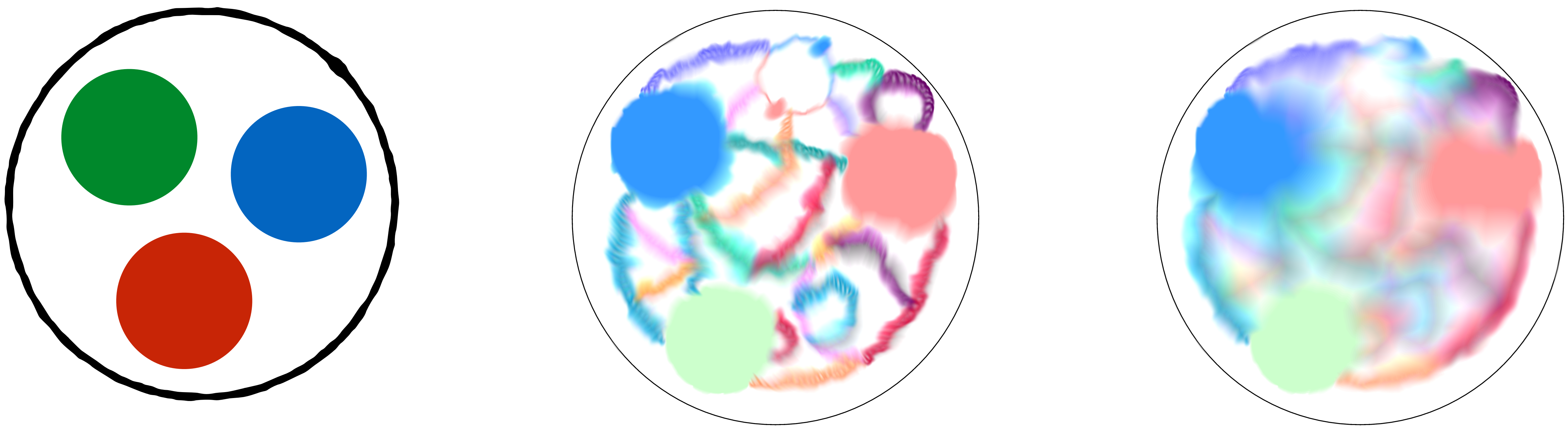
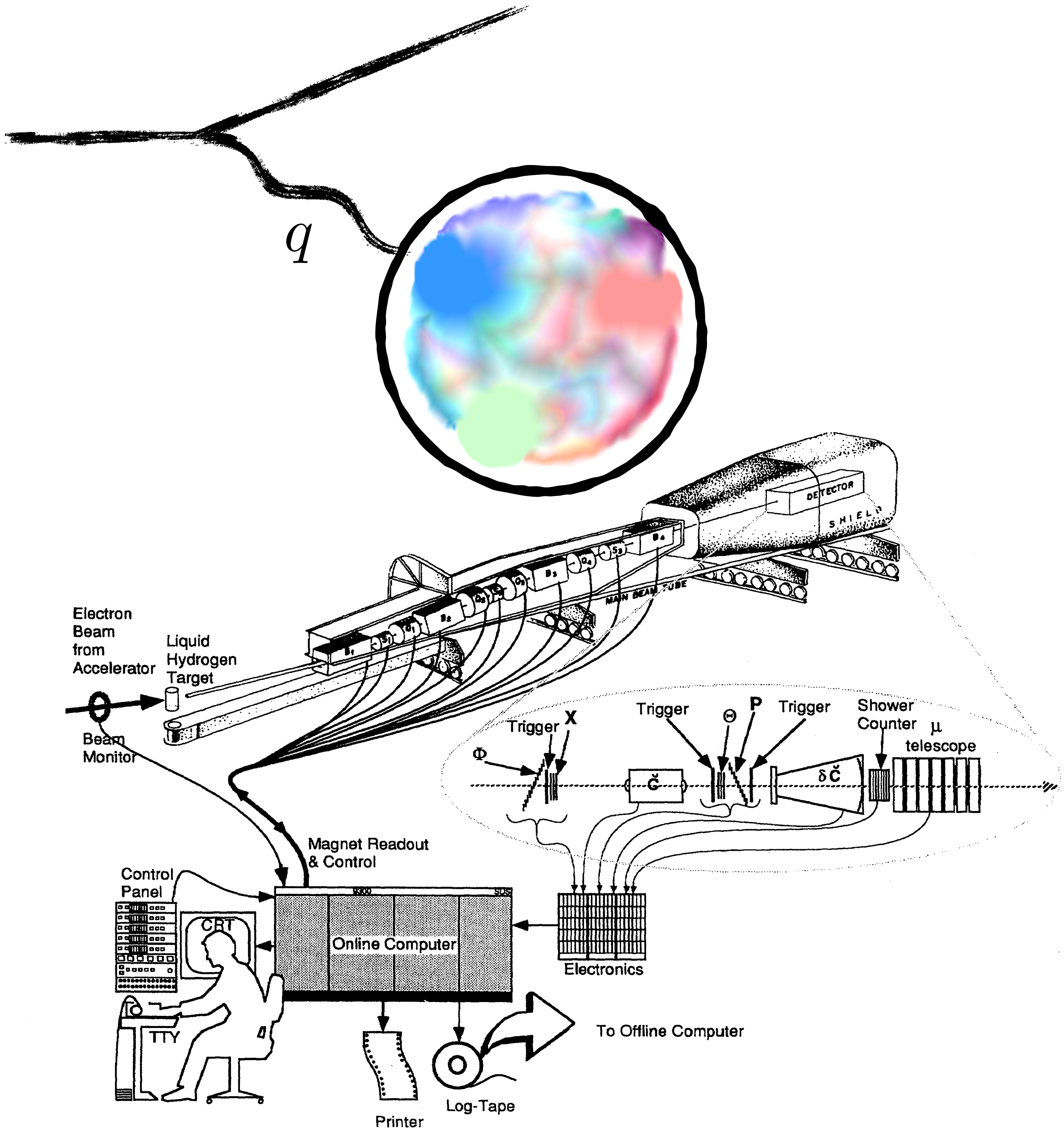
Hadron as a many-body parton system

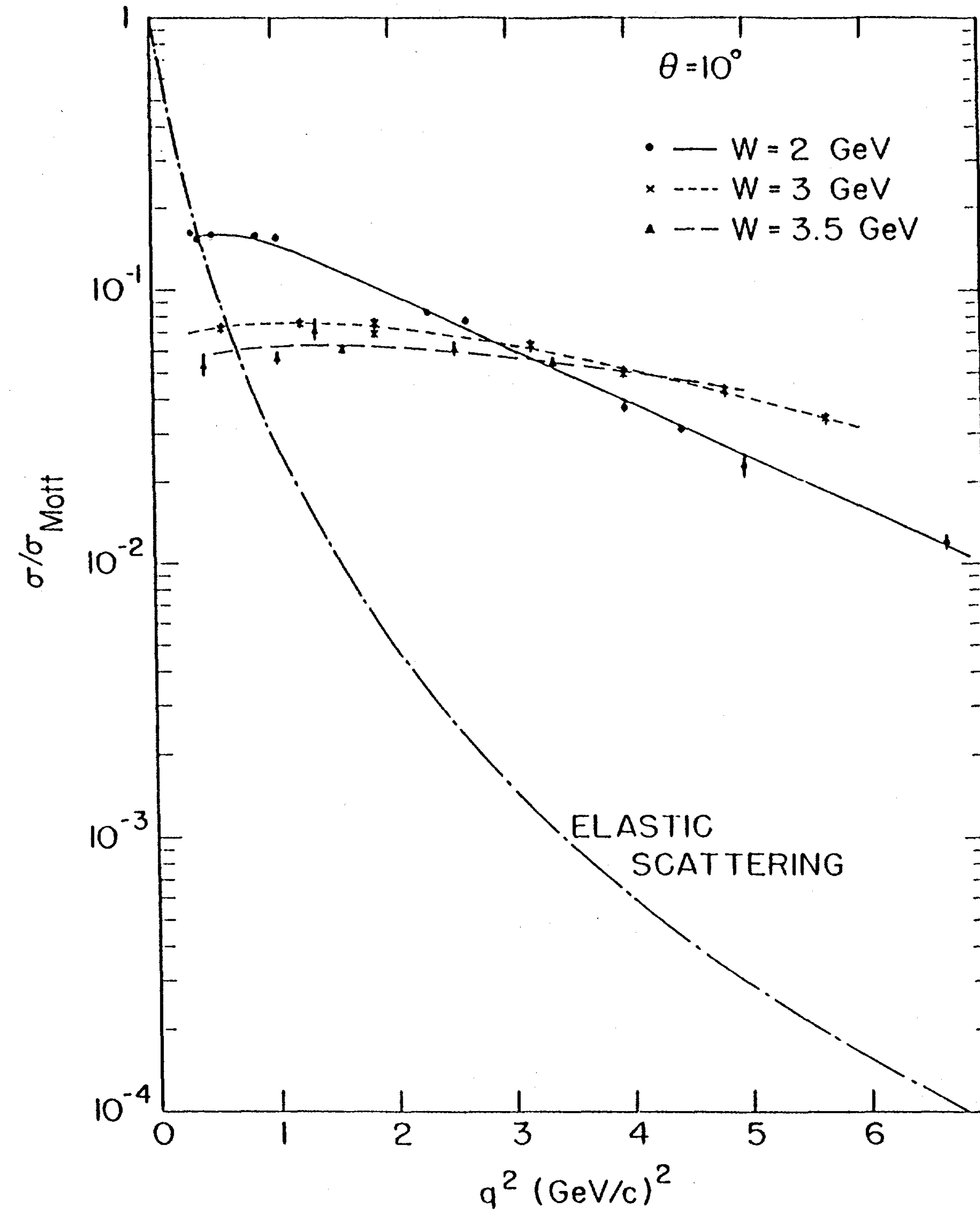
$1\ fm \sim 5\ GeV^{-1}$



- The typical hadron scale is small
- Due to running of the coupling constant one has to understand the hadron as a many-body parton system which is strongly bonded
- Hadron is characterized by complex dynamics of parton interactions
- How do we study this system?

High-energy probe (external scale)

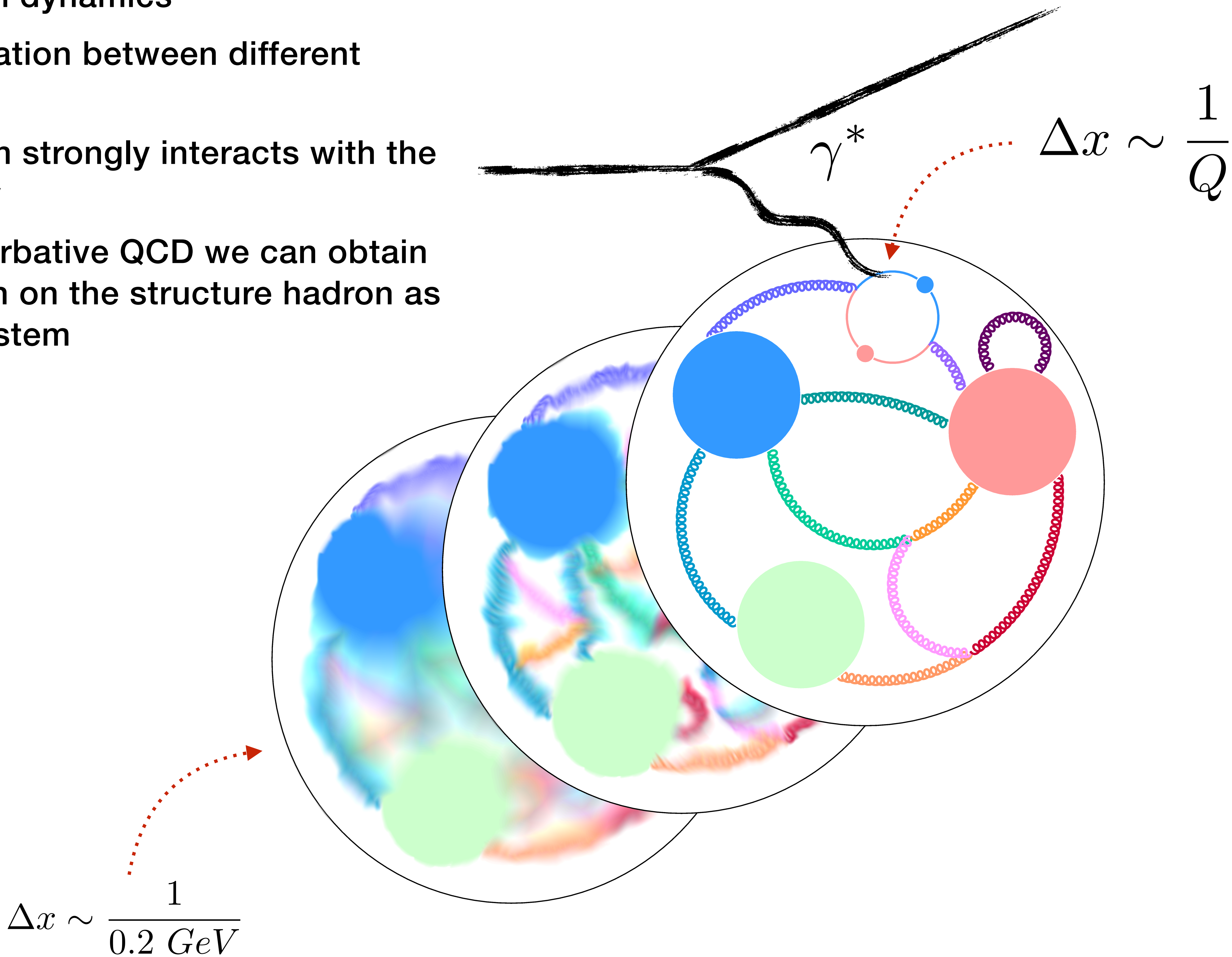




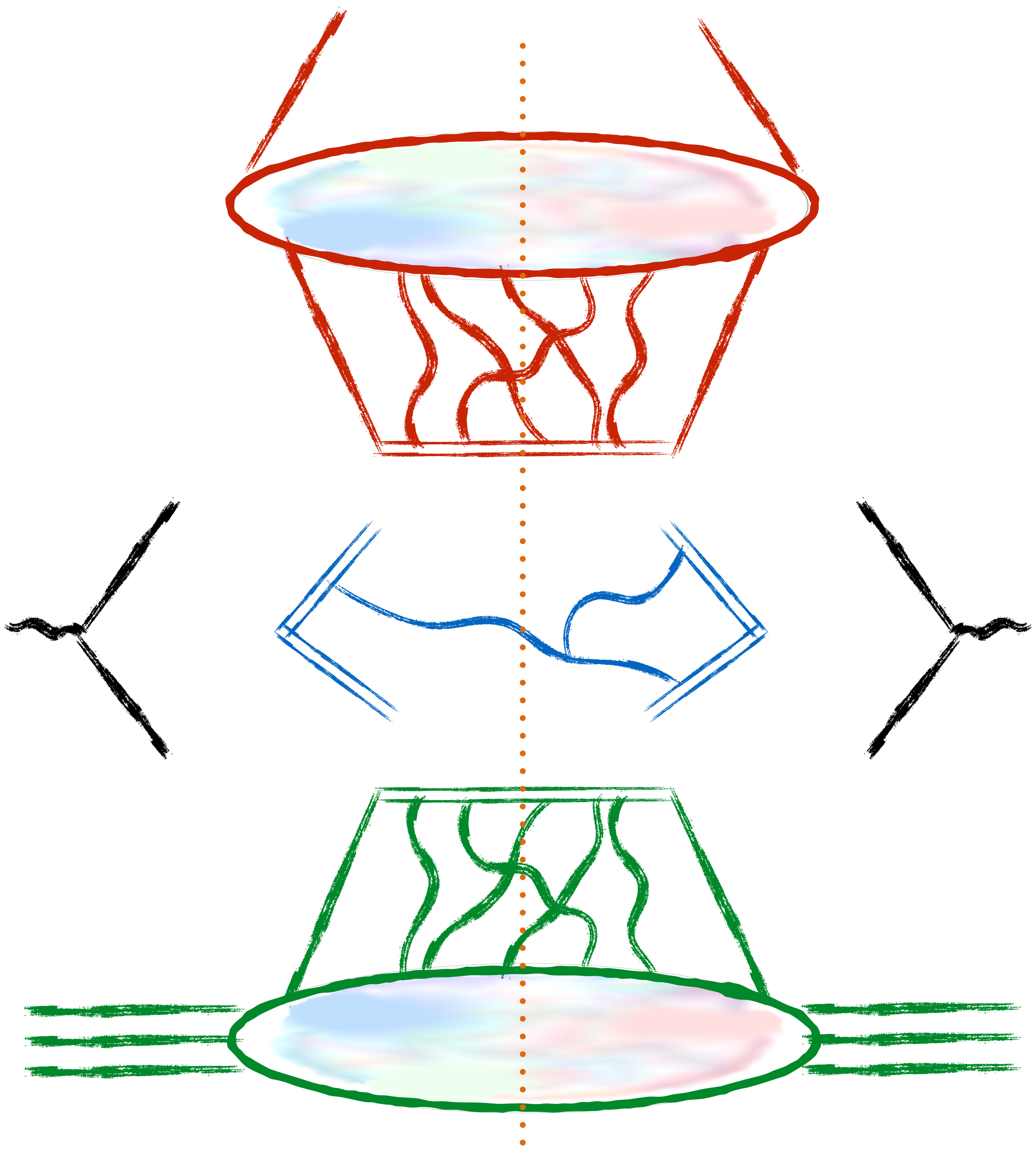
- How does the strongly bounded system respond to a high-energy probe?
- The existence of individual quarks in the system was discovered
- It became the foundation of the parton model and perturbative QCD

Hadron as a many-body parton system

- Interacting with external probe the hadron reveals different types of parton dynamics
- There is a strong correlation between different phases
- The dense QCD medium strongly interacts with the probe in dynamical way
- Using methods of perturbative QCD we can obtain very precise information on the structure hadron as a many-body parton system



Factorization and different types of dynamics



- Separation of different phases is based on TMD factorization

J.C. Collins, D.E. Soper and G. Sterman, Phys. Lett. B 109 (1982) 388;
J.C. Collins, D.E. Soper and G.F. Sterman, Nucl. Phys. B 250 (1985) 199;
G.T. Bodwin, Phys. Rev. D 31 (1985) 10;
X.-d. Ji, J.-p. Ma and F. Yuan, Phys. Rev. D 71 (2005) 034005;
M.G. Echevarria, A. Idilbi and I. Scimemi, JHEP 07 (2012) 002

Background field method

L.F. Abbott, Acta Phys. Polon. B13 (1982) 33

$$S_{bQCD}(A, B) = S_{QCD}(A + B) - S_{QCD}(B)$$

$$\mathcal{L} = -\frac{1}{2} A_\mu^a \Big(- g^{\mu\nu} \mathcal{D}^2 - 2[\mathcal{D}^\mu, \mathcal{D}^\nu] \Big)^{ab} A_\nu^b + \text{int. terms}$$

$$\mathcal{D}_\mu = \partial_\mu - igB_\mu$$

- We can separate different phases of the many body parton system at the level of the QCD Lagrangian
- The method provides a consistent way to take into account interaction of the perturbative phase with a non-perturbative background (many body interactions)

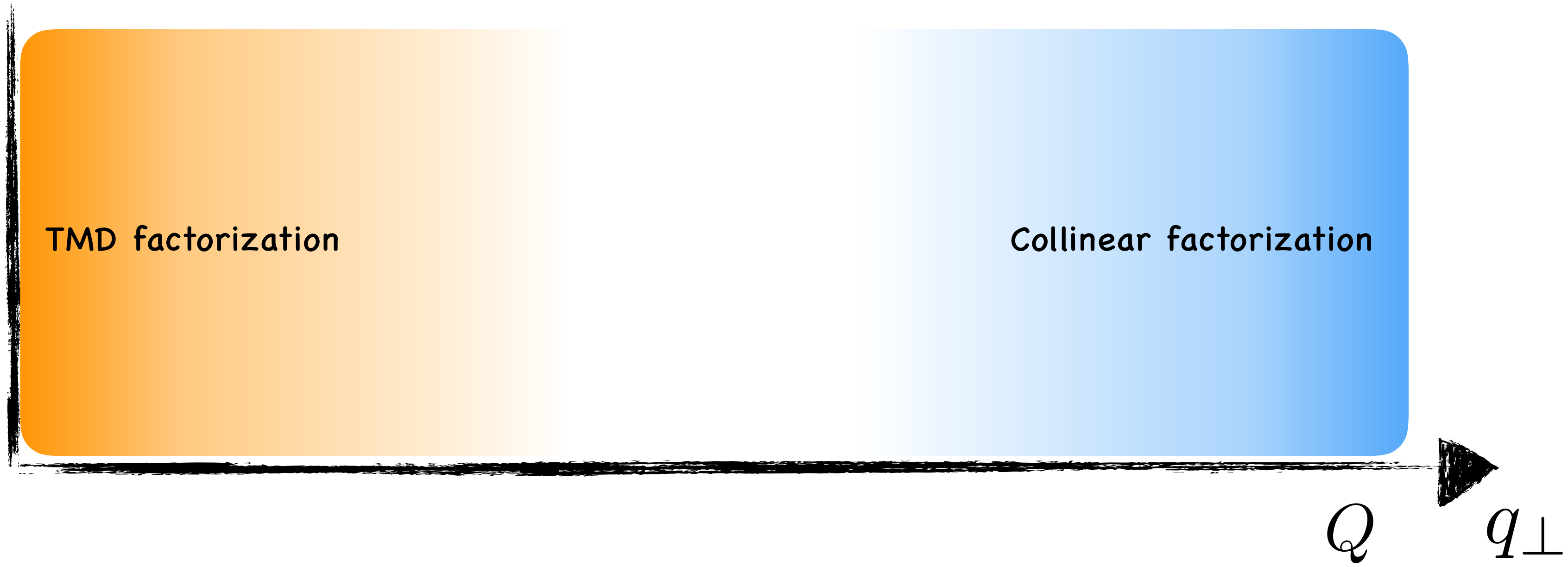
Background field method

$$S_{bQCD}(A,B) = S_{QCD}(A+B) - S_{QCD}(B)$$

$$(x|\frac{1}{\mathcal{P}^2+i\epsilon}|y) = (x|\frac{1}{p^2+i\epsilon} - \frac{1}{p^2+i\epsilon}\{p,B\}\frac{1}{p^2+i\epsilon} + \dots |y)$$

- We can precisely describe interaction between phases using expansions in the background field. An expansion parameter is required
- We can can consider different types of interaction with the non-perturbative parton system
- The expansion can be rearrange into explicitly gauge invariant form

Power corrections to TMD factorization



$$W(\alpha_z,\beta_z,q_\perp) \simeq -\frac{e^2}{8s_W^2c_W^2N_c}\int d^2k_\perp \frac{1}{k_\perp^2(q-k)_\perp^2}\Big[1-2\frac{(k,q-k)_\perp}{Q^2}\Big]$$
$$\times\Big[\Big\{(1+a_u^2)[f_u(\alpha_z)\bar{f}_u(\beta_z)+\bar{f}_u(\alpha_z)f_u(\beta_z)]\Big\}+\Big\{u\leftrightarrow c\Big\}+\Big\{u\leftrightarrow d\Big\}+\Big\{u\leftrightarrow s\Big\}\Big]$$

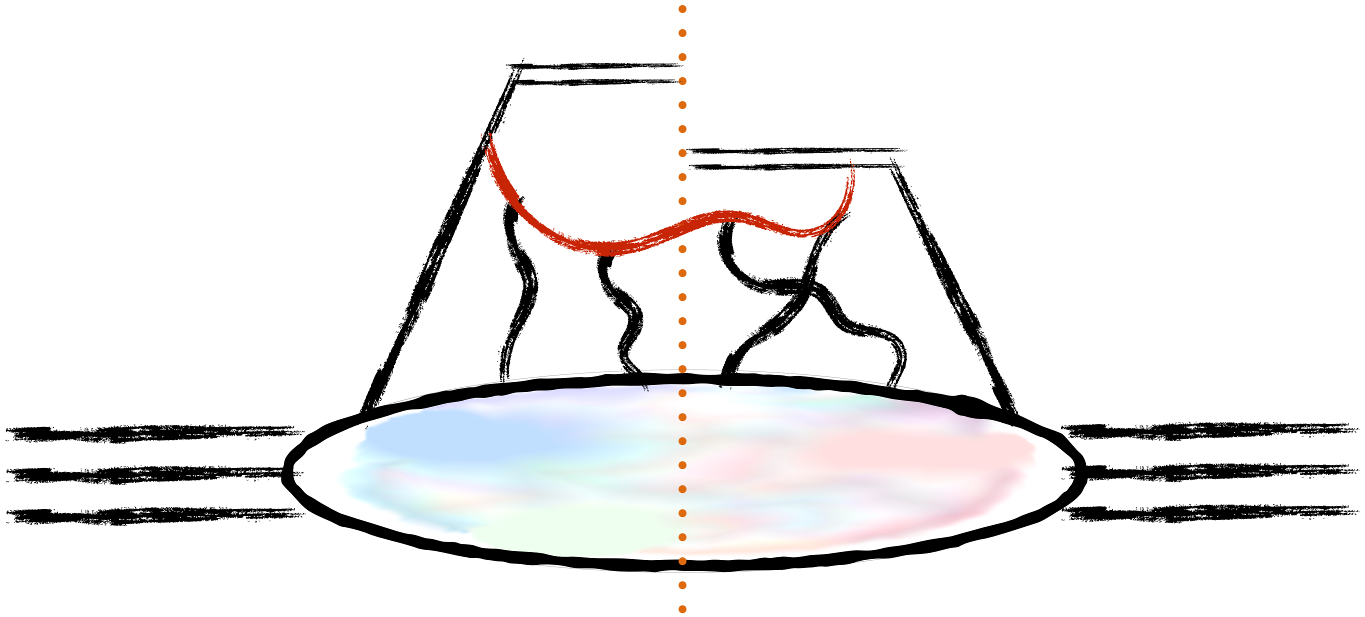
I. Balitsky, A. Tarasov, JHEP 05 (2018) 150

- Hard part of the Drell-Yan process in the background field was calculated.
- We introduced parametrization of the background field
- With certain approximations the structure of corrections gets a very simple form
- We estimate that effects become important at $q_\perp \sim \frac{1}{4}Q$
- The method can be used for analysis of factorization breaking effects in polarized observables

TMD distributions and scale parameters

$$\frac{d}{d\ln\mu^2}F(x,b;\zeta,\mu)=\frac{\gamma(\mu,\zeta)}{2}F(x,b;\zeta,\mu)$$

$$\frac{d}{d\ln\zeta}F(x,b;\zeta,\mu)=-\mathcal{D}(\mu,b)F(x,b;\zeta,\mu)$$

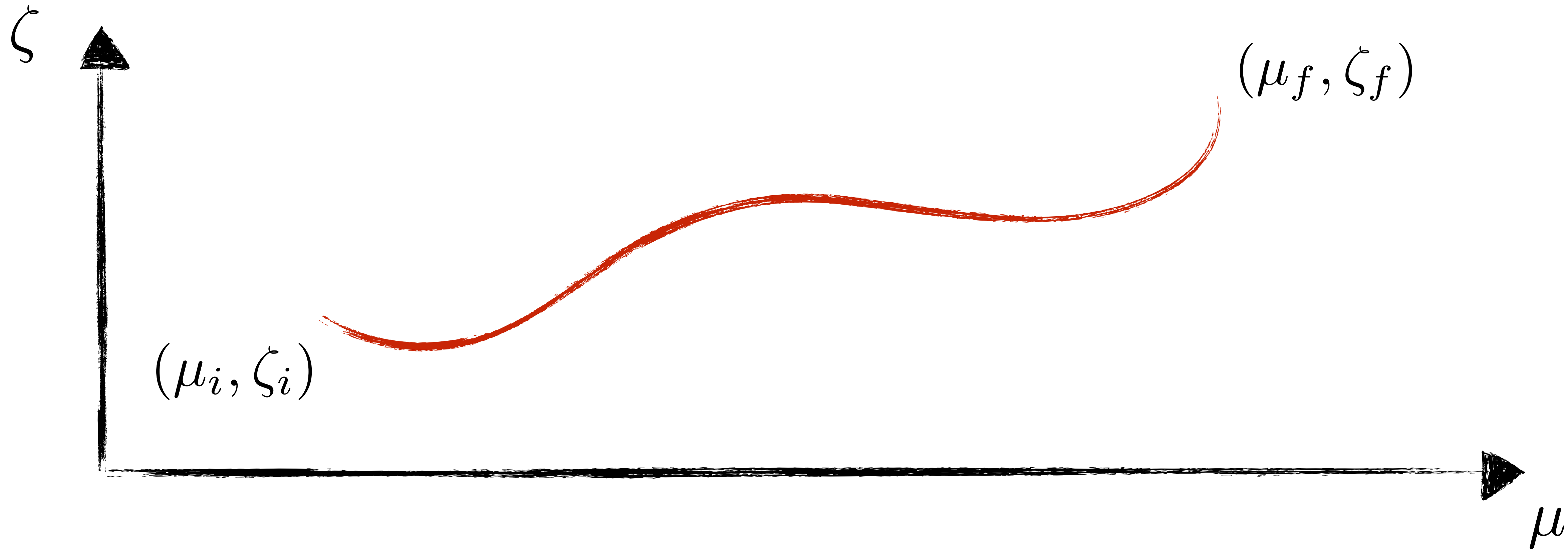


J. C. Collins, Foundations of perturbative QCD, 2011

- We look at interaction between different phases
- Interaction of perturbative and non-perturbative phases can be described through evolution equations
- Dependence of TMD distributions on scales can be found by analysis of perturbative emission in the non-perturbative background
- Anomalous dimensions are known up to three loops
- The fitting of distributions is highly constrained due to strong correlation between perturbative and non-perturbative phases
- We check our predictions for properties of the hadron as many body parton system

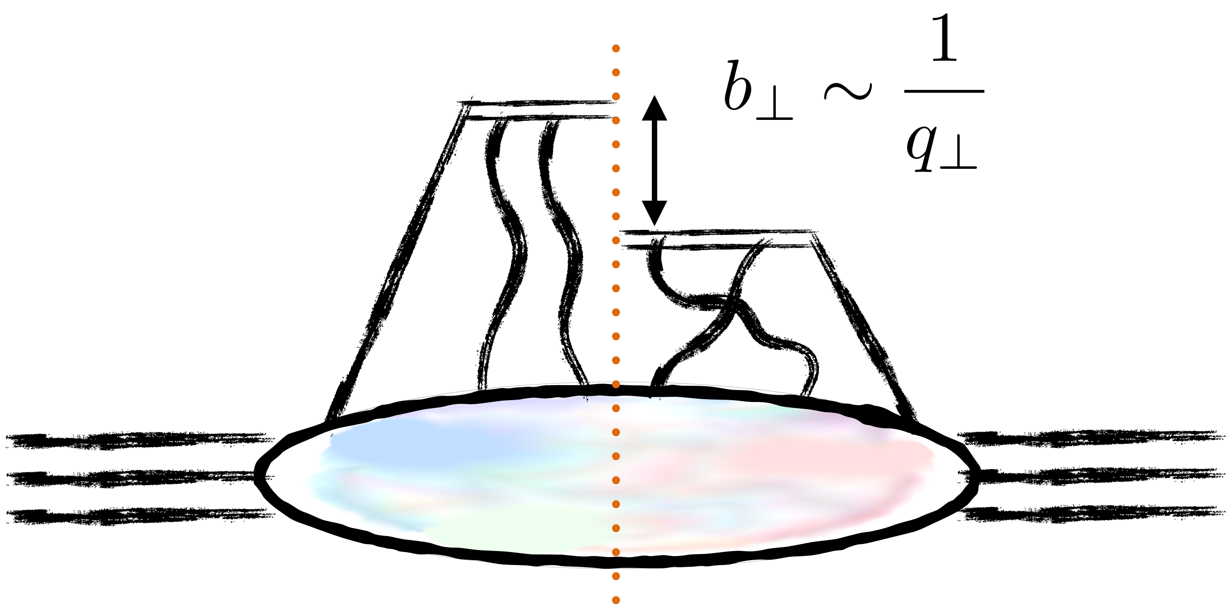
Solution of evolution equations

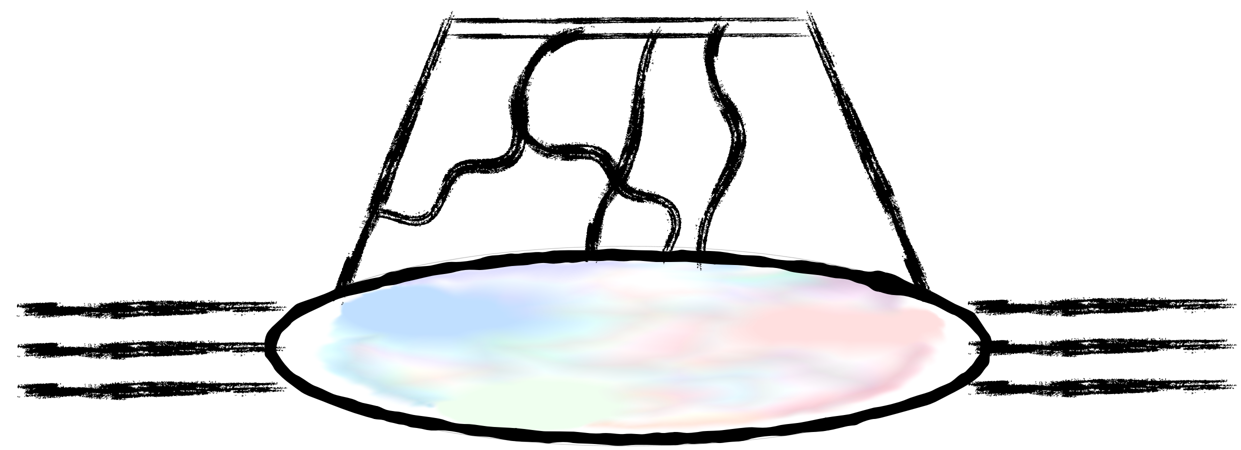
$$F(x,b;\zeta_f,\mu_f) = F(x,b;\zeta_i,\mu_i)\exp\left\{\int_{\mu_i}^{\mu_f}\frac{d\mu}{\mu}\gamma\Big(\alpha_s(\mu),\ln\frac{\zeta_f}{\mu^2}\Big)\right\}\Big(\frac{\zeta_f}{\zeta_i}\Big)^{-\mathcal{D}(\mu_i,b)}$$



- Evolution equations predict how the non-perturbative system evolves from one scale to another. We can predict this transition with very high accuracy from pQCD.
- There are non-perturbative effects in the evolution as well
- TMD distribution is a complex function which is difficult to extract
- Initial condition can be defined by the collinear distributions

TMD vs. collinear distributions



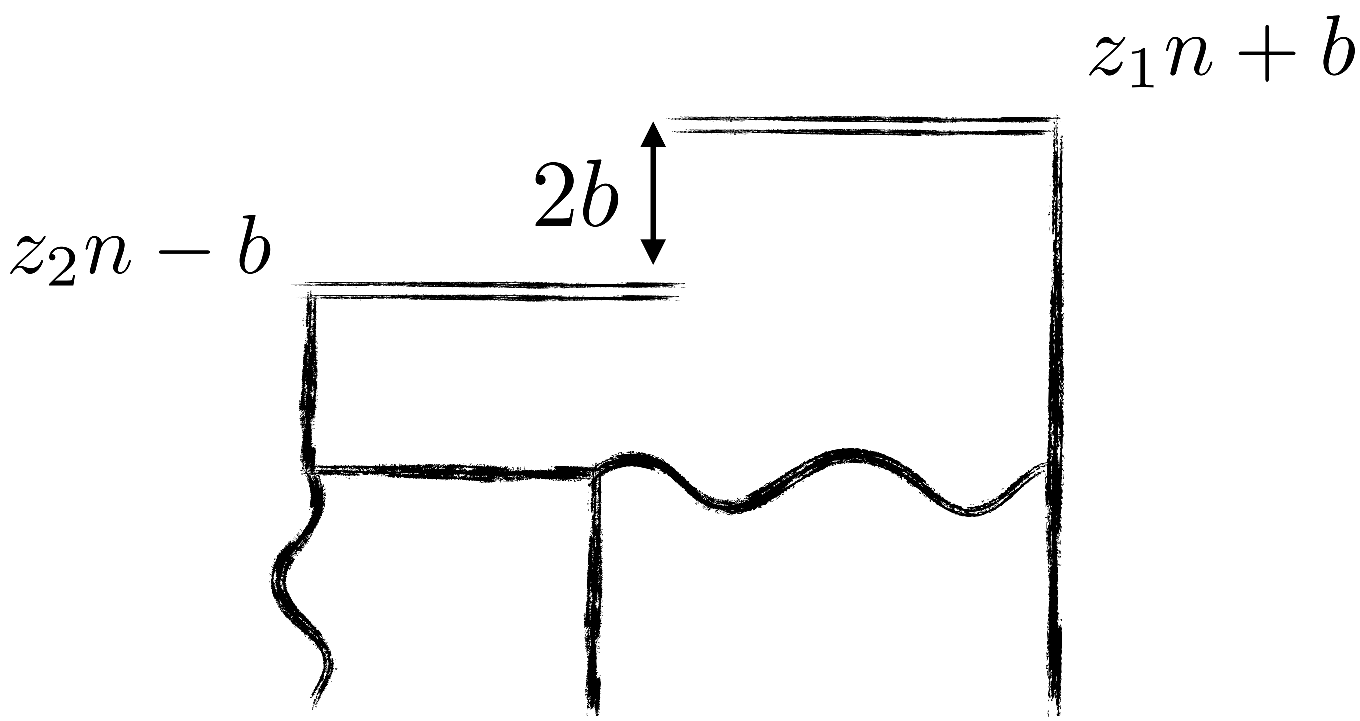


$$F(x, b; \zeta_f, \mu_f) = F(x, b; \zeta_i, \mu_i) \exp\left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma\Big(\alpha_s(\mu), \ln \frac{\zeta_f}{\mu^2}\Big) \right\} \Big(\frac{\zeta_f}{\zeta_i}\Big)^{-\mathcal{D}(\mu_i, b)}$$

- Collinear distributions can be used as an initial condition for TMD evolution
- In the region of small transverse separation they should coincide
- Using calculations in the background field we can construct projection of TMDs onto collinear distributions
- This is another example of how perturbative QCD defines the non-perturbative structure

Operator definition

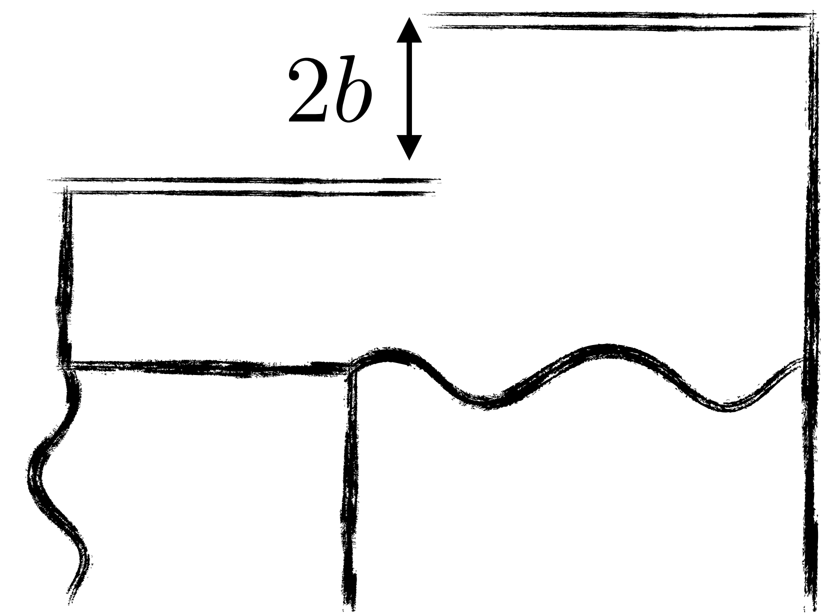
$$\mathcal{U}^{\gamma^+}_{\text{DIS}}(z_1, z_2, \mathbf{b}) = \bar{q}(z_1 n + \mathbf{b})[z_1 n + \mathbf{b}, +\infty n + \mathbf{b}]\,\gamma^+\,[+\infty n - \mathbf{b}, z_2 n - \mathbf{b}]q(z_2 n - \mathbf{b})$$



- We start our derivation from the operator which generates Sivers function, but the method can be applied to an operator of arbitrary structure
- Emission at the NLO level is analyzed in the limit of small b where TMDs match collinear distributions

Collinear matching

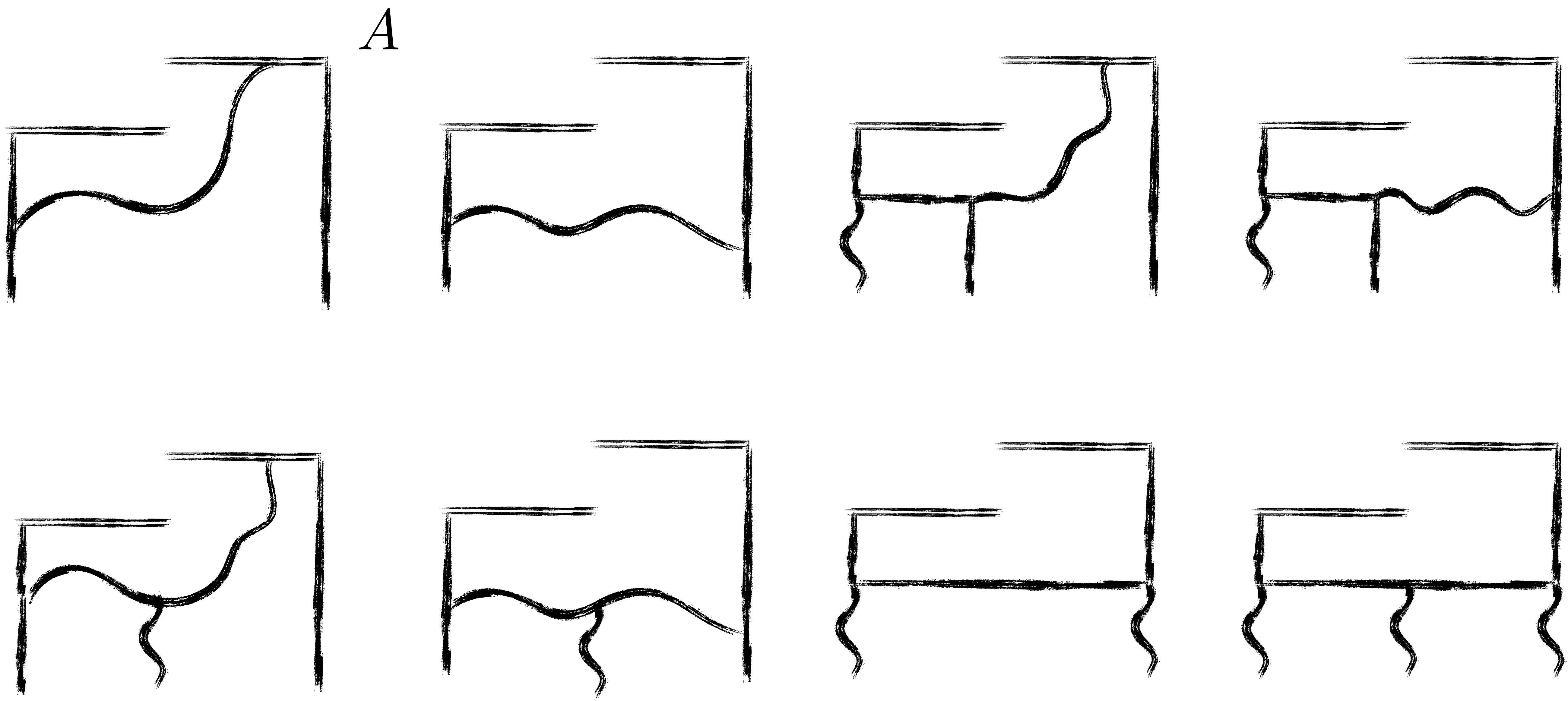
$$\mathcal{U}(z,\vec{b}) = \sum_n C_n^{\text{tw-2}}(z,\mathbf{L}_\mu,a_s(\mu)) \otimes \mathcal{O}_n^{\text{tw2}}(z;\mu)$$
$$+b_\nu \sum_n C_n^{\text{tw-3}}(z,\mathbf{L}_\mu,a_s(\mu)) \otimes \mathcal{O}_n^{\nu,\text{tw3}}(z;\mu) + O(\vec{b}^2)$$



- We construct expansion of the TMD operator onto collinear operators of twist two and three
- The expansion is defined by matching coefficients which we want to find
- The matching coefficients depend on two types of logarithms

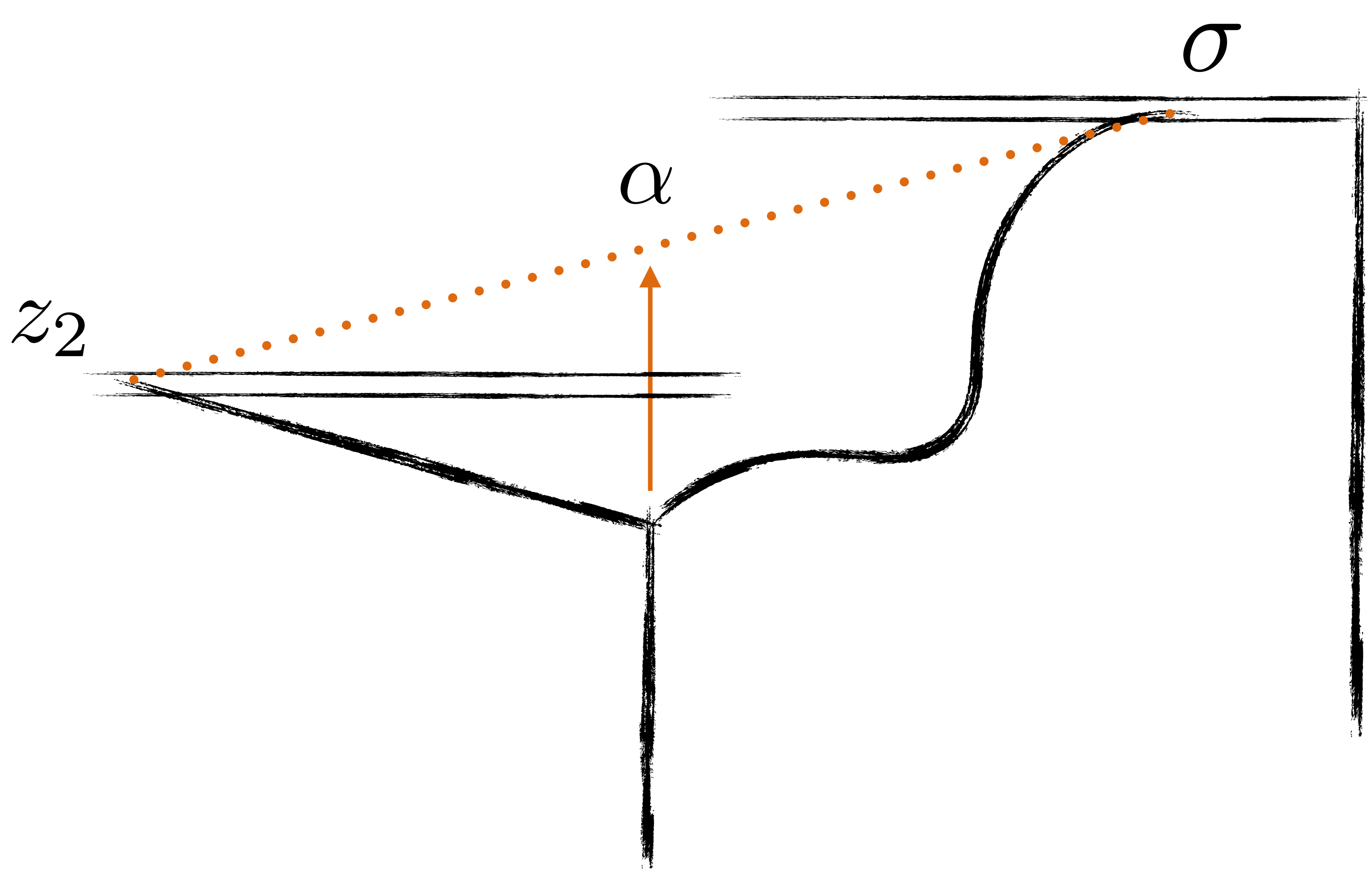
$$\mathbf{L}_\mu = \ln\left(\frac{\mu^2\mathbf{b}^2}{4e^{-2\gamma_E}}\right) \qquad \mathbf{l}_\zeta = \ln\left(\frac{\mu^2}{\zeta}\right)$$

Diagrams



- There are both quark-quark and quark-gluon channels
- We use the light-cone gauge for the background field and background-Feynman gauge for the perturbative phase

Diagram A



- The matching formula can be obtained by expansion of the NLO diagramming the transverse space
- It is natural to perform expansion onto a straight line between emission and absorption points

$$\xi^\mu(u) = \bar{\alpha}(z_2 n^\mu - \mathbf{b}^\mu) + \alpha(\sigma n^\mu + \mathbf{b}^\mu)$$

Diagram A

$$\mathcal{U}_{DY}^{[\gamma^+]}(z_1, z_2, \mathbf{b}) = 2C_F g^2 \int_0^\infty ds \int_0^1 du \; s \int_{-\infty}^{z_1} dz \bar{q}(z_1 n + \mathbf{b}) \gamma^+ (z_2 n - \mathbf{b} | P^+ e^{is\bar{u}P^2} q e^{isuP^2} | zn + \mathbf{b})$$

$$q = q(\xi) + (X-\xi)_\alpha \partial^\alpha q(\xi) + \frac{1}{2}(X-\xi)_\alpha (X-\xi)_\beta \partial^\alpha \partial^\beta q(\xi) + \dots$$

- Variable s is a Feynman parameter
- Use first two orders of expansion
- P is an operator

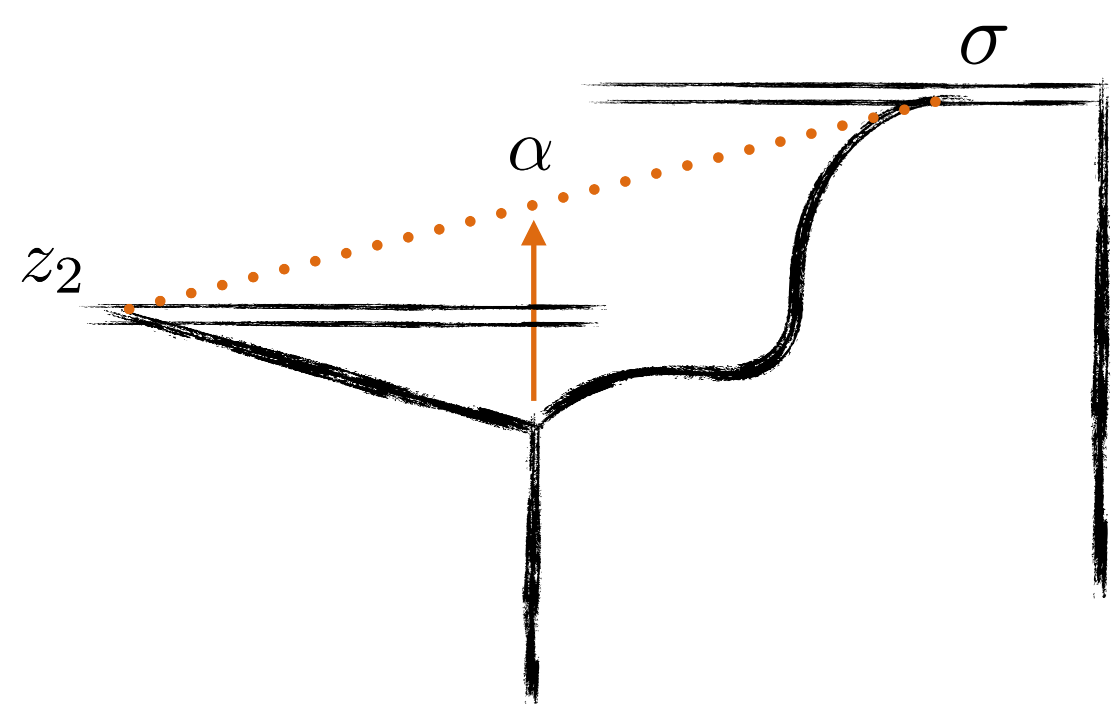


Diagram A

- **Expansion on to the trajectory. Order of operators is important**

$$\mathcal{U}_{DY}^{[\gamma^+]}(z_1,z_2,\mathbf{b}) = 2C_F g^2 \int_0^\infty ds \int_0^1 du\ s \int_{-\infty}^{z_1} dz \bar{q}(z_1 n+\mathbf{b})\gamma^+(z_2 n-\mathbf{b}|P^+ e^{is\bar{u}P^2} e^{isuP^2}|zn+\mathbf{b})q(\xi_u)$$
$$+2C_F g^2 \int_0^\infty ds \int_0^1 du\ s \int_{-\infty}^{z_1} dz \bar{q}(z_1 n+\mathbf{b})\gamma^+(z_2 n-\mathbf{b}|P^+ e^{is\bar{u}P^2}(X-\xi)_\alpha e^{isuP^2}|zn+\mathbf{b})\partial^\alpha q(\xi_u)$$

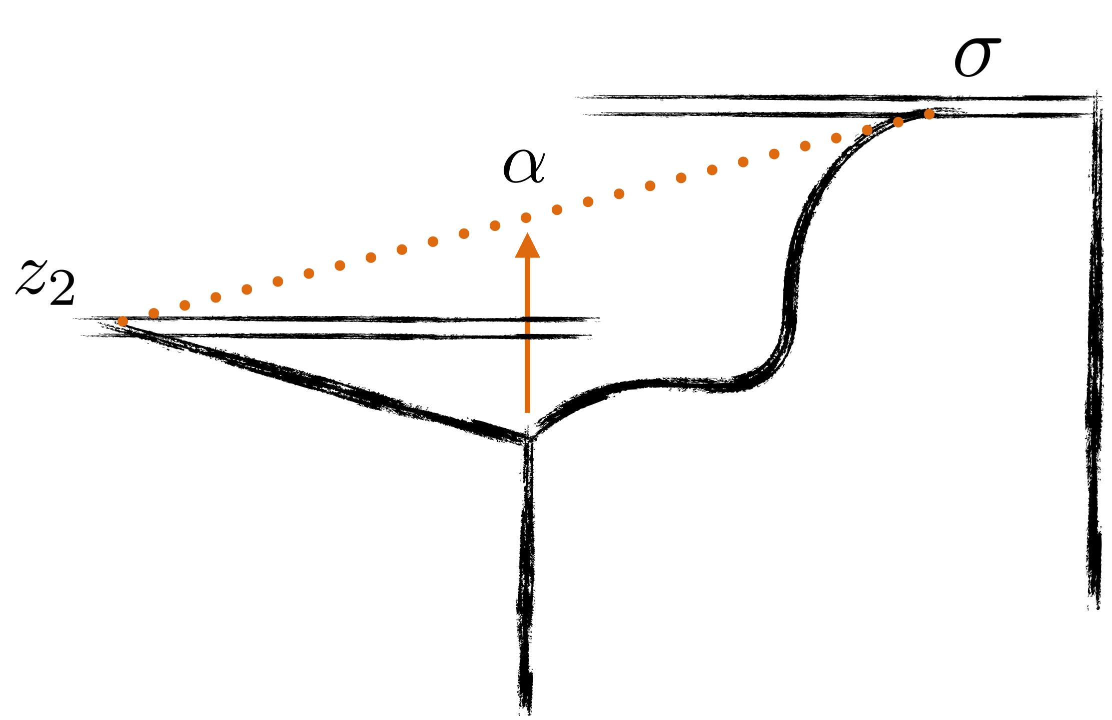
- **Properties of operator X**

$$X|x) = x|x) \qquad [P_\mu, X_\nu] = i g_{\mu\nu}$$

$$P|p) = p|p)$$

- **First we commute X, then calculate Fourier transformation**

$$(x|f(p)|y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} f(p)$$



- **Do the same for all diagrams**

Rapidity divergence

$$\widetilde{\mathcal{U}}_A = 2a_s C_F \Gamma(-\epsilon)\mathbf{b}^{2\epsilon} \int_{-\infty}^{z_1} d\sigma \int_0^1 d\alpha \, \frac{\bar{\alpha}}{\alpha} \; \bar{q}(nz_1)\gamma^+ \frac{\partial}{\partial\sigma} q(nz_{2\sigma}^{\alpha})$$

- We observe rapidity divergence at $\alpha \to 0$
- Regularization is performed using rapidity regulator
- It can be introduced by redefinition of Wilson lines

$$P\exp\left(ig\int_{-\infty}^{z} d\sigma A_+(n\sigma + x)\right) \to P\exp\left(ig\int_{-\infty}^{z} d\sigma A_+(n\sigma + x)e^{-\delta|\sigma|}\right)$$

Delta regulator

$$\widetilde{\mathcal{U}}_A^{\text{sing}} = 2a_s C_F \Gamma(-\epsilon)\mathbf{b}^{2\epsilon} \int_{-\infty}^{0} d\tau \int_0^1 d\alpha\, e^{\delta\frac{\tau}{\alpha}} \frac{\bar{\alpha}}{\alpha}\; \bar{q}(nz_1)\gamma^+ \frac{\partial}{\partial\tau} q(n(z_2+\tau))$$

- The regulator passes unchanged into the rapidity divergent diagram A
- Logarithm of δ represents rapidity singularity

$$\int_0^1 d\alpha\, e^{\delta\frac{\tau}{\alpha}} \frac{\bar{\alpha}}{\alpha} \sim \ln\delta$$

M. G. Echevarria, A. Idilbi and I. Scimemi, JHEP 1207 (2012) 002

M. G. Echevarria, I. Scimemi and A. Vladimirov, Phys. Rev. D 93, 054004 (2016)

Final result for diagram A

$$\widetilde{\mathcal{U}}_A = 2a_s C_F \Gamma(-\epsilon)\mathbf{b}^{2\epsilon}\Big\{\int_0^1 d\alpha \frac{\bar{\alpha}}{\alpha}\Big[\mathcal{U}^{\gamma^+}(z_1, z_{21}^{\alpha}; \bar{\alpha}\mathbf{b}) - \mathcal{U}^{\gamma^+}(z_1, z_2; \mathbf{b})\Big] - \Big(1 + \ln\Big(\frac{\delta}{p^+}\Big)\Big)\mathcal{U}^{\gamma^+}(z_1, z_2; \mathbf{b})\Big\} + O(\mathbf{b}^2\partial^2 q)$$

- After implementation of the rapidity regulator the diagram becomes finite
- Diagram A is the only diagram with rapidity divergence
- It has both twist-2 and twist-3 contributions

$$\mathcal{U}^{\gamma^+}(z_1, z_2, \mathbf{b}) = \mathcal{U}^{\gamma^+}(z_1, z_2, \vec{0}) + b^{\mu}\frac{\partial}{\partial b^{\mu}}\mathcal{U}^{\gamma^+}(z_1, z_2, \mathbf{b})\Big|_{\mathbf{b}=0} + O(\mathbf{b}^2)$$

Collinear operators

$$\widetilde{\mathcal{U}}(z_1,z_2;\mathbf{b}) = \sum_i \Big[1_i + a_s\Gamma(-\epsilon)\mathbf{b}^{2\epsilon}\tilde{C}_i^{\text{tw2}} + O(a_s^2)\Big] \otimes \mathcal{O}_{i,\text{tw2}}(z_1,z_2)$$
$$+ b_\mu \sum_i \Big[1_i + a_s\Gamma(-\epsilon)\mathbf{b}^{2\epsilon}\tilde{C}_i^{\text{tw3}} + O(a_s^2)\Big] \otimes \mathcal{O}^\mu_{i,\text{tw3}}(z_1,z_2) + O(\vec{b}^2)$$

- **We represent the result of calculation in terms of collinear operators**
- **There are three quark and two gluon operators**

$$\mathcal{O}_{\gamma^+}(z_1,z_2) = \bar{q}(z_1 n)[z_1 n, z_2 n]\gamma^+ q(z_2 n),$$

$$\mathcal{T}^\mu_{\gamma^+}(z_1,z_2,z_3) = g\bar{q}(z_1 n)[z_1 n, z_2 n]\gamma^+ F^{\mu+}(z_2 n)[z_2 n, z_3 n]q(z_3 n),$$

$$\mathcal{T}^\nu_{\gamma^+\gamma_T^{\nu\mu}}(z_1,z_2,z_3) = g\bar{q}(z_1 n)[z_1 n, z_2 n]\gamma^+\gamma_T^{\nu\mu} F^{\nu+}(z_2 n)[z_2 n, z_3 n]q(z_3 n)$$

TMD vs. collinear distributions

$$\Phi^{[\gamma^+]}(x, \mathbf{b}) = \int \frac{dz}{2\pi} e^{-2ixzp^+} \langle p, S | \mathcal{U}^{\gamma^+}(z, -z; \frac{\vec{b}}{2}) | p, S \rangle$$

$$\Phi^{[\gamma^+]}(x, \mathbf{b}) = f_1(x, \mathbf{b}) + i\epsilon_T^{\mu\nu} b_\mu s_{T\nu} M f_{1T}^\perp(x, \mathbf{b})$$

- **The result of calculation gives us connection between TMD and collinear distributions**

$$\langle p, S | O_{\gamma^+}(z_1, z_2) | p, S \rangle = 2p^+ \int dx e^{ix(z_1 - z_2)p_+} f_1(x)$$

$$\langle p, S | \mathcal{T}^\mu_{\gamma^+}(z_1, z_2, z_3) | p, S \rangle = 2\tilde{s}^\mu (p^+)^2 M \int [dx] e^{-ip^+(x_1 z_1 + x_2 z_2 + x_3 z_3)} T(x_1, x_2, x_3)$$

Bare result

$$f_1(x,\mathbf{b}) + i\epsilon_T^{\mu\nu} b_\mu s_{T\nu} M f_{1T}^{\perp}(x,\mathbf{b})\Big|_{A+A^*}$$

$$= 2a_s C_F \Gamma(-\epsilon)\mathbf{B}^\epsilon \int d\xi \int_0^1 dy \delta(x-y\xi)\Big\{$$

$$\Big[\Big(\frac{2y}{1-y}\Big)_+ - 2\delta(\bar{y})\Big(1+\ln\Big(\frac{\delta}{p^+}\Big)\Big)\Big](f_1(\xi) + \mathbf{s}T(-\xi,0,\xi)) - 2y\mathbf{s}T(-\xi,0,\xi)\Big\}$$

$$C(y,\delta,\epsilon) \qquad\qquad \mathcal{O}(\xi)$$

$$\mathbf{s} = i\pi\epsilon^{\mu\nu} b_\mu \tilde{s}_\nu M, \qquad \mathbf{B} = \frac{\mathbf{b}^2}{4}$$

- The result has a structure of the matching formula
- It is obtained from expansion of the NLO diagram in the transverse space
- The result depends on rapidity and UV regulators and should be renormalized

Renormalization

$$\mathcal{U}_f(x, \mathbf{b}; \mu, \zeta) = Z_i^{-1} Z_f^{TMD}\left(\frac{\mu^2}{\zeta}\right) R_f\Big(\mathbf{b}; \mu, \zeta\Big) \mathcal{U}_f^{bare}(x, \mathbf{b})$$

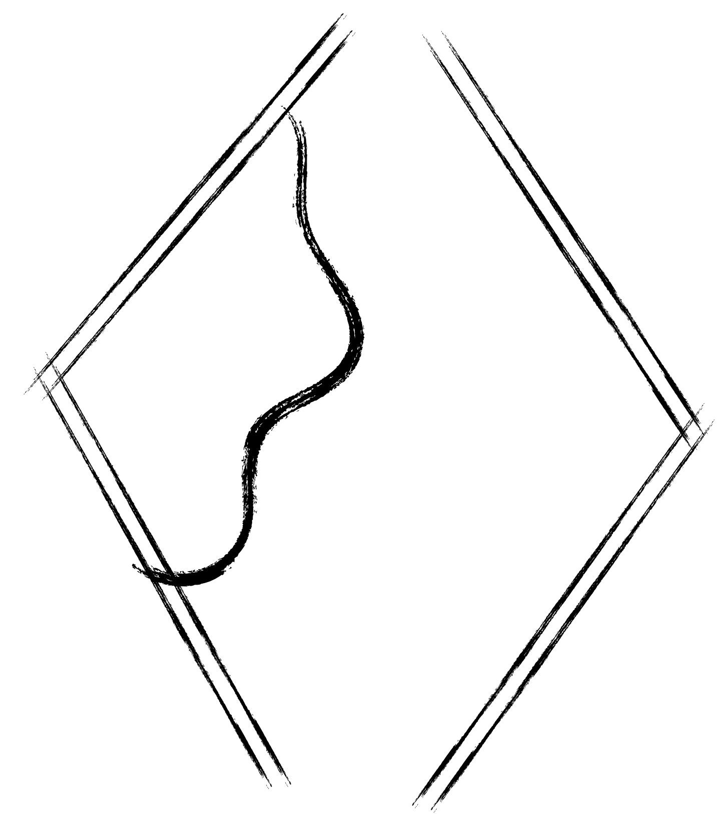
- We multiply the bare result by renormalization constants: wave-function renormalization, TMD renormalization and rapidity renormalization
- The form of the constants is know though their explicit form depends on the regularization scheme

Rapidity renormalization and the soft factor

$$R_q(\mathbf{b};\mu,\zeta) = S^{-1/2}(\mathbf{b};\mu,\zeta)$$

- An explicit form of renormalization constants is defined by the regularization scheme in use
- The rapidity renormalization factor is given by the soft function

$$S\Big(\mathbf{b};\ln\Big(\frac{\mu^2}{\delta^+\delta^-}\Big)\Big)$$
$$= S^{1/2}\Big(\mathbf{b};\ln\Big(\frac{\mu^2}{(\delta^+/p^+)^2\zeta_+}\Big)\Big)S^{1/2}\Big(\mathbf{b};\ln\Big(\frac{\mu^2}{(\delta^-/p^-)^2\zeta_-}\Big)\Big)$$



Renormalization constants

- Rapidity renormalization constant

$$R_q(\vec{b};\mu,\zeta) = 1 + 2a_s C_F \mathbf{B}^{\epsilon}\mu^{2\epsilon}e^{-\epsilon\gamma_E}\Gamma(-\epsilon)\Big(\ln\Big(\mathbf{B}\delta^2\frac{\zeta}{(p^+)^2}\Big) - \psi(-\epsilon) + \gamma_E\Big) + O(a_s^2)$$

- UV renormalization constants

$$Z_2^{-1}Z_q^{TMD}\Big(\frac{\mu^2}{\zeta}\Big) = 1 - a_s C_F\Big(\frac{2}{\epsilon^2} + \frac{3 + 2\ln(\mu^2/\zeta)}{\epsilon}\Big) + O(a_s^2)$$

- Dependence on regularization parameters in our matching formula vanishes when we multiply it by the renormalization constants

Matching formula

$$f_{1T}^{\perp}(x,\mathbf{b};\mu,\zeta)=\sum_f C_{1T}^{\perp}(x_1,x_2,x_3,\mathbf{b},\mu,\zeta)\otimes T(x_1,x_2,x_3,\mu)+O(\mathbf{b}^2)$$

- Dependence on regularization parameters in our matching formula vanishes when we multiply it by the renormalization constants
- Both the collinear function and the matching coefficient depend on the UV scale
- The coefficient depends on rapidity renormalization scale

Matching formula for the unpolarized distribution

$$f_1(x,\vec{b};\mu,\zeta) = f_1(x) + a_s(\mu)\Big\{ -2\mathbf{L}_\mu P\otimes f_1 + C_F\Big(-\mathbf{L}_\mu^2 + 2\mathbf{l}_\zeta\mathbf{L}_\mu + 3\mathbf{L}_\mu - \frac{\pi^2}{6}\Big)f_1(x)$$
$$+\int d\xi \int_0^1 dy\delta(x-y\xi)\Big[C_F 2\bar{y}f_1(\xi) + 2y\bar{y}g(\xi)\Big]\Big\} + O(a_s^2) + O(\vec{b}^2)$$

- There are leading order (LO) and next-to-leading (NLO) parts
- Matching at LO is simple and is given by the unpolarized collinear distribution
- The first term of the NLO part is given by the DGLAP evolution kernel
- The second term originates in the rapidity divergence
- The third term is a finite, logarithm independent part
- This formula is in agreement with known results, which provides a consistency check

J. C. Collins, Foundations of perturbative QCD, 2011

Matching formula for the Sivers function

$$f^{\perp}_{1T;q\leftarrow h;\text{DY}}(x,\boldsymbol{b};\mu,\zeta)=\pi T(-x,0,x)+\pi a_s(\mu)\Big\{$$
$$-2\mathbf{L}_\mu P\otimes T+C_F\left(-\mathbf{L}_\mu^2+2\mathbf{l}_\zeta\mathbf{L}_\mu+3\mathbf{L}_\mu-\frac{\pi^2}{6}\right)T(-x,0,x)$$
$$+\int d\xi\int_0^1 dy\delta(x-y\xi)\Big[\left(C_F-\frac{C_A}{2}\right)2\bar{y}T(-\xi,0,\xi)+\frac{3y\bar{y}}{2}\frac{G_+(-\xi,0,\xi)+G_-(-\xi,0,\xi)}{\xi}\Big]\Big\}$$

- **The structure of the matching formula for Sivers function is similar**
- **Matching at LO is simple and is given by the ETQS (Efremov-Teryaev-Qui-Sterman) distribution**
- **The first term of the NLO part is given by the collinear evolution for the twist-3 ETQS. The second term originates in the rapidity divergence. The third term is a finite, logarithm independent part**
- **This formula is in agreement with known results**

V. M. Braun, A. N. Manashov and B. Pirnay, Phys. Rev. D80 (2009) 114002;

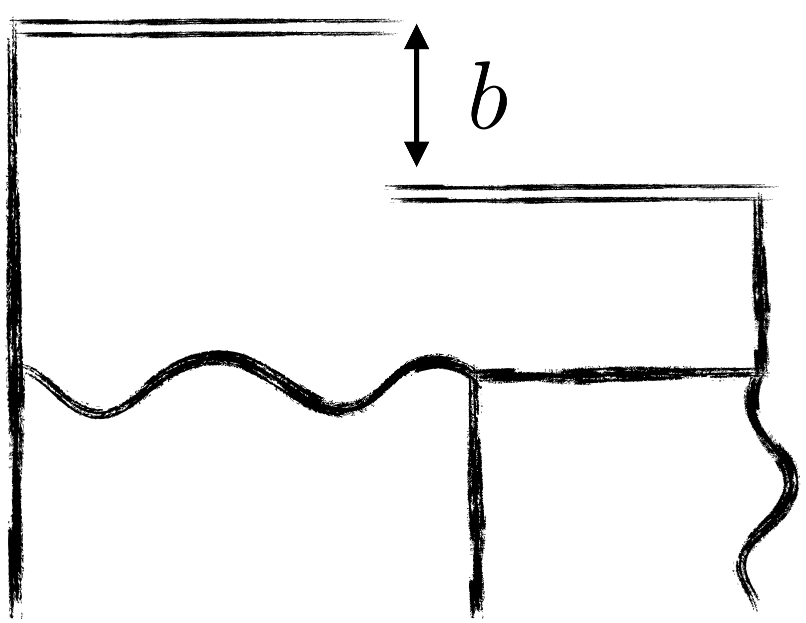
Z.-B. Kang and J.-W. Qiu, Phys. Lett. B713 (2012) 273–276

P. Sun and F. Yuan, Phys. Rev. D88 (2013) 114012

Collinear matching

$$f_{1T;q\leftarrow h;\mathrm{DY}}^{\perp}(x,\boldsymbol{b};\mu,\zeta)=\pi T(-x,0,x)+\pi a_s(\mu)\Big\{$$
$$-2\mathbf{L}_\mu P\otimes T+C_F\left(-\mathbf{L}_\mu^2+2\mathbf{l}_\zeta\mathbf{L}_\mu+3\mathbf{L}_\mu-\frac{\pi^2}{6}\right)T(-x,0,x)$$
$$+\int d\xi\int_0^1 dy\delta(x-y\xi)\Big[\left(C_F-\frac{C_A}{2}\right)2\bar{y}T(-\xi,0,\xi)+\frac{3y\bar{y}}{2}\frac{G_+(-\xi,0,\xi)+G_-(-\xi,0,\xi)}{\xi}\Big]\Big\}$$

I. Scimemi, A. Tarasov, A. Vladimirov, arXiv:1901.04519



- We derive matching coefficient for the Sivers function at the next-to-leading order
- We use background field method to calculate emission in the many body parton background
- We perform expansion in powers of b
- The structure of the result is dictated by strong interaction between perturbative and non-perturbative phases
- It is easy to generalize calculation to other operators and matrix elements (Collins function)
- The results will be implemented in extraction of the Sivers function