Parton Energy Loss and the Generalized Jet transport Coefficient

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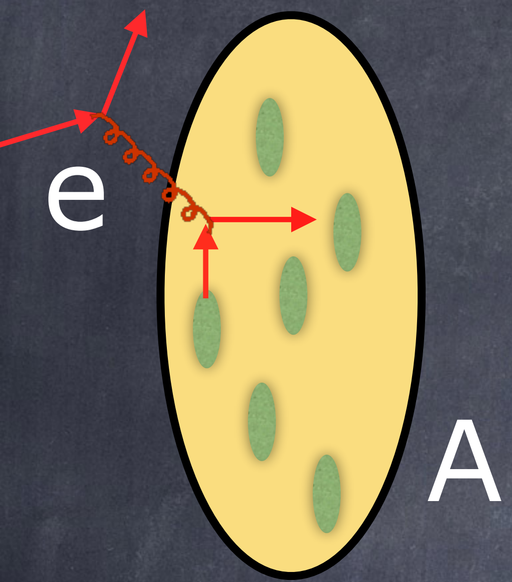
arXiv:1905.12699

POETIC 2019

Outline

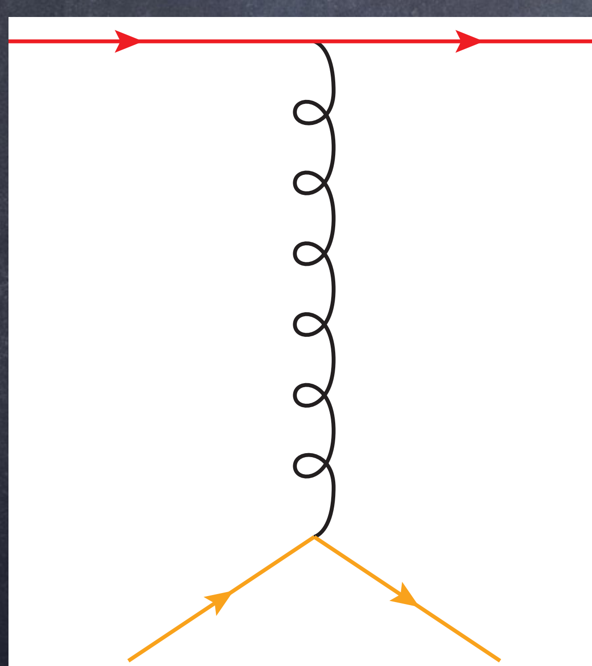
- Introduction
- Approach
- Results and approximations
- Summary

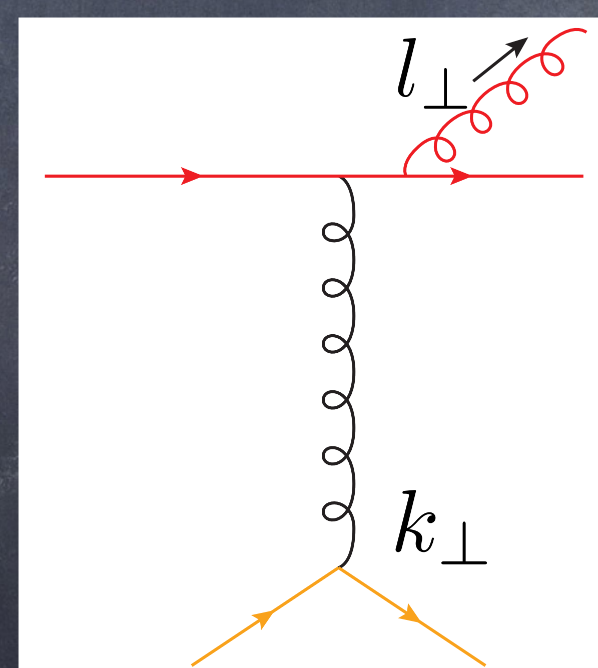
Parton energy loss in medium



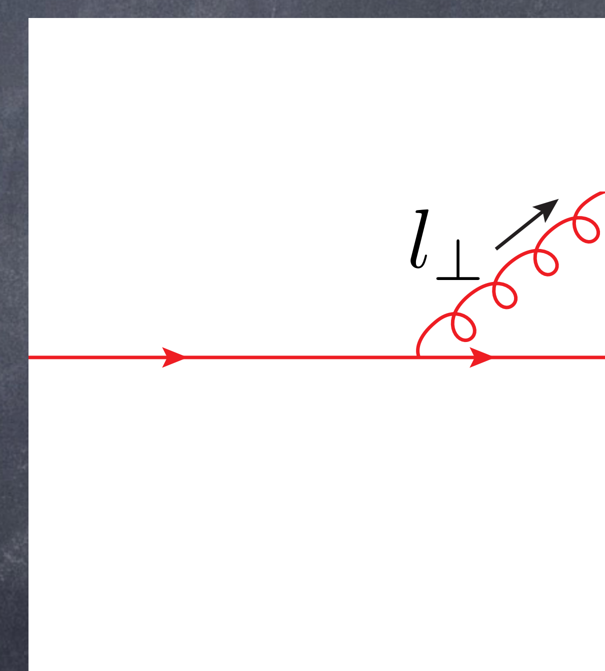
- High energy partons travel through hot QGP or cold nuclei (eA DIS process)
- Energy loss mechanisms reveal medium properties

Parton energy loss mechanisms





medium induced



vacuum

collisional energy loss

radiative energy loss

Radiative energy loss:assumptions

Approaches to radiative energy loss : BDMPS-Z, GLV, AMY, SCET, High Twist

- Scattering Center : Static or Dynamic?

  Static: no energy transfer (BDMPS-Z, GLV)

  Extension of GLV to dynamic S.C.
  Djordjevic,Heinz PRL 101.2 (2008): 022302

  Dynamic: both momentum and energy transfer ✓

- Radiated Gluon : Soft or hard ?

  z → 0 (BDMPS-Z, GLV)

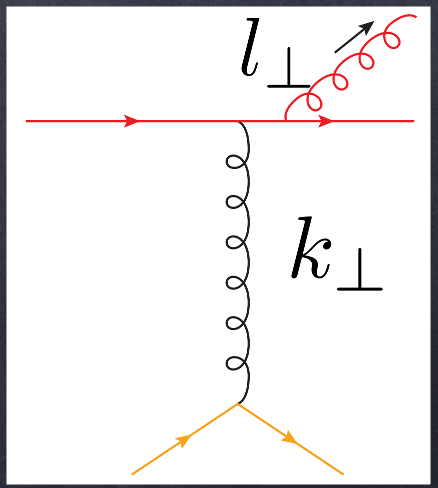
  z finite ✓

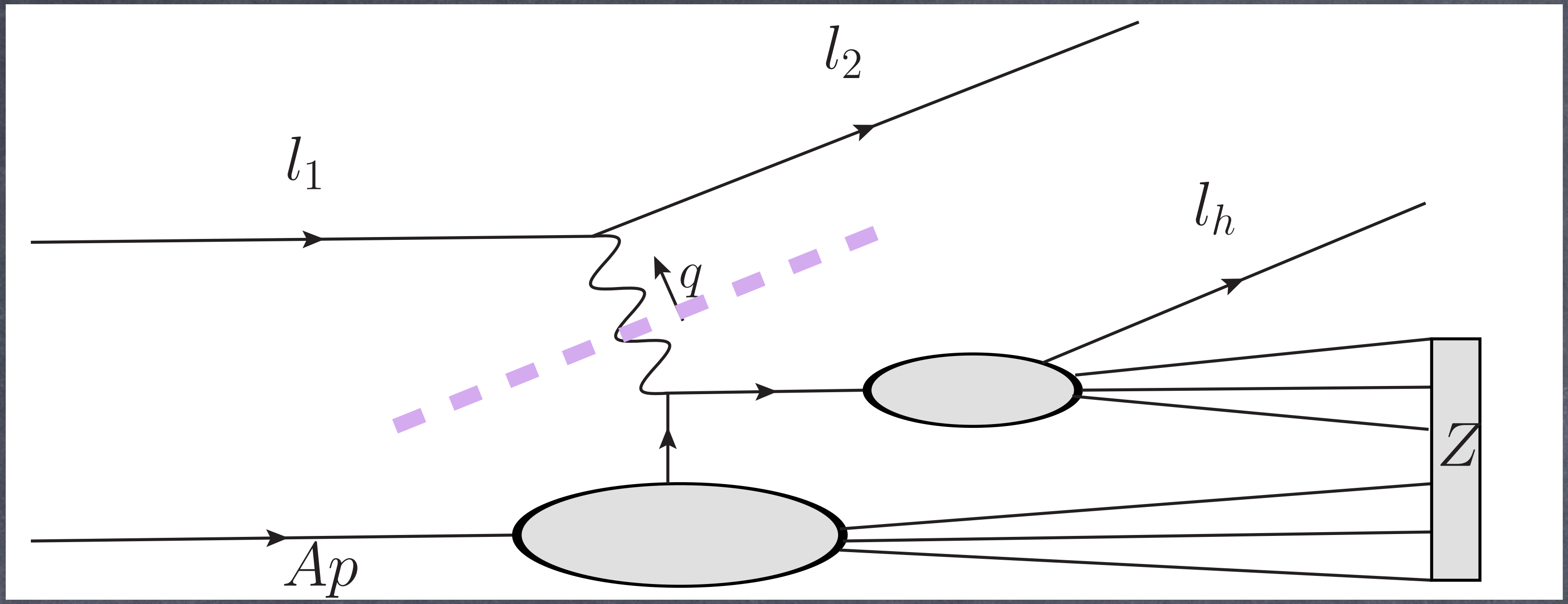
  Beyond soft appr. of GLV
  Blagojevic *et al*. arXiv:1804.07593
  Beyond soft appr. of SCET
  Ovanesyan, Vitev JHEP *2011*(6), 80

- Transverse momentum transfer $k_\perp$: smaller or same order $l_\perp$?

  $k_\perp \ll l_\perp$ (High Twist)

  $k_\perp \sim l_\perp$ ✓





- lepton-nucleus scattering

$$e(l_1) + A(Ap) \to e(l_2) + h(l_h) + Z$$

identify one hadron in final state

- cross section $d\sigma$ and hadronic tensor $W^{\mu\nu}$

$$d\sigma = \frac{e^4}{2s}\frac{\sum_q e_q^2}{q^4}\int \frac{d^4 l_2}{(2\pi)^4} 2\pi\delta(l_2^2)\frac{1}{2}L_{\mu\nu}W^{\mu\nu}$$

Collinear Factorization for SIDIS

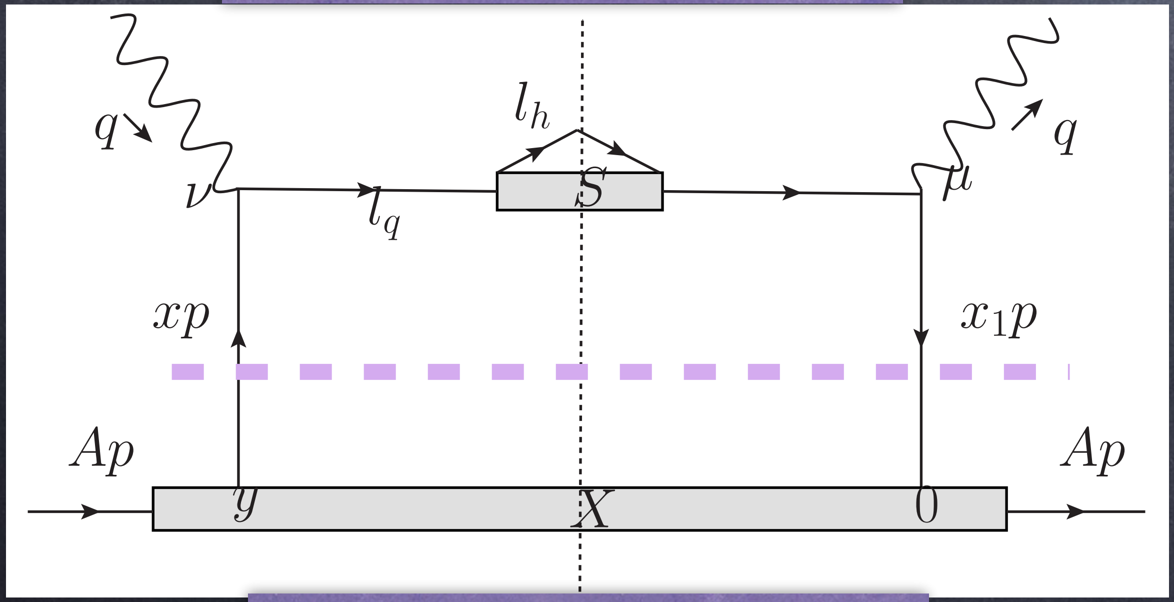
- Factorization

Separate non-perturbative part (pdf, fragmentation function) from perturbative part (hard scattering)

- SIDIS process

Collinear factorization when final hadron $l_{h\perp}$ integrated

hadronic tensor



handbag diagram

$$\frac{dW^{\mu\nu}_{S(0)}}{dz_h} = \int dx\, f_q^A(x) H^{\mu\nu}_{(0)} D_{q\to h}(z_h)$$

$f_q^A(x)$ nuclear quark distribution function

$D_{q\to h}(z_h)$ quark fragmentation function

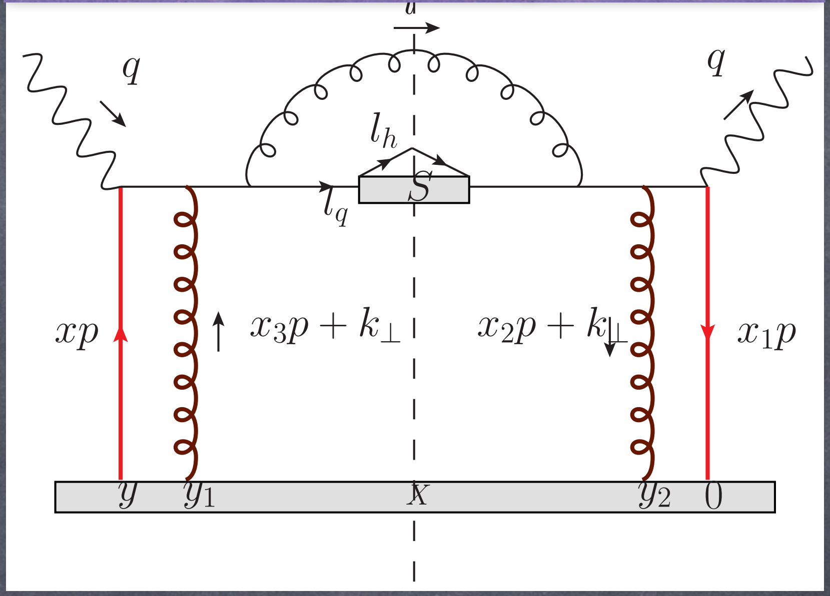
Factorization of Medium Induced Radiation

- High Twist approach

Collinear factorization

XF Guo, XN Wang (2000) PRL *85*(17), 3591
XN Wang and *XF Guo (2001)* Nucl. Phys. A 696, 788

example diagram



$$\frac{dW^{\mu\nu}_{D(1)q}}{dz_h} = \int_{z_h}^{1} \frac{dz}{z} D_{q\to h}(z_h/z) \int \frac{dy^-}{2\pi} dy_1^- dy_2^- \frac{d^2 y_{1\perp}}{(2\pi)^2} d^2 y_{2\perp} d^2 k_\perp e^{-i\vec{k}_\perp\cdot(\vec{y}_{1\perp}-\vec{y}_{2\perp})}$$
$$\frac{1}{2} < A|\bar{\psi}_q(0)\gamma^+ A^+(y_2^-,\vec{y}_{2\perp})A^+(y_1^-,\vec{y}_{1\perp})\psi_q(y^-)|A> H_D^{\mu\nu}(k_\perp, y^-, y_1^-, y_2^-, p, q, z)$$

Collinear expansion of hard part

$k_\perp \ll l_\perp$ approximation

$$H_D^{\mu\nu}(k_\perp, y^-, y_1^-, y_2^-, p, q, z) = H_D^{\mu\nu}(k_\perp = 0) + \left.\frac{\partial H_D^{\mu\nu}}{\partial k_\perp^\alpha}\right|_{k_\perp=0} k_\perp^\alpha$$
$$+ \frac{1}{2} \left.\frac{\partial^2 H_D^{\mu\nu}}{\partial k_\perp^\alpha k_\perp^\beta}\right|_{k_\perp=0} k_\perp^\alpha k_\perp^\beta + \cdots$$

- Our approach

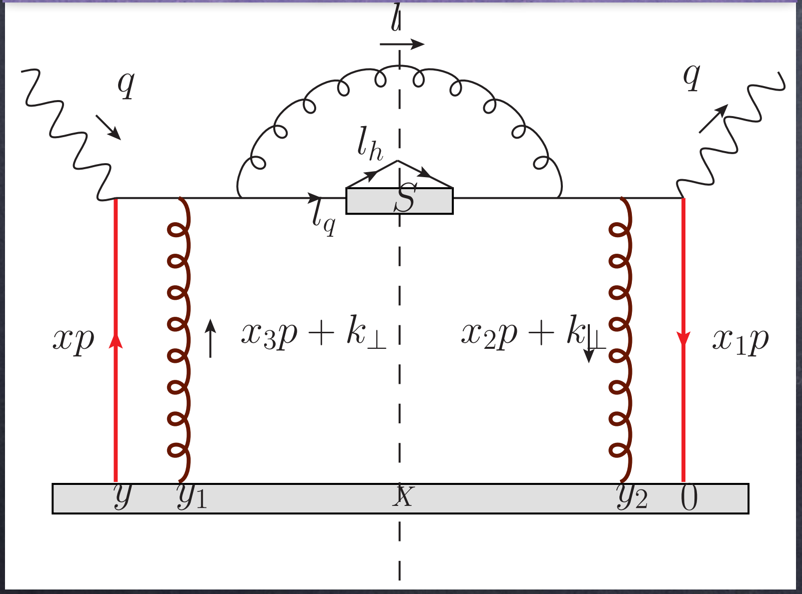
relax $k_\perp \ll l_\perp$ keep Q^2 large, without collinear expansion

Receive contribution from more diagrams

YY Zhang, GY Qin, XN Wang
arXiv:1905.12699

$$\frac{dW^{\mu\nu}_{D(1)q}}{dz_h} = \frac{\alpha_s}{2\pi}\frac{2\pi\alpha_s}{N_c} \otimes \mathcal{F}_c(\vec{l}_\perp, \vec{k}_\perp) \otimes P_{qg}(z) \otimes H^{\mu\nu}_{(0)}(x) \otimes \frac{D_{q\to h}}{z}(z_h/z) \otimes T^A_{qg}(x, x_1, x_2)$$

example diagram



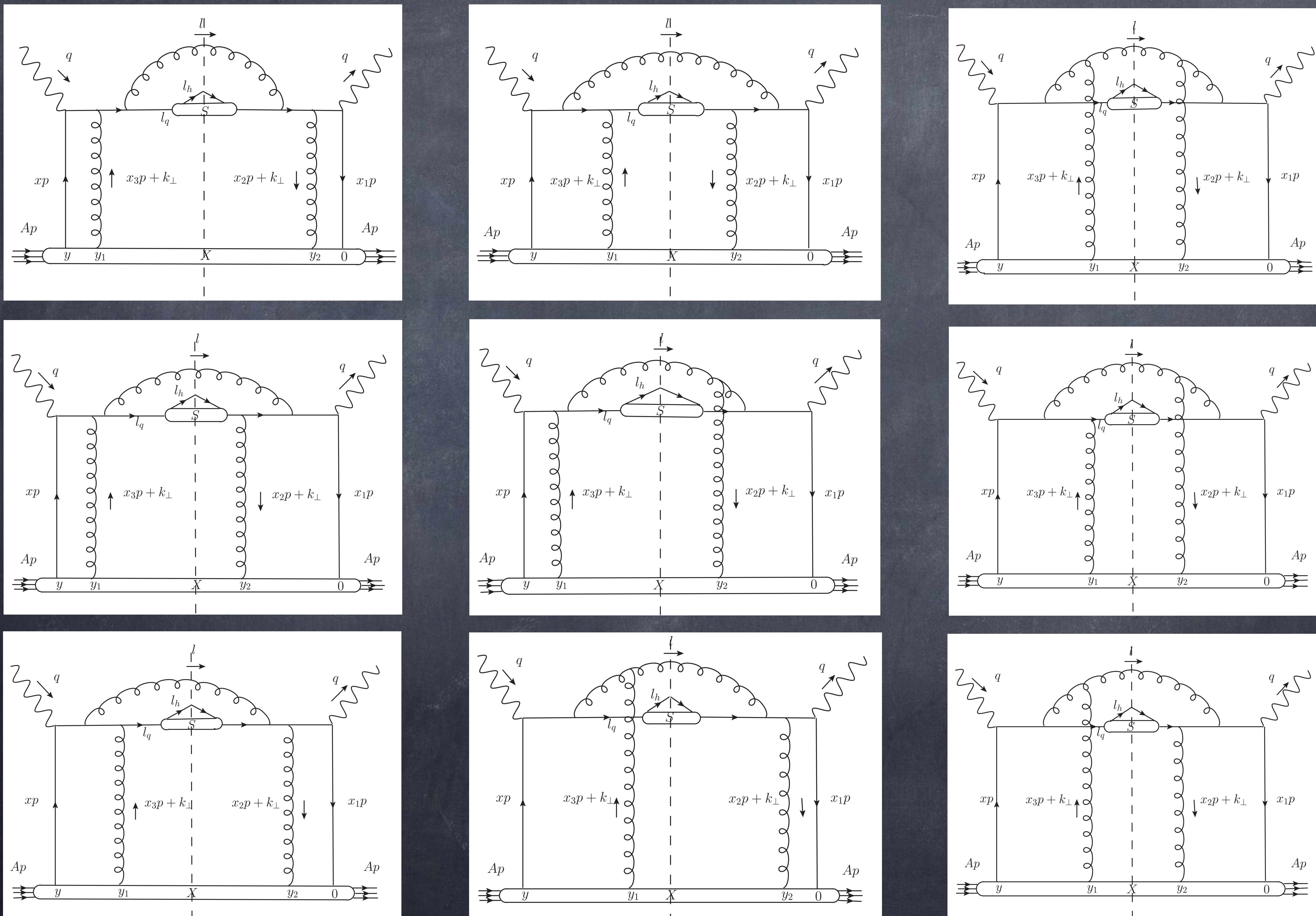
For this specific diagram

$$\mathcal{F}_c(\vec{l}_\perp, \vec{k}_\perp) = \frac{C_F}{l^2_\perp}$$

$$[T^A_{qg}(x + x_L, x, x_D) - T^A_{qg}(x + x_L, x - x_L, x_L + x_D)$$
$$- T^A_{qg}(x, x + x_L, x_D) + T^A_{qg}(x, x, x_L + x_D)]$$

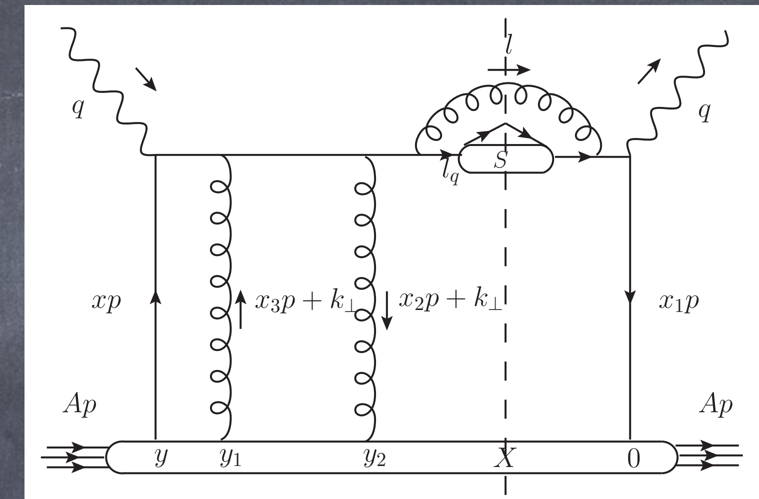
quark-gluon correlation function

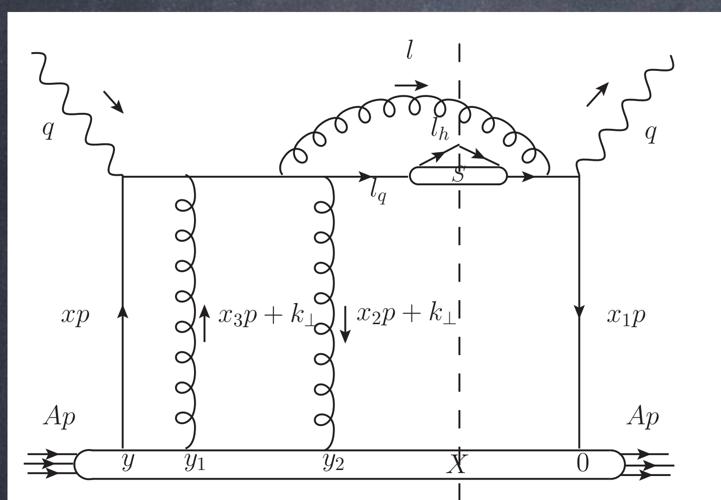
Central cut diagrams

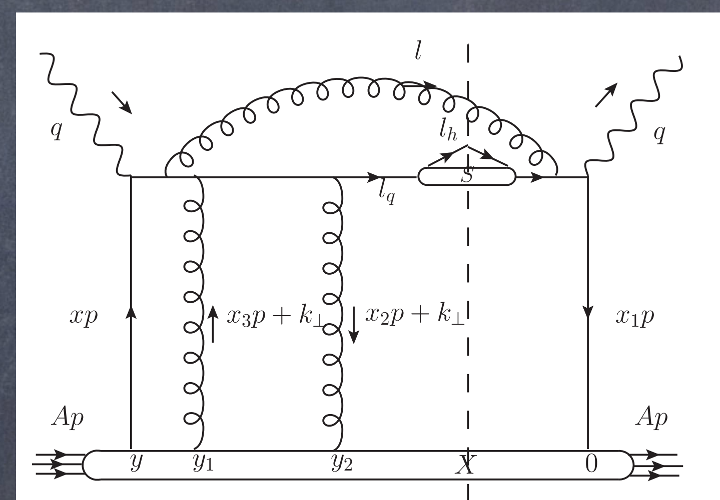


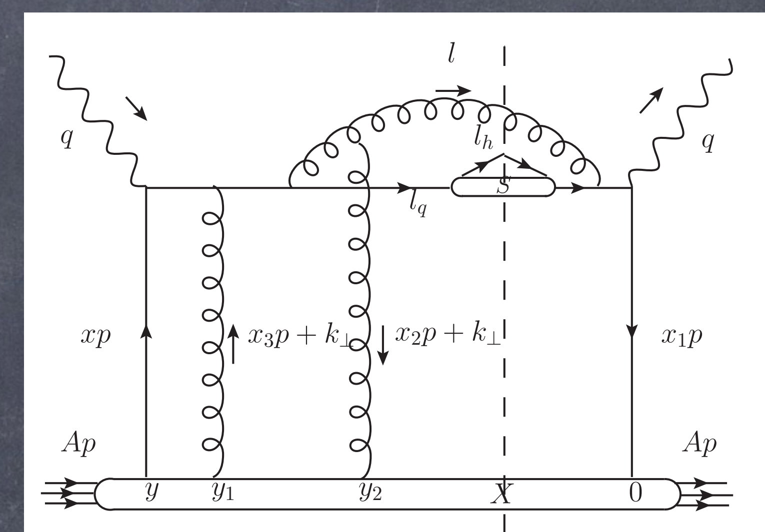
Right cut diagrams

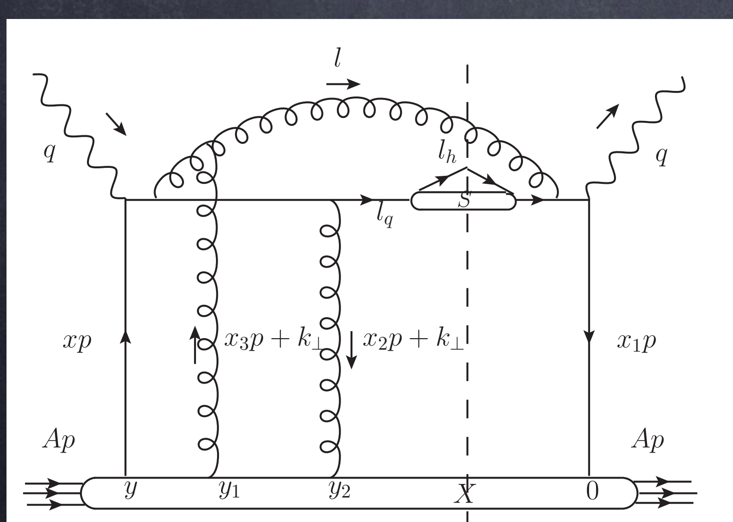
- Interference between vacuum radiation and medium induced radiation
- Symmetry between right cut and left cut diagrams

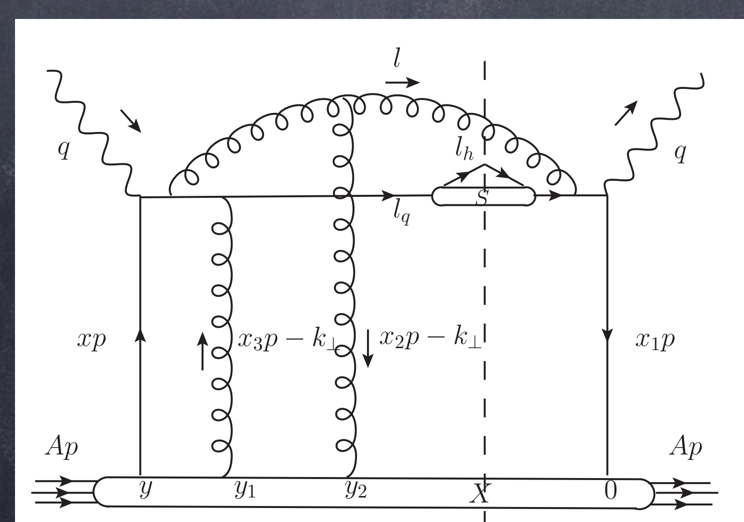


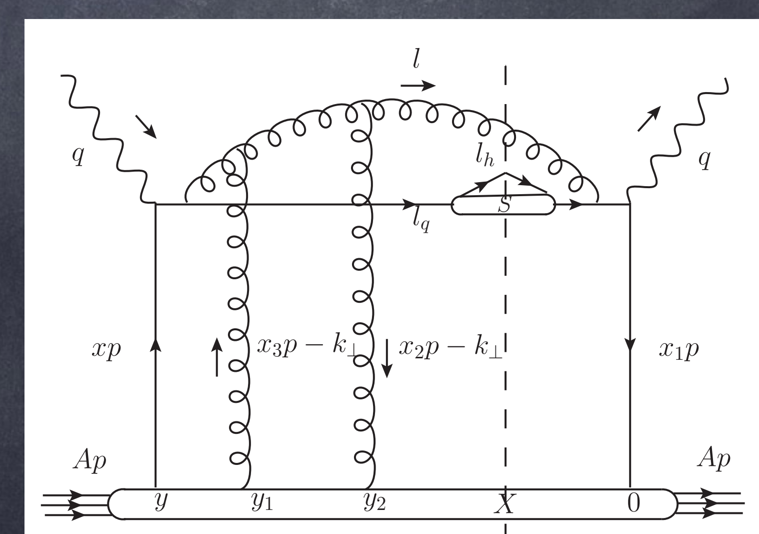








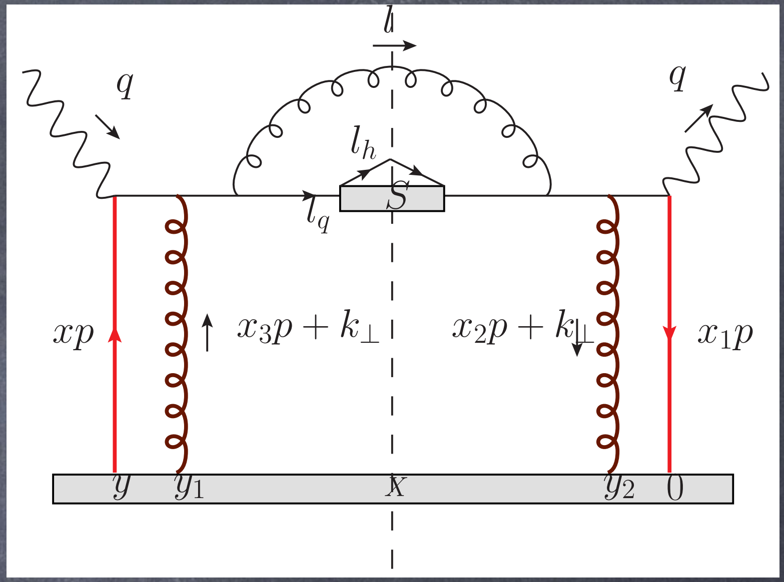




Nuclear enhancement and contact term

The θ function in every term (qg correlation func.) constrain the integration range of y_1^- and y_2^-

$$T_{qg}^A(x,x_1,x_2) = \int \frac{dy^-}{2\pi} dy_1^- dy_2^- \int d^2\vec{y}_{12\perp} e^{-ixp^+y^-} e^{-ix_2p^+(y_1^- - y_2^-)} e^{i(x-x_1)p^+y_1^-} e^{i\vec{k}_\perp\cdot\vec{y}_{12\perp}}$$
$$\times \langle A|\bar{\psi}(y^-)\frac{\gamma^+}{2}A^+(y_1^-,\vec{y}_{1\perp})A^+(y_2^-,\vec{y}_{2\perp})\psi(0)|A\rangle\theta(f_1)\theta(f_2)$$



$$\theta(f_1)\theta(f_2) = \begin{cases} \theta(y_2^-)\theta(y_1^- - y^-): & \text{central}, \\ \theta(y_2^- - y_1^-)\theta(y_1^- - y^-): & \text{left}, \\ \theta(y_1^- - y_2^-)\theta(y_2^-): & \text{right}. \end{cases}$$

$$|y_1^- - y_2^-|, |y^-| < r_N \qquad \text{(confinement)}$$

Nuclear enhancement : y_1^- integrate over the whole nuclei $\int_{y^-}^{\infty} dy_1^- \int_{-r_N}^{r_N} dy_{12}^-$

Contact term : quark, gluon from same nucleon $\int_0^{y^-} dy_1^- \int_0^{y_1^-} dy_2^-$

negligible

TMD Quark-gluon correlation function

Neglect contact term, further neglect the interaction between two parent nucleons. Approximately factorize T_{qg}^{A} into quark pdf and TMD gluon distribution

$$T_{qg}^{A}(x,x_1,x_2) = C\ f_q^A(x) \int dy_1^- \rho(y_1^-, \vec{y}_\perp) e^{i(x-x_1)p^+ y_1^-} \frac{\phi(x_2, \vec{k}_\perp)}{k_\perp^2}$$

TMD gluon distribution

$$\phi\left(x_2, \vec{k}_\perp\right) \equiv \int \frac{dy^-}{2\pi p^+} \int d^2 y_\perp e^{ix_2 p^+ y^- - i\vec{k}_\perp \cdot \vec{y}_\perp} \langle p \left| F_\alpha^+(0) F^{+\alpha}\left(y^-, y_\perp\right) \right| p \rangle$$

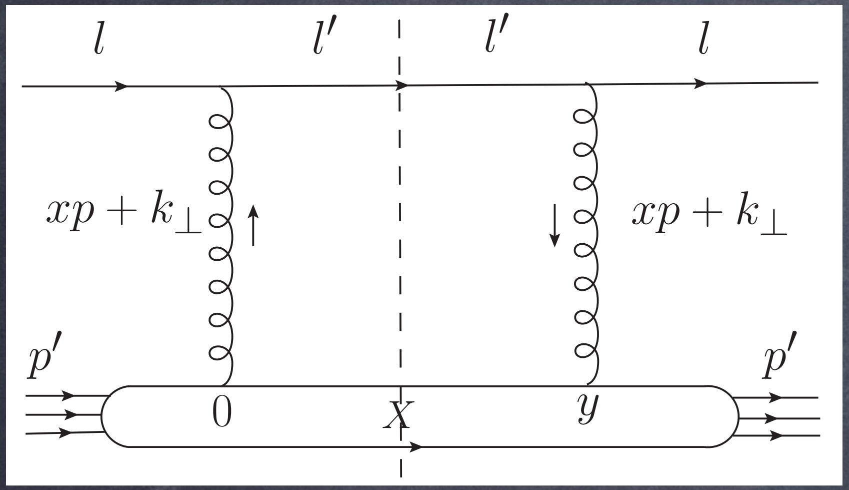
- finite x_2 entails energy/momentum transfer $x_2 p^+$ with medium
- TMD gluon pdf relate to TMD jet transport coefficient $\hat{q}(\vec{k}_\perp)$

TMD gluon distribution function

- TMD gluon distribution function emerge from TMD (generalized) transport coefficient $\hat{q}(\vec{k}_\perp)$

$$\hat{q} = \rho \int dk_\perp^2 \frac{\langle d\sigma \rangle}{dk_\perp^2} k_\perp^2$$

color source density*average kT broadening per scattering



$$\hat{q} \equiv \int \frac{d^2k_\perp}{(2\pi)^2} \int dx \delta(x - \frac{k_\perp^2}{2p^+l^-}) \frac{4\pi\alpha_s C_2(R)}{N_c^2 - 1} \rho(y) \phi(x, \vec{k}_\perp)$$

$$\equiv \int \frac{d^2k_\perp}{(2\pi)^2} \hat{q}(\vec{k}_\perp)$$

$\phi(x, \vec{k}_\perp)$ emerge naturally

$$\phi(x, \vec{k}_\perp) \equiv \int \frac{dy^-}{2\pi p^+} \int d^2y_\perp e^{ixp^+y^- - i\vec{k}_\perp \cdot \vec{y}_\perp} \langle p | F_\alpha{}^+(0) F^{+\alpha}(y^-, y_\perp) | p \rangle$$

$\phi(x, \vec{k}_\perp)$ depend on the parton energy l^- and medium color source energy p^+ (=<p’>) via $x = k_\perp^2 / 2p^+l^-$

Gluon Spectrum Result: Full Result

We get radiated gluon spectrum from the hadronic tensor

$$\frac{dW^{\mu\nu}_{D(1)g}}{dz_h} = \int dx f_q^A(x) H^{\mu\nu}_{(0)}(x) \int \frac{dz}{z} D_{q\to h}(z_h/z) \int dl_\perp^2 \frac{dN_g}{dl_\perp^2\, dz}.$$

Gluon spectrum : probability of quark to radiate one gluon with longitudinal momentum zq^- transverse momentum $l_\perp$

$$\frac{dN_g}{dl_\perp^2\, dz} = \frac{\pi}{f_q^A(x)} \frac{\alpha_s}{2\pi} \frac{1+(1-z)^2}{z} \frac{2\pi\alpha_s}{N_c} \int \frac{d^2k_\perp}{(2\pi)^2} \int_{y^-}^{\infty} dy_1^- \rho_A(y_1^-, \vec{y}_\perp) \left[\tilde{H}_C^D + \frac{1}{2}\tilde{H}_L^D + \frac{1}{2}\tilde{H}_R^D \right],$$

$\tilde{H}_C^D = \cdots$ $\quad$ $\tilde{H}_L^D = \cdots$ $\quad$ $\tilde{H}_R^D = \cdots$

1.Static Scattering Center Approximations

no energy transfer for medium scattering

no x dependence in TMD gluon distribution function

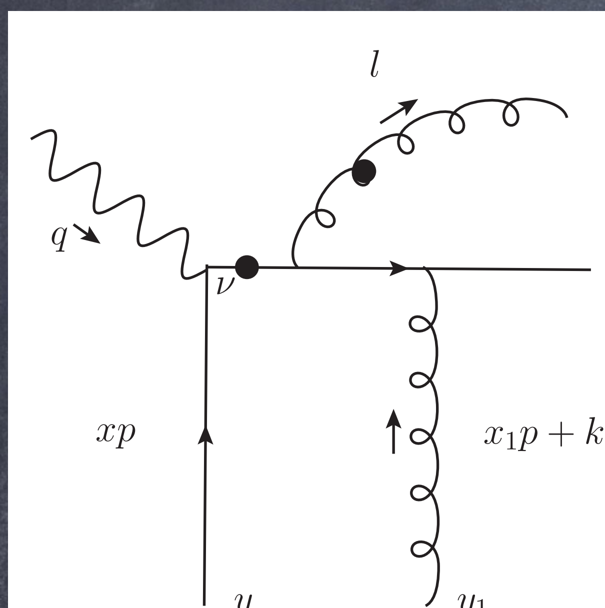
$$\begin{cases} \phi(x_L + \frac{1-z}{z} x_T, \vec{k}_\perp) \approx \phi(x_T, \vec{k}_\perp) \approx \cdots \approx \phi(0, \vec{k}_\perp) \\ f_q^A(x_B + x_L + \frac{x_T}{z}) \approx f(x_B + x_L) \approx f_q^A(x_B) \end{cases}$$

equivalent to $Q^2 \gg \frac{l_\perp^2}{z(1-z)}, \frac{k_\perp^2}{z(1-z)}$ or $x_B \gg x_L, \frac{x_D}{1-z}$

$$\begin{aligned} \frac{dN_g^{static}}{dl_\perp^2\, dz} =& \pi \frac{\alpha_s}{2\pi} \frac{1+(1-z)^2}{z} \frac{2\pi\alpha_s}{N_c} \int \frac{d^2k_\perp}{(2\pi)^2} \int dy_1^- \rho(y_1^-, \vec{y}_{1\perp}) \left\{ C_F \left[\frac{1}{(\vec{l}_\perp - z\vec{k}_\perp)^2} - \frac{1}{l_\perp^2} \right] \right. \\ &+ C_A \left[\frac{2}{(\vec{l}_\perp - \vec{k}_\perp)^2} - \frac{\vec{l}_\perp \cdot (\vec{l}_\perp - \vec{k}_\perp)}{l_\perp^2 (\vec{l}_\perp - \vec{k}_\perp)^2} - \frac{(\vec{l}_\perp - \vec{k}_\perp)\cdot(\vec{l}_\perp - z\vec{k}_\perp)}{(l_\perp - k_\perp)^2 (\vec{l}_\perp - z\vec{k}_\perp)^2} \right] \times (1 - \cos[(x_L + \frac{x_D}{1-z}) p^+ y_1^-]) \\ &\left. + \frac{1}{N_c} \left[\frac{\vec{l}_\perp \cdot (\vec{l}_\perp - z\vec{k}_\perp)}{l_\perp^2 (\vec{l}_\perp - z\vec{k}_\perp)^2} - \frac{1}{l_\perp^2} \right] \times (1 - \cos[x_L p^+ y_1^-]) \right\} \frac{\phi(0, \vec{k}_\perp)}{k_\perp^2} \end{aligned}$$

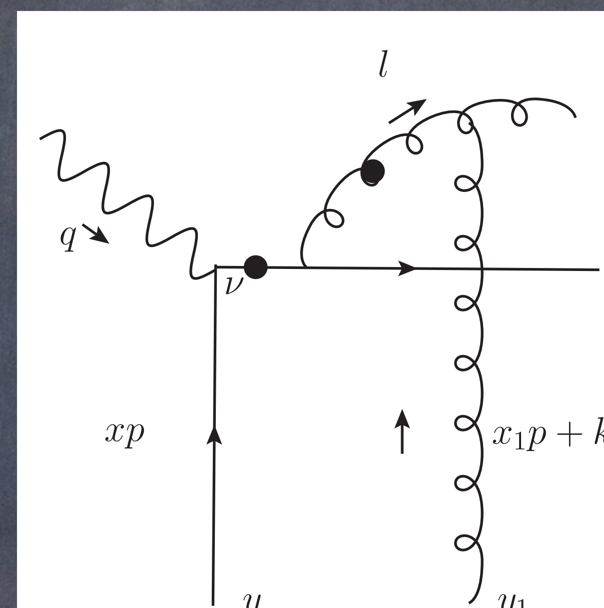
L Zhang, DF Hou, GY Qin
PRC 98.3 (2018): 034913

Divergences in gluon spectrum with static s.c.



$$\vec{l}_\perp = 0$$

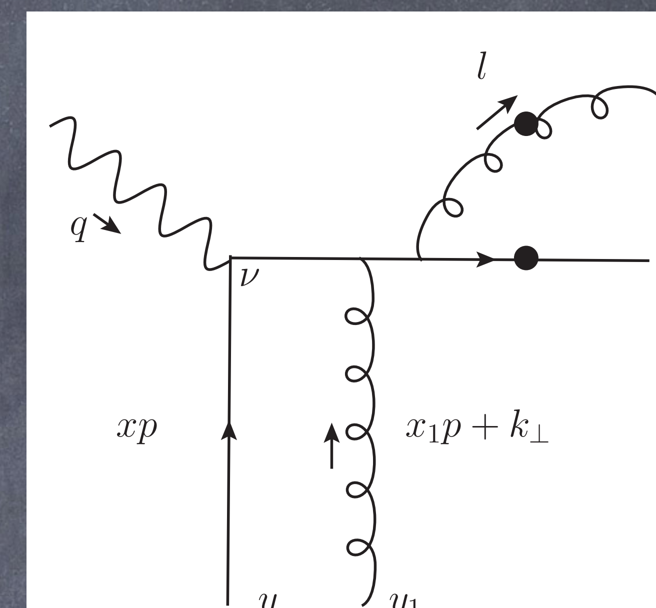
Renormalization of quark distribution function /q-g correlation function



$$\vec{l}_\perp - \vec{k}_\perp = 0$$

LPM effect

$$\frac{1}{(\vec{l}_\perp - \vec{k}_\perp)^2}(1 - cos[\frac{(\vec{l}_\perp - \vec{k}_\perp)^2}{2p^+q^-z(1-z)}p^+y_1^-])$$



$$\vec{l}_\perp - z\vec{k}_\perp = 0$$

Renormalization of quark fragmentation function

2.Soft radiated gluon approximation

the radiated gluon momentum fraction z ⟶ 0

$$\frac{dN_g^{\rm soft}}{dl_\perp^2\,dz} = \frac{\pi}{f_q^A(x)}\frac{\alpha_s}{2\pi}P_{qg}(z)\frac{2\pi\alpha_s}{N_c}\int\frac{d^2k_\perp}{(2\pi)^2}\int dy_1^-\rho_A(y_1^-,\vec{y}_\perp)\left[(\tilde{H}_C^D)_{\rm soft}+\frac{1}{2}(\tilde{H}_L^D)_{\rm soft}+\frac{1}{2}(\tilde{H}_R^D)_{\rm soft}\right]$$

$$\begin{aligned}(\tilde{H}_C^D)_{\rm soft} = \Bigg\{ & \left[\left(\frac{C_A}{(\vec{l}_\perp-k_\perp)^2}f_q^A(x+x_L+\frac{x_D}{1-z})-\frac{C_A}{l_\perp^2}f_q^A(x+x_L)\right)\frac{\phi(\frac{z}{1-z}x_D,\vec{k}_\perp)}{k_\perp^2}\right]\\ & +\left[C_A\frac{k_\perp^2}{l_\perp^2(\vec{l}_\perp-\vec{k}_\perp)^2}\frac{\phi(x_L+\frac{z}{1-z}x_D,\vec{k}_\perp)}{k_\perp^2}f_q^A(x)\right]\\ & +\left[-C_A\frac{\vec{k}_\perp\cdot\vec{l}_\perp}{l_\perp^2(\vec{l}_\perp-\vec{k}_\perp)^2}\frac{\phi(x_L+\frac{z}{1-z}x_D,\vec{k}_\perp)}{k_\perp^2}e^{i(x_L+\frac{x_D}{1-z})p^+y_1^-}f_q^A(x+x_L+\frac{x_D}{1-z})\right]\\ & +\left[-C_A\frac{\vec{k}_\perp\cdot\vec{l}_\perp}{l_\perp^2(\vec{l}_\perp-\vec{k}_\perp)^2}\frac{\phi(x_D,\vec{k}_\perp)}{k_\perp^2}e^{-i(x_L+\frac{x_D}{1-z})p^+y_1^-}f_q^A(x)\right]\Bigg\}\end{aligned}$$

$(\tilde{H}_L^D)_{\rm soft} = \cdots$ $\quad$ $(\tilde{H}_R^D)_{\rm soft} = \cdots$

no dependence on z except splitting function $P_{qg}(z)\approx 2/z$, ϕ and f_q^A

3. Static+Soft Approximations

$Q^2 \gg \frac{l_\perp^2}{z(1-z)}, \frac{k_\perp^2}{z(1-z)}$ ($x_B \gg x_L, \frac{x_T}{z}$) as z → 0

the gluon spectrum is

$$\frac{dN_g^{static+soft}}{dl_\perp^2\, dz} = \pi \frac{\alpha_s}{2\pi} P_{qg}(z) \frac{2\pi\alpha_s}{N_c} \int \frac{d^2k_\perp}{(2\pi)^2} \int dy_1^- \rho_A(y_1^-, \vec{y}_\perp)$$
$$\times C_A \frac{2\vec{k}_\perp \cdot \vec{l}_\perp}{l_\perp^2 (\vec{l}_\perp - \vec{k}_\perp)^2} \left(1 - \cos[(x_L + \frac{x_D}{1-z})p^+ y_1^-]\right) \frac{\phi(0, \vec{k}_\perp)}{k_\perp^2}$$

$\phi(x, \vec{k}_\perp)$ **non-perturbative**

TMD gluon pdf relation to transport parameter $\hat{q}$

$$\hat{q} = \frac{4\pi\alpha_s C_2(R)}{N_c^2 - 1}\rho(y)\int \frac{d^2k_\perp}{(2\pi)^2}\phi(0,\vec{k}_\perp)$$ ★

One method : Static potential model to calculate $\hat{q}$

$$< d\sigma >= \frac{C_2(R)C_2(T)}{d_A}\frac{4\pi\alpha_s^2}{t^2}dt$$

$$\hat{q} = \rho\int dk_\perp^2 \frac{C_2(R)C_2(T)}{d_A}\frac{4\pi\alpha_s^2}{k_\perp^2 + \mu_0^2}$$ ★

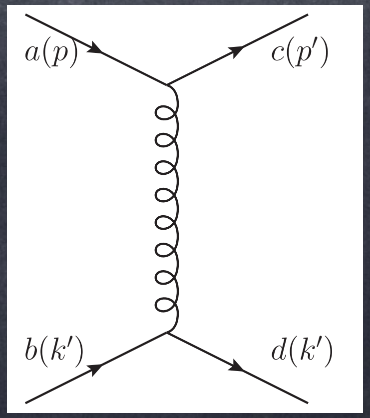
Casimir $C_2(R)$ $C_2(T)$

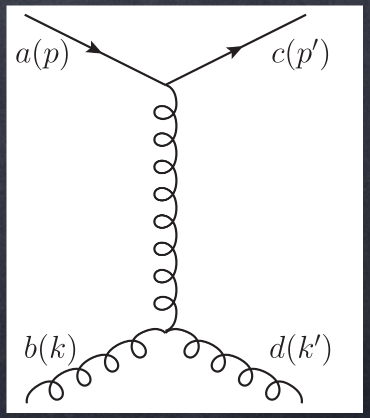
Mandelstam variable t

$$t = k_\perp^2 + \mu_0^2$$

compare two $\hat{q}$

$$\phi(0,\vec{k}_\perp) = C_2(T)\frac{4\alpha_s}{k_\perp^2 + \mu_0^2}$$





3. Static+Soft Approximations

substitute $\phi(0,\vec{k}_\perp)$ into gluon spectrum, one get

$$\frac{dN_g^{static+soft}}{dl_\perp^2\,dz} = 8\pi\alpha_s^3\frac{C_2(T)C_A}{N_c}P_{qg}(z)\int\frac{d^2k_\perp}{(2\pi)^2}\int dy_1^-\rho_A(y_1^-,\vec{y}_\perp) \times \frac{\vec{k}_\perp\cdot\vec{l}_\perp}{l_\perp^2(\vec{l}_\perp-\vec{k}_\perp)^2}\left(1-\cos[(x_L+\frac{x_D}{1-z})p^+y_1^-]\right)\frac{1}{(k_\perp^2+\mu_D^2)^2}$$

agrees with of GLV result at first order in opacity

arguments in Cosine function
$$\begin{cases}\omega_1\approx\sqrt{2}(x_L+\frac{1}{z}x_T)p^+\\ y_{10}=y_1-y_0\approx\frac{y_1^-}{\sqrt{2}}\end{cases}$$

Plot the integration kernel of gluon spectrum

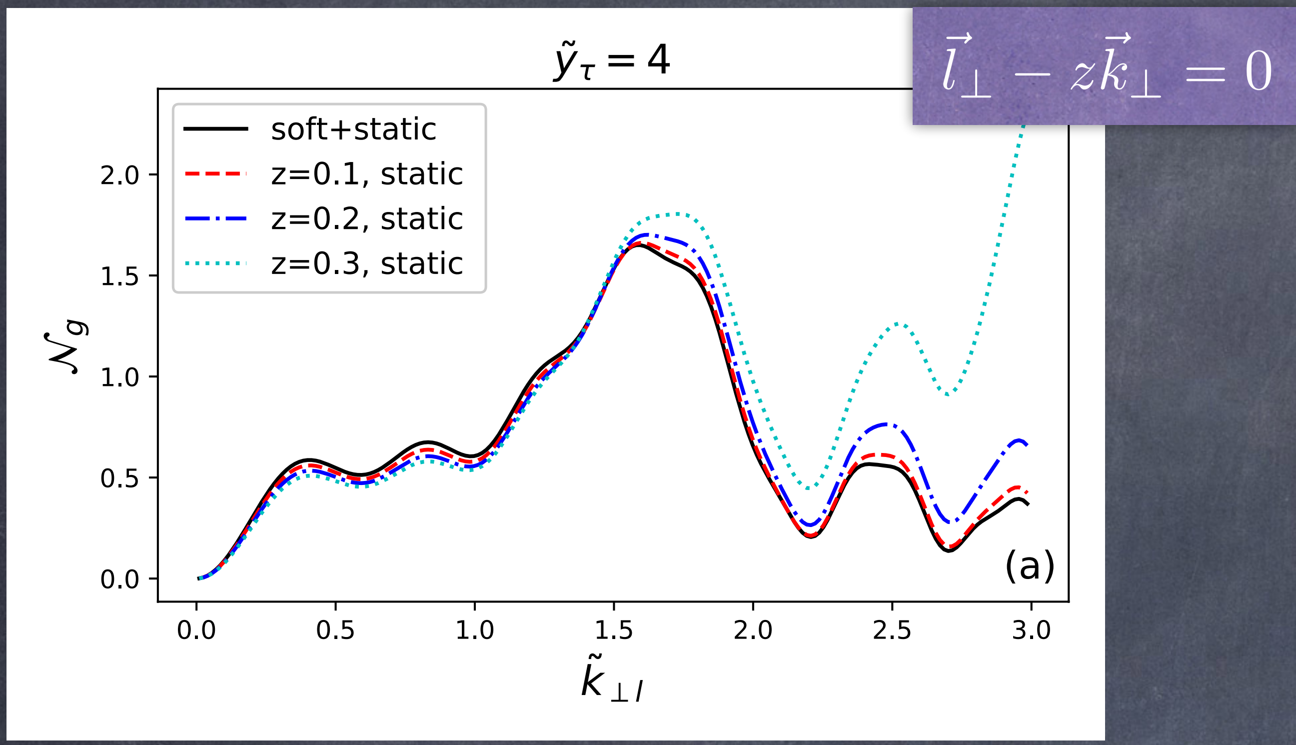
$$\frac{dN_g}{dl_\perp^2\,dz} = \int_{y^-}^{\infty} dy_1^- \left[\rho_A\left(y_1^-, \vec{y}_\perp\right)\frac{2\pi\alpha_s}{N_c}\pi\int\frac{dk_\perp^2}{(2\pi)^2}\frac{\phi\left(0,\vec{k}_\perp\right)}{k_\perp^2}\right]\pi\frac{\alpha_s}{2\pi}P_{qg}(z)\frac{C_A}{l_\perp^2}\mathcal{N}_g$$

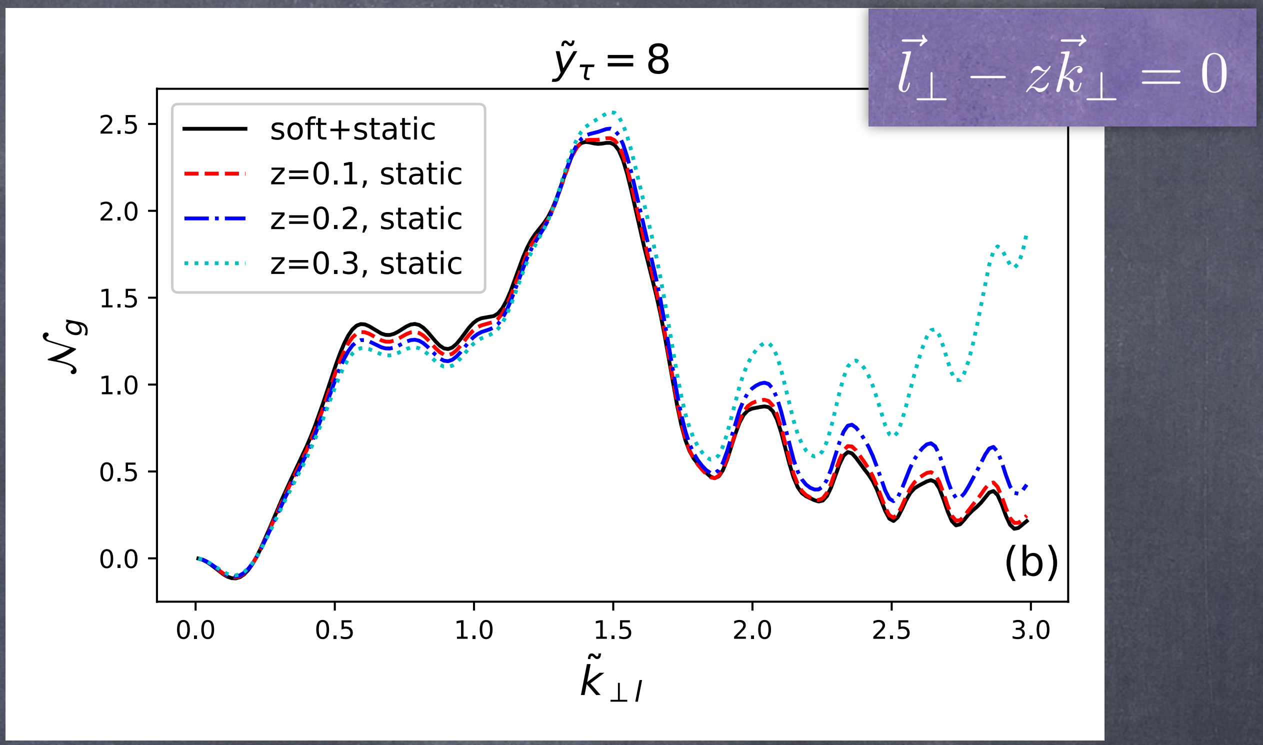
$$\mathcal{N}_g^{\text{static}} = \int\frac{d\varphi}{2\pi}\frac{l_\perp^2}{C_A}\left\{C_F\left[\frac{1}{(\vec{l}_\perp - z\vec{k}_\perp)^2} - \frac{1}{l_\perp^2}\right] + C_A\left[\frac{2}{(\vec{l}_\perp - \vec{k}_\perp)^2} - \frac{\vec{l}_\perp\cdot(\vec{l}_\perp - \vec{k}_\perp)}{l_\perp^2(\vec{l}_\perp - \vec{k}_\perp)^2} - \frac{(\vec{l}_\perp - \vec{k}_\perp)\cdot(\vec{l}_\perp - z\vec{k}_\perp)}{(l_\perp - k_\perp)^2(\vec{l}_\perp - z\vec{k}_\perp)^2}\right]\right.$$
$$\left.\times(1-\cos[(x_L + \frac{x_D}{1-z})p^+y_1^-]) + \frac{1}{N_c}\left[\frac{\vec{l}_\perp\cdot(\vec{l}_\perp - z\vec{k}_\perp)}{l_\perp^2(\vec{l}_\perp - z\vec{k}_\perp)^2} - \frac{1}{l_\perp^2}\right]\times\left(1-\cos[x_L p^+ y_1^-]\right)\right\}$$

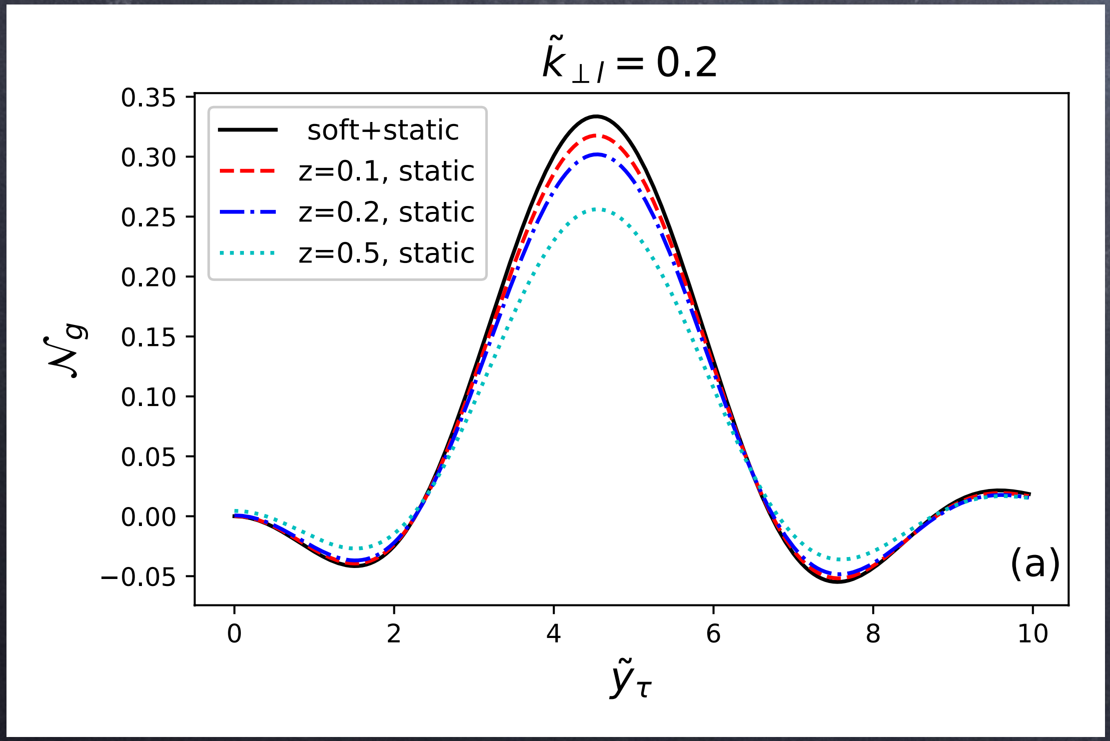
$$\mathcal{N}_g^{\text{static+soft}} = \int\frac{d\varphi}{2\pi}\frac{2\vec{k}_\perp\cdot\vec{l}_\perp}{(\vec{l}_\perp - \vec{k}_\perp)^2}\left(1-\cos[(x_L + \frac{x_D}{1-z})p^+y_1^-]\right)$$

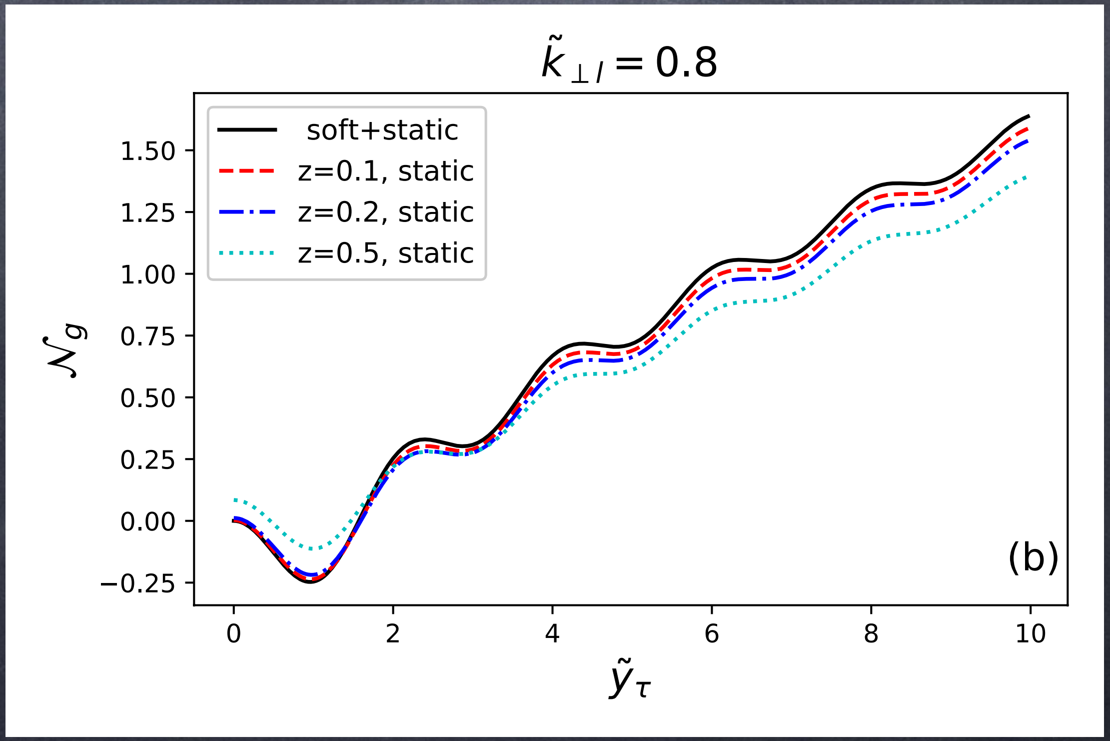
Numerical Comparisons

$$\tilde{k}_{\perp l} \equiv k_\perp / l_\perp \quad , \quad \tilde{y}_\tau \equiv \frac{y_1^- l_\perp^2}{2q^- z(1-z)} \equiv \frac{y_1^-}{\tau_f} \quad (\tau_f = 2q^- z(1-z)/l_\perp^2)$$









Summary

- Radiative gluon spectrum (energy loss) in SIDIS. Relax the $k_\perp \ll l_\perp$ approximation in HT.
- Approximately factorize TMD gluon pdf from TMD quark-gluon correlation function. TMD (generalized) transport coefficient relate to TMD gluon pdf
- Our result with static scattering center and soft radiative gluon approximations agrees with GLV result

Further work

- Investigate the divergences, other observable (single inclusive jet)
- implementation into CoLBT-Hydro Model

Thanks!