Large Transverse Momentum

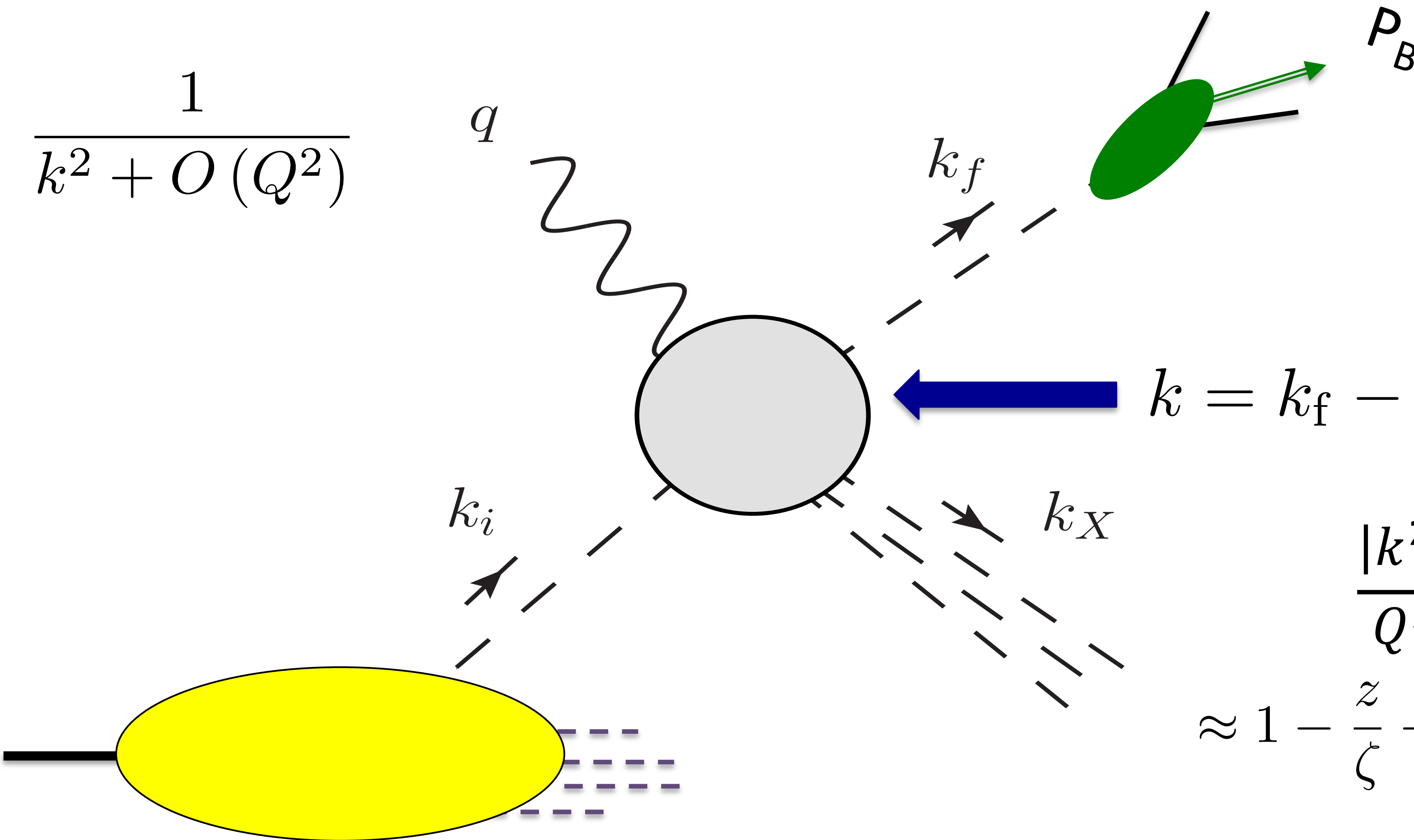
Jefferson Lab/Old Dominion University

- Motivation
- Survey of processes

TMD Collaboration Meeting 2019, September 18th

Large and Small Transverse Momentum

$$\frac{1}{k^2 + O\left(Q^2\right)}$$



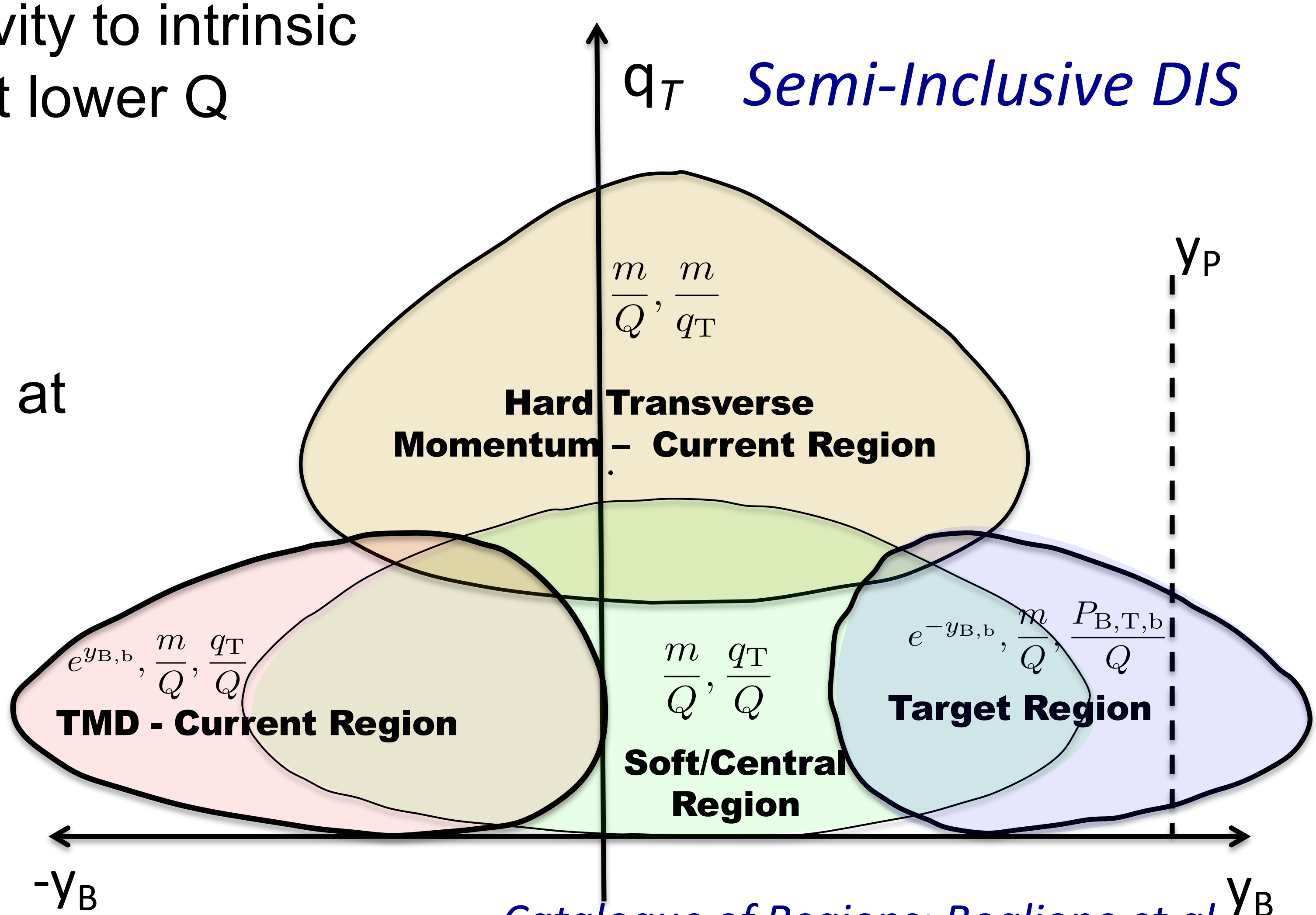
$$k = k_{\mathrm{f}} - q$$

$$\frac{|k^2|}{Q^2} \approx 1 - \frac{z}{\zeta} + \frac{z}{\zeta}\frac{q_{\mathrm{T}}^2}{Q^2}$$

Semi-Inclusive DIS

What is the relevant description?

- High sensitivity to intrinsic properties at lower Q
- Small errors at larger Q



Catalogue of Regions: Boglione et al (To appear in JHEP)

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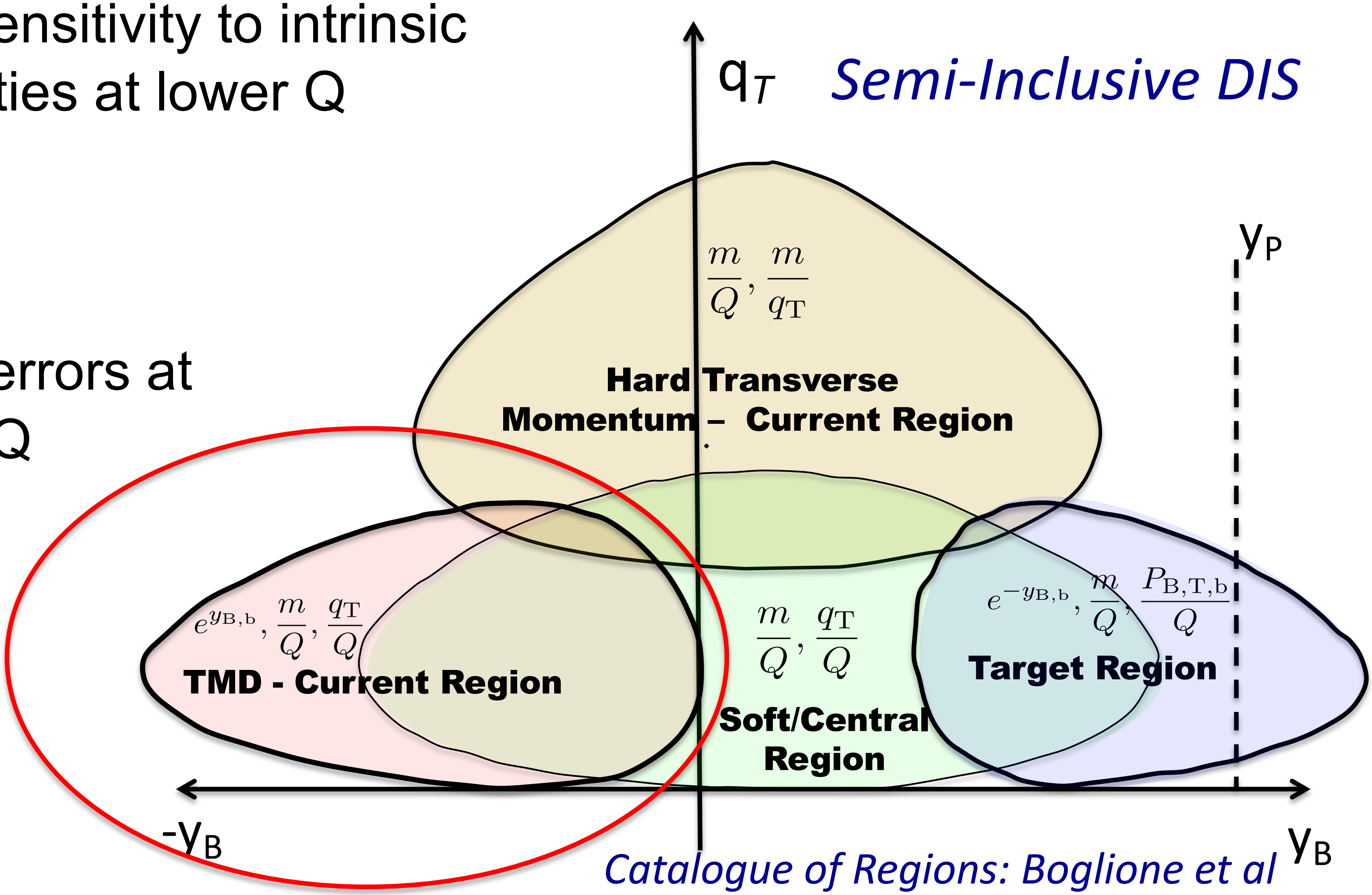


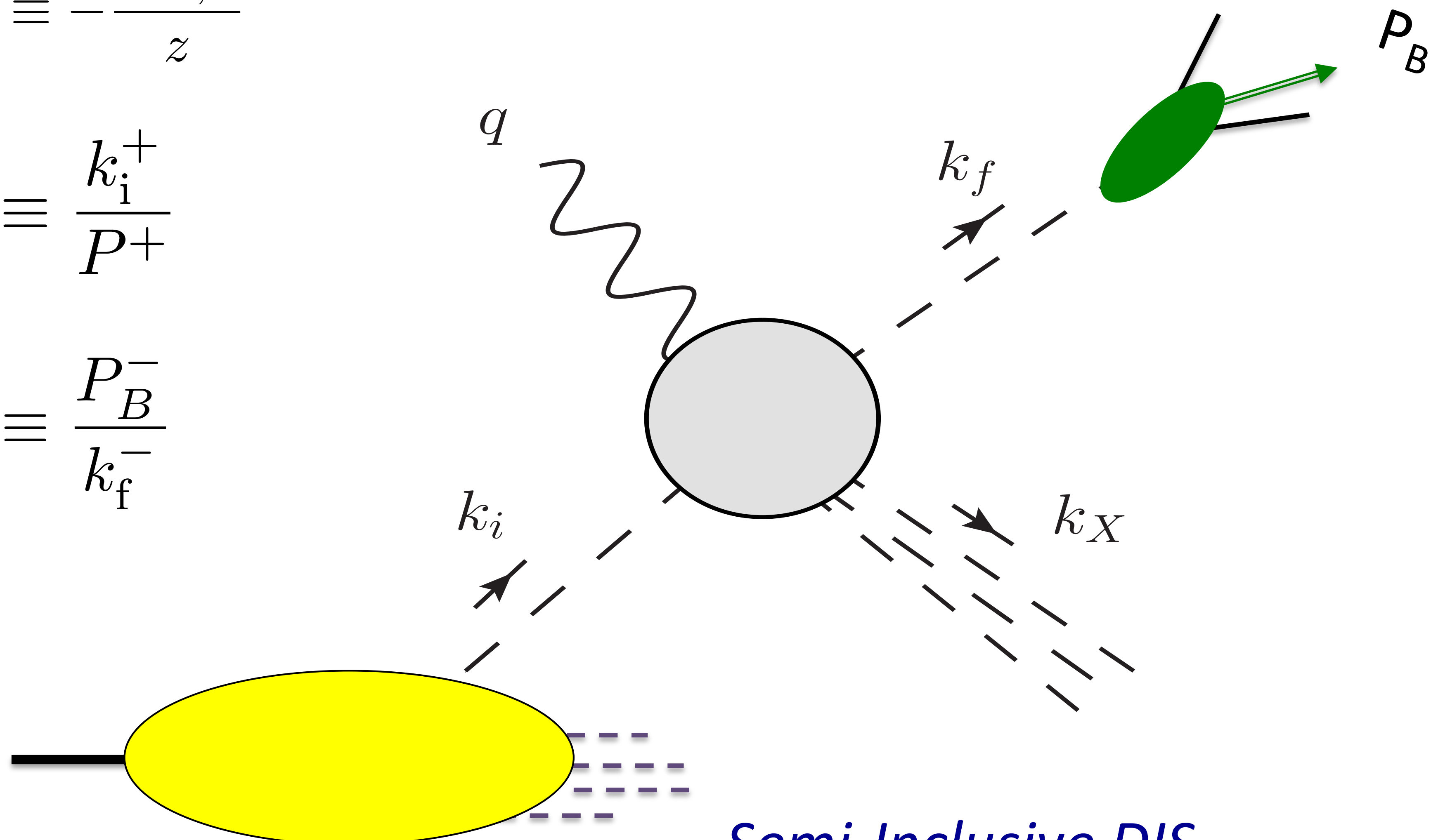
Catalogue of Regions: Boglione et al (To appear in JHEP)

Current Fragmentation

$$\boldsymbol{q}_{\mathrm{T}} \equiv -\frac{\boldsymbol{P}_{B,\mathrm{T}}}{z}$$

$$\xi \equiv \frac{k_{\mathrm{i}}^{+}}{P^{+}}$$

$$\zeta \equiv \frac{P_B^{-}}{k_{\mathrm{f}}^{-}}$$

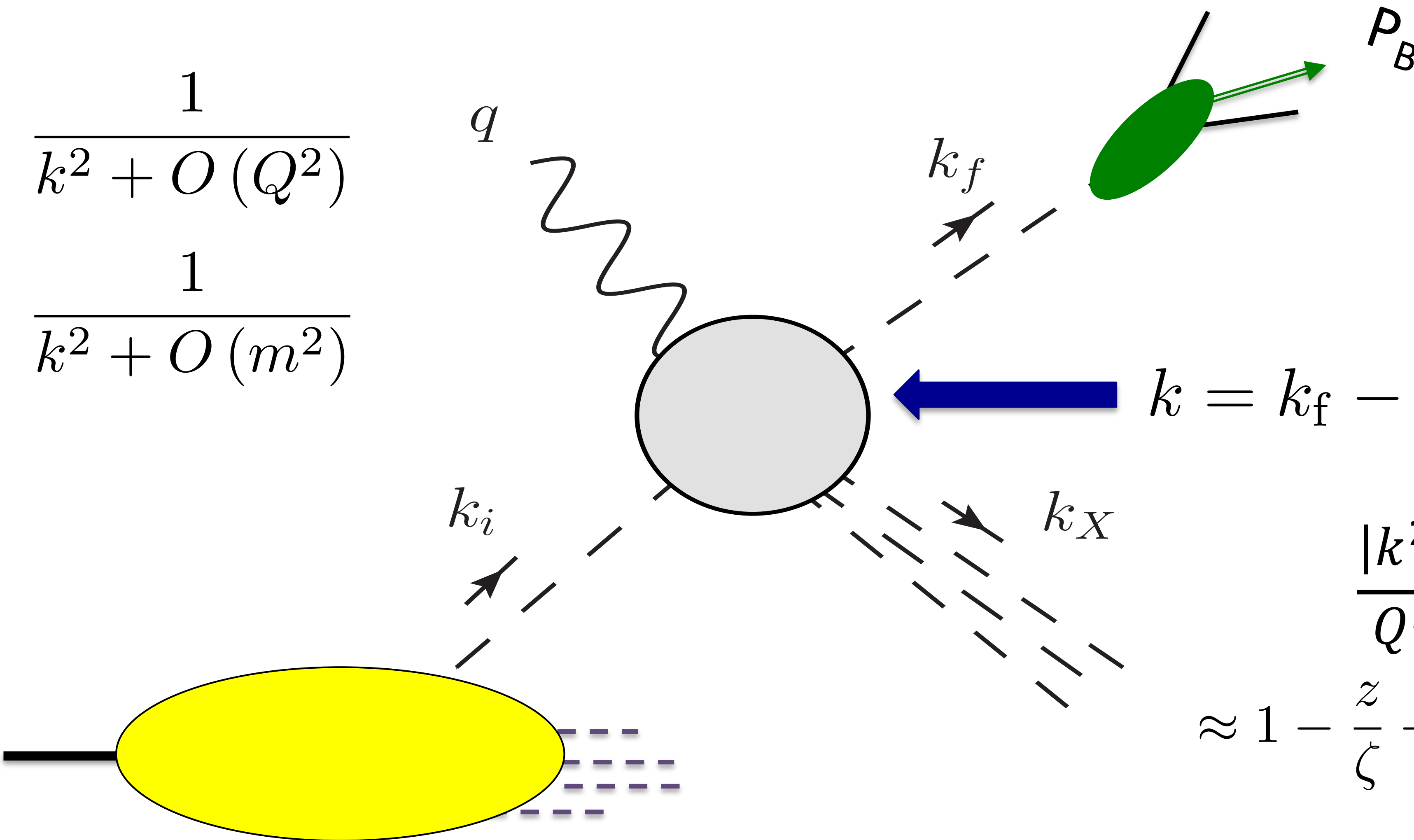


Semi-Inclusive DIS

Large and Small Transverse Momentum

$$\frac{1}{k^2 + O\left(Q^2\right)}$$

$$\frac{1}{k^2 + O\left(m^2\right)}$$



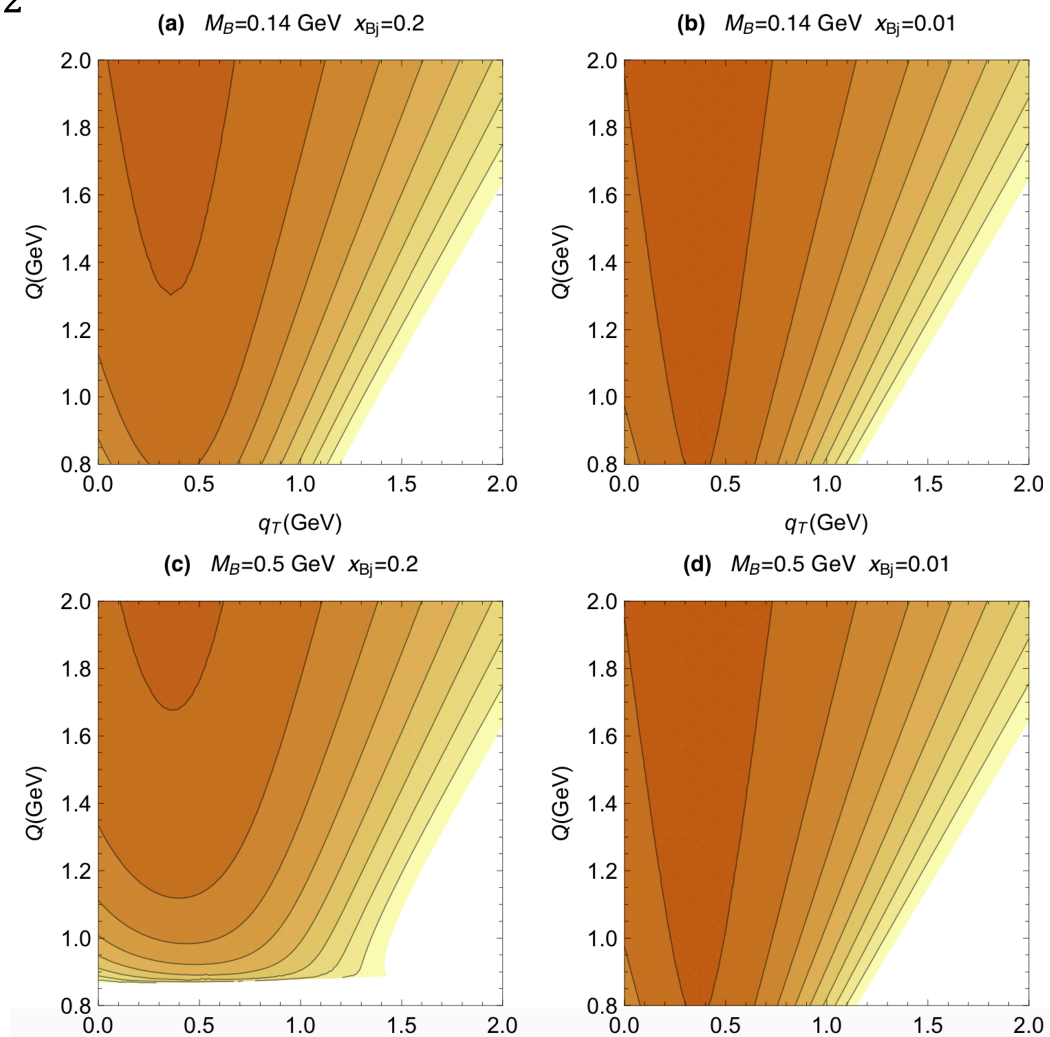
$$k = k_{\mathrm{f}} - q$$

$$\frac{|k^2|}{Q^2} \approx 1 - \frac{z}{\zeta} + \frac{z}{\zeta}\frac{q_{\mathrm{T}}^2}{Q^2}$$

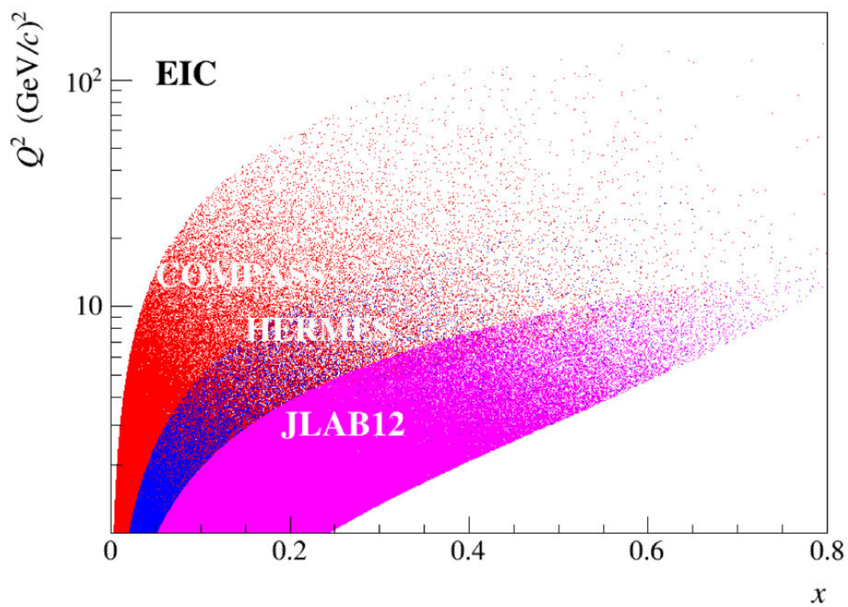
Semi-Inclusive DIS

Large and Small Transverse Momentum

$$\frac{|k^2|}{Q^2}$$



$z_h = 0.25$, ζ=0.3 and. ξ=0.2.



-H. Avakian, A. Bressan, and M. Contalbrigo, "Experimental results on TMDs" (2016)

Semi-Inclusive DIS

Large and Small Transverse Momentum

- Hadron masses

$$z_{\mathrm{N}} = \frac{P_{\mathrm{B}}^-}{q^-} \qquad\qquad z_{\mathrm{h}} = \frac{P \cdot P_{\mathrm{B}}}{P \cdot q} = 2x_{\mathrm{Bj}} \frac{P \cdot P_{\mathrm{B}}}{Q^2}$$

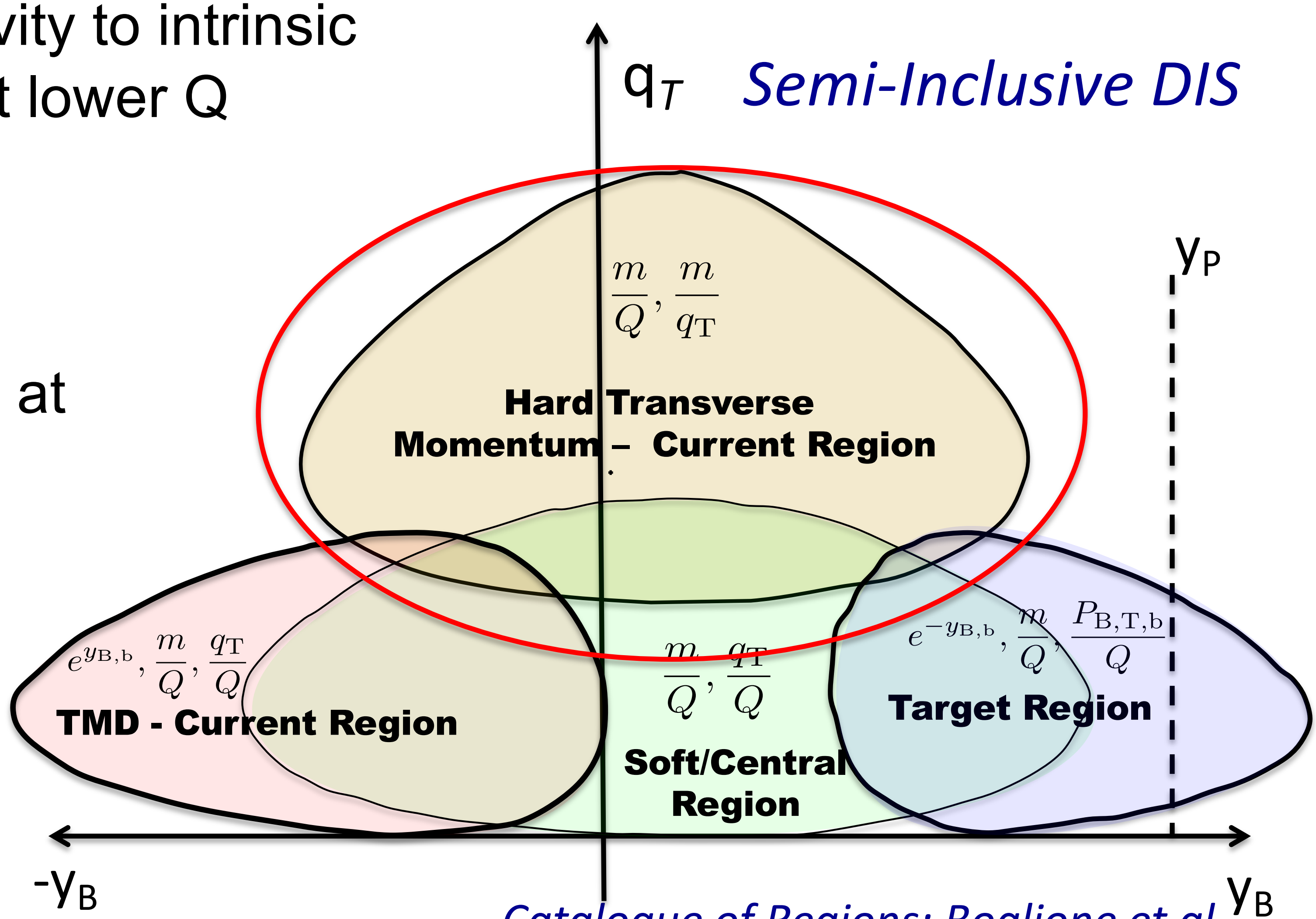
- Power corrections

$$\begin{aligned} z_{\mathrm{N}} &= \frac{x_{\mathrm{N}} z_{\mathrm{h}}}{2x_{\mathrm{Bj}}} \left(1 + \sqrt{1 - \frac{4M^2 M_{\mathrm{B,T}}^2 x_{\mathrm{Bj}}^2}{Q^4 z_{\mathrm{h}}^2}} \right) \\ &= z_{\mathrm{h}} \left(1 - \frac{x_{\mathrm{Bj}}^2 M^2}{Q^2} \left(1 + \frac{P_{\mathrm{B,T}}^2}{z_{\mathrm{h}}^2 Q^2} \right) + \left(\frac{x_{\mathrm{Bj}}^2 M^2}{Q^2} \right)^2 \left(\frac{P_{\mathrm{B,T}}^2}{z_{\mathrm{h}}^2 Q^2} - \frac{P_{\mathrm{B,T}}^4}{z_{\mathrm{h}}^4 Q^4} + 2 - \frac{M_{\mathrm{B}}^2}{z_{\mathrm{h}}^2 M^2 x_{\mathrm{Bj}}^2} \right) + O\left(\left(\frac{x_{\mathrm{Bj}}^2 M^2}{Q^2} \right)^3 \right) \right) \end{aligned}$$

What is the relevant description?

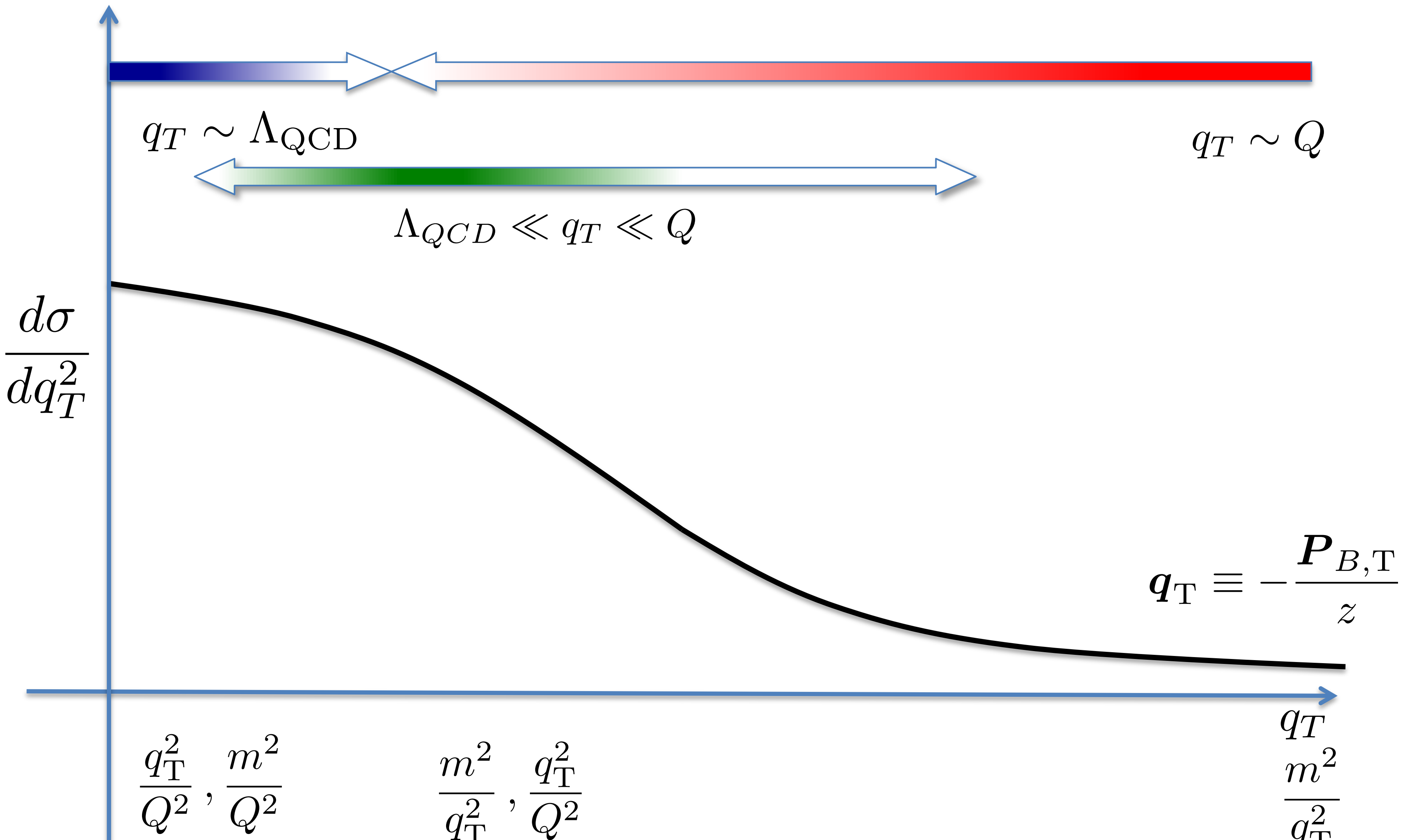
- High sensitivity to intrinsic properties at lower Q

- Small errors at larger Q



Catalogue of Regions: Boglione et al (To appear in JHEP)

Large and Small Transverse Momentum



Large and Small Transverse Momentum

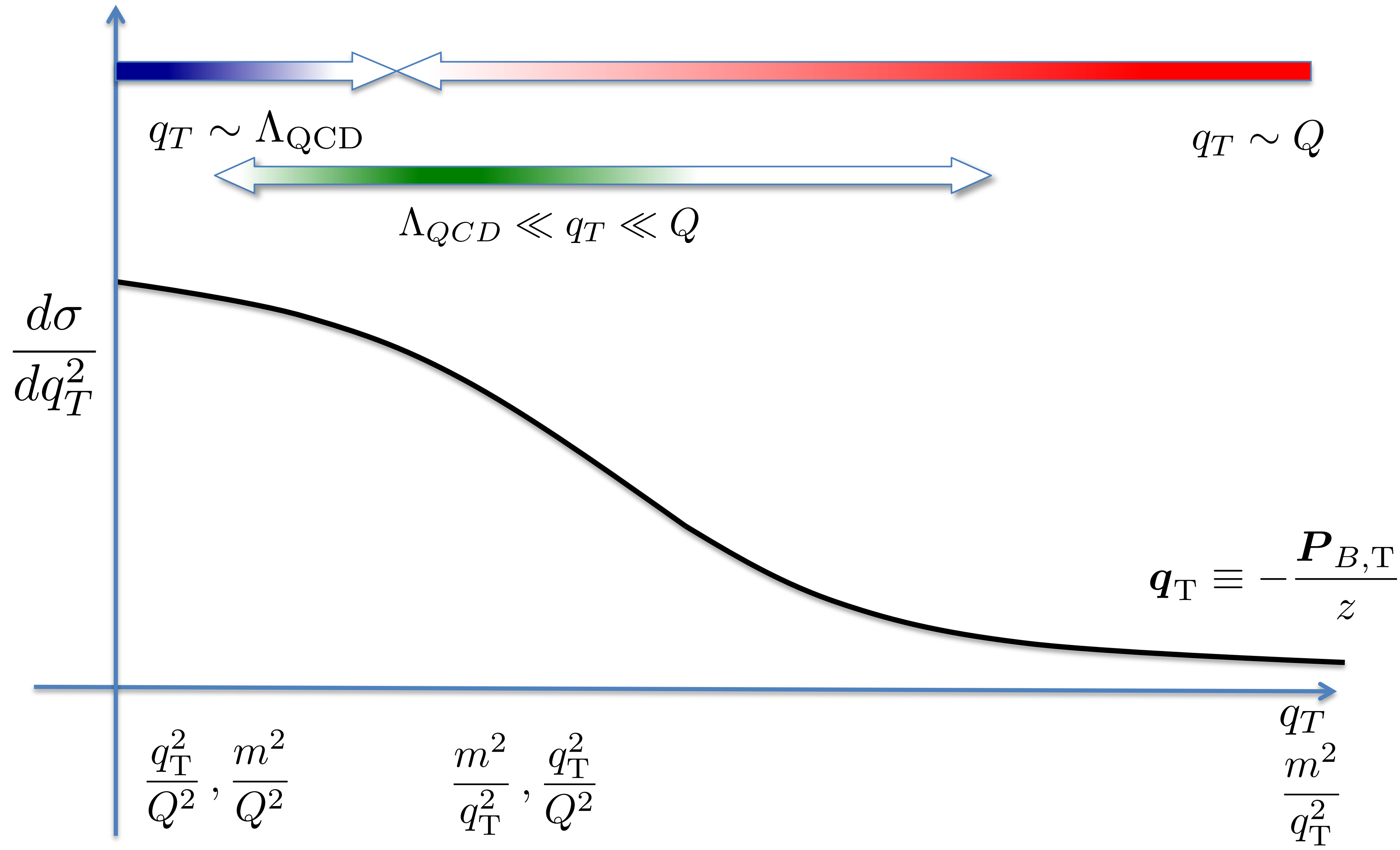
- TMD pdfs in physical processes
  - Small q_T/Q expansion
    - Need to separate small and large q_T/Q

$$\frac{d\sigma}{dx\,dz\,dQ\,dq_T} = \underbrace{H(Q) f(x, k_{1T}) \otimes d(z, z\,k_{2T})}_{\substack{\text{Small } q_T/\text{Q} \\ \text{approximation} \\ \text{(TMD factorization)}}} + \underbrace{Y(x, z, q_T)}_{\substack{q_T \sim Q \\ \text{correction} \\ \text{(collinear factorization)}}} + \mathrm{P.S.C.}$$

- Works best if there is a region of overlap between large and small q_T/Q methods:

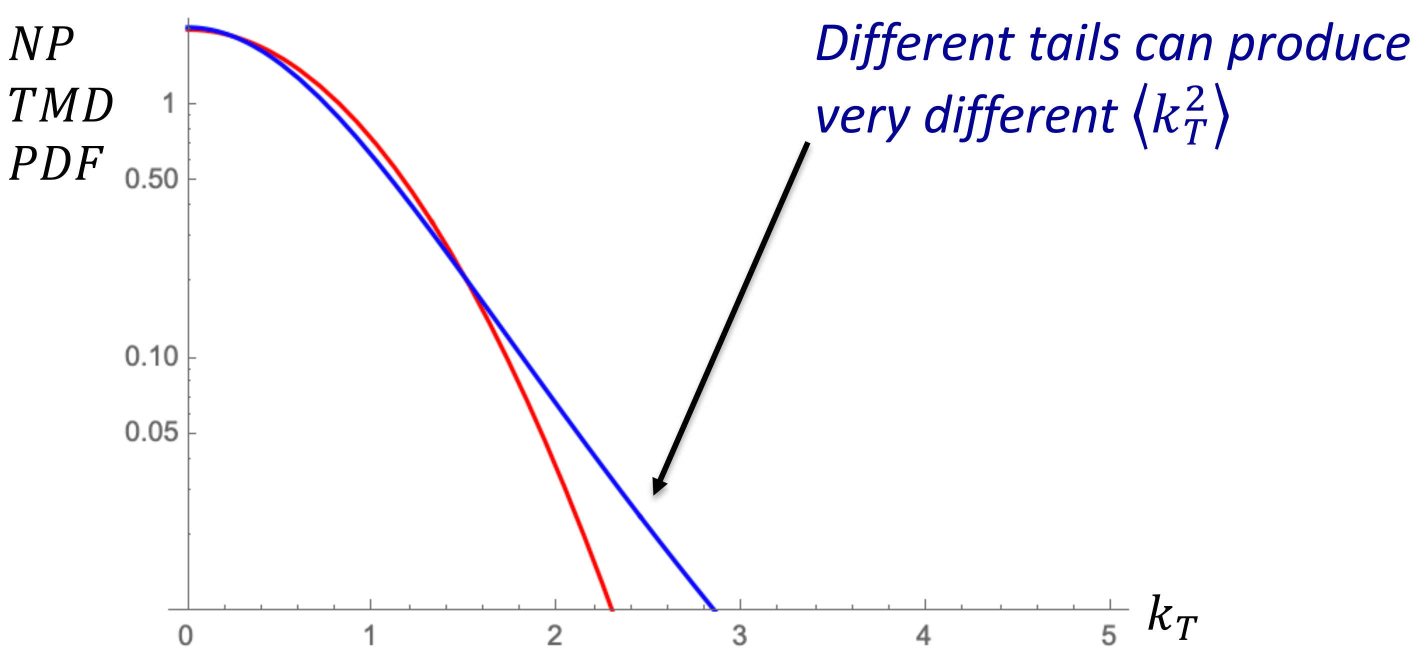
$$Y(x, z, q_T) = \text{Large } \; q_T/Q \; \text{ Approx.} \; - \text{Small and Large } \; q_T/Q \; \text{ Approx.}$$

Why Large Transverse Momentum?



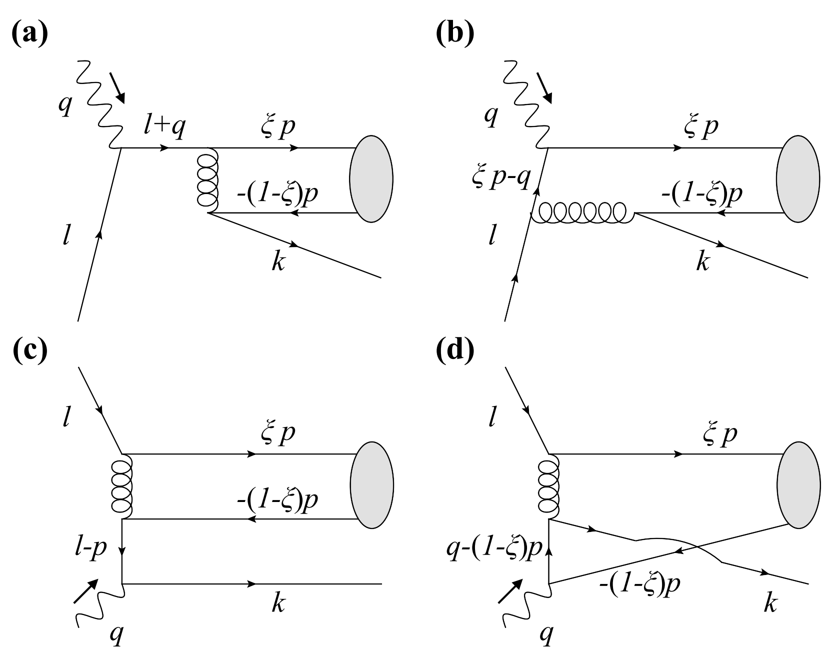
Why Large Transverse Momentum?

- Example: Shapes of TMD distributions:
  - Sea versus valence?
  - Different flavors?

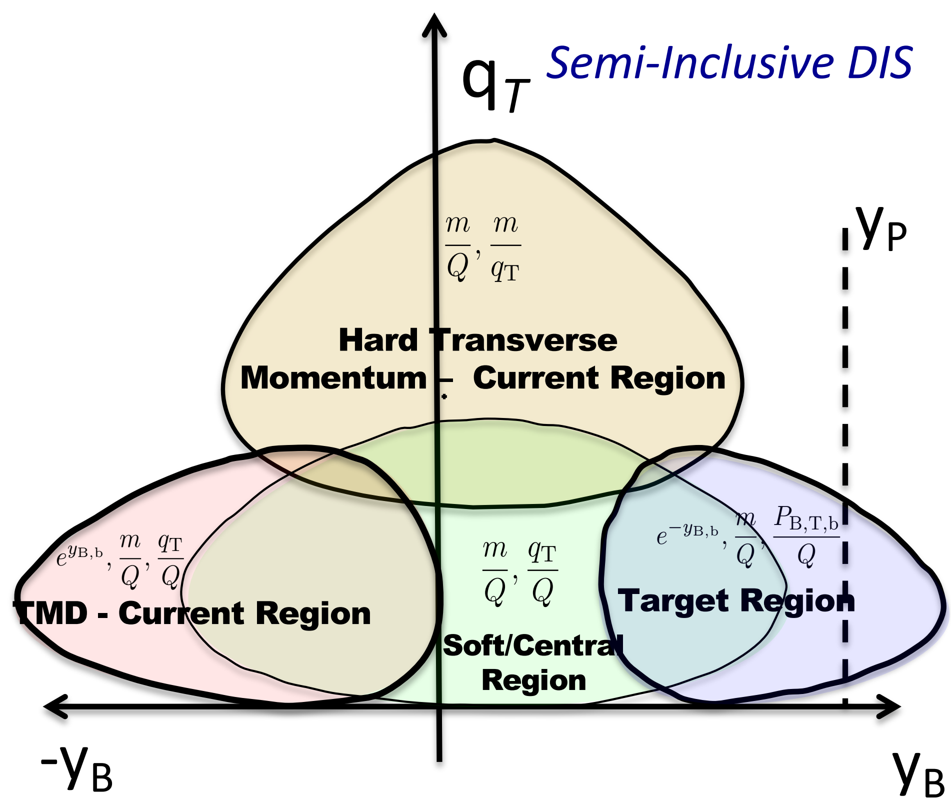


Why Large Transverse Momentum?

- What is the mechanism for hadron production when hard scales are not that hard?



T. Liu, J.-W. Qiu (2019)



Why Large Transverse Momentum?

- Interpretation of integrals

$$\int d^2\mathbf{k}_T f(x,k_T) = f(x), \quad \int d^2\mathbf{k}_T \frac{k_T^2}{2M^2} f_{1\perp}(x,k_T) = f_{1\perp}^{(1)}(x), \quad \dots$$

- In generalized parton model:

$$\frac{d\sigma}{dx\,dz\,dQ\,dq_T} = H(Q) f(x,k_{1T}) \otimes d(z, z\,k_{2T}) + \cancel{Y(x,z,q_T)} + \cancel{\mathrm{P.S.C.}}$$

$$= H(Q) \int d^2\mathbf{k}_{1T} \int d^2\mathbf{k}_{2T}\, f(x,k_{1T}) d(z,k_{2T}) \delta^{(2)}(\mathbf{k}_{2T} - \mathbf{k}_{1T} - \mathbf{q}_T)$$

Assume all contributions to transverse momentum dependence are intrinsic

Why Large Transverse Momentum?

- Integrated cross section in generalized parton model

$$\int d^2\boldsymbol{q}_T \left(\frac{d\sigma}{dx\,dz\,dQ\,dq_T}\right) = H(Q)\left(\int d^2\mathbf{k}_{1T}\, f(x, k_{1T})\right)\left(\int d^2\mathbf{k}_{2T}\, d(z, z\, k_{2T})\right)$$

$$= H(Q) f(x) d(z)$$

- Full integral

$$\int d^2\boldsymbol{q}_T \left(\frac{d\sigma}{dx\,dz\,dQ\,dq_T}\right) = \int d^2\boldsymbol{q}_T \left(H(Q) f(x, k_{1T}) \otimes d(z, z\, k_{2T}) + Y(x, z, q_T)\right)$$

Cutoff dependence cancels between terms

Simplest Processes with Transverse Momentum

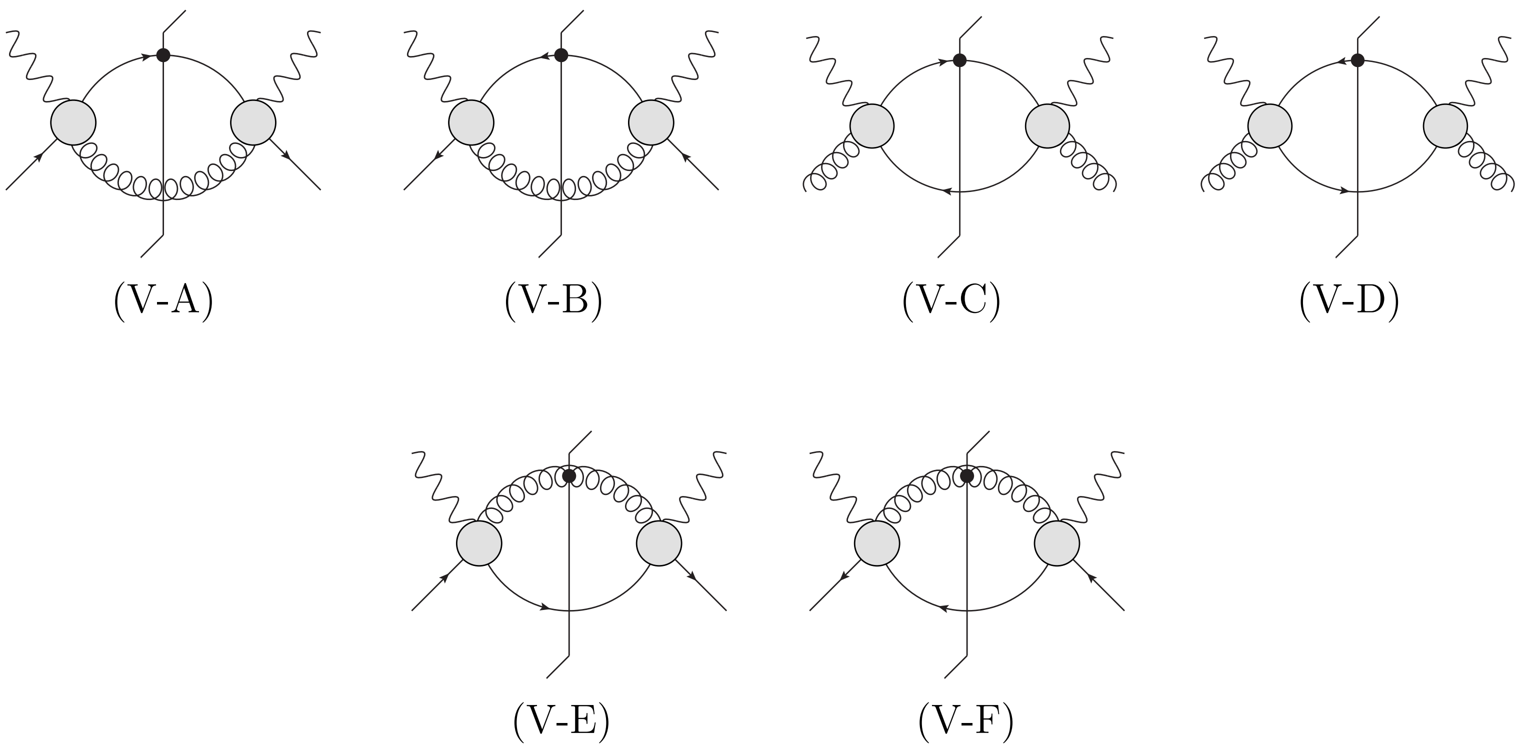
- Semi-inclusive DIS (JLab, EIC,…)

- Drell-Yan

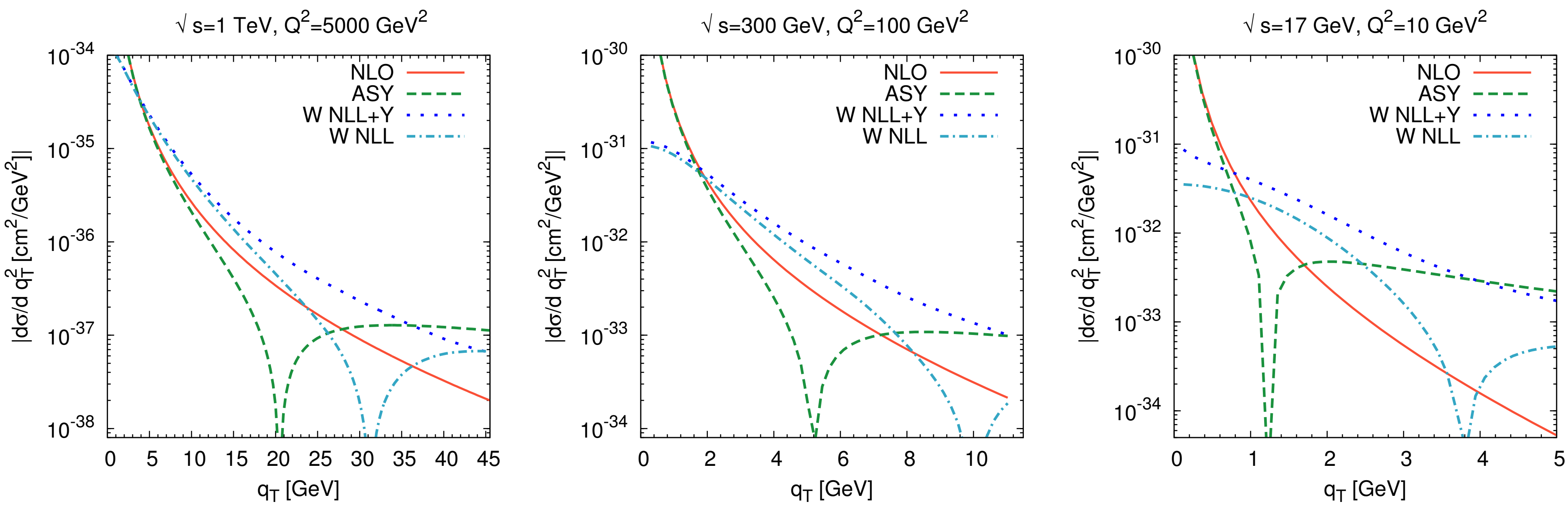
- e^+e^- annihilation (Belle)

SIDIS

- Leading order contribution to large transverse momentum production



SIDIS



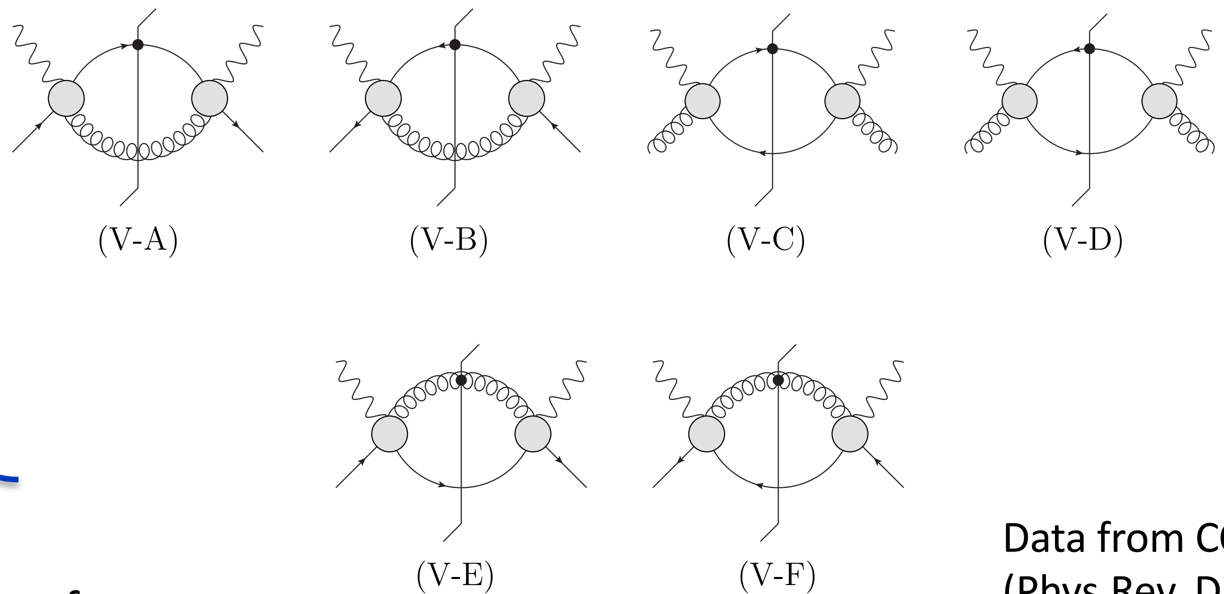
Boglione et al, JHEP 1502 (2015) 095:

- Directly implement Y-term

- Standard Y-correction diverges for both $q_T \to 0$ and $q_T \to \infty$
- For moderate Q, $m \ll q_T$ << Q region gets squeezed
- There are details in the treatment of both large and small q_T that can improve the situation *Collins et al, Phys.Rev. D (2016)*
- How is the matching in practice?

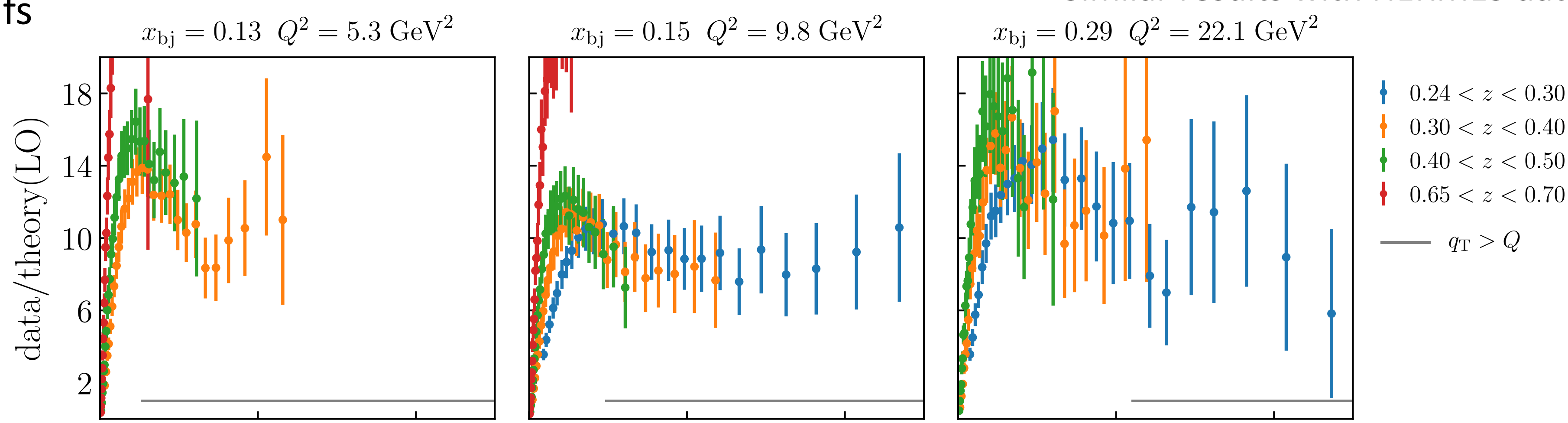
SIDIS: Leading Order

Hard part
at large q_T,
lowest order



DSS fragmentation funcs.
CJ pdfs

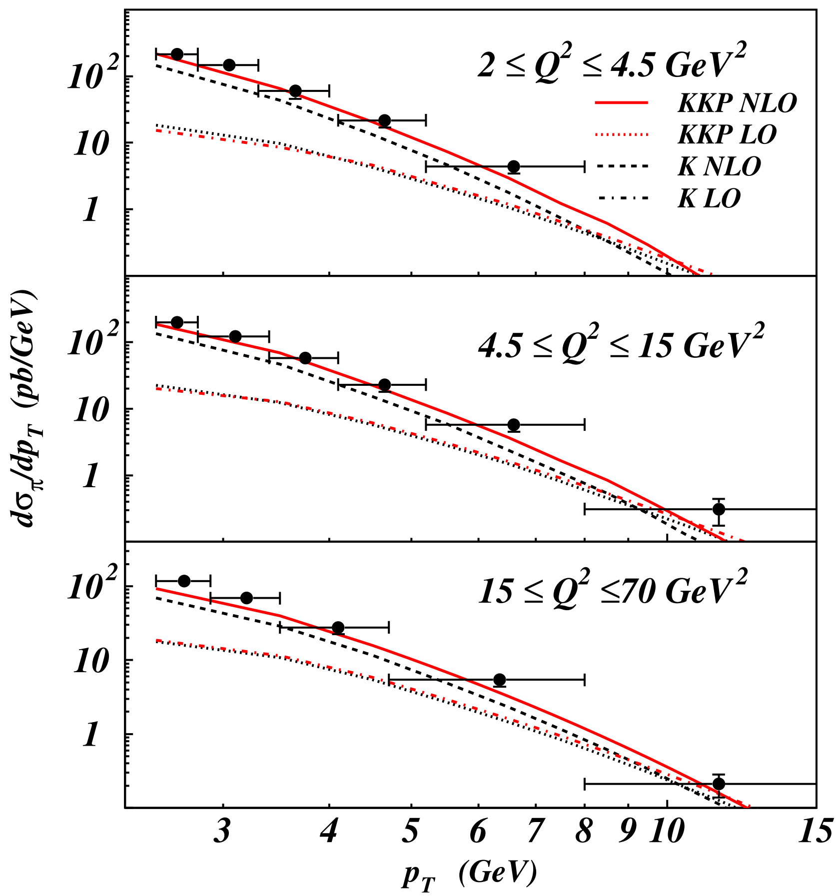
Data from COMPASS:
(Phys.Rev. D 97, 032006 (2018))
Similar results with HERMES data



J. O. Gonzalez-Hernandez, TR, N. Sato, and B. Wang Phys. Rev. D 98, 114005

SIDIS

- Large corrections from NLO



- Collinear functions tend to be constrained by higher energy / less detailed observables

- Refit collinear pdfs and ffs using large TM?

E. Berger et al, 1998 (For Drell-Yan)

Leading order

$$\frac{d\sigma}{dQ^2\, dx\, dz\, dq_T^2} \sim \int_{\xi_{\min}}^{1} d\xi\, f(\xi)\, d\left(\zeta = z\left(1 + \frac{x q_T^2}{(\xi - x)Q^2}\right)\right)$$

$$\xi_{\min} = x\left(1 + \frac{z q_T^2}{(1-z)Q^2}\right)$$

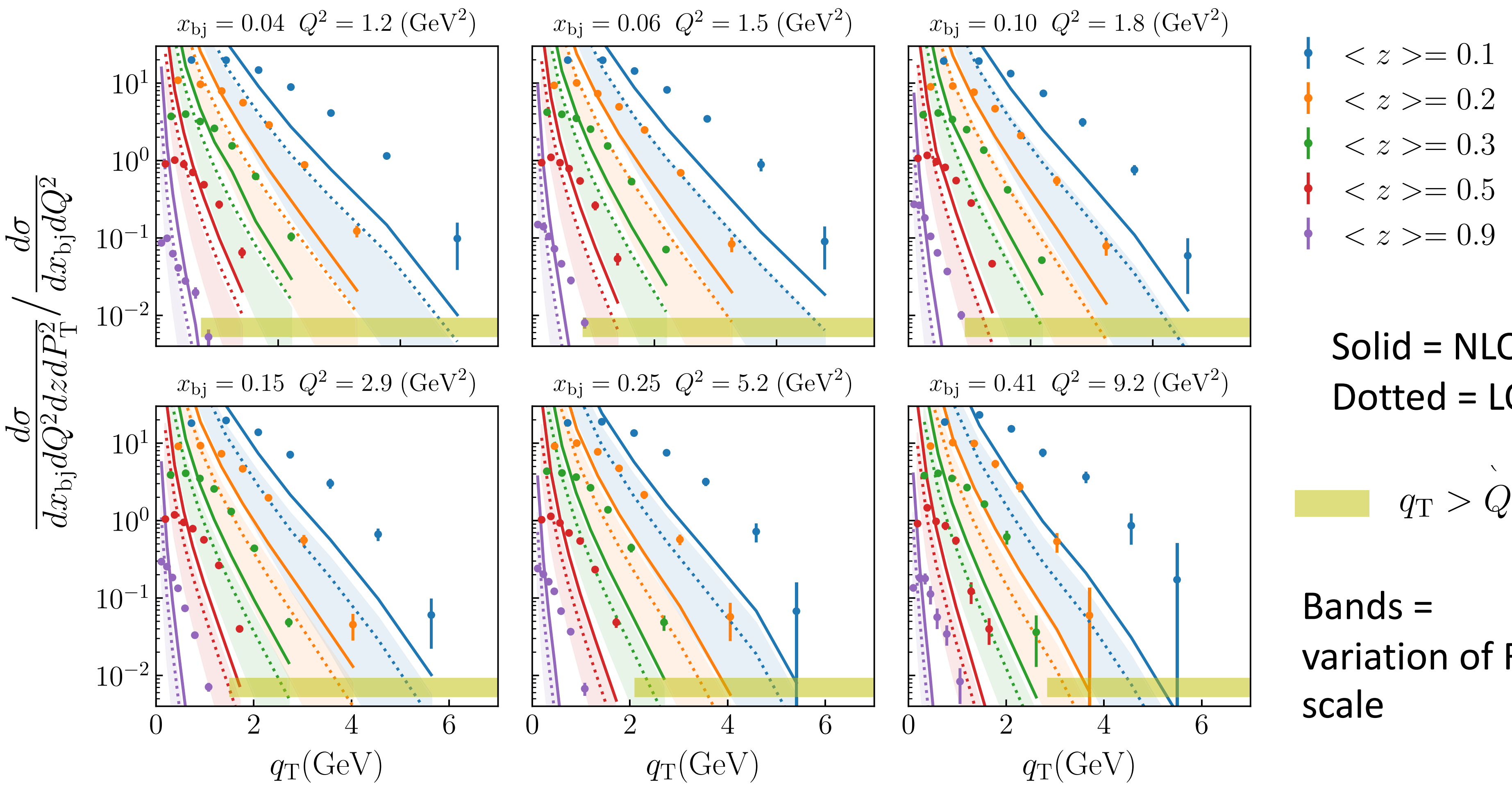
A. Daleo, D. de Florian, and R. Sassot, Phys. Rev. D 71, 034013 (2005)

SIDIS

- NLO study near the valence region.

B. Wang, J. O. Gonzalez-Hernandez, TR, N. Sato
Phys. Rev. D 99, 094029

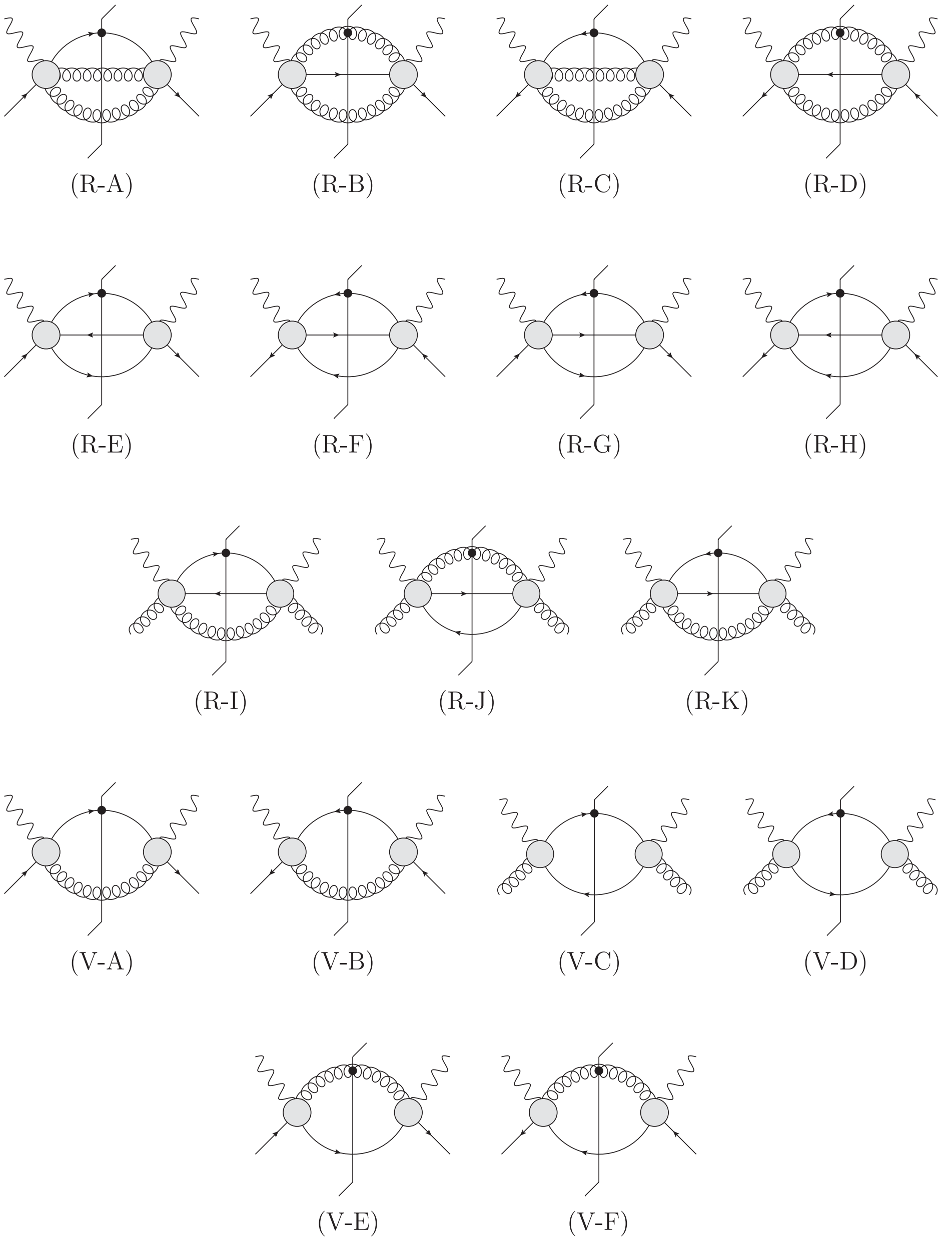
https://jeffersonlab.github.io/BigTMD/_build/html/index.html



Solid = NLO
Dotted = LO

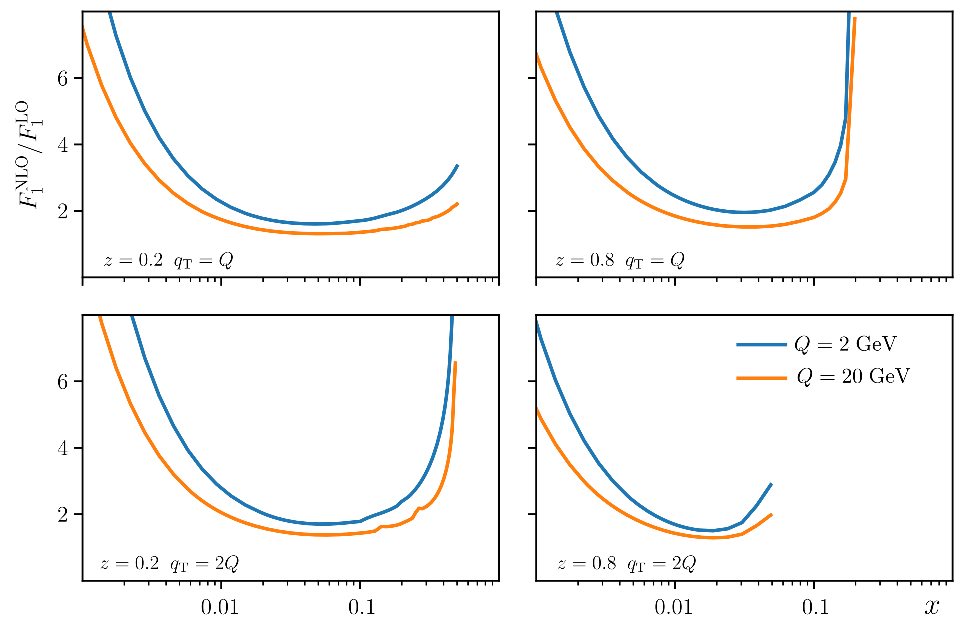
Bands = variation of RG scale

SIDIS

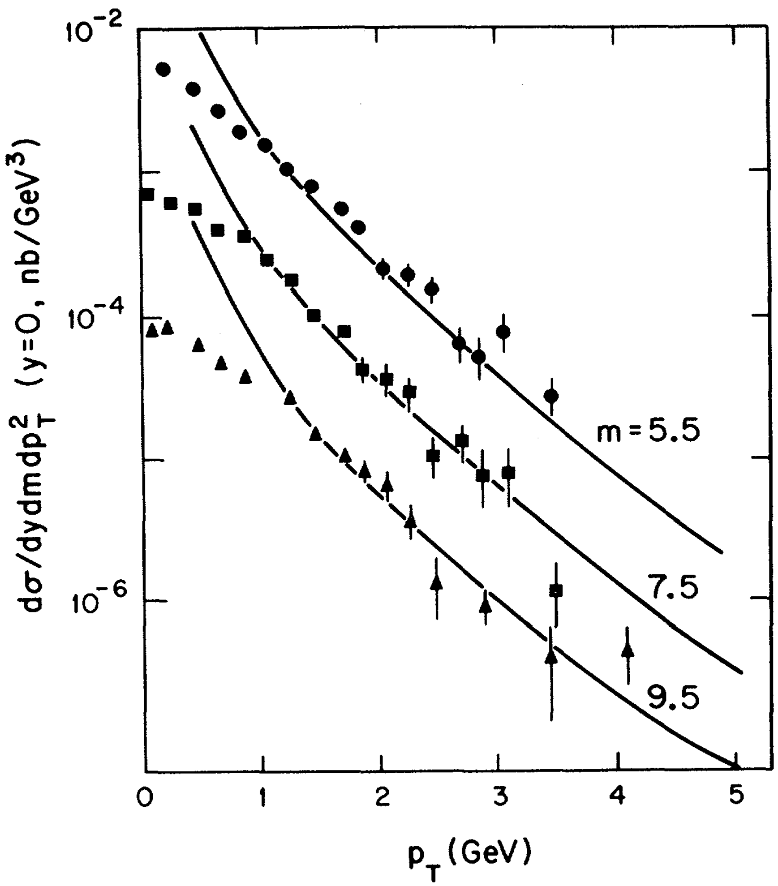


- NLO study of the valence region.

B. Wang, J. O. Gonzalez-Hernandez, TR, N. Sato
Phys. Rev. D 99, 094029

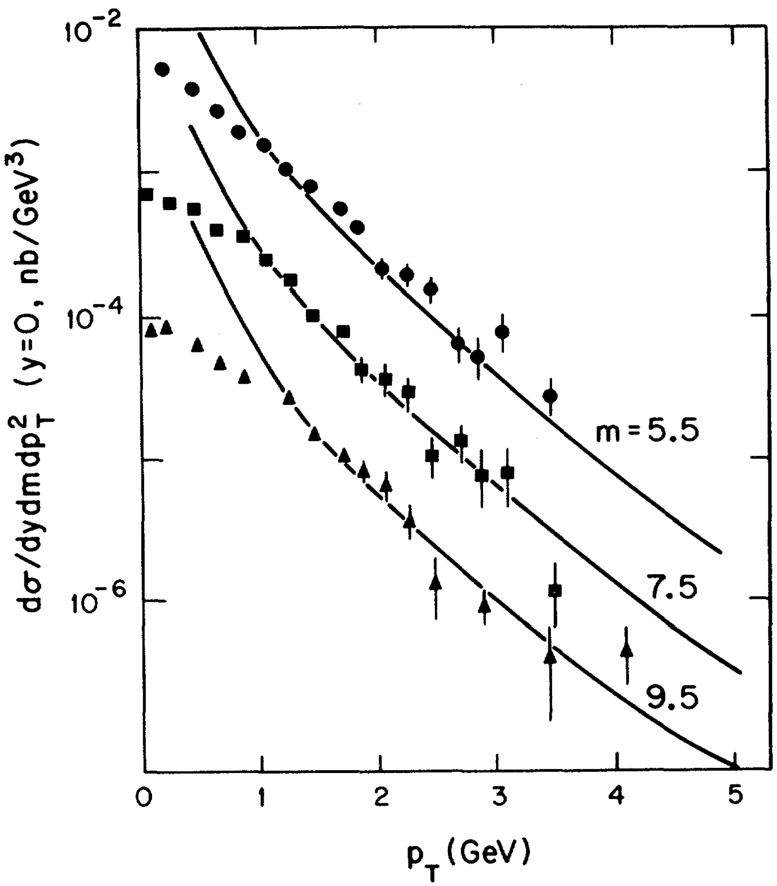


Drell-Yan: Old Results

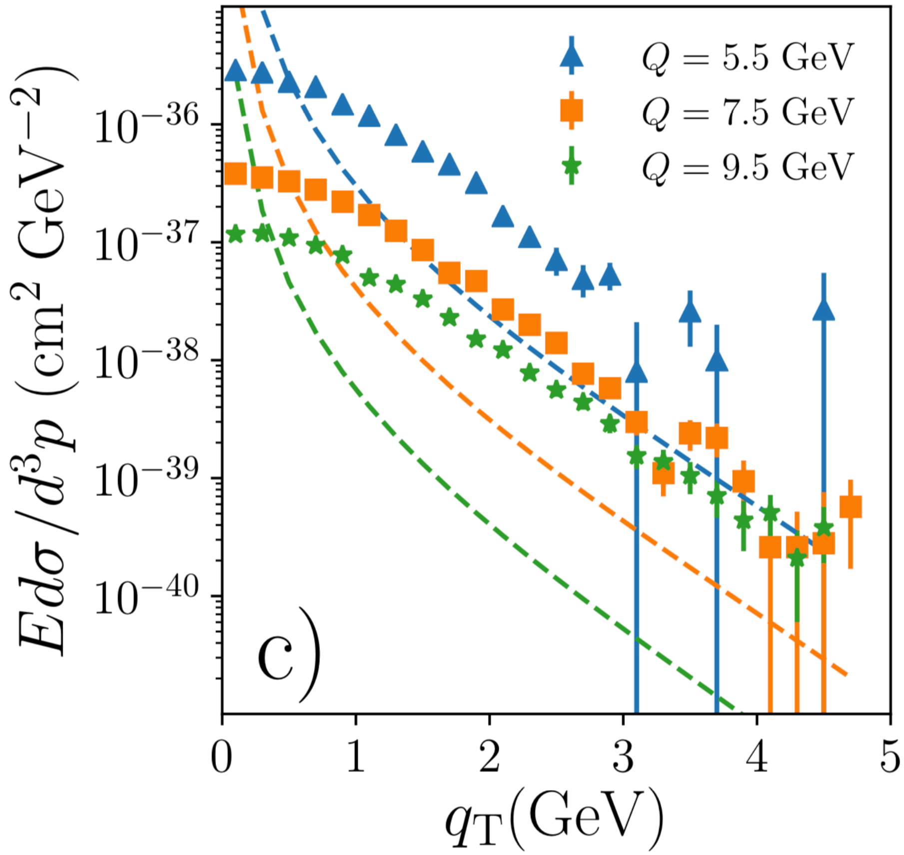


- Halzen & Scott, Phys. Rev. Lett. 40, 1117 (1978)

Drell-Yan: Old Results



- Halzen & Scott, Phys. Rev. Lett. 40, 1117 (1978)

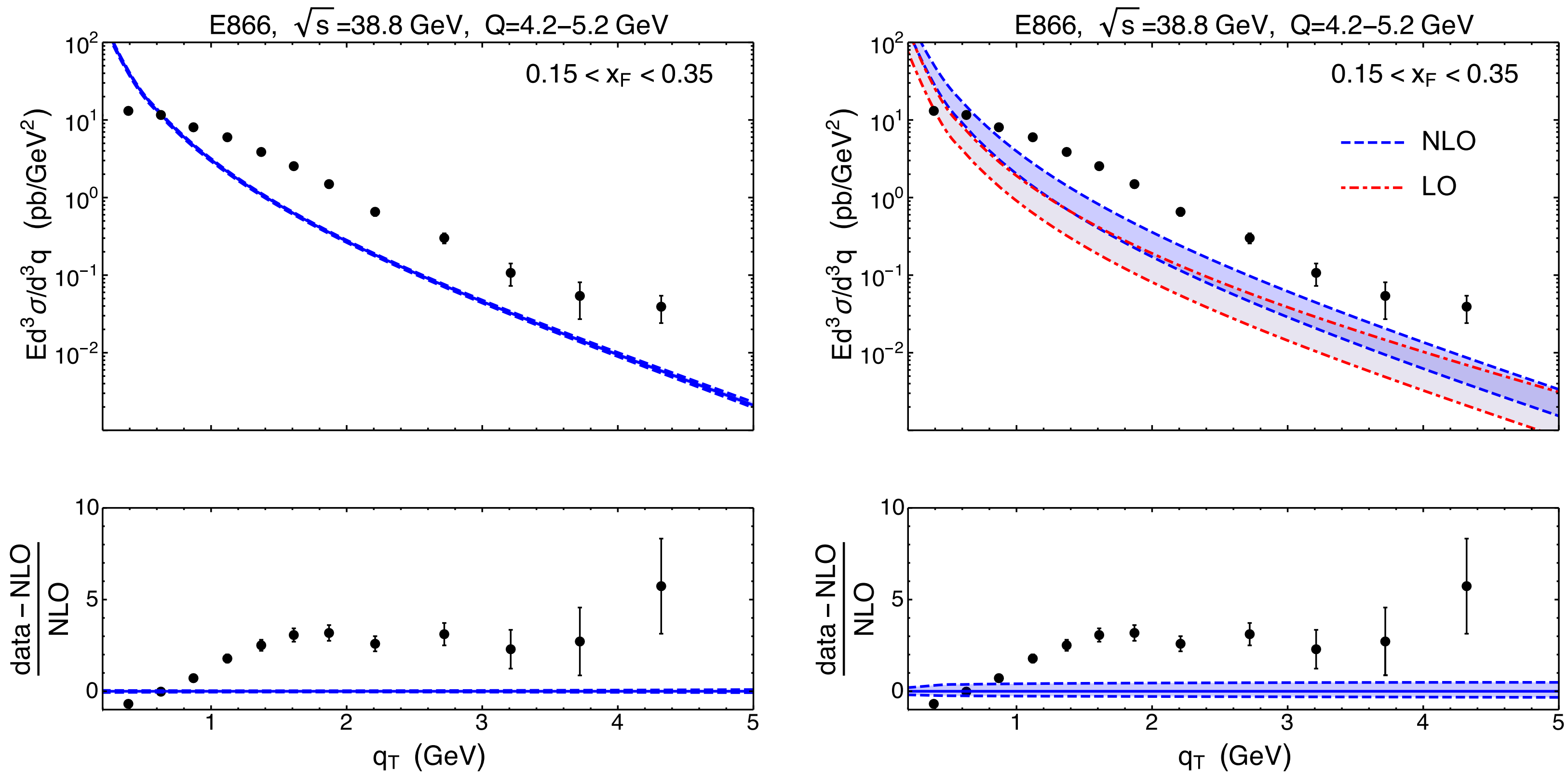


Corrected data + modern pdf set

Data from A. S. Ito et al., Phys. Rev. D23, 604 (1981)

Drell-Yan: New Results

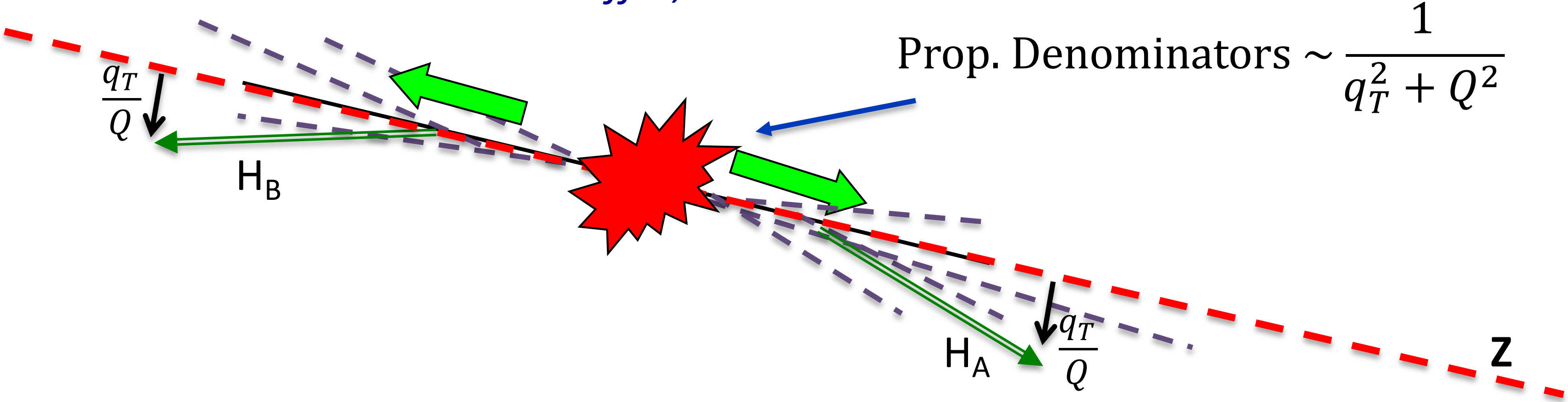
- Current status: Similar trend



Bacchetta et al., Phys.Rev. D100 (2019) no.1, 014018

Dihadron Production in e⁺e⁻ Annihilation

E. Moffat, et al: arXiv:1909.02951



- If q_T/Q is small, TMD fragmentation functions are needed.
- If q_T/Q is large, there is recoil against a high momentum parton and collinear factorization is needed:

$$z_A = \frac{p_A \cdot p_B}{q \cdot p_B}$$

$$\hat{z}_A = z_A/\zeta_A$$

$$z_B = \frac{p_A \cdot p_B}{q \cdot p_A}$$

$$\hat{z}_B = z_B/\zeta_B$$

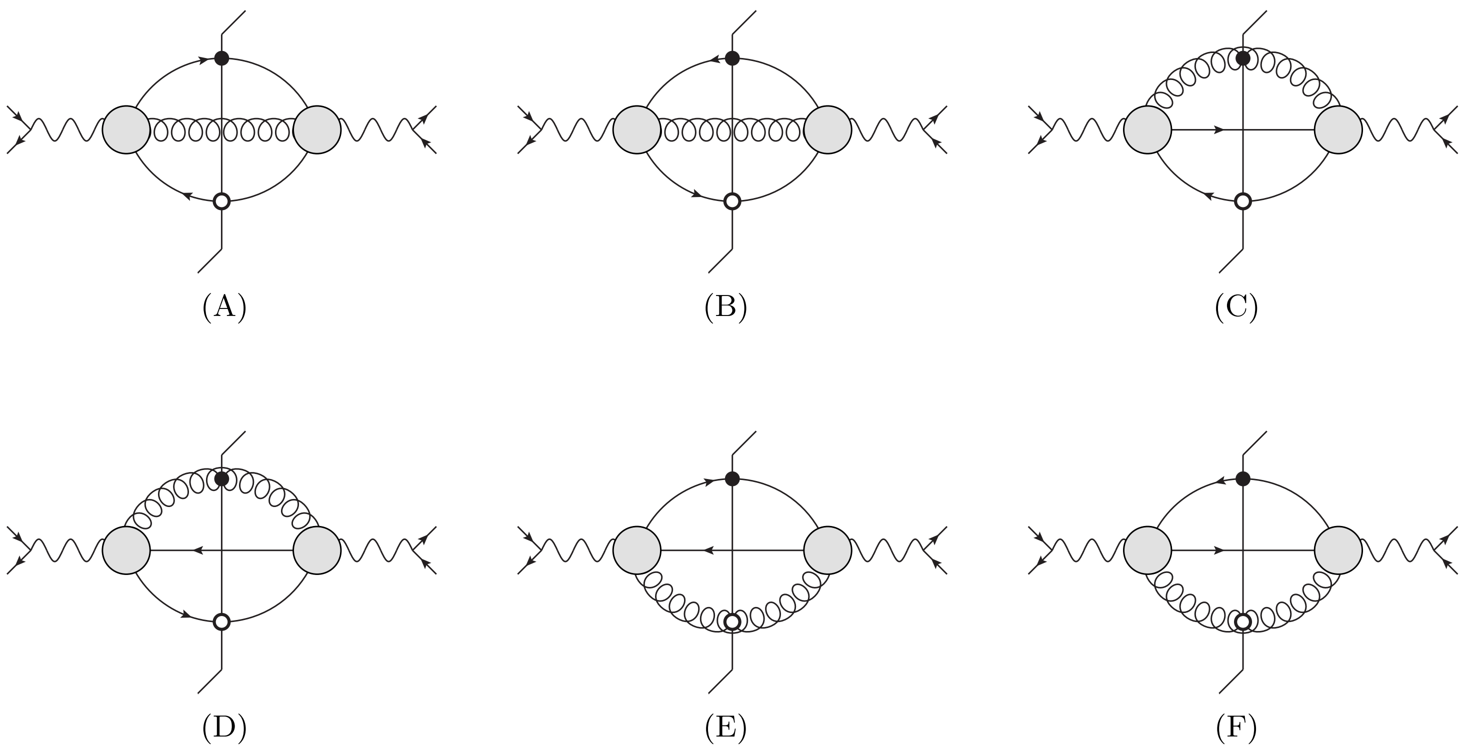
$$E_A E_B \frac{\mathrm{d}\sigma_{AB}}{\mathrm{d}^3\boldsymbol{p}_A \mathrm{d}^3\boldsymbol{p}_B} = \sum_{i,j} \int_{z_A}^{1} \mathrm{d}\zeta_A \int_{z_B}^{1} \mathrm{d}\zeta_B \left(E_A E_B \frac{\mathrm{d}\hat{\sigma}_{ij}(\hat{z}_A, \hat{z}_B)}{\mathrm{d}^3\boldsymbol{p}_A \mathrm{d}^3\boldsymbol{p}_B} \right) d_{H_A/i}(\zeta_A) d_{H_B/j}(\zeta_B)$$

Dihadron Production in e$^+$e$^-$ Annihilation

Channels

● = Hadron A

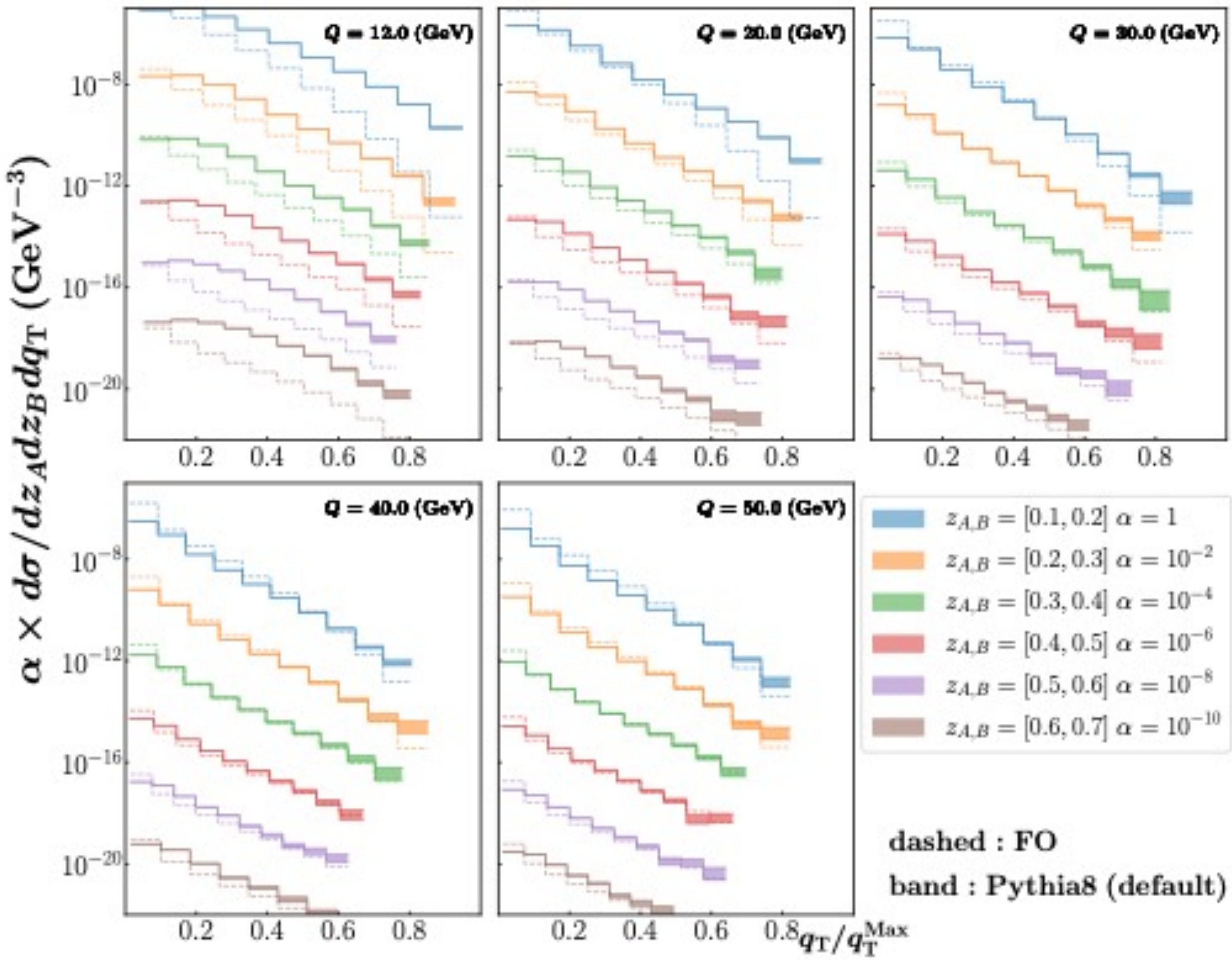
○ = Hadron B



$$\frac{\mathrm{d}\sigma_{AB}}{\mathrm{d}z_A \mathrm{d}z_B \mathrm{d}q_\mathrm{T}} = \sum_{i,j} \int_{z_A}^{1} \frac{\mathrm{d}\zeta_A}{\zeta_A} \int_{z_B}^{1} \frac{\mathrm{d}\zeta_B}{\zeta_B} \left(\frac{\mathrm{d}\hat{\sigma}_{ij}(\hat{z}_A, \hat{z}_B)}{\mathrm{d}\hat{z}_A \mathrm{d}\hat{z}_B \mathrm{d}q_\mathrm{T}} \right) d_{H_A/i}(\zeta_A) d_{H_B/j}(\zeta_B)$$

Dihadron Production in e^+e^- Annihilation

- Validate with event generator

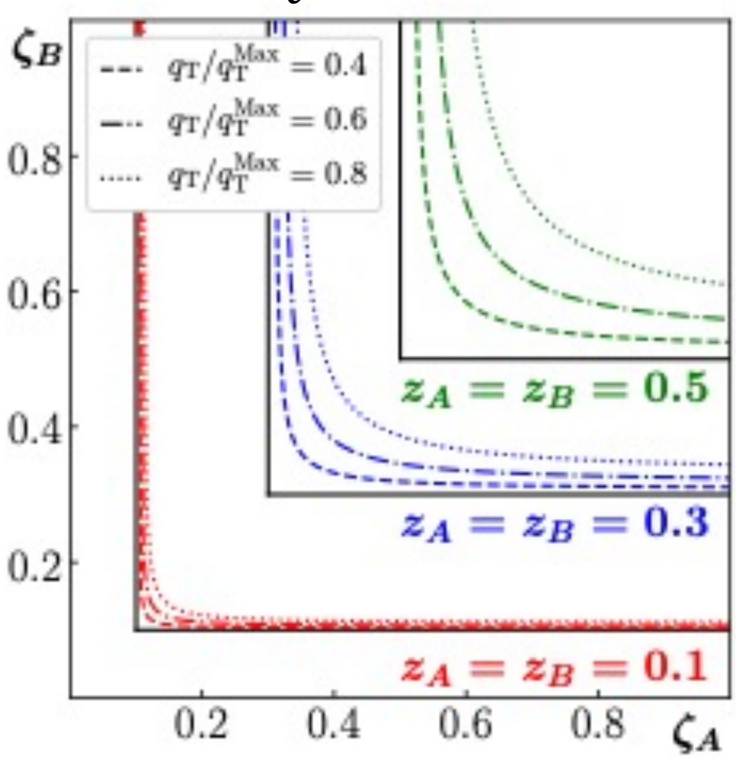


Small Transverse Momentum Approximations

- $q_T/Q \to 0$ asymptotic limit of large q_T computations

$$\frac{\mathrm{d}\sigma_{AB}^{ASY}}{\mathrm{d}z_A \mathrm{d}z_B \mathrm{d}q_\mathrm{T}} = \frac{4\alpha_{\mathrm{em}}^2 \alpha_s}{Q^2 q_\mathrm{T}} \sum_q e_q^2 \left\{ 2C_F \left[\ln\left(\frac{Q^2}{q_\mathrm{T}^2}\right) - \frac{3}{2} \right] \left(d_{H_A/q}(z_A) d_{H_B/\bar{q}}(z_B) + d_{H_A/\bar{q}}(z_A) d_{H_B/q}(z_B) \right) \right.$$
$$+ d_{H_A/q}(z_A) \left[(P_{\bar{q}\bar{q}} \otimes d_{H_B/\bar{q}})(z_B) + (P_{g\bar{q}} \otimes d_{H_B/g})(z_B) \right]$$
$$+ d_{H_A/\bar{q}}(z_A) \left[(P_{qq} \otimes d_{H_B/q})(z_B) + (P_{gq} \otimes d_{H_B/g})(z_B) \right]$$
$$+ d_{H_B/q}(z_B) \left[(P_{\bar{q}\bar{q}} \otimes d_{H_A/\bar{q}})(z_A) + (P_{g\bar{q}} \otimes d_{H_A/g})(z_A) \right]$$
$$\left. + d_{H_B/\bar{q}}(z_B) \left[(P_{qq} \otimes d_{H_A/q})(z_A) + (P_{gq} \otimes d_{H_A/g})(z_A) \right] \right\},$$

$Q = 12\ GeV$



$$q_\mathrm{T}^{\mathrm{Max}\,2} \leq \frac{Q^2(1-z_A)(1-z_B)}{1-(1-z_A)(1-z_B)}$$

$$(p_A + p_B)^2 = z_A z_B \left(Q^2 + q_\mathrm{T}^2\right)$$

$$\zeta_B = z_B \frac{(Q^2 + q_\mathrm{T}^2)(z_A - \zeta_A)}{q_\mathrm{T}^2 z_A + Q^2(z_A - \zeta_A)}$$

Transverse Momentum Dependent Fragmentation Functions

- Factorization for $q_T/Q \approx 0$:

$$d\sigma = H_{ij} \int d^2\boldsymbol{k}_{1T} d^2\boldsymbol{k}_{2T}\, D_{A/i}(z_A, z_A\boldsymbol{k}_{1T}) D_{B/j}(z_B, z_B\boldsymbol{k}_{2T})\ \delta^{(2)}(\boldsymbol{q}_T - \boldsymbol{k}_{1T} - \boldsymbol{k}_{2T})$$

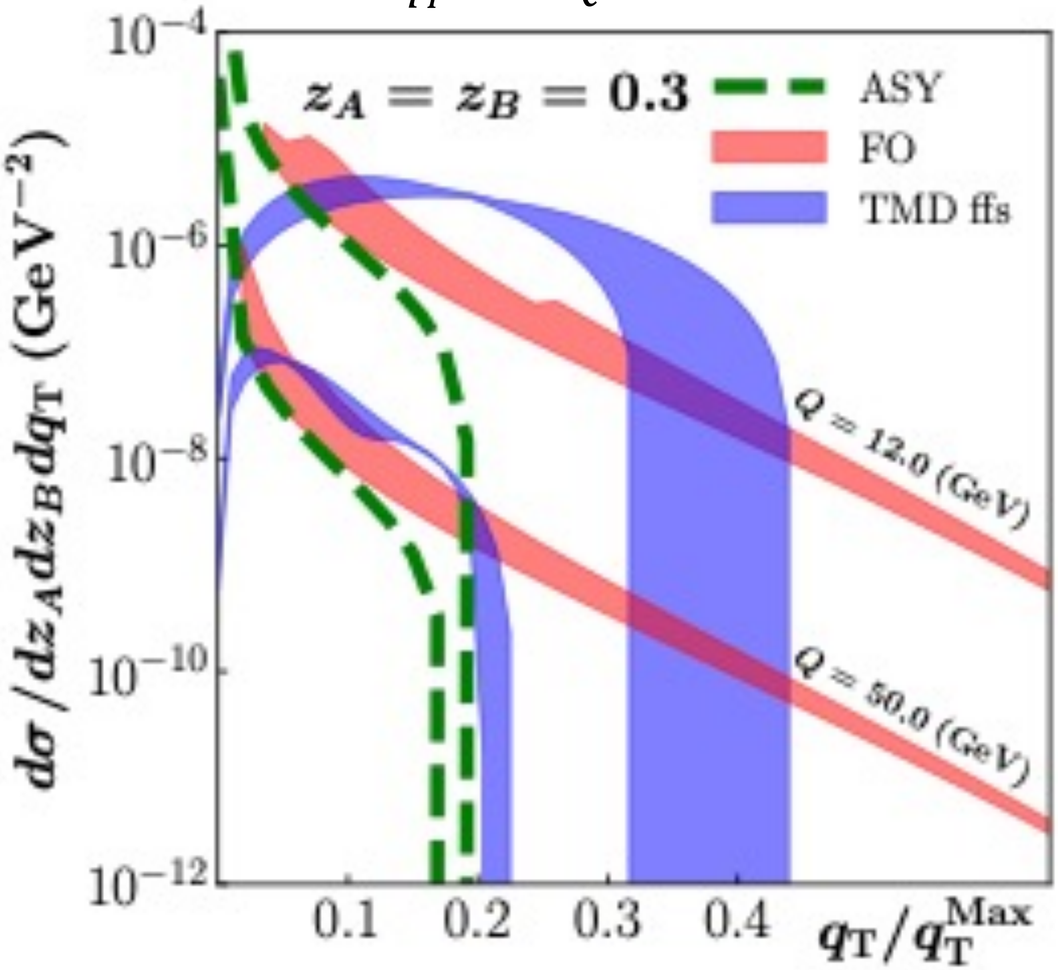
- TMD ffs and evolution

$$\tilde{D}_{H/q}(z, \boldsymbol{b}_T; \mu, \zeta_D) = \sum_j \int_z^1 \frac{d\hat{z}}{\hat{z}^3} \tilde{C}_{j/q}(z/\hat{z}, b_*; \zeta_D, \mu) d_{H/j}(\hat{z}, \mu_b)$$
$$\times \exp\left\{ \ln\frac{\sqrt{\zeta_D}}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu} \left[\gamma(\mu'; 1) - \ln\frac{\sqrt{\zeta_D}}{\mu'} \gamma_K(\mu') \right] + g_{H/j}(z, b_T) + \frac{1}{2} g_K(b_T) \ln\frac{\zeta_D}{\zeta_{D,0}} \right\}$$

Non-perturbative parts

Electron-Positron Annihilation

$q_T^{Max} \approx Q$



- Blue band:
  - from survey of non-perturbative fits
- Pink band:
  - Large TM calculation, width from varying RG scale
- Green:
  - Small $q_T/Q \to 0$ asymptote
- No overlap in the transition region for smaller Q

Summary

- Evidence for tension with collinear factorization in three processes

- A handle on the collinear factorization at large transverse momentum is needed for complete treatment of transverse momentum spectrum