Quasi-PDFs and Quasi-GPDs

(A. Metz, Temple University)

- Introduction: parton quasi-distributions
- Quasi-GPDs
  - Definition and motivation
  - Results in scalar diquark model
  - Power corrections
- Model-independent results
  - Moments of quasi-distributions and spin sum rule
  - Definition of quasi-distributions
  - ξ-symmetry of quasi-GPDs
- Summary

Based on: S. Bhattacharya, C. Cocuzza, A.M., arXiv:1808.01437, arXiv:1903.05721

(related talks by Stewart, Zhao, Xu, Liu, Zhu, Constantinou)

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Quasi-PDFs

- Light-cone unpolarized quark PDF (support: $-1 \leq x \leq 1$)

$$f_1(x) = \frac{1}{2} \int \frac{dz^-}{2\pi} \, e^{ik \cdot z} \, \langle P | \bar{\psi}(-\tfrac{z}{2}) \, \gamma^+ \, \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}) \, \psi(\tfrac{z}{2}) | P \rangle \Big|_{z^+=0, \vec{z}_\perp = \vec{0}_\perp}$$

  - correlator depends on time $t = z^0 = \frac{1}{\sqrt{2}} z^-$ $\rightarrow$ cannot be computed in LQCD

- Suggestion: consider quasi-PDF instead (Ji, 2013) (support: $-\infty < x < \infty$)

$$f_{1,\mathrm{Q}(3)}(x, P^3) = \frac{1}{2} \int \frac{dz^3}{2\pi} \, e^{ik \cdot z} \, \langle P | \bar{\psi}(-\tfrac{z}{2}) \, \gamma^3 \, \mathcal{W}_\mathrm{Q}(-\tfrac{z}{2}, \tfrac{z}{2}) \, \psi(\tfrac{z}{2}) | P \rangle \Big|_{z^0=0, \vec{z}_\perp = \vec{0}_\perp}$$

  - correlator depends on position z^3 $\rightarrow$ can be computed in LQCD
  - quasi-PDF depends on $x = k^3/P^3$, and on hadron momentum P^3
  - quasi-PDF and lc-PDF contain same non-pert. physics, but different UV physics
  - at large P^3, difference in UV behavior is dealt with via perturbative matching
    (e.g., Xiong, Ji, Zhang, Zhao, 2013 / Stewart, Zhao, 2017 / Izubuchi, Ji, Jin, Stewart, Zhao, 2018)

- Generic structure of matching formula (scale-dependence omitted)

$$f_{1,\mathrm{Q}(3)}(x, P^3) = \int_{-1}^{1} \frac{dy}{|y|}\, C\left(\frac{x}{y}\right) f_1(y) + \mathcal{O}\left(\frac{M^2}{(P^3)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(P^3)^2}\right)$$

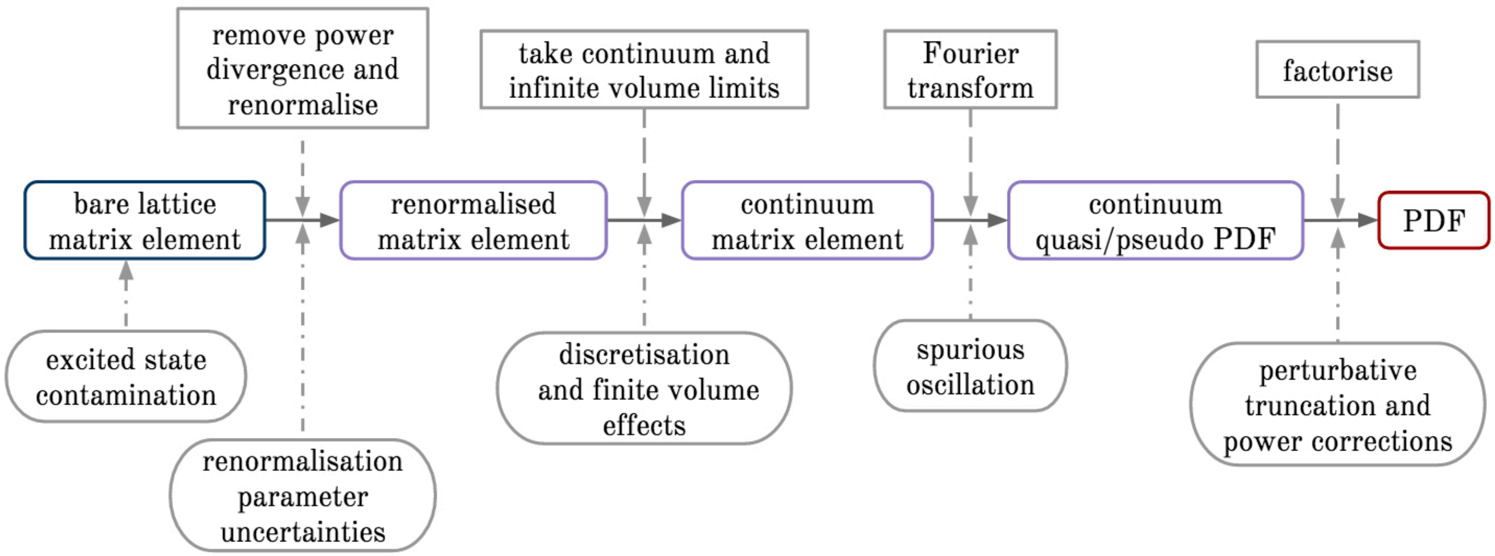
  - C is matching coefficient (presently known to one-loop order)
  - several works on power corrections available
  - quasi-PDFs can be considered as "good lattice cross section" (Ma, Qiu, 2014)

- Choosing γ^0 (instead of γ^3) for unpolarized quasi-PDF (Radyushkin, 2016)

$$f_{1,\mathrm{Q}(0)}(x, P^3) = \frac{1}{2} \int \frac{dz^3}{2\pi}\, e^{ik \cdot z} \langle P|\bar{\psi}(-\tfrac{z}{2})\, \gamma^0\, \mathcal{W}_{\mathrm{Q}}(-\tfrac{z}{2}, \tfrac{z}{2})\, \psi(\tfrac{z}{2})|P\rangle\Big|_{z^0=0, \vec{z}_\perp = \vec{0}_\perp}$$

  - in principle, any linear combination of γ^3 and γ^0 would work (except γ^-)
  - $f_{1,\mathrm{Q}(0)}$ better behaved w.r.t. renormalization (Constantinou, Panagopoulos, 2017)

- Steps needed to obtain x-dependent PDFs from lattice QCD

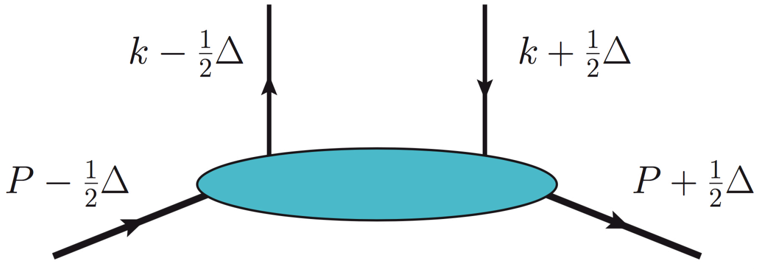


(figure from Monahan, 2018)

- Several other suggestions for computing PDFs and related quantities (Braun, Müller, 2008 / Ma, Qiu, 2014 / Radyushkin, 2017 / ...)
  - some of them were proposed before quasi-PDFs and/or are related to quasi-PDFs
  - presently unclear if certain method(s) will “win”

Definition of (Quasi-) GPDs

- GPD correlator: graphical representation



$$P = \frac{p + p'}{2} \qquad \Delta = p' - p$$

- Correlator for light-cone GPDs of quarks

$$F^{[\Gamma]}(x, \Delta) = \frac{1}{2} \int \frac{dz^-}{2\pi} \, e^{ik \cdot z} \, \langle p' | \bar{\psi}(-\tfrac{z}{2}) \, \Gamma \, \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}) \, \psi(\tfrac{z}{2}) | p \rangle \Big|_{z^+ = 0, \vec{z}_\perp = \vec{0}_\perp}$$

correlator parameterized through GPDs $X(x, \xi, t)$

$$x = \frac{k^+}{P^+} \qquad \xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+} \qquad t = \Delta^2$$

- Kinematic relation

$$t = -\frac{1}{1 - \xi^2} \left(4\xi^2 M^2 + \vec{\Delta}_\perp^2 \right)$$

- (Spatial) correlator for quasi-GPDs of quarks (Ji, 2013)

$$F_{\mathrm{Q}}^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi}\, e^{ik\cdot z}\, \langle p'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}_{\mathrm{Q}}(-\tfrac{z}{2},\tfrac{z}{2})\,\psi(\tfrac{z}{2})|p\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

- Definition of twist-2 vector quasi-GPDs H_{Q} and E_{Q}

$$F_{\mathrm{Q}}^{[\gamma^0]}(x,\Delta;P^3) = \frac{1}{2P^0}\,\bar{u}(p')\Big[\gamma^0\, H_{\mathrm{Q}(0)} + \frac{i\sigma^{0\mu}\Delta_\mu}{2M}\, E_{\mathrm{Q}(0)}\Big]u(p)$$

$$F_{\mathrm{Q}}^{[\gamma^3]}(x,\Delta;P^3) = \frac{1}{2P^3}\,\bar{u}(p')\Big[\gamma^3\, H_{\mathrm{Q}(3)} + \frac{i\sigma^{3\mu}\Delta_\mu}{2M}\, E_{\mathrm{Q}(3)}\Big]u(p)$$

  - we have explored both definitions of quasi-GPDs
  - in forward limit, definitions of quasi-GPDs reduce to most frequently used definitions of quasi-PDFs
  - quasi-GPDs depend on

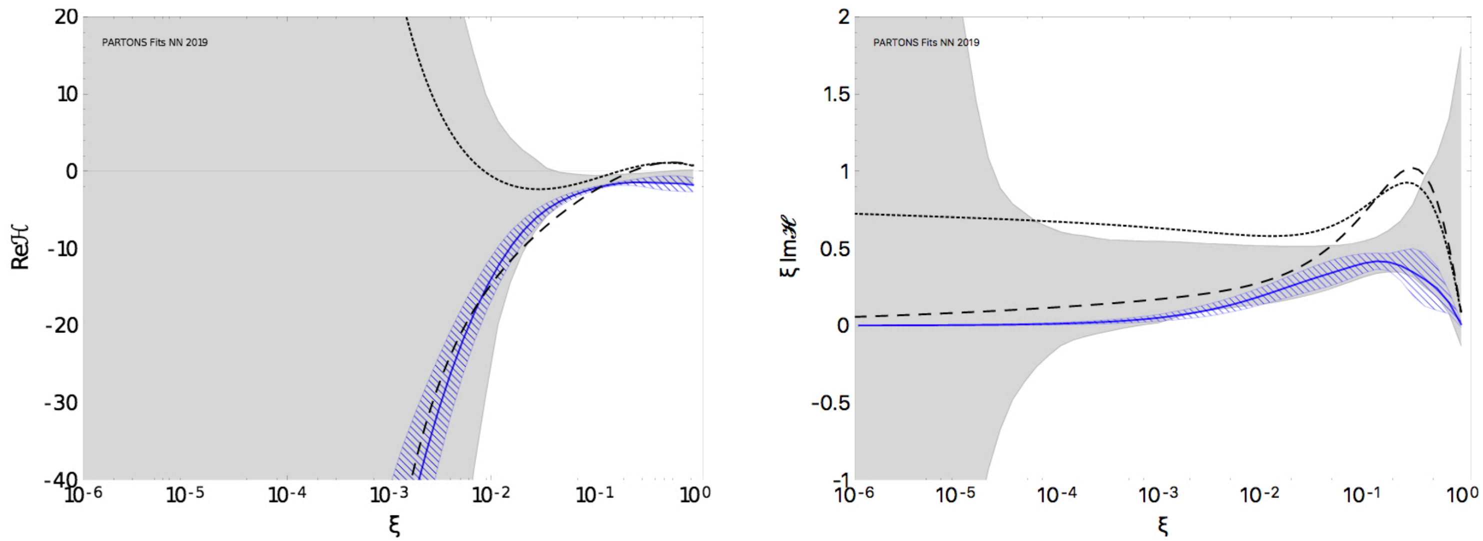
$$x = \frac{k^3}{P^3} \neq \frac{k^+}{P^+} \qquad \xi \qquad t = \Delta^2 \qquad P^3$$

Why Studying Quasi-GPDs ?

- Non-trivial behavior of quasi-GPDs at $x = \pm\,\xi$?

- Extraction of GPDs from experimental data is difficult
  - very recent example

(Moutarde, Sznajder, Wagner, 2019)



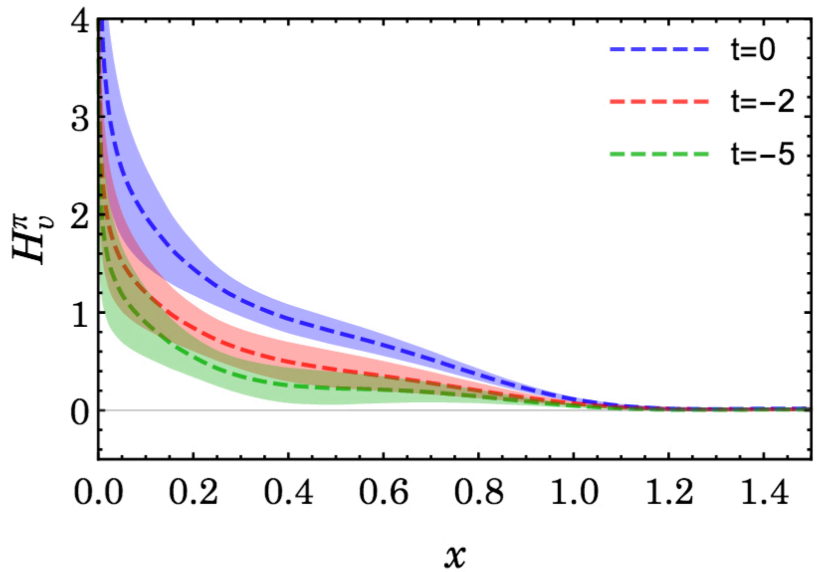
  - real and imaginary part of Compton form factor $\mathcal{H}$ using neural network approach
  - at present, errors are still (very) large

- In the future, combination of experimental data (also from EIC) and input from LQCD may be used to pin down GPDs

Available Studies on Quasi-GPDs

- Matching calculations for quasi-GPDs (→ see also talk by Liu)
  (Ji, Schäfer, Xiong, Zhang, 2015 / Xiong, Zhang, 2015 /
  Liu, Wang, Xu, Zhang, Zhang, Zhao, Zhao, 2019)
- Model calculations, etc
  (Bhattacharya, Cocuzza, AM, 2018, 2019)
- Exploratory LQCD calculation for pion

(Chen, Lin, Zhang, 2019)



- calculation of H for π^+ for $u_{\mathrm{val}} - d_{\mathrm{val}}$
- calculation for $\xi = 0$ and $m_\pi = 310\,\mathrm{MeV}$

Diquark Spectator Model

- Idea: describe spectator partons as diquarks (of spin-0 or spin-1)
  (e.g., Jakob, Mulders, Rodrigues, 1997)
- Graphical representation of two-quark correlator

- Often phenomenological nucleon-quark-diquark vertices with form factors used
- Previous studies of quasi-PDFs in diquark spectator model
  (Gamberg, Kang, Vitev, Xing, 2014 / Bacchetta, Radici, Pasquini, Xiong, 2016)
- We (mostly) use scalar diquark model (SDM)

$$\begin{aligned}\mathcal{L}_{\rm SDM} \quad &= \quad \bar{\Psi}(i\,\not{\partial} - M)\Psi + \bar{\psi}(i\,\not{\partial} - m_q)\psi + \frac{1}{2}(\partial_\mu\varphi\,\partial^\mu\varphi - m_s^2\,\varphi^2) \\ &\quad + g(\bar{\Psi}\,\psi\,\varphi + \bar{\psi}\,\Psi\,\varphi)\end{aligned}$$

- Cut-graph (diquark on-shell) can be used to compute PDFs, but care has to be taken for quasi-PDFs (Bhattacharya, Cocuzza, AM, 2018)

Analytical Results in Scalar Diquark Model

- Considered all eight leading-twist quark GPDs

- For light-cone GPDs, agreement with results extracted from calculation of GTMDs
  (Meißner, AM, Schlegel, 2009)

- Correlator for quasi-GPDs

$$F_{\mathrm{Q}}^{[\Gamma]}(x,\Delta;P^3) = \frac{i\,g^2}{2(2\pi)^4} \int dk^0\, d^2\vec{k}_\perp \, \frac{\bar{u}(p')\left(\not{k} + \frac{\not{\Delta}}{2} + m_q\right)\Gamma\left(\not{k} - \frac{\not{\Delta}}{2} + m_q\right)u(p)}{D_{\mathrm{GPD}}}$$

$$D_{\mathrm{GPD}} = \left[\left(k+\frac{\Delta}{2}\right)^2 - m_q^2 + i\varepsilon\right]\left[\left(k-\frac{\Delta}{2}\right)^2 - m_q^2 + i\varepsilon\right]\left[(P-k)^2 - m_s^2 + i\varepsilon\right]$$

- Quasi-GPDs are continuous at $x = \pm\,\xi$ (even beyond leading twist); situation differs from (higher-twist) GPDs
  (Aslan, Burkardt, Lorcé, AM, Pasquini, 2018 / Aslan, Burkardt, 2018)

- For $P^3 \to \infty$, all quasi-GPDs reduce to corresponding light-cone GPDs

Numerical Results in Scalar Diquark Model

- Parameter choice
  - coupling (exact value of g irrelevant for our purpose)

$$g = 1$$

  - masses must satisfy $M < m_s + m_q$; we mostly use

$$m_s = 0.7\,\mathrm{GeV} \qquad m_q = 0.35\,\mathrm{GeV}$$

    values similar to previous work (Gamberg, Kang, Vitev, Xing, 2014)
    "optimal choice" for minimizing difference btw quasi and light-cone distributions

  - momentum transfer

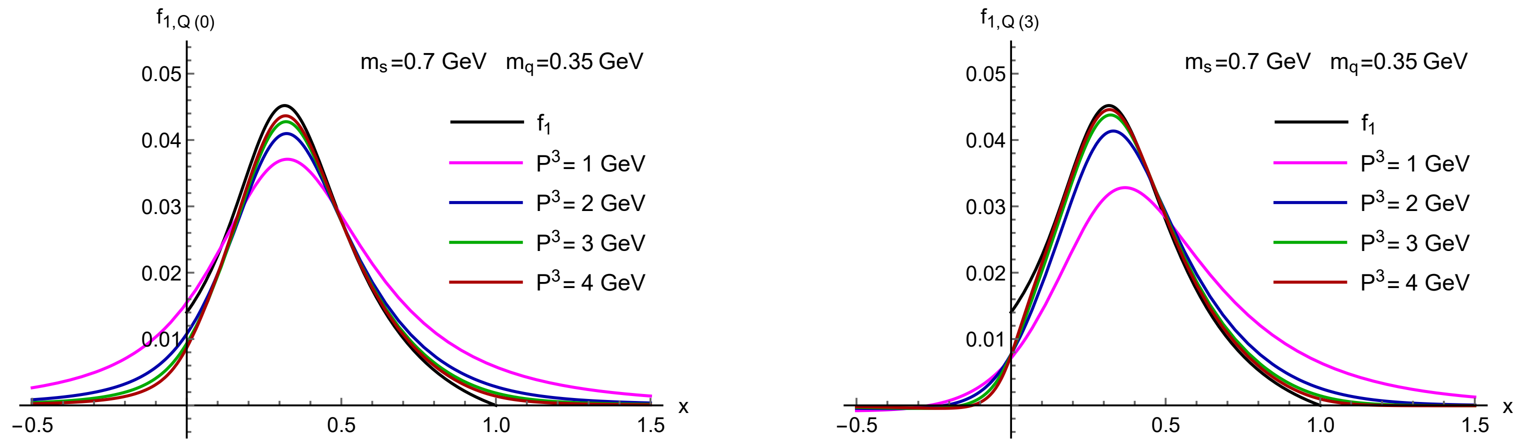
$$|\vec{\Delta}_\perp| = 0$$

  - cutoff for $k_\perp$ integration

$$\Lambda = 1\,\mathrm{GeV}$$

  - variations of $|\vec{\Delta}_\perp|$ and Λ do not affect general results
  - using form factor (rather than $k_\perp$ cutoff) does not affect general results

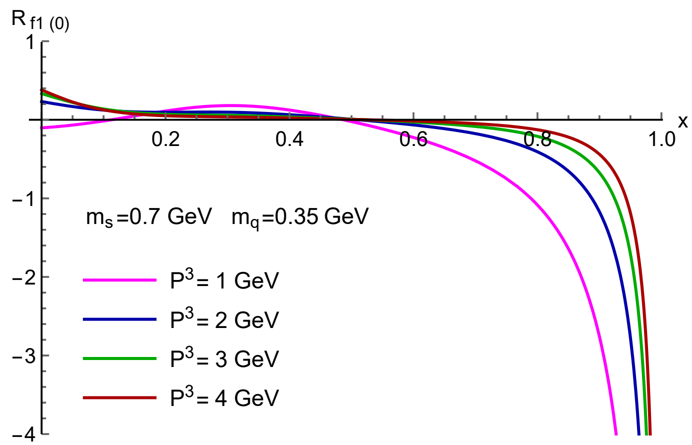
- Quasi-PDFs

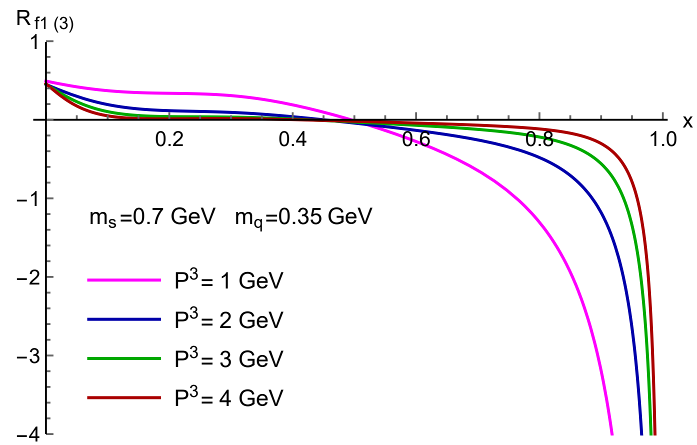


  - for larger P^3 ($\gtrsim 2\,\mathrm{GeV}$), quasi-PDFs are close to f_1 in wide x range
  - for larger P^3, not much difference btw $f_{1,\mathrm{Q}(0)}$ and $f_{1,\mathrm{Q}(3)}$;
    this is general feature for all cases
  - considerable discrepancies btw quasi-PDFs and f_1 at large x
    (compare also, Gamberg, Kang, Vitev, Xing, 2014)
  - considerable discrepancies btw quasi-PDFs and f_1 at small x
    f_1 is discontinuous at $x = 0$ $(f_1(x < 0) = 0)$
    quasi-PDFs are continuous at $x = 0$ and must change rapidly around $x = 0$
    discontinuity is probably not a model artifact $(f_1^q(x < 0) = -f_1^{\bar{q}}(x > 0))$

- Relative difference for quasi-PDFs

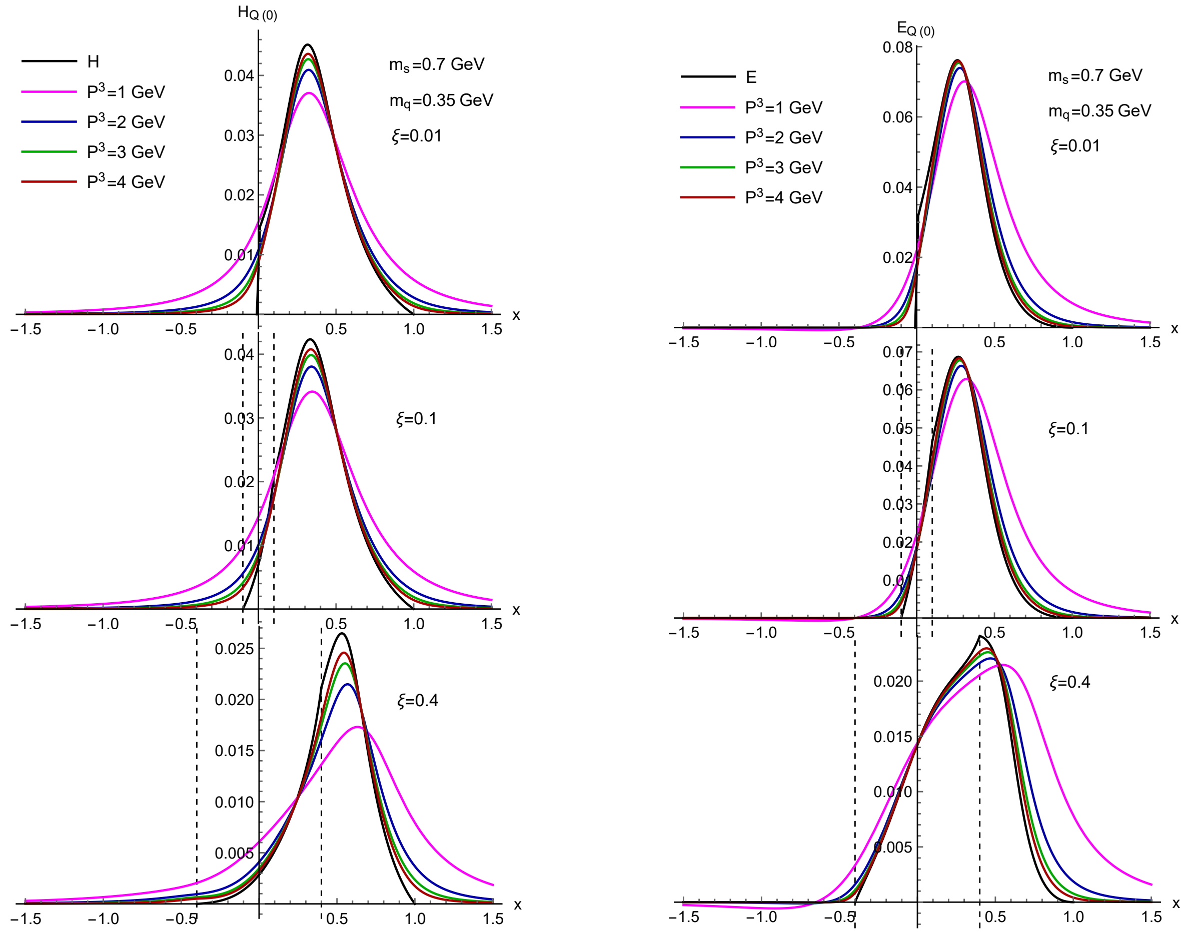
$$R_{f1(0/3)}(x;P^3) = \frac{f_1(x) - f_{1,\mathrm{Q}(0/3)}(x;P^3)}{f_1(x)}$$





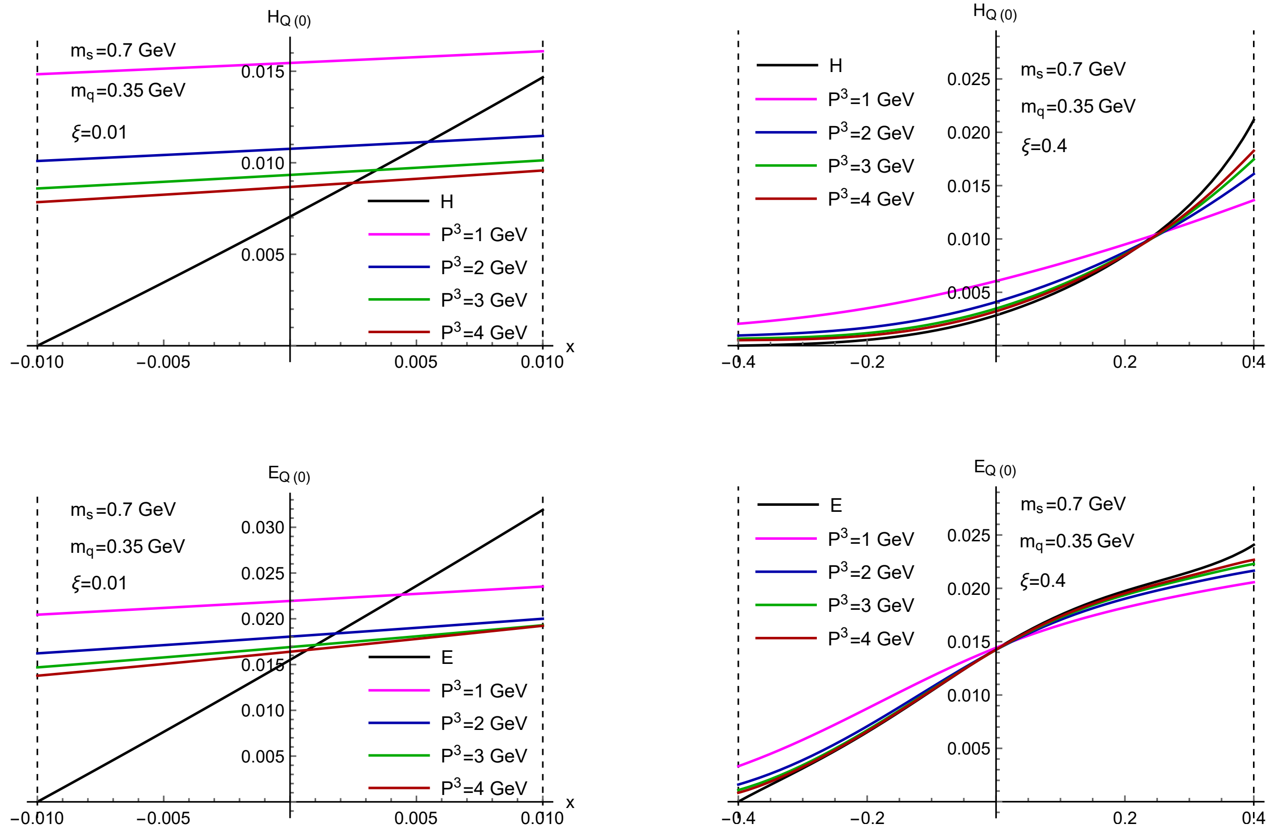
  - relative difference makes discrepancies very explicit (especially at large x)
  - for $P^3 \gtrsim 2\,\mathrm{GeV}$, good results for $0.1 < x < 0.8$
  - at large x, problem partly due to mismatch btw k^+/P^+ and k^3/P^3 for finite P^3
  - calculations of quasi-PDFs in LQCD also lead to discrepancies (at large x)

- Quasi-GPDs: sample plots



- for larger P^3 ($\gtrsim 2\,\mathrm{GeV}$), quasi-GPDs are close to light-cone GPDs in wide x range; agreement can depend on ξ
- considerable discrepancies btw quasi and light-cone GPDs for large x; issue tends to become more severe as ξ increases
- same qualitative results for all leading-twist GPDs

- Quasi-GPDs in ERBL region



  - light-cone (twist-2) GPDs are continuous in entire x range (unlike PDFs)
  - for small ξ, large discrepancies btw quasi and light-cone GPDs in ERBL region (compare region around $x = 0$ for PDFs)
  - for large ξ, good agreement btw quasi and light-cone GPDs in large part of ERBL region $\rightarrow$ potentially nice opportunity for LQCD calculations

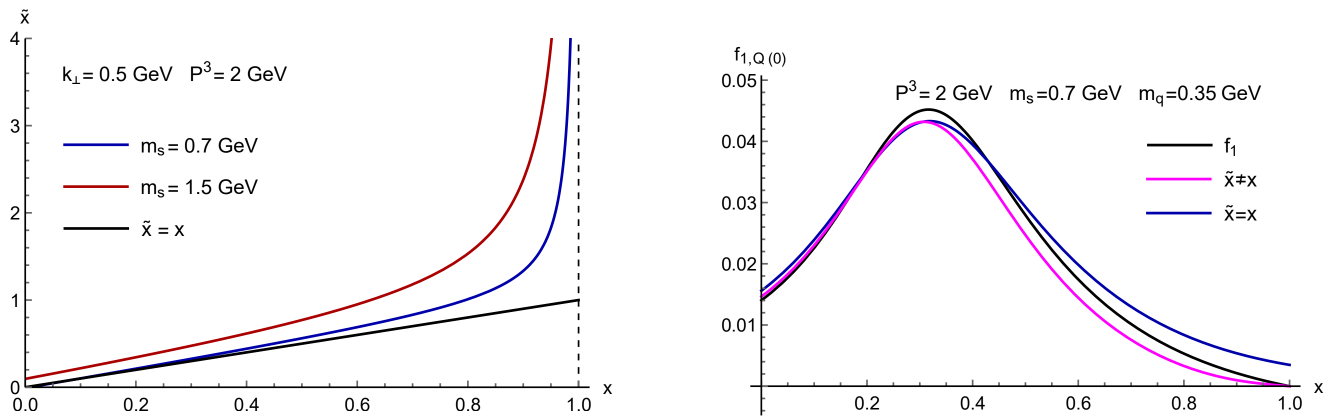
Parton Momenta and Power Corrections

- Recall: parton momentum fractions of standard and quasi PDFs are different; no model-independent relation btw momentum fractions

- Relation in SDM in cut-graph approximation (see also, Gamberg, Kang, Vitev, Xing, 2014)

$$\tilde{x} = \frac{k^3}{P^3} = x + \frac{1}{4(P^3)^2}\left(\frac{\vec{k}_\perp^{\,2} + m_s^2}{1-x} - (1-x)M^2\right) + \mathcal{O}\left(\frac{1}{(P^3)^4}\right)$$

  - difference btw $\tilde{x}$ and x is power correction, but $\tilde{x} - x \to \infty$ as $x \to 1$
  - some numerics



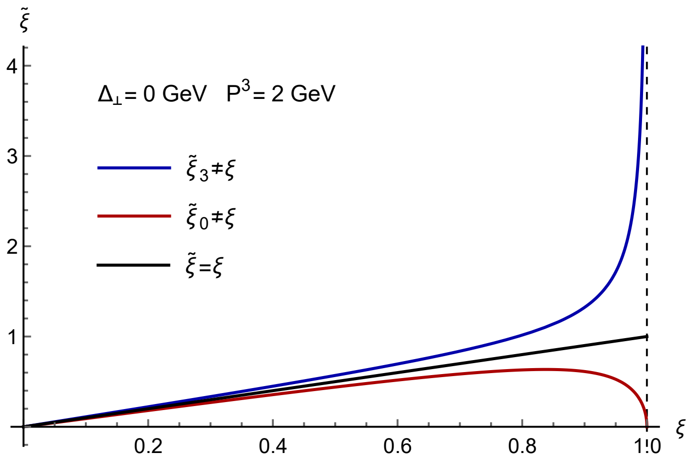
  - improved results at (very) large x if one distinguishes btw $\tilde{x}$ and x
  - in renormalon approach one also finds corrections $\sim 1/\left[(P^3)^2(1-x)\right]$ (Braun, Vladimirov, Zhang, 2018)

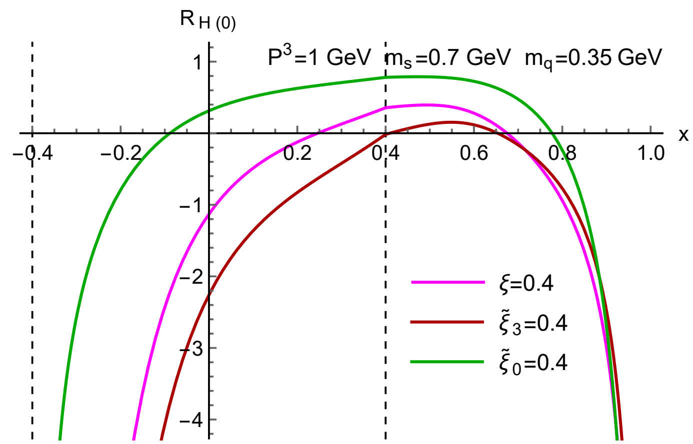
Skewness and Power Corrections

- Quasi-GPDs can be computed using standard skewness ξ
- Other definitions for skewness could be used. Examples:

$$\tilde{\xi}_3 = -\frac{\Delta^3}{2P^3} = \delta\, \xi \,, \qquad \tilde{\xi}_0 = -\frac{\Delta^0}{2P^0} = \frac{\xi}{\delta} \,, \qquad \text{with } \delta = \frac{P^0}{P^3} = 1 + \mathcal{O}\left(\frac{1}{(P^3)^2}\right)$$

  - δ describes power correction, but diverges as $\xi \to 1$
  - some numerics





  - different skewness variables can lead to considerable differences
  - which skewness variable works best depends on x and GPD

Moments of Quasi-Distributions and Spin Sum Rule

- Lowest moment

$$\int_{-1}^{1} dx\, H^q(x,\xi,t) = F_1^q(t)$$
$$= \int_{-\infty}^{\infty} dx\, \frac{1}{\delta}\, H^q_{\mathrm{Q}(0)}(x,\xi,t;P^3) = \int_{-\infty}^{\infty} dx\, H^q_{\mathrm{Q}(3)}(x,\xi,t;P^3)$$

$$\int_{-1}^{1} dx\, E^q(x,\xi,t) = F_2^q(t)$$
$$= \int_{-\infty}^{\infty} dx\, \frac{1}{\delta}\, E^q_{\mathrm{Q}(0)}(x,\xi,t;P^3) = \int_{-\infty}^{\infty} dx\, E^q_{\mathrm{Q}(3)}(x,\xi,t;P^3)$$

  - corresponding relations for other GPDs (and PDFs)
  - moments do not depend on P^3 (for f_1 see also, Radyushkin, 2018)

- Second moment of quasi-GPDs and Ji's spin sum rule

$$\int_{-\infty}^{\infty} dx\, x\, \frac{1}{\delta} \left(H^q_{\mathrm{Q}(0)} + E^q_{\mathrm{Q}(0)}\right) \;=\; \frac{1}{2}(\delta^2+1)\big(A^q(t)+B^q(t)\big) + \frac{1}{2}(\delta^2-1)D^q(t)$$

$$\int_{-\infty}^{\infty} dx\, x \left(H^q_{\mathrm{Q}(3)} + E^q_{\mathrm{Q}(3)}\right) \;=\; A^q(t)+B^q(t)$$

  - A, B, D are form factors of energy momentum tensor
  - $A^q(0)+B^q(0)=J^q$

- Second moment of quasi-PDFs

$$\int_{-\infty}^{\infty} dx\, x\, \frac{1}{\delta_0}\, f^q_{1,\mathrm{Q}(0)} \;=\; A^q(0) = \int_{-1}^{1} dx\, x\, f^q_1(x) \qquad\qquad (\delta_0 = \delta|_{t=0})$$

$$\int_{-\infty}^{\infty} dx\, x\, f^q_{1,\mathrm{Q}(3)} \;=\; A^q(0) - (\delta_0^2-1)\bar{C}^q(0)$$

- In general, for moments P^3 dependence either absent or calculable; moment relations may help to study systematics of LQCD calculations

Definition of Quasi-Distributions and Symmetry in ξ

- Moment analysis and definition of quasi-distributions
  - analysis suggests preferred definition of quasi-PDFs and quasi-GPDs

$$\tilde{f}_{1,\mathrm{Q}(0)} \equiv \frac{1}{\delta_0} f_{1,\mathrm{Q}(0)} \qquad \tilde{g}_{1,\mathrm{Q}(3)} \equiv \frac{1}{\delta_0} g_{1,\mathrm{Q}(3)} \qquad \tilde{h}_{1,\mathrm{Q}(0)} \equiv \frac{1}{\delta_0} h_{1,\mathrm{Q}(0)}$$

  - so far, most of the literature used $f_{1,\mathrm{Q}(0)}$, $g_{1,\mathrm{Q}(3)}$, $h_{1,\mathrm{Q}(0)}$
  - strictly speaking, both definitions suitable since difference is power-suppressed
  - for instance: $\delta_0(P^3 = 1\,\mathrm{GeV}) = 1.37$

- Symmetry of quasi-GPDs under $\xi \to -\xi$
  - behavior of light-cone GPDs (based on hermiticity and time-reversal)

$$X(x,-\xi,t) \;=\; +\,X(x,\xi,t) \;\; \text{for all quark GPDs } X \text{ but } \widetilde{E}_T$$
$$\widetilde{E}_T(x,-\xi,t) \;=\; -\,\widetilde{E}_T(x,\xi,t)$$

  - corresponding quasi-GPDs have the exact same behavior
  - ξ-symmetry may be exploited in LQCD calculations

Summary

- Partonic quasi-distributions have attracted considerable interest;
  first encouraging LQCD results exist

- Quasi-GPDs in scalar diquark model
  - for $P^3 \to \infty$, all quasi-GPDs agree with respective light-cone GPDs
  - for $P^3 \gtrsim 2\,\mathrm{GeV}$, quasi-GPDs are close to light-cone GPDs in wide x range
  - large discrepancies btw quasi and light-cone GPDs at large x,
    issue tends to become more severe as ξ increases
  - for large ξ, good agreement btw quasi and light-cone GPDs in ERBL region
  - obtained same qualitative results for vector diquark for some PDFs/GPDs

- Model-independent results
  - quasi-GPDs can be computed for standard skewness ξ, but other variables possible
  - (lowest) moments of quasi-distributions have no or calculable P^3 dependence
  - moment analysis suggests preferred definition for several quasi-distributions
  - quasi-GPDs and light-cone GPDs have same behavior under $\xi \to -\xi$

- Overall qualitative outcome

Computing GPDs through quasi-distributions in lattice QCD may be promising

$\longrightarrow$ see talk by Martha Constantinou for first LQCD results for nucleon