Optimizing coupled magnetodynamic FEM models of HTS tapes for magnet applications

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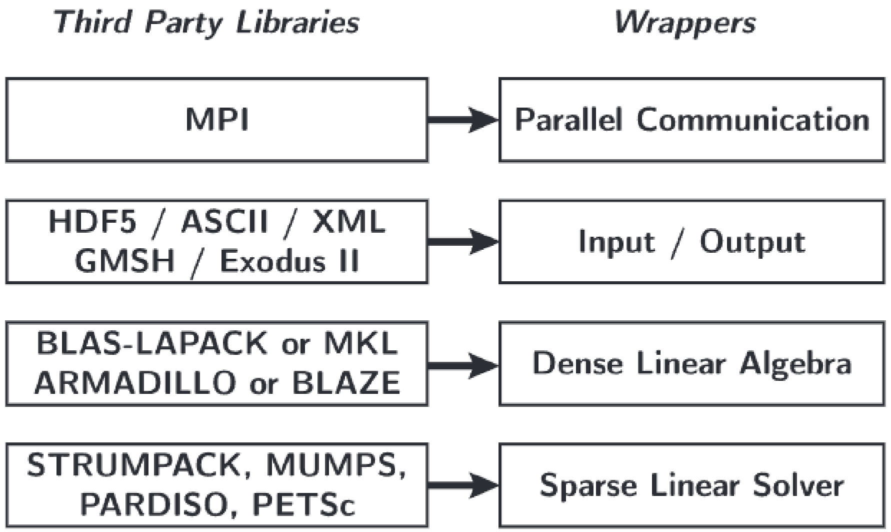
INTRODUCTION

BELFEM: BErkeley Lab Finite EleMent framework

- Integrated platform to study magnetization effects and quench in HTS tapes, cables and magnet

Includes:

- EM formulations, thin-shell model
- Lumped-mass thermal model
- Circuit solver
- High-performance linear algebra solvers
- Parallel computing (OpenMP, MPI)



Recently went through code refactoring to make easier implementation of new physics

OUTLINE

- Introduction
- **Magnetodynamic FEM simulations**
  - **The $H - \phi$ formulation**
  - **Automatic cohomology cuts**
  - **Thin-shell simplifications**
- Performance comparisons
- Benchmarks
  - The undulator benchmark (2-D)
  - Towards 3-D benchmarks
- Conclusion

THE $H-\phi$ FORMULATION

H-conform vs. B-conform formulations

Governing equations are domain specific

- Need for interface conditions

Challenges:

- User-friendliness
- Mesh generation
- Advantages of each formulation not always easy to assess

	Air / Vacuum	Conductor	Ferromagnetic Alloy
Governing Equation	$\nabla \times h = 0$ Ampére-Maxwell	$\nabla \times h = j$ Ampére-Maxwell	$\nabla \times e = \dot{b}$ Faraday's Law
Degree of Freedom	$h = -\nabla \phi$ Magnetic Scalar Potential	h Magnetic Field	$b = \nabla \times a$ Magnetic Vector Potential
Transport Law	none	$e = \rho \cdot j$ Ohm's Law	$h = \nu \cdot b$ Magnetic Law
Comment	minimal number of dofs	need edge elements	simple material law implementation

Mixed $h-\phi$ is usually the most efficient formulation to model HTS !

THE $H - \phi$ FORMULATION

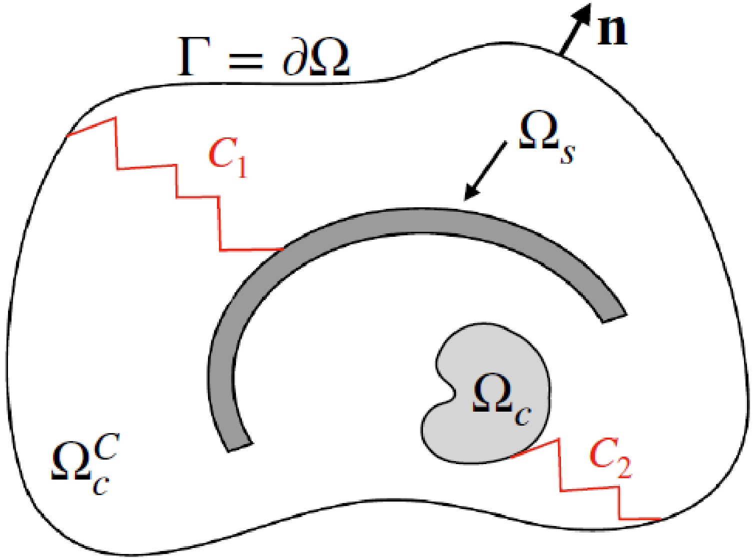
Problem: Ampere's law cannot be respected in a non-conducting domain if ϕ is continuous

$$\oint_L (-\nabla\phi) \cdot d\boldsymbol{l} = 0$$

... but: the integral solution should be I if L surrounds a conductor!

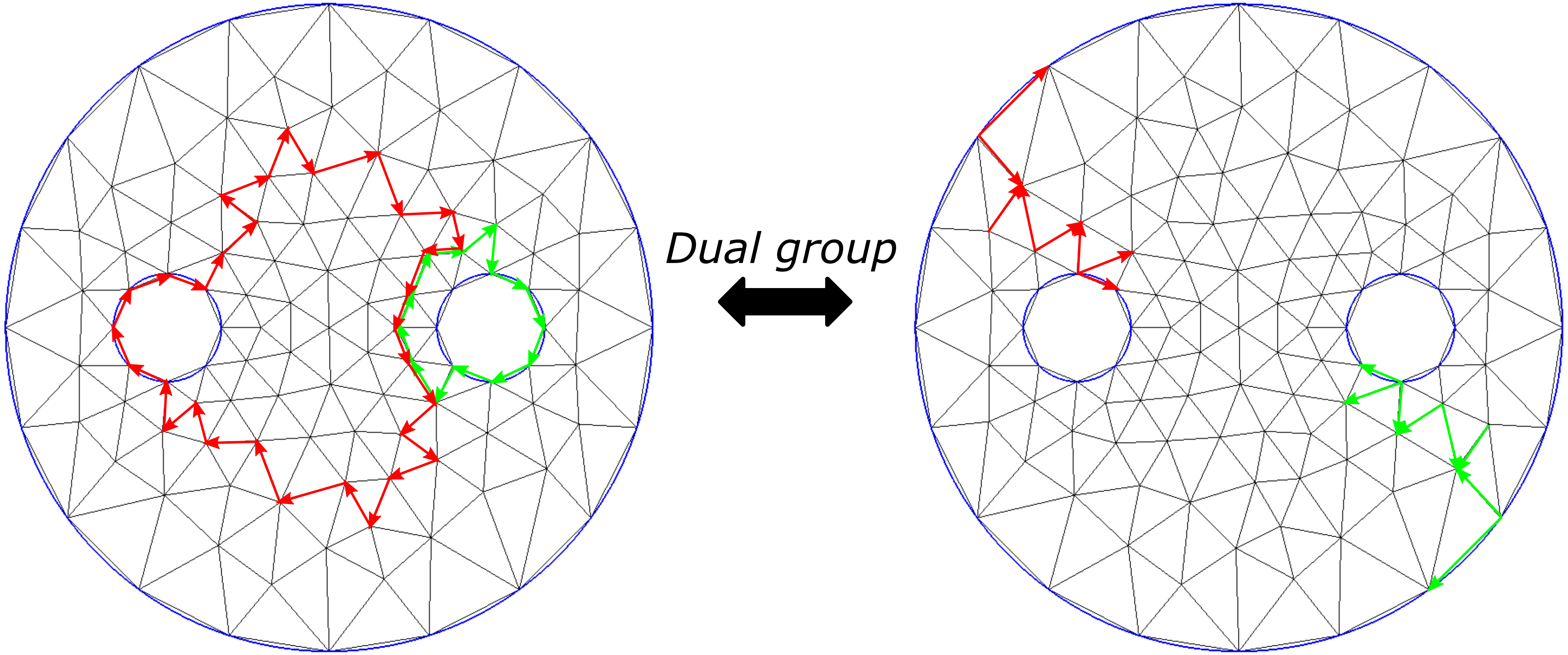
Solutions:

1. Make the domain simply connected (thin cuts)
2. Use cohomology generators (thick cuts)



THE $H - \phi$ FORMULATION

Consider this simple problem in 2-D: two conductors with a current $\pm I$. We want every integral over a loop in the air domain to be 0, **except** if it circles a conductor



1-homology

1-cohomology

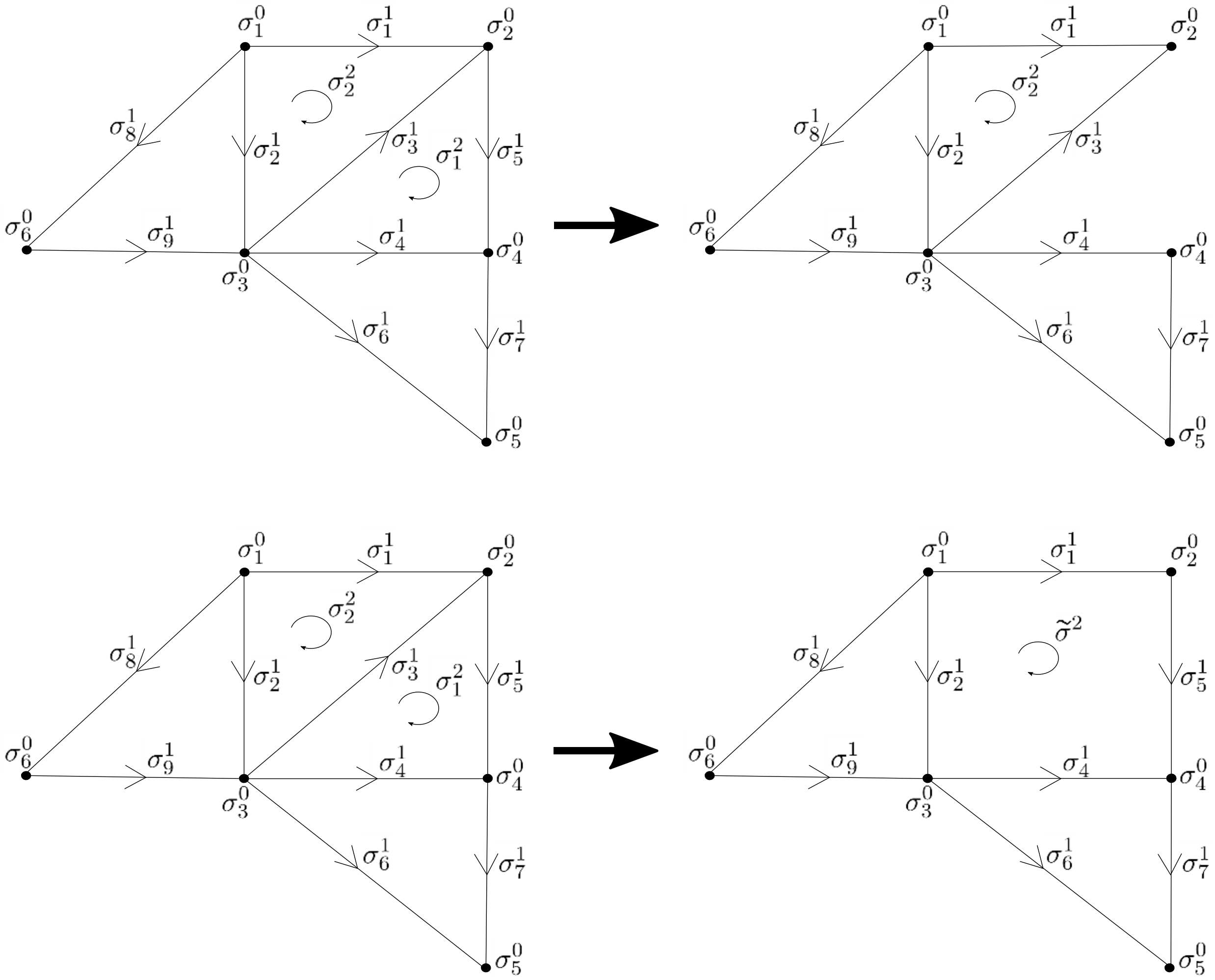
<u>General idea:</u>
add a discontinuity along the cohomology cuts in the FEM discretization to impose the currents

AUTOMATIC COHOMOLOGY CUTS

Computing these cohomology cuts automatically can be a challenging task

The Smith normal form (specific SVD decomposition of a matrix) is the standard method used, but it is very expensive numerically (unpractical for >1000 elements).

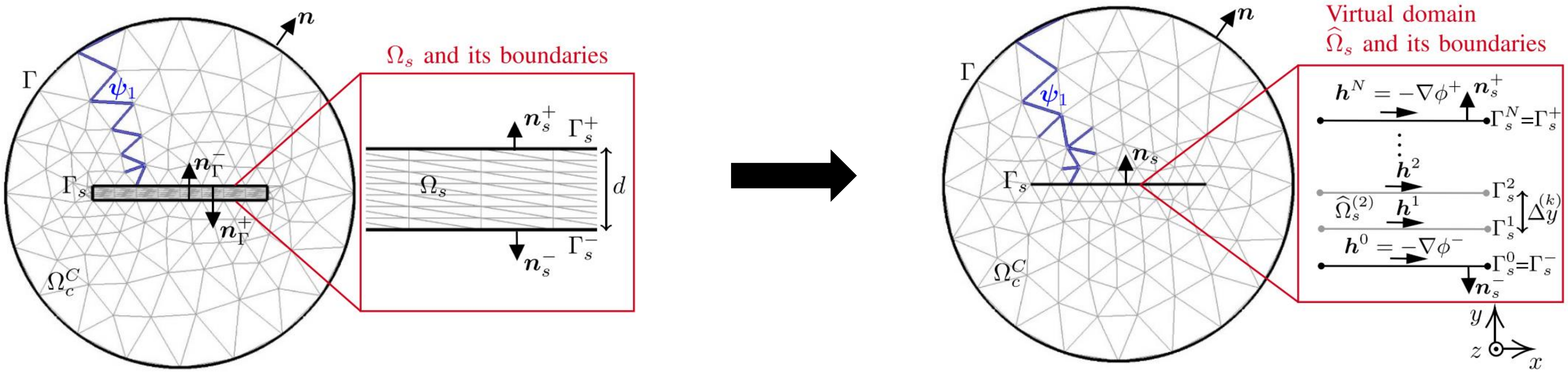
Fortunately, there exists some homology-preserving operations on a FEM mesh to significantly reduce the size of the matrices



POLYTECHNIQUE MONTRÉAL

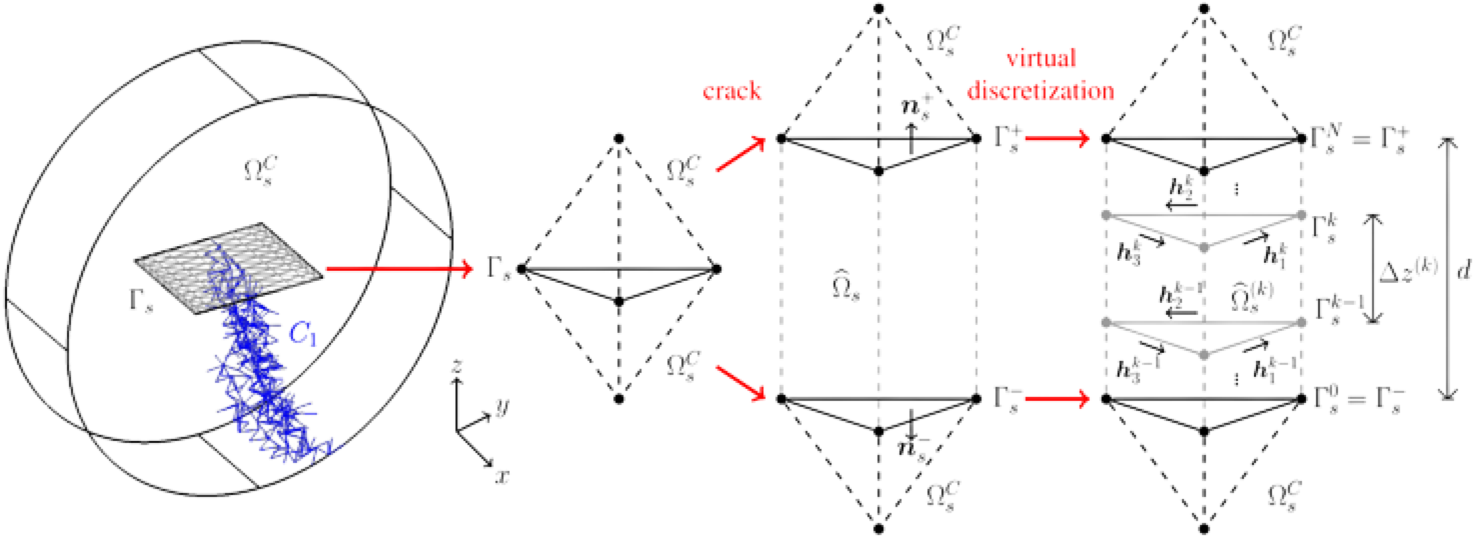
THIN-SHELL SIMPLIFICATIONS IN $H-\phi$ FORMULATION (2-D AND 3-D)

2-D:



B. De Sousa Alves et al., SuST 35 (2), 024001 (2021)

3-D:



B. De Sousa Alves et al., IEEE Trans. Appl. Supercond. 32 (5), 7500411 (2022)

In-plane current

$$\boldsymbol{j} = \nabla \times \boldsymbol{h} = \begin{bmatrix} -\partial_z h_y \\ \partial_z h_x \\ \partial_x h_y - \partial_y h_x \end{bmatrix}$$

Out-of-plane current

➔Current sharing!

THIN-SHELL SIMPLIFICATIONS IN T-A FORMULATION (2-D)

T-A formulation: widely used in the literature to model superconducting tapes

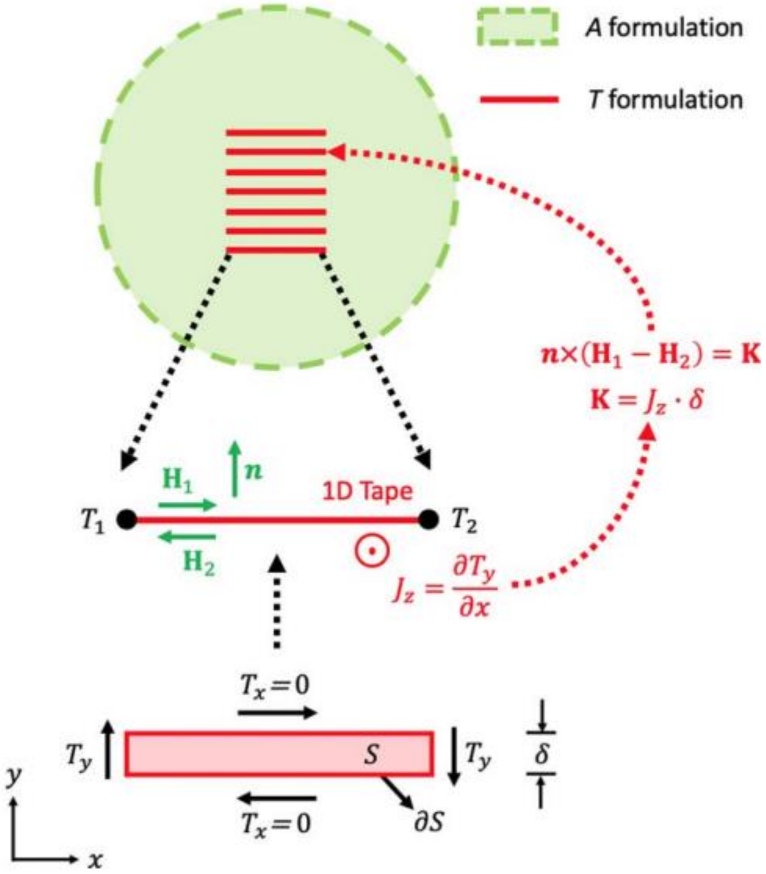
In the non-conducting domain: $\mathbf{A}$ formulation ($\mathbf{B}$-conform)

On the conductor: Faraday's law with the current vector potential $\mathbf{T}$

$$\nabla \times (\rho \nabla \times \mathbf{T}) = -\partial_t \mathbf{B}$$

Condition on the current:

$$I = (T_1 - T_2)\delta$$



E. Berrospe-Juarez et al.,
SuST. 32(6), 065003 (2019)

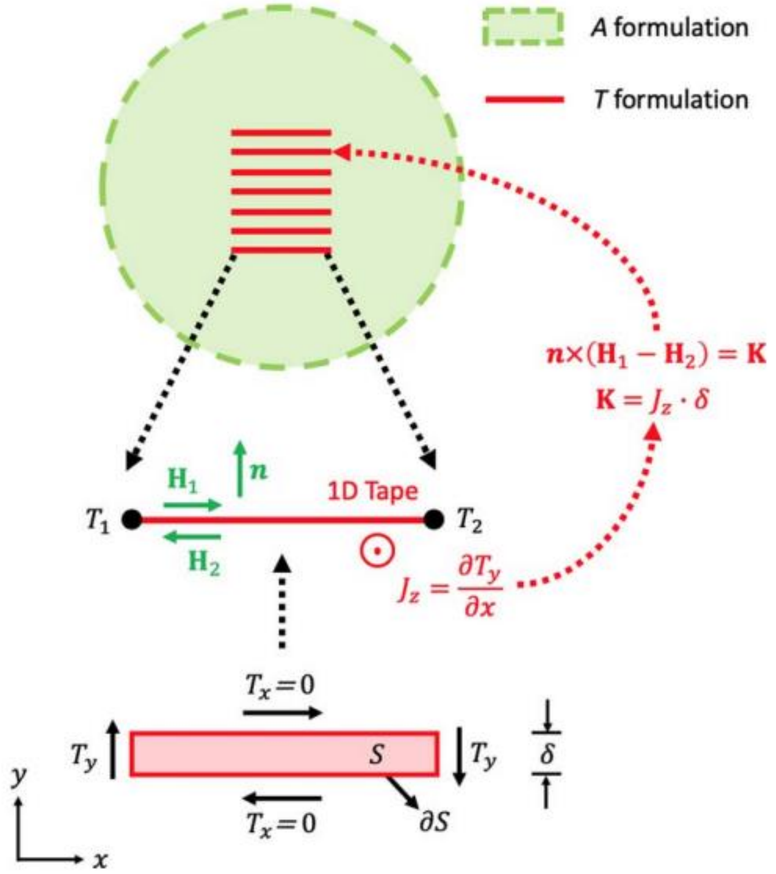
THIN-SHELL SIMPLIFICATIONS IN T-A FORMULATION (2-D)

Advantages of T-A formulation

- Documented and well-known
- Simple to implement (in COMSOL, for example)
- No need for cohomology cuts

Disadvantage of T-A formulation

- Can only model one layer of tape
- No current sharing possible
- Bad performance in 3-D ($\mathbf{A}$ has 3 spatial components in the whole space)



E. Berrospe-Juarez et al., SuST. 32(6), 065003 (2019)

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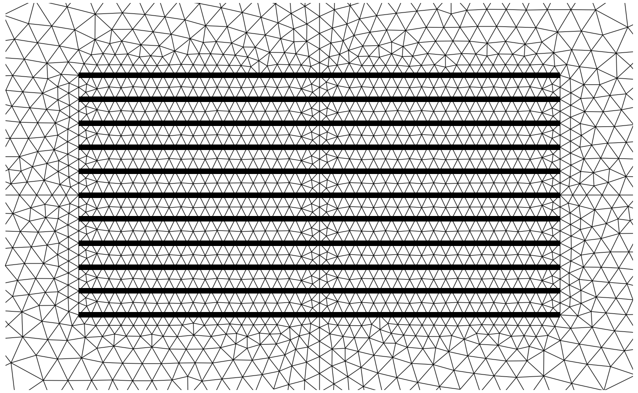
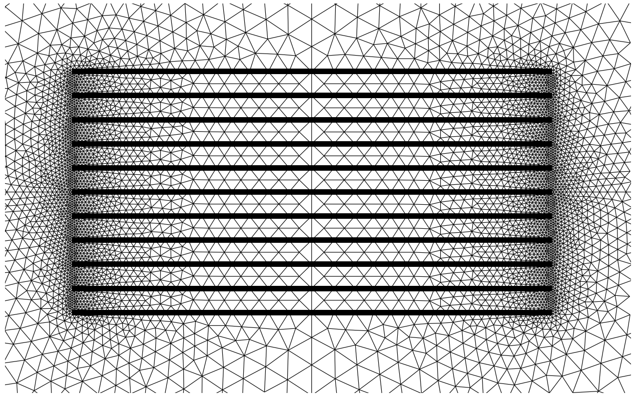
PERFORMANCE COMPARISONS (PHYSICS COMPUTATION)

Coarse Mesh
(6407 DoFs)



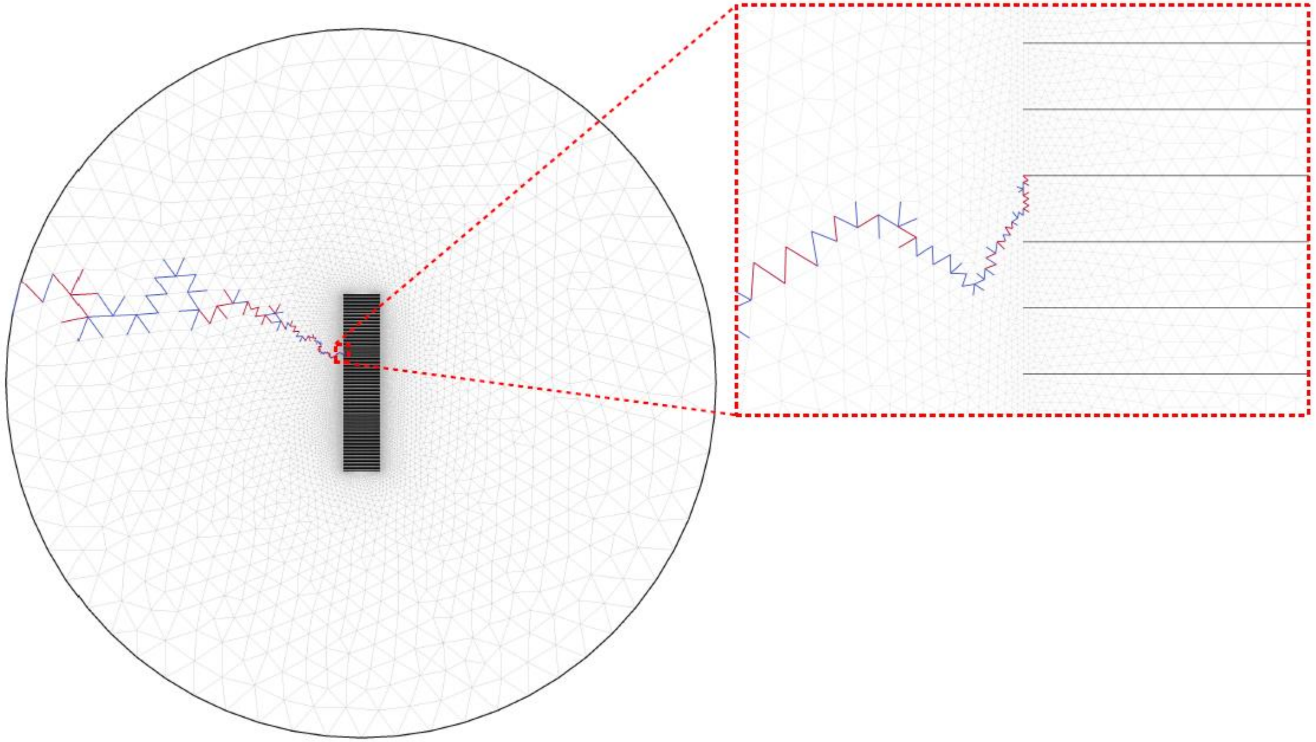
Fine Mesh
(10620 DoFs)

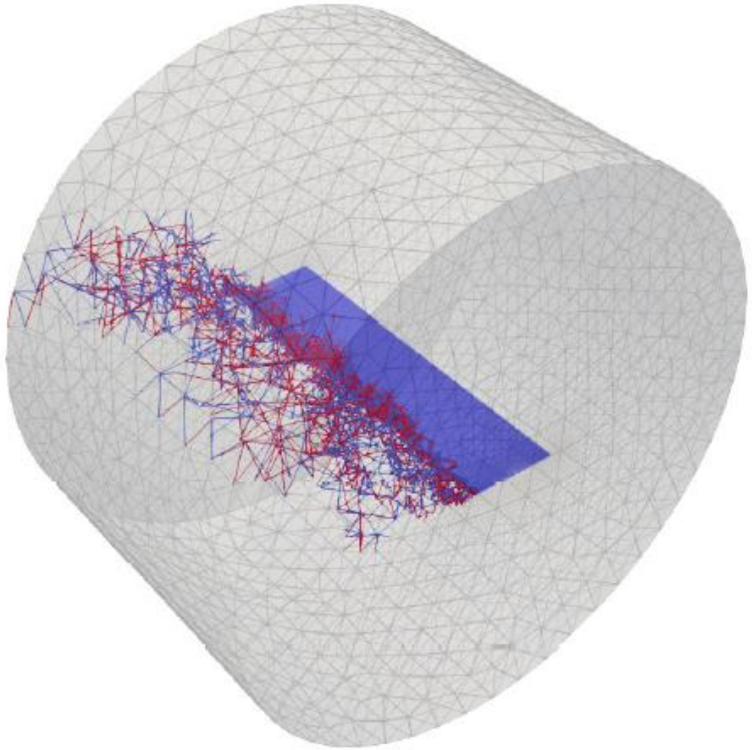


		GetDP	BELFEM	BELFEM	COMSOL
formulation		h-ϕ	h-ϕ	h-ϕ	t-a
solver		MUMPS	MUMPS	STRUMPACK	MUMPS
constant timestep	coarse mesh	5:13	1:56	0:55	2:21
	fine mesh	10:57	7:15	2:54	4:04
adaptive timestep	coarse mesh	2:36	**0:23**	**0:11**	1:10
	fine mesh	7:58	**1:09**	**0:25**	3:17

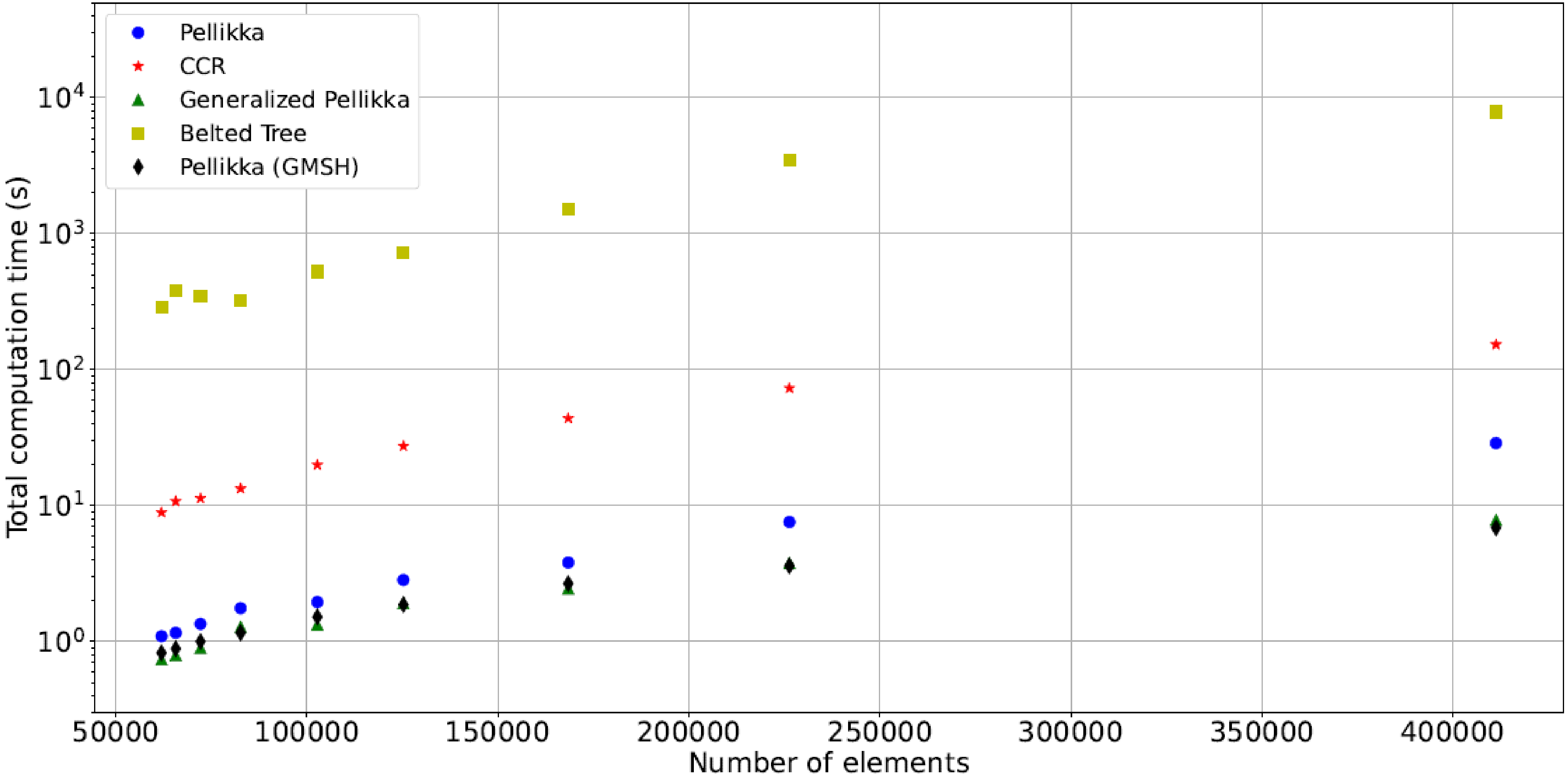
PERFORMANCE COMPARISONS (COHOMOLOGY COMPUTATION)

Multiple cohomology computation algorithms from the literature were implemented in BELFEM





PERFORMANCE COMPARISONS (COHOMOLOGY COMPUTATION)



3-D benchmark

Two conductors

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BENCHMARKING METHODOLOGY

Goal: Provide a normalized strategy to model various HTS benchmark problems with different simulation tools and accurately compare the solutions and performances obtained

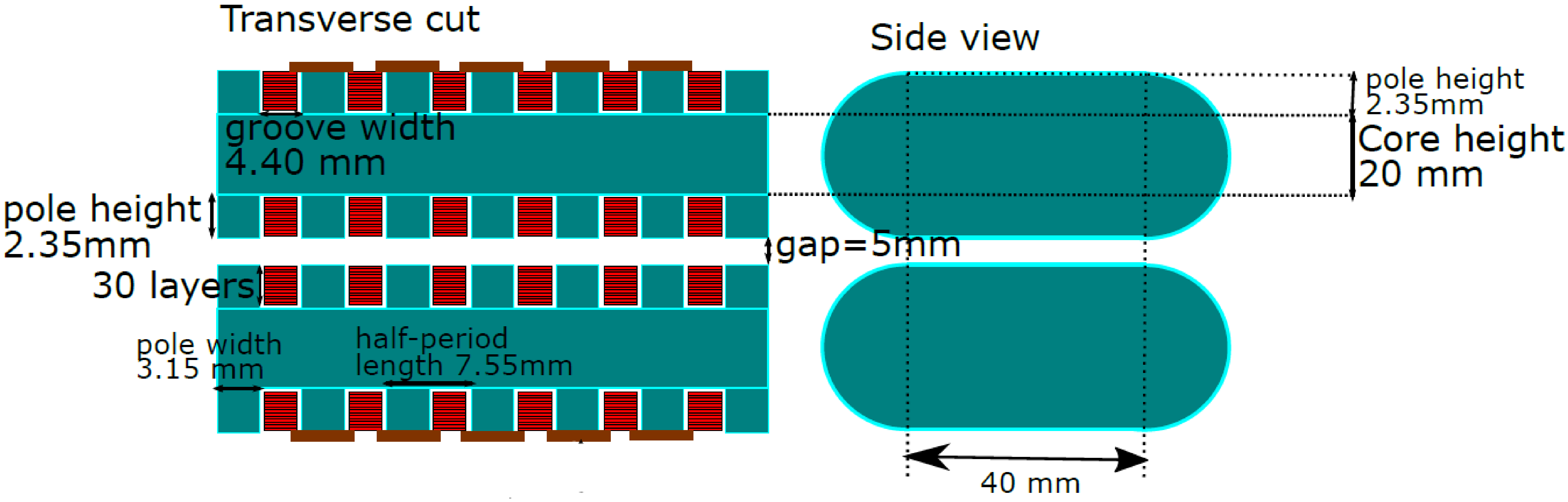
Geometry and meshing tool: **GMSH**

Simulation software for now: **COMSOL, GetDP** and **BELFEM**

Simple procedure to generate equivalent models within all simulation tools

THE UNDULATOR BENCHMARK (2-D)

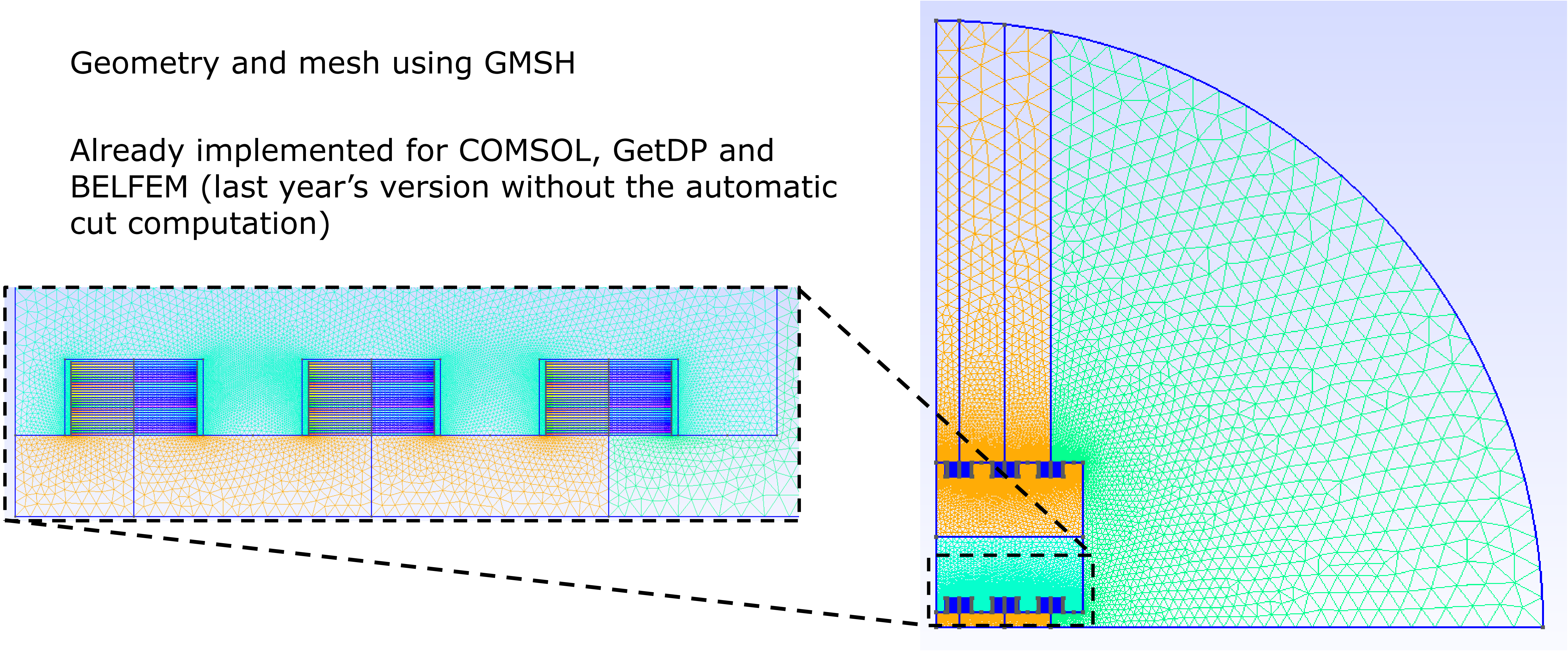
2-D Undulator benchmark from a collaboration with European XFEL's Undulator Systems group (special thanks to Dr. Vanessa Grattoni and Dr. Sara Casalbuoni)



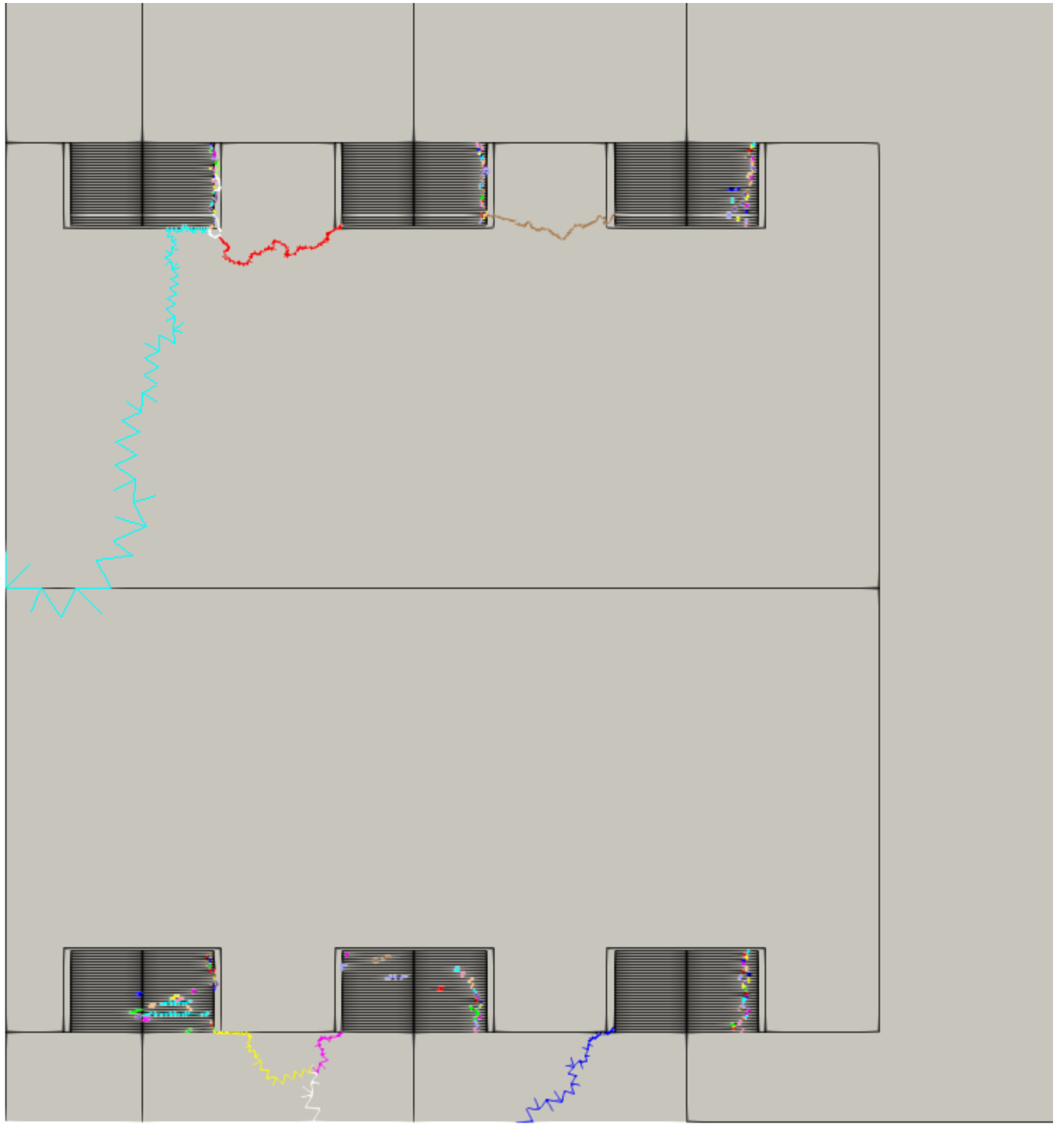
THE UNDULATOR BENCHMARK (2-D)

Geometry and mesh using GMSH

Already implemented for COMSOL, GetDP and BELFEM (last year's version without the automatic cut computation)



THE UNDULATOR BENCHMARK (2-D)



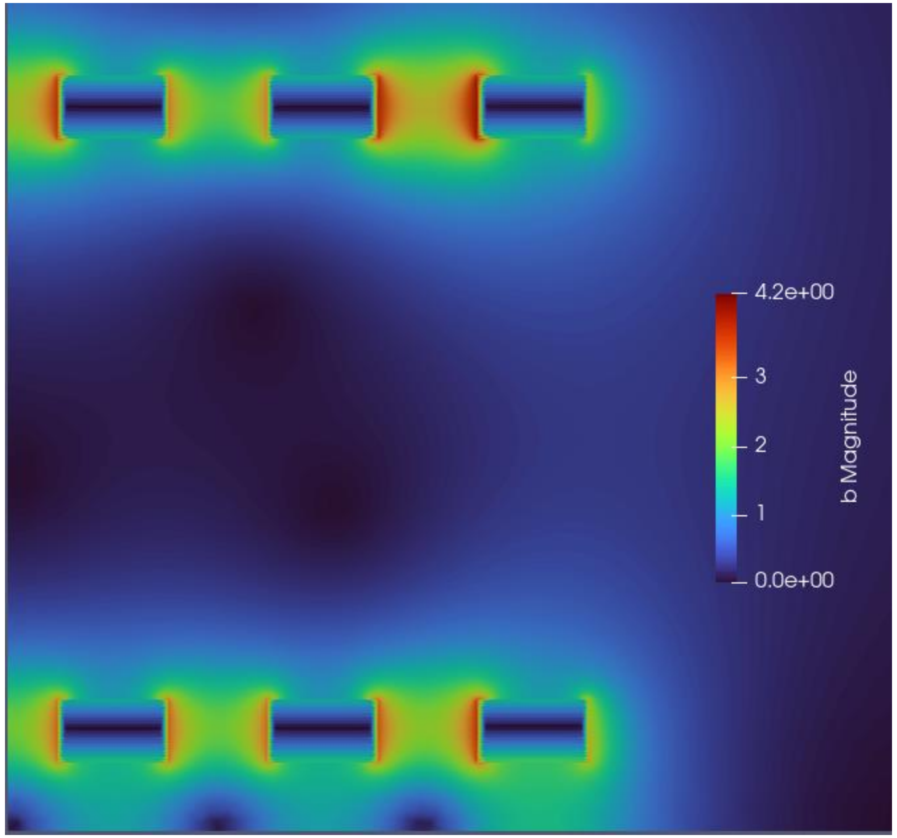
It is now very easy to create the cuts in BELFEM*, and we don’t need those transverse lines in the geometry anymore (see previous slide)

For example in this case:
~112 000 elements, ~0.5 s to compute the cuts

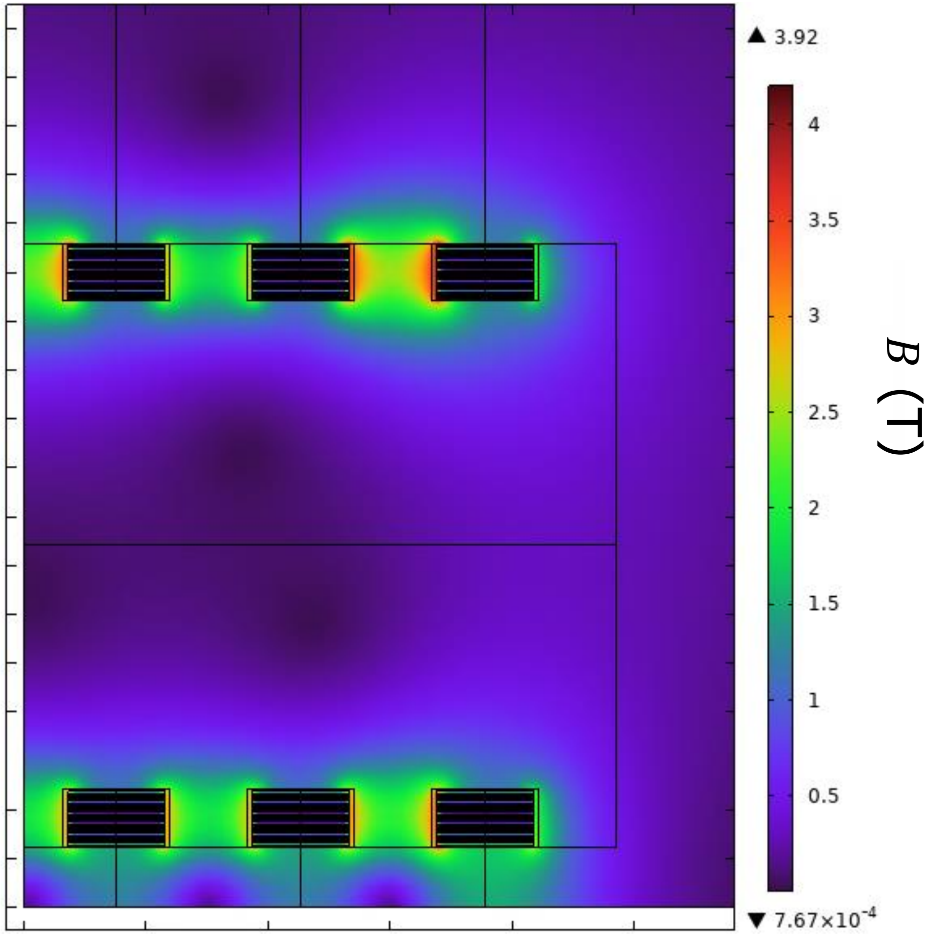
* The user does not even need to know about their existence!

THE UNDULATOR BENCHMARK (2-D)

Result examples (700 A, 5 min ramp)



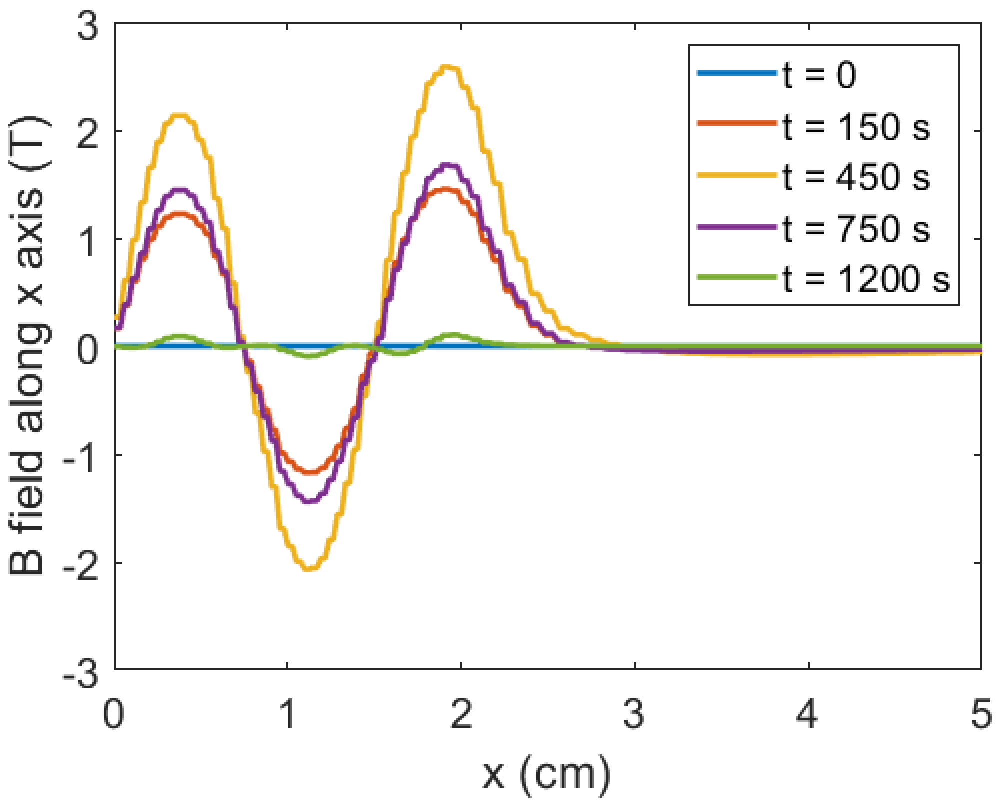
BELFEM (H-ϕ, with manual cuts)

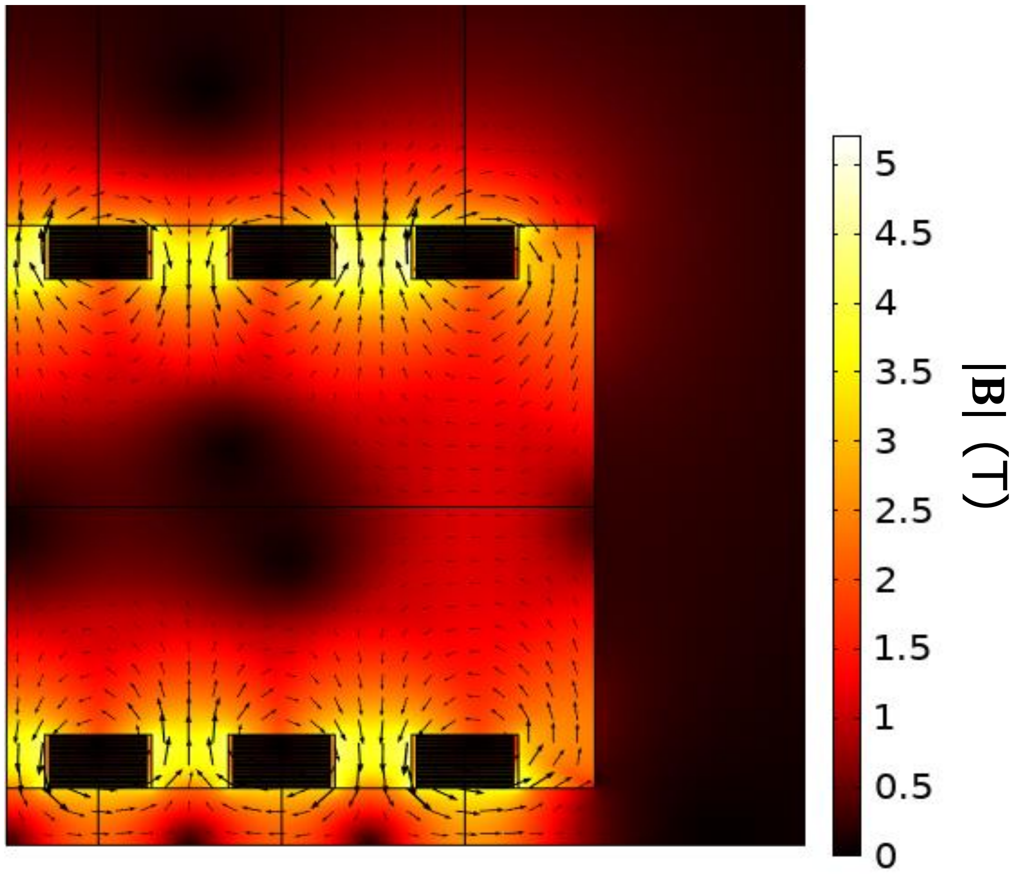


COMSOL (T-A)

THE UNDULATOR BENCHMARK (2-D)

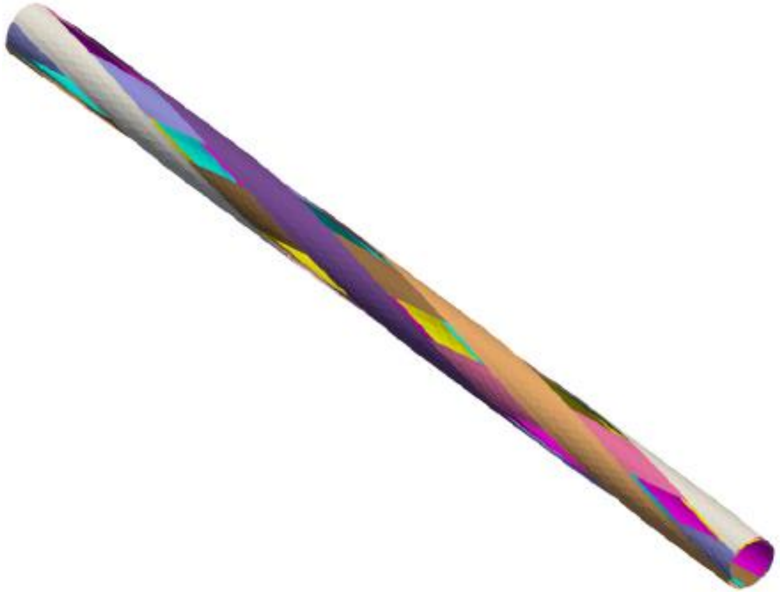
B field distribution considering the full physic (Iron core, $J_c(T,B)$, $n(T,B)$)

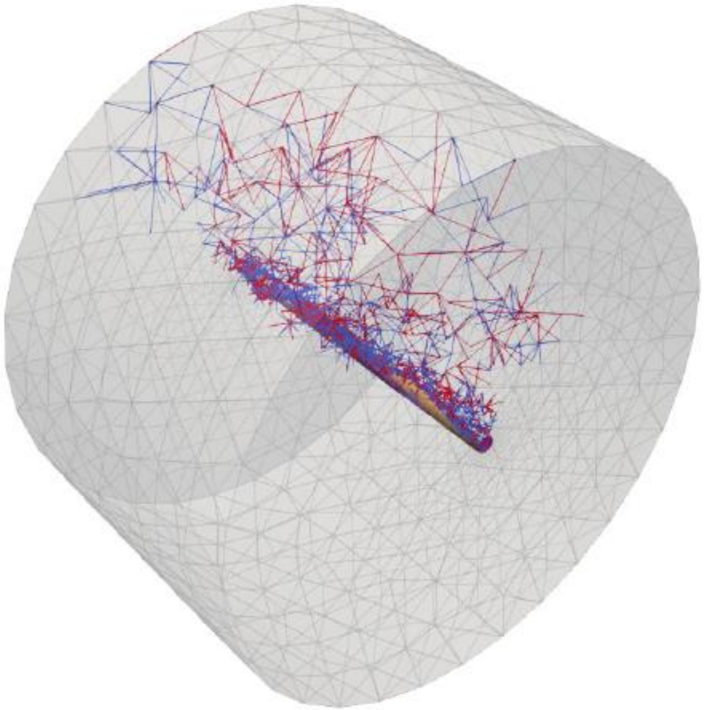


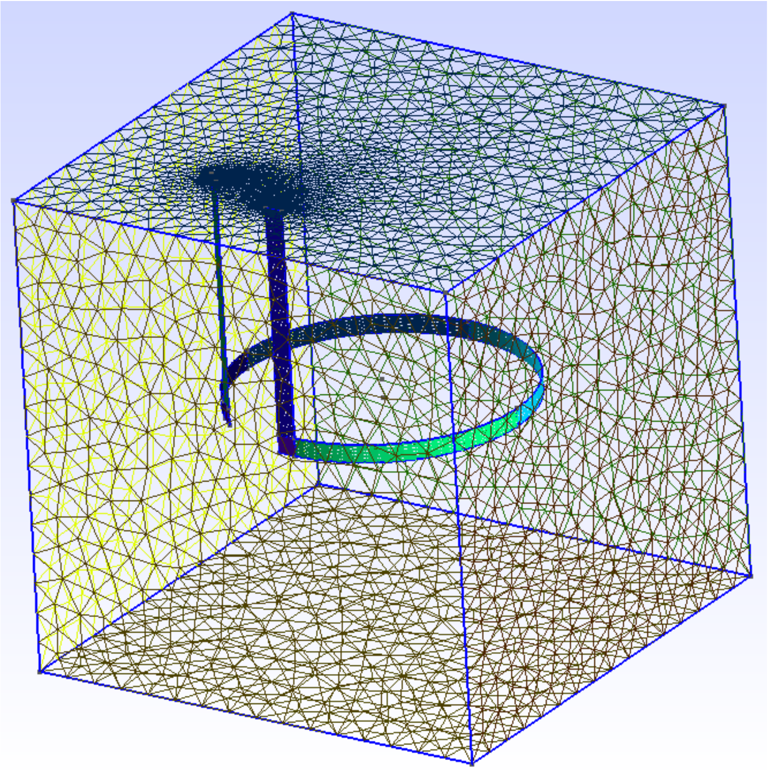


TOWARDS 3-D BENCHMARKS

CORC® cable, pancake coil with current leads







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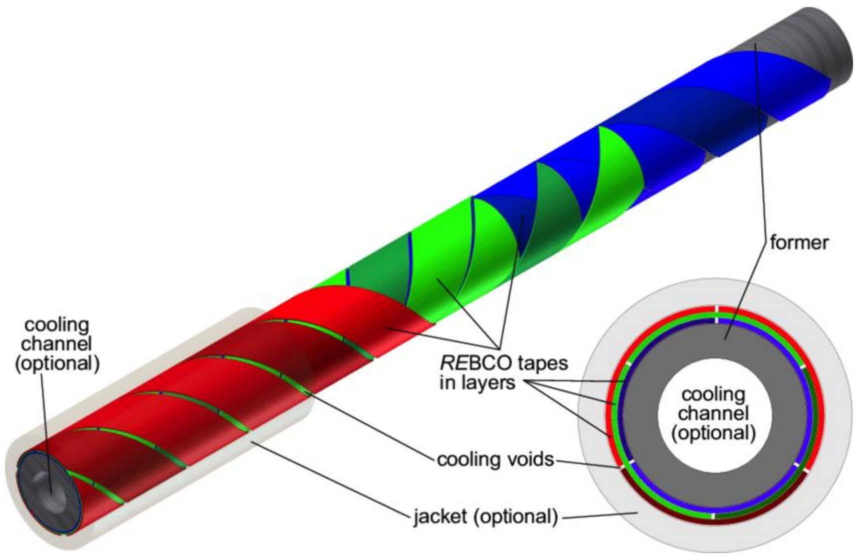
CONCLUSION

BELFEM is being built from the ground up with all the best numerical methods and formulations for HTS problems:

- $H - \phi$ with thin-shells, which is mathematically the most optimized formulation for HTS **(not fully available in COMSOL)**
- Automatic cuts generation, designed to avoid any user input **(not available in COMSOL)**
- State-of-the-art direct and iterative solvers such as STRUMPACK **(not available in COMSOL or GetDP)**

Being contributors and the first users, our group at Polytechnique Montreal can efficiently provide feedback to the LBNL group about the implementation of complex benchmarks.



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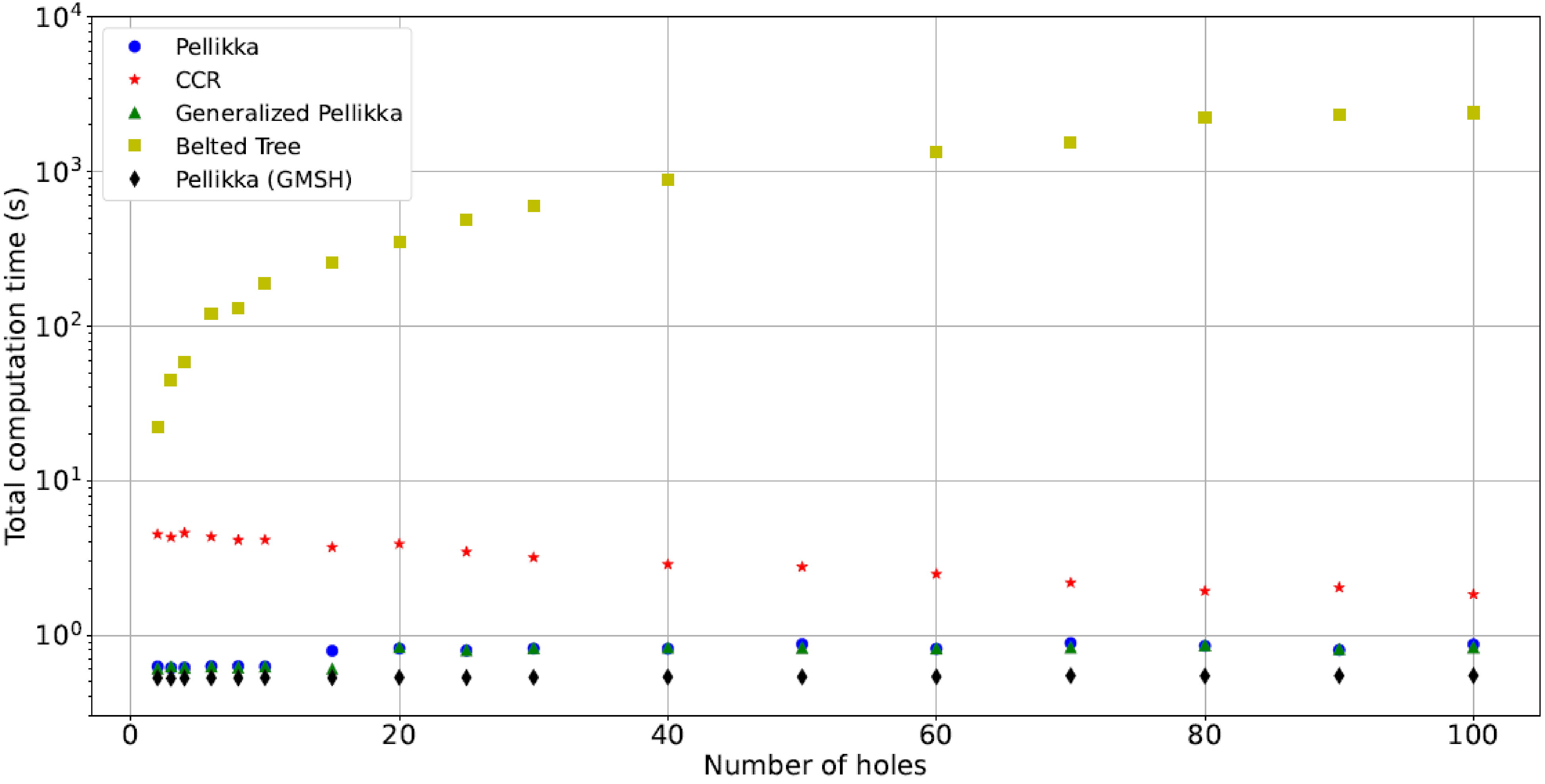
BONUS SLIDES

Bonus slides

PERFORMANCE COMPARISONS (COHOMOLOGY COMPUTATION)



2-D benchmark

92 686 elements