AI-enhanced analysis of $\pi\pi$ scattering

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The pion-pion scattering amplitude. IV: Improved analysis with once subtracted Roy-like equations up to 1100 MeV

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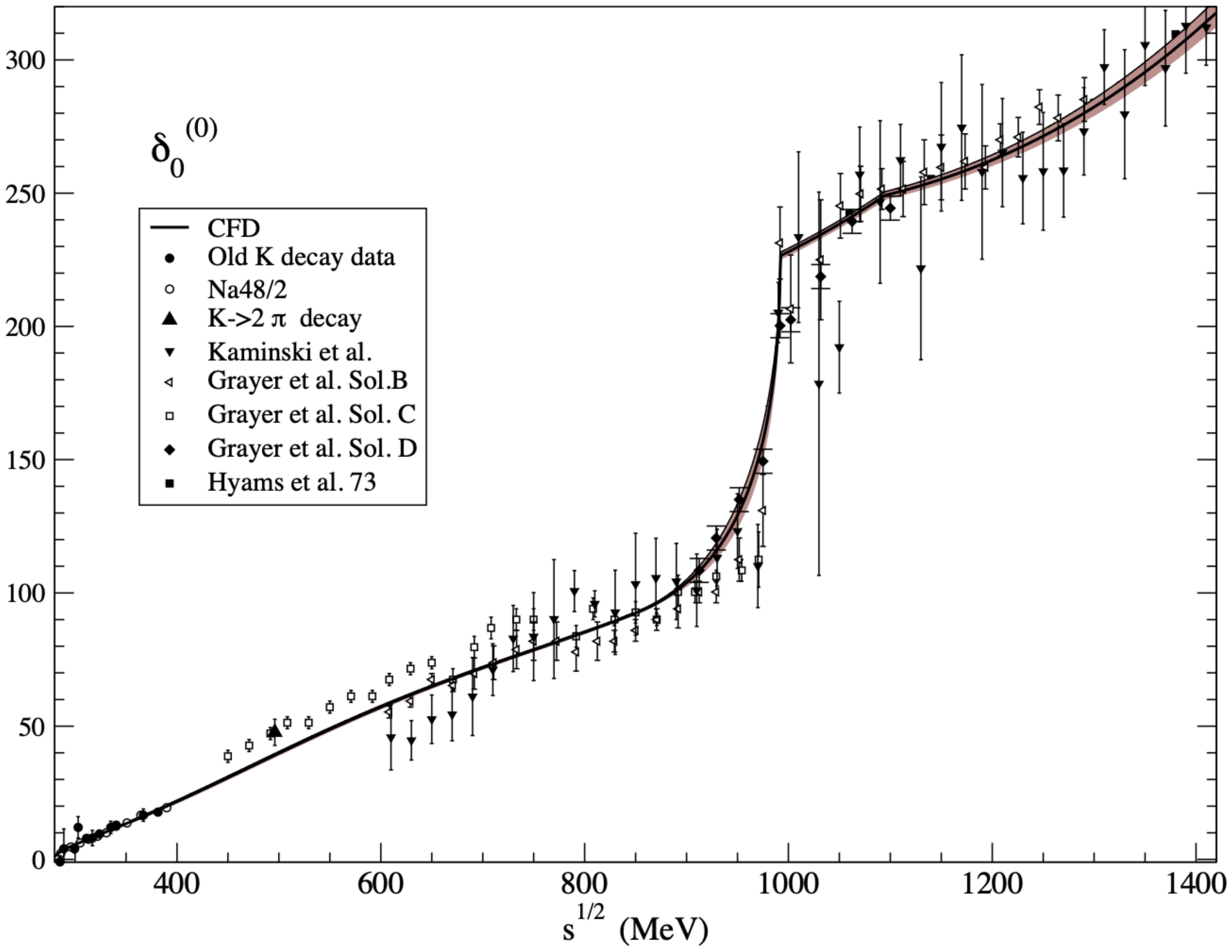


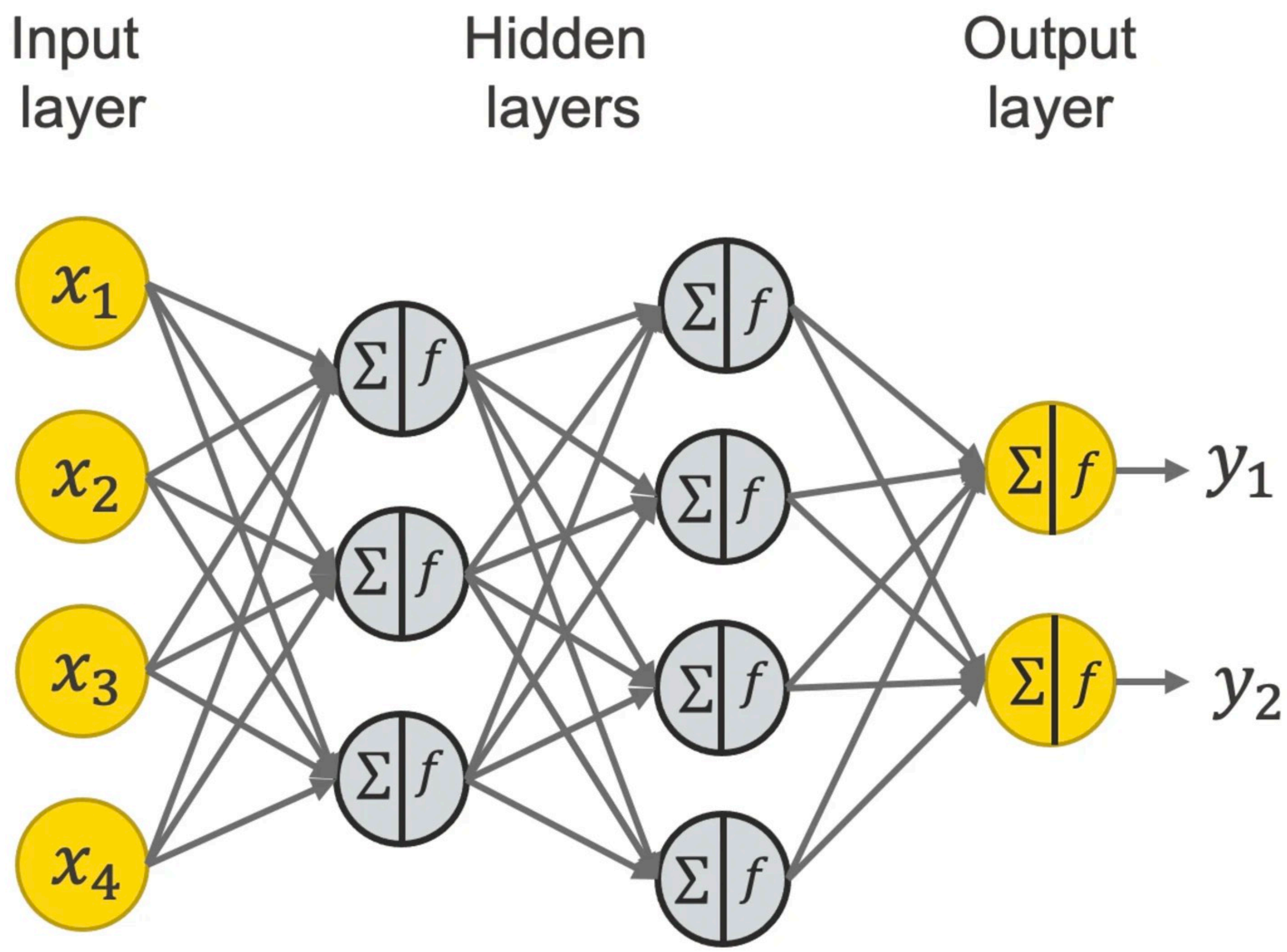
- Pion-Pion scattering is extremely important in modern particle experiments!
- Heroic efforts to parameterize pion-pion scattering - still ongoing, see Pablo Rabán's poster tomorrow!
- Data comes from several older experiments
- The shaded region is the error band—It's extremely narrow!

Experimental data is old, data sets are incompatible, and errors are likely underestimated!

Neural Networks

Neural Network = Complicated, arbitrarily scalable function

NN's provide extremely flexible models for fitting data



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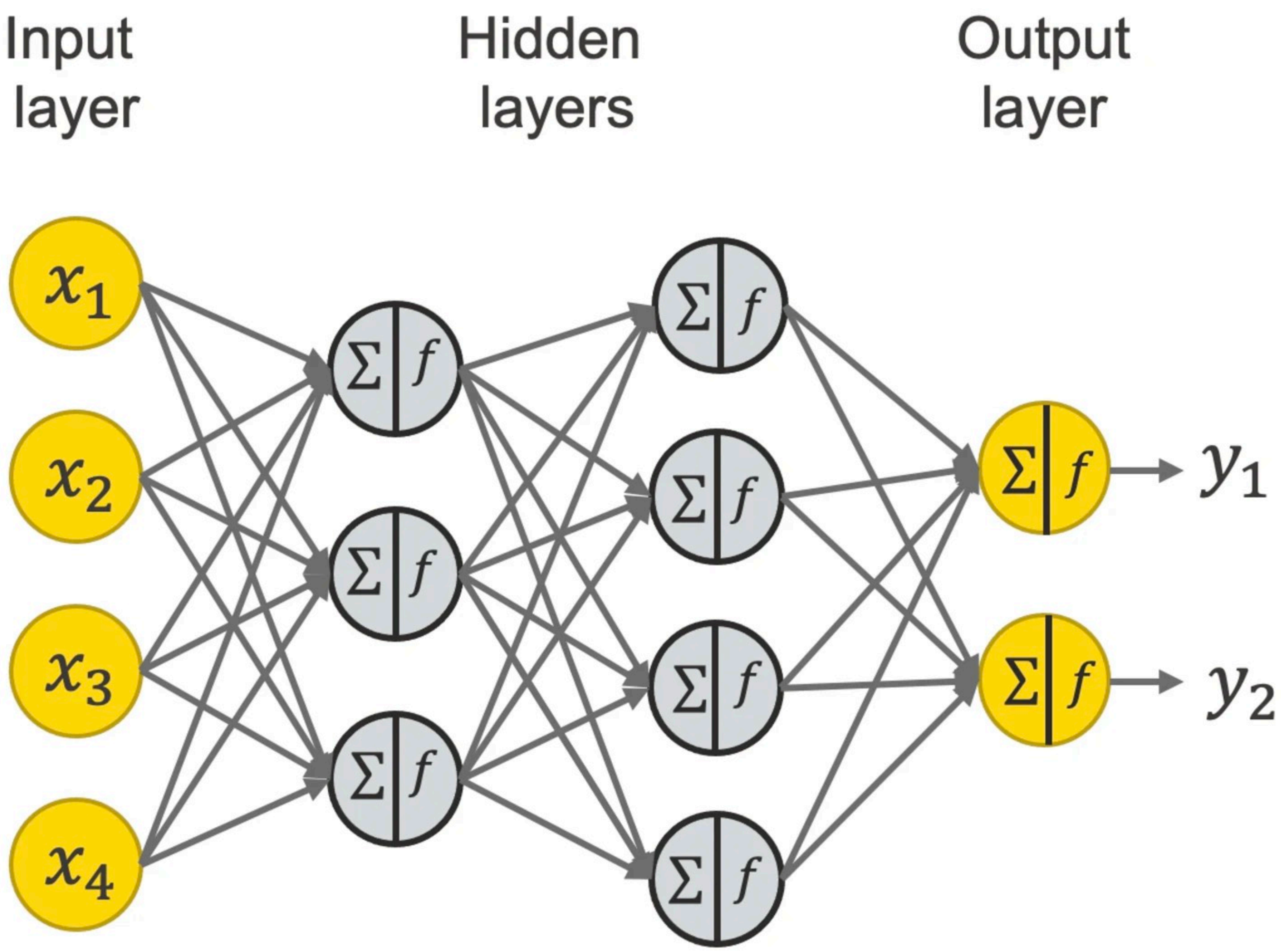


Why use NNs for physics?

- Less model dependence - we don't know the right answer!
- Lower computational costs (one and done?)
- Established codebases - efficient use of computing resources
- $$$

Overall Strategy

$$F^{(I)}(s,t) = \frac{8}{\pi}\sum_{\ell}(2\ell+1)P_\ell(\cos\theta)t_\ell^{(I)}(s),$$

$$t_\ell^{(I)}(s) = \frac{\sqrt{s}}{2k}\hat{f}_\ell^{(I)}(s), \quad \hat{f}_\ell^{(I)}(s) = \frac{\eta_\ell^{(I)}(s)e^{2i\delta_\ell^{(I)}(s)} - 1}{2i}$$

- Use neural network in place of human-writable model:

$$NN: \ E \to \{(\delta_\ell^{(I)}, \eta_\ell^{(I)})\}$$

- Train many neural networks on re-sampled data to produce a Monte Carlo-esque sample of models (think NNPDF)

- Error bars determined from the ensemble of the neural networks rather than from a single fit

- All physics content must be enforced by carefully constructed loss function!

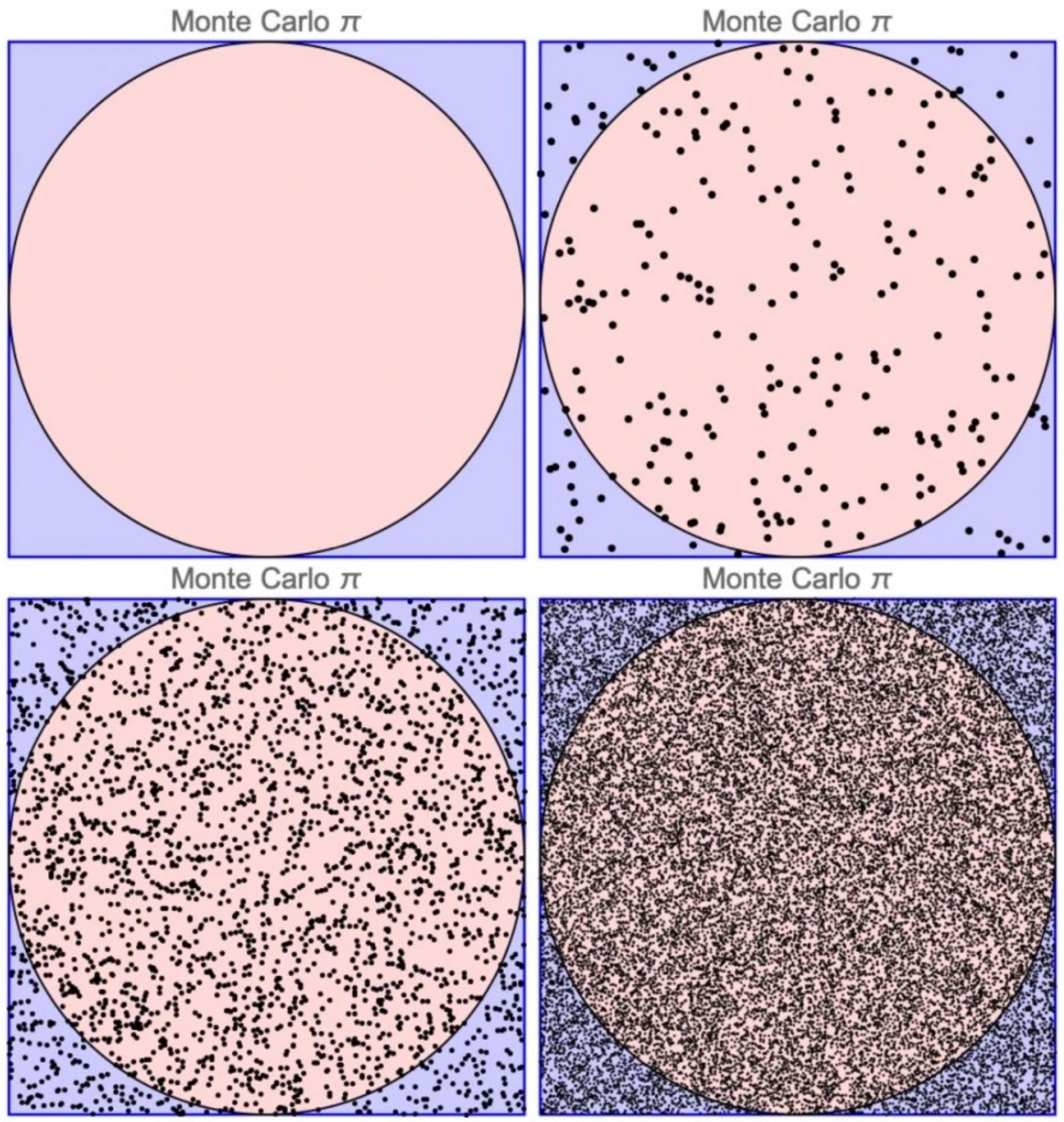


Figure 2: Squares with zero, 250, 2500 and 25000 points.

https://thatsmaths.com/2020/05/28/the-monte-carlo-method/

Loss function

$$\text{LOSS} = \text{MSE} + \lambda_{Roy}\Delta_{Roy} + \lambda_{Regge}\,\text{MSE}_{Regge}$$

Fit the data

Satisfy physics

Match high-energy Regge

- MSE easily minimized due to the flexibility of even a simple NN (beware overfitting!)
- Δ_{Roy} contains discrepancies from Roy-like equations (enforcement of dispersion relation, analyticity, crossing symmetry)
- MSE_{Regge} forces the neural network to respect high-energy dominance of regge physics
- λ's are Lagrange multipliers that can be tuned/adjusted on the fly.

Loss function

$$\text{LOSS} = \text{MSE} + \lambda_{Roy}\Delta_{Roy} + \lambda_{Regge}\,\text{MSE}_{Regge}$$

- The dispersive integral is in the loss function

$$\Delta_\ell^{(I)} = \left| \text{Re}\ t_\ell^I(s) - ST_\ell^I(s) - DT_\ell^I(s) - \sum_{I',\ \ell'} \text{P.V.} \int_{4M_\pi^2}^{s_{\max}} ds' K_{\ell\ell'}^{II'}\, \text{Im}\ t_{\ell'}^{I'}(s') \right|$$

- Subtraction terms need to be predicted as well!

- Data from different experiments, or from different isospins/partial waves can be turned on/off or multiplied by additional λ's at will

- Additional constraints are no problem! Sum rules, continuity at transition to Regge, etc.

$$\text{LOSS} += \lambda_{SR}\ \Delta_{SR}$$

How to train a NN?

- Curriculum-based learning: Slowly increase weighting of Roy part of loss

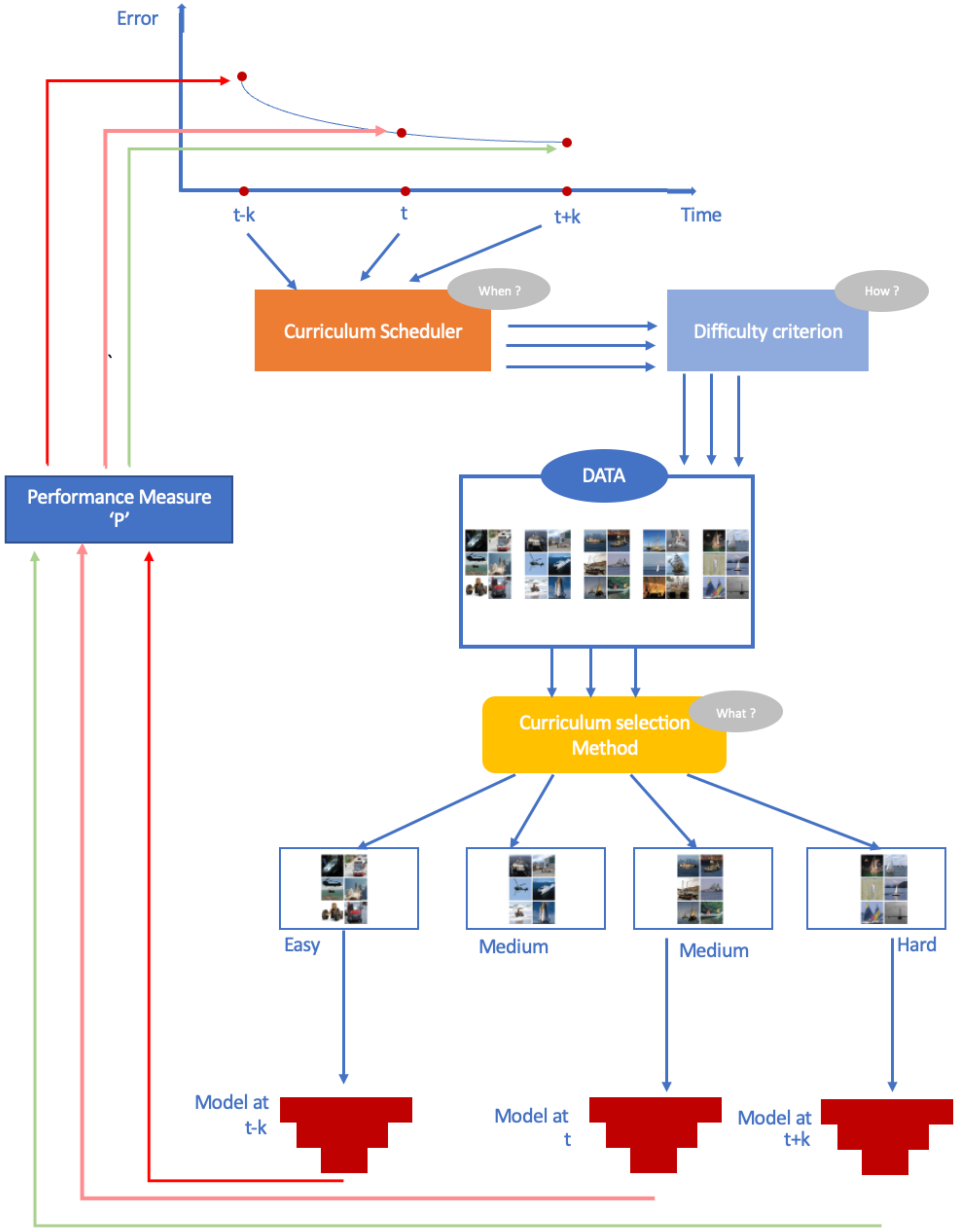
- Force model weights to decay to combat overfitting

- Partial waves not made equal! Allow model to adaptively prioritize underperforming waves by readjusting weights

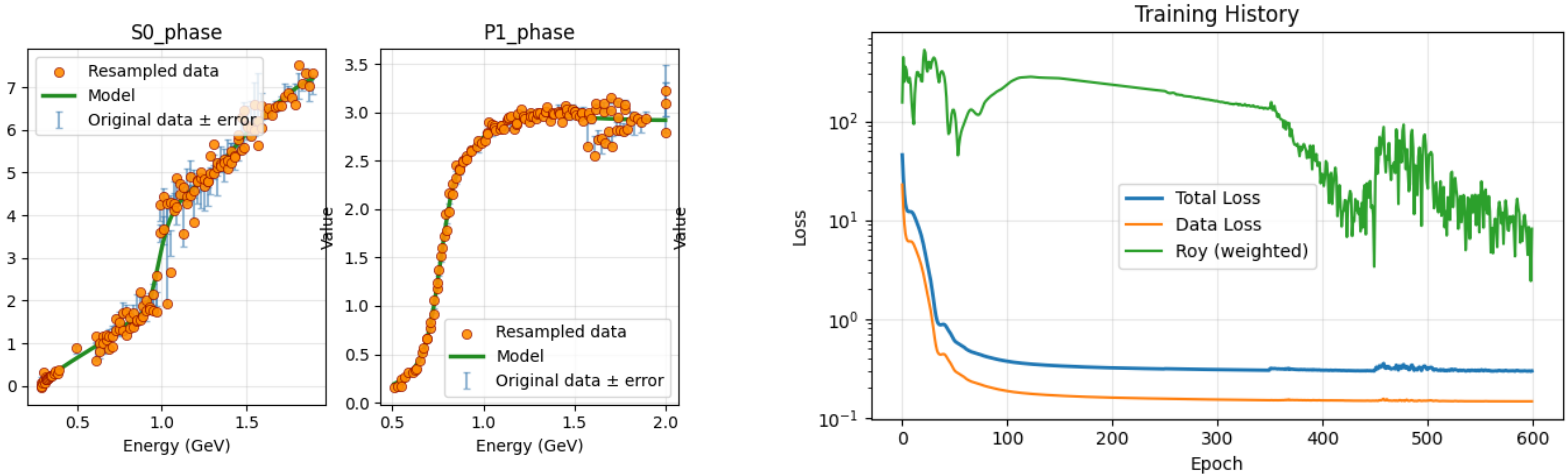
- Avoid large perturbations in weights when we're doing well -> Lower learning rate (velocity of parameter change)

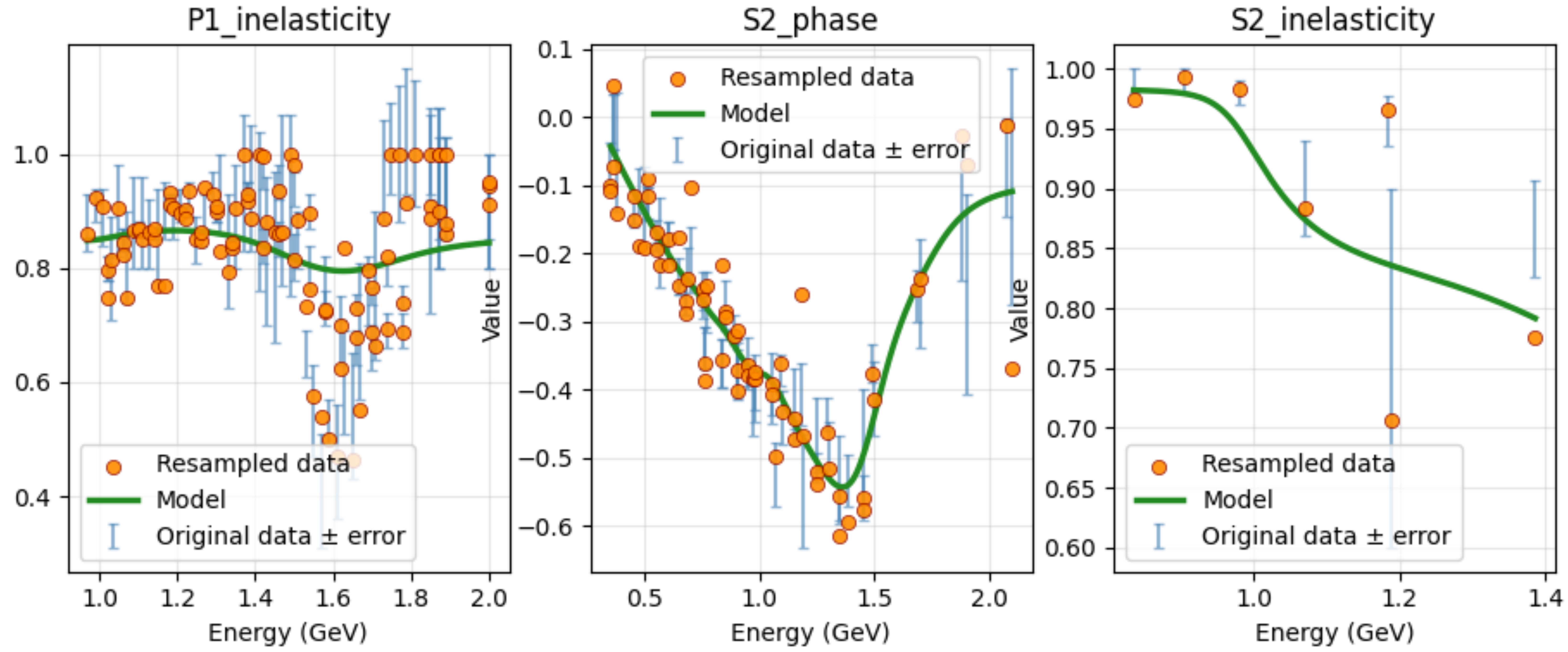
- Not currently interested in high-energy description:

$$\text{LOSS} = \text{MSE} + \lambda_{Roy}\Delta_{Roy} + \cancel{\lambda_{Regge}\text{MSE}_{Regge}}$$



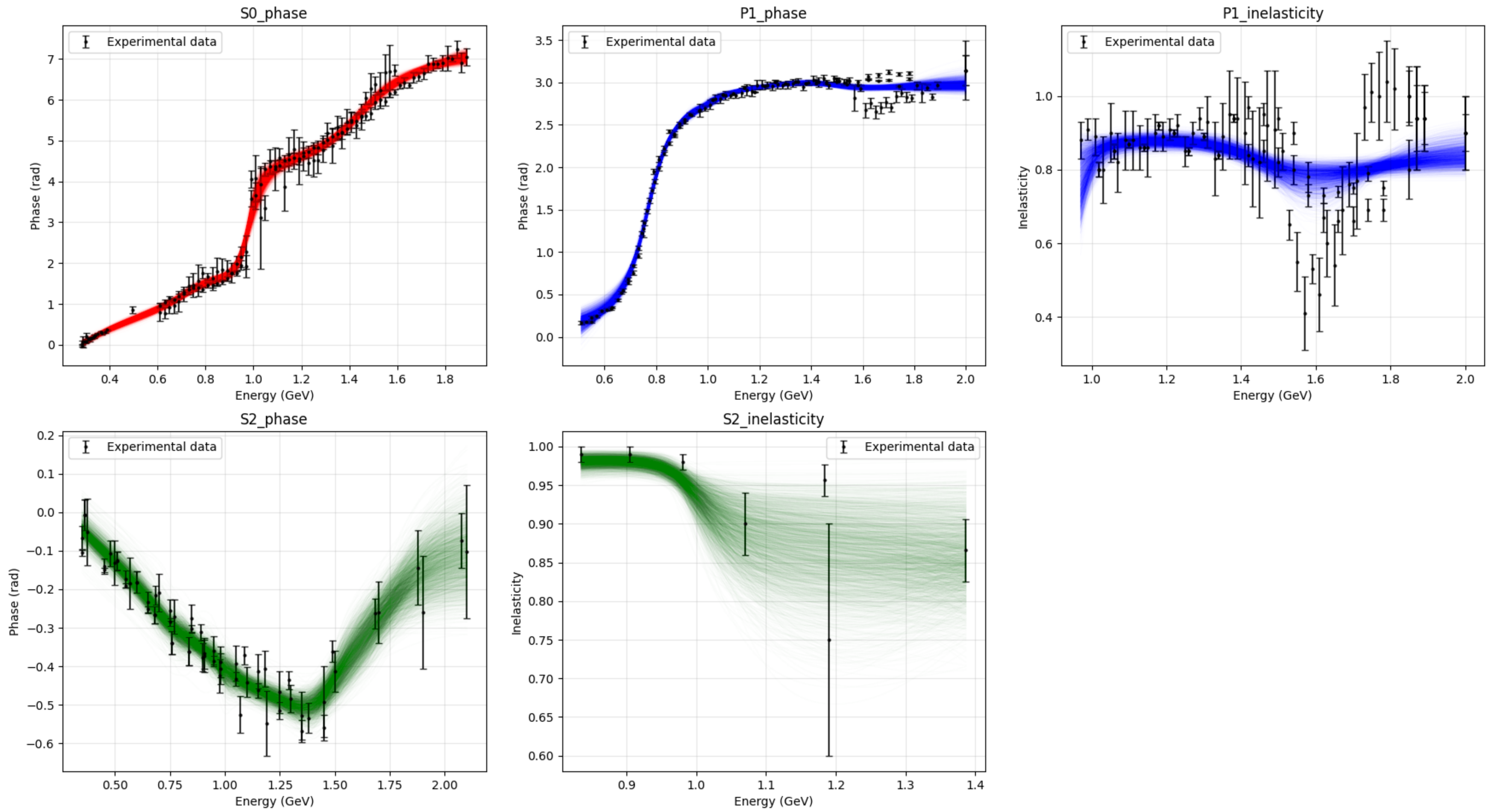
Noha Nekamiche: https://medium.com/aiguys/curriculum-learning-83b1b2221f33



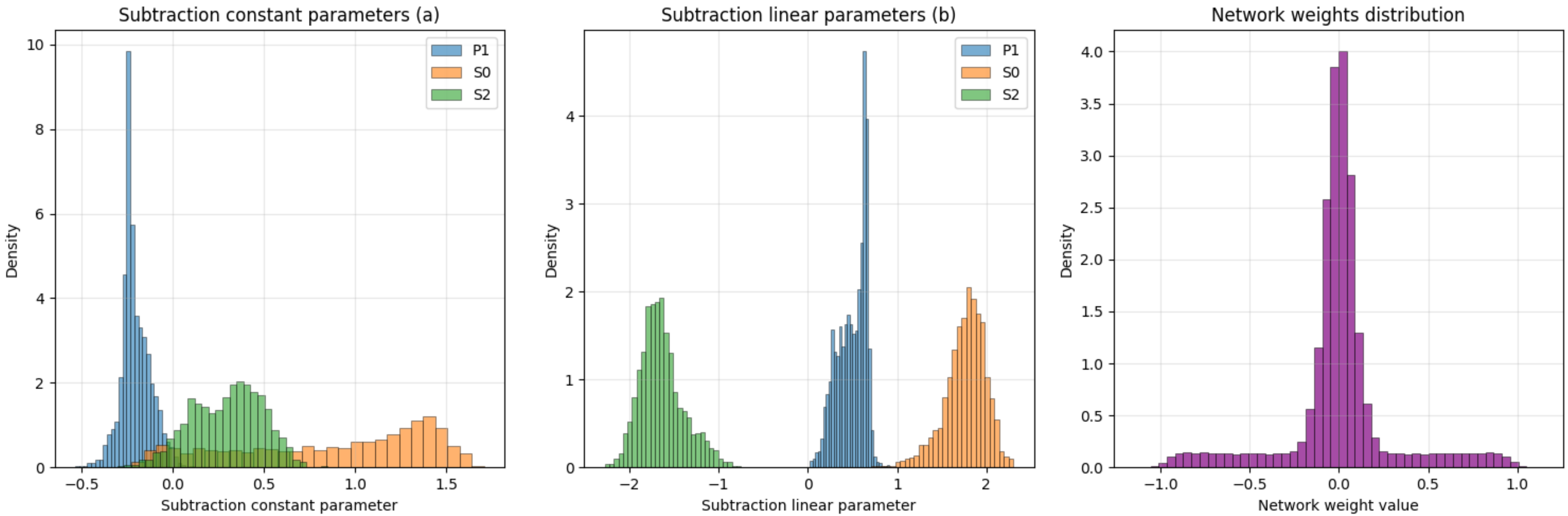


- 50k parameters!
- Replicas re-sampled assuming gaussian errors
- Curriculum phases apparent from loss vs epoch

Preliminary ensemble (~2k NNs)



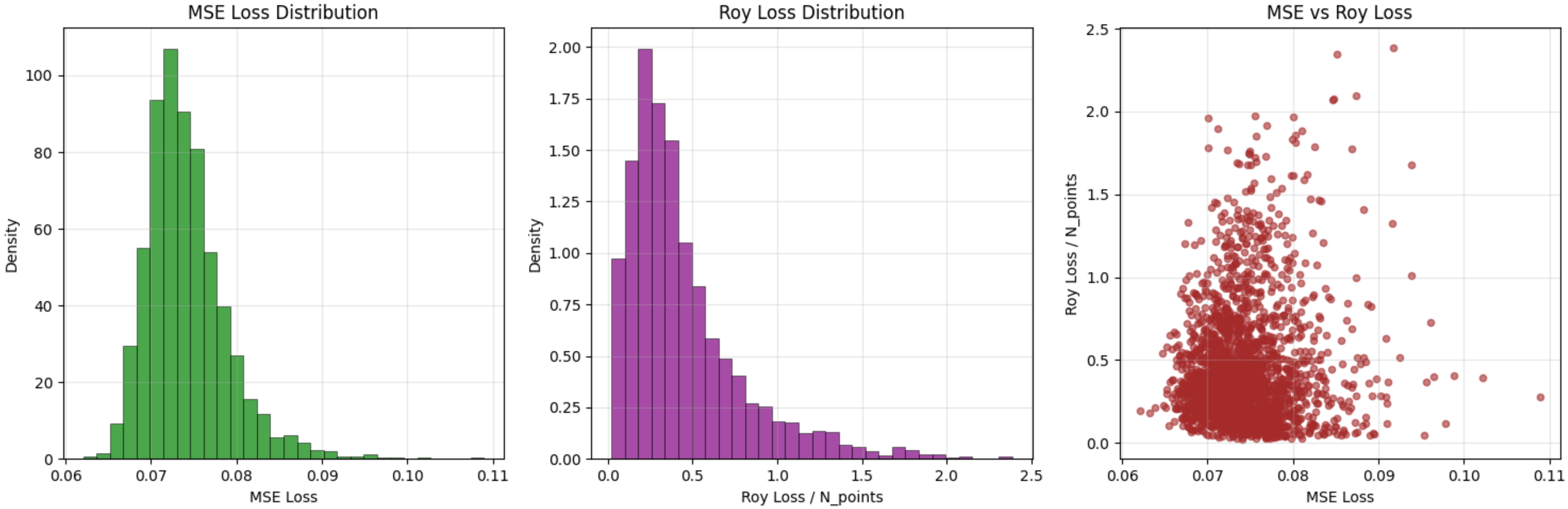
Diagnostics: Subtractions and Weights



- Clustering of subtraction terms - capturing something real? (ChiPT predictions of scattering lengths?)

- Many weights near zero: We have more than enough parameters to span the space of solutions!

Diagnostics: MSE and Roy Losses



- Unweighted MSE and Roy histograms look reasonable
- Potential cuts in Roy loss?
- No major correlations in the MSE and Roy losses

Takeaways

- (Almost) fully model-independent description of $\pi\pi$ scattering is possible

- Error estimates from Monte-Carlo replicas are significantly increased due to model flexibility

- Discrimination of incompatible experimental data may be possible

- Resonance extraction soon!