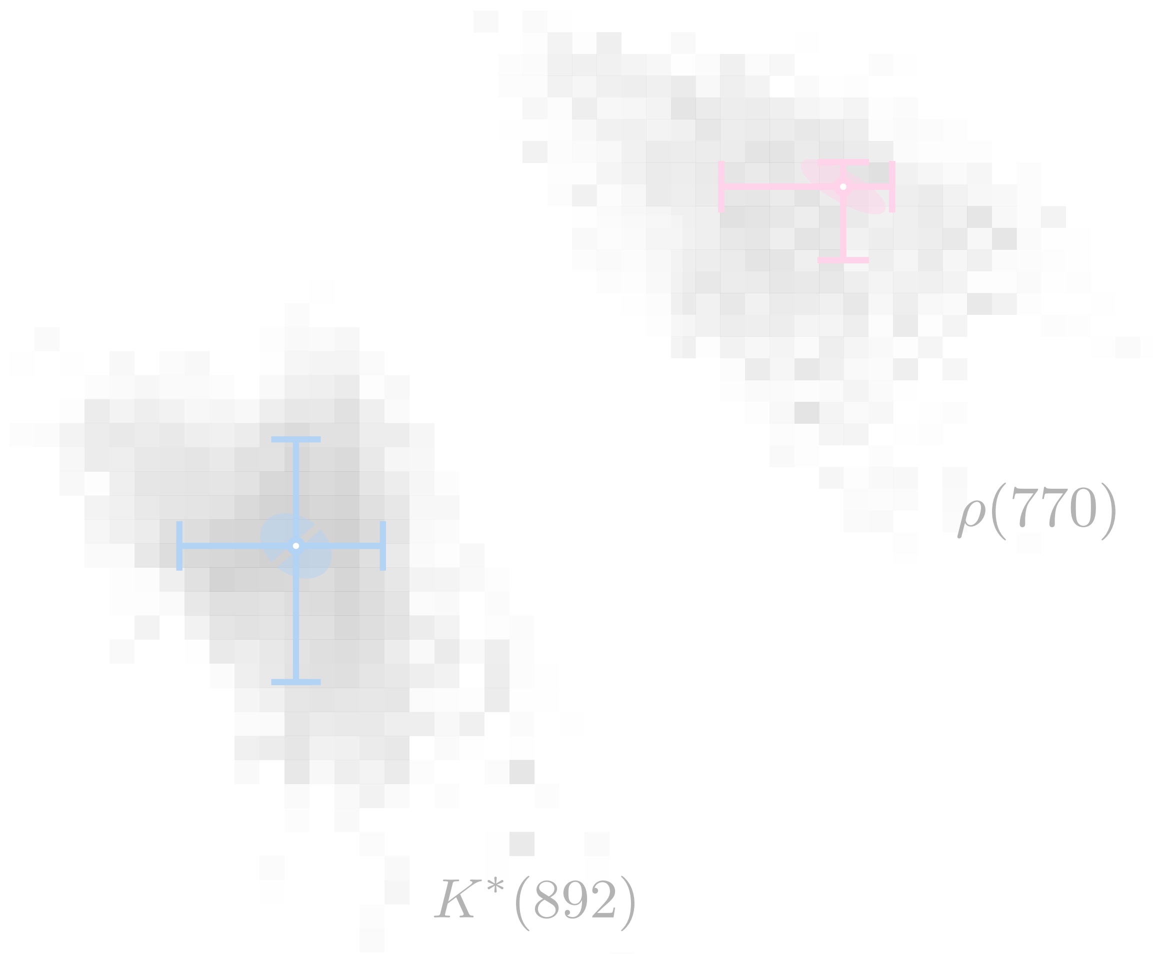
The ρ and K^* resonances from lattice QCD at physical quark masses

PRL (2406.19194) & PRD (2406.19193)

Nelson P. Lachini

in collaboration with:
P. Boyle, F. Erben, V. Gülpers, M. T. Hansen, F. Joswig, M. Marshall, A. Portelli



28 July 2025, Berkeley

MULTI-PARTICLE 25

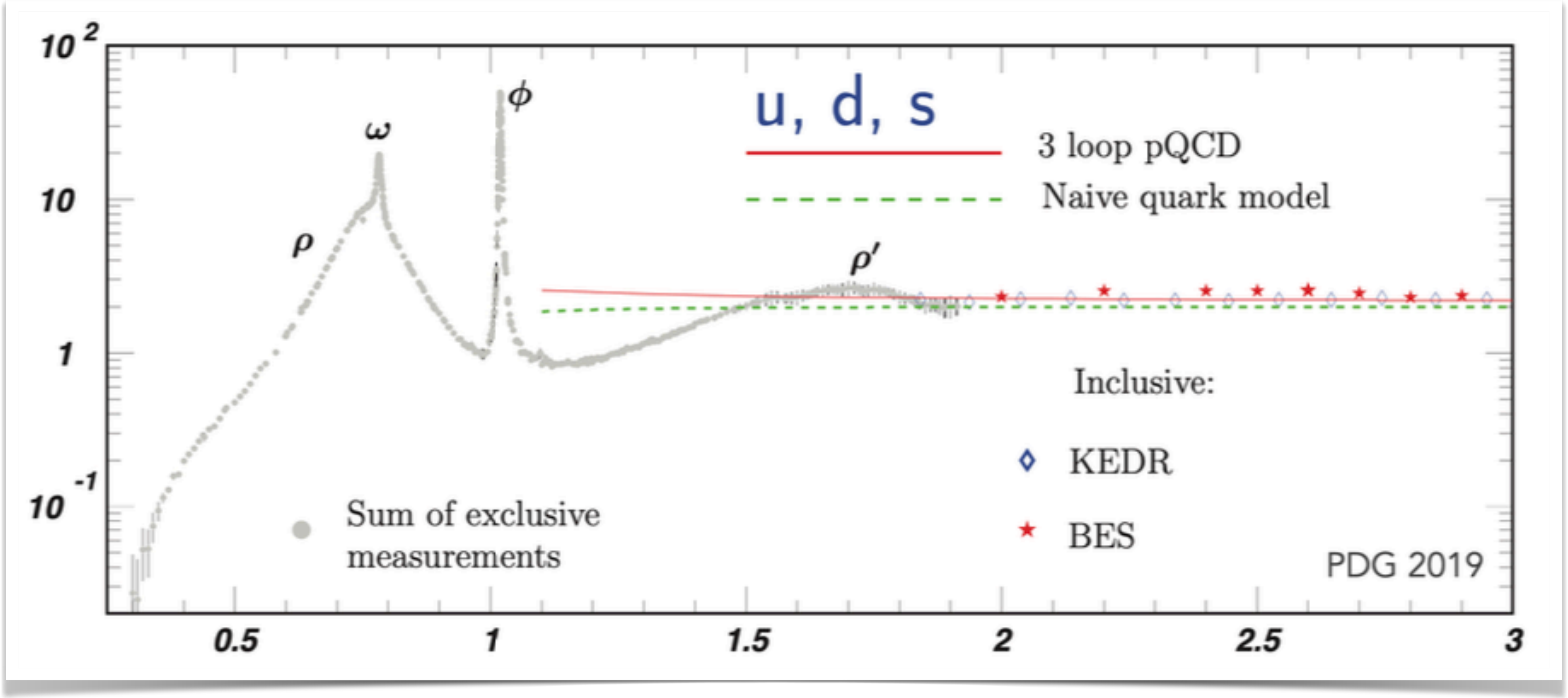
Science and Technology Facilities Council

UNIVERSITY OF CAMBRIDGE

Resonance Phenomenology

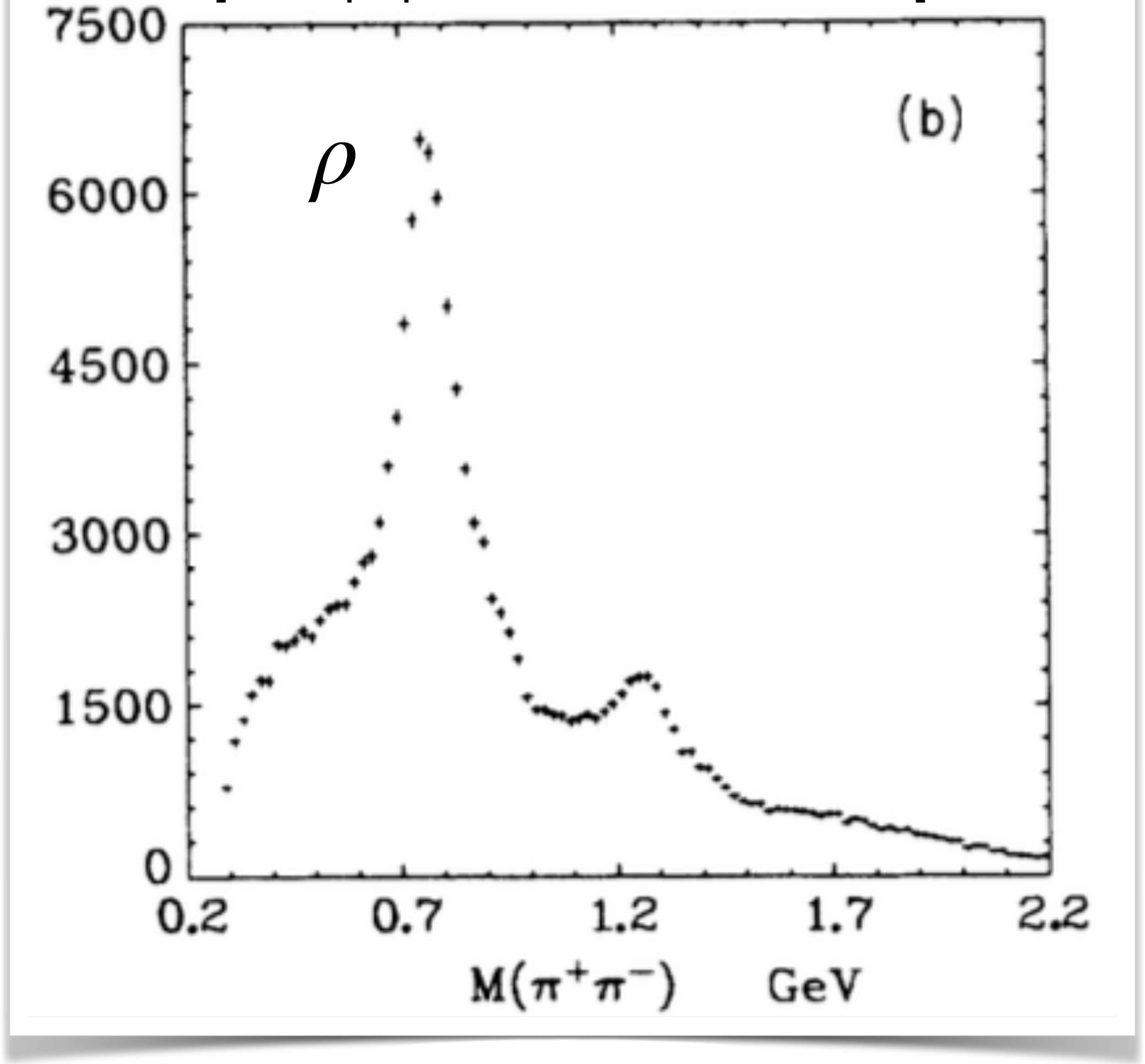
- cross-section “enhancements”
- multi-hadron states
- process dependent

$e^+e^- \to \pi\pi \; (\to \rho \to \pi\pi)$



$\pi p \to \Delta\pi\pi \; (\to \rho \to \pi\pi)$

[Protopopescu et al, PRD, 1973]



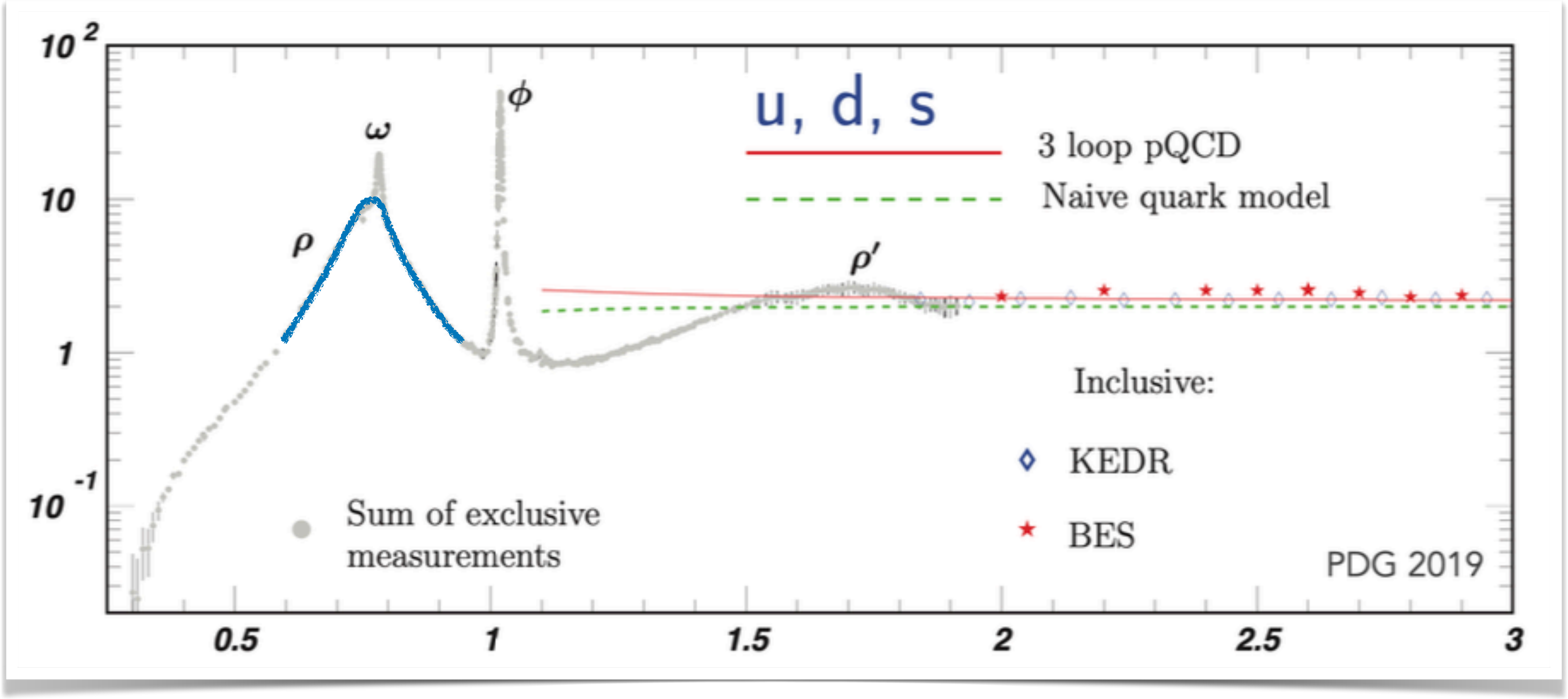
Resonance Phenomenology

- cross-section “enhancements”
- multi-hadron states
- process dependent

eg, Breit-Wigner

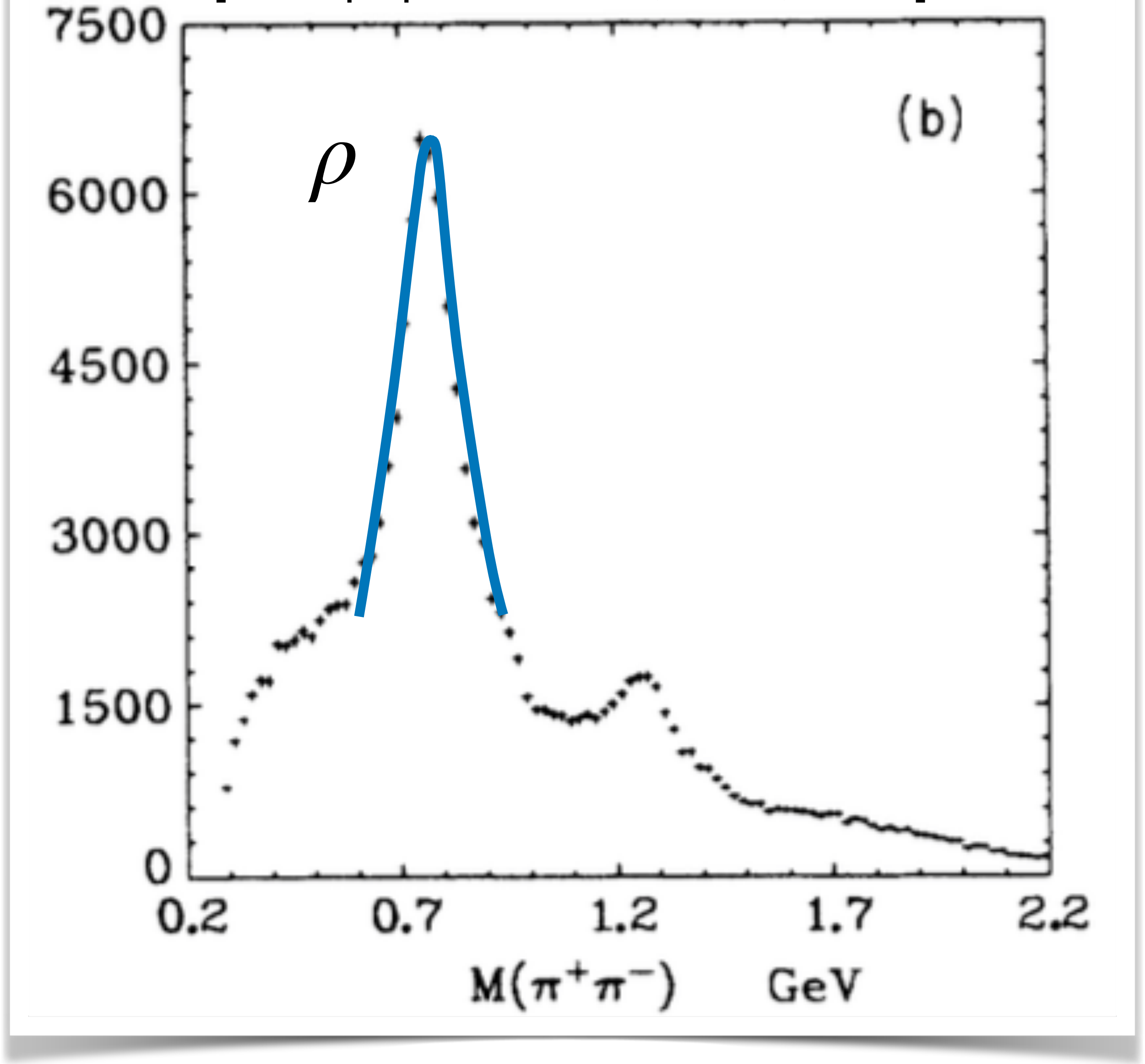
$$\sigma \propto \frac{1}{(s - m_{bw}^2)^2 + \Gamma_{bw}^2 m_{bw}^2}$$

$e^+e^- \to \pi\pi \; (\to \rho \to \pi\pi)$

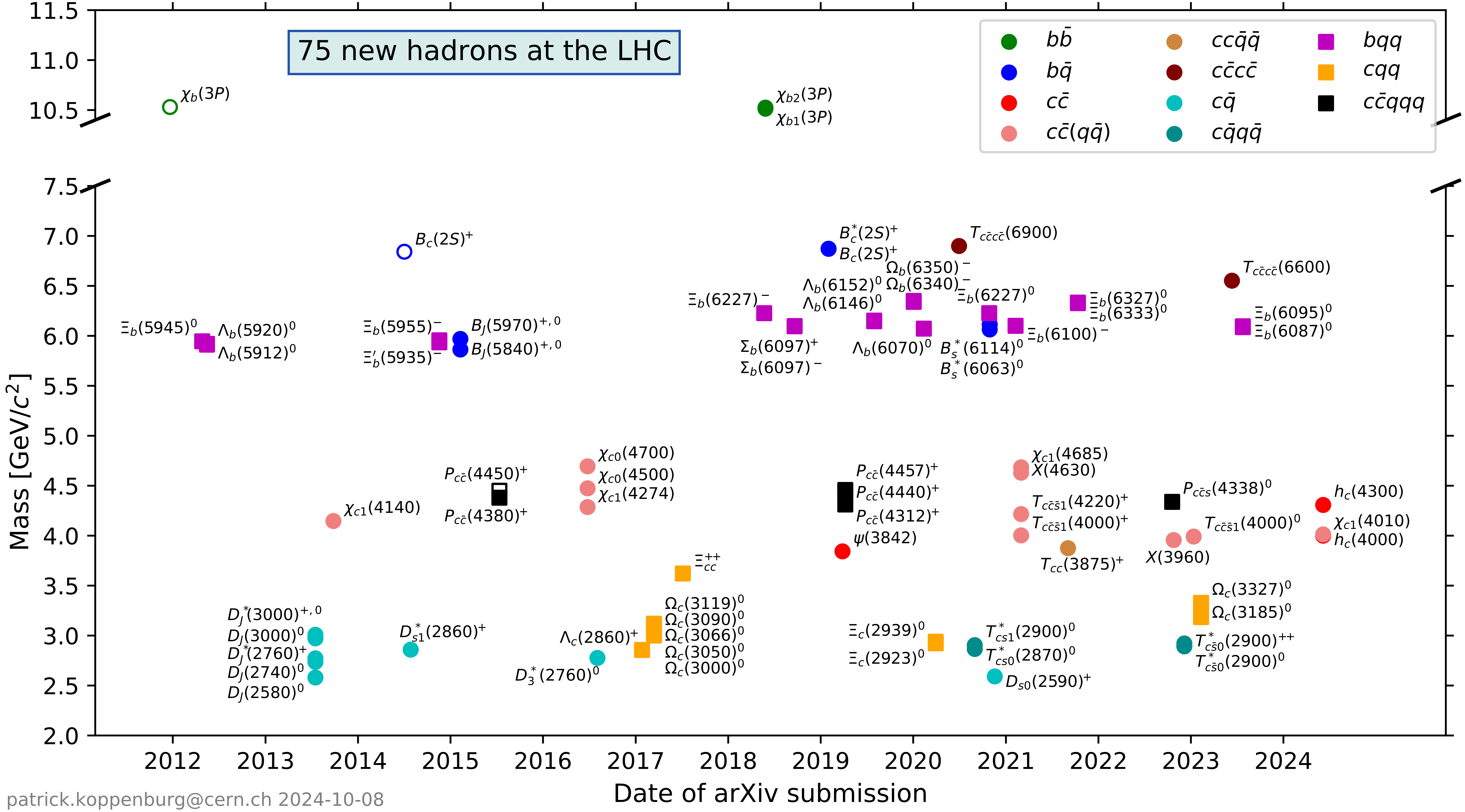


$\pi p \to \Delta\pi\pi \; (\to \rho \to \pi\pi)$

[Protopopescu et al, PRD, 1973]



Resonance Spectroscopy



(Beyond) Standard Model

Experiments

(Beyond) Standard Model

LFUV

$B \to K^* \ell^+ \ell^-$

$B \to \rho \ell \nu$

LHCb [Aaij et al, JHEP, 2016] [Aaij et al, Nature, 2022] [Aaij et al, PRD,2023]

⋮

Belle II [Abudinén, 2206.05946v4, 2020] [Bernlochner, PRD, 2014]

⋮

Experiments

⋮

(Beyond) Standard Model

LFUV

$B \to K^* \ell^+ \ell^-$

LHCb [Aaij et al, JHEP, 2016] [Aaij et al, Nature, 2022] [Aaij et al, PRD,2023]

$\vdots$

$B \to \rho \ell \nu$

Belle II [Abudinén, 2206.05946v4, 2020] [Bernlochner, PRD, 2014]

$\vdots$

CP violation in strange, charm

$K \to \pi\pi \quad (\sigma?)$

LHCb [Aaij et al, PRL, 2019]

$\vdots$

$D \to \pi\pi, K\bar{K} \quad (f_0(1710)?)$

BaBar [Aubert et al, PRL, 2008]

Experiments

$\vdots$

(Beyond) Standard Model

LFUV

$B \to K^* \ell^+ \ell^-$

$B \to \rho \ell \nu$

LHCb [Aaij et al, JHEP, 2016] [Aaij et al, Nature, 2022] [Aaij et al, PRD,2023]

⋮

Belle II [Abudinén, 2206.05946v4, 2020] [Bernlochner, PRD, 2014]

⋮

CP violation in strange, charm

$K \to \pi\pi \quad (\sigma?)$

$D \to \pi\pi, K\bar{K} \quad (f_0(1710)?)$

LHCb [Aaij et al, PRL, 2019]

⋮

BaBar [Aubert et al, PRL, 2008]

Muon $g - 2$

$e^+e^- \to \rho \to \pi\pi$

Muon g-2 [Aguillard et al, 2506.03069, 2025]

Experiments

⋮

(Beyond) Standard Model

LFUV

$B \to K^* \ell^+ \ell^-$

$B \to \rho \ell \nu$

LHCb [Aaij et al, JHEP, 2016] [Aaij et al, Nature, 2022] [Aaij et al, PRD,2023]

⋮

Belle II [Abudinén, 2206.05946v4, 2020] [Bernlochner, PRD, 2014]

⋮

Experiments

⋮

CP violation in strange, charm

$K \to \pi\pi \quad (\sigma?)$

$D \to \pi\pi, K\bar{K} \quad (f_0(1710)?)$

LHCb [Aaij et al, PRL, 2019]

⋮

BaBar [Aubert et al, PRL, 2008]

Muon $g-2$

$e^+e^- \to \rho \to \pi\pi$

Muon g-2 [Aguillard et al, 2506.03069, 2025]

nonperturbative understanding from QCD

[Gurbernari et al, PRL, 2019]

[Schacht & Soni, PRB, 2022]

⋮

→Precision & reliability

(Beyond) Standard Model

LFUV

$B \to K^* \ell^+ \ell^-$

$B \to \rho \ell \nu$

LHCb [Aaij et al, JHEP, 2016] [Aaij et al, Nature, 2022] [Aaij et al, PRD,2023]

⋮

Belle II [Abudinén, 2206.05946v4, 2020] [Bernlochner, PRD, 2014]

⋮

Experiments

⋮

CP violation in strange, charm

$K \to \pi\pi \quad (\sigma?)$

$D \to \pi\pi, K\bar{K} \quad (f_0(1710)?)$

LHCb [Aaij et al, PRL, 2019]

⋮

BaBar [Aubert et al, PRL, 2008]

Muon $g-2$

$e^+e^- \to \rho \to \pi\pi$

Muon g-2 [Aguillard et al, 2506.03069, 2025]

nonperturbative understanding from QCD

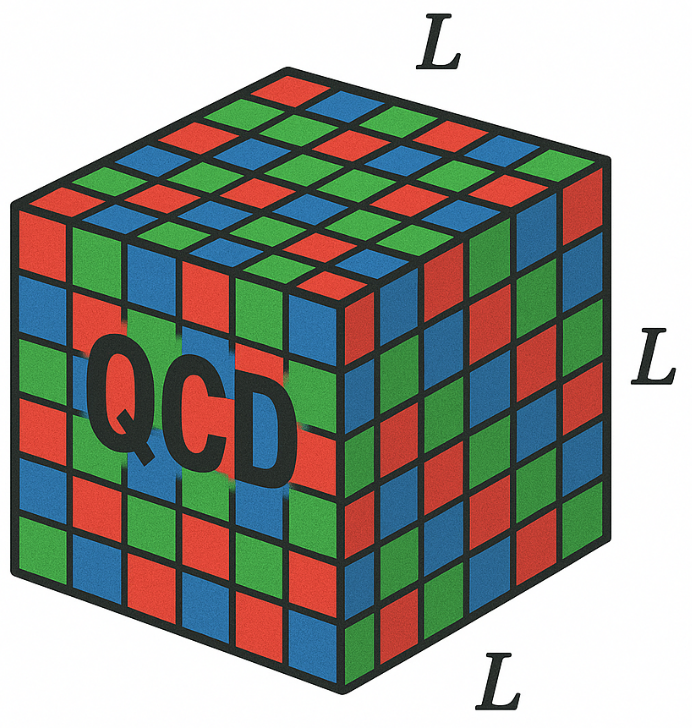
[Gurbernari et al, PRL, 2019]

[Schacht & Soni, PRB, 2022]

⋮

→Precision & reliability

[g-2 Theory Whitepaper, 2025]



$\pi\pi \to \rho \to \pi\pi$ and $K\pi \to K^* \to K\pi$ from lattice QCD

$\pi\pi \to \rho \to \pi\pi$ and $K\pi \to K^* \to K\pi$ from lattice QCD

Scattering is expensive: $m_\pi > 139\ \mathrm{MeV}$ (besides other reasons)

[Fischer et al - PLB, 2021][Rendon et al - PRD, 2021] [Wilson et al - PRL, 2019] [Bali et al - PRD, 2016] [Bret et al - Nuc.Phys.B, 2018] [Aoki et al - PRD, 2011] [Feng et al - PRD, 2011] [Lang et al - PRD, 2011] [Pelissier et al - PRD, 2013] [Dudek et al - PRD, 2013] [Bulava et al - Nuc.Phys.B, 2016] [Fu et al - PRD, 2016] [Andersen et al - Nuc.Phys.B, 2019] [Erben et al - PRD, 2020] [Alexandrou et al - PRD, 2017] ...

$\pi\pi \to \rho \to \pi\pi$ and $K\pi \to K^* \to K\pi$ from lattice QCD

Scattering is expensive: $m_\pi > 139\ \mathrm{MeV}$ (besides other reasons)

[Fischer et al - PLB, 2021][Rendon et al - PRD, 2021] [Wilson et al - PRL, 2019] [Bali et al - PRD, 2016] [Bret et al - Nuc.Phys.B, 2018] [Aoki et al - PRD, 2011] [Feng et al - PRD, 2011] [Lang et al - PRD, 2011] [Pelissier et al - PRD, 2013] [Dudek et al - PRD, 2013] [Bulava et al - Nuc.Phys.B, 2016] [Fu et al - PRD, 2016] [Andersen et al - Nuc.Phys.B, 2019] [Erben et al - PRD, 2020] [Alexandrou et al - PRD, 2017] ...

Non-trivial m_π dependence of resonance poles: QCD dynamics

- Extrapolations $m_\pi \to m_\pi^{phys}$: one more systematic

$\pi\pi \to \rho \to \pi\pi$ and $K\pi \to K^* \to K\pi$ from lattice QCD

Scattering is expensive: $m_\pi > 139\ \mathrm{MeV}$ (besides other reasons)

[Fischer et al - PLB, 2021][Rendon et al - PRD, 2021] [Wilson et al - PRL, 2019] [Bali et al - PRD, 2016] [Bret et al - Nuc.Phys.B, 2018] [Aoki et al - PRD, 2011] [Feng et al - PRD, 2011] [Lang et al - PRD, 2011] [Pelissier et al - PRD, 2013] [Dudek et al - PRD, 2013] [Bulava et al - Nuc.Phys.B, 2016] [Fu et al - PRD, 2016] [Andersen et al - Nuc.Phys.B, 2019] [Erben et al - PRD, 2020] [Alexandrou et al - PRD, 2017] ...

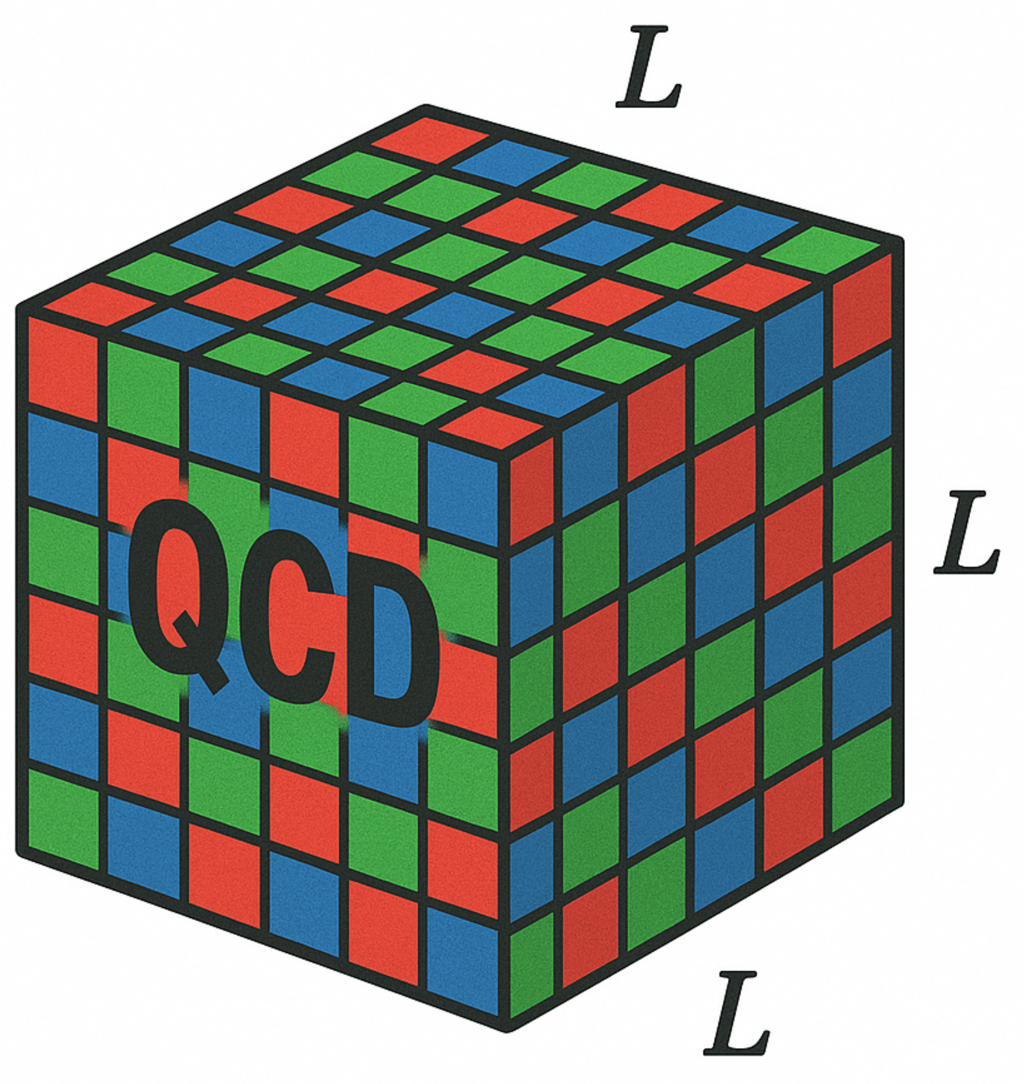
Non-trivial m_π dependence of resonance poles: QCD dynamics

- Extrapolations $m_\pi \to m_\pi^{phys}$: one more systematic

Example: $\rho \to \pi\pi$ contributions to muon $g-2$ at $\approx m_\pi^{phys}$

[g-2 Theory Whitepaper, 2025]

"Standing on the shoulders of giants"



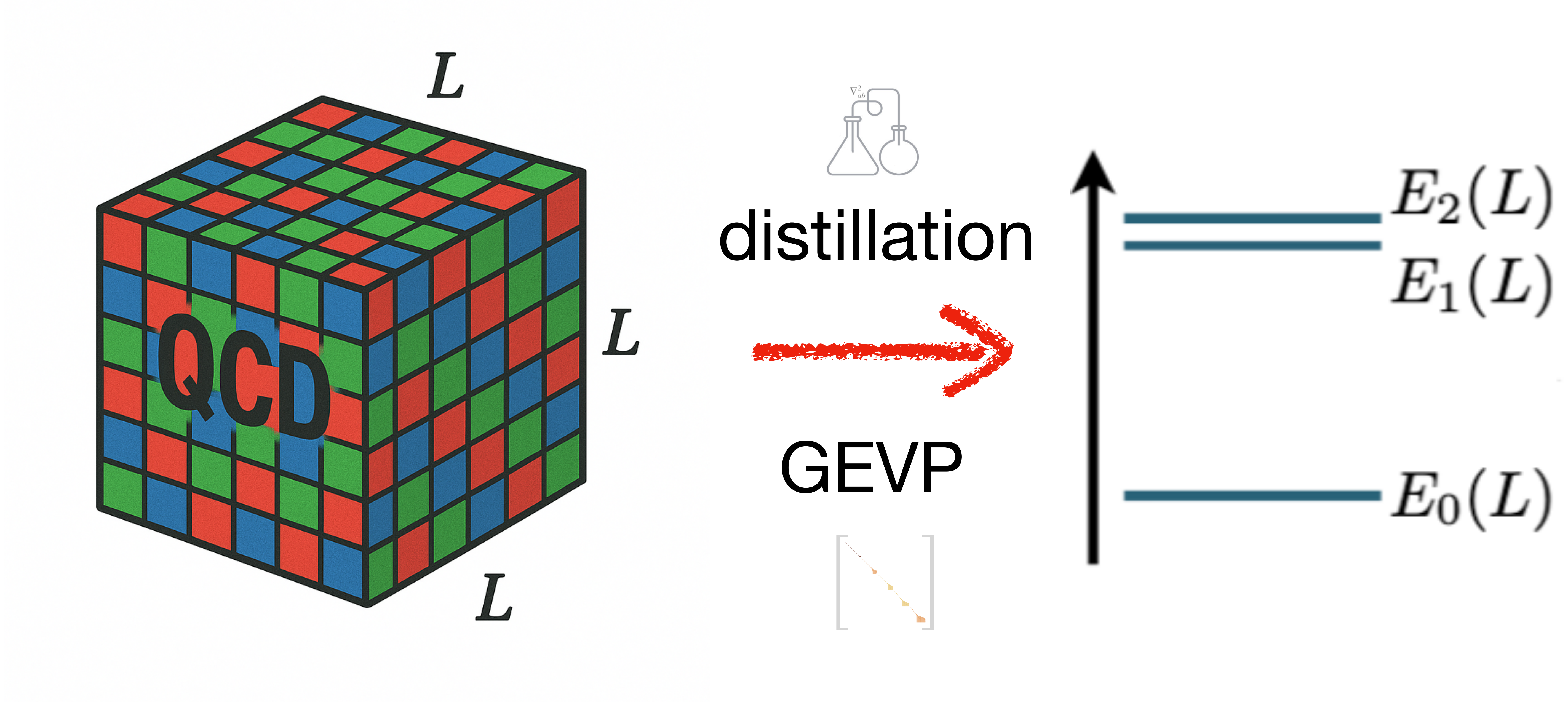
DWF $N_f = 2+1 \begin{cases} m_u = m_d \\ m_s \end{cases}$

RBC-UKQCD

volume	$48^3 \times 96$
a	≈ 0.114 fm
L	≈ 5.5 fm
$m_\pi L$	≈ 3.8
$\boldsymbol{m_\pi}$ m_K	$\approx$ **139** MeV ≈ 499 MeV

[Blum et al, PRD, 2016]

"Standing on the shoulders of giants"



RBC-UKQCD

DWF $N_f = 2+1 \begin{cases} m_u = m_d \\ m_s \end{cases}$

volume	$48^3 \times 96$
a	≈ 0.114 fm
L	≈ 5.5 fm
$m_\pi L$	≈ 3.8
$\boldsymbol{m_\pi}$	$\approx$ **139** MeV
m_K	≈ 499 MeV

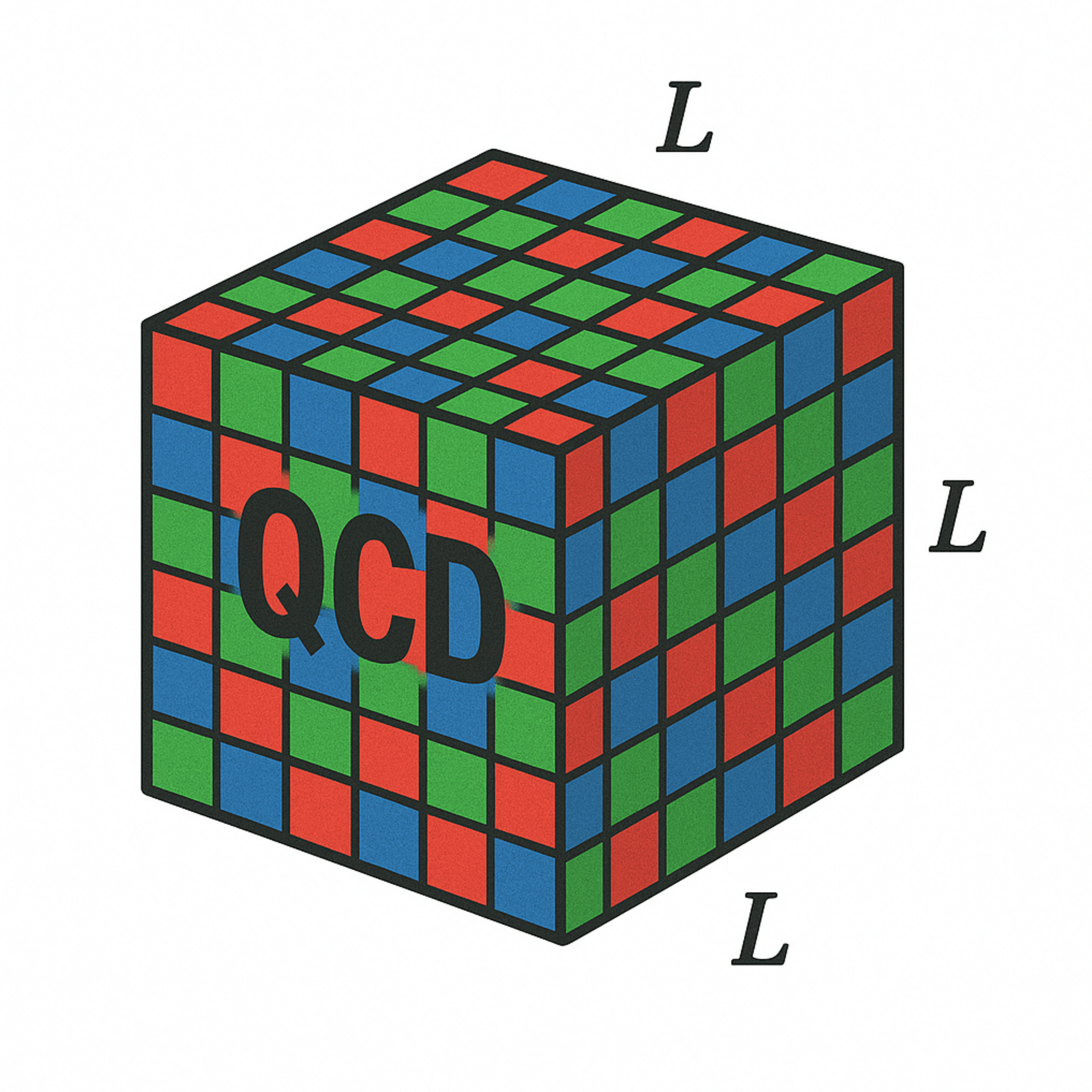
[Blum et al, PRD, 2016]

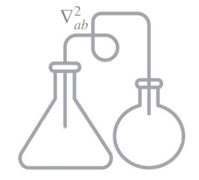
bilinear $O_{\bar{q}\gamma_i q}(\mathbf{P}) \begin{cases} q = s, d \\ q = u, d \end{cases}$ + two-bilinear $\begin{cases} O_{K\pi}^{I=1/2}(\mathbf{p}_1, \mathbf{p}_2) \\ O_{\pi\pi}^{I=1}(\mathbf{p}_1, \mathbf{p}_2) \end{cases}$

	$\Lambda[\mathbf{d}_{\text{ref}}]$	
Group	$K\pi^{I=1/2}$ and $\pi\pi^{I=1}$	$\pi\pi^{I=1}$ only
O_h	$T_{1u}[000]$	-
C_{4v}	$E[001]$, $E[002]$	$A_1[001]$, $A_1[002]$
C_{2v}	$B_1[110]$	$A_1[110]$
	$B_2[110]$	
C_{3v}	$E[111]$	$A_1[111]$

$(\mathbf{d}_{\text{ref}} =\equiv \frac{L}{2\pi}\mathbf{P}_{\text{ref}})$

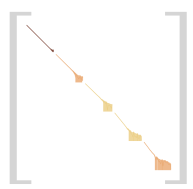
"Standing on the shoulders of giants"

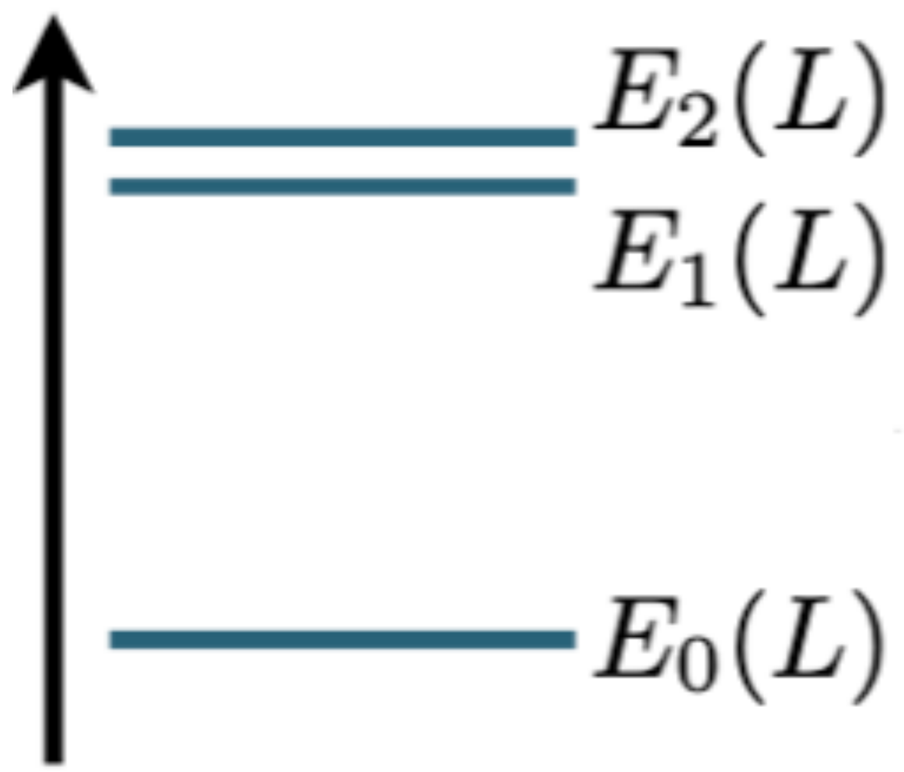




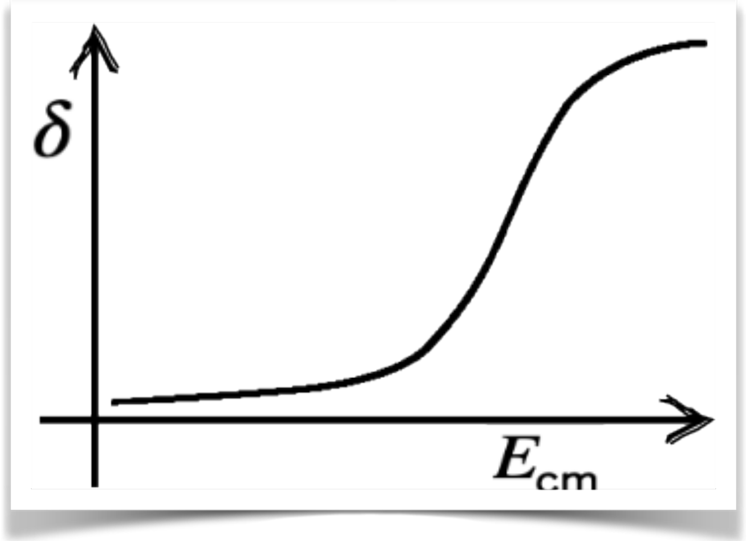
distillation

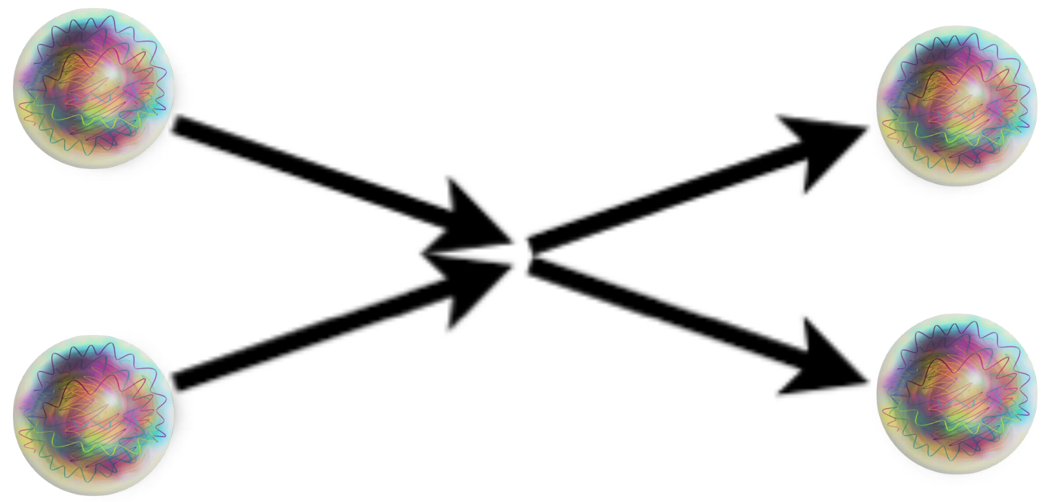
GEVP

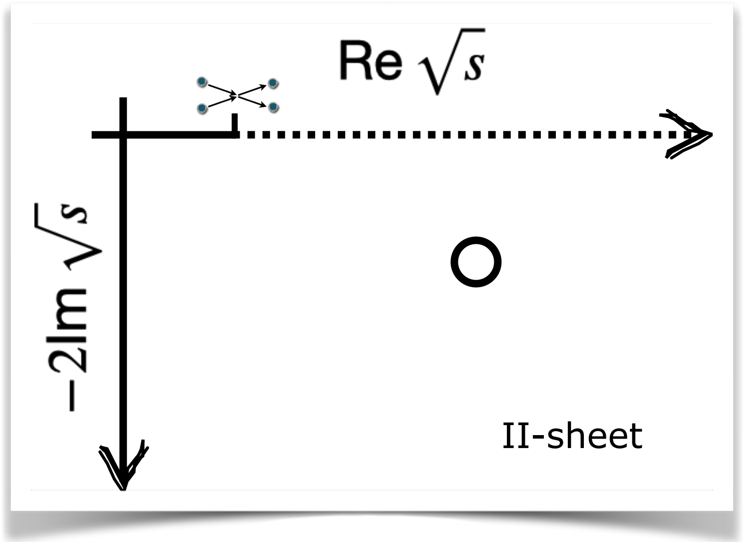




QC







DWF $N_f = 2+1 \begin{cases} m_u = m_d \\ m_s \end{cases}$

RBC-UKQCD

volume	$48^3 \times 96$
a	≈ 0.114 fm
L	≈ 5.5 fm
$m_\pi L$	≈ 3.8
$\boldsymbol{m_\pi}$	$\approx$ **139 MeV**
m_K	≈ 499 MeV

[Blum et al, PRD, 2016]

bilinear $O_{\bar{q}\gamma_i q}(\mathbf{P}) \begin{cases} q = s, d \\ q = u, d \end{cases}$ + two-bilinear $\begin{cases} O^{I=1/2}_{K\pi}(\mathbf{p}_1, \mathbf{p}_2) \\ O^{I=1}_{\pi\pi}(\mathbf{p}_1, \mathbf{p}_2) \end{cases}$

	$\Lambda[\mathbf{d}_{\mathrm{ref}}]$	
Group	$K\pi^{I=1/2}$ and $\pi\pi^{I=1}$	$\pi\pi^{I=1}$ only
O_h	$T_{1u}[000]$	-
C_{4v}	$E[001], E[002]$	$A_1[001], A_1[002]$
C_{2v}	$B_1[110]$ $B_2[110]$	$A_1[110]$
C_{3v}	$E[111]$	$A_1[111]$

$(\mathbf{d}_{\mathrm{ref}} =\equiv \frac{L}{2\pi}\mathbf{P}_{\mathrm{ref}})$

Physical-mass ρ and K^*

Main decay products ($J = \ell = 1$)

$$K^*(892) \to K\pi, K\gamma, K\pi\pi, \ldots$$

$\lesssim 0.1\,\%$

$$\rho(770) \to \pi\pi, \pi\gamma, 4\pi, \ldots$$

[PDG, 2024]

PHYSICAL REVIEW LETTERS **134,** 111901 (2025)

Light and Strange Vector Resonances from Lattice QCD at Physical Quark Masses

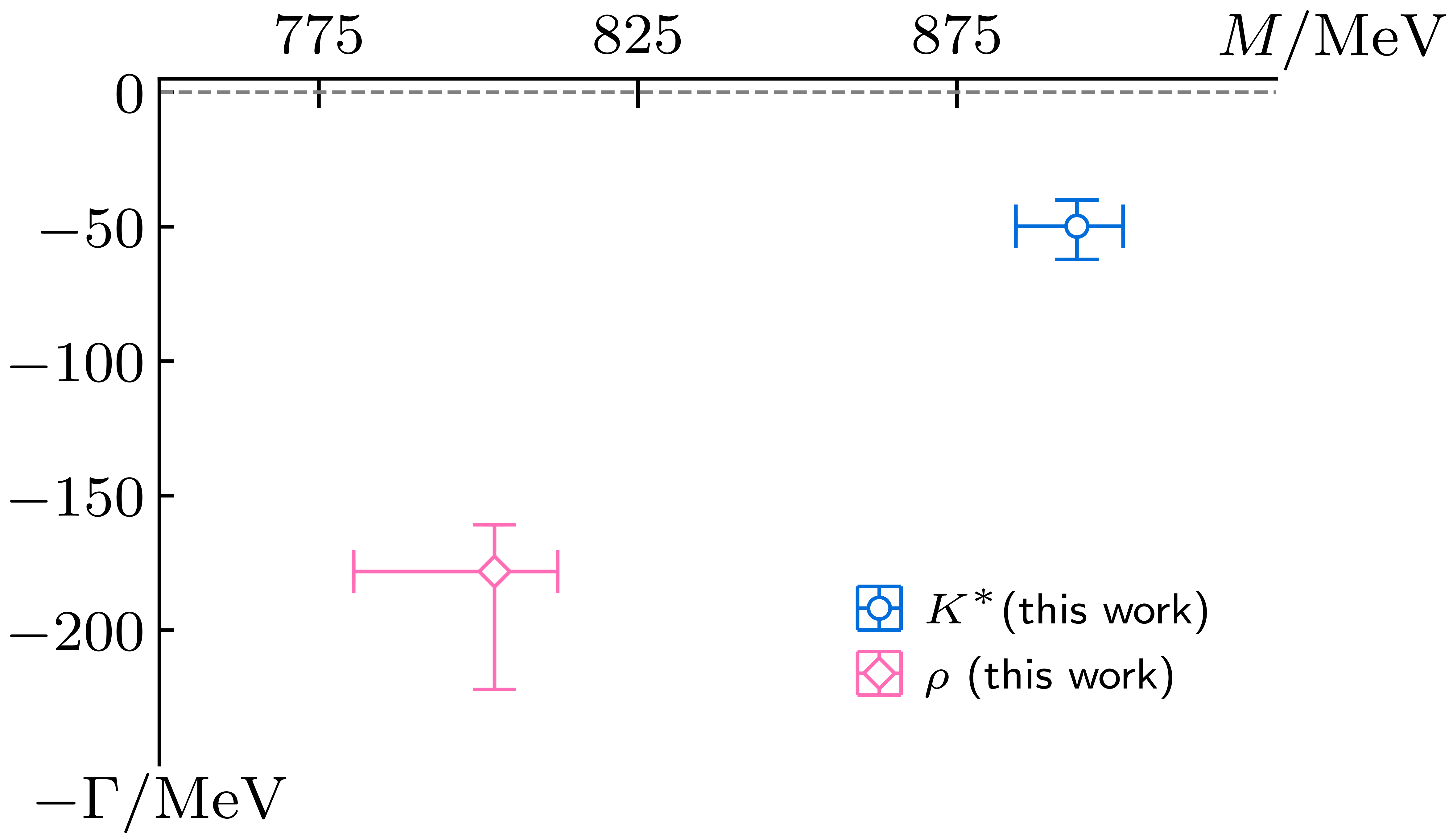
Peter Boyle,[1,2] Felix Erben,[3,2] Vera Gülpers,[2] Maxwell T. Hansen,[2] Fabian Joswig,[2] Michael Marshall,[2]
Nelson Pitanga Lachini,[4,2,*] and Antonin Portelli[2,3,5]

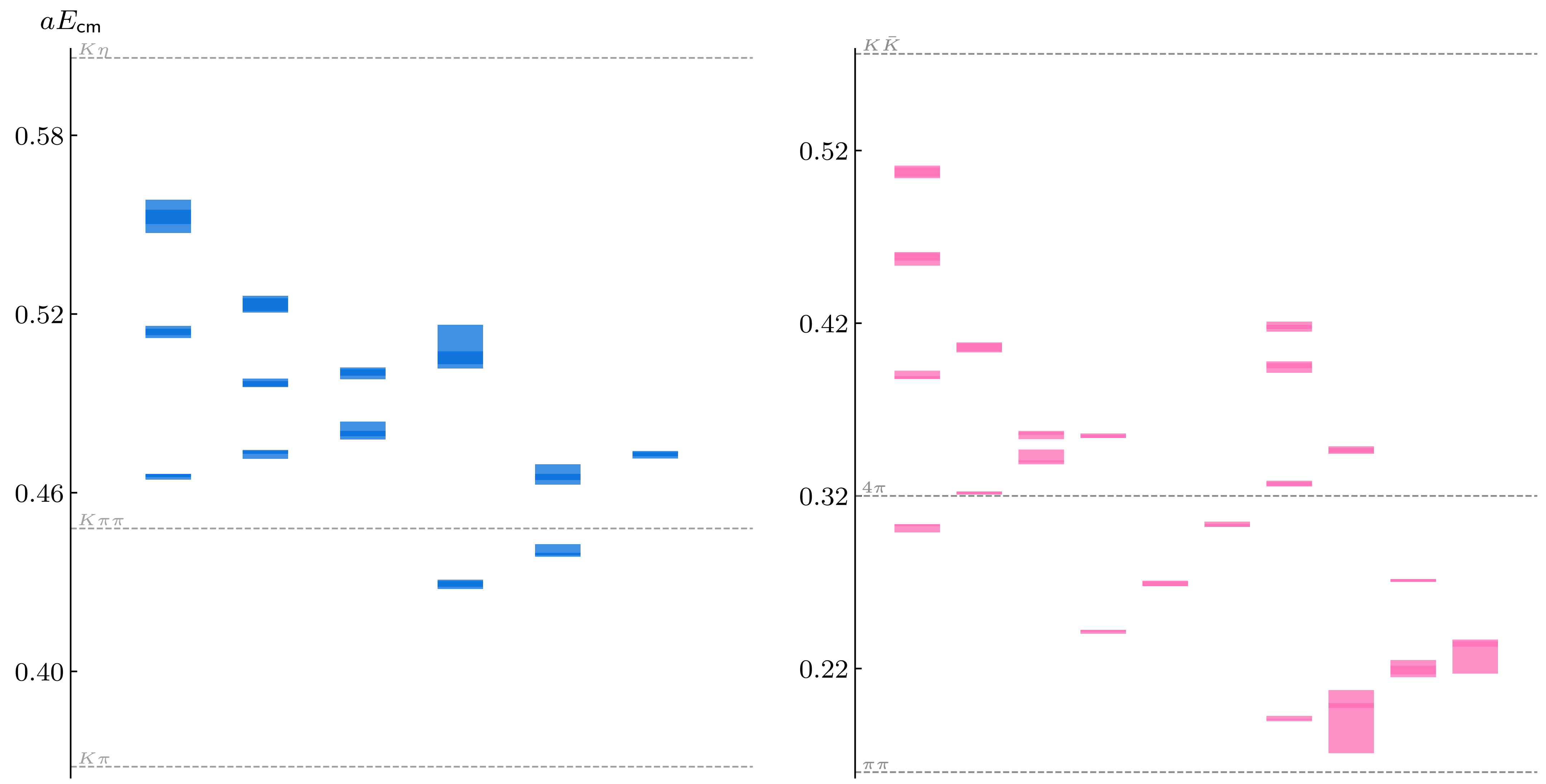
PHYSICAL REVIEW D **111,** 054510 (2025)

Physical-mass calculation of $\rho(770)$ and $K^*(892)$ resonance parameters via $\pi\pi$ and $K\pi$ scattering amplitudes from lattice QCD

Peter Boyle,[1,2] Felix Erben,[3,2] Vera Gülpers,[2] Maxwell T. Hansen,[2] Fabian Joswig,[2]
Michael Marshall,[2] Nelson Pitanga Lachini,[4,2,*] and Antonin Portelli[2,3,5]

[NPL et al, 2025]





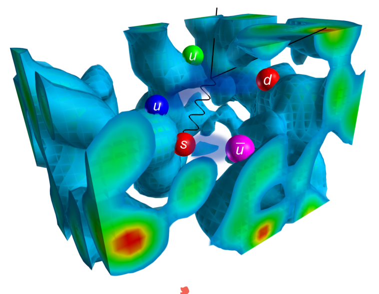
Implementation

Implementation

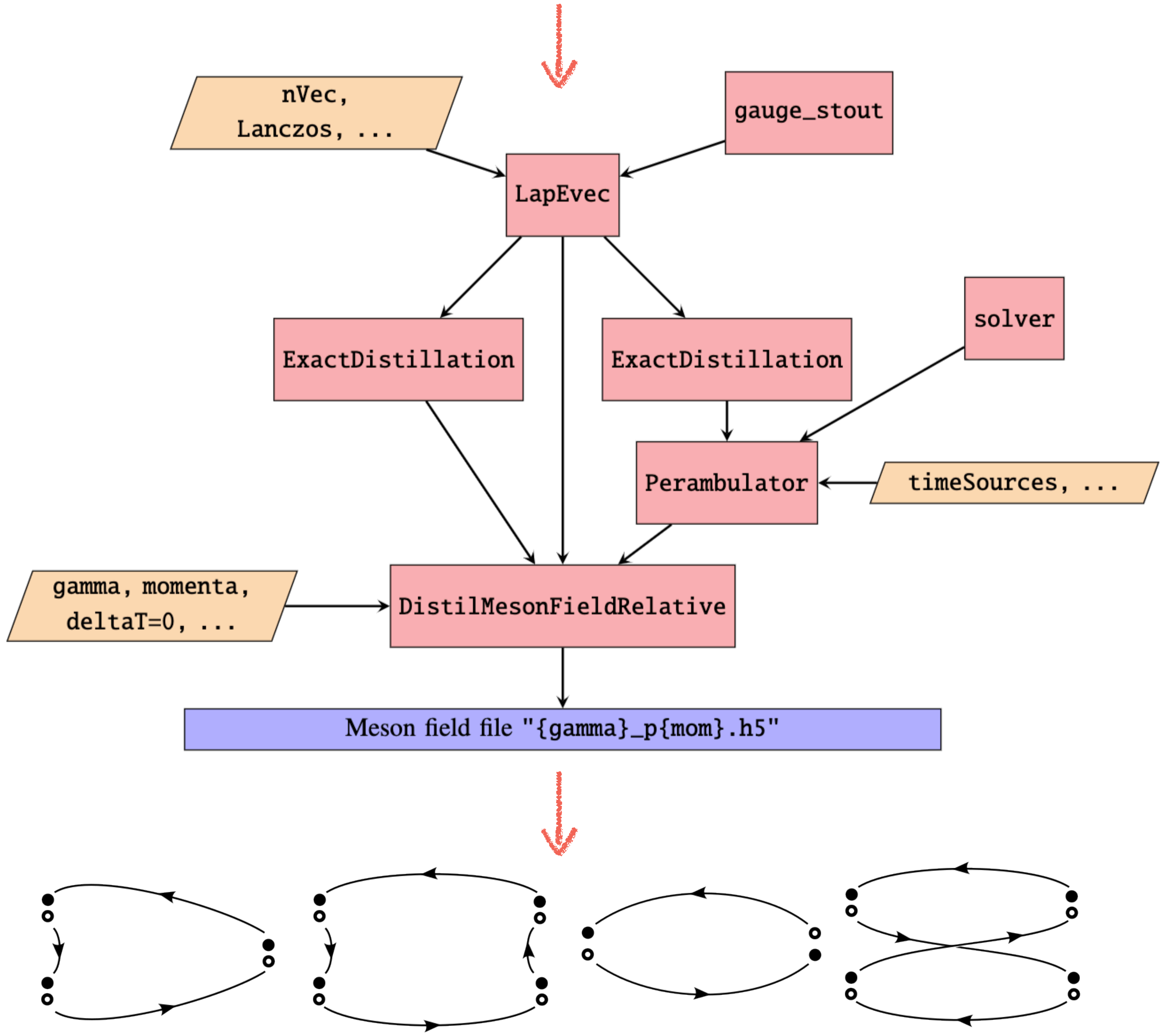
Open-source and free software

- *Grid*: data parallel C++ lattice library
- *Hadrons*: workflow management for lattice simulations

Implementation



workflow example



Open-source and free software

- *Grid*: data parallel C++ lattice library
- *Hadrons*: workflow management for lattice simulations

Distillation within *Grid* and *Hadrons*

- agnostic to action
- stochastic/diluted sources

Implementation

Open-source and free software

- *Grid*: data parallel C++ lattice library
- *Hadrons*: workflow management for lattice simulations

Distillation within *Grid* and *Hadrons*

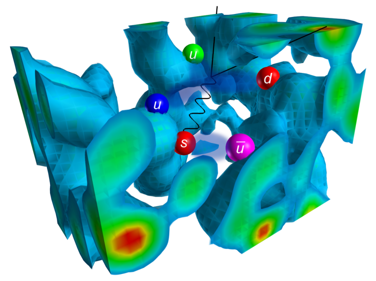
- agnostic to action
- stochastic/diluted sources

Running

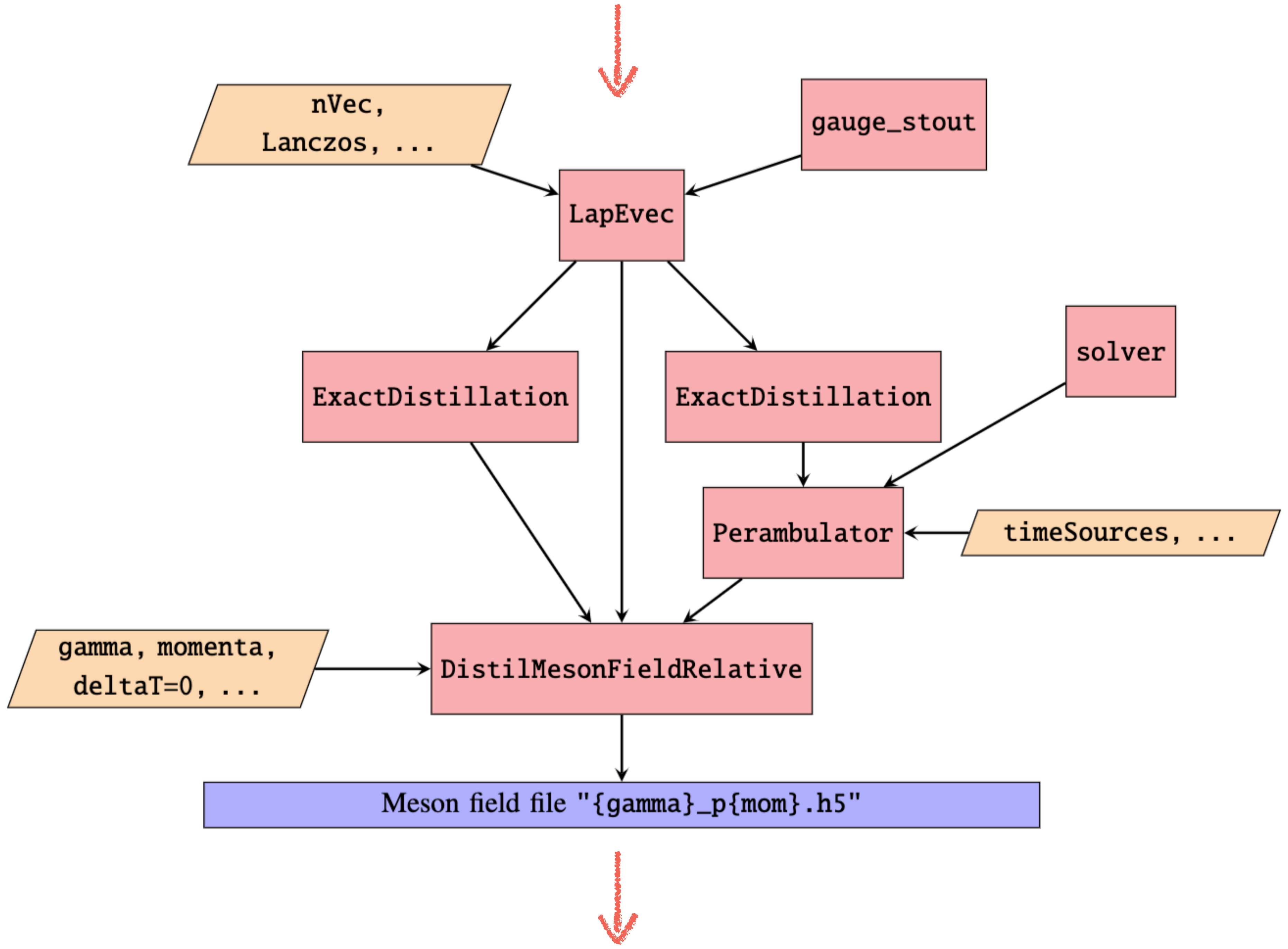
[dirac.ac.uk/extreme-scaling-edinburgh]

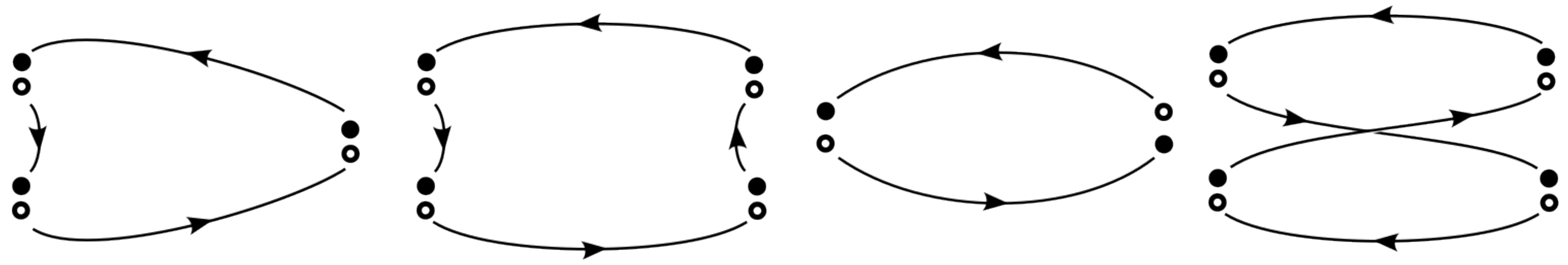
- 2 DiRAC machines, same high-level code
- 'raw' correlators publicly shared

[repository.cern/records/vy9x7-bzn92]



workflow example

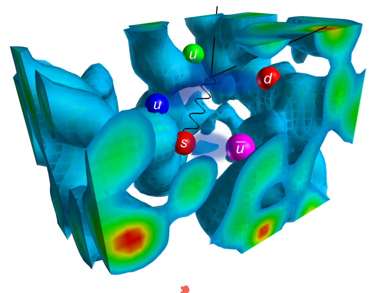




Tesseract (CPU)

Tursa (GPU)

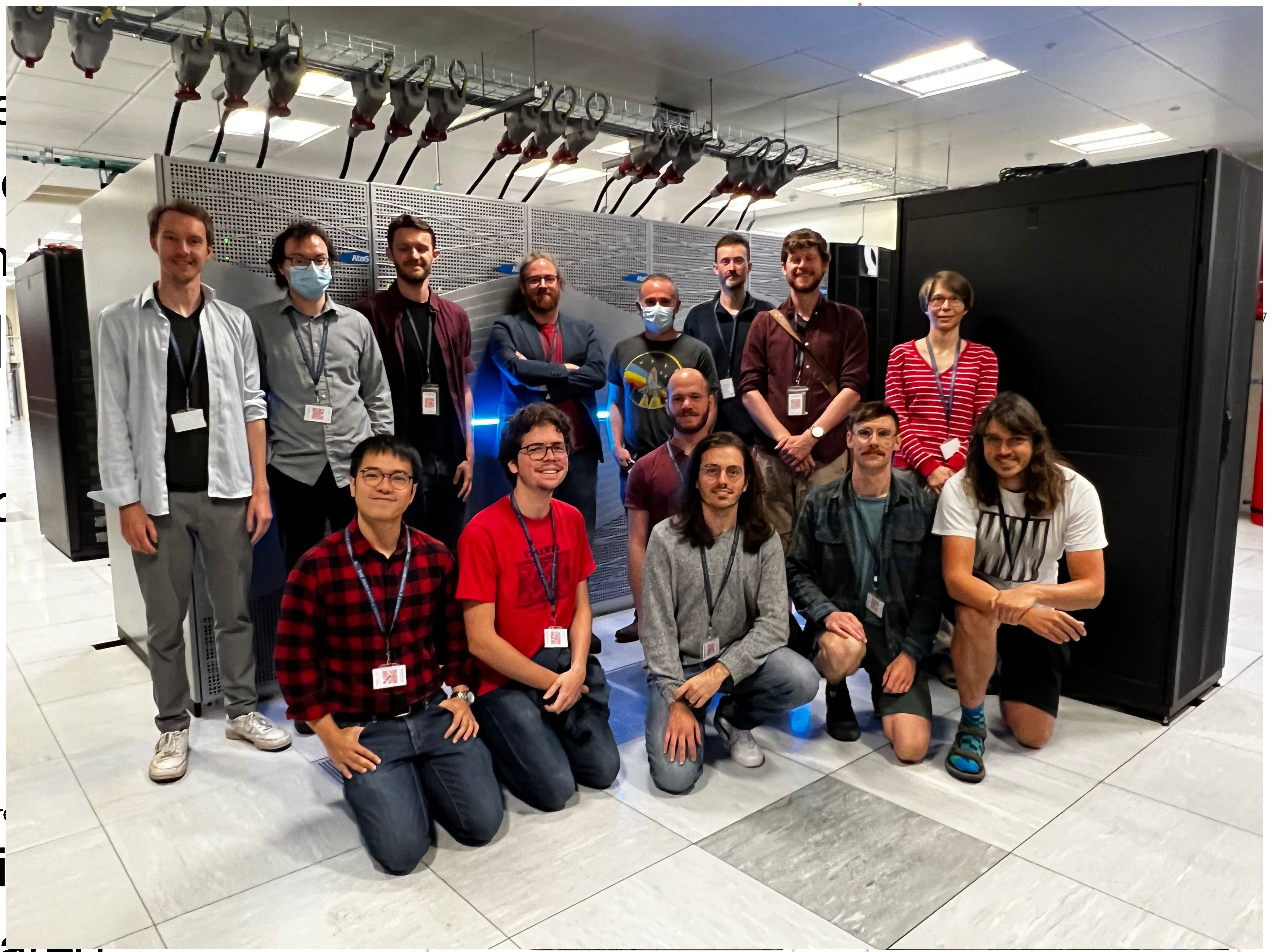
Implementation



workflow example

Open-source and free software

- *Grid*: data parallel C++ lattic
- *Hadrons*: workflow managen lattice simulations Had



Distillation within *Grid* and *Hac*

- agnostic to action
- stochastic/diluted sources

Running

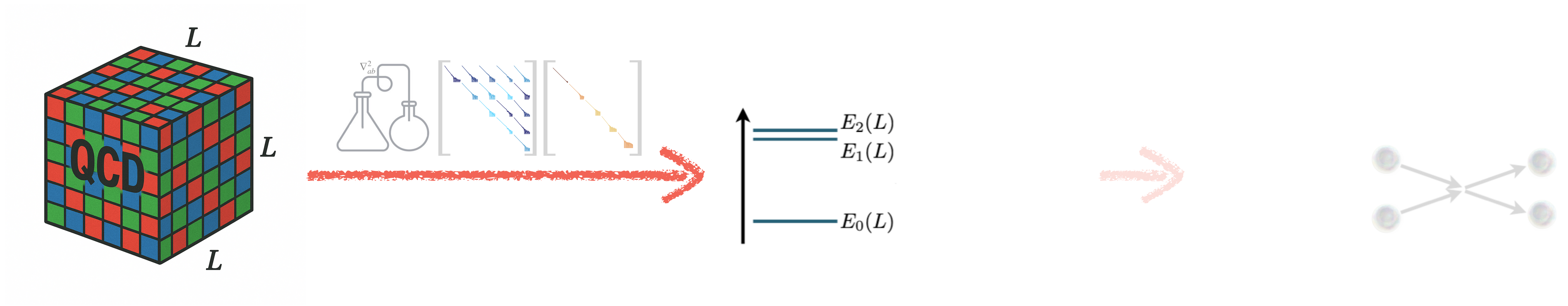
[dirac.ac.uk/extr

- 2 DiRAC machines, same hi
- 'raw' correlators publicly shared

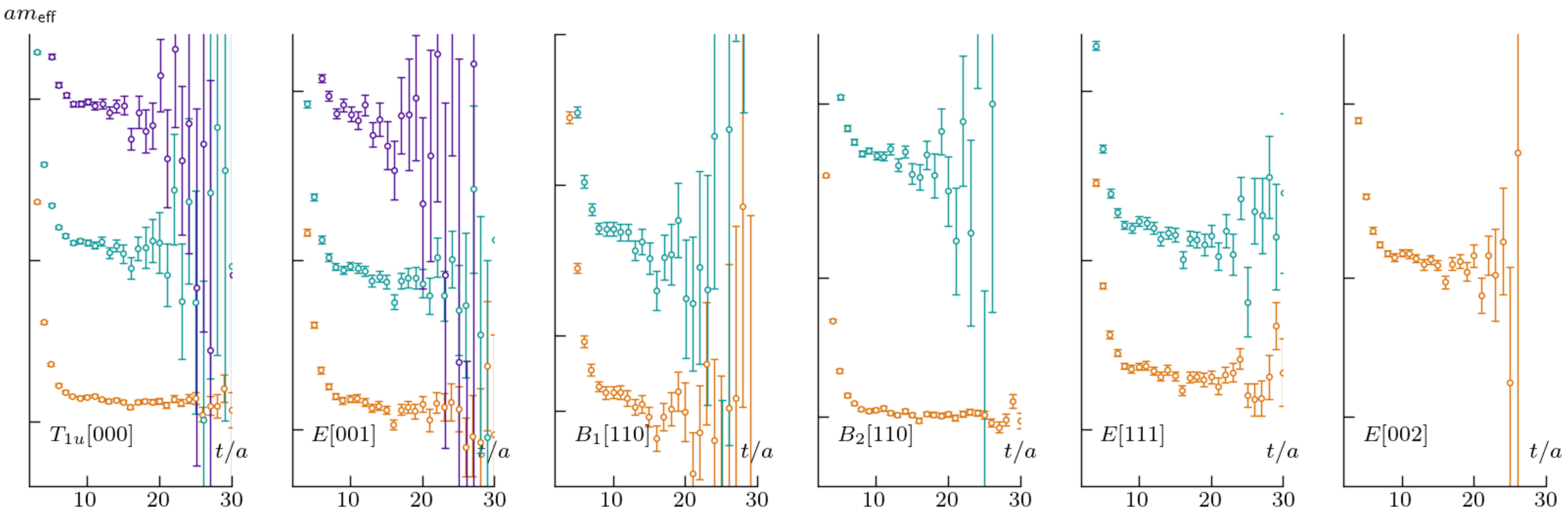
[repository.cern/records/vy9x7-bzn92]

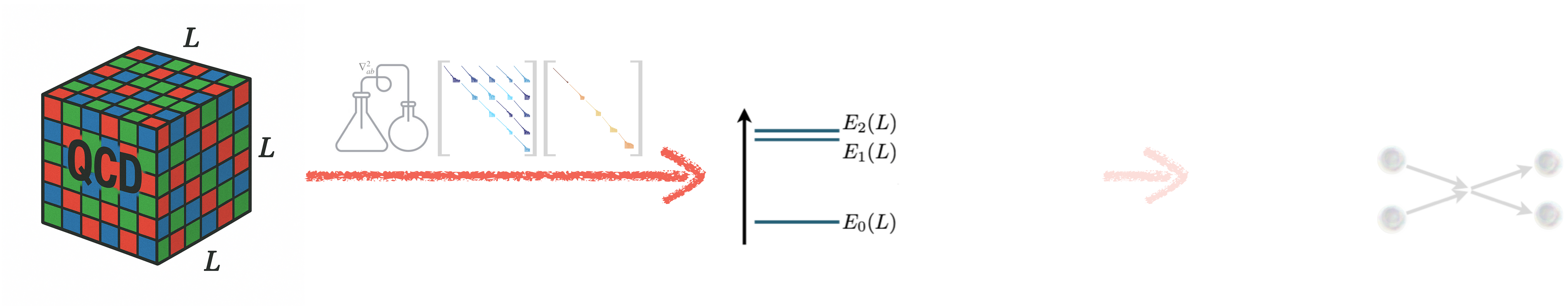
Tesseract (CPU)

Tursa (GPU)

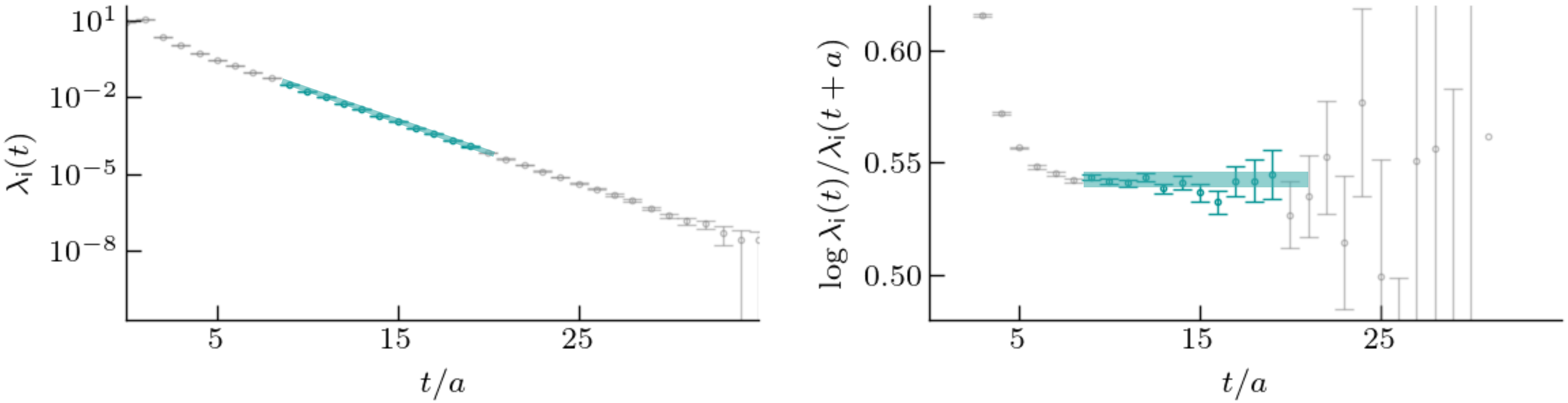

L
L
L
QCD
$E_2(L)$
$E_1(L)$
$E_0(L)$

$K\pi, I = 1/2:$

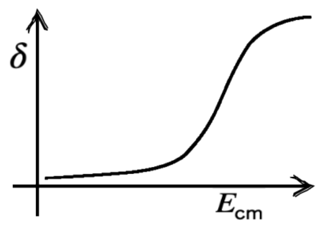

am_{eff}
$T_{1u}[000]$
$E[001]$
$B_1[110]$
$B_2[110]$
$E[111]$
$E[002]$
t/a
10
20
30


L
L
L
QCD
$E_2(L)$
$E_1(L)$
$E_0(L)$

$K\pi, I = 1/2:$


10^{1}
10^{-2}
10^{-5}
10^{-8}
$\lambda_i(t)$
5
15
25
t/a
0.60
0.55
0.50
$\log \lambda_i(t)/\lambda_i(t+a)$
5
15
25
t/a

Phase-shift model



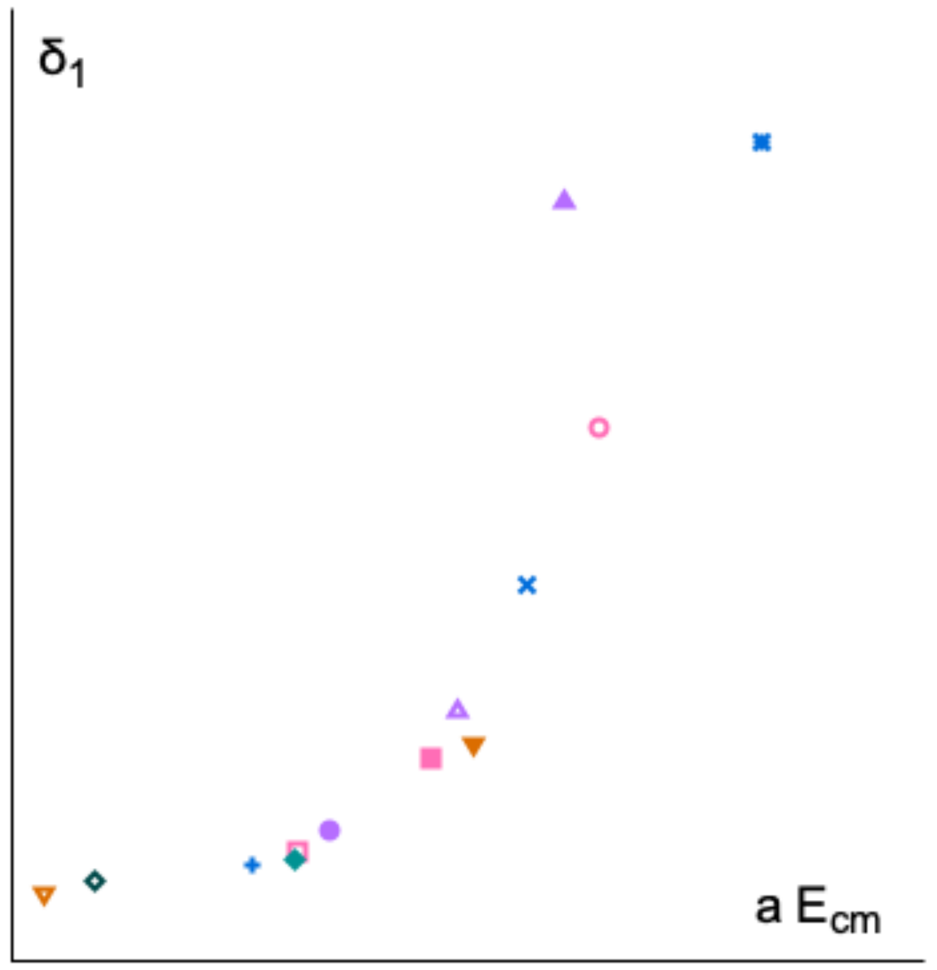
QC reminder:

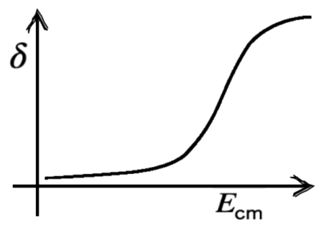
[Lüscher, 1986]
[Lüscher, 1991]

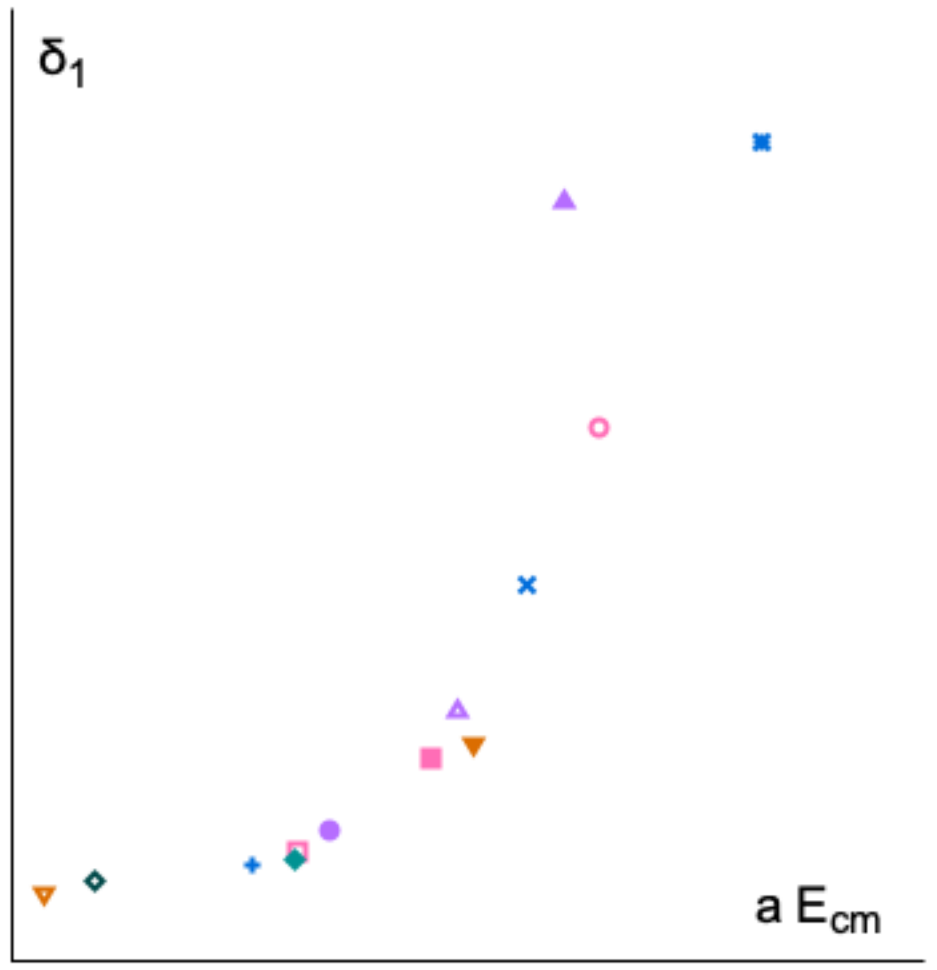
$$\delta\big(E_{\text{cm}}(L)\big) = n\pi - \phi^{\Lambda}\big(E_{\text{cm}}(L), L\big), \quad n \in \mathbb{Z}$$

$i \equiv (n, \Lambda, \text{flavour})$

Allows computation of $\delta_1(E^i_{\text{cm}})$, but **poles** inaccessible



Phase-shift model 



QC reminder:

[Lüscher, 1986]
[Lüscher, 1991]

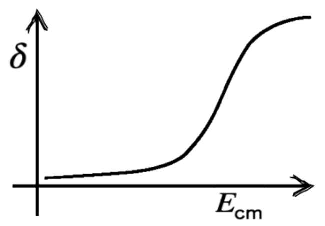
$$\delta^{\text{mod}}\big(\mathscr{E}_{\text{cm}}(L)\big) = n\pi - \phi^{\Lambda}\big(\mathscr{E}_{\text{cm}}(L), L\big), \quad n \in \mathbb{Z}$$

$i \equiv (n, \Lambda, \text{flavour})$

Allows computation of $\delta_1(E^i_{\text{cm}})$, but **poles** inaccessible

Reverse: given model δ^{mod} with parameters α^{mod}, find $\mathscr{E}^i_{\text{cm}}$

Phase-shift model



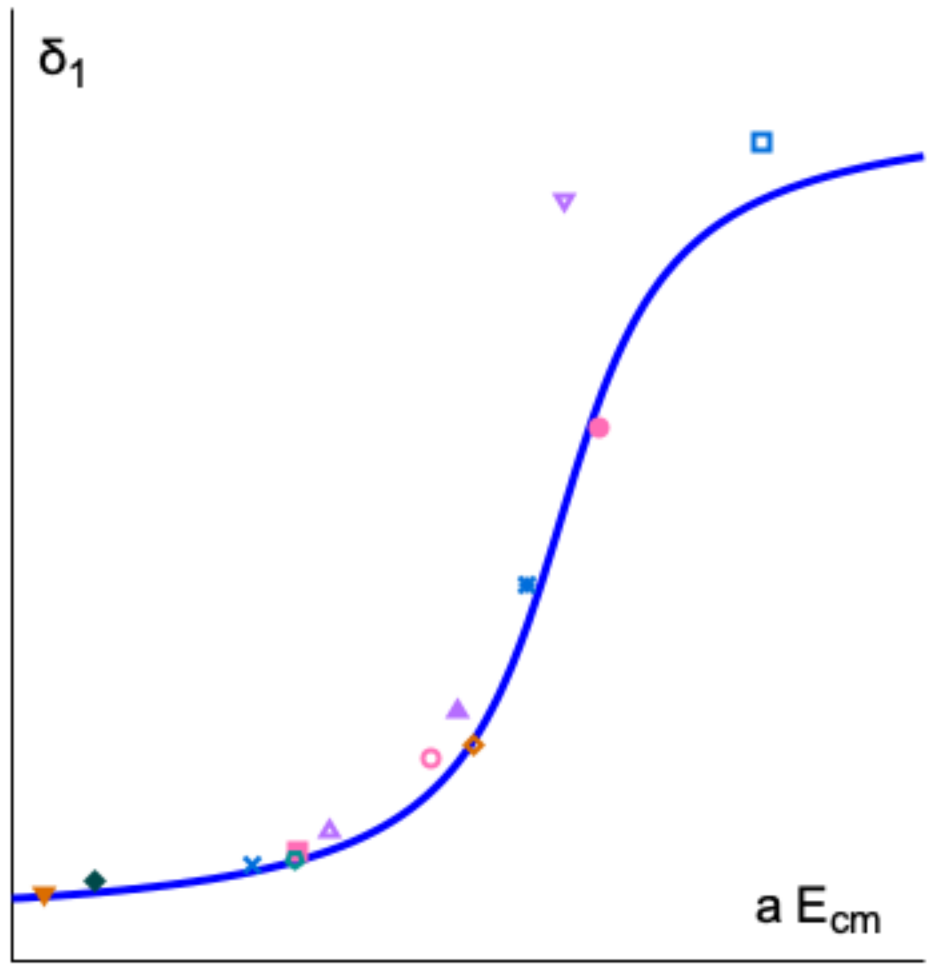
QC reminder:

[Lüscher, 1986]
[Lüscher, 1991]

$$\delta^{\text{mod}}\big(\mathscr{E}_{\text{cm}}(L)\big) = n\pi - \phi^{\Lambda}\big(\mathscr{E}_{\text{cm}}(L), L\big), \quad n \in \mathbb{Z}$$

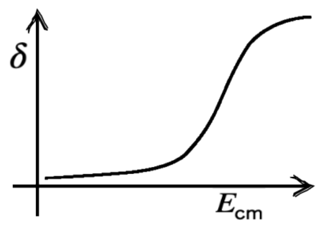
$i \equiv (n, \Lambda, \text{flavour})$

Allows computation of $\delta_1(E^i_{\text{cm}})$, but **poles** inaccessible



Reverse: given model δ^{mod} with parameters α^{mod}, find $\mathscr{E}^i_{\text{cm}}$

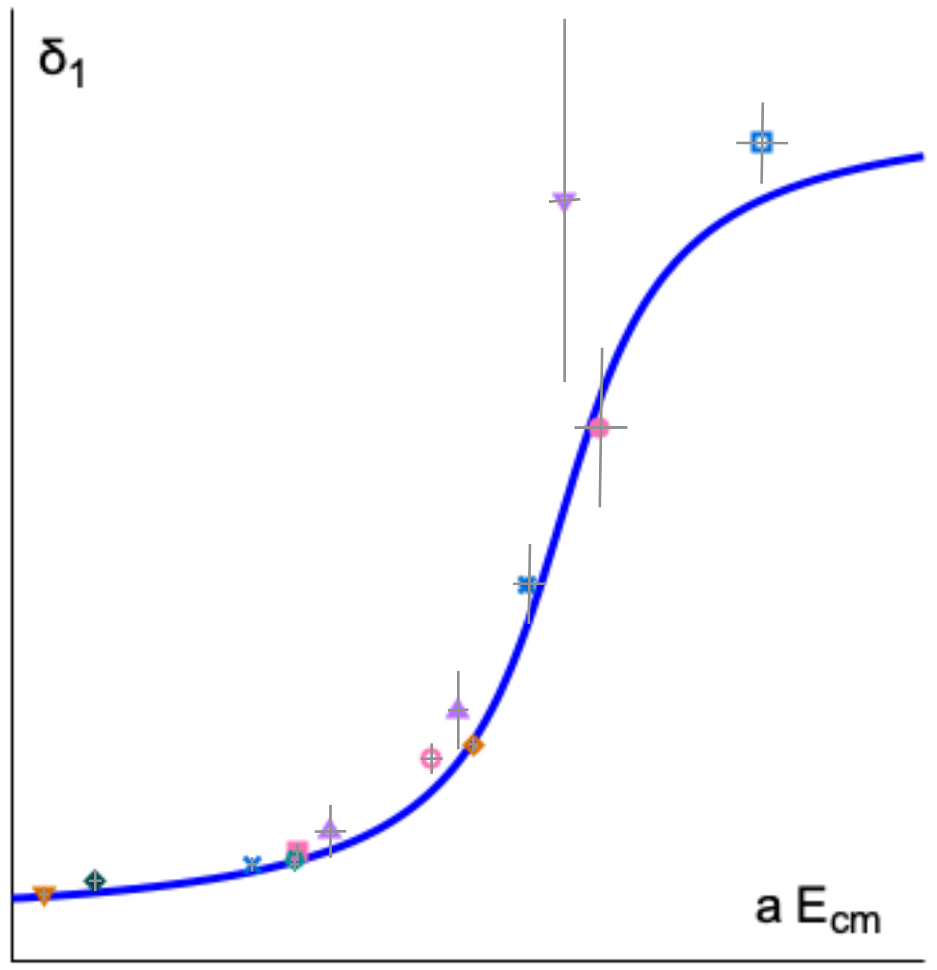
Phase-shift model



QC reminder:

[Lüscher, 1986]
[Lüscher, 1991]

$$\delta^{\text{mod}}\big(\mathscr{E}_{\text{cm}}(L)\big) = n\pi - \phi^{\Lambda}\big(\mathscr{E}_{\text{cm}}(L), L\big), \quad n \in \mathbb{Z}$$



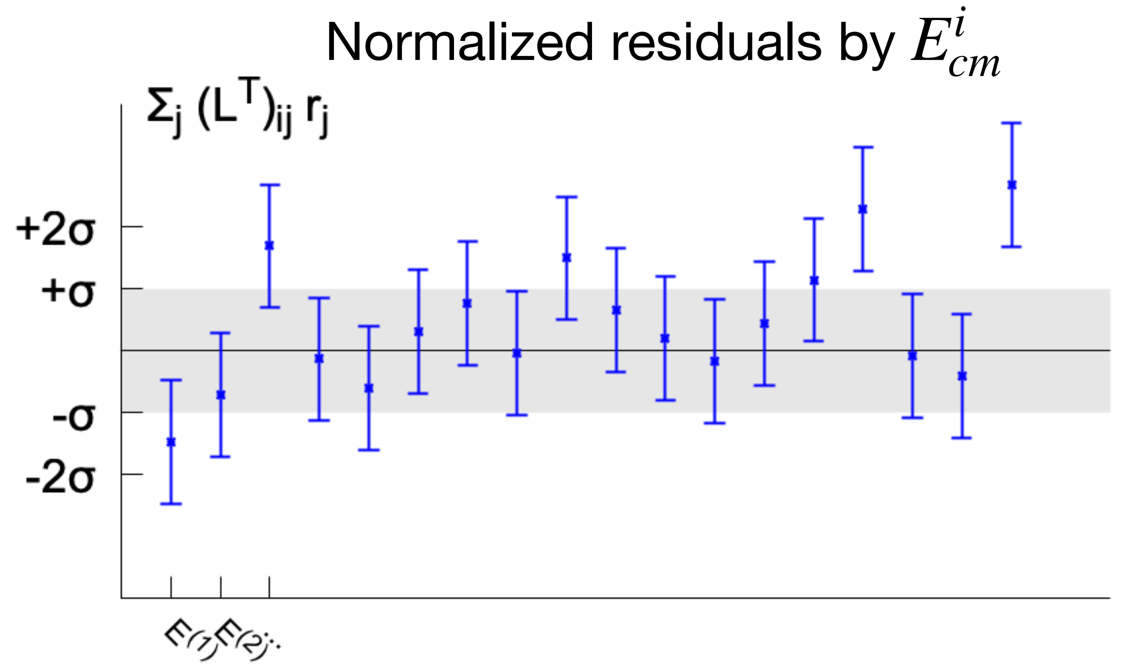
$i \equiv (n, \Lambda, \text{flavour})$

Allows computation of $\delta_1(E^i_{\text{cm}})$, but **poles** inaccessible

Reverse: given model δ^{mod} with parameters α^{mod}, find $\mathscr{E}^i_{\text{cm}}$

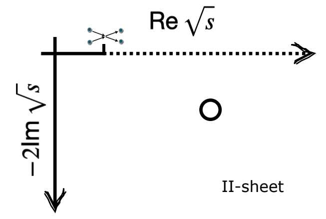
Minimise correlated

$$\chi^2_{\text{PS}}(\alpha^{\text{mod}}) = \sum_{i,j} \left[E^i_{\text{cm}} - \mathscr{E}^i_{\text{cm}}(\alpha^{\text{mod}})\right](\text{Cov}^{-1})_{ij}\left[E^j_{\text{cm}} - \mathscr{E}^j_{\text{cm}}(\alpha^{\text{mod}})\right]$$



to constrain δ^{mod}

Resonance pole



Substitute and analytically-continue

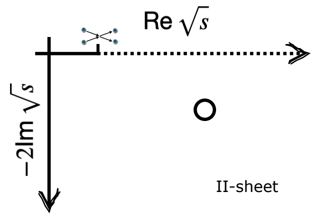
$$T^{\mathrm{mod}}(\sqrt{s}) = \frac{1}{\cot\delta^{\mathrm{mod}}(\sqrt{s}) - i}$$

find T_1^{mod} complex pole

⬍

find root $\cot\delta^{\mathrm{mod}} - i$

Resonance pole



Substitute and analytically-continue

$$T^{\text{mod}}(\sqrt{s}) = \frac{1}{\cot \delta^{\text{mod}}(\sqrt{s}) - i}$$

find T_1^{mod} complex pole

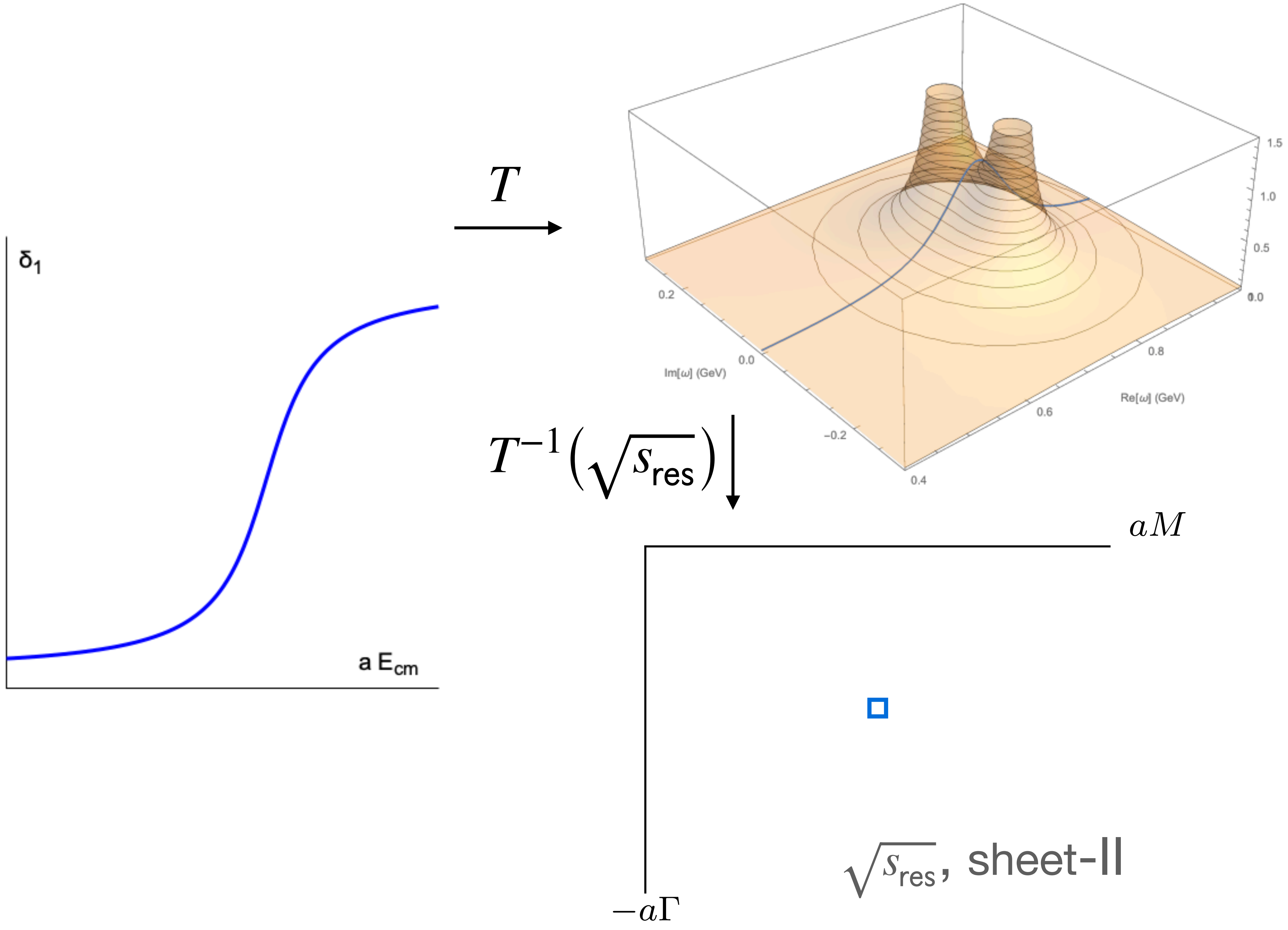
⬍

find root $\cot \delta^{\text{mod}} - i$

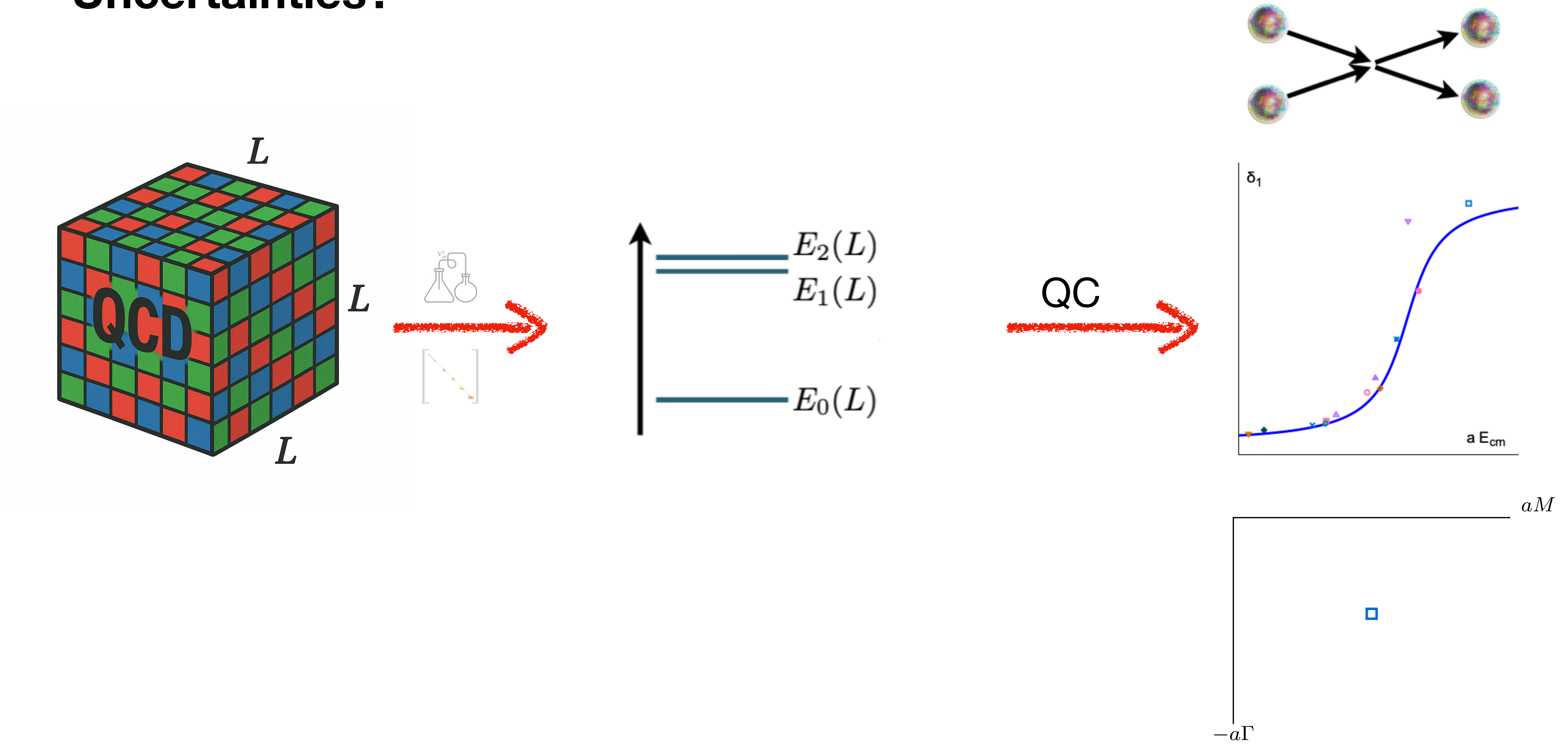
Resonance → sheet-II → Im $p < 0$

$$\sqrt{s(p_{\text{res}})} = M - \frac{i}{2}\Gamma$$

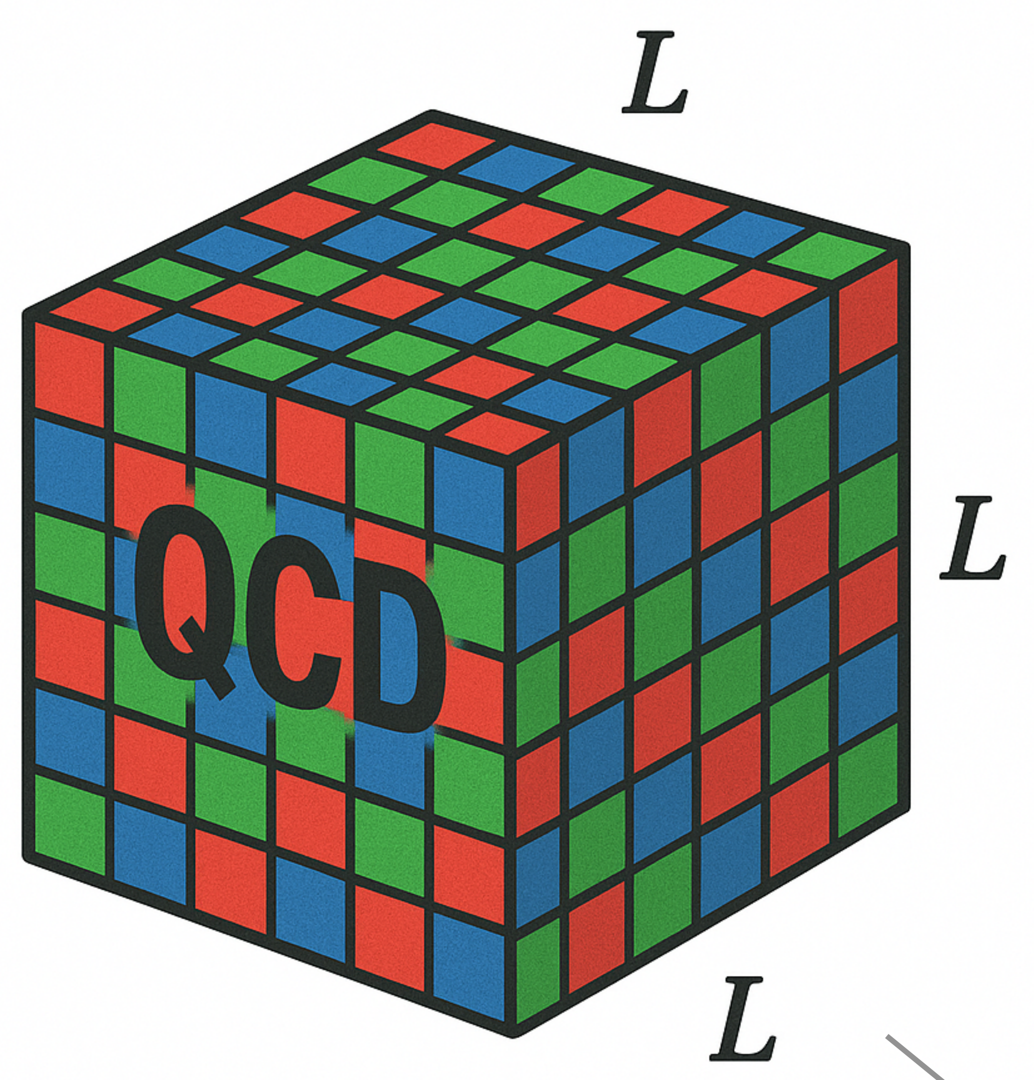
If not possible exactly, numerically minimise $|T^{-1}|^2$ and ensure it's root

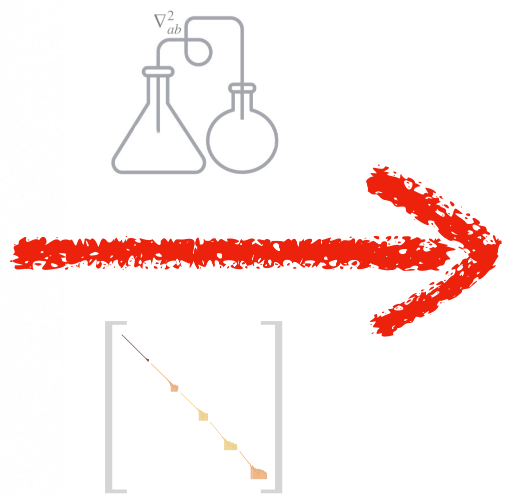


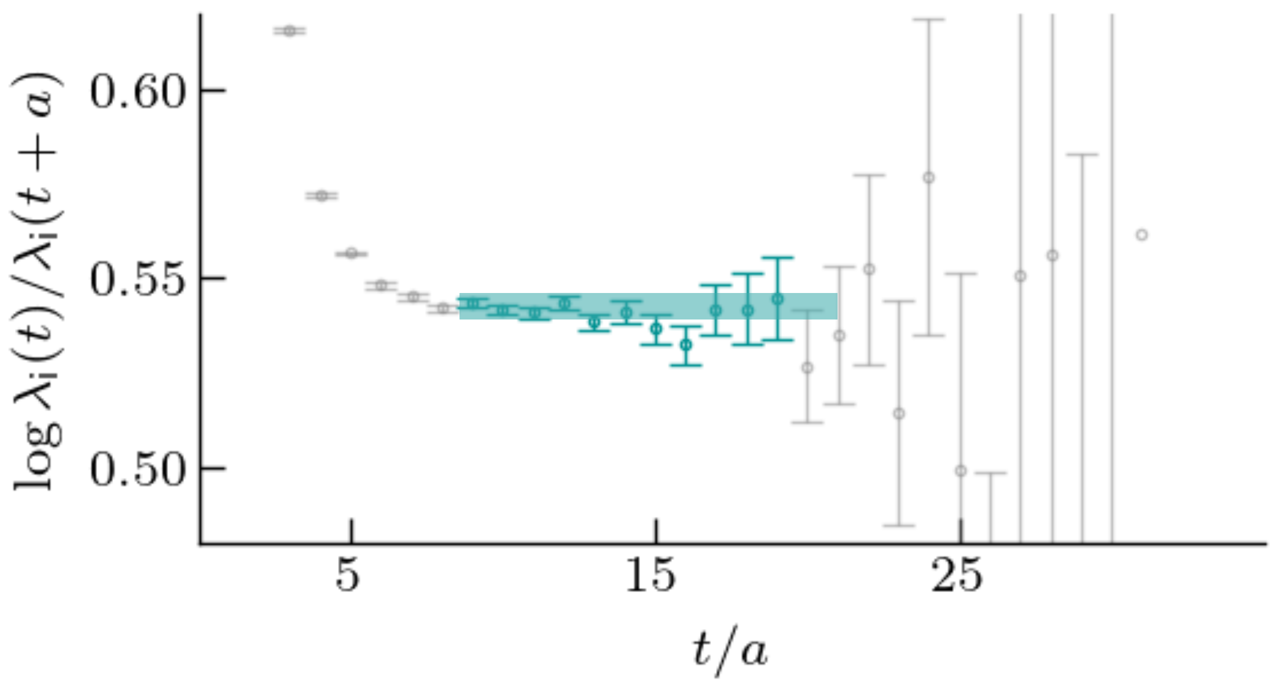
Uncertainties?



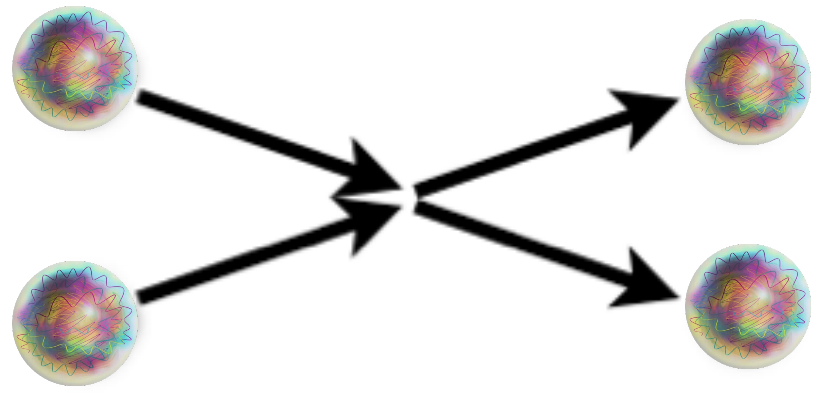
Uncertainties?

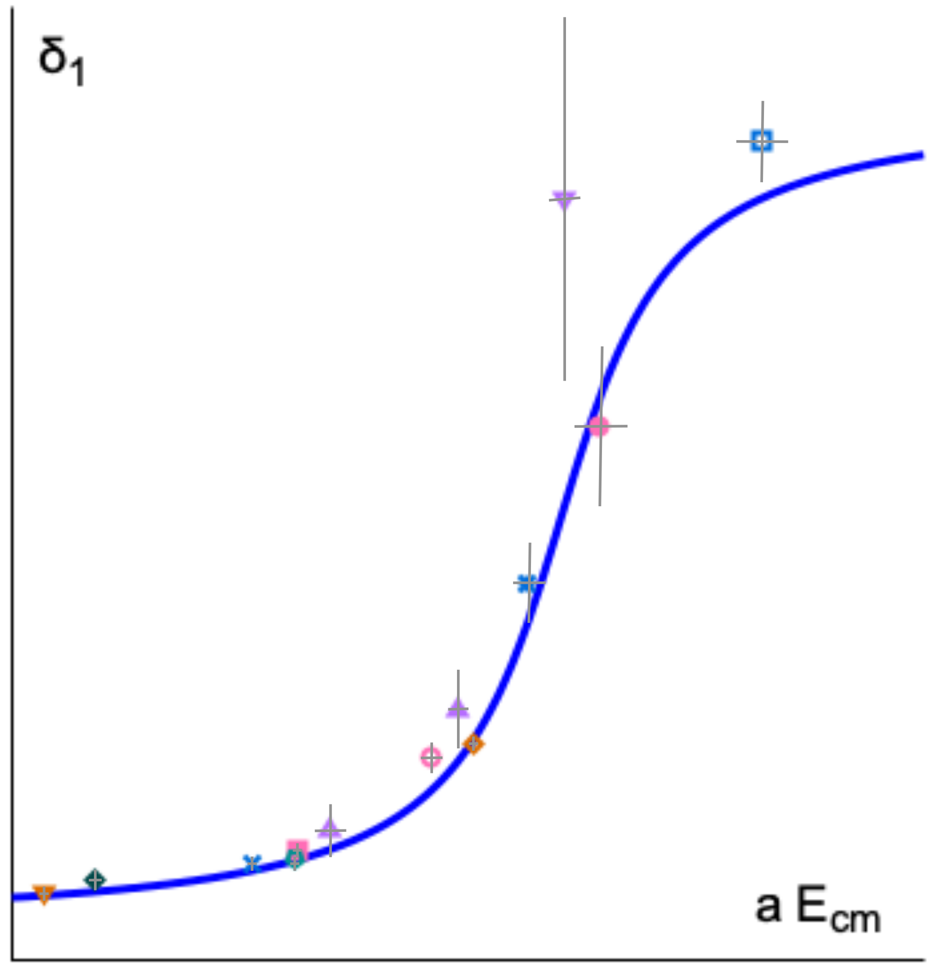


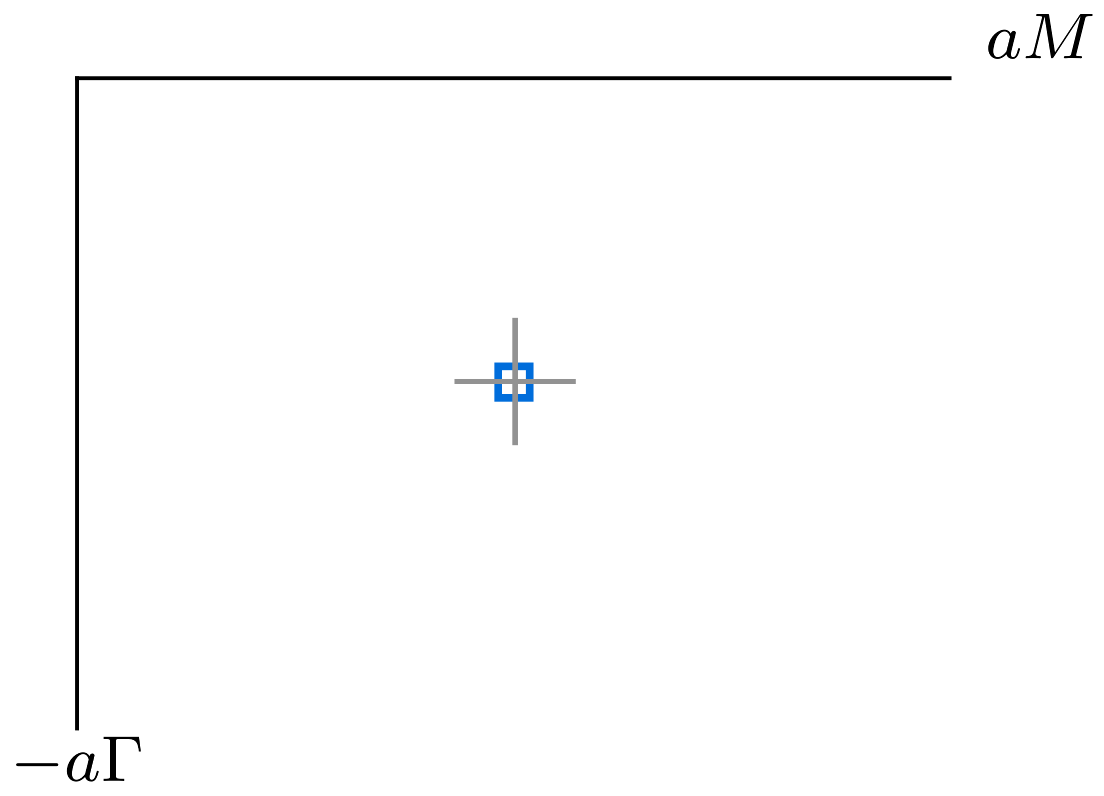




QC

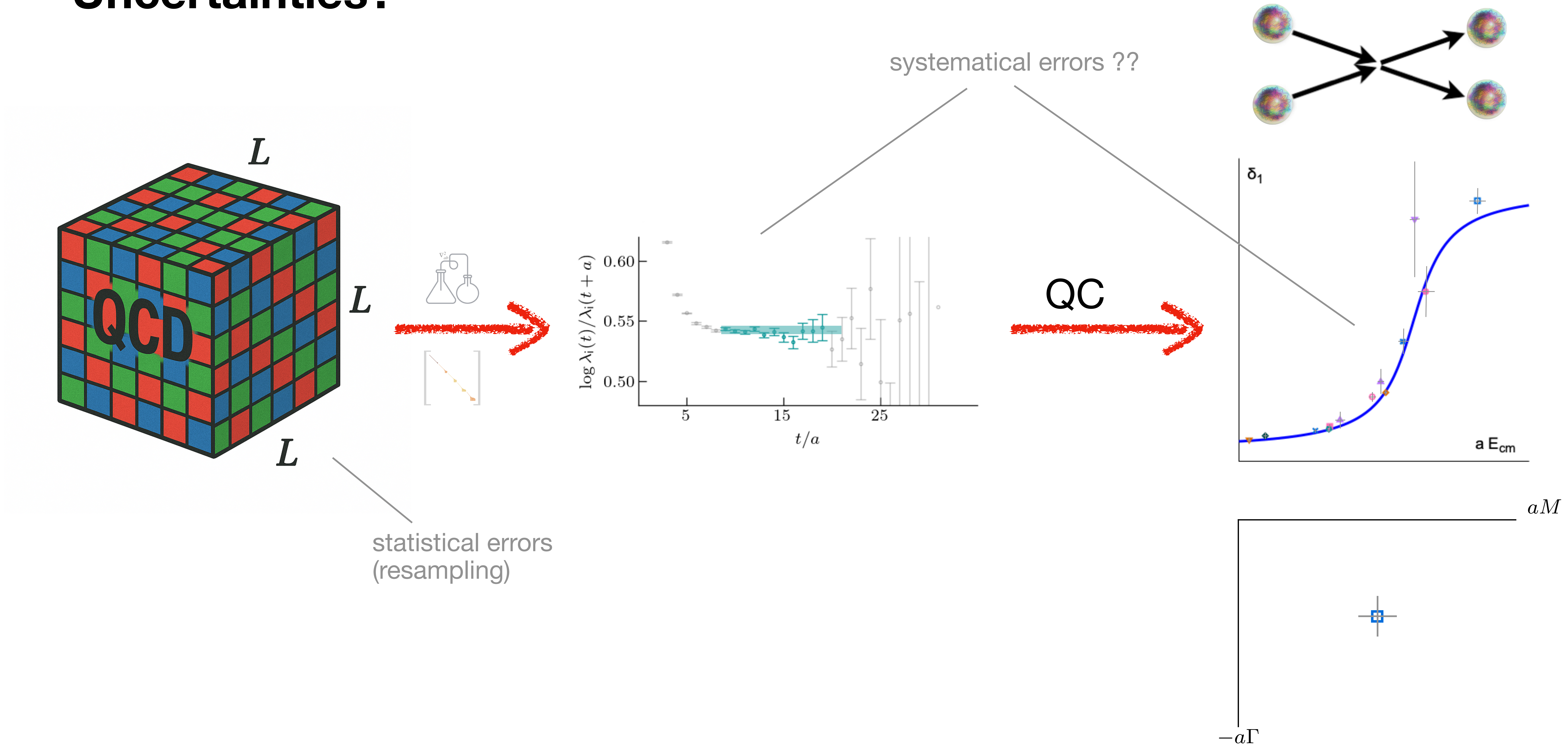






statistical errors
(resampling)

Uncertainties?



Model Averaging

Akaike information criterion (AIC)

- probabilities for different models
- model comparison
- spread of weighted distribution

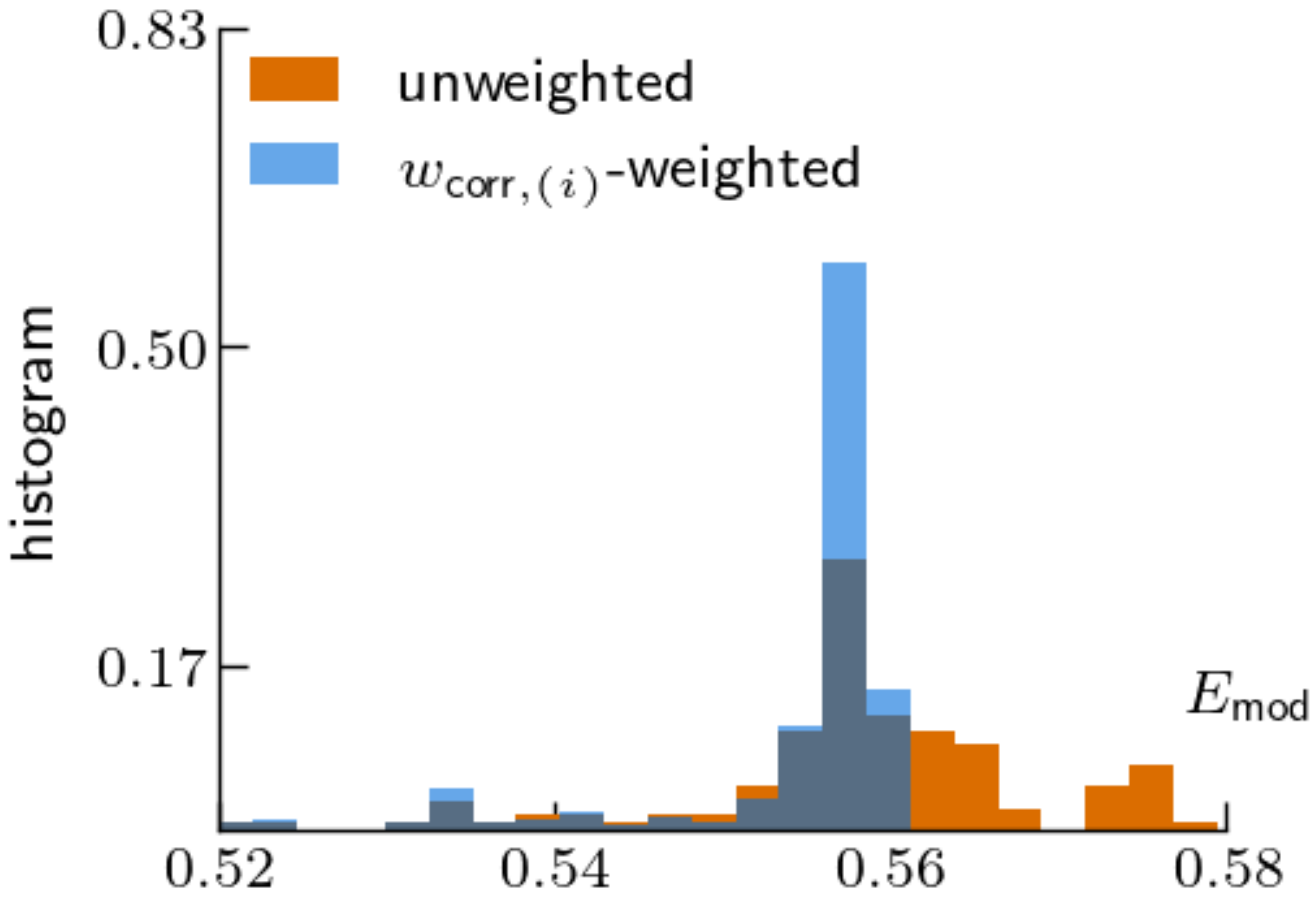
$$w \propto \exp -\frac{1}{2}\Big[\overbrace{\chi^2 + 2n^{\text{par}} - 2n^{\text{data}}}^{\text{AIC}}\Big]$$

[Borsanyi, Fodor, Guenther et al - Nature, 2021] [Neil & Sitison - PRE, 2023]

Different $[t^i_{\text{min}}, t^i_{\text{max}}] = \mathsf{f}_\mathsf{i} \sim$ different models

$$\downarrow$$

$$w^i_{\text{corr}}(\mathsf{f}_i) = \exp -\frac{1}{2}\text{AIC}_{\text{corr}}(\mathsf{f}_i)$$



Model Averaging

Akaike information criterion (AIC)

- probabilities for different models
- model comparison
- spread of weighted distribution

$$w \propto \exp -\frac{1}{2}\big[\overbrace{\chi^2 + 2n^{\text{par}} - 2n^{\text{data}}}^{\text{AIC}}\big]$$

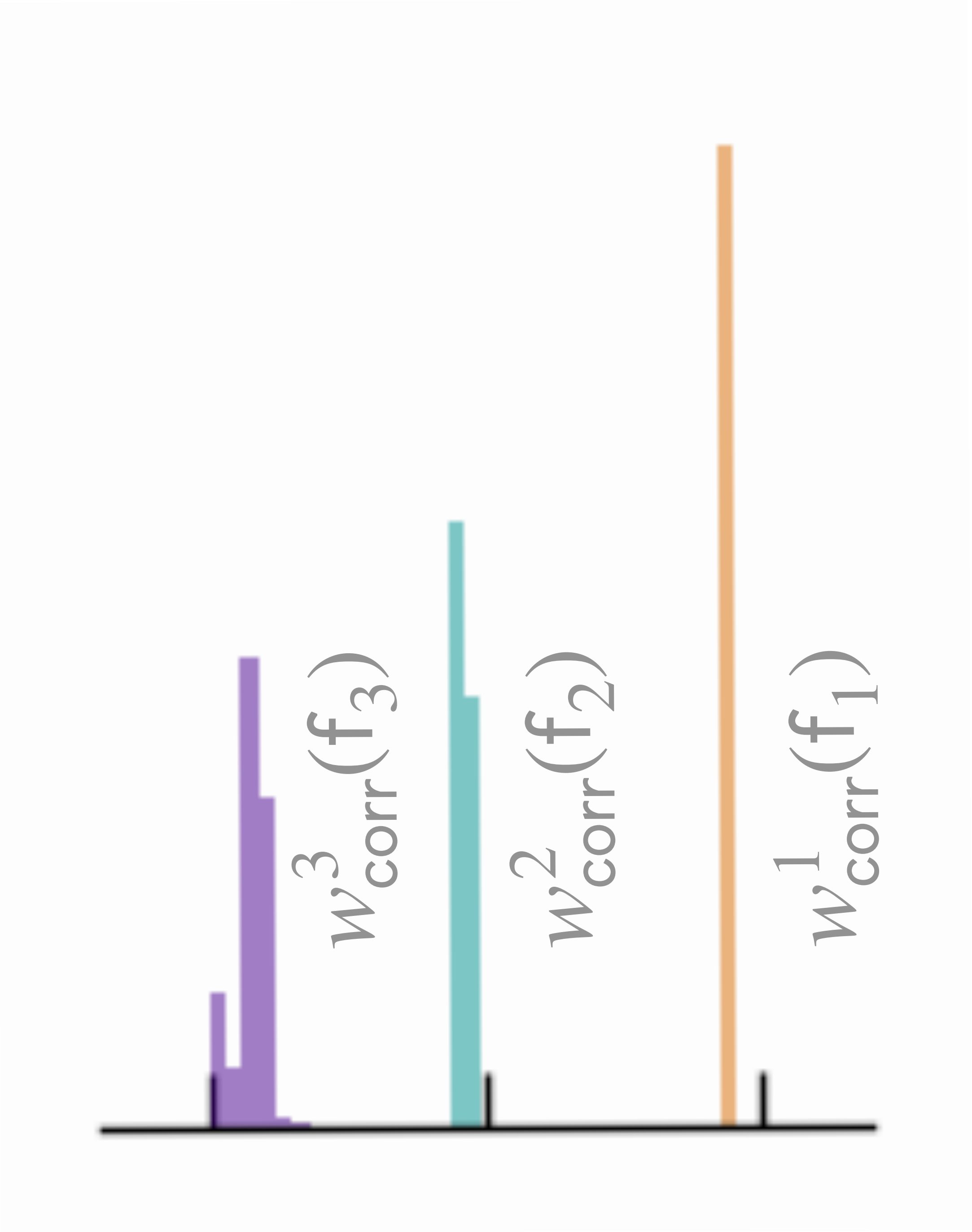
[Borsanyi, Fodor, Guenther et al - Nature, 2021] [Neil & Sitison - PRE, 2023]

Different $[t^i_{\text{min}}, t^i_{\text{max}}] = \text{f}_\text{i} \sim$ different models

$$\downarrow$$

$$w^i_{\text{corr}}(\text{f}_i) = \exp -\frac{1}{2}\text{AIC}_{\text{corr}}(\text{f}_i)$$

$$\{\text{f}_1, \text{f}_2, \ldots\} = \text{f} \longrightarrow \{E_{\text{cm}}\}$$



Model Averaging

Akaike information criterion (AIC)

- probabilities for different models
- model comparison
- spread of weighted distribution

$$w \propto \exp -\frac{1}{2}\left[\overbrace{\chi^2 + 2n^{\mathsf{par}} - \quad n^{\mathsf{data}}}^{\text{AIC}}\right]$$

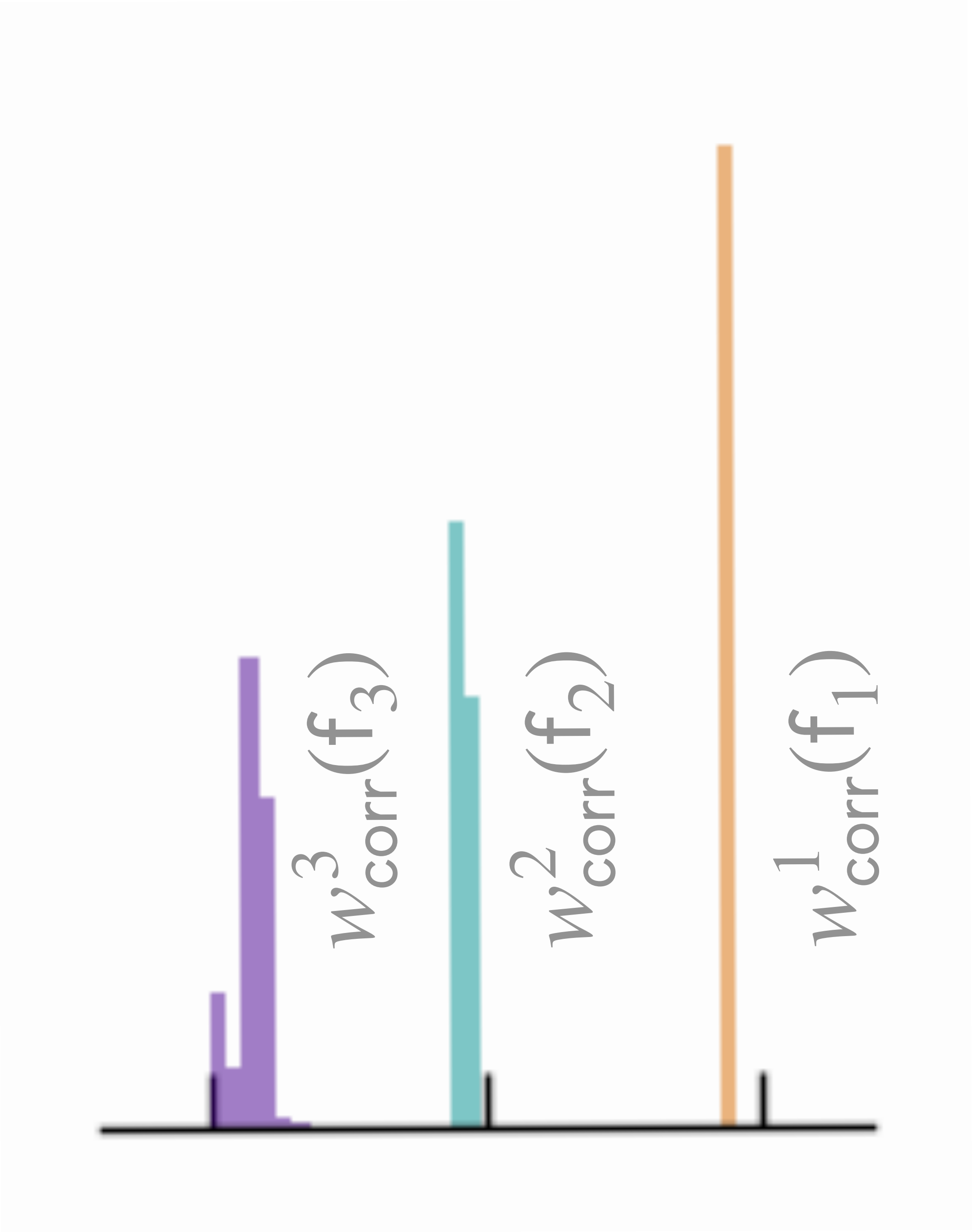
[Borsanyi, Fodor, Guenther et al - Nature, 2021] [Neil & Sitison - PRE, 2023]

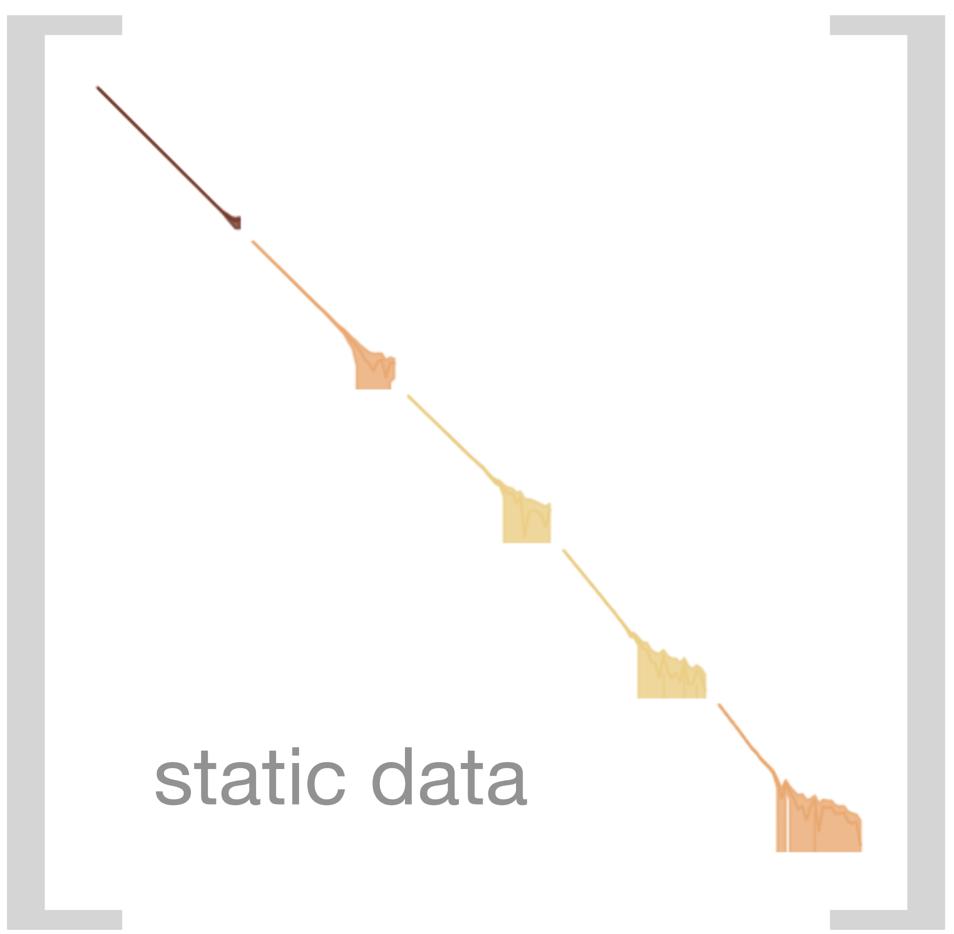
Different $[t^i_{\mathsf{min}}, t^i_{\mathsf{max}}] = \mathsf{f_i} \sim$ different models

$$\downarrow$$

$$w^i_{\mathsf{corr}}(\mathsf{f}_i) = \exp -\frac{1}{2}\text{AIC}_{\mathsf{corr}}(\mathsf{f}_i)$$

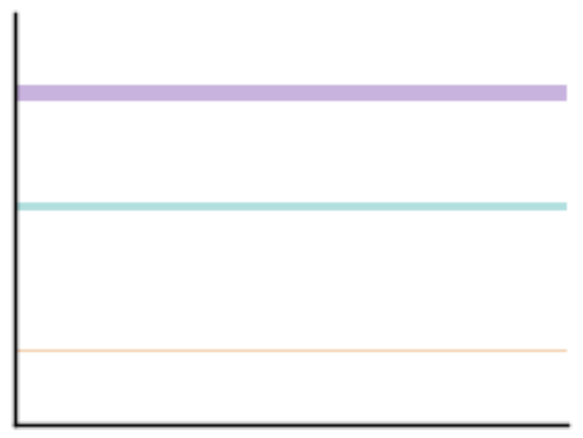
$$\{\mathsf{f}_1, \mathsf{f}_2, \ldots\} = \mathsf{f} \longrightarrow \{E_{\mathsf{cm}}\}$$





n^{lev} fits

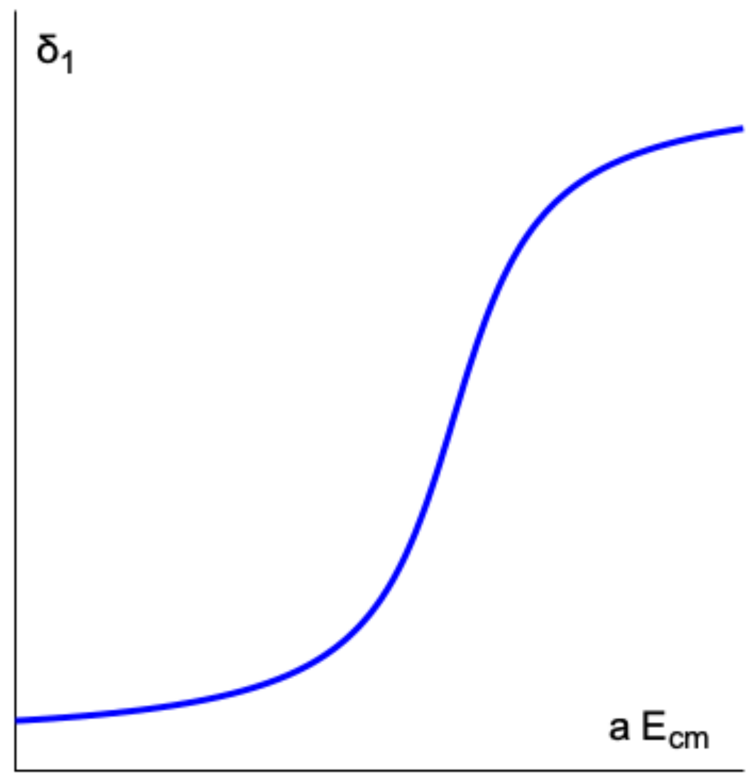
$\lambda_i(t), \mathsf{f}_i \to E^i_{\text{cm}}$

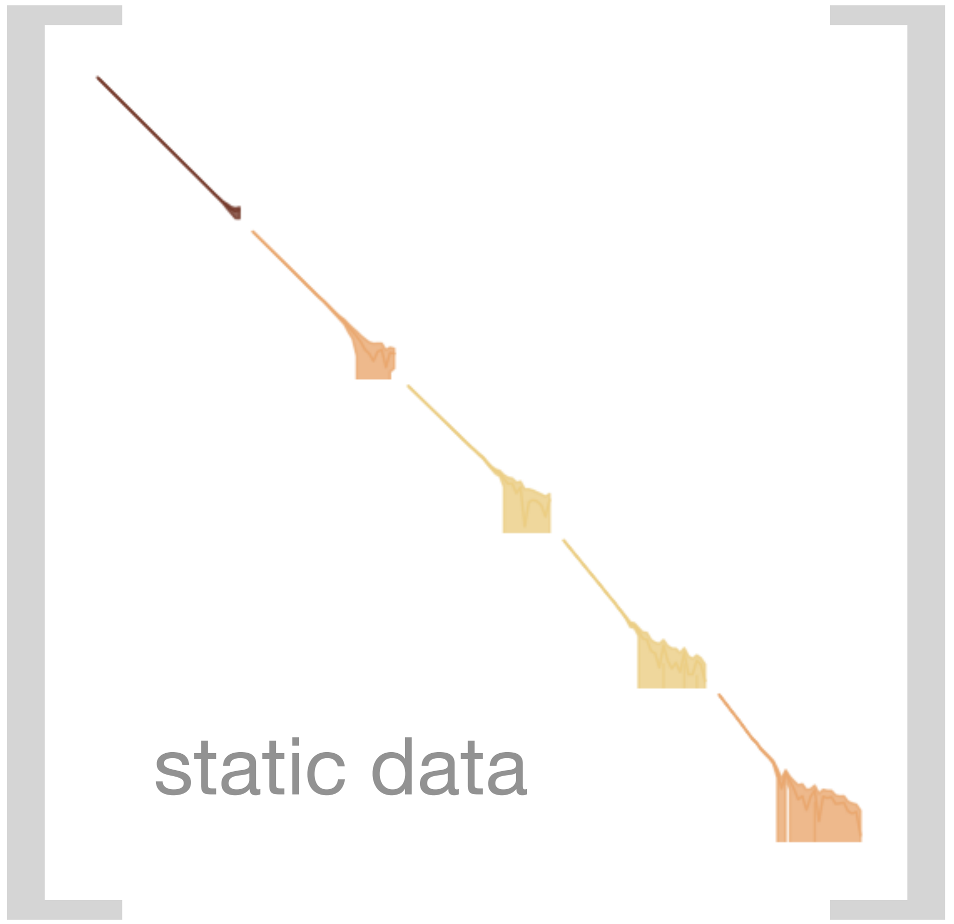


static data

one fit

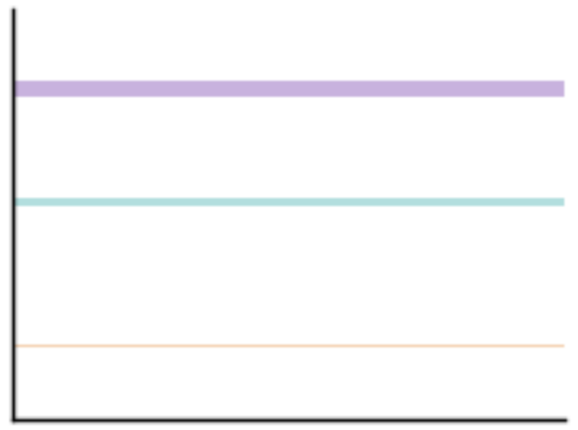
$\{E_{\text{cm}}\} \to \delta^{\text{mod}}$





n^{lev} fits

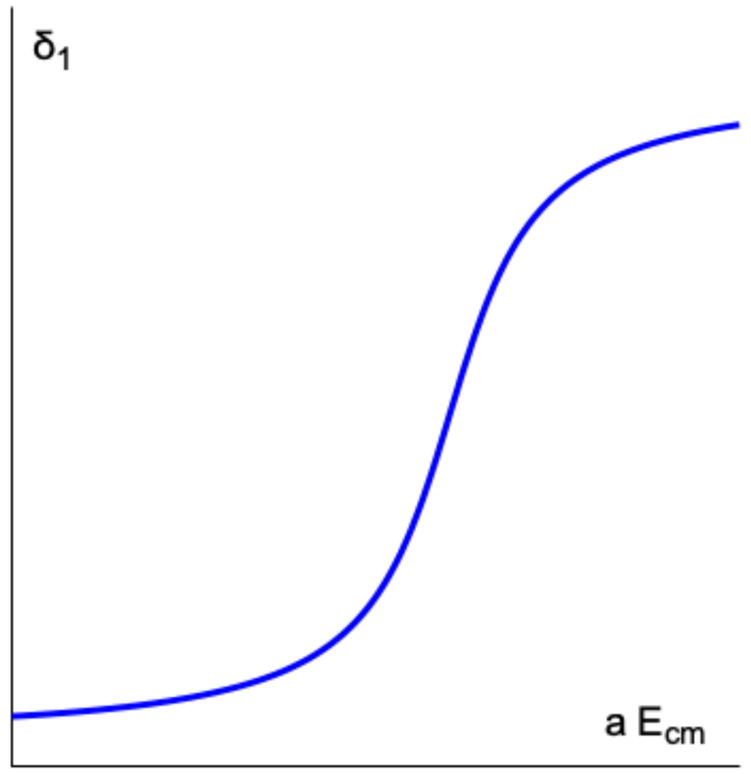
$\lambda_i(t), \mathsf{f}_i \rightarrow E^i_{\text{cm}}$



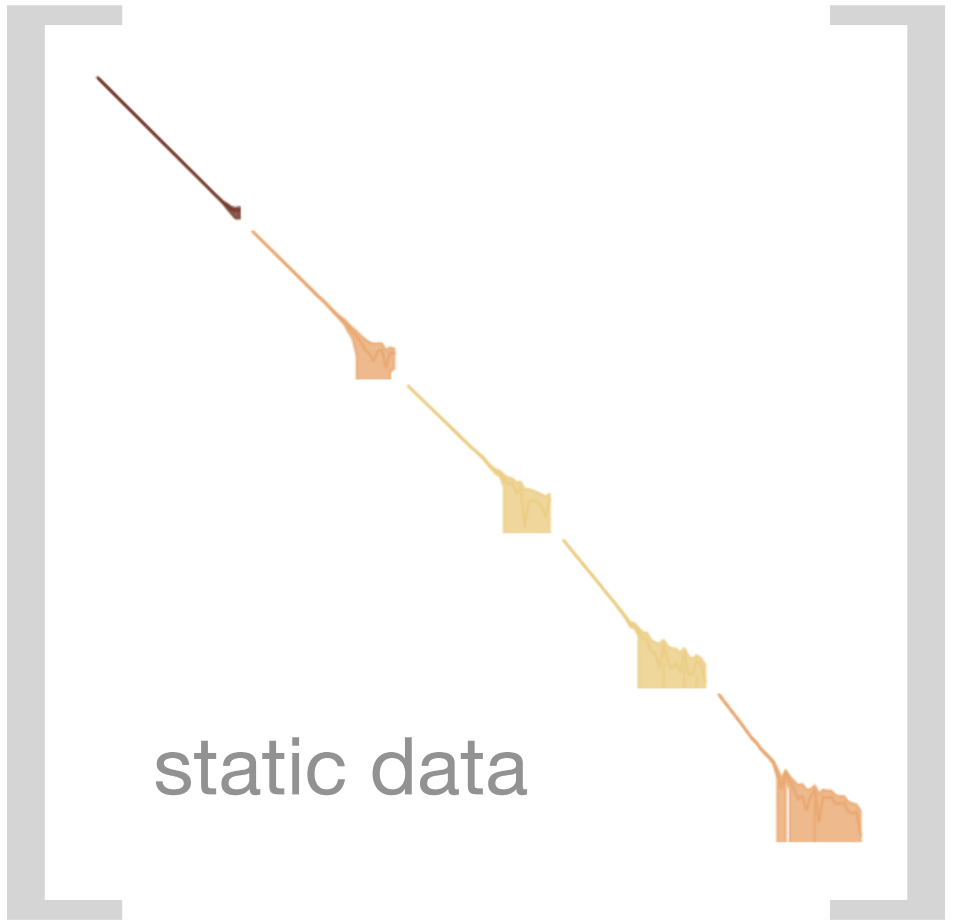
static data

one fit

$\{E_{\text{cm}}\} \rightarrow \delta^{\text{mod}}$

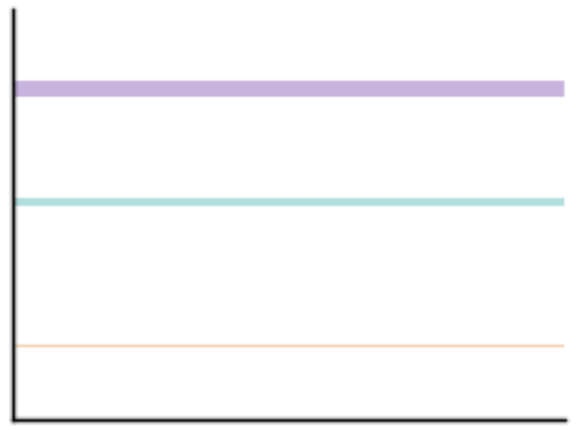


…systematics propagation into scattering?



n^{lev} fits

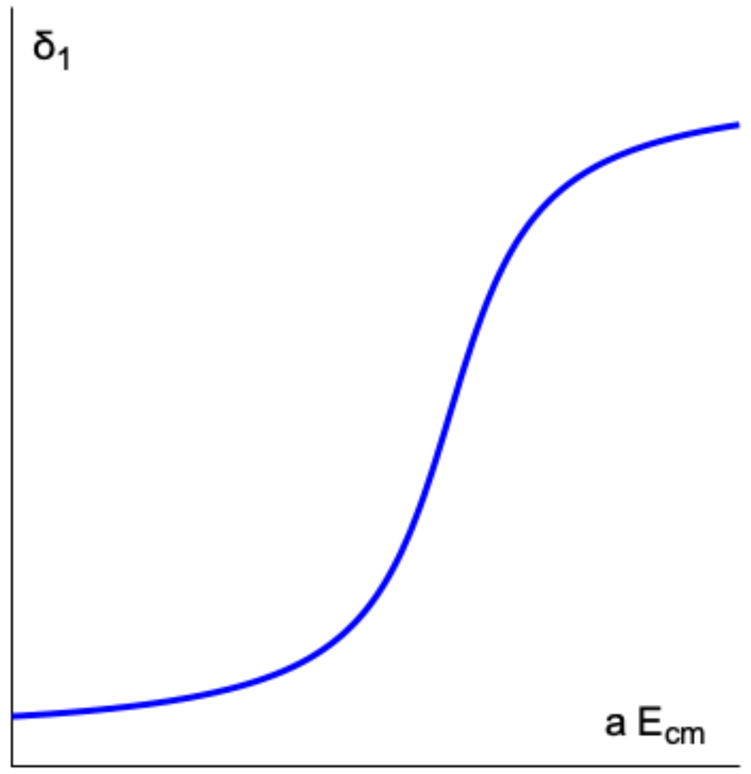
$\lambda_i(t), \mathsf{f}_i \to E^i_{\text{cm}}$



static data

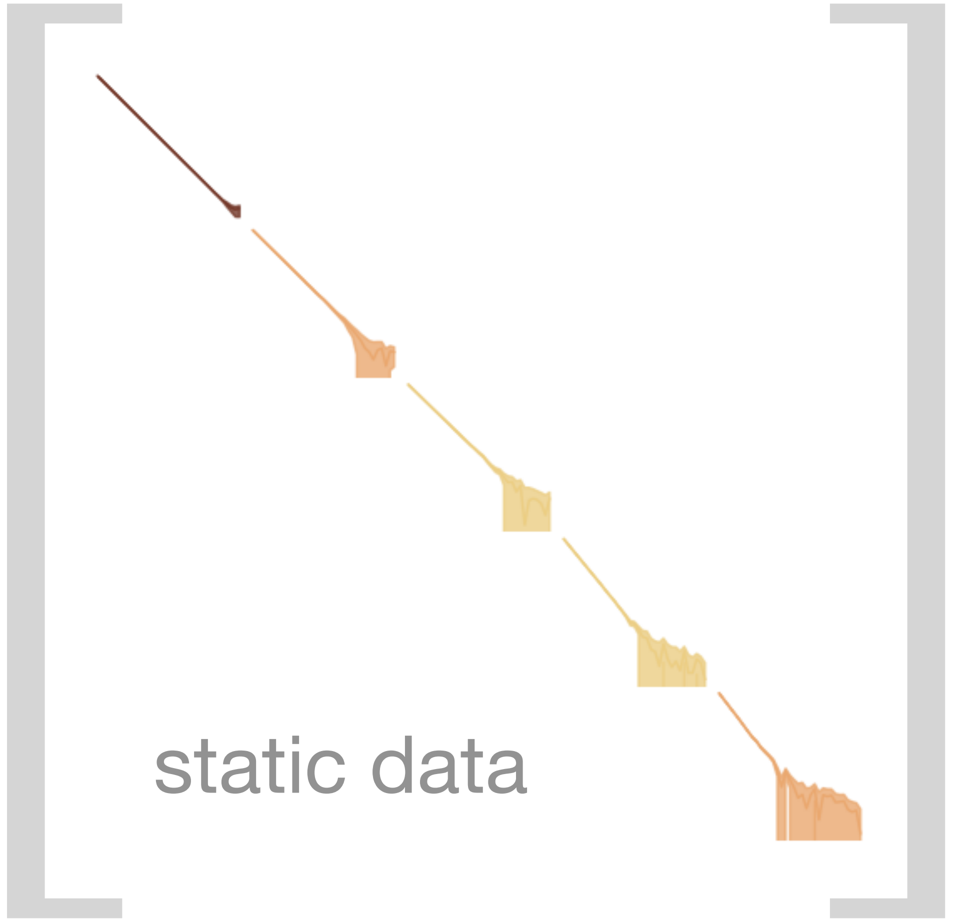
one fit

$\{E_{\text{cm}}\} \to \delta^{\text{mod}}$

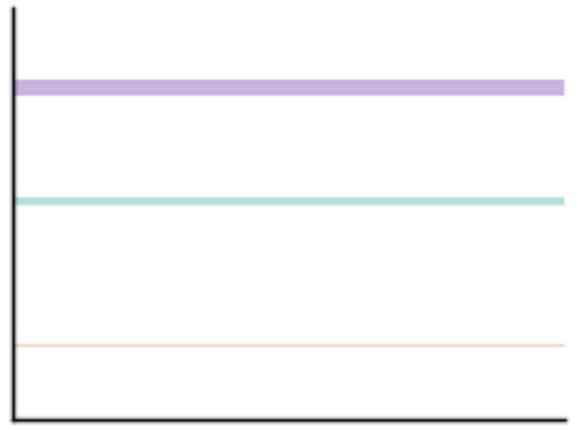


…systematics propagation into scattering?

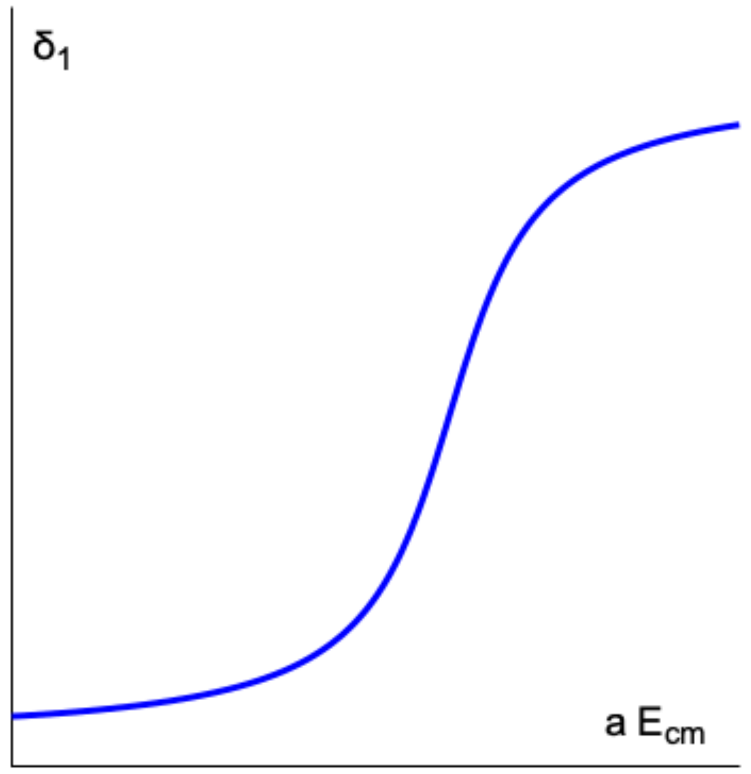
First, imagine



global minimisation unfeasible

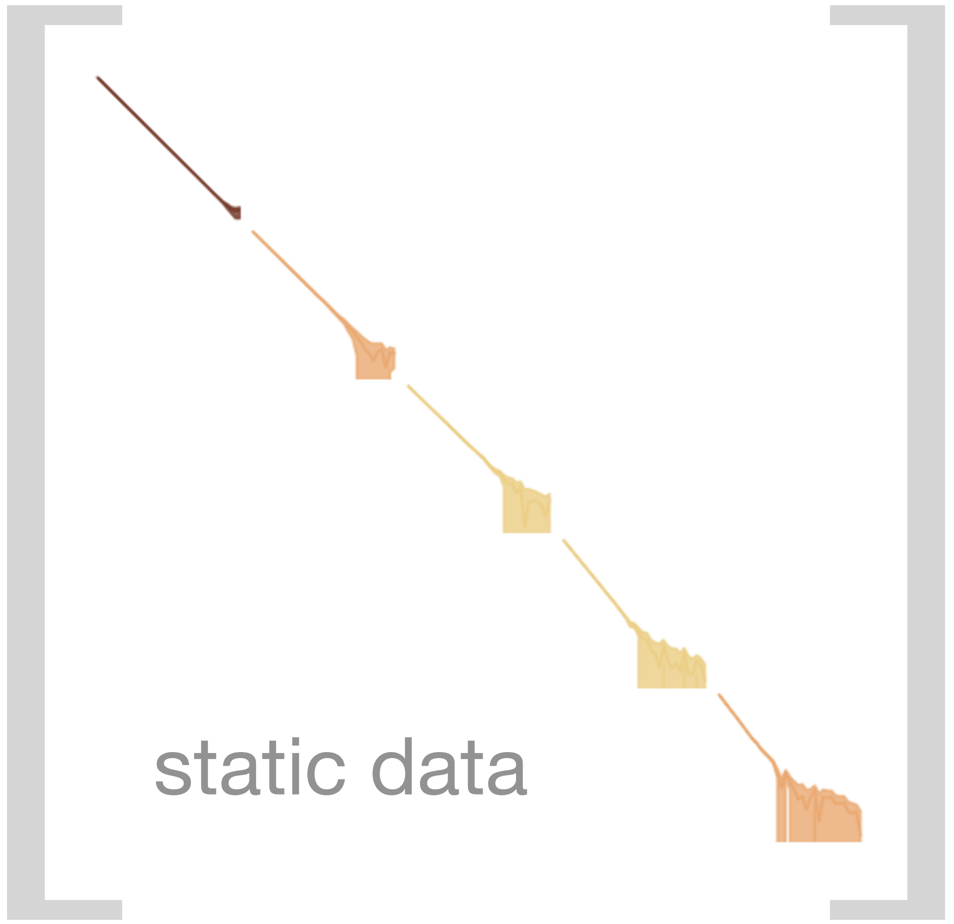


intermediate



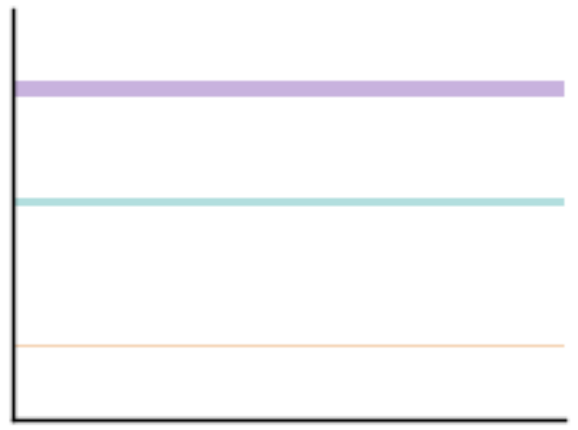
$w_{\text{ideal}}(\mathsf{f}, \delta^{\text{mod}}) \propto e^{-\text{AIC}_{\text{ideal}}(\mathsf{f},\delta^{\text{mod}})/2}$

one fit $\{\lambda(t), \mathsf{f}\} \to \delta^{\text{mod}}$



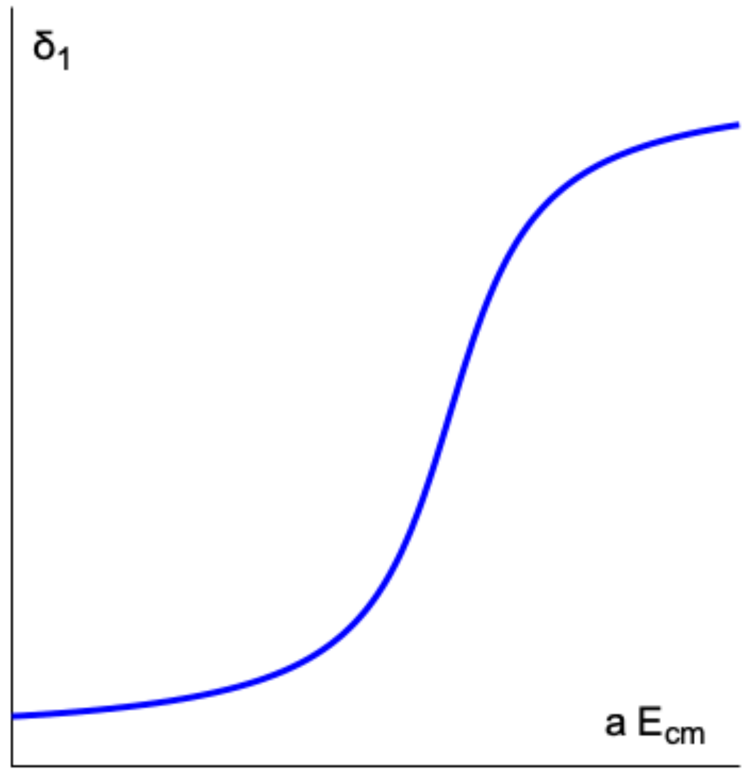
n^{lev} fits

$\lambda_i(t), \mathsf{f}_i \rightarrow E^i_{\text{cm}}$



one fit

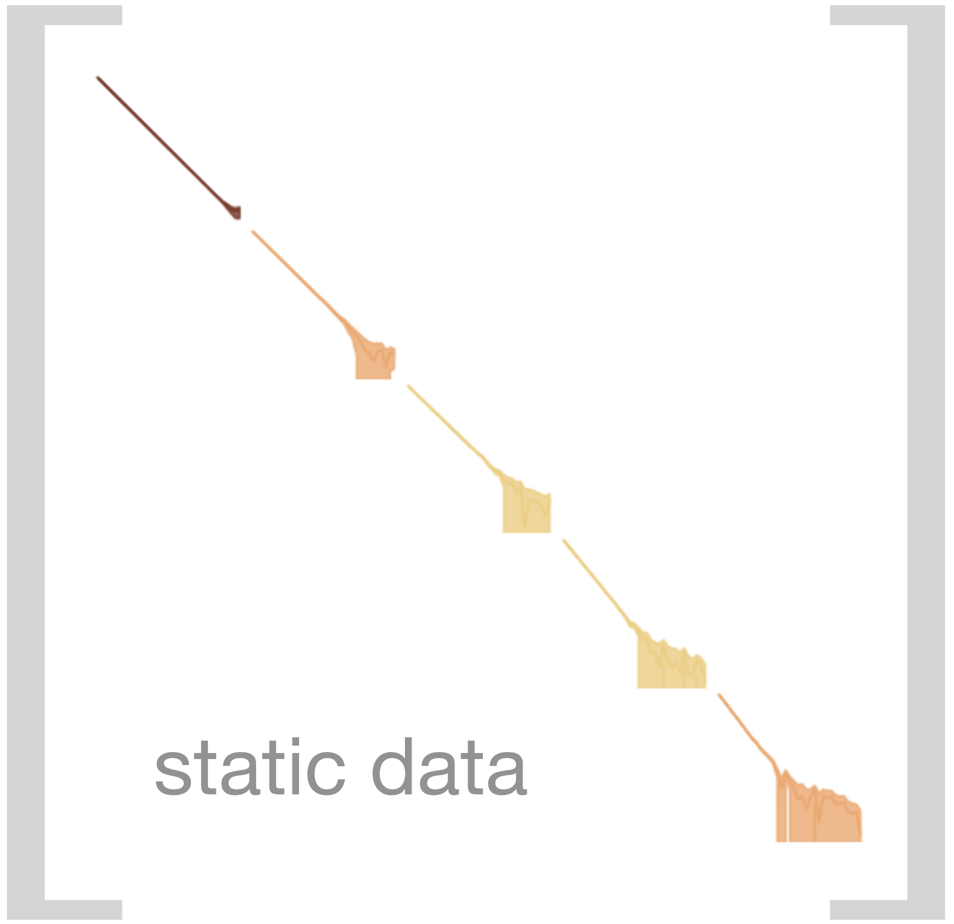
$\{E_{\text{cm}}\} \rightarrow \delta^{\text{mod}}$



back to blocked procedure

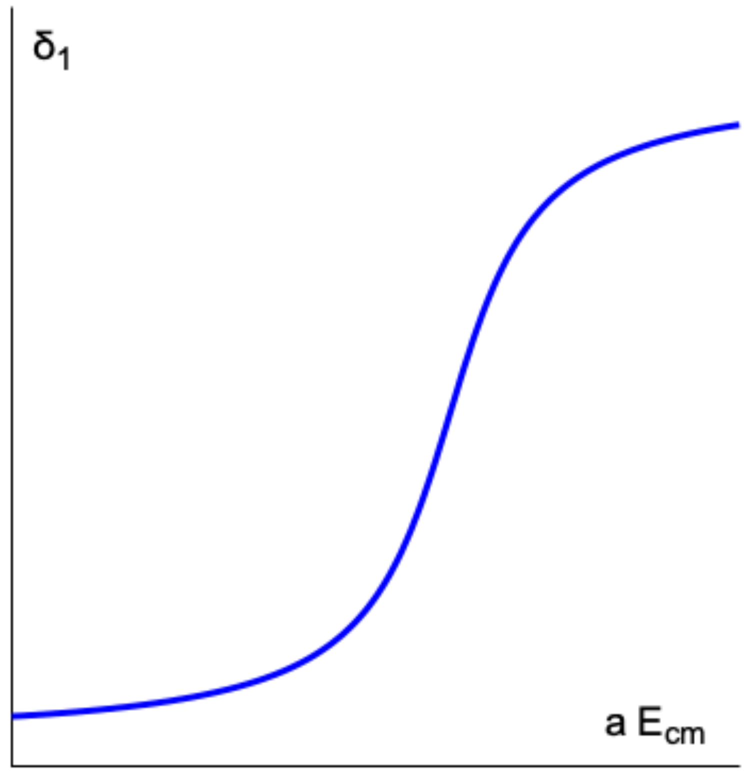
$$w_{\text{ideal}}(\mathsf{f}, \delta^{\text{mod}}) \approx$$

$$w_{\text{t}}(\mathsf{f}, \delta^{\text{mod}}) \propto e^{-\text{AIC}_{\text{PS}}(\mathsf{f},\delta^{\text{mod}})/2} \prod_i e^{-\text{AIC}_{\text{corr}}(\mathsf{f}_i)/2}$$



n^{lev} fits
$\lambda_i(t), \mathsf{f}_i \rightarrow E^i_{\text{cm}}$

one fit
$\{E_{\text{cm}}\} \rightarrow \delta^{\text{mod}}$

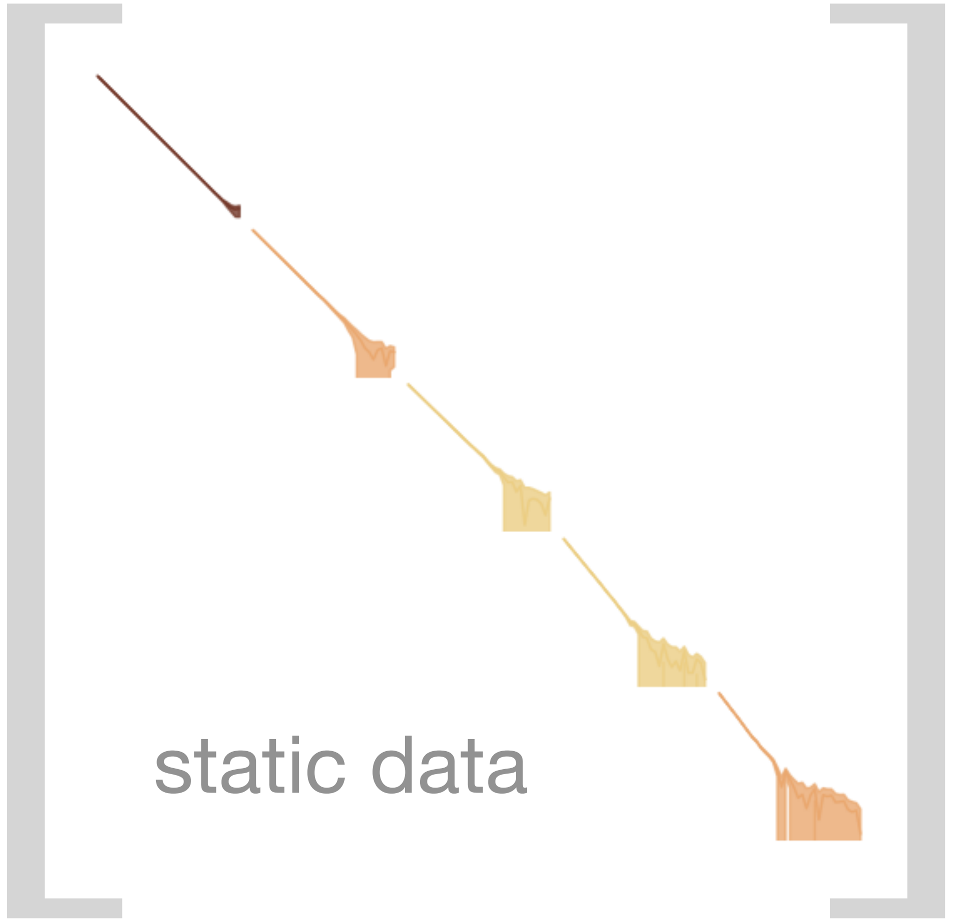


back to blocked procedure

$$w_{\text{ideal}}(\mathsf{f}, \delta^{\text{mod}}) \approx$$

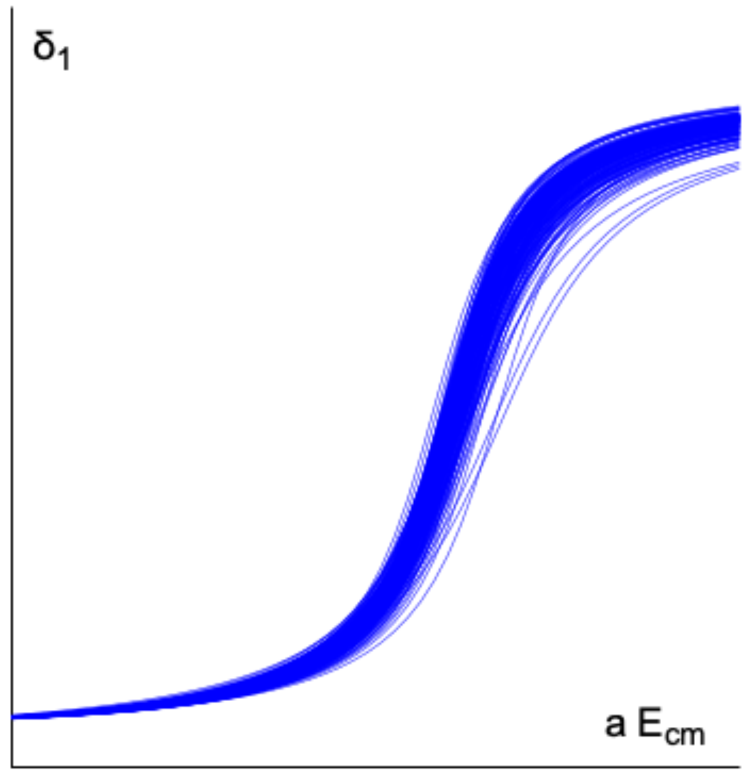
$$w_{\text{t}}(\mathsf{f}, \delta^{\text{mod}}) \propto e^{-\text{AIC}_{\text{PS}}(\mathsf{f}, \delta^{\text{mod}})/2} \prod_i e^{-\text{AIC}_{\text{corr}}(\mathsf{f}_i)/2}$$

Still, too many fit range combinations

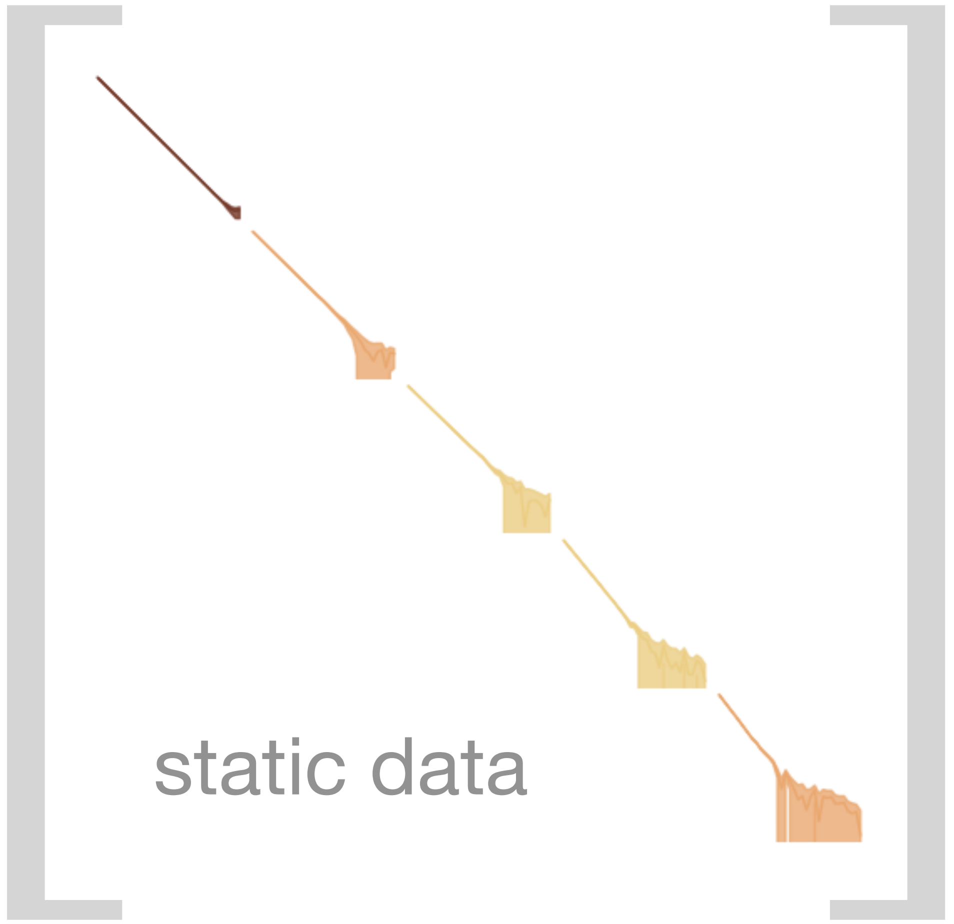


$$\dim\, \{\mathsf{f}\} \sim \mathcal{O}(10^2)^{n^{\text{lev}}} \sim \mathcal{O}(10^{30})$$

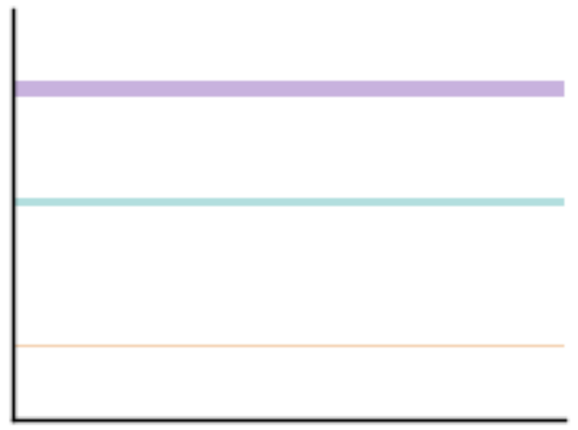
$\vdots$



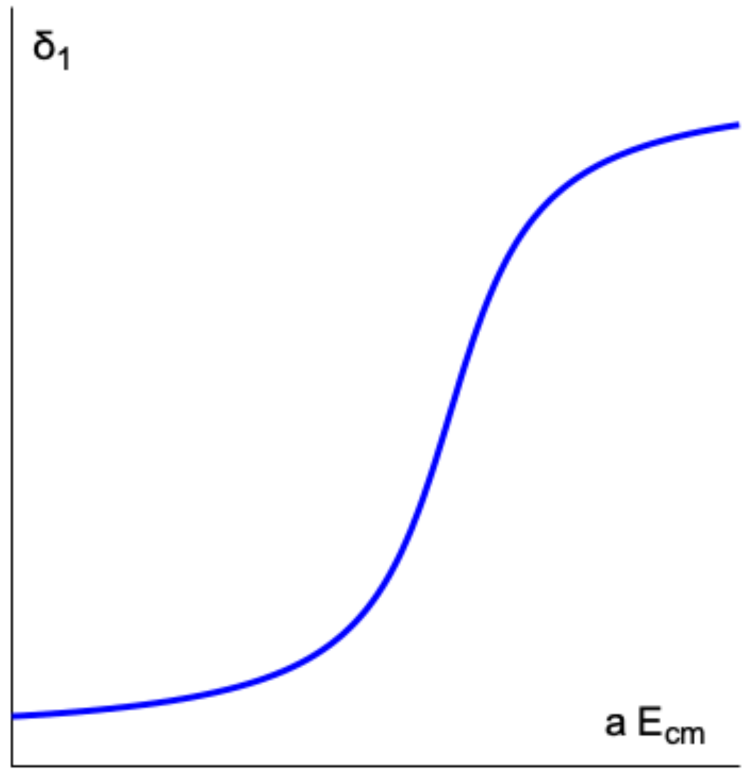
$$\big\{\{\lambda(t)\}, \{\mathsf{f}\}\big\} \rightarrow \{E_{\text{cm}}\} \rightarrow \delta^{\text{mod}}$$



n^{lev} fits
$\lambda_i(t), \mathsf{f}_i \to E^i_{\text{cm}}$



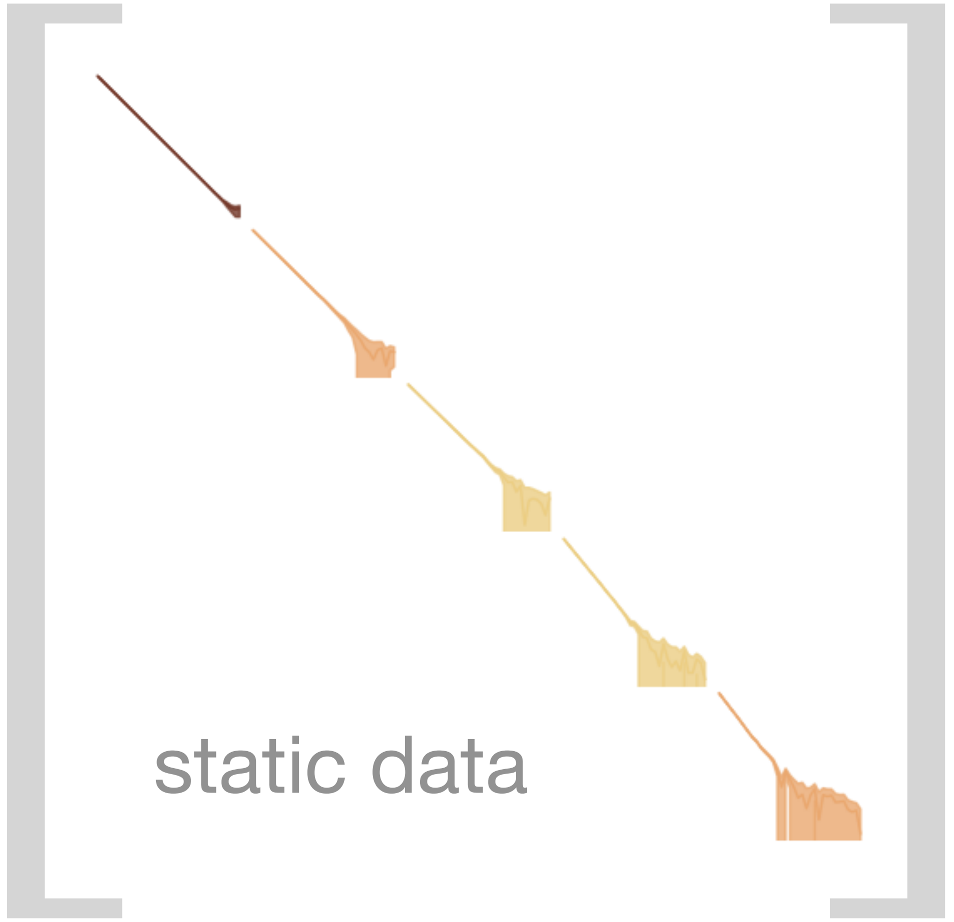
one fit
$\{E_{\text{cm}}\} \to \delta^{\text{mod}}$



back to blocked procedure

$$w_{\text{ideal}}(\mathsf{f}, \delta^{\text{mod}}) \approx$$
$$w_{\text{t}}(\mathsf{f}, \delta^{\text{mod}}) \propto e^{-\text{AIC}_{\text{PS}}(\mathsf{f},\delta^{\text{mod}})/2} \prod_i e^{-\text{AIC}_{\text{corr}}(\mathsf{f}_i)/2}$$

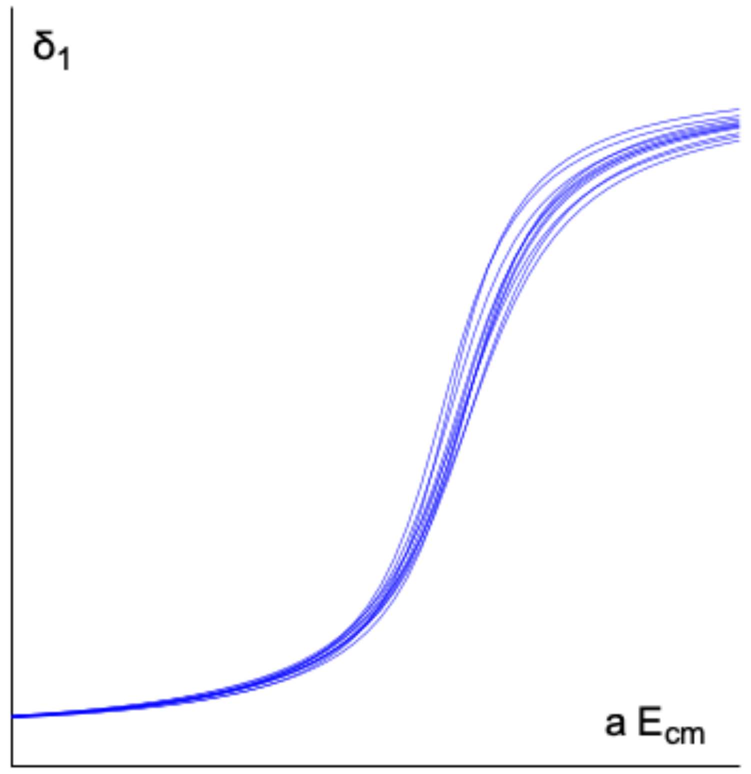
Still, too many fit range combinations



$$\dim\ \{\mathsf{f}\} \sim \mathcal{O}(10^2)^{n^{\text{lev}}} \sim \mathcal{O}(10^{30})$$

⋮

$$\{\{\lambda(t)\}, \{\mathsf{f}\}\} \to \{E_{\text{cm}}\} \to \delta^{\text{mod}}$$

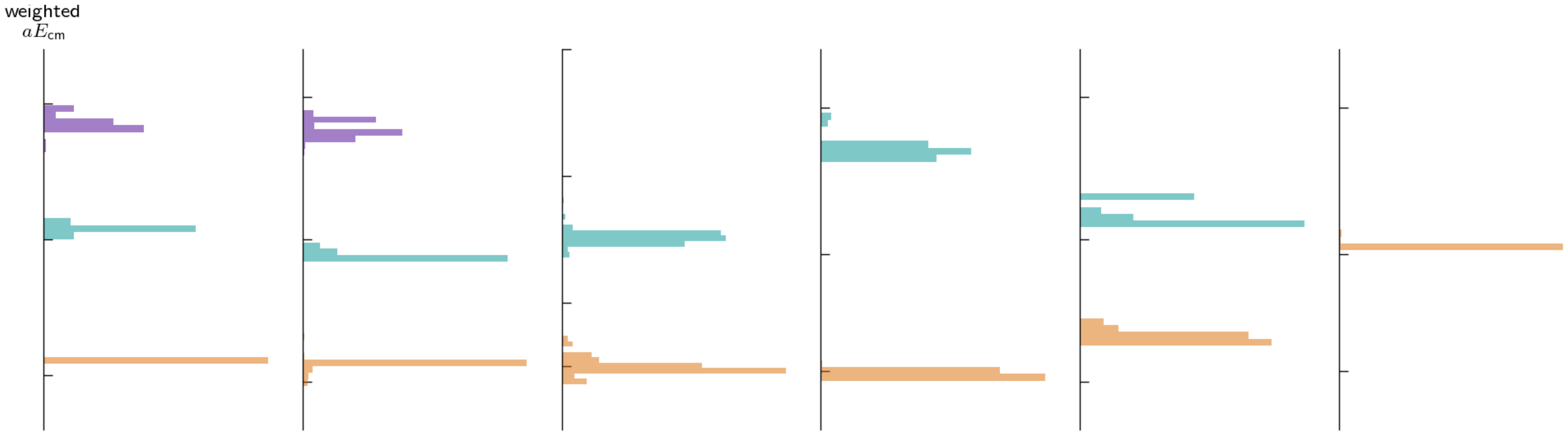


sampling?

Importance Sampling

Proposal dist.

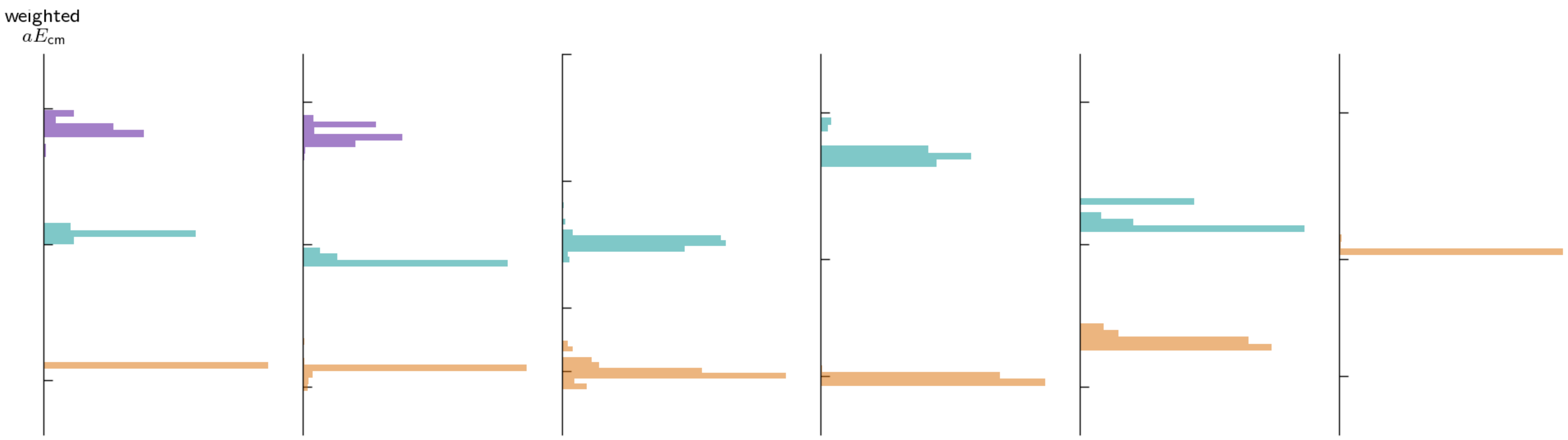
$$w_{\text{corr}}(\mathsf{f}) = \prod_i w_{\text{corr}}^{(i)}(\mathsf{f}_i)$$

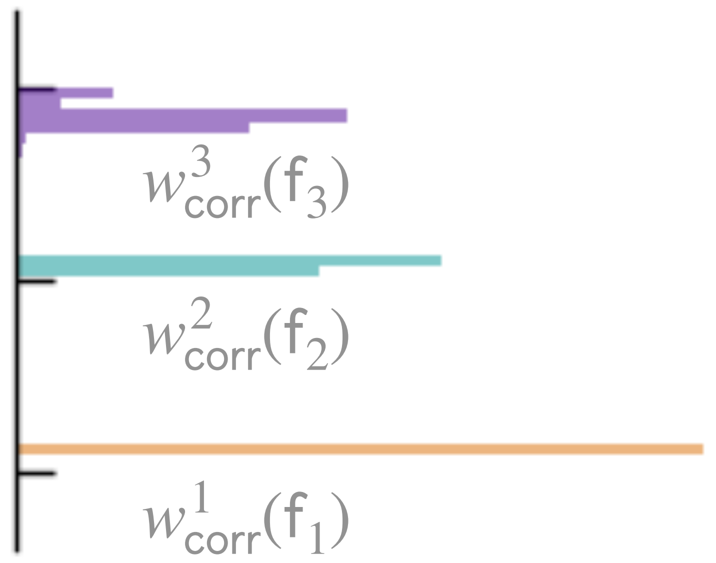


Importance Sampling

Proposal dist.

$$w_{\text{corr}}(\mathsf{f}) = \prod_i w^{(i)}_{\text{corr}}(\mathsf{f}_i)$$



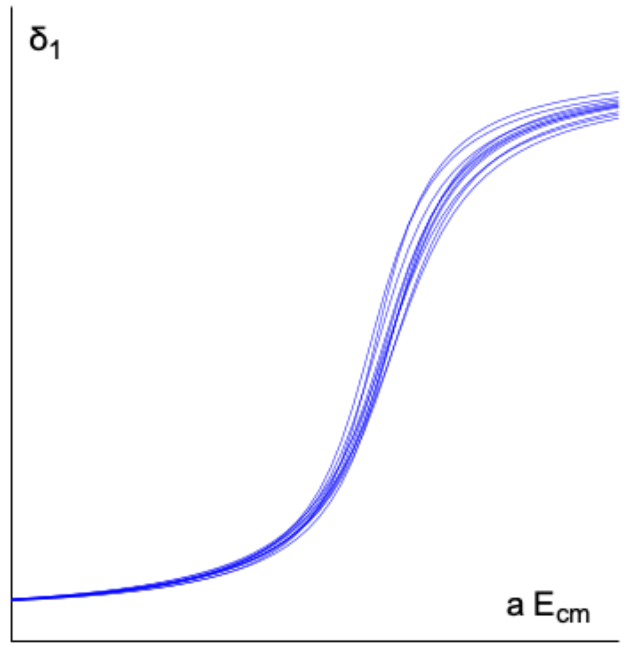


Sample & fit $\lambda_i(t)$

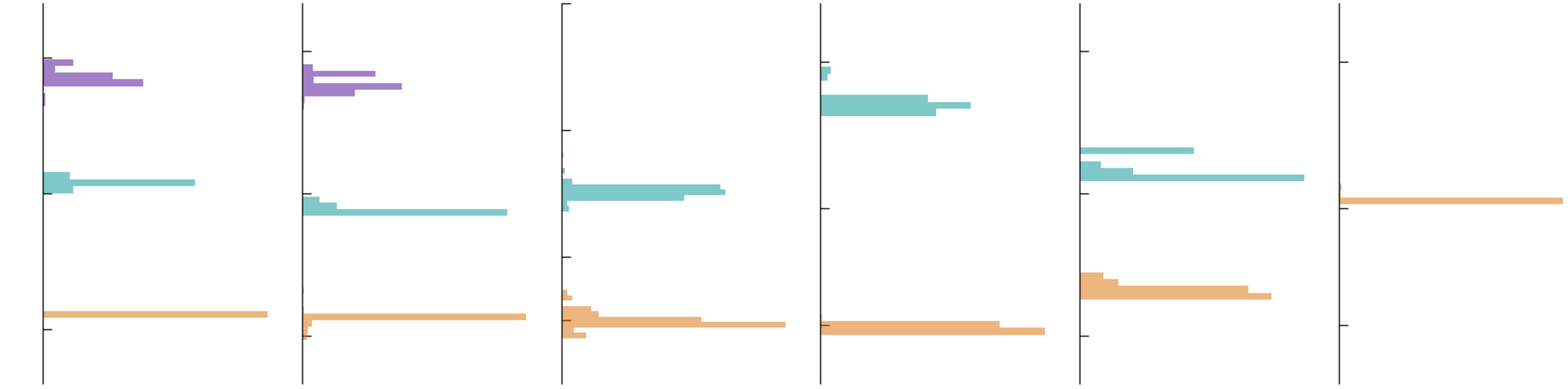
$\{\mathsf{f}_1, \mathsf{f}_2, \ldots, \mathsf{f}_{n_{\text{lev}}}\} \equiv \mathsf{s}$

$$\{E_{\text{cm}}\}^{\mathsf{s}^1}, \{E_{\text{cm}}\}^{\mathsf{s}^2}, \ldots$$

QC fit

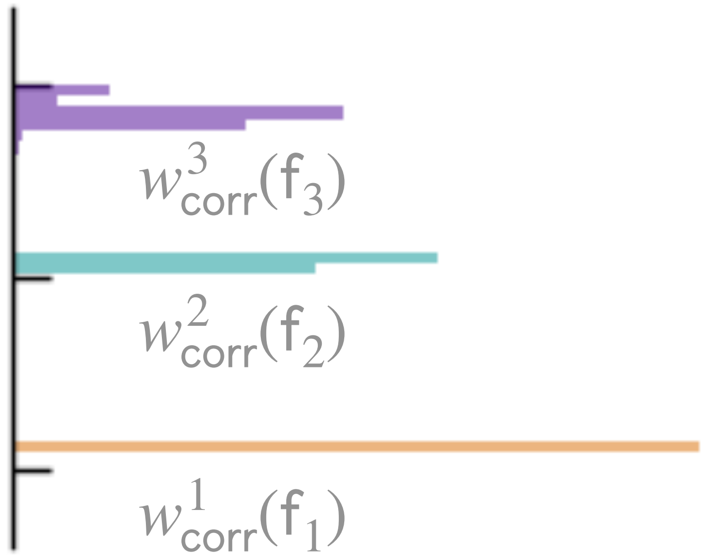


Importance Sampling



Proposal dist.

$$w_{\text{corr}}(\text{f}) = \prod_i w^{(i)}_{\text{corr}}(\text{f}_i)$$

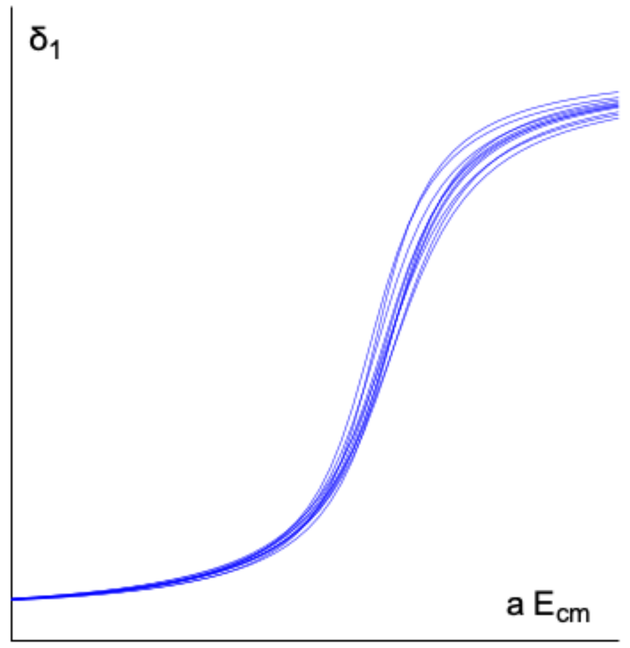


Sample & fit $\lambda_i(t)$

$\{\text{f}_1, \text{f}_2, \ldots, \text{f}_{n_{\text{lev}}}\} \equiv \text{s}$

$\{E_{\text{cm}}\}^{\text{s}^1}, \{E_{\text{cm}}\}^{\text{s}^2}, \ldots$

QC fit

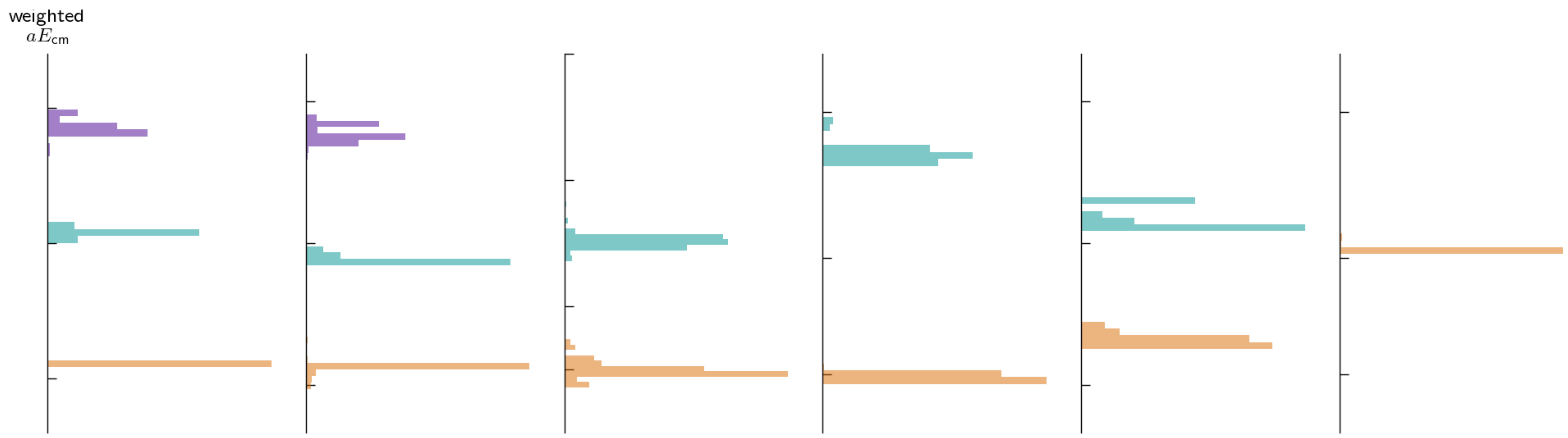


Target $w_t(\text{f}, \delta^{\text{mod}}) = w_{\text{PS}}(\text{f}, \delta^{\text{mod}})w_{\text{corr}}(\text{f})$ ➜ Reweight $w_{\text{PS}}(\text{f}, \delta^{\text{mod}})$

Model-average estimate

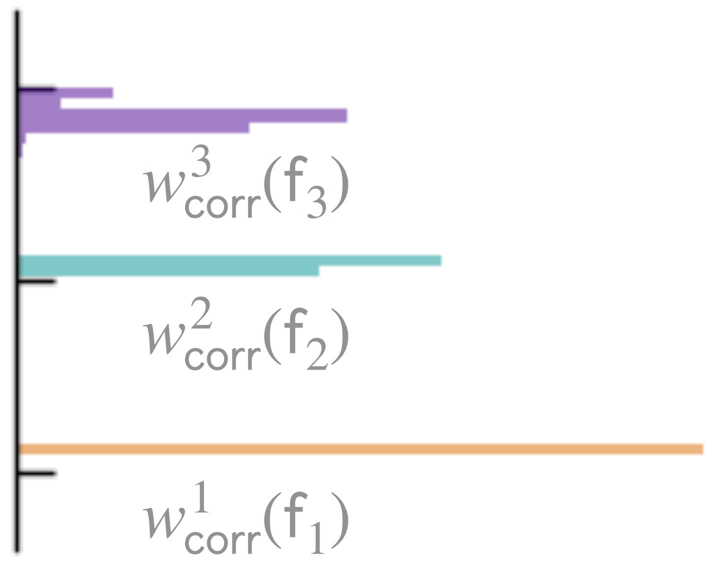
$$\hat{\alpha}^{\text{mod}} = \sum_k \alpha^{\text{mod},\text{s}^k} w^{\text{mod}}_{\text{PS}}(\text{s}^k)$$

Importance Sampling



Proposal dist.

$$w_{\text{corr}}(\mathsf{f}) = \prod_i w_{\text{corr}}^{(i)}(\mathsf{f}_i)$$

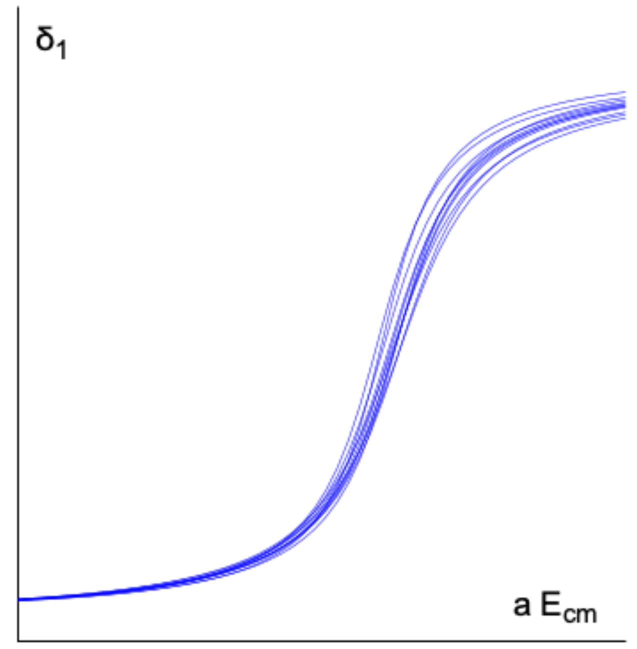


Sample & fit $\lambda_i(t)$

$\{\mathsf{f}_1, \mathsf{f}_2, \ldots, \mathsf{f}_{n_{\text{lev}}}\} \equiv \mathsf{s}$

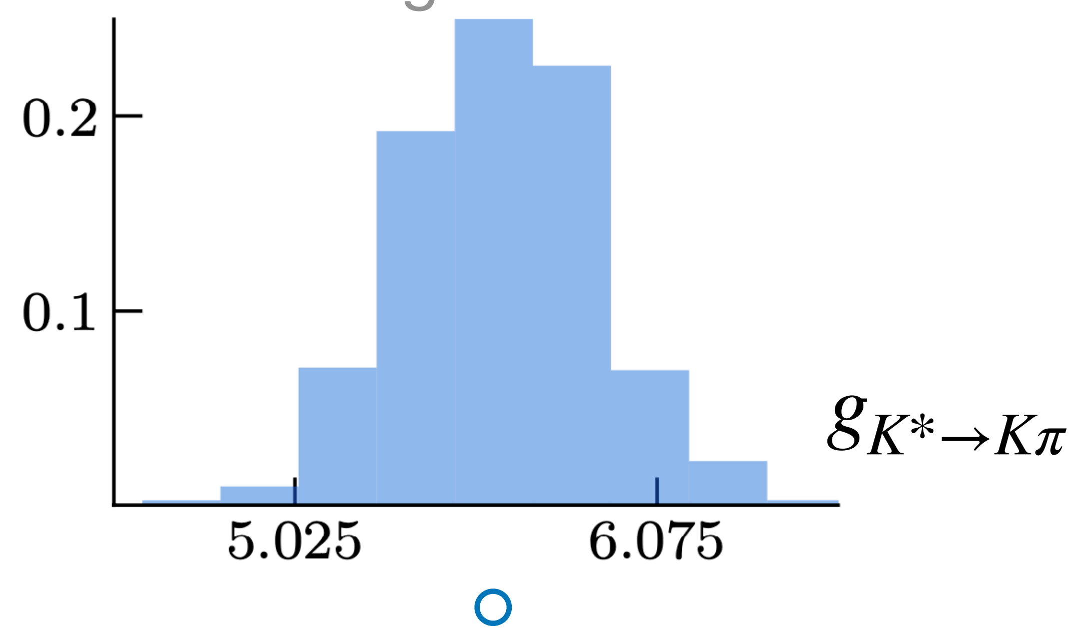
$$\{E_{\text{cm}}\}^{\mathsf{s}^1}, \{E_{\text{cm}}\}^{\mathsf{s}^2}, \ldots$$

QC fit



Target $w_t(\mathsf{f}, \delta^{\text{mod}}) = w_{\text{PS}}(\mathsf{f}, \delta^{\text{mod}})w_{\text{corr}}(\mathsf{f})$ ➡ Reweight $w_{\text{PS}}(\mathsf{f}, \delta^{\text{mod}})$

central value histogram



Model-average estimate

$$\hat{\alpha}^{\text{mod}} = \sum_k \alpha^{\text{mod},\mathsf{s}^k} w_{\text{PS}}^{\text{mod}}(\mathsf{s}^k)$$

e.g. Breit-Wigner

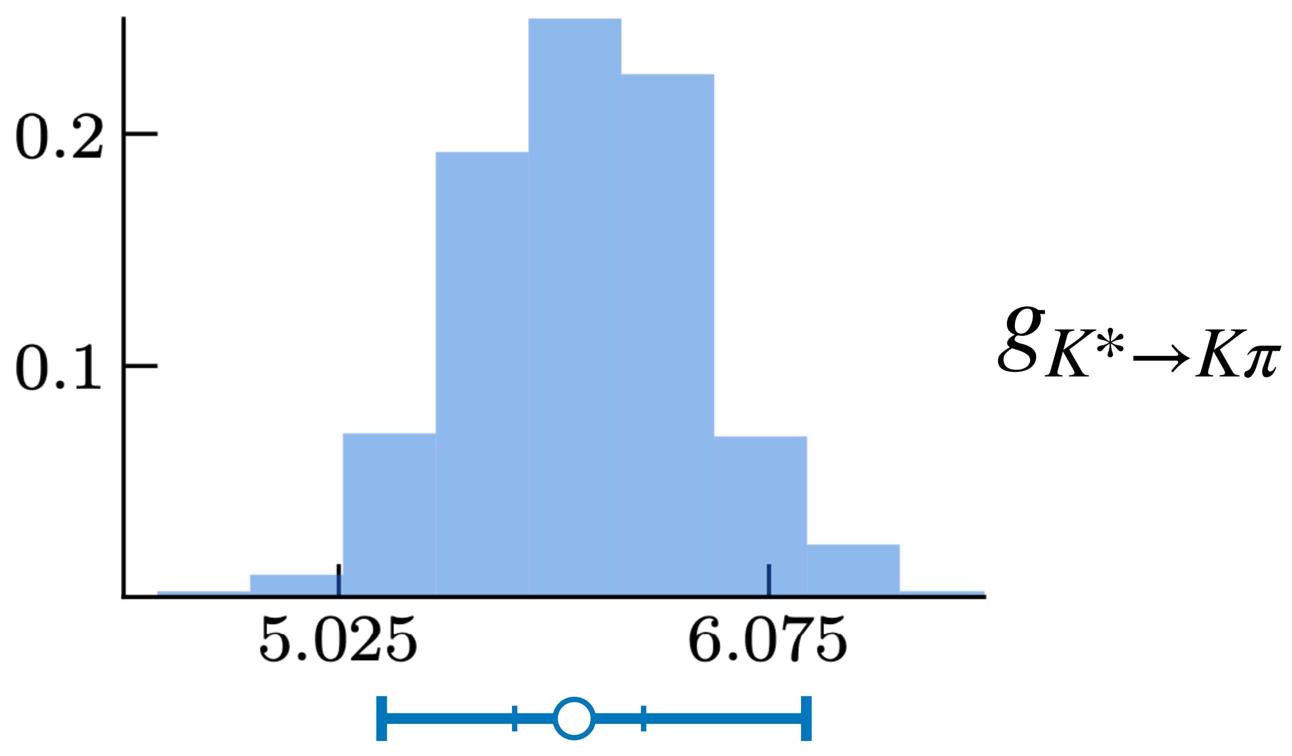
Result prescription

- data-driven systematic: weighted $95\,\%$ confidence interval of (central) weighted mean

- statistical: fluctuation of above over replicas

50000 fit-ranges

central value
histogram

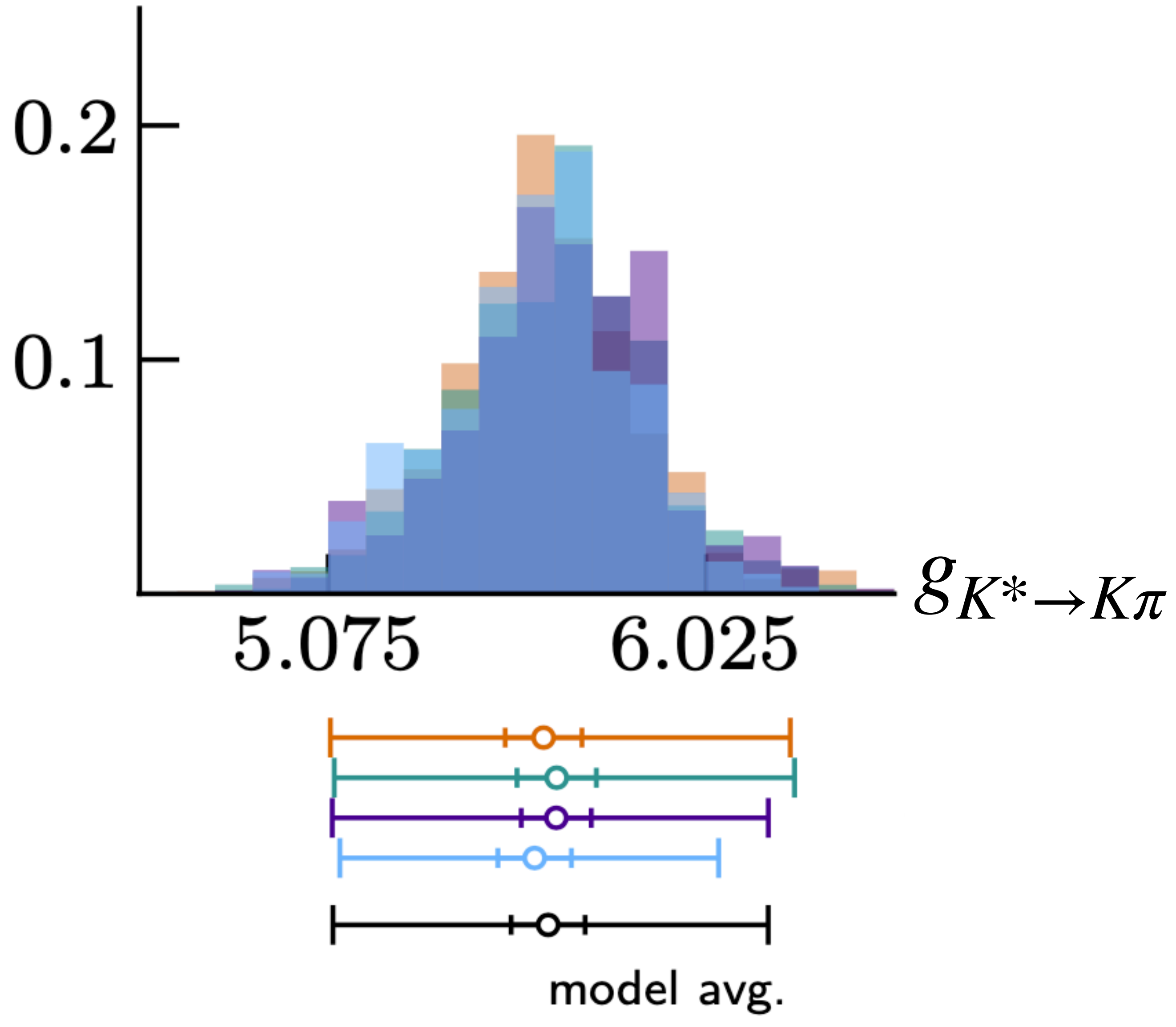


Result prescription

- data-driven systematic: weighted $95\ \%$ confidence interval of (central) weighted mean

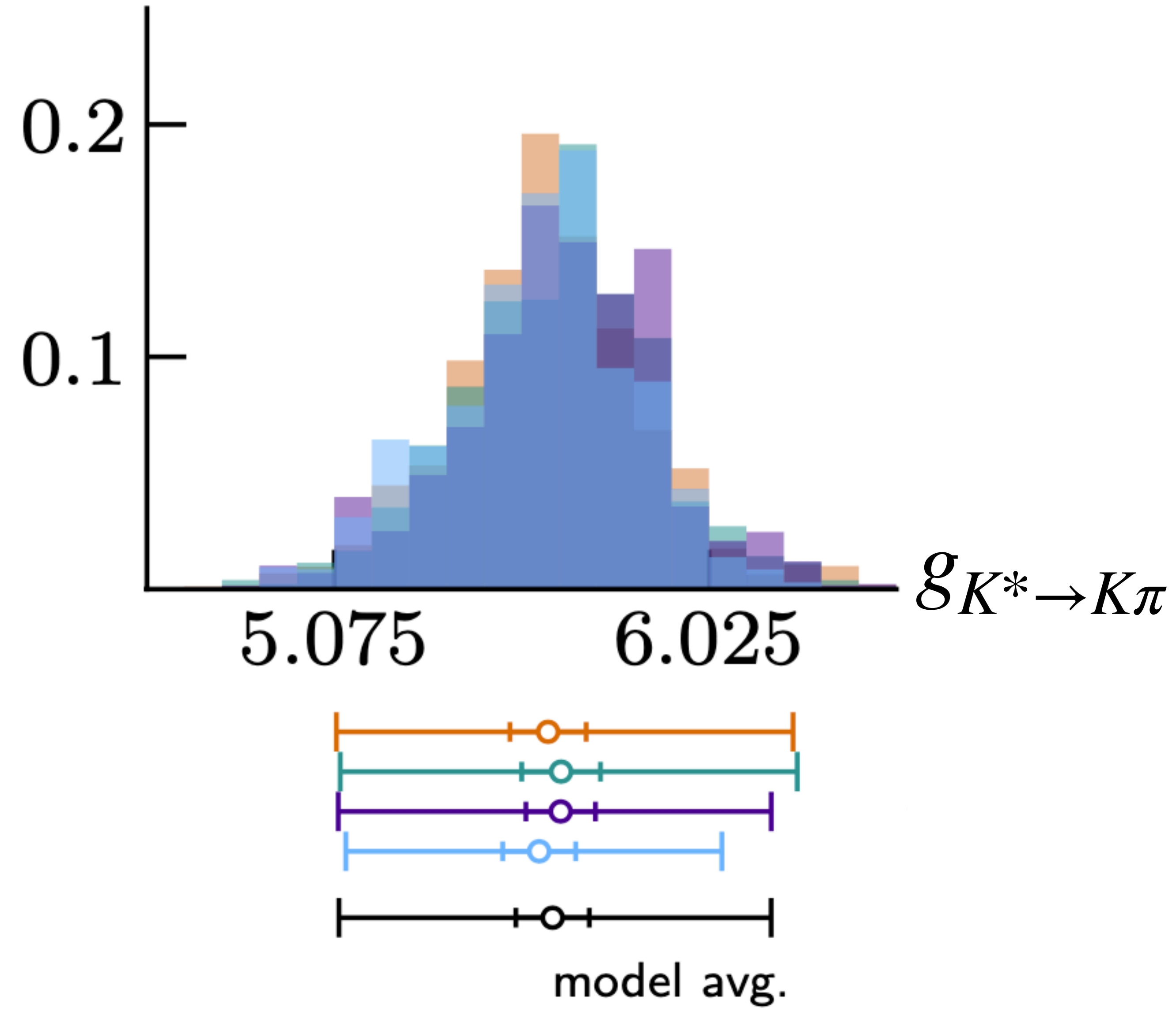
- statistical: fluctuation of above over replicas

50000 fit-ranges $\times$ 4 cuts



Result prescription

- data-driven systematic: weighted $95\,\%$ confidence interval of (central) weighted mean
- statistical: fluctuation of above over replicas



50000 fit-ranges $\times$ 4 cuts

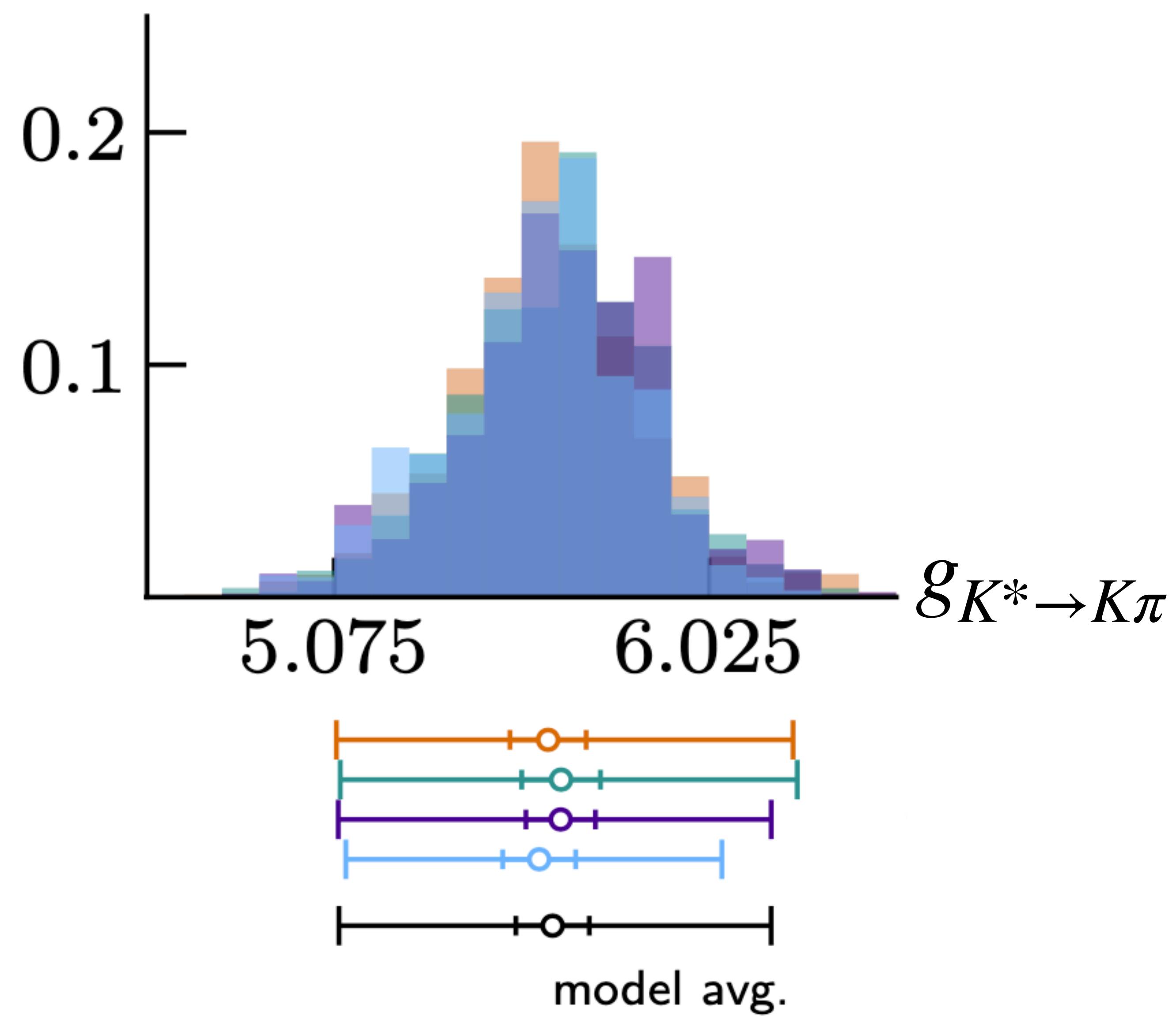
$\times$ 2000 replicas

$\times$ δ^{BW} Breit-Wigner, δ^{ERE} effective range

Result prescription

- data-driven systematic: weighted $95\ \%$ confidence interval of (central) weighted mean

- statistical: fluctuation of above over replicas

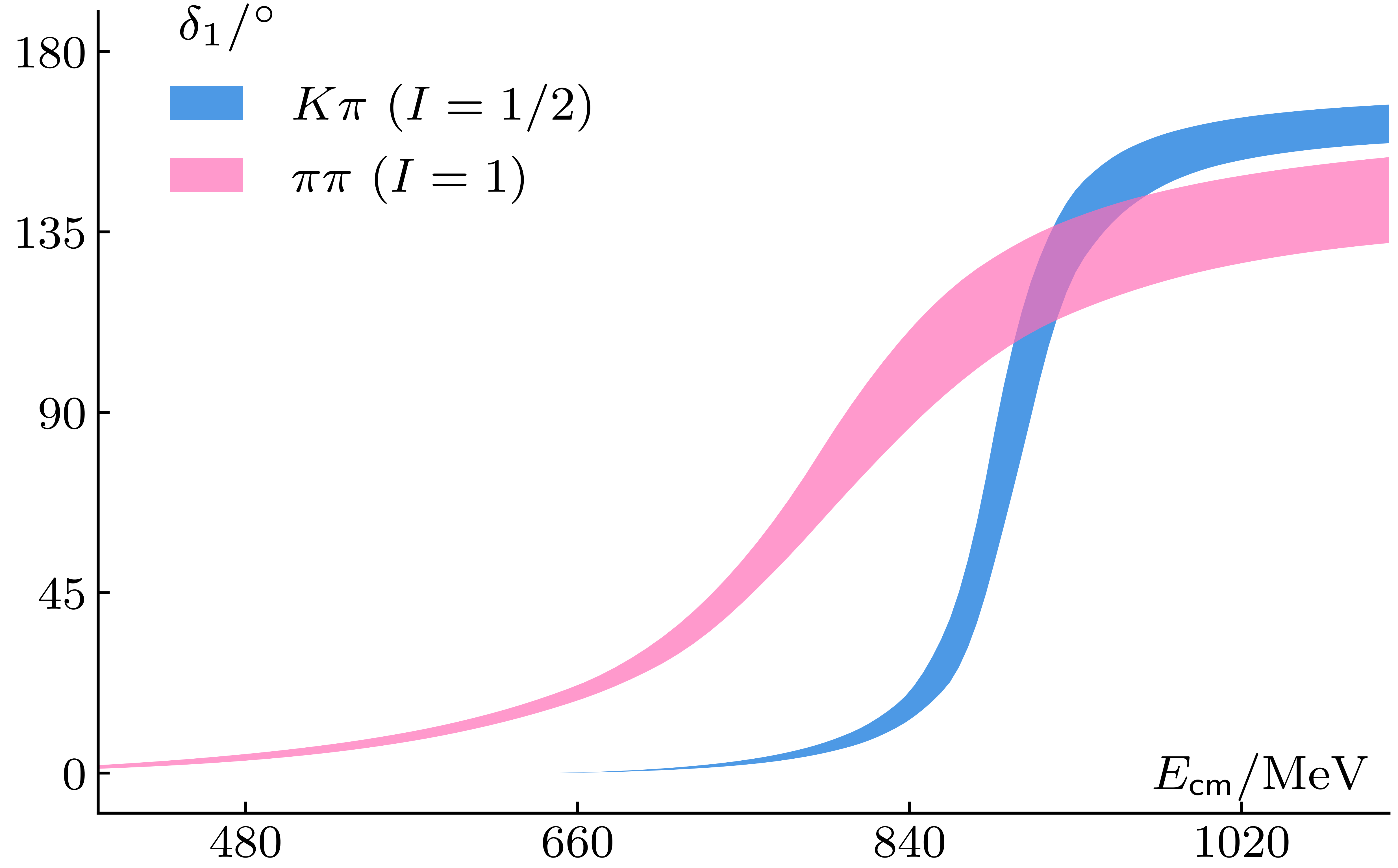


50000 fit-ranges $\times$ 4 cuts

$\times$ 2000 replicas

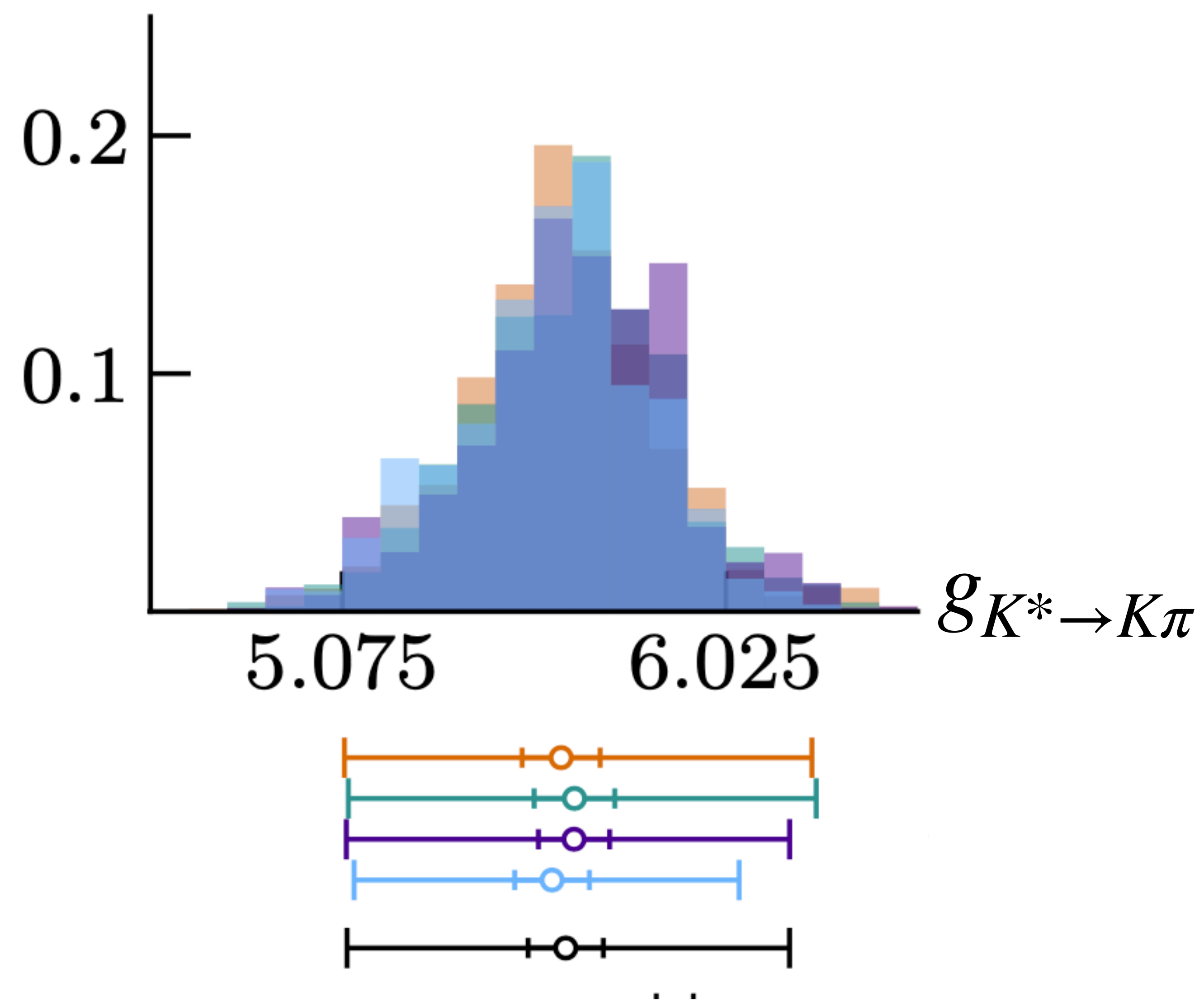
$\times$ δ^{BW} Breit-Wigner, δ^{ERE} effective range

$$\sum_{\text{fit ranges}} \to \sum_{\text{mod}} \sum_{\text{cuts}} \sum_{\text{fit ranges}}$$



Result prescription

- data-driven systematic: weighted $95\ \%$ confidence interval of (central) weighted mean
- statistical: fluctuation of above over replicas

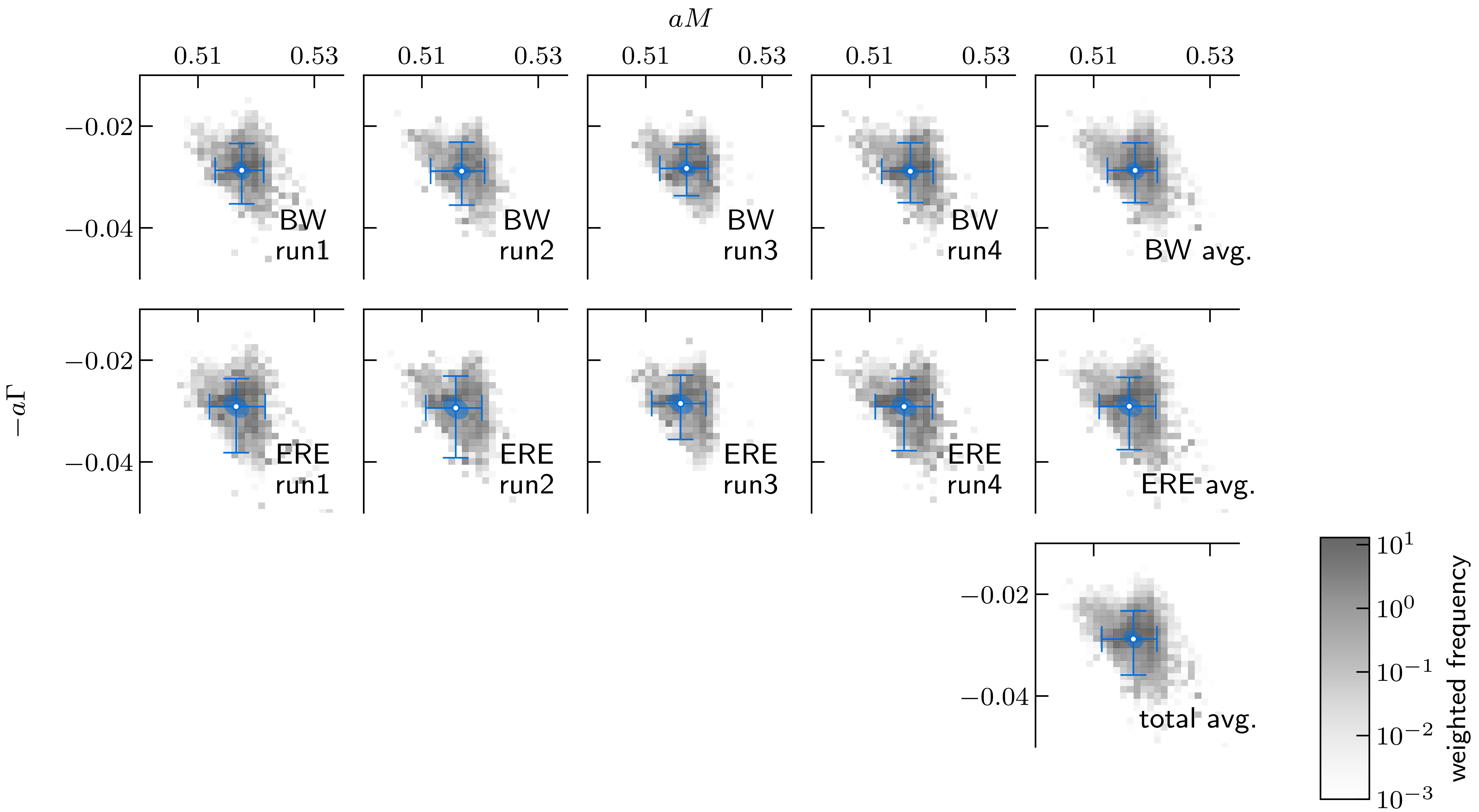


50000 fit-ranges $\times$ 4 cuts

$\times$ 2000 replicas

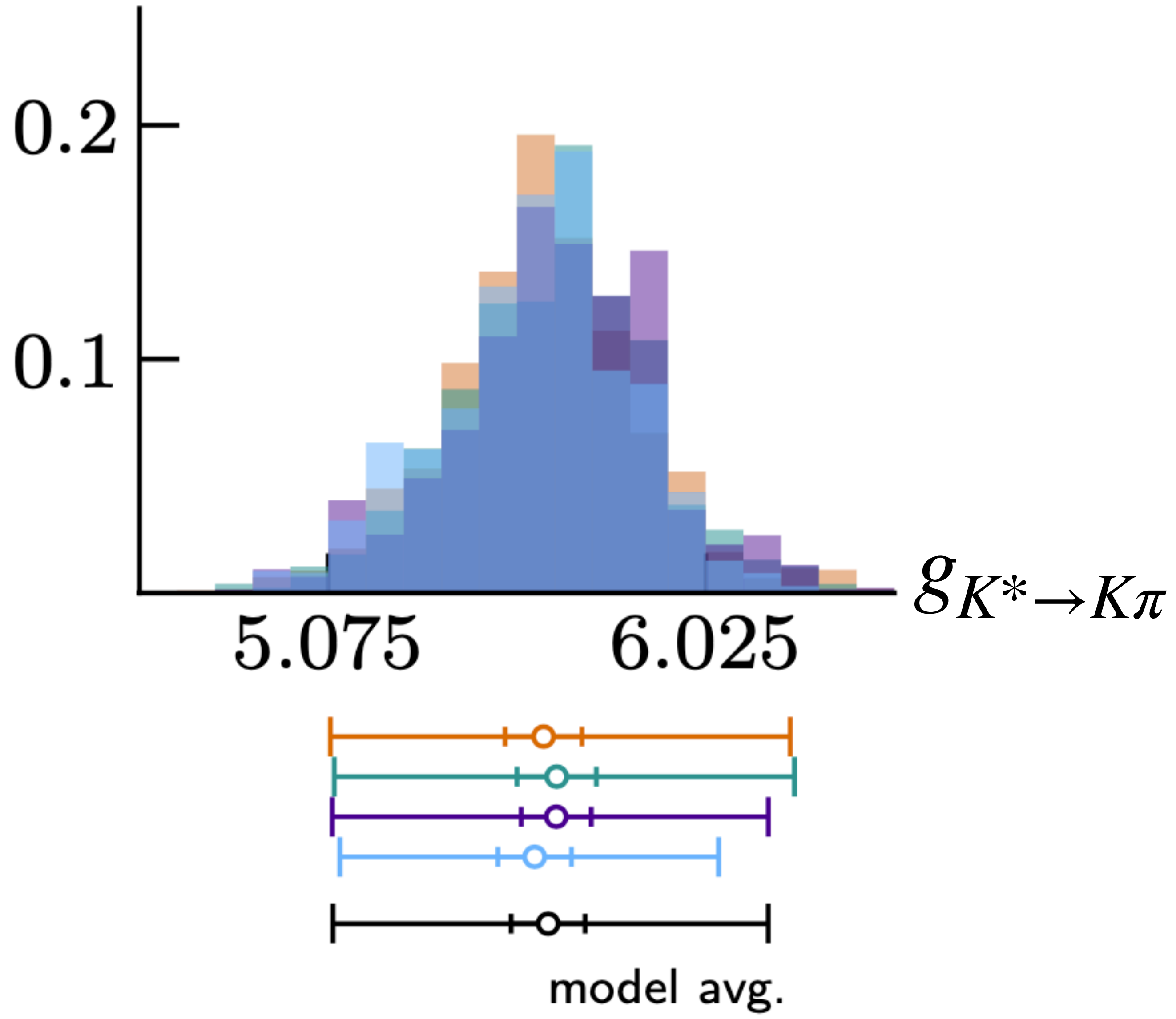
$\times$ δ^{BW} Breit-Wigner, δ^{ERE} effective range

$$\sum_{\text{fit ranges}} \rightarrow \sum_{\text{mod}} \sum_{\text{cuts}} \sum_{\text{fit ranges}}$$



Result prescription

- data-driven systematic: weighted $95\ \%$ confidence interval of (central) weighted mean
- statistical: fluctuation of above over replicas

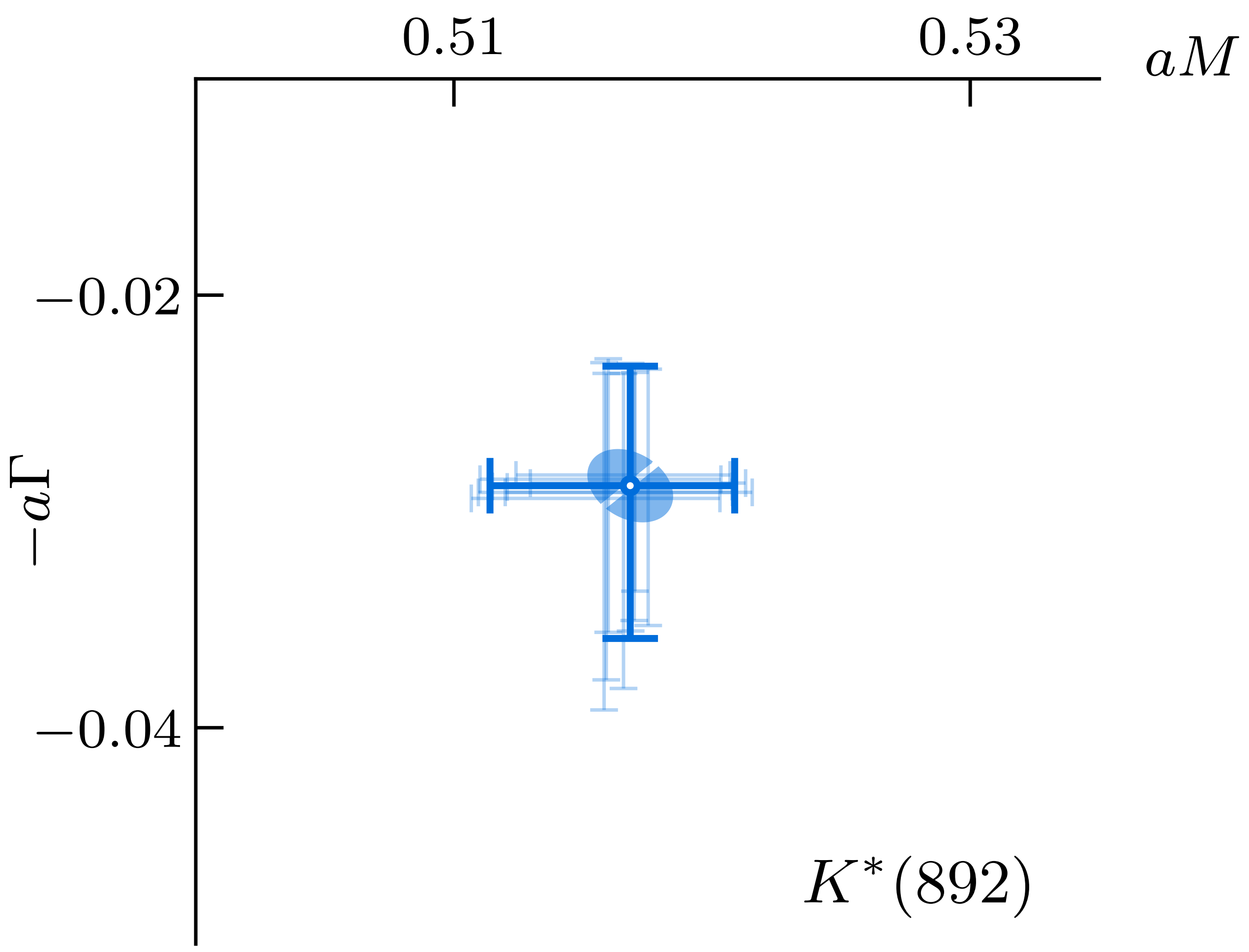


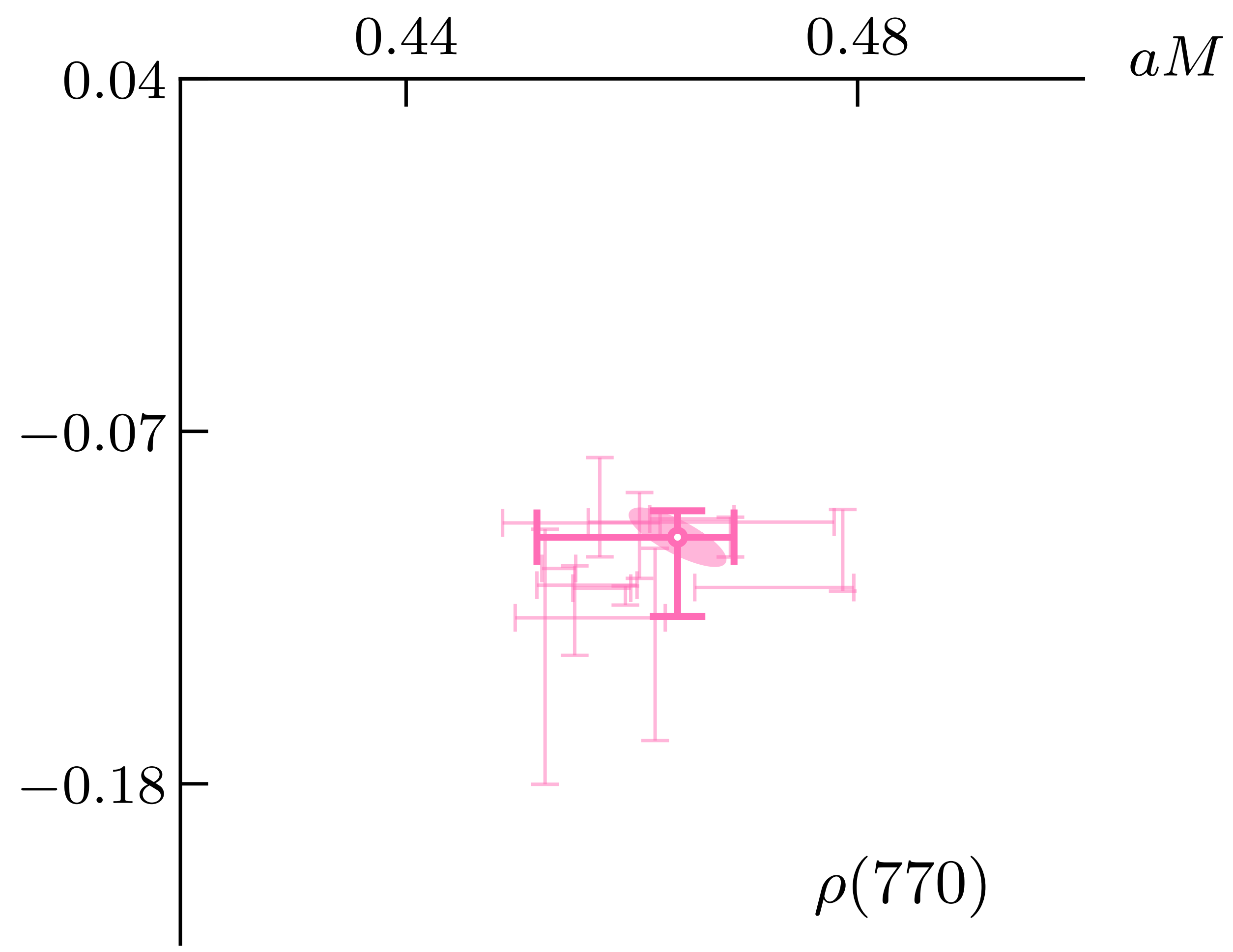
50000 fit-ranges $\times$ 4 cuts

$\times$ 2000 replicas

$\times$ δ^{BW} Breit-Wigner, δ^{ERE} effective range

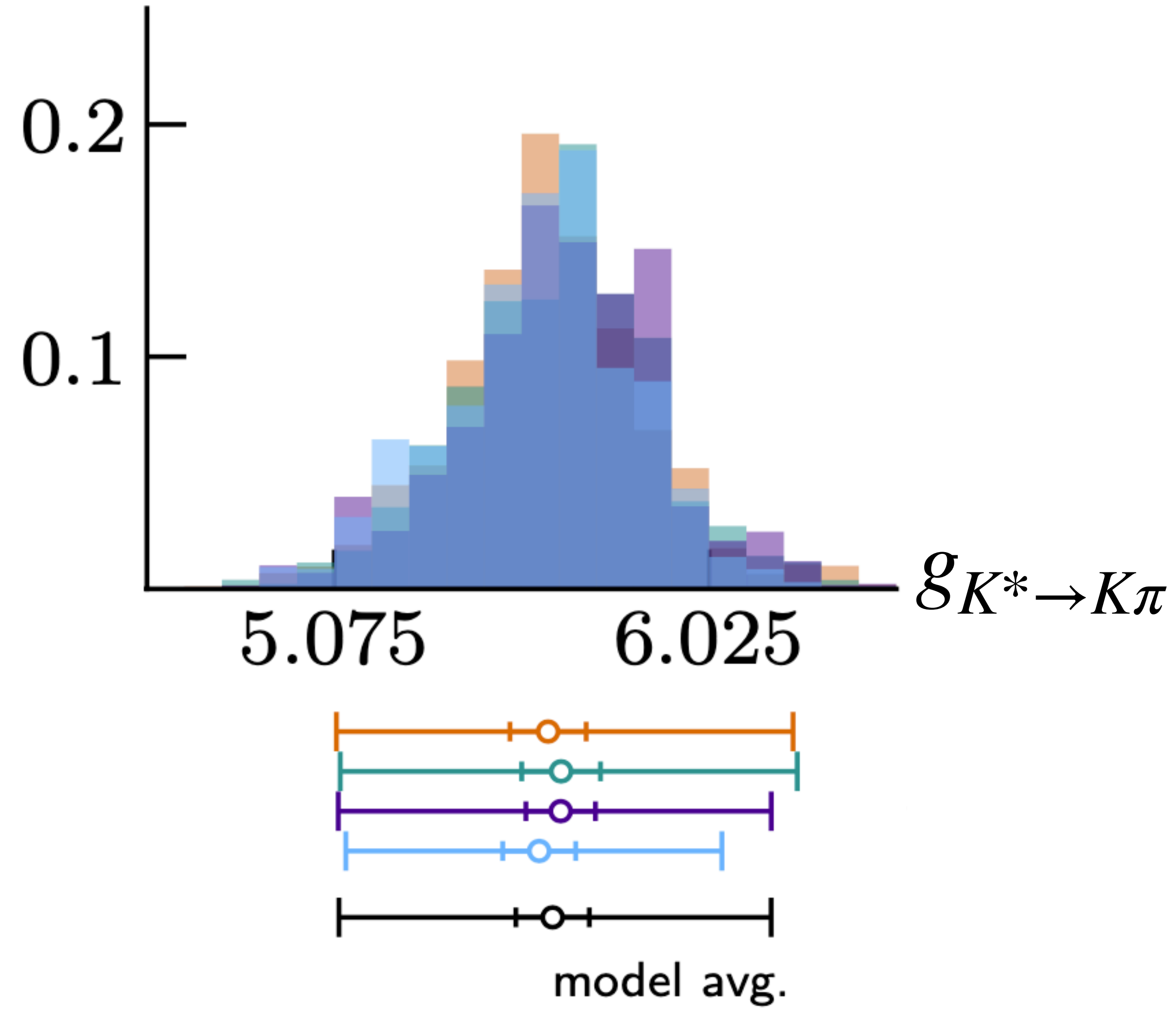
$$\sum_{\text{fit ranges}} \to \sum_{\text{mod}} \sum_{\text{cuts}} \sum_{\text{fit ranges}}$$





Result prescription

- data-driven systematic: weighted $95\ \%$ confidence interval of (central) weighted mean
- statistical: fluctuation of above over replicas

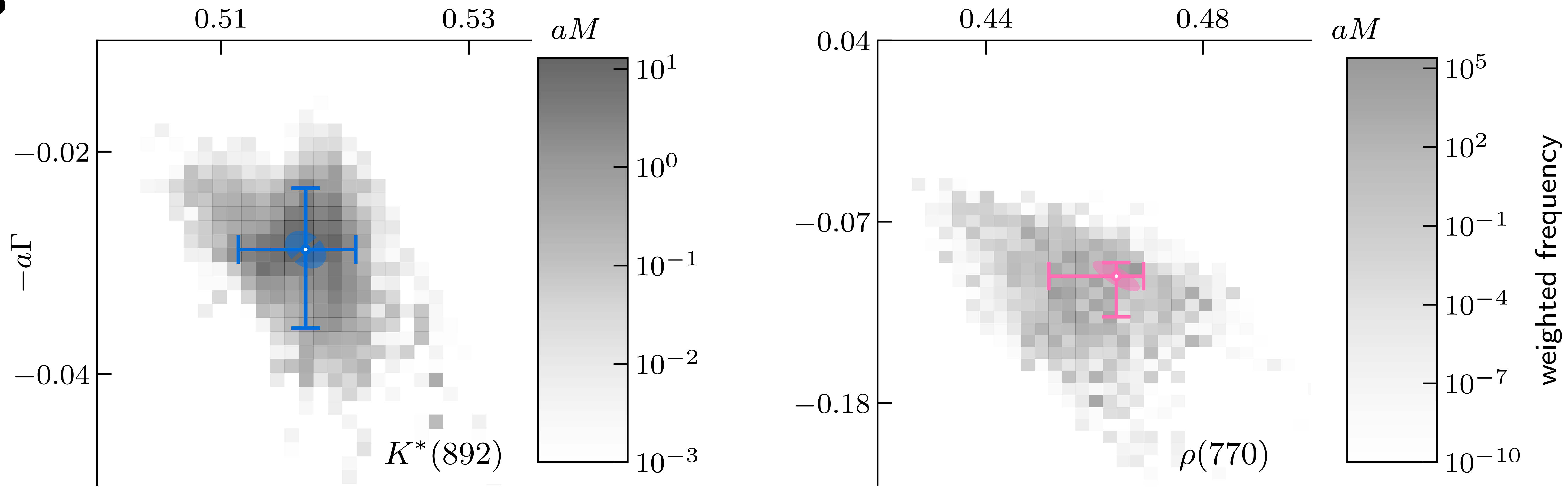


50000 fit-ranges $\times$ 4 cuts

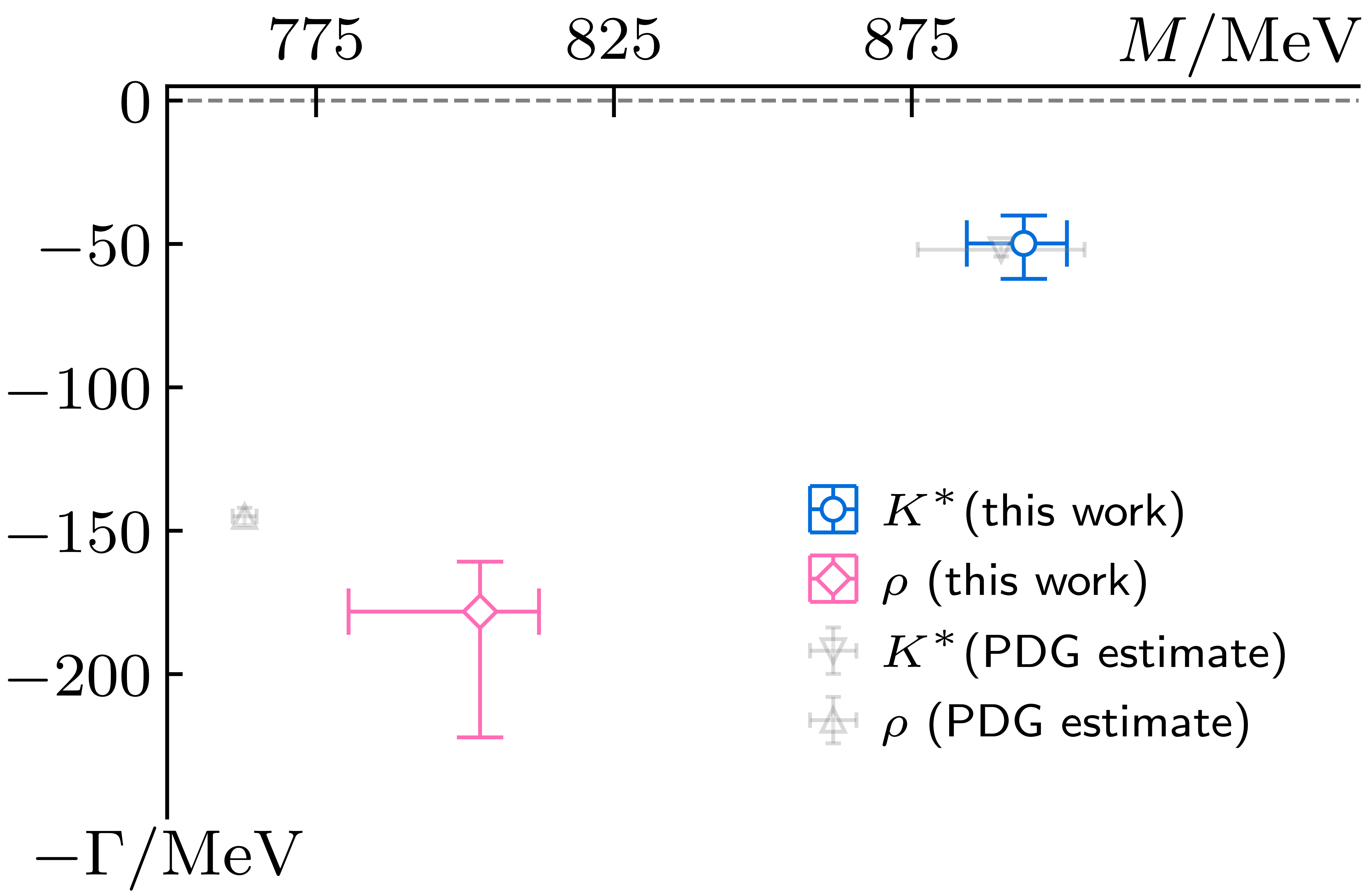
$\times$ 2000 replicas

$\times$ δ^{BW} Breit-Wigner, δ^{ERE} effective range

$$\sum_{\text{fit ranges}} \rightarrow \sum_{\text{mod}} \sum_{\text{cuts}} \sum_{\text{fit ranges}}$$



Physical units

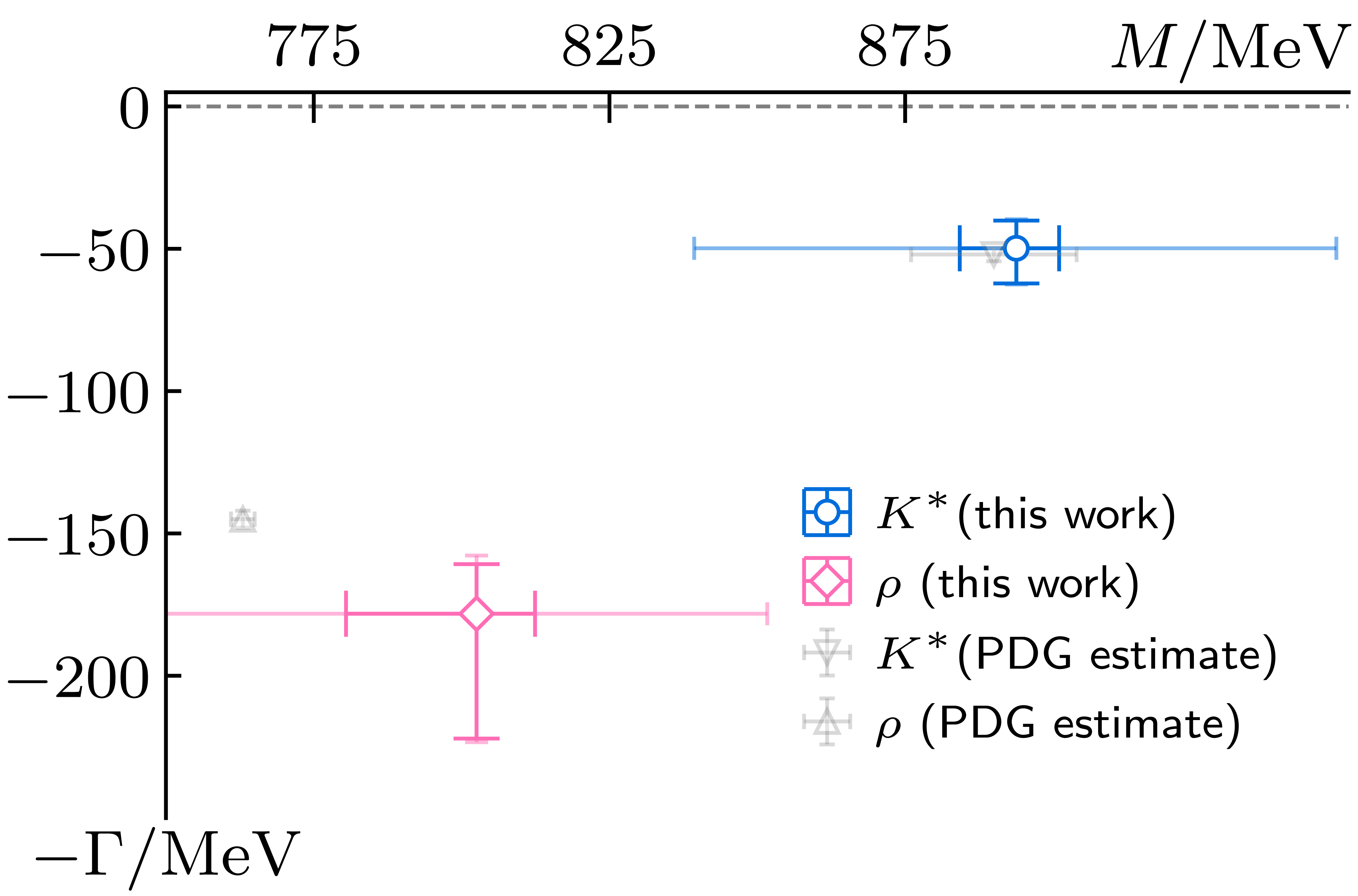


$$K^*(892) \begin{cases} M &= 893(2)(8)\ \text{MeV} \\ \Gamma &= 51(2)(11)\ \text{MeV} \end{cases}$$

$$\rho(770) \begin{cases} M &= 796(5)(15)\ \text{MeV} \\ \Gamma &= 192(10)(28)\ \text{MeV} \end{cases}$$

Statistical and data-driven systematic (quadrature in plot)

Physical units



Statistical and data-driven systematic (quadrature in plot)

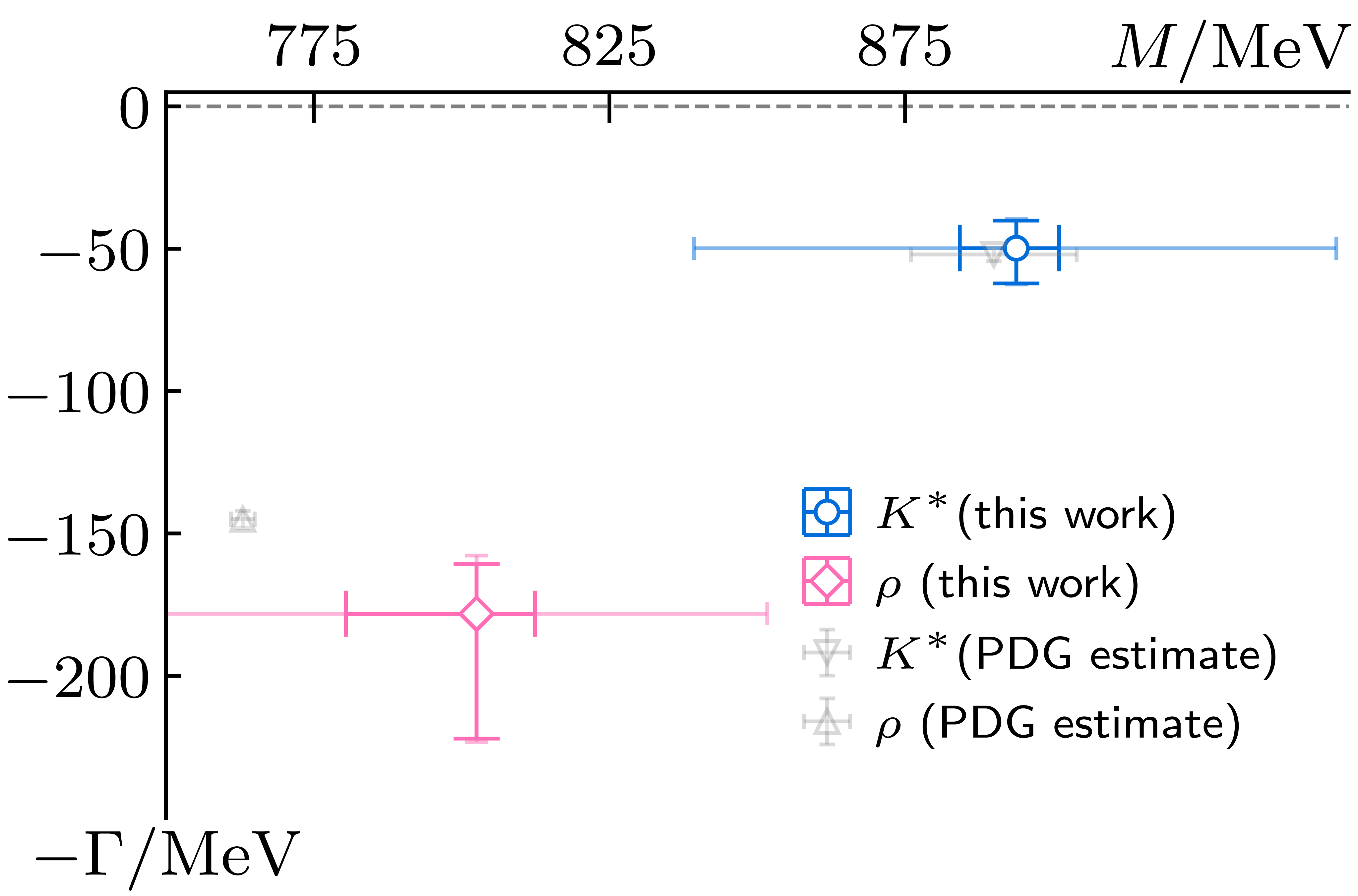
$$K^*(892)\begin{cases} M &= 893(2)(8)(54)\ \mathrm{MeV} \\ \Gamma &= 51(2)(11)(3)\ \mathrm{MeV}\end{cases}$$

$$\rho(770)\begin{cases} M &= 796(5)(15)(48)\ \mathrm{MeV} \\ \Gamma &= 192(10)(28)(12)\ \mathrm{MeV}\end{cases}$$

Other: single lattice spacing and naive power counting :

- assume $(a\Lambda_{QCD})^2 \approx 5\,\%$ conservative *discretisation* uncertainty + other estimated extra systematics $\sim 6\,\%$ total

Physical units



Statistical and data-driven systematic (quadrature in plot)

$$K^*(892) \begin{cases} M = 893(2)(8)(54)\ \text{MeV} \\ \Gamma = 51(2)(11)(3)\ \text{MeV} \end{cases}$$

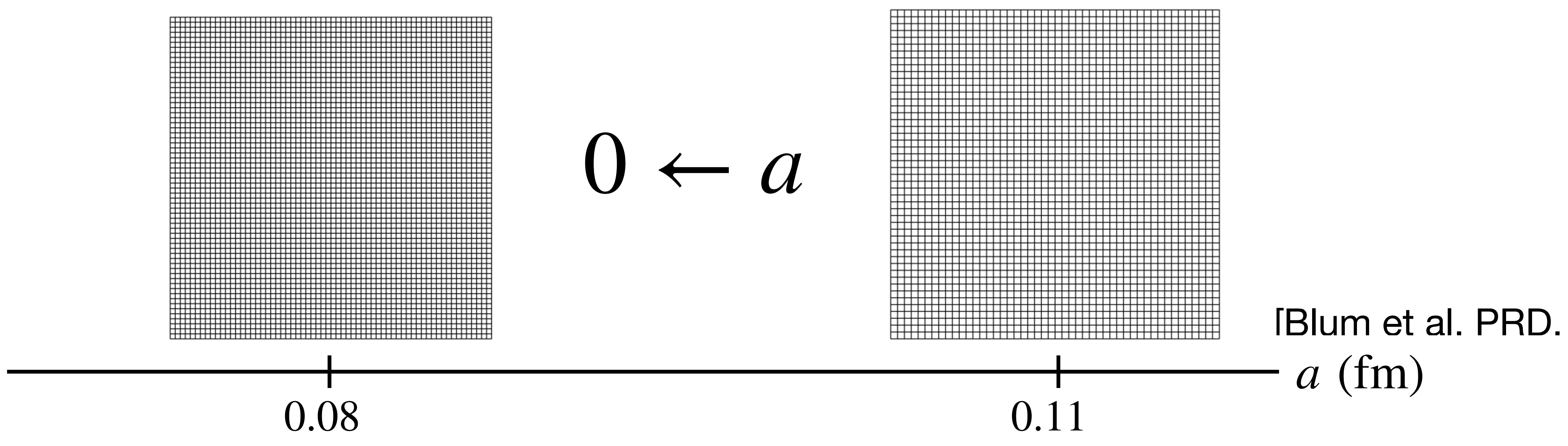
$$\rho(770) \begin{cases} M = 796(5)(15)(48)\ \text{MeV} \\ \Gamma = 192(10)(28)(12)\ \text{MeV} \end{cases}$$

Other: single lattice spacing and naive power counting :

- assume $(a\Lambda_{QCD})^2 \approx 5\,\%$ conservative *discretisation* uncertainty + other estimated extra systematics $\sim 6\,\%$ total

next frontier:
continuum limit

[Green et al, PRL, 2021]
[Peterken & Hansen, 2408.07062, 2024]



Outlook I

- Hadronic $D \to K\pi$ decays at $SU(3)_f$ point [Hansen, Black, Mukherjee, et al]

$$A(D \to h_1 h_2) = \mathcal{C}^{\rm LL}_{n,L,h_1,h_2}\left[\lim_{a\to 0} Z^{\overline{\rm MS}}\langle n, L|\mathcal{H}_W|D, L\rangle\right]$$

- Non-perturbative renormalisation of four-quark operators
- Reliable creation of excited multi-hadron final states
- Removal of discretisation effects
- Formalism to relate finite-volume matrix elements to the infinite volume
- Extraction of the matrix element from three-point correlation functions

["Charming Lattice QCD, 2025" talks: M. Black, R. Mukherjee]

- Heavy-flavour weak decays into resonant scattering states [Erben, et al]

$m_\pi = 230$ MeV
Domain-wall fermions good chirality
Supercomputer time for 2025

3pt-functions

$$\langle n, \mathbf{P}\,|\,J^\mu(0,\mathbf{q})\,|\,B, \mathbf{p}_B\rangle$$

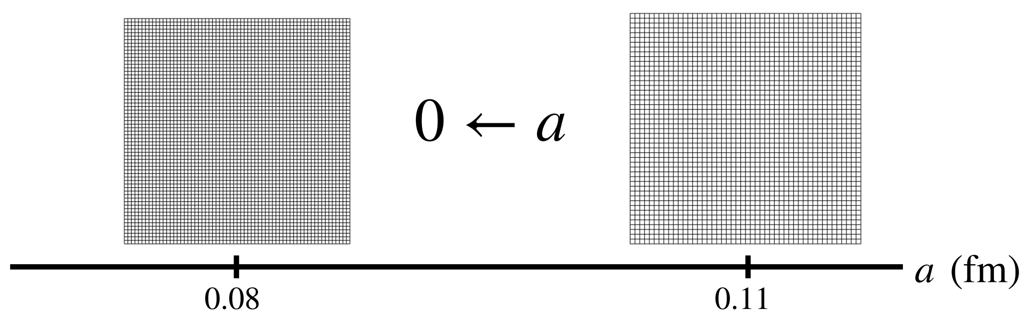
e.g. see [Erben, Lattice2024 plenary]
[Leskovec et al, PRL, 2025]

$B_{(s)} \to K^* \ell^+ \ell^-$
$B \to \rho \ell \nu$
⋮

Conclusions

$\begin{cases} K^*(892) \text{ and } \rho(770) \text{ at } m_\pi \approx 139 \text{ MeV} \\ \text{Data-driven systematics via sampling method in finite-volume analysis} \end{cases}$

Important towards precision



- continuum limit
- reliable errors $\begin{cases} \text{analysis systematics ✓} \\ \Rightarrow \text{improve operators, } \geq \text{ 3-body, …} \end{cases}$

Science and Technology Facilities Council

DiRAC

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 813942

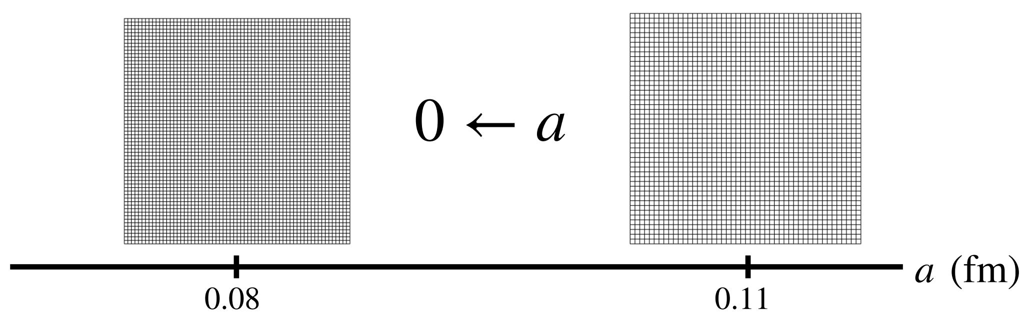
Conclusions

$K^*(892)$ and $\rho(770)$ at $m_\pi \approx 139\ \mathrm{MeV}$

Data-driven systematics via sampling method in finite-volume analysis

Important towards precision

- continuum limit



- reliable errors
  - analysis systematics ✓
  - ⇒improve operators, $\geq$ 3-body, …

Use data/code/infrastructure

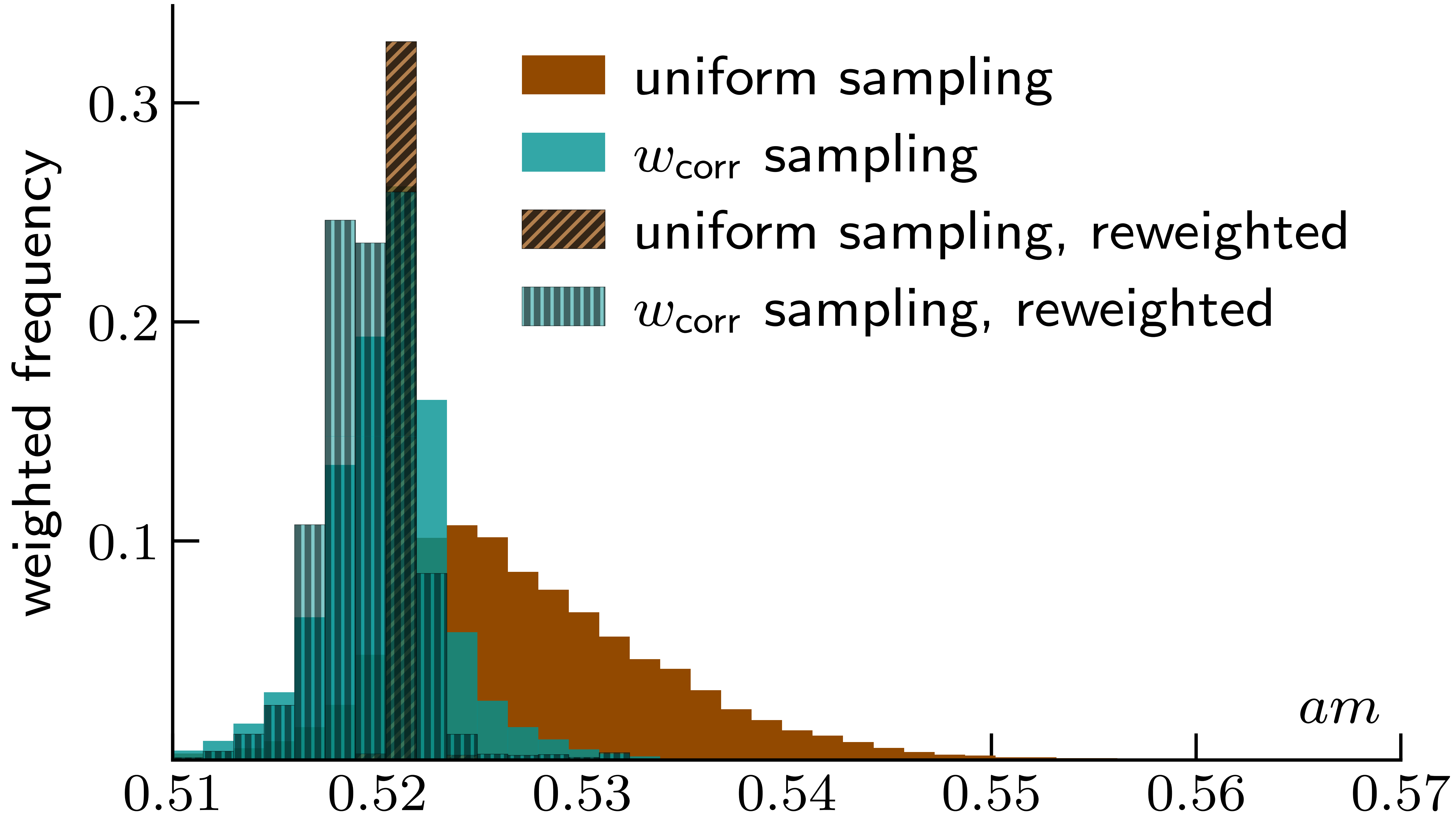
- hadronic decays $D \to K\pi, \ldots$
- heavy flavour weak decays $B_{(s)} \to K^* \ell^+ \ell^-$, $B \to \rho \ell \nu$

Thanks for the attention!

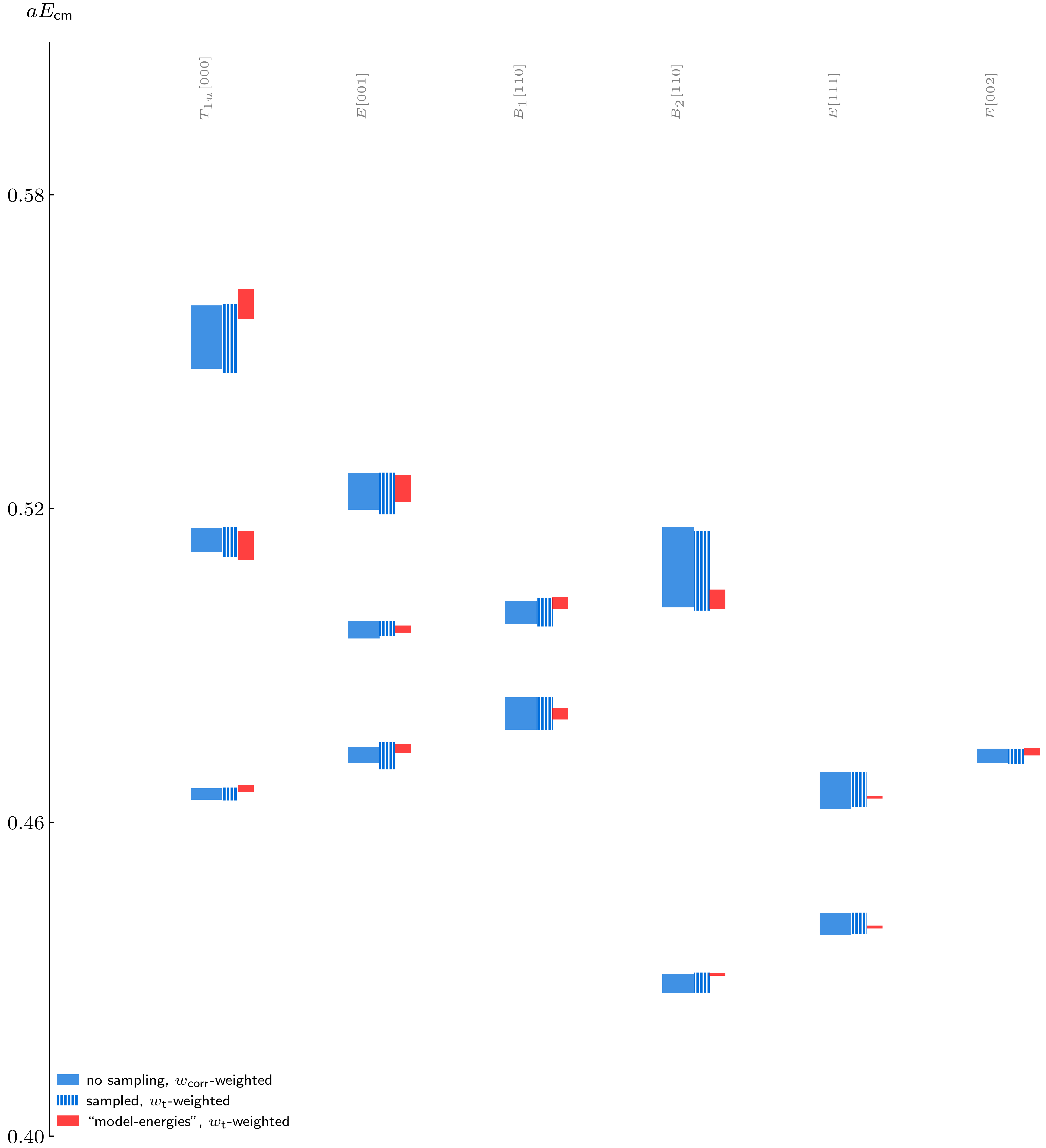
Science and Technology Facilities Council
DiRAC
This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 813942

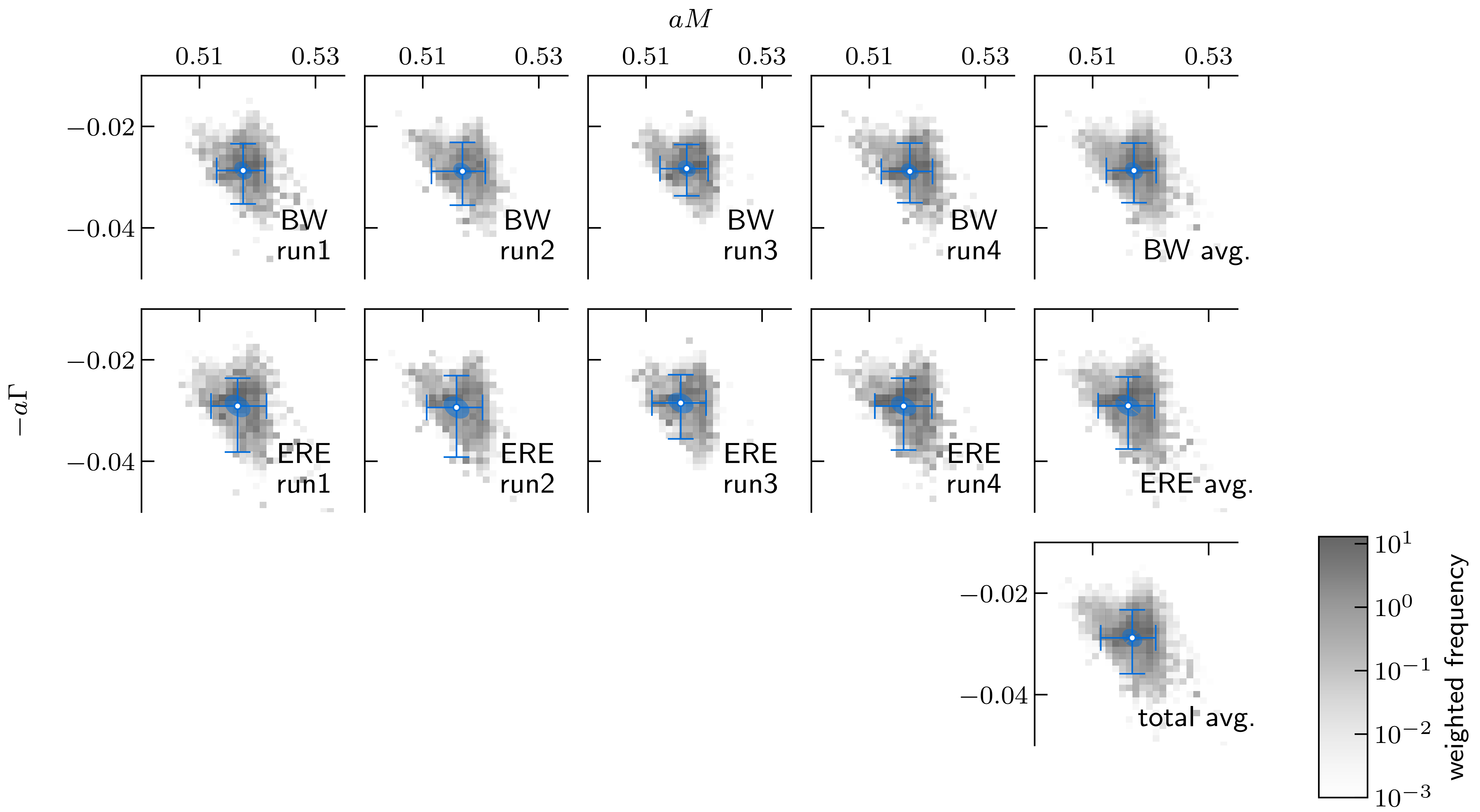
Extra

Uniform vs w_{corr}

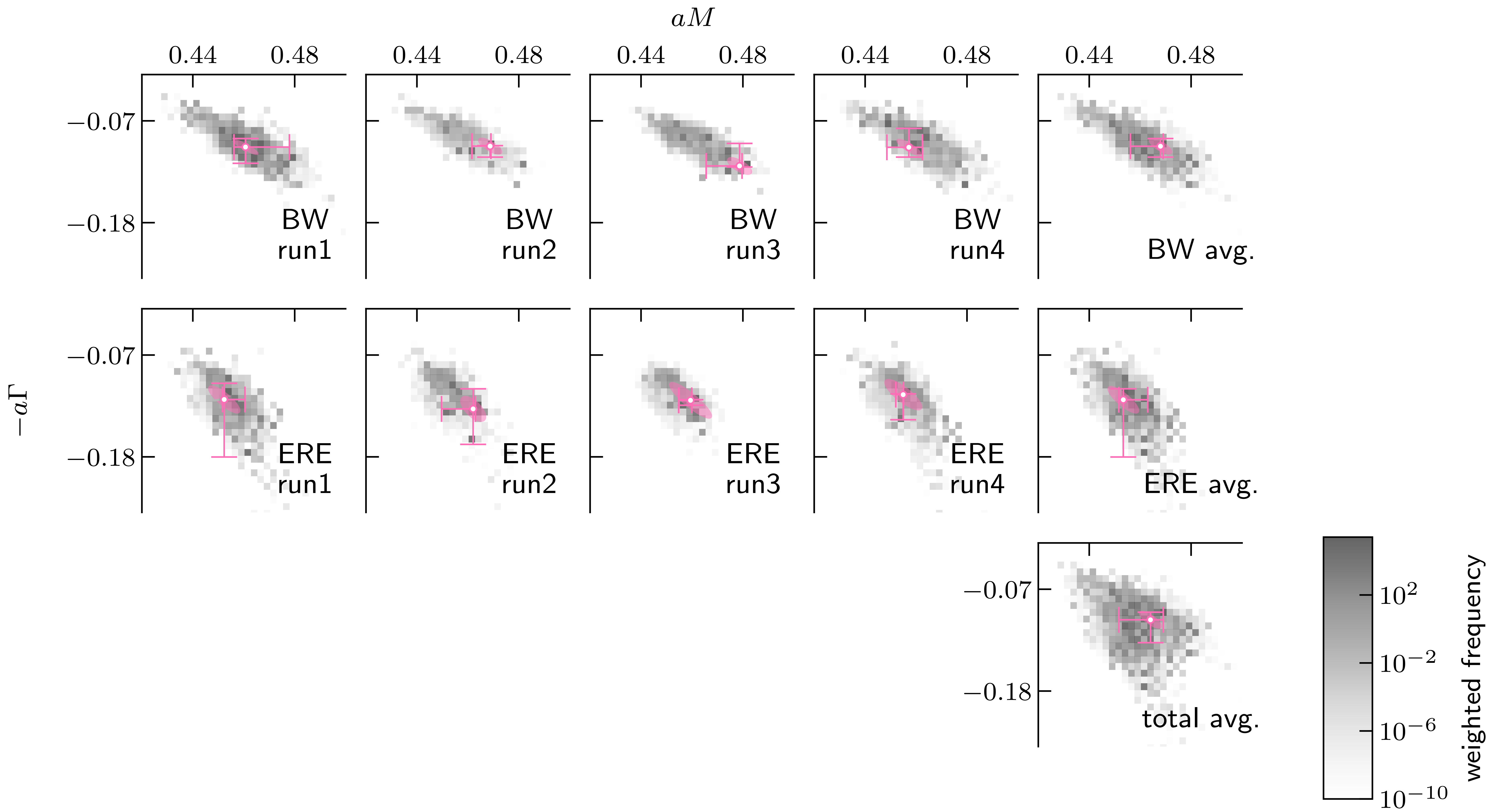


Spectrum-scattering consistency

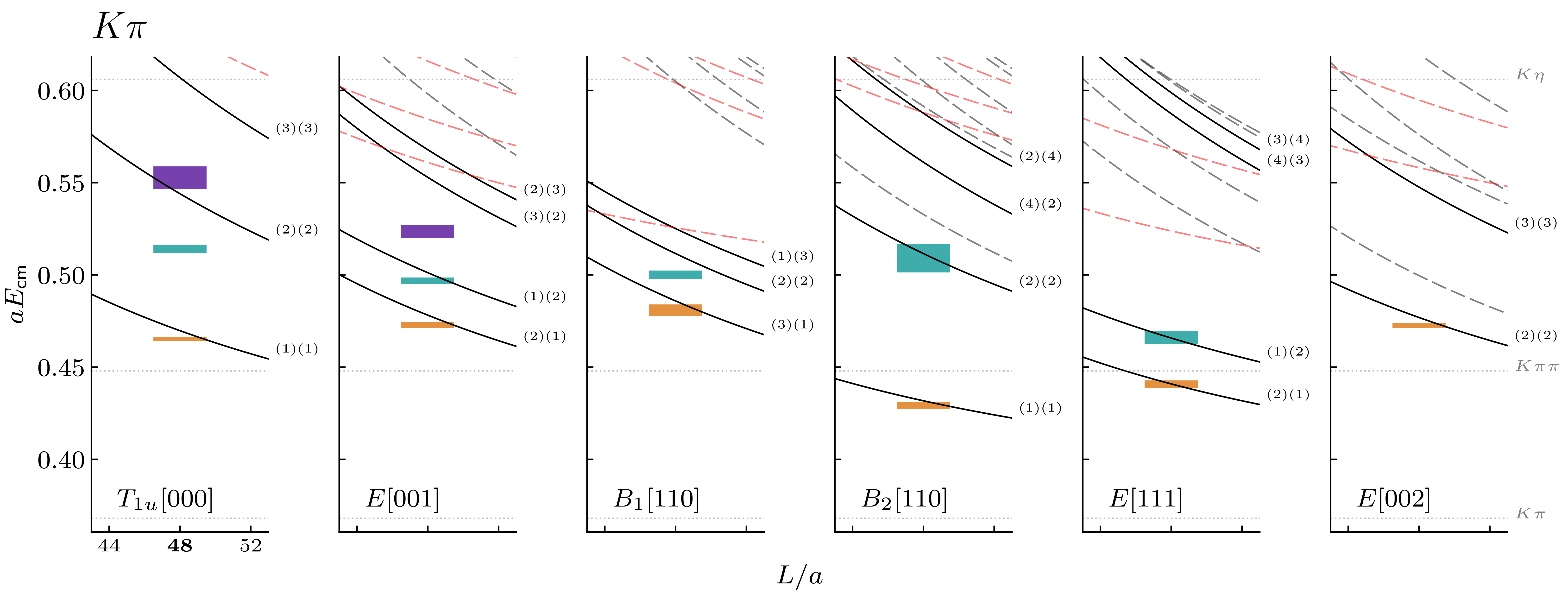




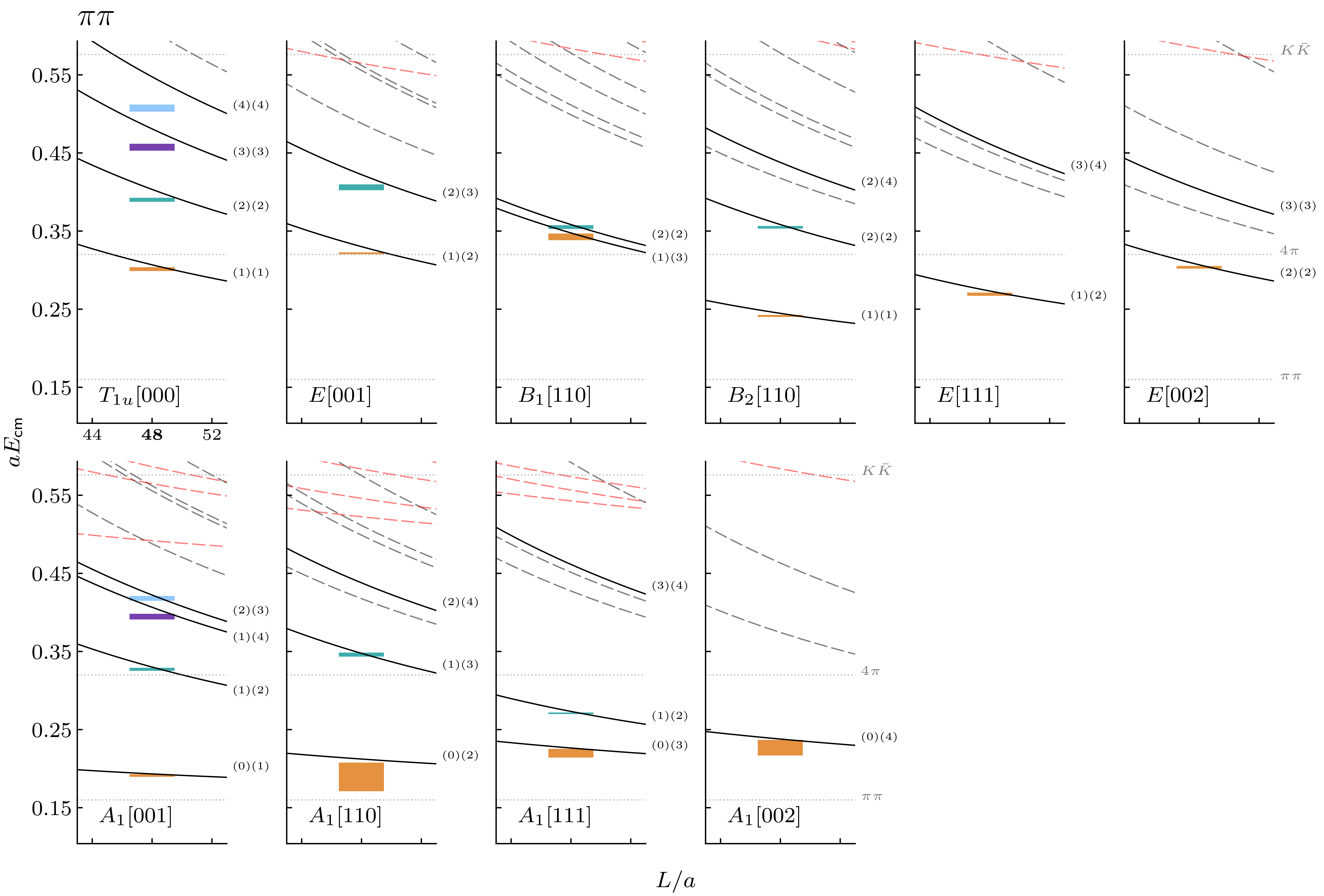
aM
0.51
0.53
−0.02
−0.04
BW run1
BW run2
BW run3
BW run4
BW avg.
ERE run1
ERE run2
ERE run3
ERE run4
ERE avg.
total avg.
$-a\Gamma$
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
weighted frequency



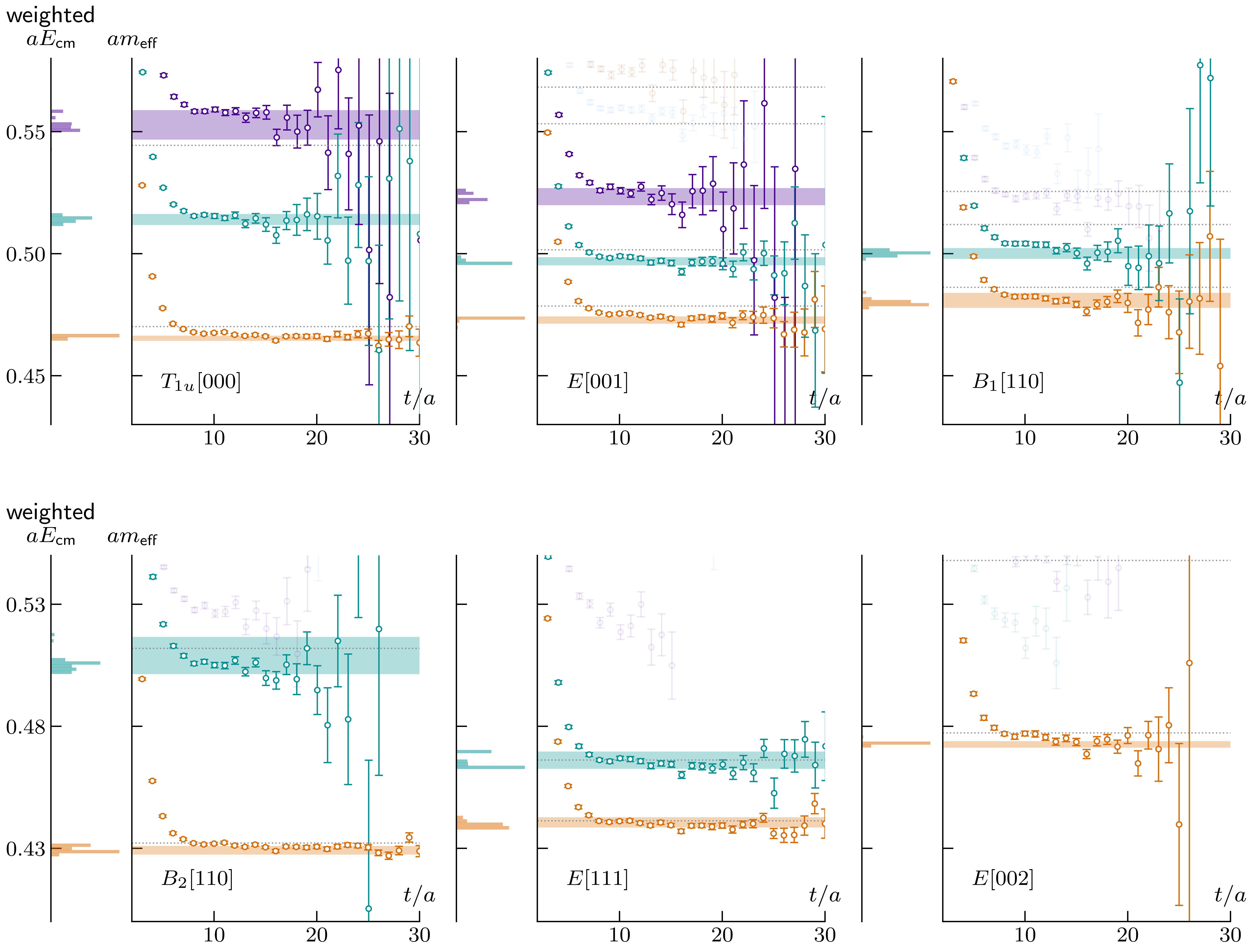
aM
0.44
0.48
−0.07
−0.18
$-a\Gamma$
BW run1
BW run2
BW run3
BW run4
BW avg.
ERE run1
ERE run2
ERE run3
ERE run4
ERE avg.
total avg.
10^{2}
10^{-2}
10^{-6}
10^{-10}
weighted frequency

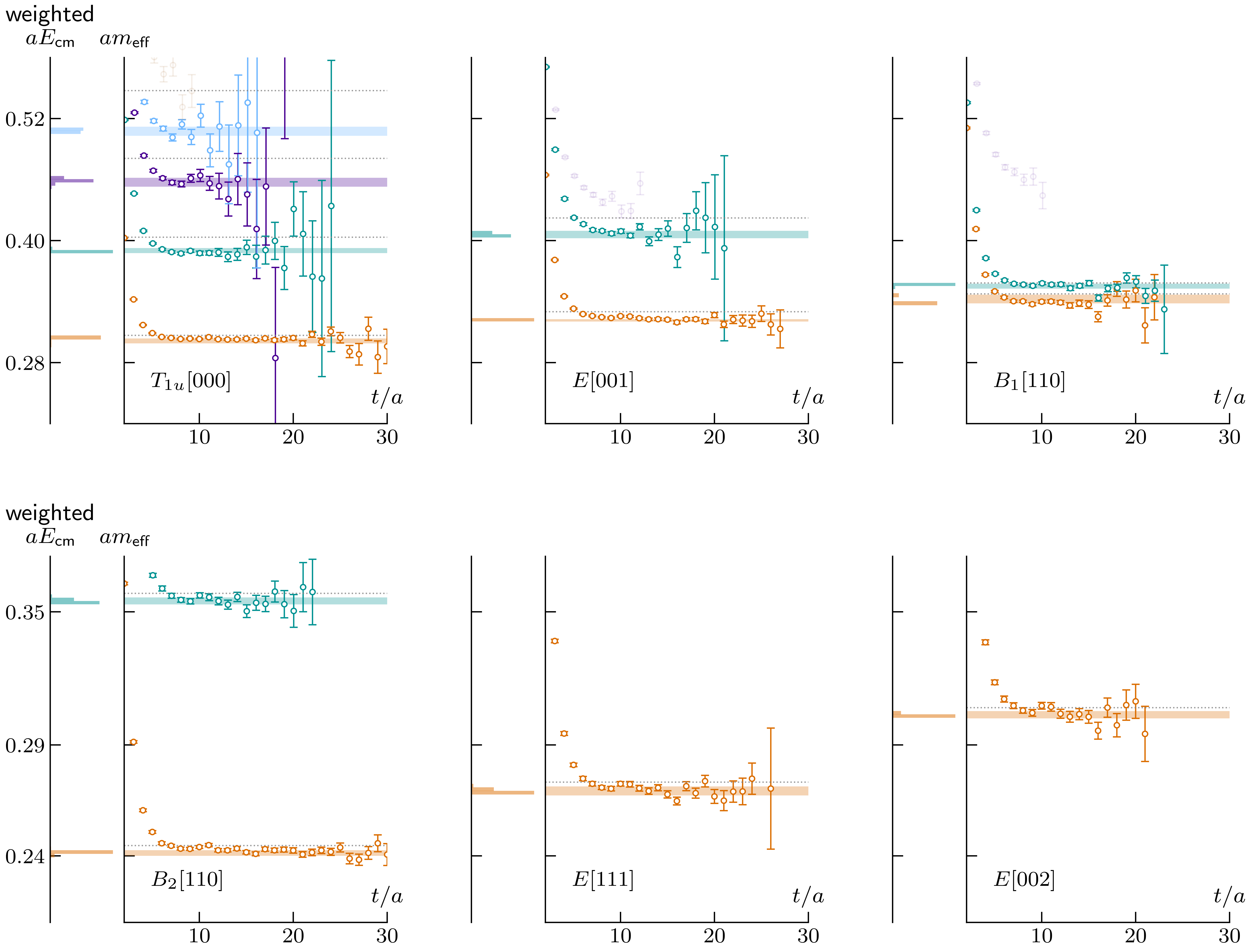


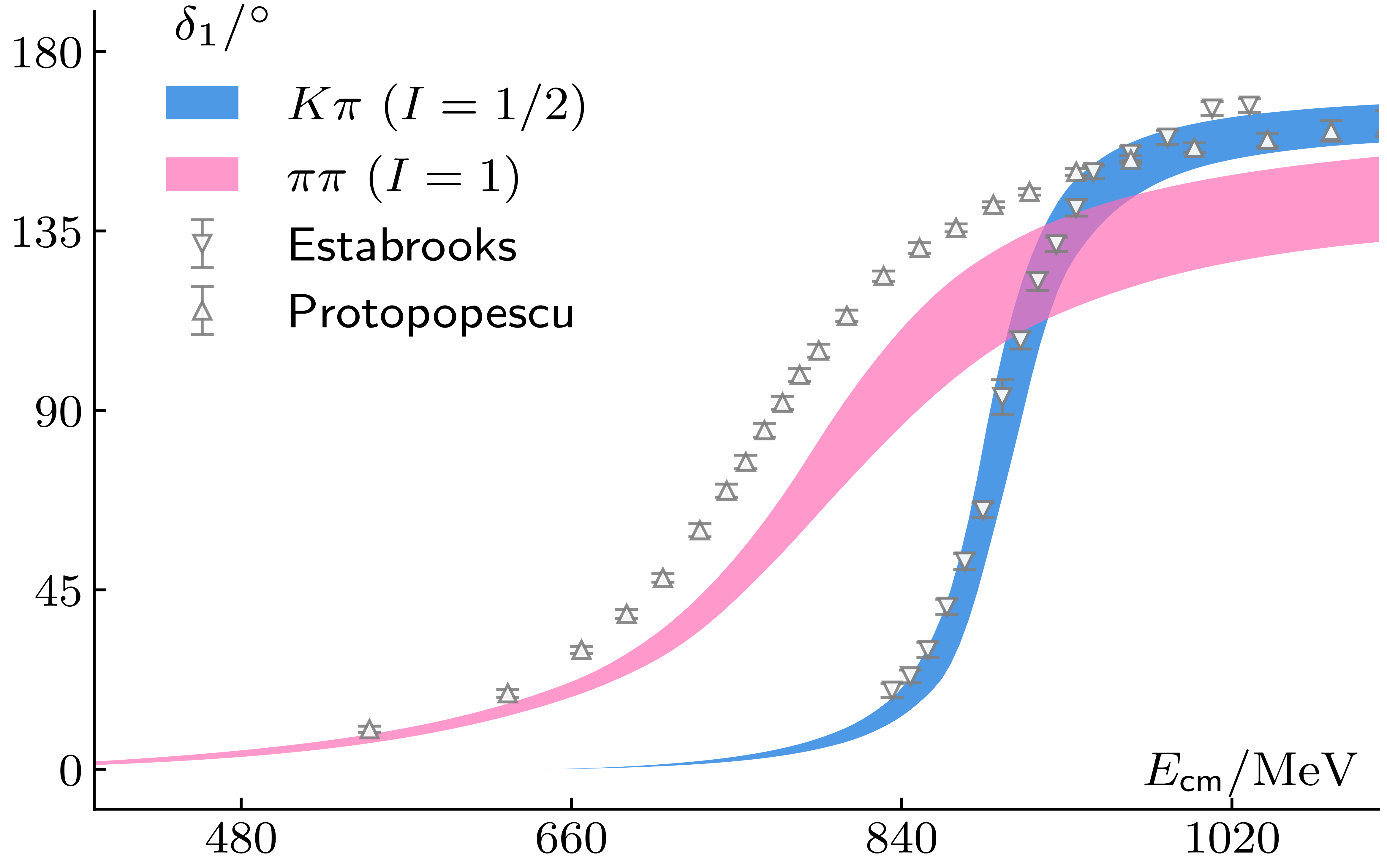
$K\pi$
0.60
0.55
0.50
0.45
0.40
aE_{cm}
(3)(3)
(2)(2)
(1)(1)
(2)(3)
(3)(2)
(1)(2)
(2)(1)
(1)(3)
(2)(2)
(3)(1)
(2)(4)
(4)(2)
(2)(2)
(1)(1)
(3)(4)
(4)(3)
(1)(2)
(2)(1)
(3)(3)
(2)(2)
$K\eta$
$K\pi\pi$
$K\pi$
$T_{1u}[000]$
$E[001]$
$B_1[110]$
$B_2[110]$
$E[111]$
$E[002]$
44
48
52
L/a



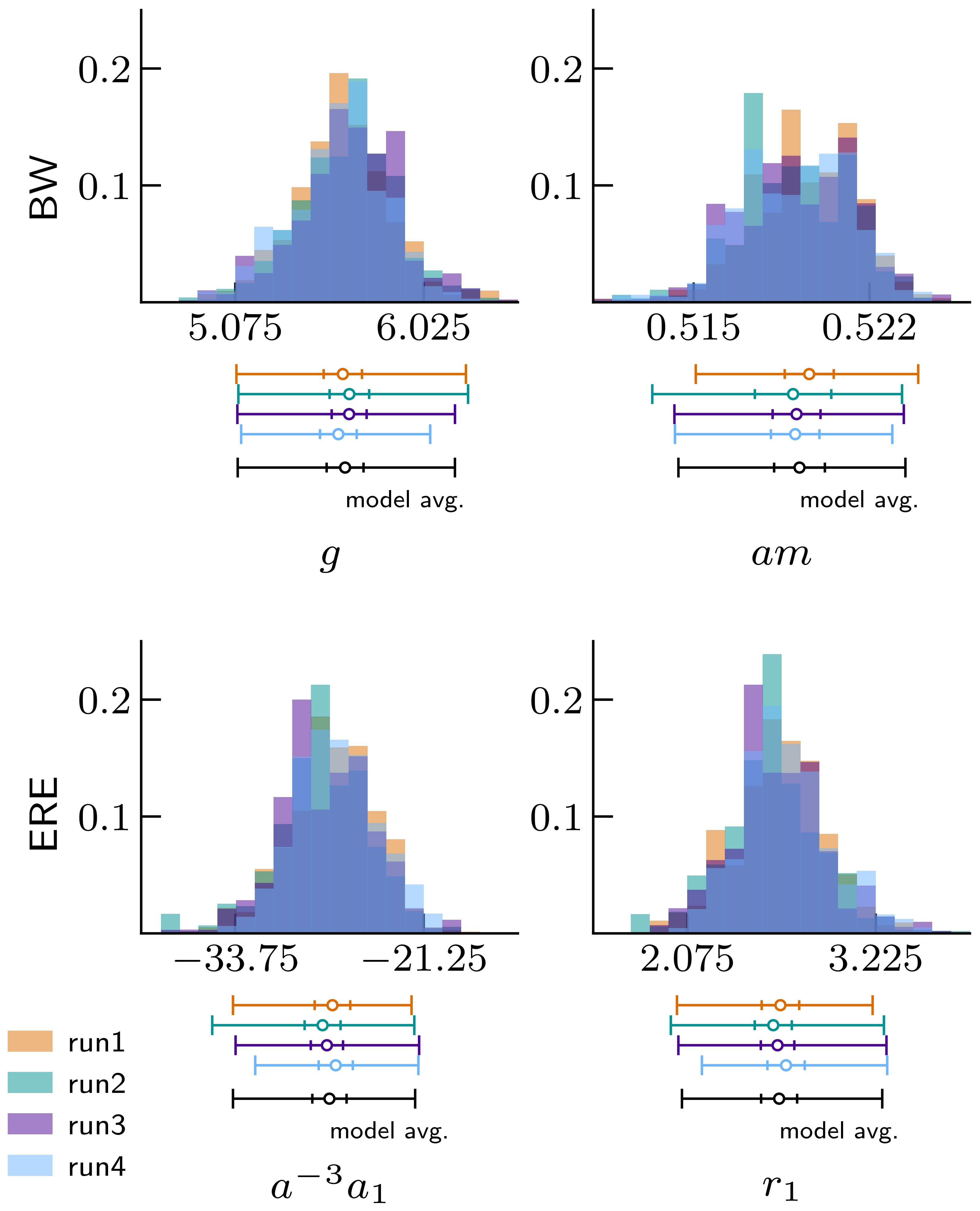
ππ
aE_{cm}
L/a
0.55
0.45
0.35
0.25
0.15
44
48
52
$T_{1u}[000]$
$E[001]$
$B_1[110]$
$B_2[110]$
$E[111]$
$E[002]$
$A_1[001]$
$A_1[110]$
$A_1[111]$
$A_1[002]$
$K\bar{K}$
4π
ππ
(4)(4)
(3)(3)
(2)(2)
(1)(1)
(2)(3)
(1)(2)
(1)(3)
(2)(4)
(3)(4)
(1)(4)
(0)(1)
(0)(2)
(0)(3)
(0)(4)


weighted
aE_{cm}
am_{eff}
0.55
0.50
0.45
$T_{1u}[000]$
$E[001]$
$B_1[110]$
t/a
10
20
30
weighted
aE_{cm}
am_{eff}
0.53
0.48
0.43
$B_2[110]$
$E[111]$
$E[002]$
t/a
10
20
30

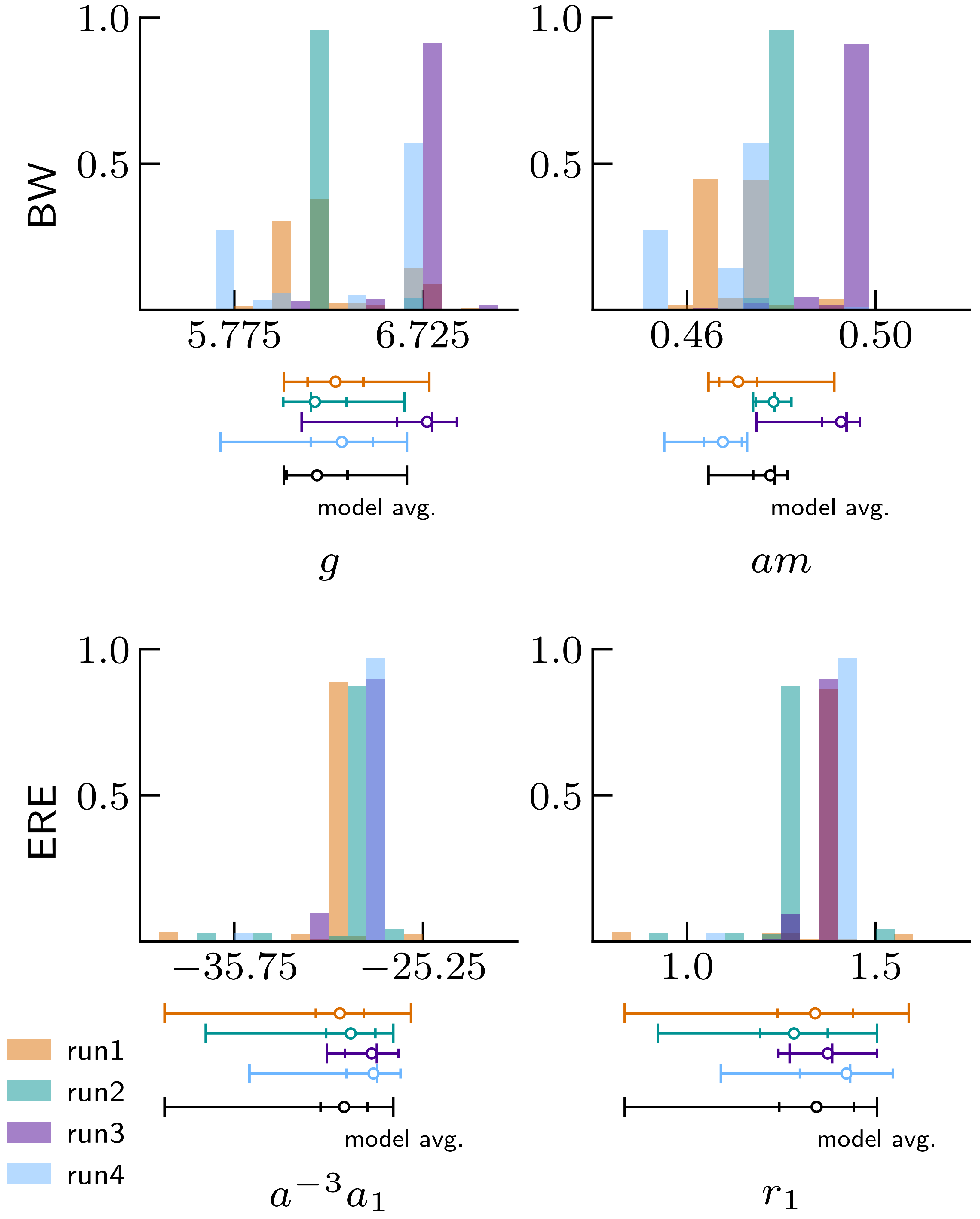

weighted
aE_{cm}
am_{eff}
0.52
0.40
0.28
$T_{1u}[000]$
$E[001]$
$B_1[110]$
t/a
10
20
30
weighted
aE_{cm}
am_{eff}
0.35
0.29
0.24
$B_2[110]$
$E[111]$
$E[002]$
t/a
10
20
30



$\delta_1/°$
180
135
90
45
0
$K\pi$ $(I = 1/2)$
$\pi\pi$ $(I = 1)$
Estabrooks
Protopopescu
E_{cm}/MeV
480
660
840
1020



BW
0.2
0.1
5.075
6.025
0.515
0.522
model avg.
model avg.
g
am
ERE
0.2
0.1
−33.75
−21.25
2.075
3.225
run1
run2
run3
run4
model avg.
model avg.
$a^{-3}a_1$
r_1



BW
1.0
0.5
5.775
6.725
0.46
0.50
model avg.
model avg.
g
am
ERE
1.0
0.5
−35.75
−25.25
1.0
1.5
run1
run2
run3
run4
model avg.
model avg.
$a^{-3}a_1$
r_1