Baryon Charges from Lattice QCD

Direct Determination of Octet Baryon Sigma Terms

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RQCD

Baryon Charges
baryon ground-state matrix elements

$$g_J^B = \langle B|J|B\rangle$$

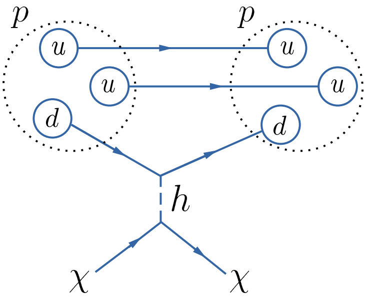
- $|B\rangle$ refers to the ground state of a baryon B at rest
- current J can be flavour diagonal or flavour changing
  e.g. $J = \bar{q}\Gamma q$ or $J = \bar{u}\Gamma d$
- Γ - choice of γ-matrices

Baryon Charges - isosinglet vs. isovector?

isosinglet (flavour diagonal)

$$J = \bar{q}\Gamma q$$

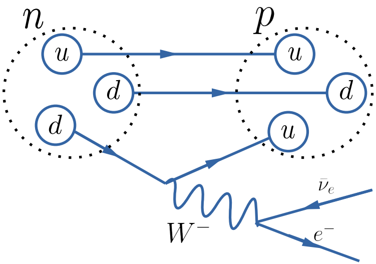
- electromagnetic (γ), weak neutral interactions (Z), dark matter mediated:



isovector (flavour changing)

$$J = \bar{u}\Gamma d$$

- $W^{\pm}$ mediated weak interactions e.g. neutron β-decay



What can these charges tell us about hadron structure?

The ultimate question - How do quarks and gluons account for the mass, spin and distribution of momenta within a baryon?

Let's focus on the isosinglet charges $g_{\Gamma}^{q} = \langle B|\bar{q}\Gamma q|B\rangle$

For different current insertions Γ :

1. **axial** $(\gamma_\mu\gamma_5)$ – $g_{\mathrm{A}}^{q} \rightarrow$ **quark contribution to the baryon's intrinsic spin**
2. **tensor** (σ_{ij}) – $g_{\mathrm{T}}^{q} \rightarrow$ **quark contribution to the baryon's electric dipole moment (EDM)**
3. **scalar** $(\mathbb{1})$ – $m_q g_{\mathrm{S}}^{q} =$ **quark mass contribution** to a baryon
   $\Rightarrow$ so-called **sigma terms** (what I have worked on)

How to determine (baryon octet) sigma terms from Lattice QCD

To get an idea of the general procedure for any of the charges

Want to determine

$$\sigma_q^B = m_q \langle B | \bar{q} \, \mathbb{1} \, q | B \rangle$$

$\Leftarrow$ B refers to the ground state of the baryon B at rest

INDIRECT

Feynman-Hellman theorem

$$\sigma_q^B = m_q \frac{\partial m_B}{\partial m_q}$$

DIRECT

"matrix element straight from the lattice"

DIRECT DETERMINATION – THE SCALAR CHARGE

RECAP: Spectral Decompositions

$$C_{\text{2pt}}(t_{\text{f}}) = \sum_{\vec{x}} \left\langle \mathcal{O}_{\text{snk}}(\vec{x}, t_{\text{f}}) \bar{\mathcal{O}}_{\text{src}}(\vec{0}, 0) \right\rangle = \sum_{n} |Z_n|^2 e^{-E_n t_{\text{f}}}$$

where $Z_n = \langle \Omega | \mathcal{O}_{\text{snk}} | n \rangle$ (vacuum state Ω) is the overlap of the interpolator $\mathcal{O}_{snk}$ onto the state n.

$$\begin{aligned} C_{\text{3pt}}(t_{\text{f}}, t) &= \sum_{\vec{x},\vec{y}} \left\langle \mathcal{O}_{\text{snk}}(\vec{x}, t_{\text{f}}) J(\vec{y}, t) \bar{\mathcal{O}}_{\text{src}}(\vec{0}, 0) \right\rangle - \sum_{\vec{x},\vec{y}} \langle J(\vec{y}, t) \rangle \left\langle \mathcal{O}_{\text{snk}}(\vec{x}, t_{\text{f}}) \bar{\mathcal{O}}_{\text{src}}(\vec{0}, 0) \right\rangle \\ &= \sum_{n,n'} Z_{n'} Z_n^* \langle \mathbf{n}' | \mathbf{J} | \mathbf{n} \rangle e^{-E_n t} e^{-E_{n'}(t_{\text{f}} - t)} \end{aligned}$$

t_{f} is the source-sink separation & t is the insertion time of the current

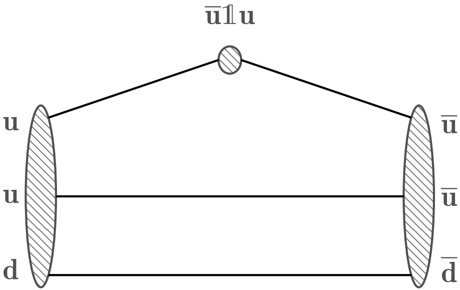
How to access the ground-state matrix element

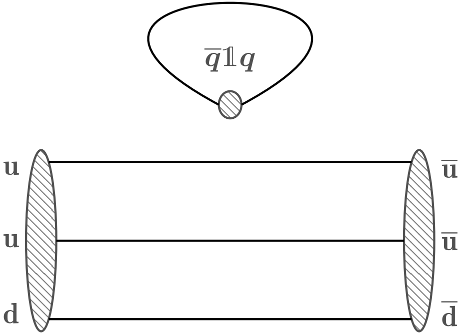
spectral decomposition

ratio method

$$R(t_\mathrm{f}, t) = \frac{C_{3\mathrm{pt}}(t_\mathrm{f}, t)}{C_{2\mathrm{pt}}(t_\mathrm{f})} \to \langle B|\bar{q}\,\mathbb{1}\,q|B\rangle + \mathcal{O}(e^{-\Delta E_1 t}) + \mathcal{O}(e^{-\Delta E_1 (t_f - t)}) + \mathcal{O}(e^{-\Delta E_1 t_f})$$

($\Delta E_1 = E_1 - E_0$, energy gap between ground & first excited state)





How to access the ground-state matrix element

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SIGNAL-TO-NOISE PROBLEM

$$R(t_\mathrm{f}, t) = \frac{C_{3\mathrm{pt}}(t_\mathrm{f}, t)}{C_{2\mathrm{pt}}(t_\mathrm{f})} \rightarrow \langle B|\bar{q}\,\mathbb{1}\,q|B\rangle + \boxed{\mathcal{O}(e^{-\Delta E_1 t}) + \mathcal{O}(e^{-\Delta E_1 (t_f - t)}) + \mathcal{O}(e^{-\Delta E_1 t_f})}$$

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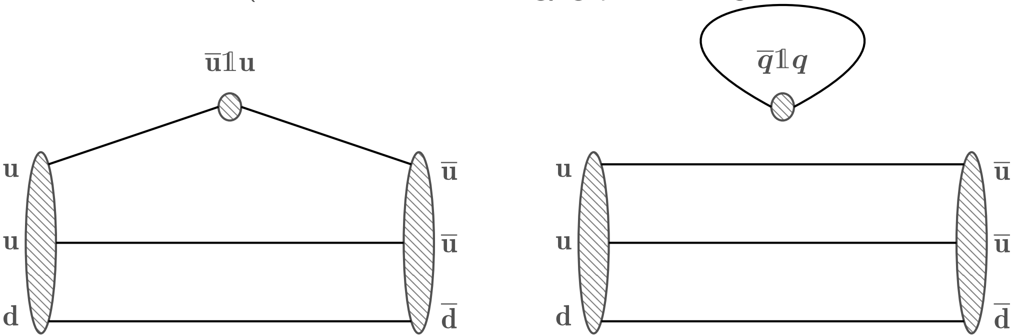

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EXCITED STATE CONTAMINATION

$$R(t_\mathrm{f}, t) = \frac{C_{3\mathrm{pt}}(t_\mathrm{f}, t)}{C_{2\mathrm{pt}}(t_\mathrm{f})} \rightarrow \langle B|\bar{q}\,\mathbb{1}\,q|B\rangle + \boxed{\mathcal{O}(e^{-\Delta E_1 t}) + \mathcal{O}(e^{-\Delta E_1 (t_f - t)}) + \mathcal{O}(e^{-\Delta E_1 t_f})}$$

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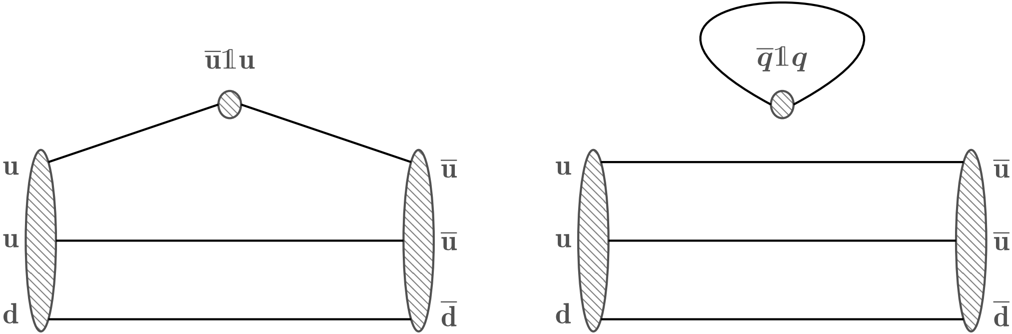

Challenges

- **signal-to-noise ratio** - large enough src-snk separations for ground state dominance ***x***
  - correlation functions: smearing to increase overlap with ground state
- **excited state contamination**
  - identification: sign/size of transition matrix elements $\langle 0|J|n\rangle$ **e.g.** $\boldsymbol{B_0 \leftrightarrow B_1 : c_{0\leftrightarrow 1}}$
  - all states with the same quantum numbers as the baryon interpolators can contribute
  - lowest lying non-interacting energy eigenstates $B(0)\pi(0)\pi(0)$ and $B(1)\pi(-1)$ [arXiv:1812.10574]
- **noisy disconnected contributions** (cancel in isovector case for N if $m_{\mathrm{u}} = m_{\mathrm{d}}$)

+ ...

Our approach:

Varied treatment of excited states to estimate systematics
arising from any residual excited state contamination

Challenges

- **signal-to-noise ratio** - large enough src-snk separations for ground state dominance ***x***
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+ ...

Our approach:

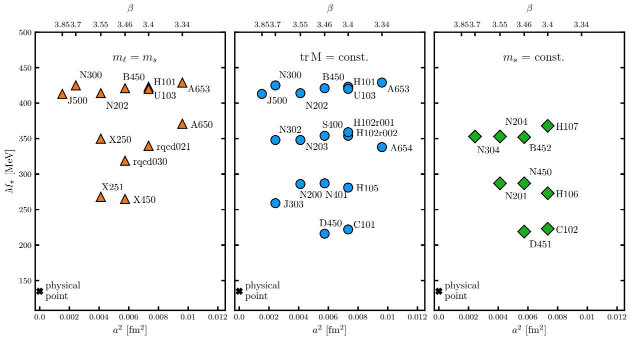
High statistics

different quark mass trajectories

large range of finite volumes & lattice spacings

Numerical Setup

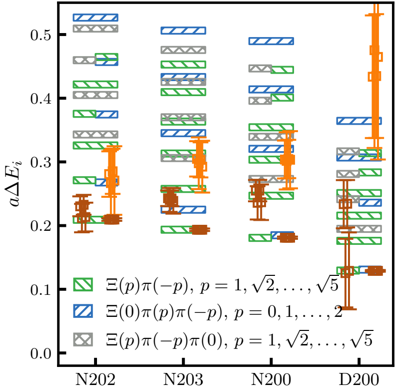
Constraining the physical point



- **three trajectories** in the quark mass plane
- **High statistics**: 34 ensembles
  - $\sim$ 1500 configurations (from 400 up to 5000)
- **five** lattice **spacings**
- **four** spatial **volumes**

- tree-level Symanzik-improved gauge action with $N_f = 2 + 1$ O(a) improved Wilson fermions
- **src-snk separation $t_f \approx 0.7\,\mathrm{fm} - 1.2\,\mathrm{fm}$**

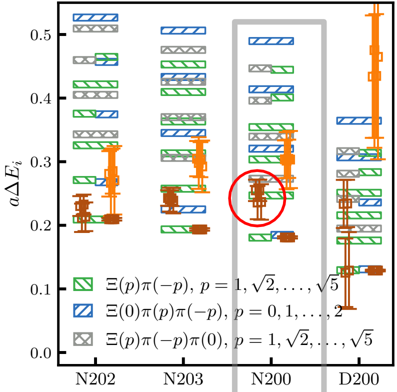
One-state fits -- ratio method with one excited state



$$R(t_\mathrm{f}, t) = g^B_{q,S} + c_{0\leftrightarrow 1}\left(\mathrm{e}^{-\Delta E_1 \cdot t} + \mathrm{e}^{-\Delta E_1 \cdot (t_\mathrm{f} - t)}\right)$$

'Octet baryon isovector charges from $N_\mathrm{f} = 2 + 1$ LQCD' [RQCD 2305.04717]

One-state fits -- ratio method with one excited state



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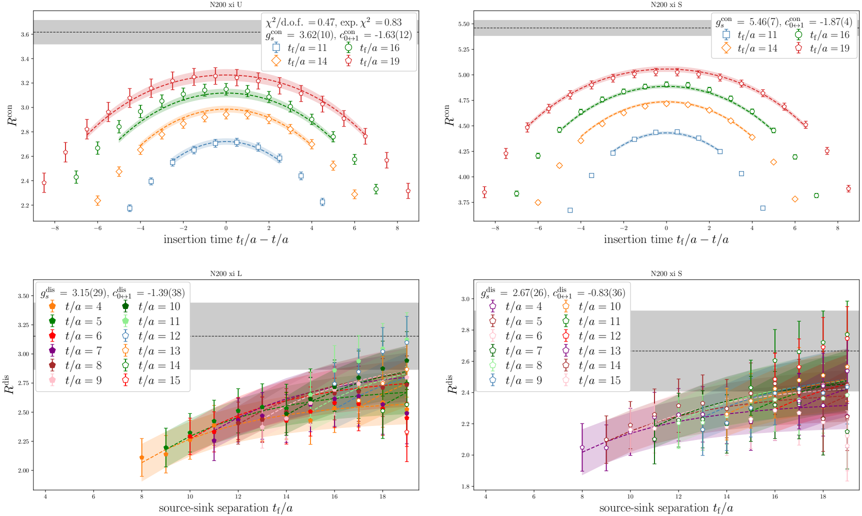
Simultaneous fits to connected & disconnected ratios



- **N200**
- **Ξ baryon**
- $a = 0.064\,\mathrm{fm}$
- $m_\pi = 286\,\mathrm{MeV}$
- **one-state fit**:
- $\chi^2/\chi^2_{\mathrm{exp}} \approx 0.83$
- $a\Delta E_1 = 0.234(12)$
- **top** (R^{con}):

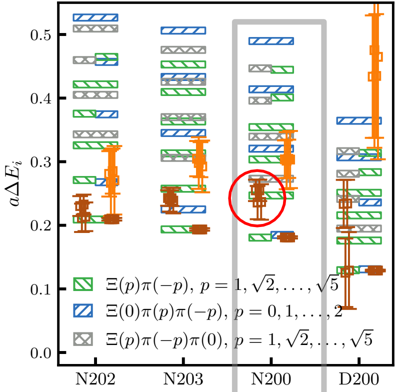
$\rightarrow$ $\bar{u}u$ current (lhs)

$\rightarrow$ $\bar{s}s$ current (rhs)
- **bottom** (R^{dis}):

$\rightarrow$ $\bar{l}l$ current (lhs)

$\rightarrow$ $\bar{s}s$ current (rhs)

One-state fits -- ratio method with one excited state

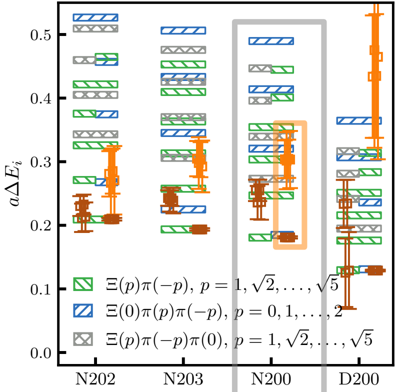


$$R(t_\mathrm{f}, t) = g^B_{q,S} + c_{0\leftrightarrow 1}\left(\mathrm{e}^{-\Delta E_1 \cdot t} + \mathrm{e}^{-\Delta E_1 \cdot (t_\mathrm{f} - t)}\right)$$

$$+ c_{0\leftrightarrow 2}\left(\mathrm{e}^{-\Delta E_2 \cdot t} + \mathrm{e}^{-\Delta E_2 \cdot (t_\mathrm{f} - t)}\right)$$

'Octet baryon isovector charges from $N_\mathrm{f} = 2 + 1$ LQCD' [RQCD 2305.04717]

Two-state fits -- ratio method with two excited states



'Octet baryon isovector charges from $N_\mathrm{f} = 2+1$ LQCD' [RQCD 2305.04717]

$$R(t_\mathrm{f}, t) = g_S^q + c_{0\leftrightarrow 1}\left(\mathrm{e}^{-\Delta E_1 \cdot t} + \mathrm{e}^{-\Delta E_1 \cdot (t_\mathrm{f} - t)}\right) + c_{0\leftrightarrow 2}\left(\mathrm{e}^{-\Delta E_2 \cdot t} + \mathrm{e}^{-\Delta E_2 \cdot (t_\mathrm{f} - t)}\right)$$

- set prior for ΔE_1: $\boldsymbol{B(1)\pi(-1)}$ or $\boldsymbol{B(0)\pi(0)\pi(0)}$ (lowest bar, green/blue)
- set prior for ΔE_2 to **a mean of the results for $\boldsymbol{\Delta E_2}$** from simultaneous fits to the four channels $J \in \{A, S, T, (V)\}$ setting a prior for ΔE_1 to the lowest multi-particle state (orange points)

Systematics

1. **Varied treatment of the excited states**
   1.1 **One-state fits**

   1.2 **Two-state fits**

2. **Varied data sets**
   2.1 **No cuts: including all 34 ensembles**

   2.2 $M_{\pi}^{<400\,\mathrm{MeV}}$: **excluding all ensembles with a pion mass** $M_{\pi} > 400\,\mathrm{MeV}$

   2.3 $a \leq 0.086\,\mathrm{fm}$: **excluding all ensembles at the coarsest lattice spacing**

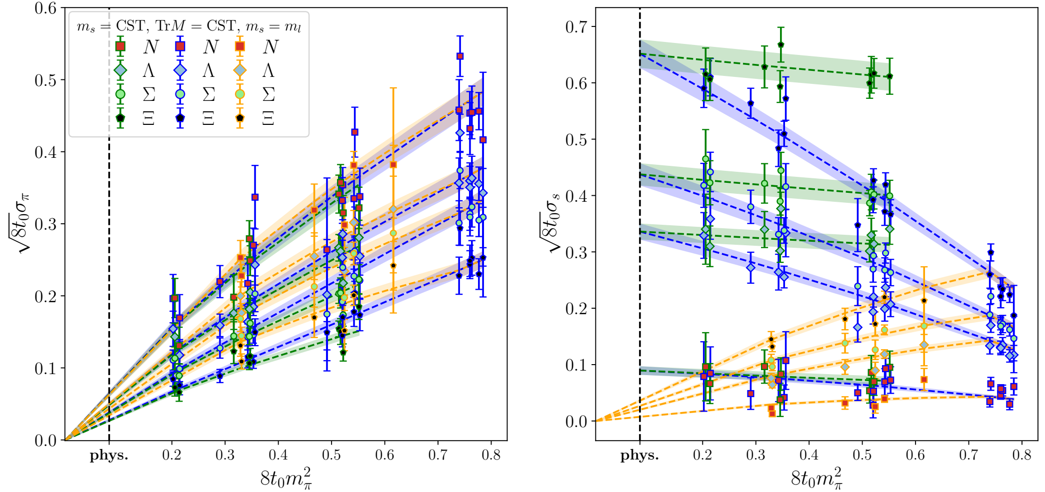
   2.4 $LM_{\pi}^{>4}$: **excluding all ensembles with** $LM_{\pi} < 4$

3. **TO DO -- Varied models for the physical point extrapolation**
   3.1 ... Model-averaging (AIC criterion)

Physical Point Extrapolation - Quark Mass Dependence (BChPT)

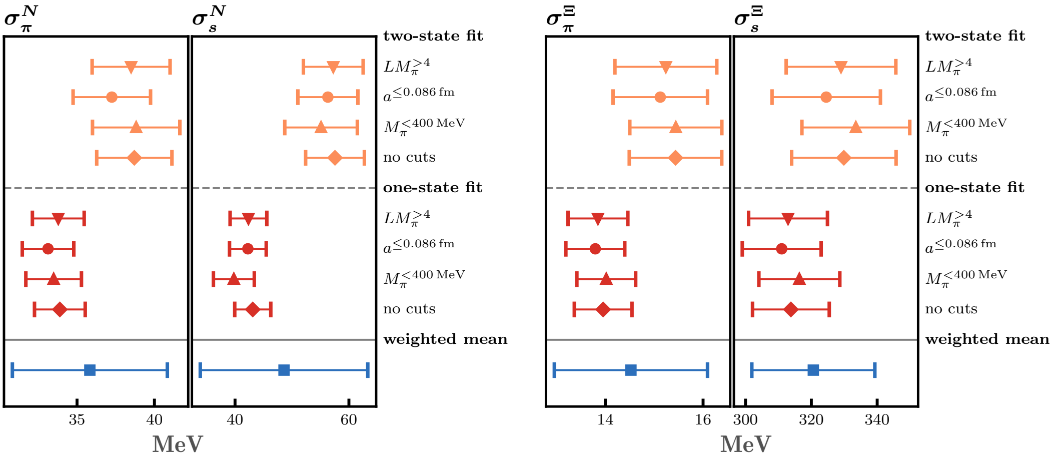
Data points based on One-state fits

$\chi^2/\chi^2_{\text{exp}} = 1.3$



Preliminary Results - Baryon Octet Sigma Terms

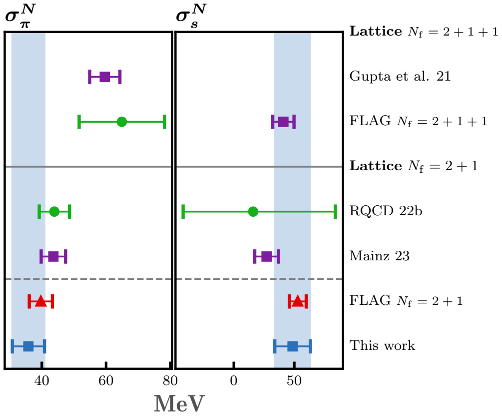
Accounting for (1.) Varied Treatment of the Excited States & (2.) Varied Data Sets



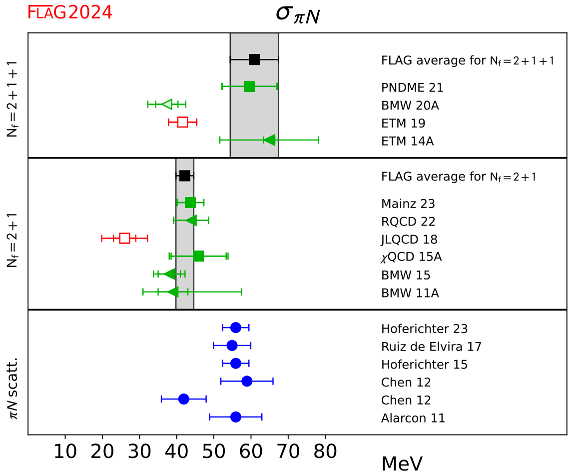
Preliminary Results Compared to Other Studies

- Direct vs. Indirect determinations and vs. Combinations (Open symbols - no continuum extrapolation)

*FLAG2021



Preliminary Results Compared to Other Studies



$\Leftarrow$ $N_\mathrm{f} = 2+1+1$ results ($60\,\mathrm{MeV}$) larger than $N_\mathrm{f} = 2+1$ expected to be due to excited state contamination

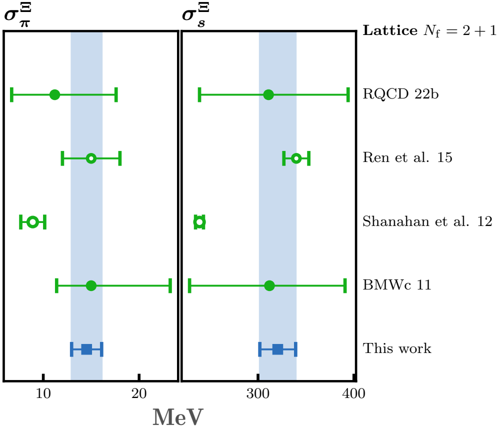
$\Leftarrow$ $N_\mathrm{f} = 2+1$ results mostly $40\,\mathrm{MeV}$
(This work too)

$\Leftarrow$ phenomenology mostly $60\,\mathrm{MeV}$

$\Rightarrow$ **tension with phenomenology for the nucleon-pion sigma term persists**

Preliminary Results Compared to Other Studies

- Direct vs. Indirect determinations and vs. Combinations
  (Open symbols - no continuum extrapolation)

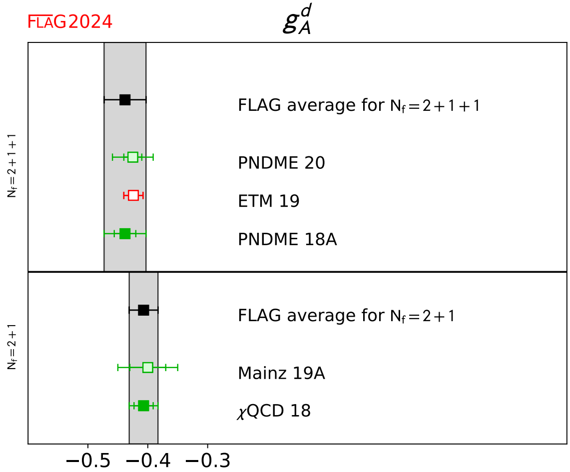


⇒ improved precision for xi sigma terms, similarly for the lambda and sigma (not for the nucleon)

- **first direct determination of all four baryon octet sigma terms** (not only N)
  ⇒ **improved precision**

Isosinglet Nucleon Charges

What's the down quark's spin contribution?

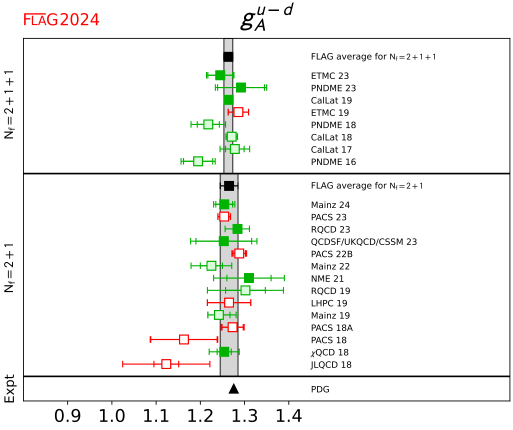


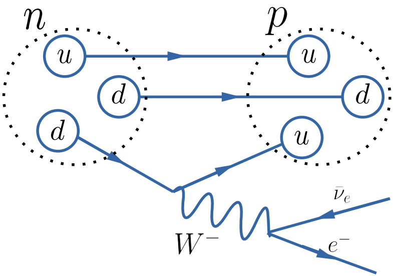
⇐ green square incl. in FLAG average, empty green square not incl. but passes criteria, empty red square others

- to find out about the up and strange quark's **spin** contribution as well
  - results for g_A^u, g_A^s (see FLAG24)
- to answer **'What are the quarks' contributions to the nucleon's electric dipole moment?'**
  - results for g_T^u, g_T^d, g_T^s (see FLAG24)

Isovector Nucleon Charges

How strong are weak interactions of nucleons?





- only show axial current here
- scalar and tensor currents have not been observed in nature but effective scalar and tensor interactions arise in SM due to loop effects
  - results for g_S^{u-d} & g_T^{u-d} (see FLAG24)

Conclusion

- baryon charges are needed can help answer the ultimate question
  - **How do quarks and gluons account for the mass, spin and distribution of momenta within a baryon?**
  - BSM
- baryon charges widely studied, ESC one of the main challenges
- **first direct determination of all four baryon octet sigma terms** (not only N)
  $\Rightarrow$ **improved precision** (AIC averaging to do)
- more work needs to be done on the pion-nucleon sigma term to understand/resolve the discrepancies with phenomenology
  - methods from spectroscopy?