Exotic T_{bc} tetraquarks from Lattice QCD

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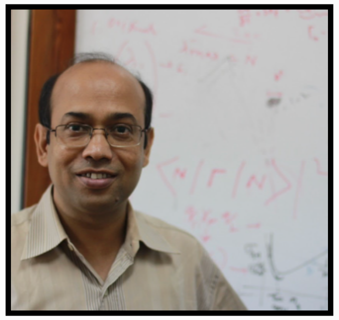
In collaboration with



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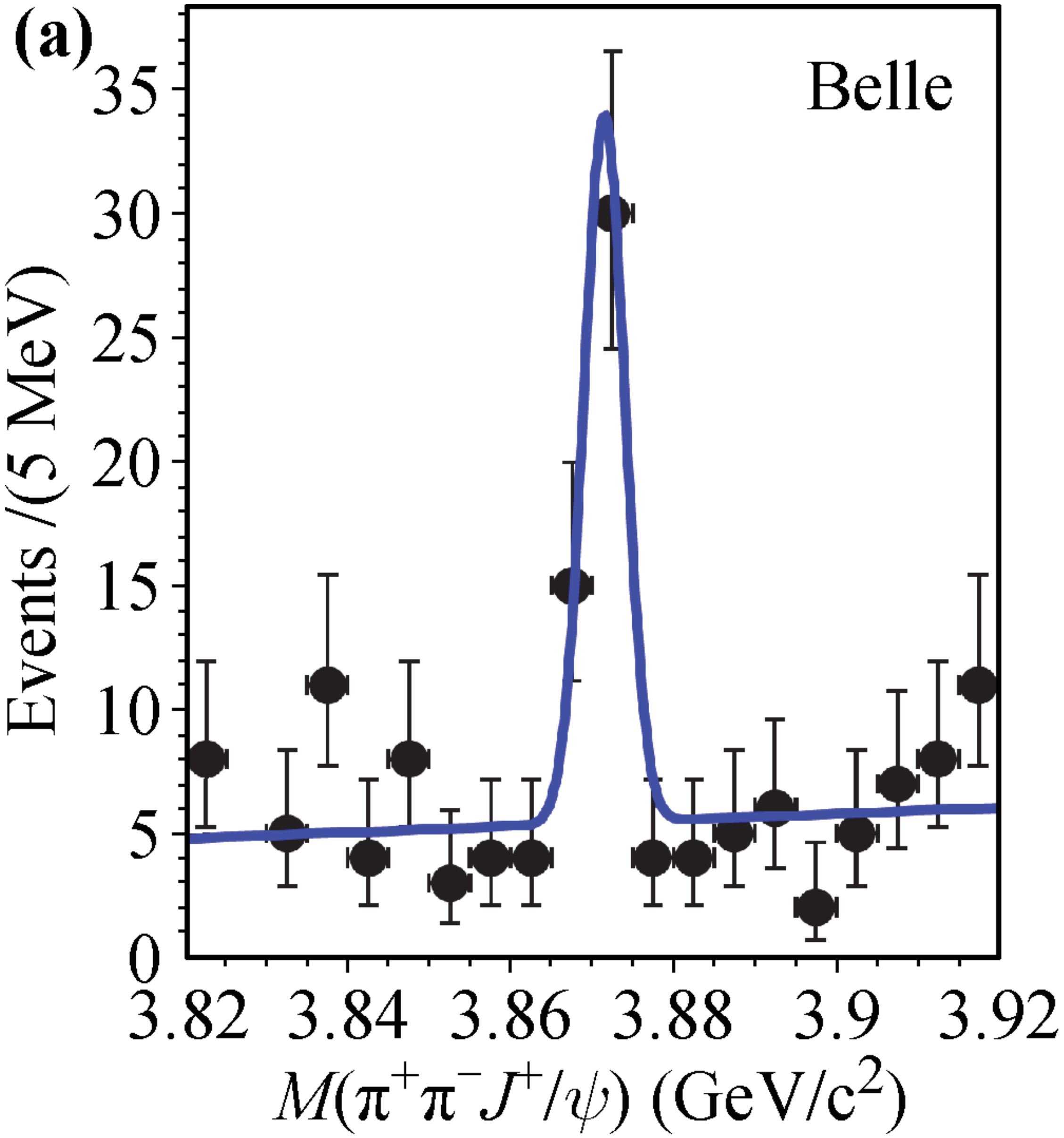
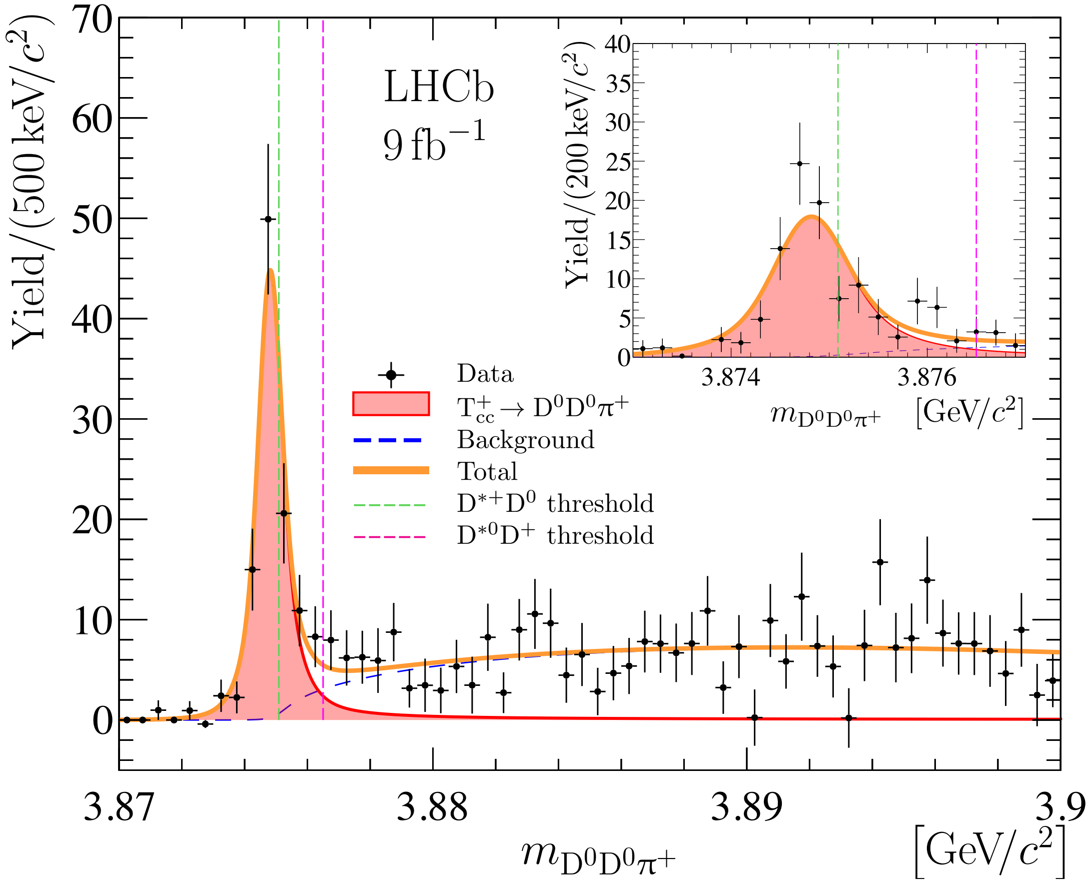
Tetraquarks

- Jaffe first proposed tetraquarks in 1977
- First experimental observation only in 2003 of X(3872) by Belle 1 MeV below *DD** threshold



- **T_{cc} seen by LHCb in 2021**
- 273 keV below *DD** threshold
- Proposals by experiments to look for other candidates



Tetraquarks

- Tetraquark candidates and their plausible valence quark content - most of them are hidden flavor candidates

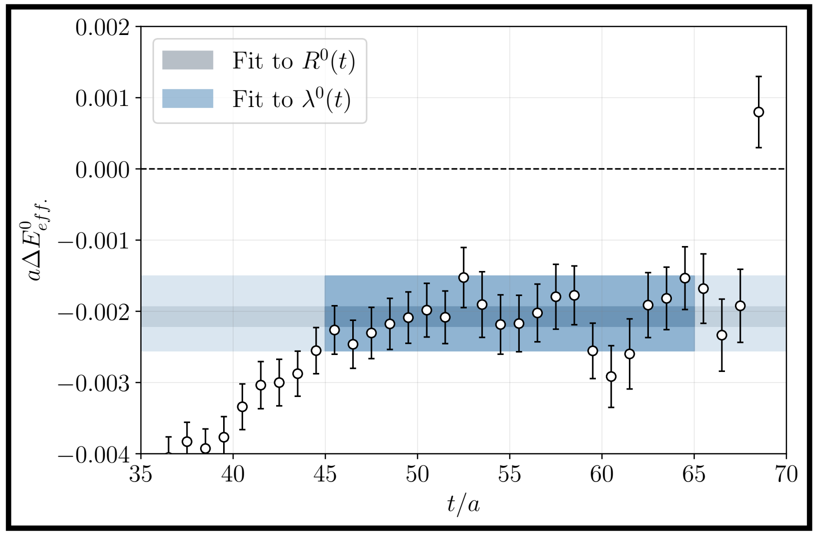
States	**Quark Content**
$X_0(2900), X_1(2900)$	$\bar{c}d\bar{u}s$
$X_{c1}(3872)$	$c\bar{c}q\bar{q}$
$Z_c(3900), Z_c(4020), Z_c(4050), X(4100), Z_c(4200), Z_c(4430), R_{c0}(4240)$	$c\bar{c}u\bar{d}$
$Z_{cs}(3985), Z_{cs}(4000), Z_{cs}(4220)$	$c\bar{c}u\bar{s}$
$X_{c1}(4140), X_{c1}(4274), X_{c0}(4500), X_{c0}(4700), X(4630), X(4685), X(4740)$	$c\bar{c}s\bar{s}$
$X(6900)$	$c\bar{c}c\bar{c}$
$Z_b(10610), Z_b(10650)$	$b\bar{b}u\bar{d}$

- Heavy-light tetraquarks ($QQ'\bar{q}\bar{q}'$) are prime candidates to be bound and therefore long-lived

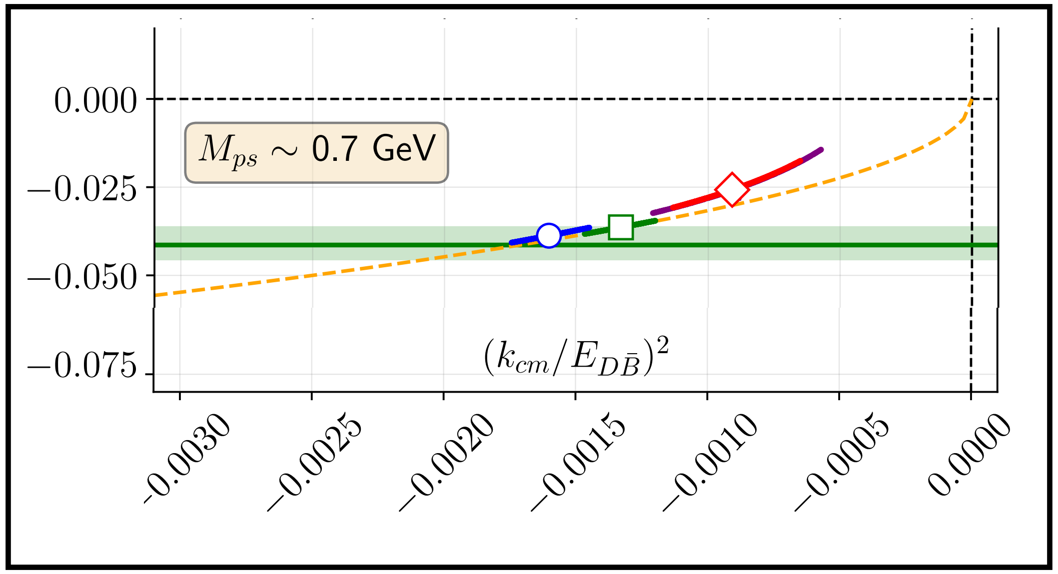
Phys. Rev. D 25, 2370

Phys. Lett., B123:449–451, 1983

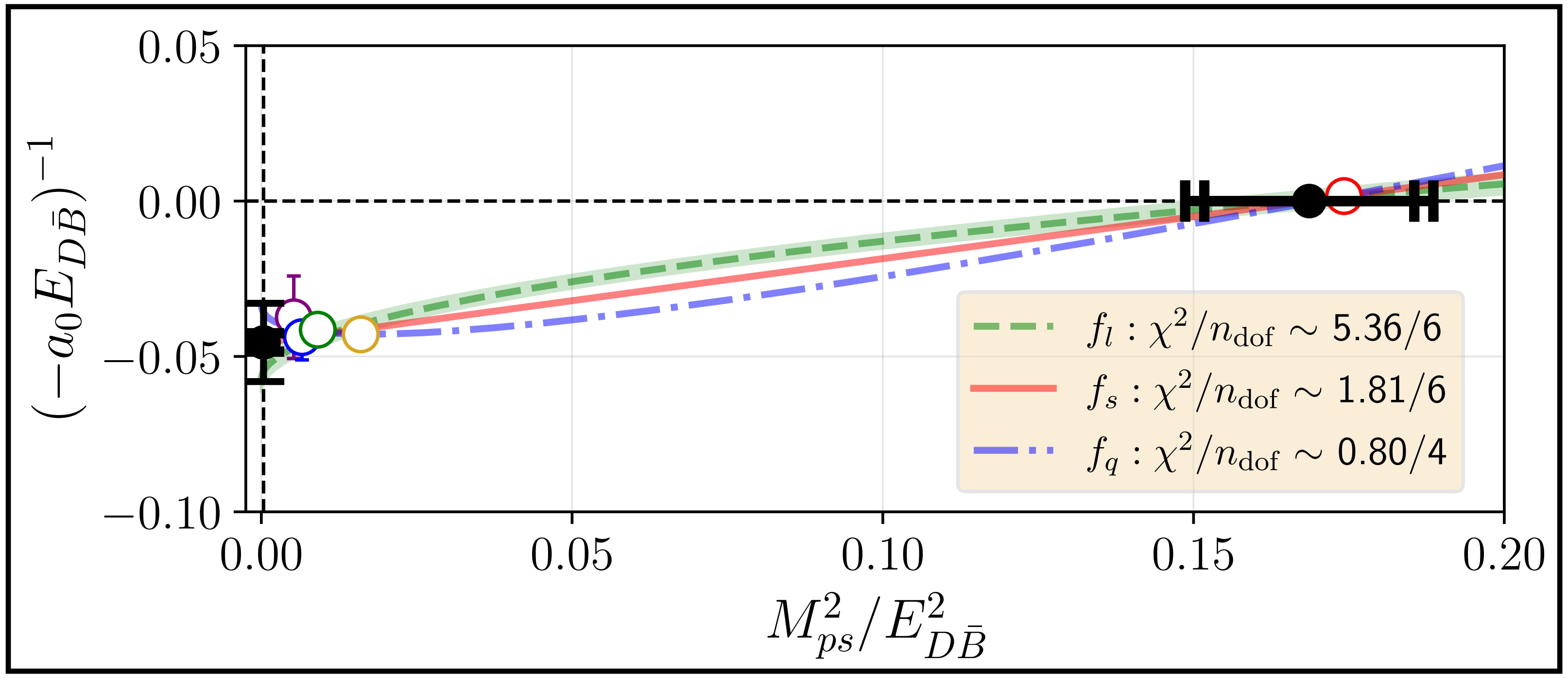
Lattice Methodology



The time dependence of two point correlation functions gives discrete energy levels that depend on the volume of the lattice



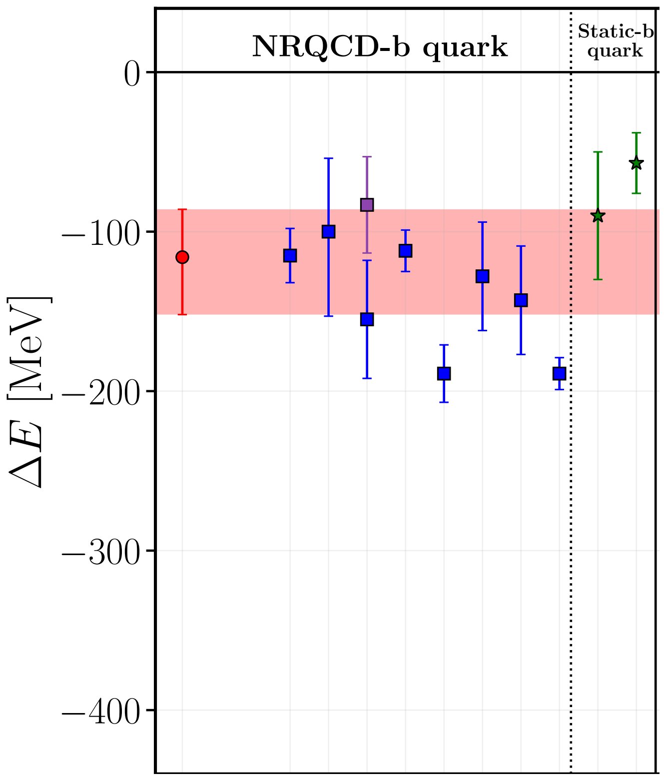
Map the discrete energy levels to the scattering amplitudes



Continuum and chiral extrapolation to get the binding energies at physical point

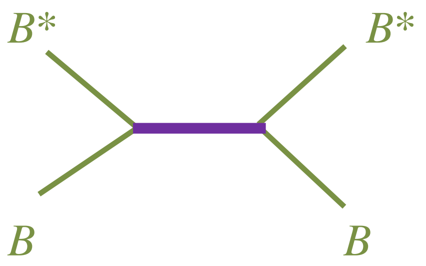
$QQ'\bar{q}\bar{q}'$ states on the lattice $[I\,(J^P)]$

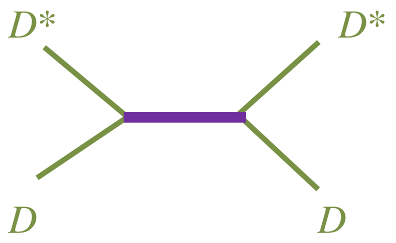
$bb\bar{u}\bar{d}$ $0(1^+)$



- Tripathy et al (2025)
- Colquhoun et al (2024)
- Alexandrou et al (2024)
- Aoki et al (2023)
- Hudspith et al (2023)
- Mohanta et al (2020)
- Leskovec et al (2019)
- Junnarkar et al (2018)
- Francis et al (2016)
- Bicudo et al (2015)
- Bicudo et al (2012)

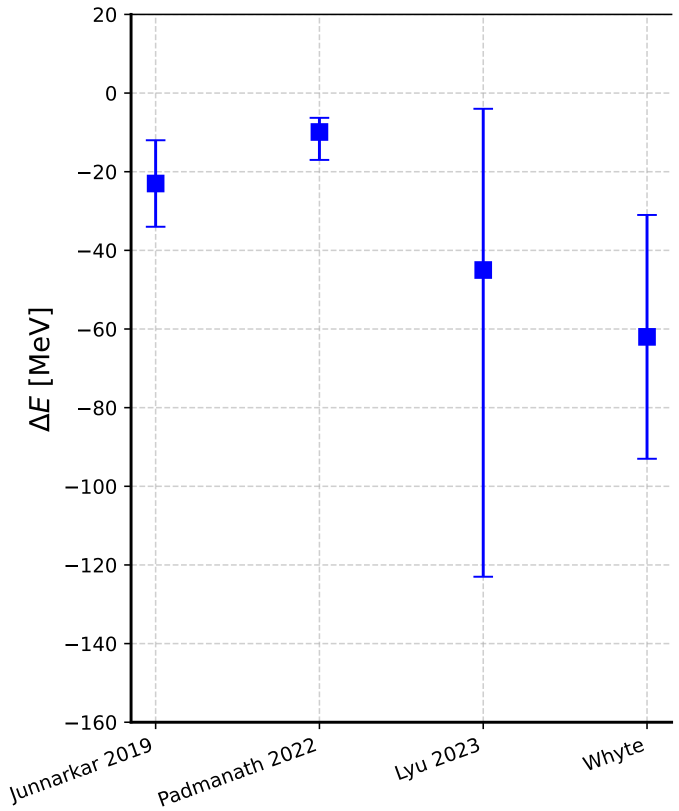
Tripathy et al. (2025)





$cc\bar{u}\bar{d}$ $0(1^+)$

Paper	Results
Padmanath et al. [PRL 129,032002 (2022)]	$-9.9^{+3.6}_{-7.1}$ MeV
Chen et al. [PLB 833,137391 (2022)]	$I = 0, 1$ (attractive,repulsive) no info about the pole
Lyu et al. [PRL 131,161901 (2023)]	$-45\left(^{+41}_{-78}\right)$ keV
Collins et al. [arXiv:2402.14715 (2024)]	quark mass dependence checked
Whyte et al. [arXiv:2405.15741 (2024)]	-62 ± 31 MeV

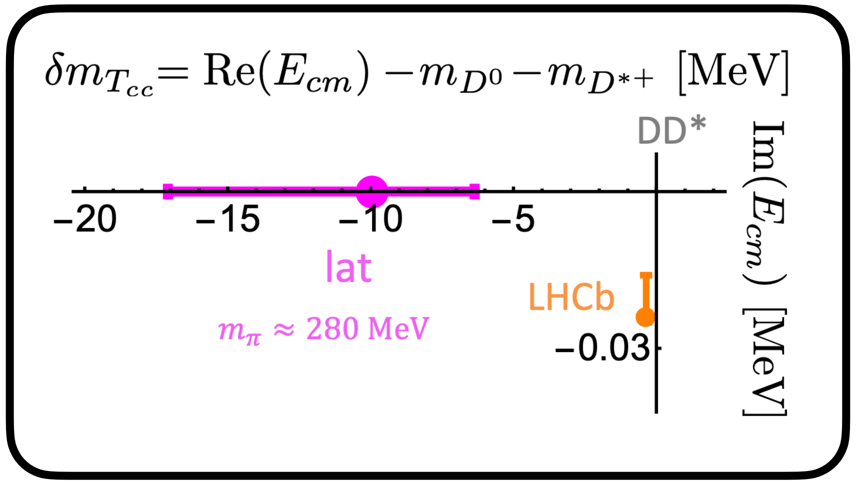


These states may be discovered in future, but the energies might be too high for current experiments to reliably explore

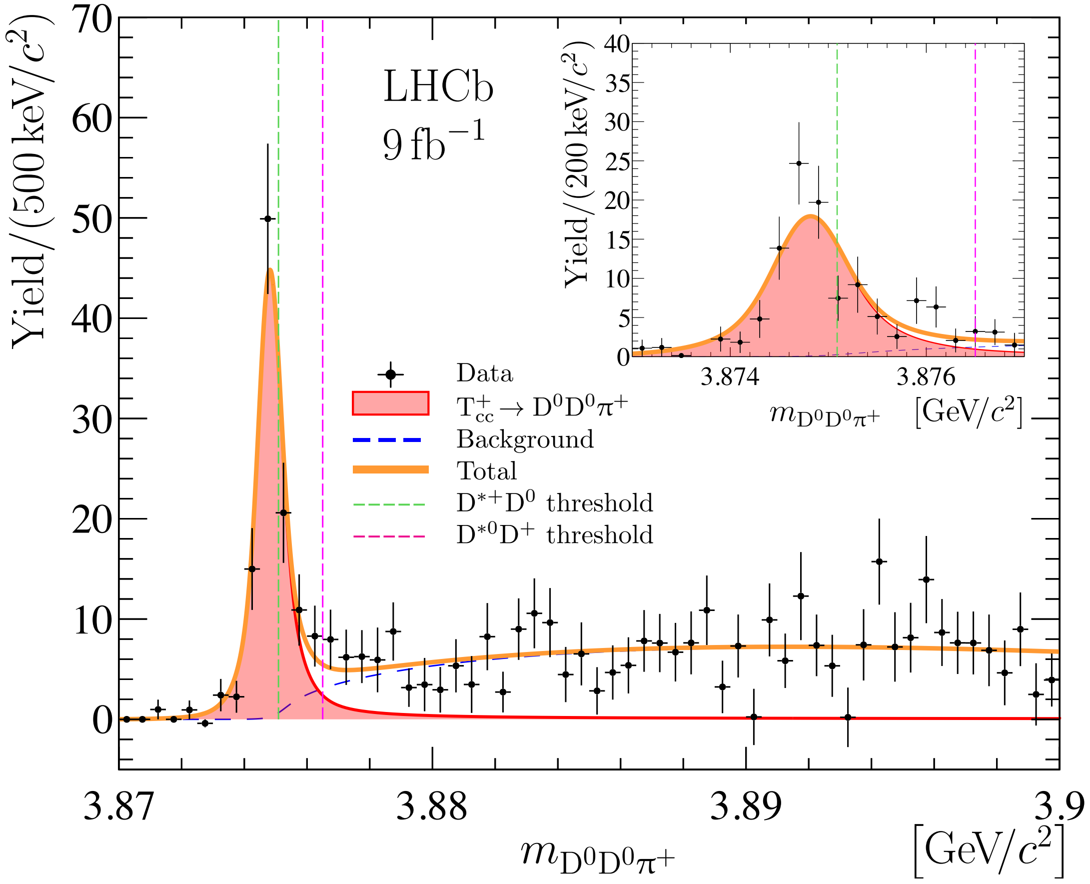
The T_{cc} $[I(J^P)]$

States near threshold

$cc\bar{u}\bar{d}$ $0(1^+)$



Padmanath et al. PRL. 129 (2022)



Exploring tetraquarks with bottom and charm may be accessible to experiments

Motivates the study of $bc\bar{u}\bar{d}$

but significant volume effects possible - close to threshold

$QQ'\bar{q}\bar{q}'$ states on the lattice

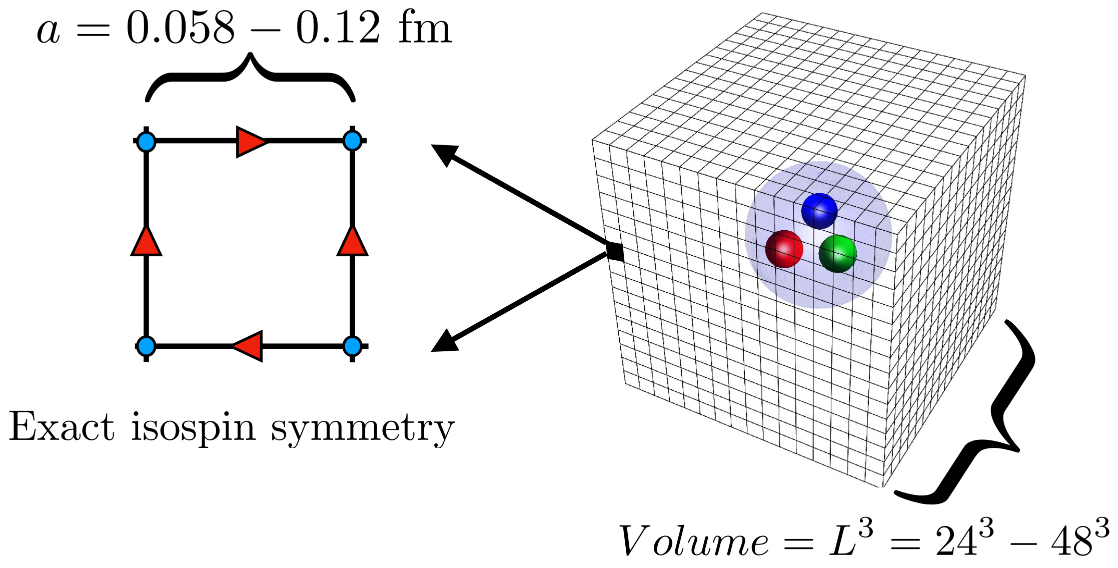
Type	$I(J^P)$	E_B[MeV]	a_0[fm]	r_0[fm]	Study	Method	Comments
T_{bb}^{ud}	$0(1^+)$	-57(19) to -30(17)			[3]	static potential	
		-90(40)			[127, 128]	static potential	single channel
		-59^{+30}_{-38}			[131]	static potential	coupled channel
		-189(10)			[4]	spectrum	
		-143(34)			[134]	spectrum	
		-128(34)			[69]	spectrum	
		-189(18)			[140]	spectrum	
		-113			[78]	spectrum	preliminary
		-112(13)			[41]	spectrum	
		-155(17)			[152]	HAL potential	single channel
		-83(10)	-0.43(5)	0.18(6)	[152]	HAL potential	coupled channel
		$-100(10)\,\left(^{+36}_{-43}\right)$			[162]	spectrum	
T_{bb}^{ud}	$0(0^+)$	-5(18)			[134]	spectrum	

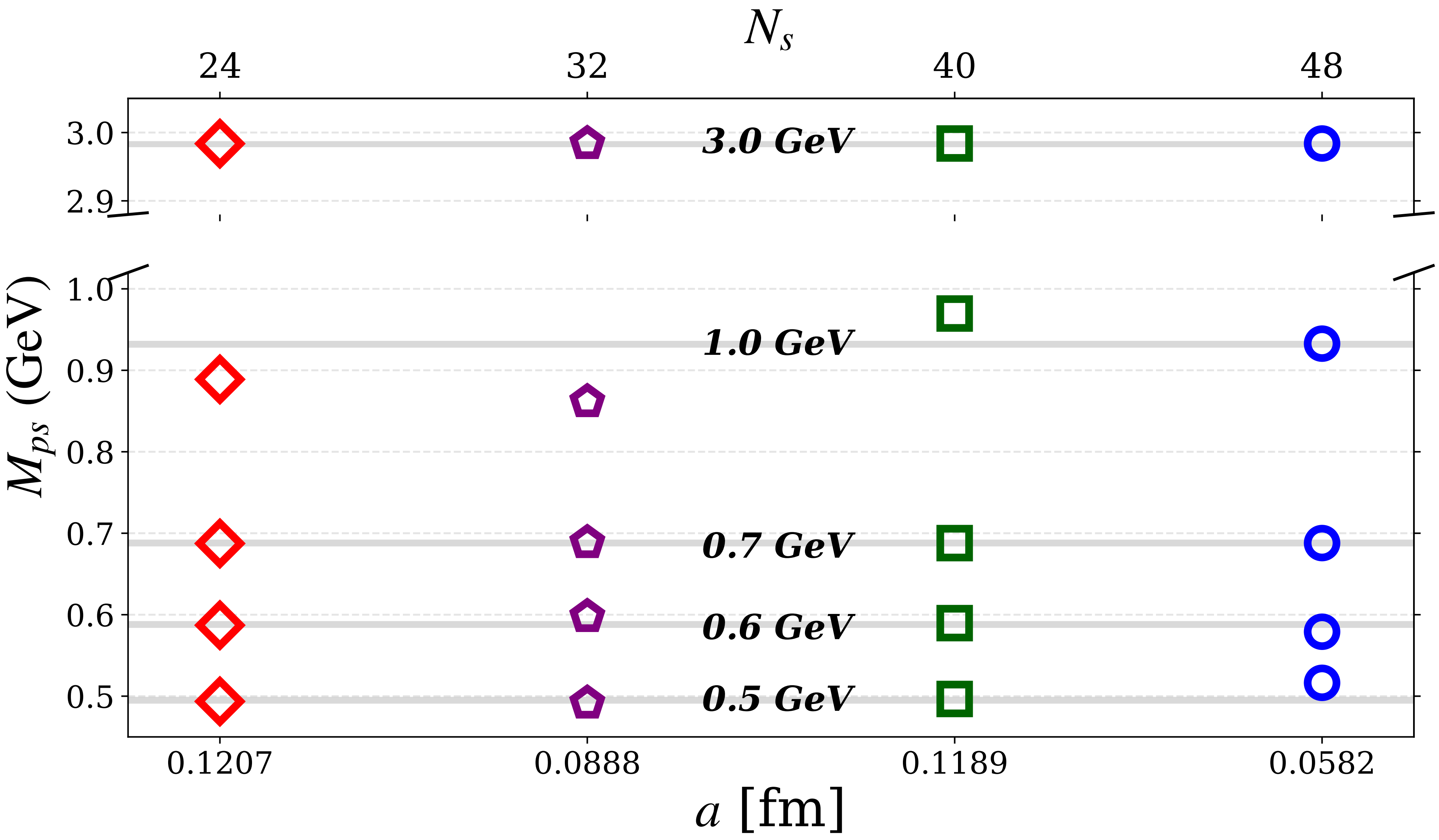
Type	$I(J^P)$	E_B[MeV]	a_0[fm]	r_0[fm]	Study	Method	Comments
T_{cc}^{ud}	$0(1^+)$	not resonant			[77]	spectrum	
		-23(11)			[134]	spectrum	
		$-9.9^{+3.6}_{-7.2}$	1.04(29)	$0.96\,\left(^{+0.18}_{-0.20}\right)$	[68]	spectrum	heavy charm
		$-15.0^{+4.6}_{-9.3}$	0.86(0.22)	$0.92\,\left(^{+0.17}_{-0.19}\right)$	[68]	spectrum	light charm
		$-59\,\left(^{+53}_{-99}\right)\,\left(^{+2}_{-67}\right)$	$20(20)\,\left(^{+8}_{-8}\right)$	$1.12(3)\,\left(^{+3}_{-8}\right)$	[147]	HAL potential	lattice m_π
		$-45\,\left(^{+41}_{-78}\right)$	$-33(44)$	1.12(3)	[147]	HAL potential	rescaled m_π
T_{bb}^{ud}	$0(0^+)$	not bound			[134]	spectrum	

A.Francis (2025) arXiv: 2502.04701

Lattice details

- MILC ensembles NF=2+1+1 HISQ action.
- valence quarks were implemented using an overlap action (u, d, s, c).
- the evolution of the bottom quark was studied within a non-relativistic QCD framework.
- b, c, s at the physical point and light quark mass is varied.





Extraction of the energy spectrum

The time dependence of Euclidean two point correlation function gives the energy,

$$C(t) = \langle 0|\mathcal{O}_f(t)\mathcal{O}_i^\dagger(0)|0\rangle$$

Adding a complete set of states,

$$C(t) = \sum_n e^{-E_n t} Z_n^f Z_n^i \qquad Z_\alpha^i = \langle \alpha|\mathcal{O}_i^\dagger|0\rangle$$

time-evolution

overlaps

at large times they are dominated by the ground state

Local 2 two-meson-like interpolators and one diquark-antidiquark-like interpolator

$$\begin{aligned}
\mathcal{O}_1(x) &= [\bar{u}\gamma_i b][\bar{d}\gamma_5 c](x) - [\bar{d}\gamma_i b][\bar{u}\gamma_5 c](x),\\
\mathcal{O}_2(x) &= [\bar{u}\gamma_5 b][\bar{d}\gamma_i c](x) - [\bar{d}\gamma_5 b][\bar{u}\gamma_i c](x),\\
\mathcal{O}_3(x) &= [(\bar{u}^T\Gamma_5\bar{d} - \bar{d}^T\Gamma_5\bar{u})(b\Gamma_i c)](x).
\end{aligned}$$

$$\begin{bmatrix} C(t)_{00} & C(t)_{01} & \cdots \\ \vdots & \ddots & \\ C(t)_{N0} & & C(t)_{NN} \end{bmatrix} v_a = \lambda_a(t,t_0) \begin{bmatrix} C(t_0)_{00} & C(t_0)_{01} & \cdots \\ \vdots & \ddots & \\ C(t_0)_{N0} & & C(t_0)_{NN} \end{bmatrix} v_a$$

$$\lambda_\alpha(t,t_0) \sim e^{-E_\alpha(t-t_0)}$$

best linear combination of the basis to interpolate a specific state

$$\mapsto \quad \Omega^\alpha = \sum_i v_\alpha^i \mathcal{O}_i$$

Optimized Operators

Extraction of the energy spectrum

The time dependence of Euclidean two point correlation function gives the energy,

$$C(t) = \langle 0|\mathcal{O}_f(t)\mathcal{O}_i^\dagger(0)|0\rangle$$

Adding a complete set of states,

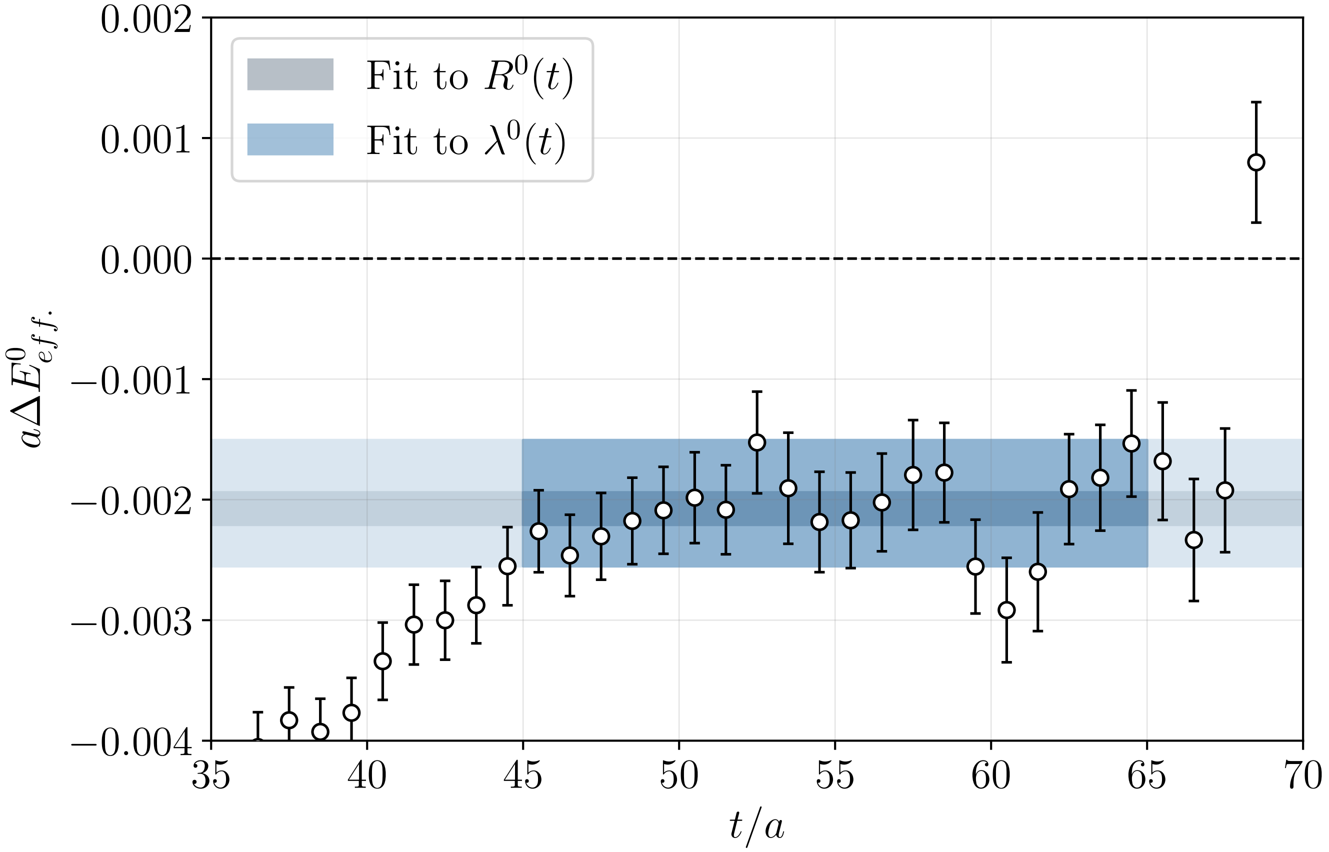
$$C(t) = \sum_n e^{-E_n t} Z_n^f Z_n^i \qquad Z_\alpha^i = \langle \alpha|\mathcal{O}_i^\dagger|0\rangle$$

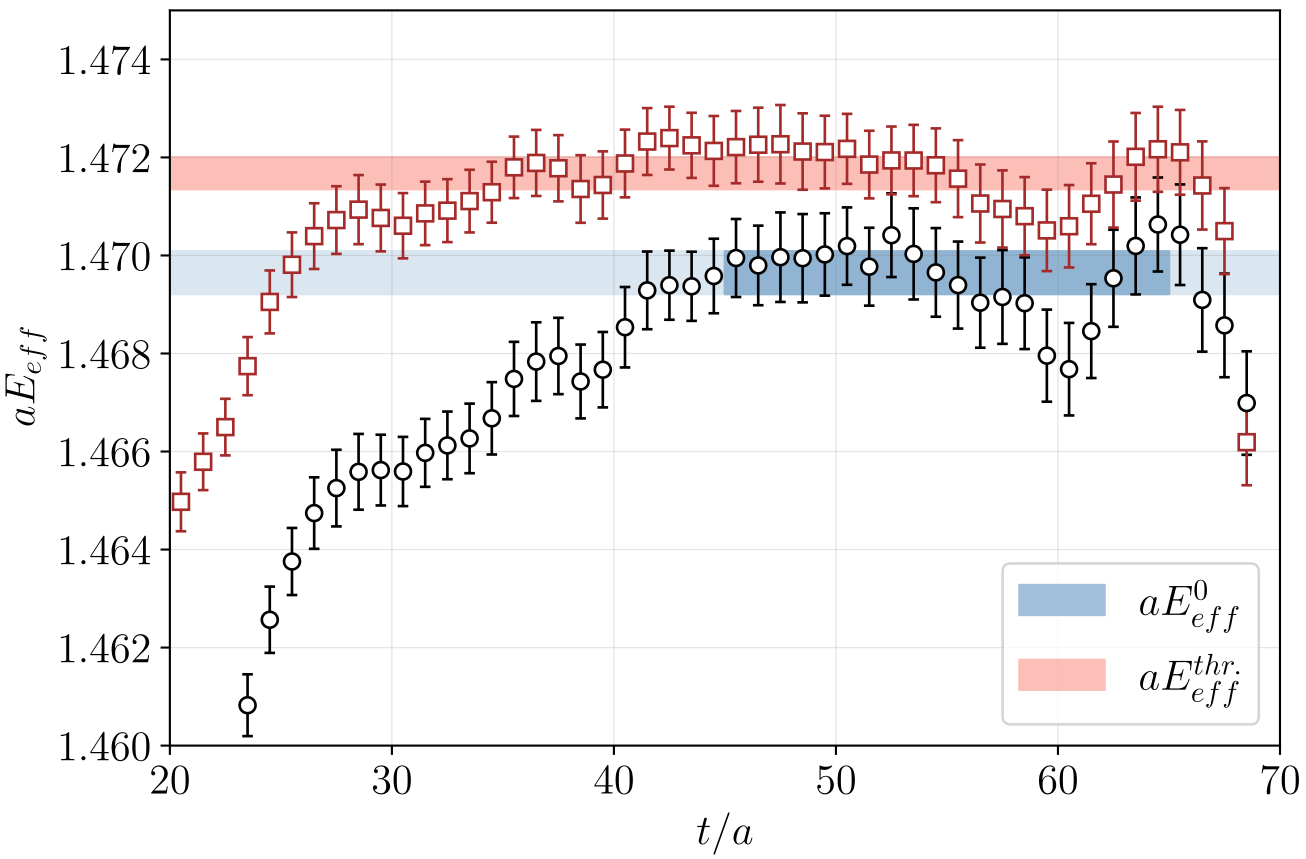
at large times they are dominated by the ground state

time-evolution

overlaps

$$m_{eff} = \ln \frac{C(t)}{C(t+1)}$$

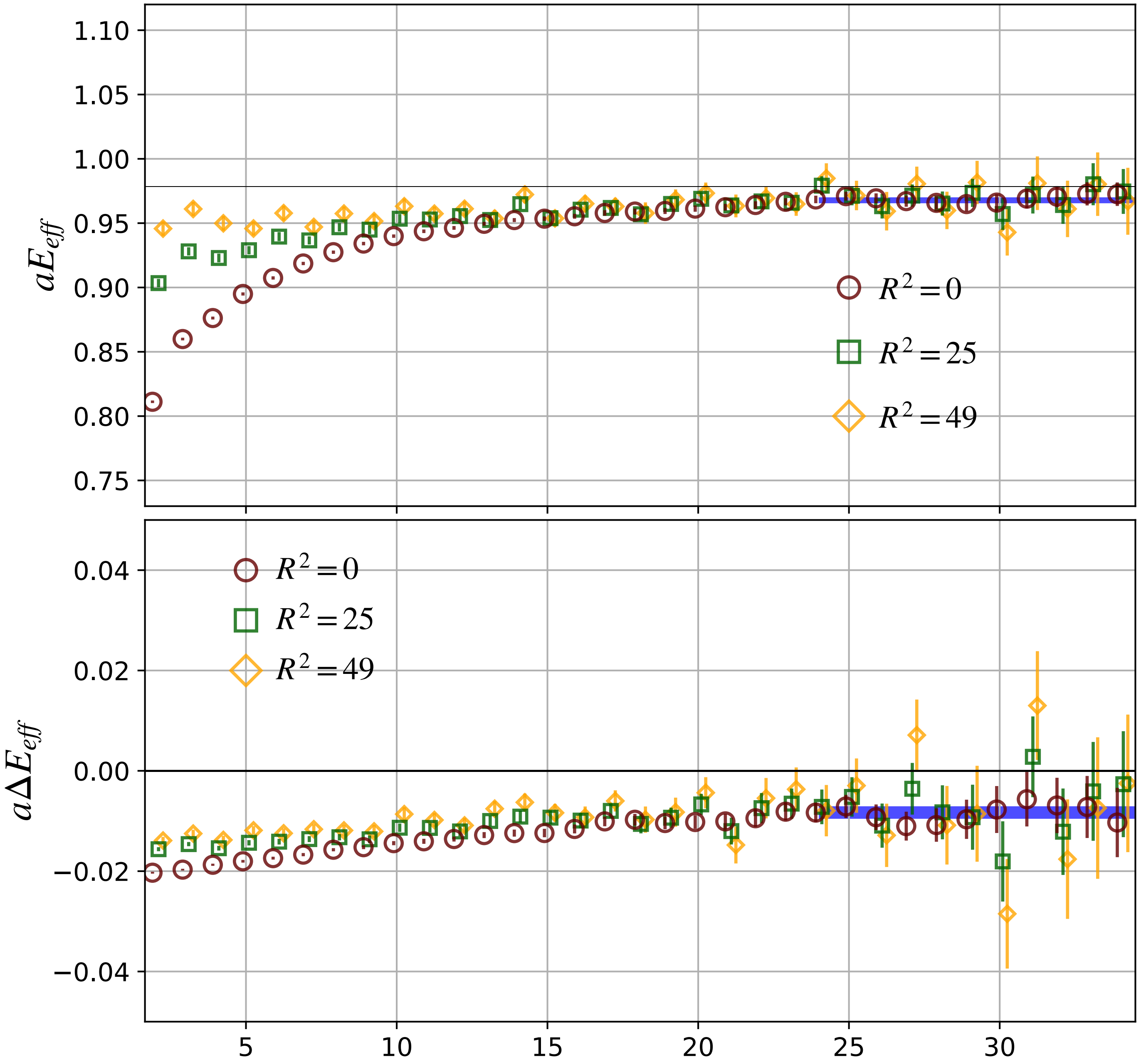


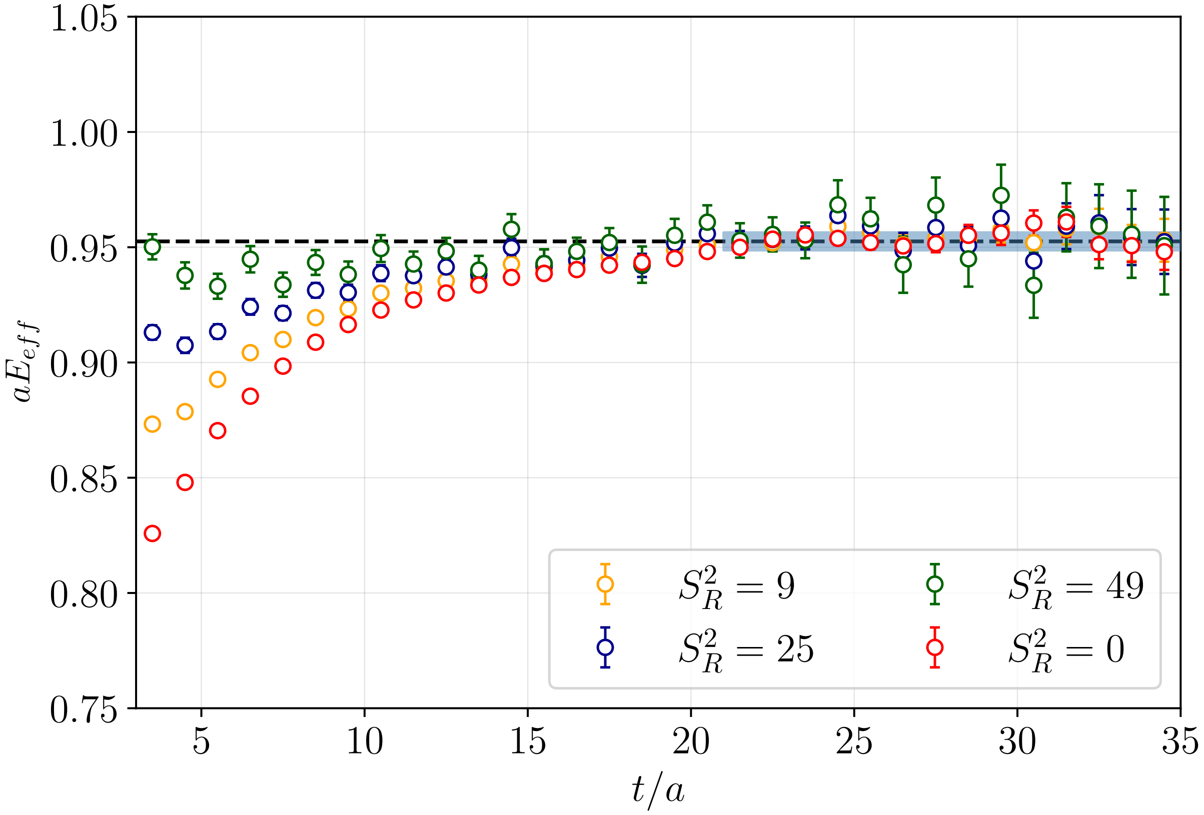


Checking the “robustness of the ground state”

1^+

0^+





Extraction of the energy spectrum

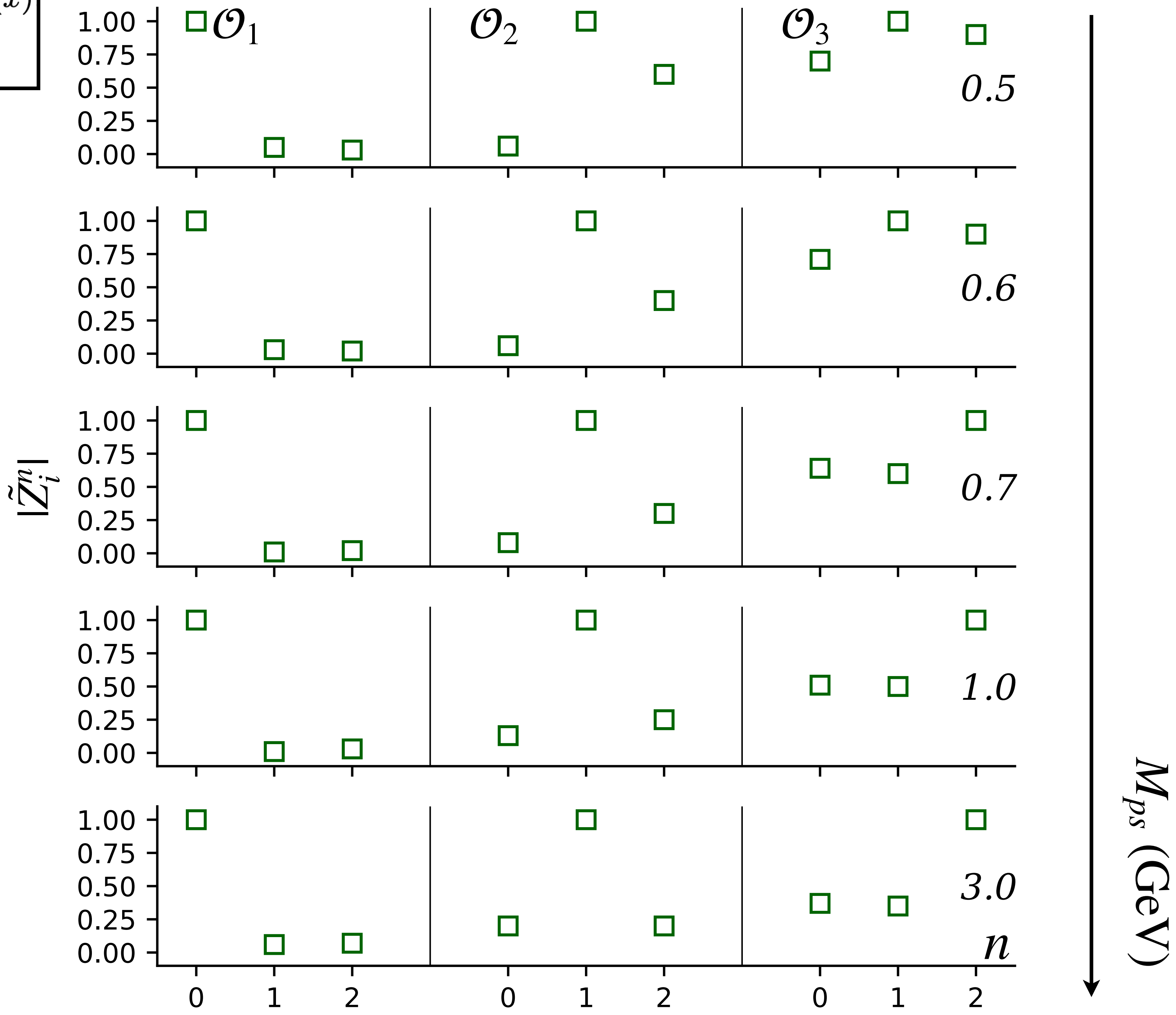
$$\begin{aligned}\mathcal{O}_1(x) &= [\bar{u}\gamma_i b][\bar{d}\gamma_5 c](x) - [\bar{d}\gamma_i b][\bar{u}\gamma_5 c](x)\\ \mathcal{O}_2(x) &= [\bar{u}\gamma_5 b][\bar{d}\gamma_i c](x) - [\bar{d}\gamma_5 b][\bar{u}\gamma_i c](x)\\ \mathcal{O}_3(x) &= [(\bar{u}^T\Gamma_5\bar{d} - \bar{d}^T\Gamma_5\bar{u})(b\Gamma_i c)](x).\end{aligned}$$

Normalized overlaps:

$$\tilde{Z}_i^\alpha = \frac{\langle\alpha|\mathcal{O}_i^\dagger|0\rangle}{\max_\alpha Z_i^\alpha}$$

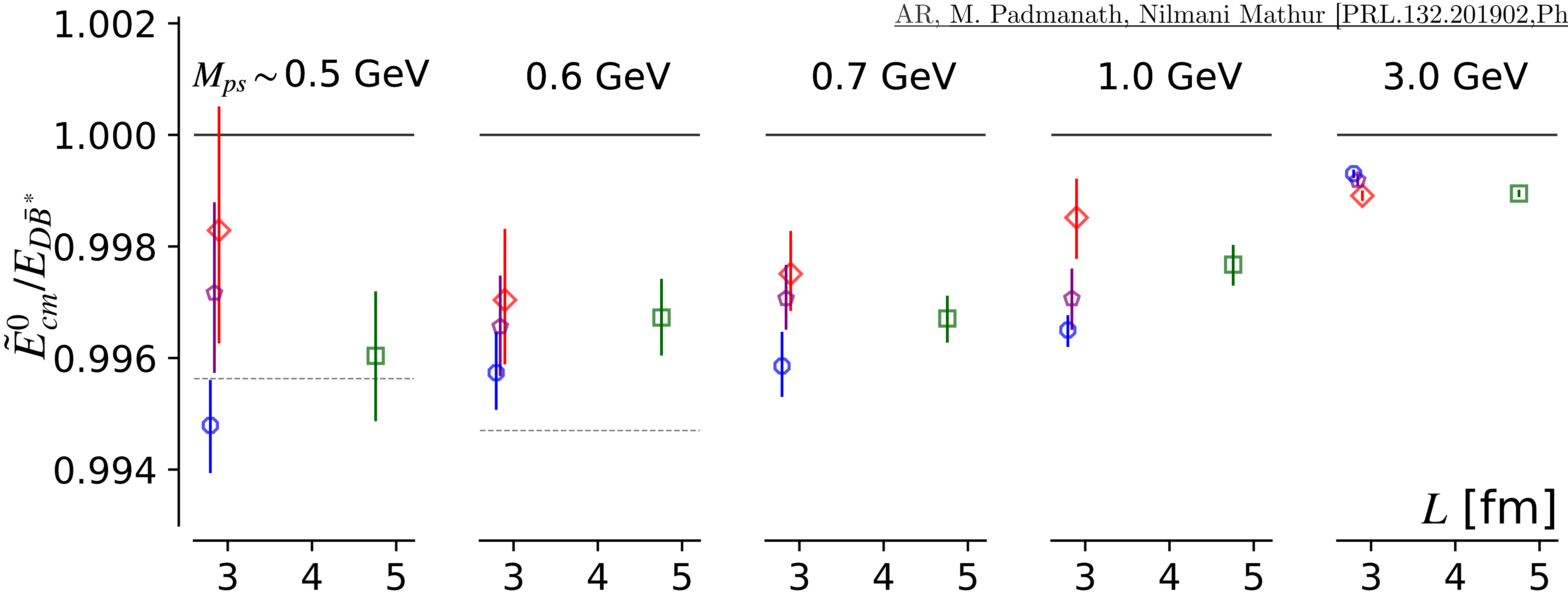
*DB** operator has consistently large overlap to ground state

Diquark-antidiquark overlap to ground state increases as light quark masses decrease

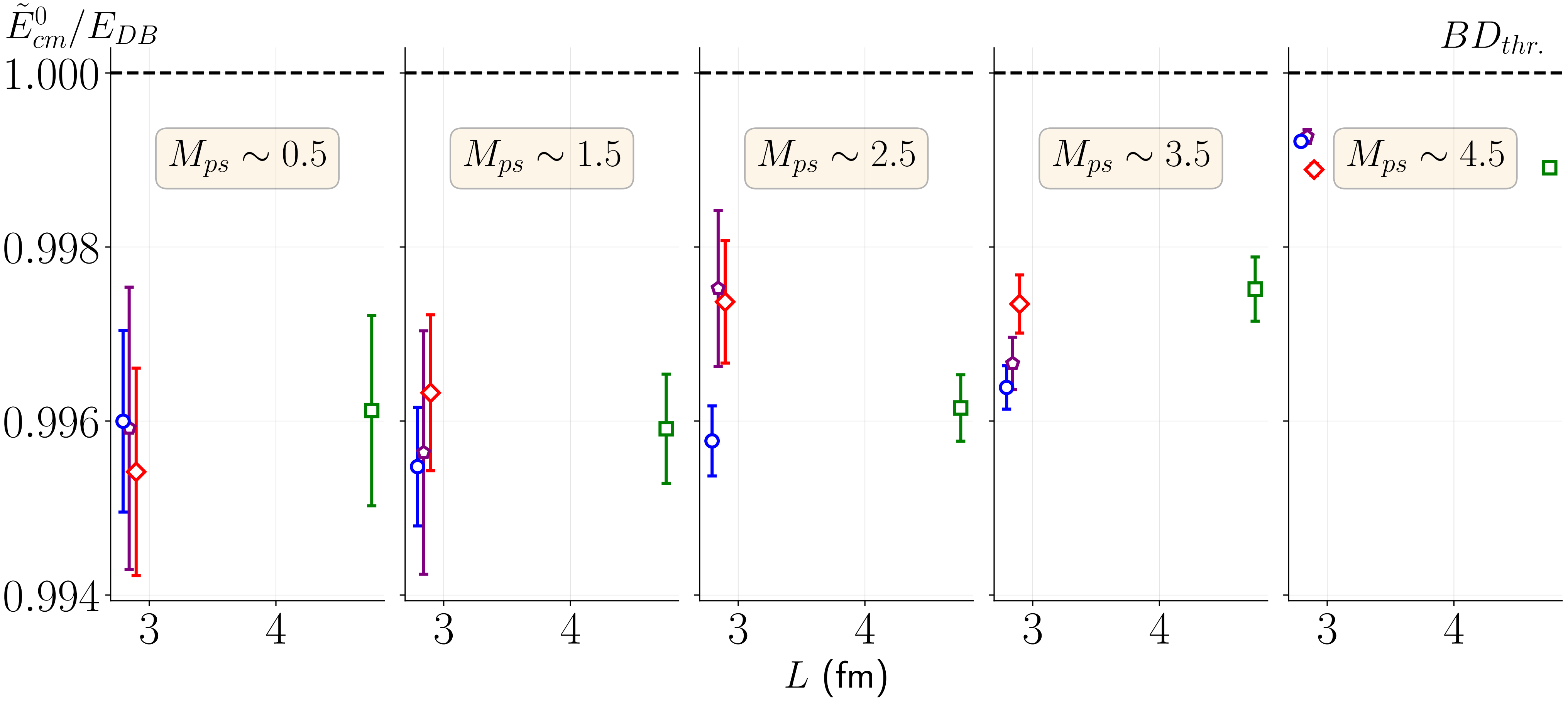


Finite Volume Spectra - $\bar{b}\bar{c}ud$

AR, M. Padmanath, Nilmani Mathur [PRL.132.201902,Phys.Rev.D 110 (2024)]



1^+

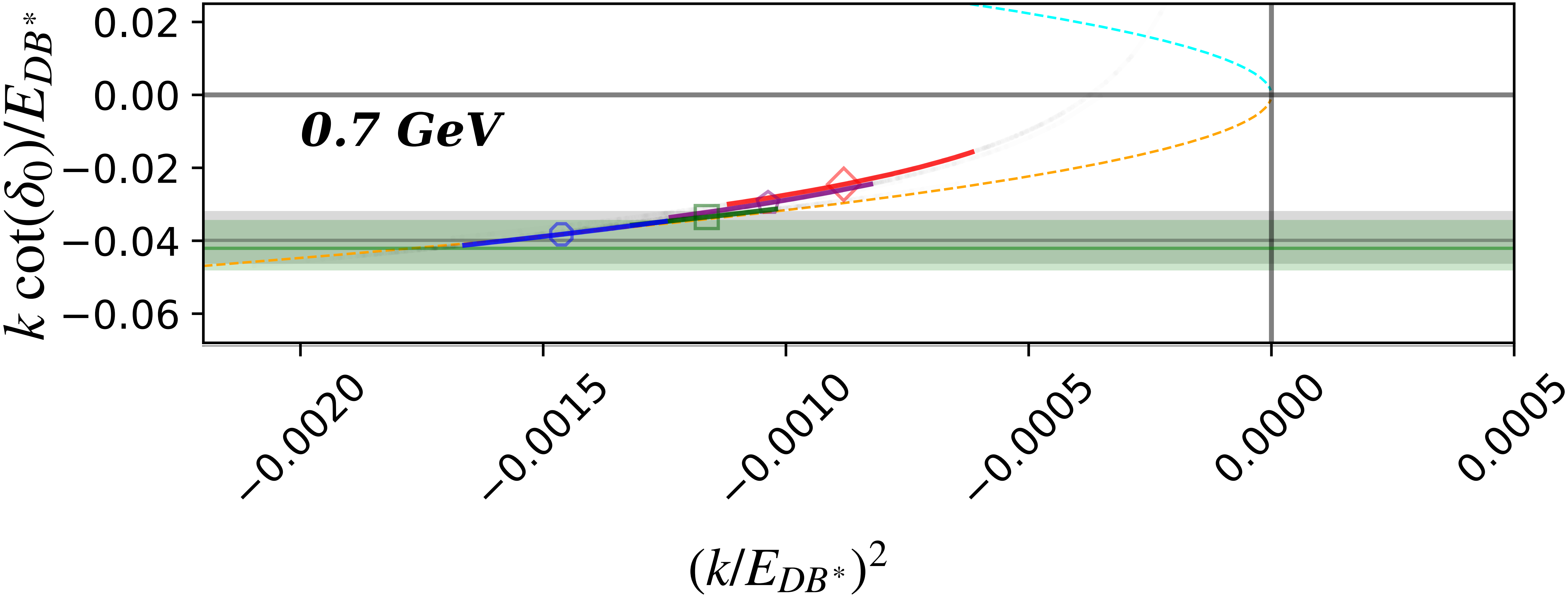


0^+

Infinite volume amplitude with Lüscher- $\bar{b}\bar{c}ud$ 0(1^+)

M. Padmanath, AR, Nilmani Mathur [PRL.132.201902]

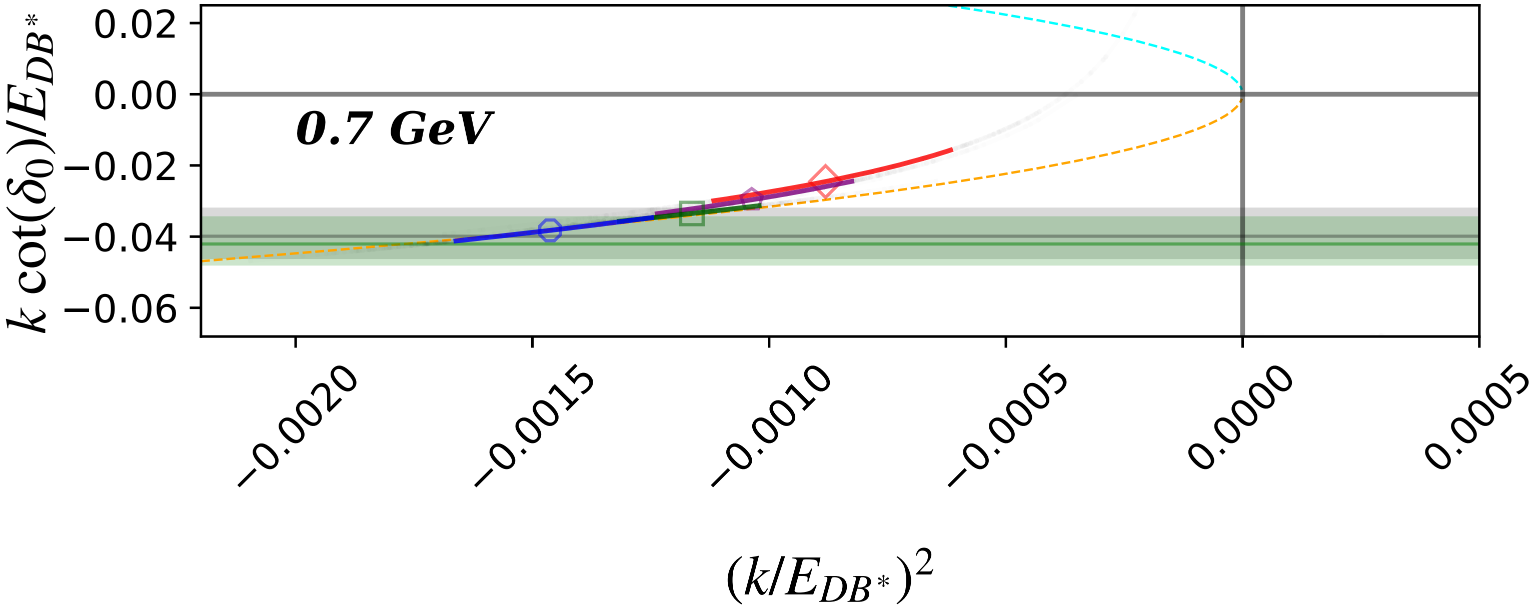
Amplitude: $k\cot\delta_0 \sim -1/a_0$



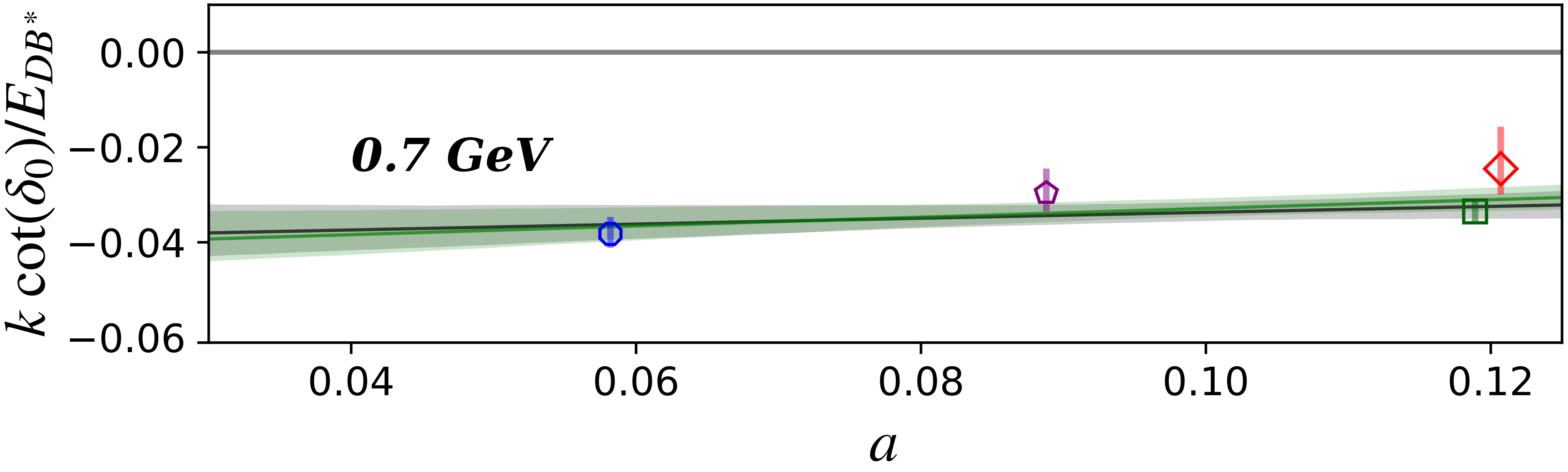
- Non-trivial quark mass dependence and lattice spacing dependence is seen.
- Perform a continuum extrapolation and chiral extrapolation to get the physical amplitudes - then find poles in the amplitude to look for bound states or resonances.

$\bar{b}\bar{c}ud$ $0(1^+)$

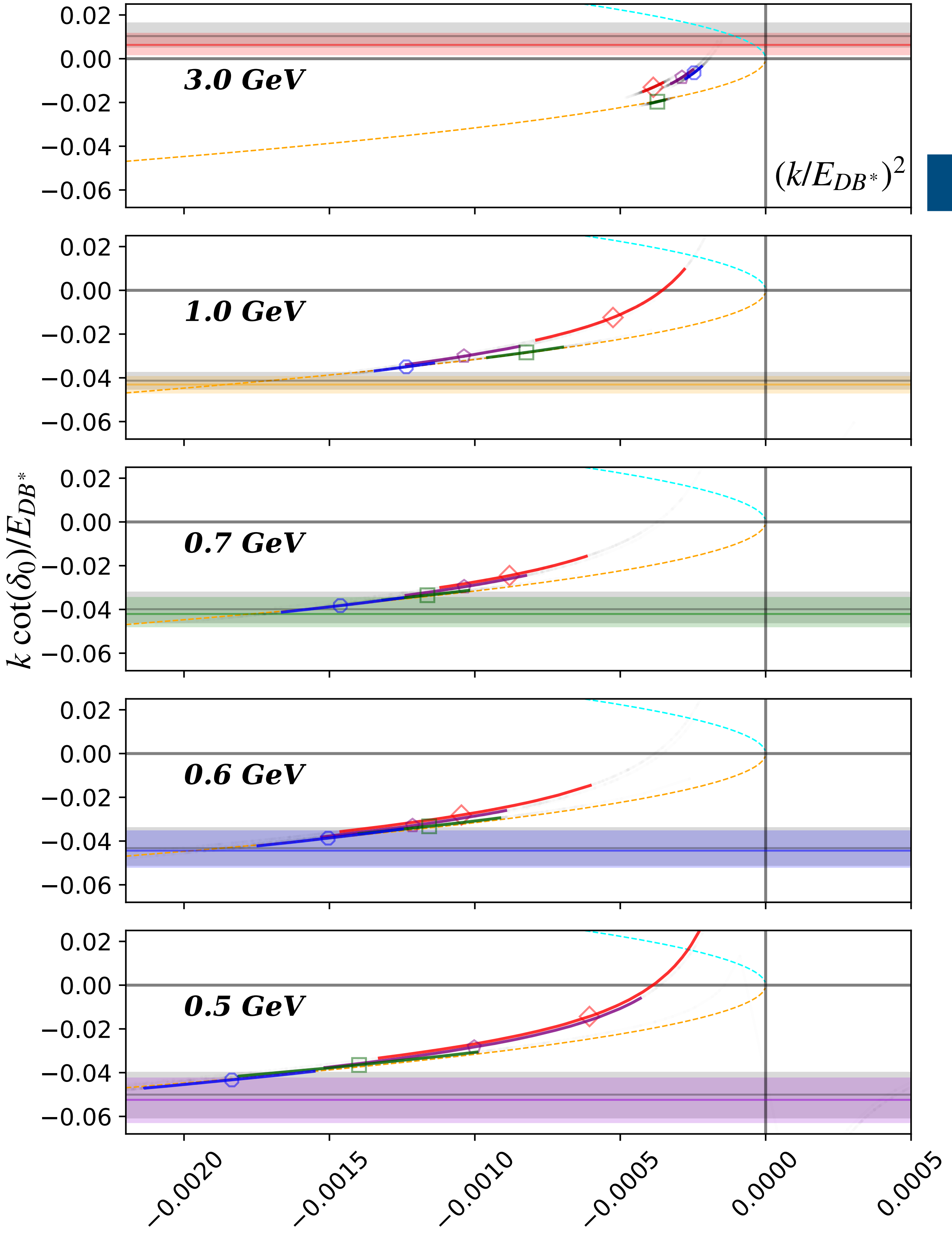
M. Padmanath, AR, Nilmani Mathur [PRL.132.201902]

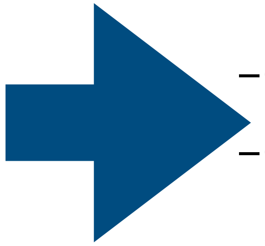


$$k\cot\delta_0 = -1/a_0 + aA^{[1]}$$

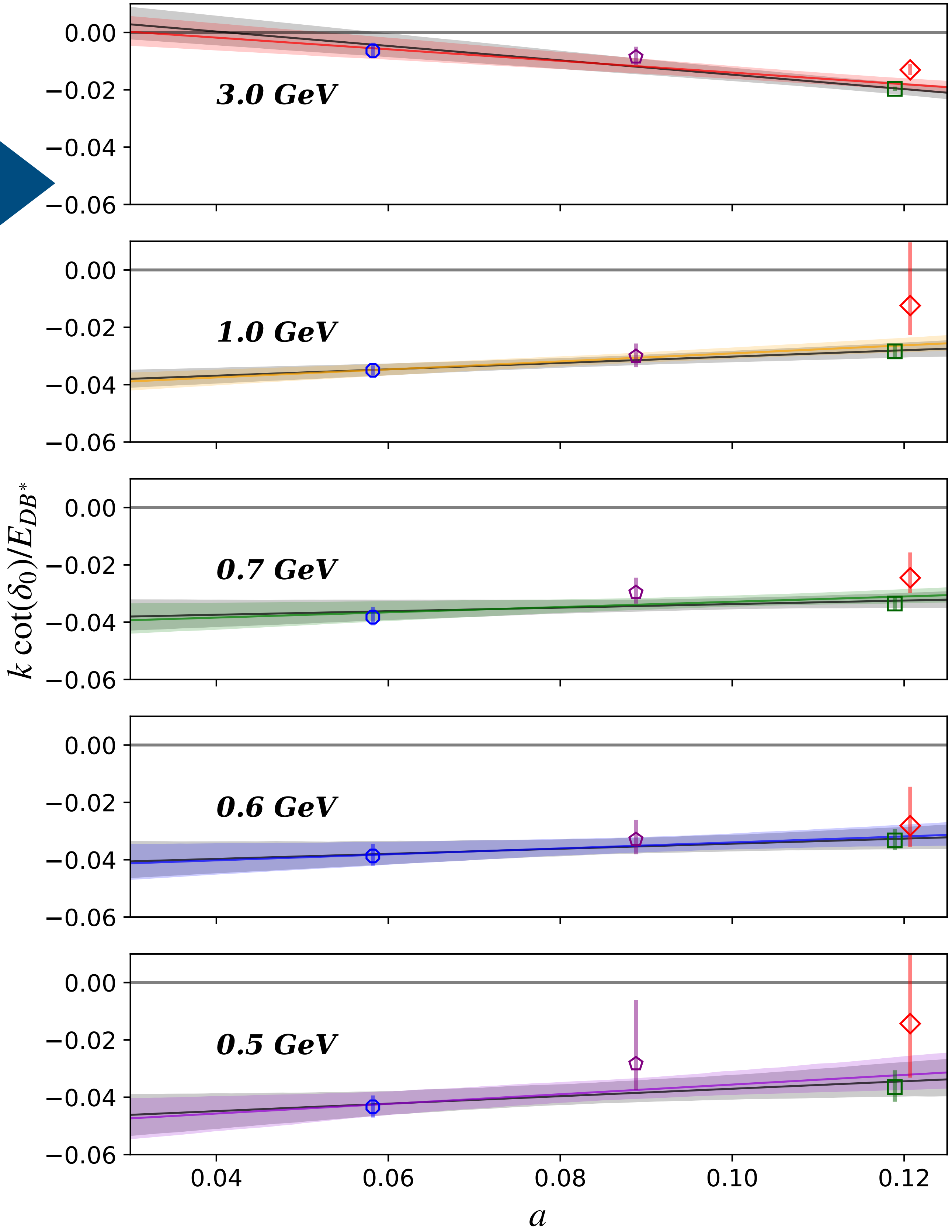


$\bar{b}\bar{c}ud$ $0(1^+)$



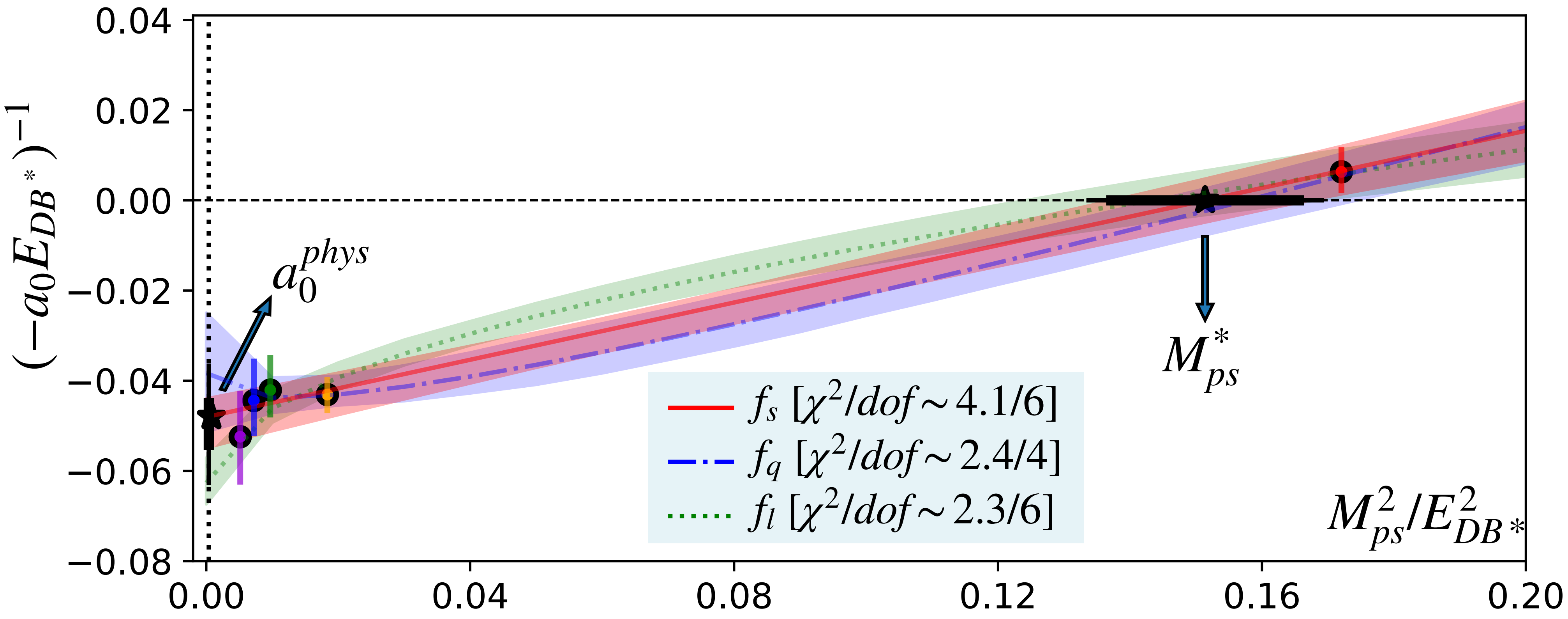


M. Padmanath, AR, Nilmani Mathur [PRL.132.201902]



$\bar{b}\bar{c}ud$ $0(1^+)$

M. Padmanath, AR, Nilmani Mathur [PRL.132.201902]



$$
\begin{aligned}
f_l(M_{ps}) &= \alpha_c + \alpha_l M_{ps}, \\
f_s(M_{ps}) &= \beta_c + \beta_s M_{ps}^2, \quad \text{and} \\
f_q(M_{ps}) &= \theta_c + \theta_l M_{ps} + \theta_s M_{ps}^2
\end{aligned}
$$

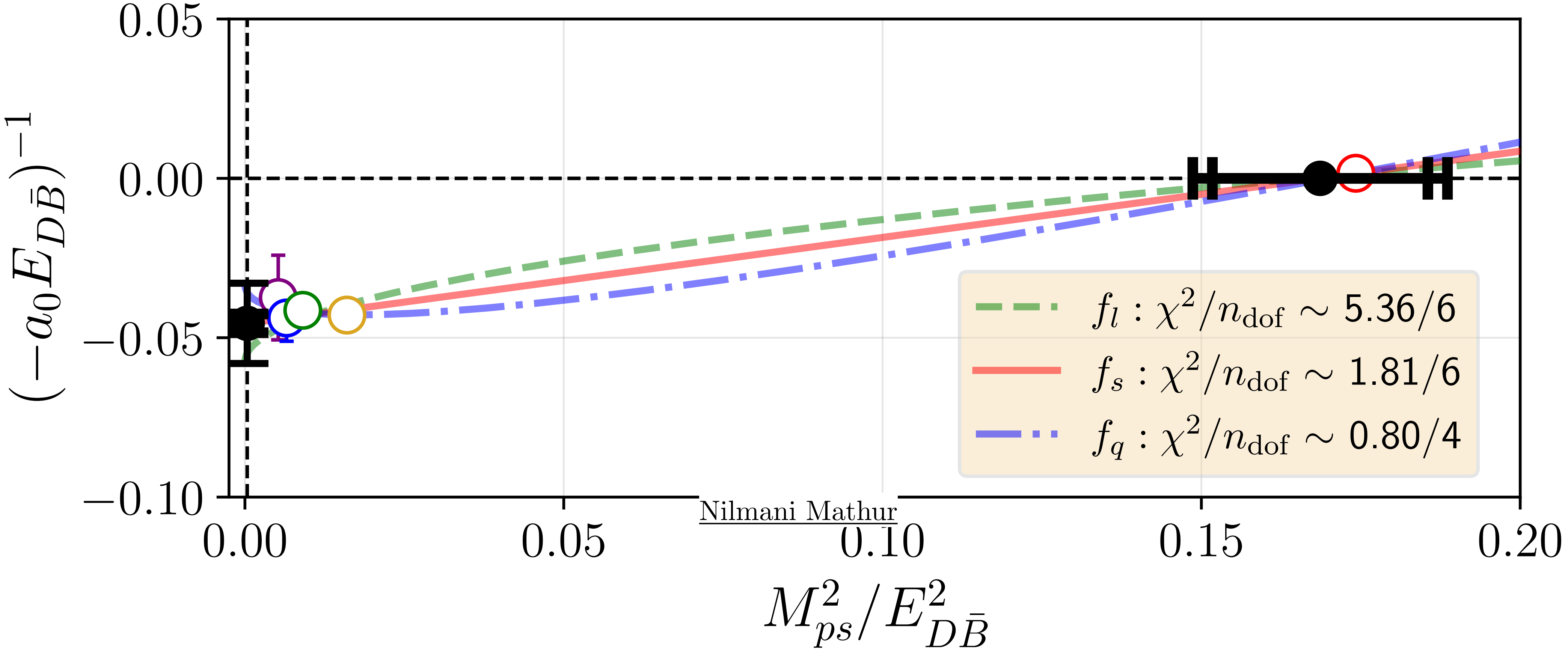
- The binding energy decreases with the increasing light quark mass.
- Indicates the **presence of a real bound state** at physical quark mass.

$a_o = 0.57(^{+4}_{-5})(17)$ fm

$\delta m_{T_{bc}} = -43(^{+6}_{-7})(^{+14}_{-24})$ MeV

$\bar{b}\bar{c}ud$ 0(0^+)

AR, M. Padmanath, Nilmani Mathur [Phys.Rev.D 110 (2024)]



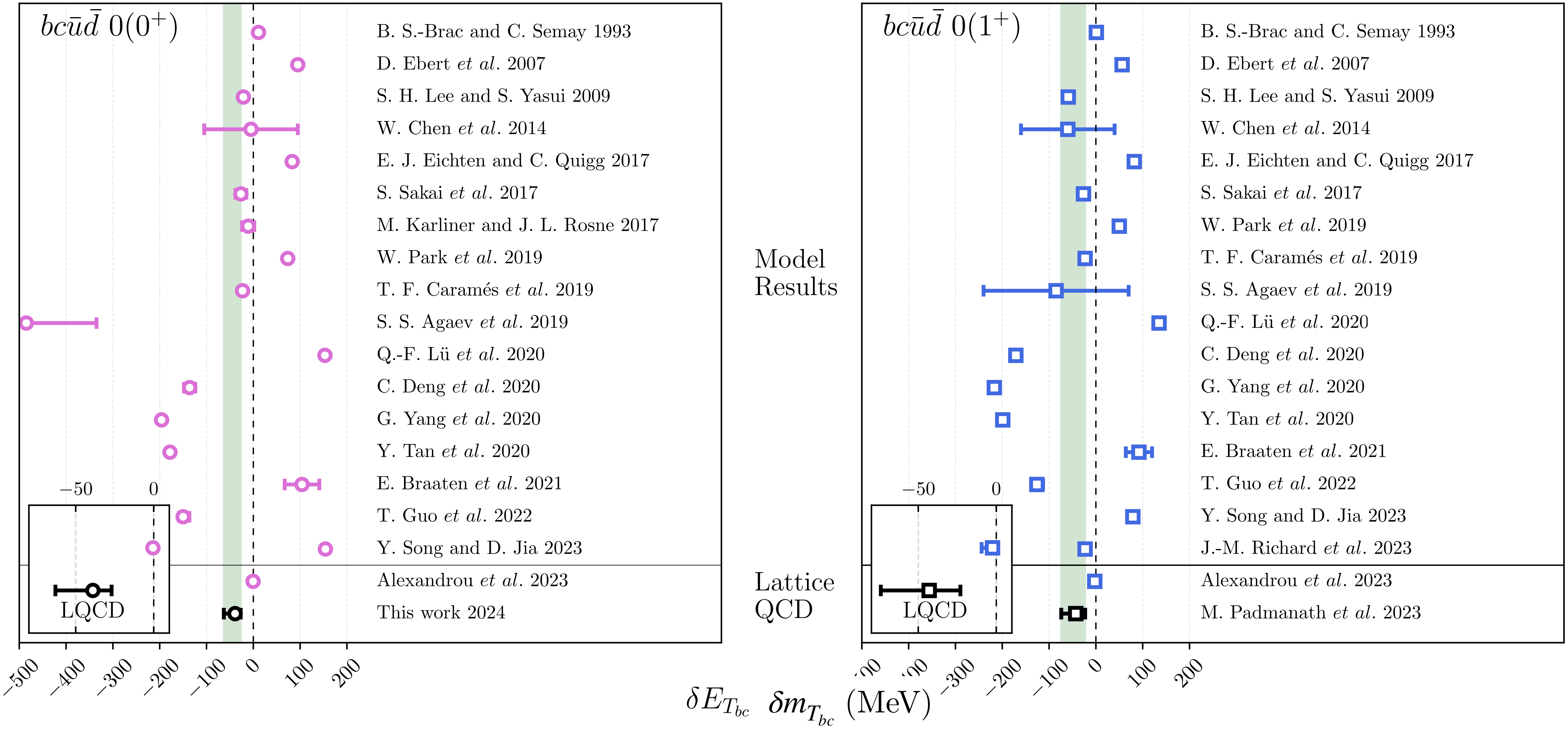
- The binding energy decreases with the increasing light quark mass.
- Indicates the **presence of a real bound state** at physics quark mass.

$a_o = 0.61(^{+3}_{-4})(18)$ fm

$\delta m_{T_{bc}} = -39(^{+4}_{-6})(^{+8}_{-18})$ MeV

Comparison of T_{bc} binding energy results

AR, M. Padmanath, Nilmani Mathur [Phys.Rev.D 110 (2024)]



Conclusion

- Multiple channel analysis, more volumes, moving frames and multiple operators to better understand these tetraquarks from the lattice perspective.
- Relativistic bottom quarks?

- Since the $0(0^+)$ T_{bc} tetraquark is a bound state, will decay through the weak interaction only and could become the first tetraquark to be observed at the LHC with this feature.

- The $0(1^+)$ T_{bc} is expected to decay electromagnetically to $BD\gamma$

- The study of radiative transition

$$B^*D \to 0(1^+)\ T_{bc} \to BD\gamma$$

can map the structure!