Meson production amplitudes from Lattice QCD

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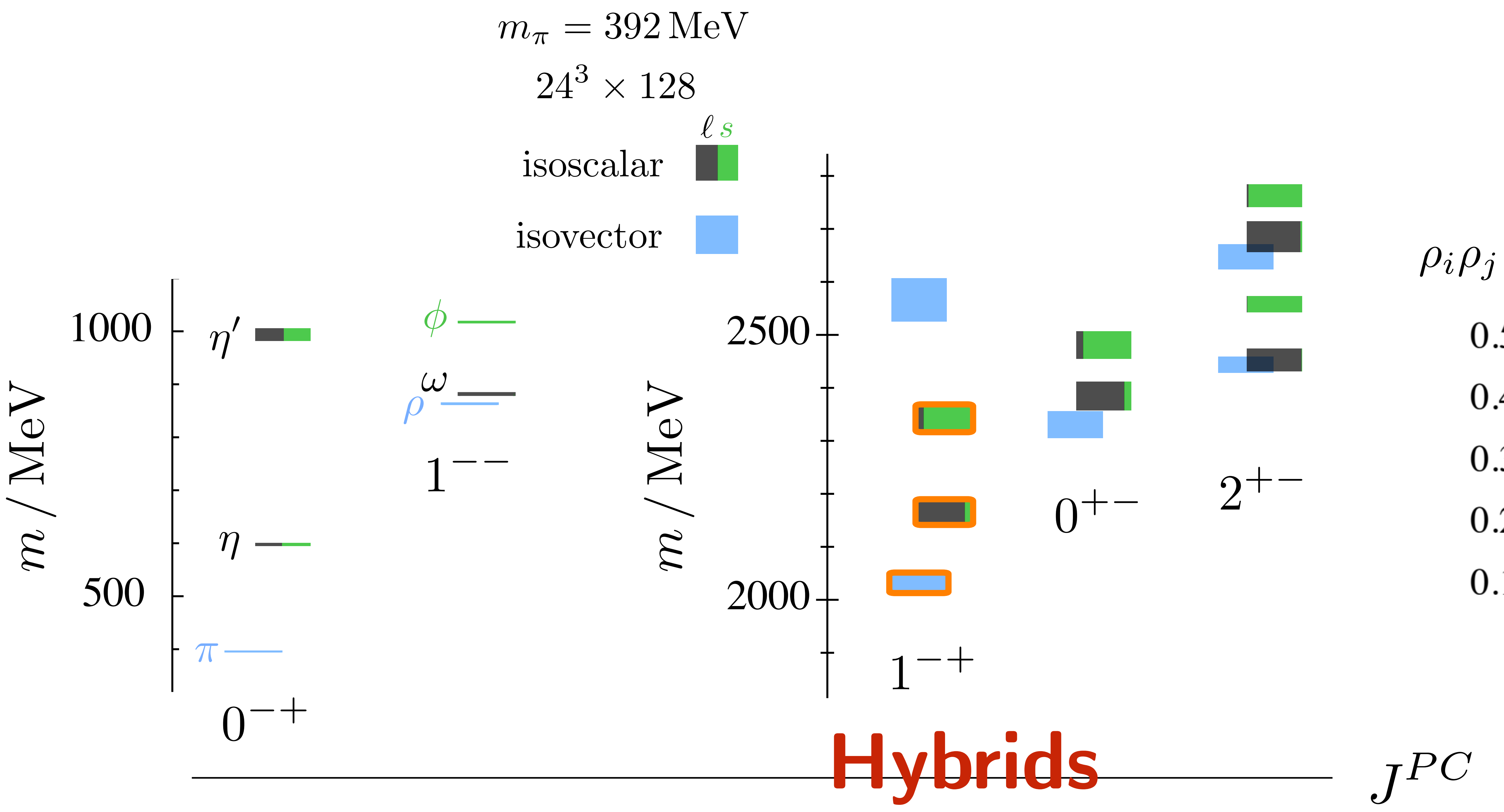
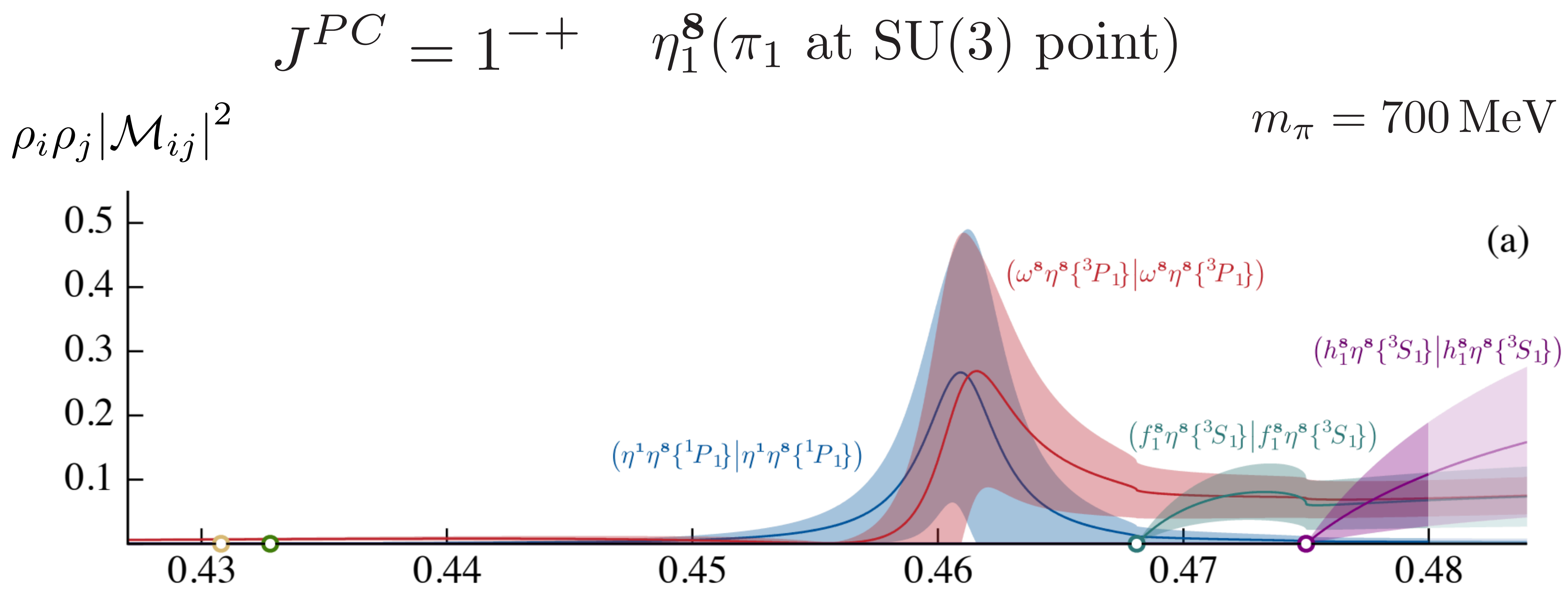
Characterization of hadrons (Lattice QCD)

[Dudek et al. 1309.2608]
[Woss et al. 2009.10034]

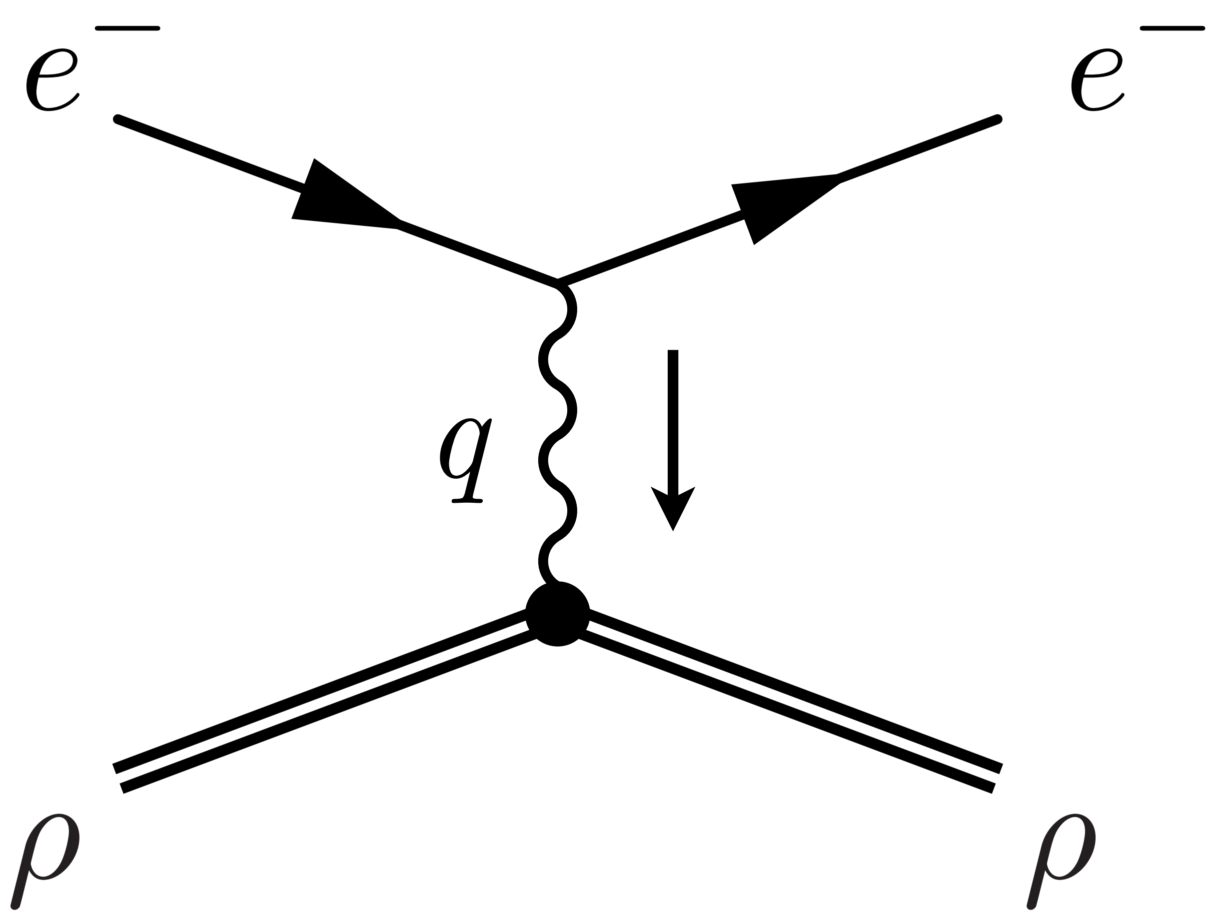




- Masses and decay widths
- Couplings to hadronic decay channels
- *Couplings to electroweak currents*

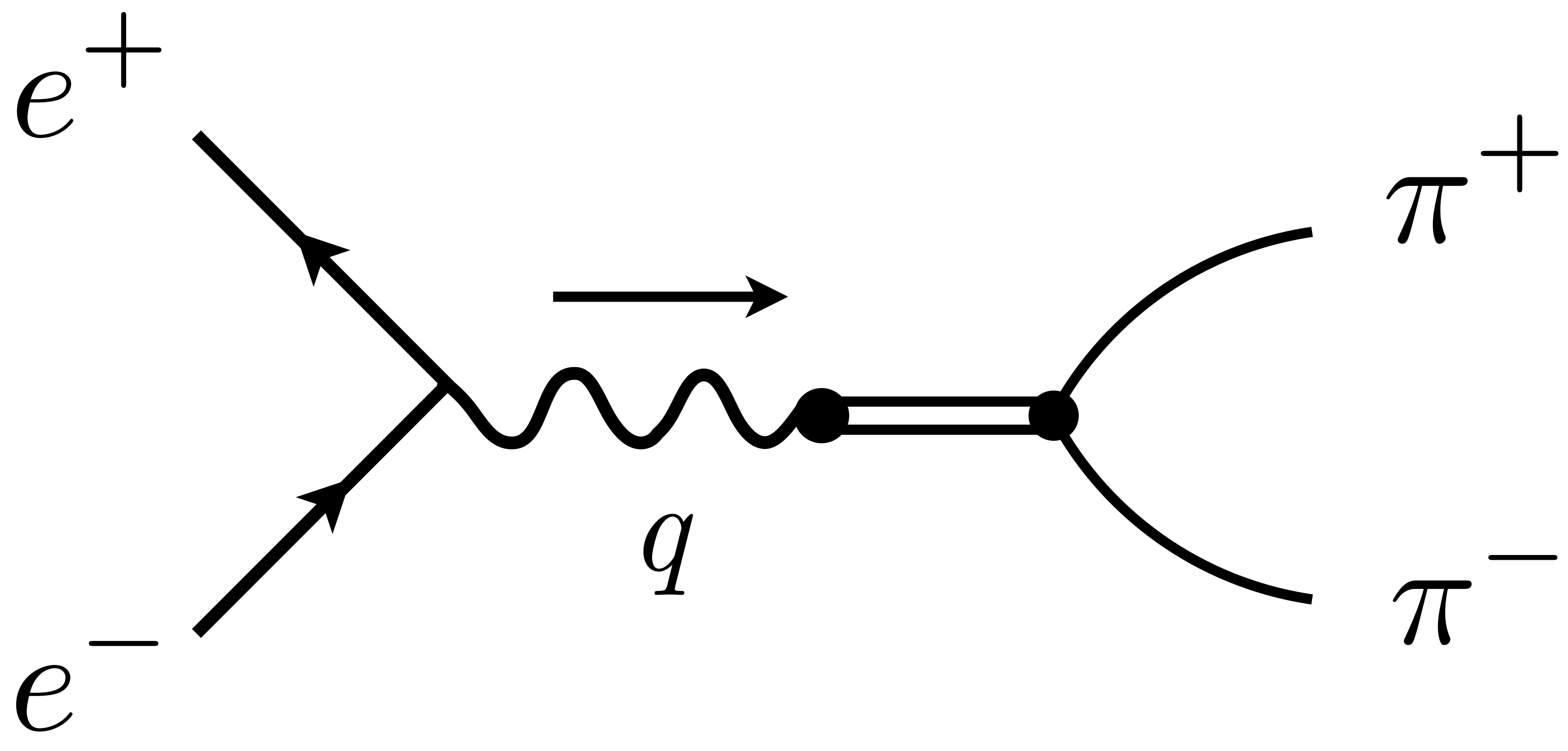
Study of resonances

Spacelike $Q^2 \equiv -q^2 > 0$



Structural information

Timelike $q^2 > 0$



Production mechanisms

Resonant meson form factors

Hadronic two-to-two

Scattering amplitude

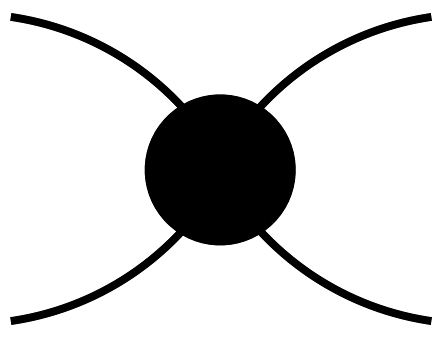
Two-to-two

Transition amplitude

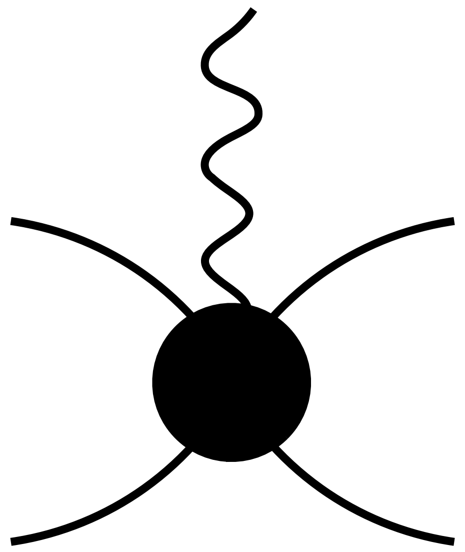
Production amplitude

with hadronic final state interactions

$\mathcal{M} =$



$\mathcal{W} =$



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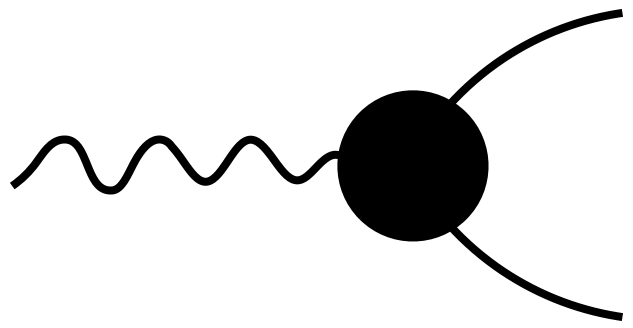
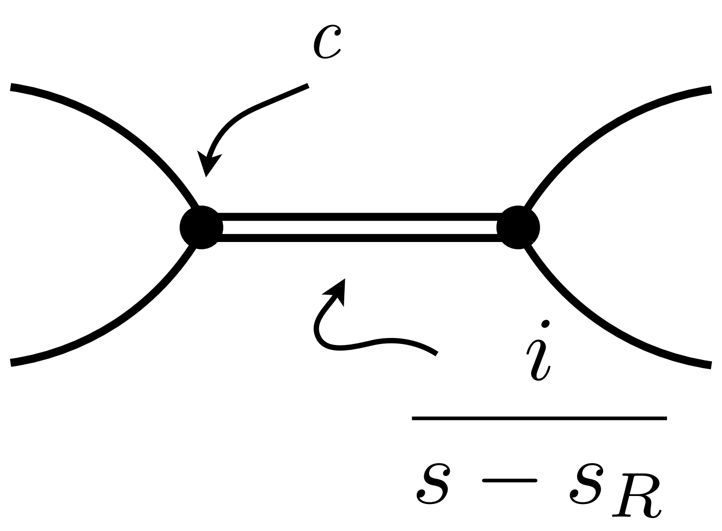
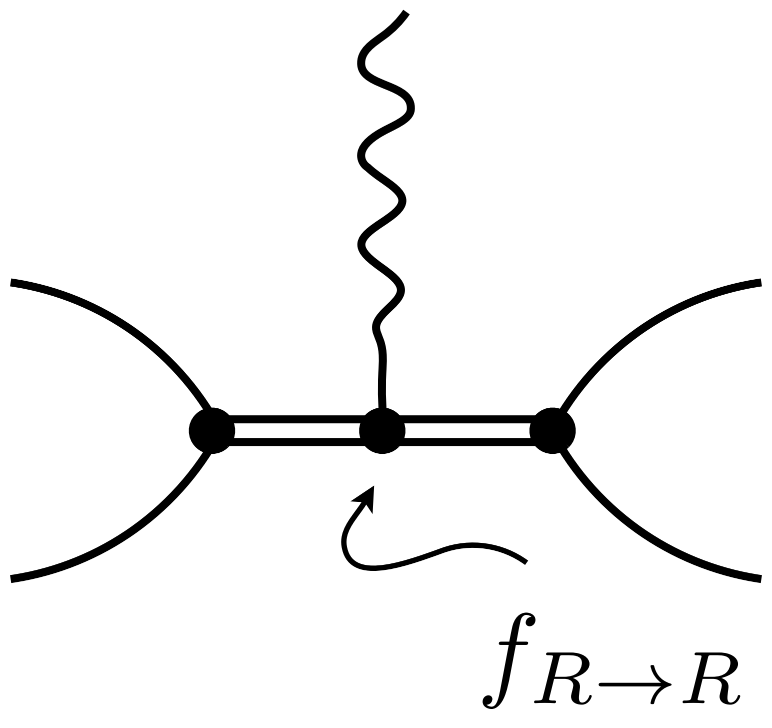
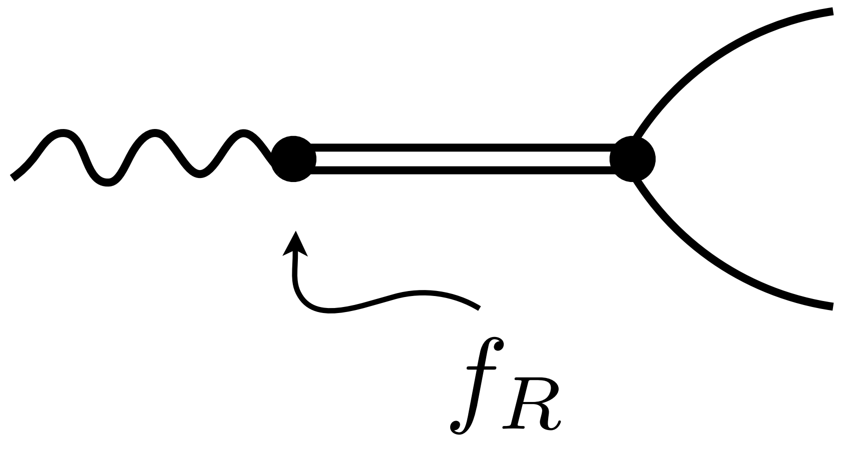






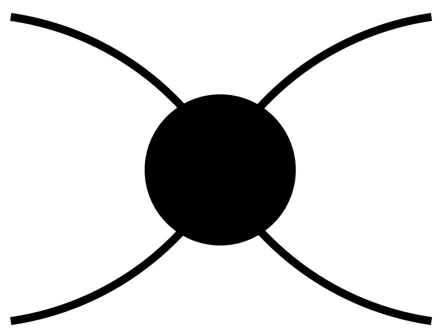


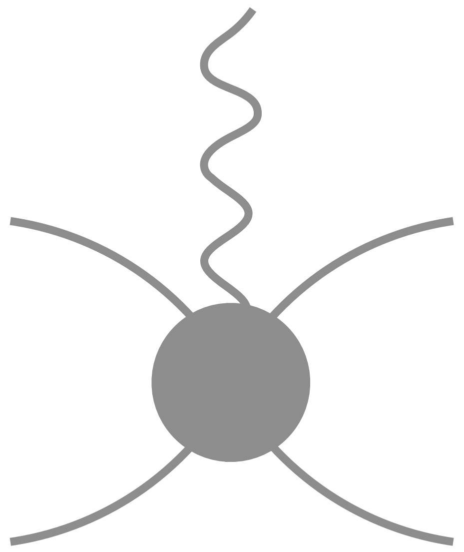
Resonant meson form factors

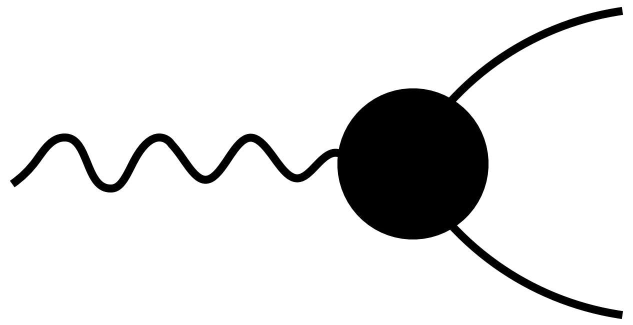
Hadronic two-to-two
Scattering amplitude

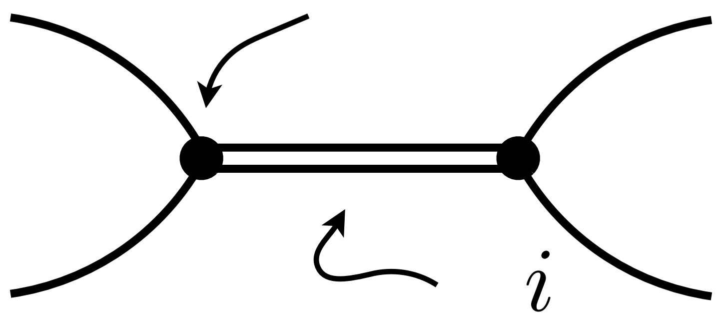
Two-to-two
Transition amplitude

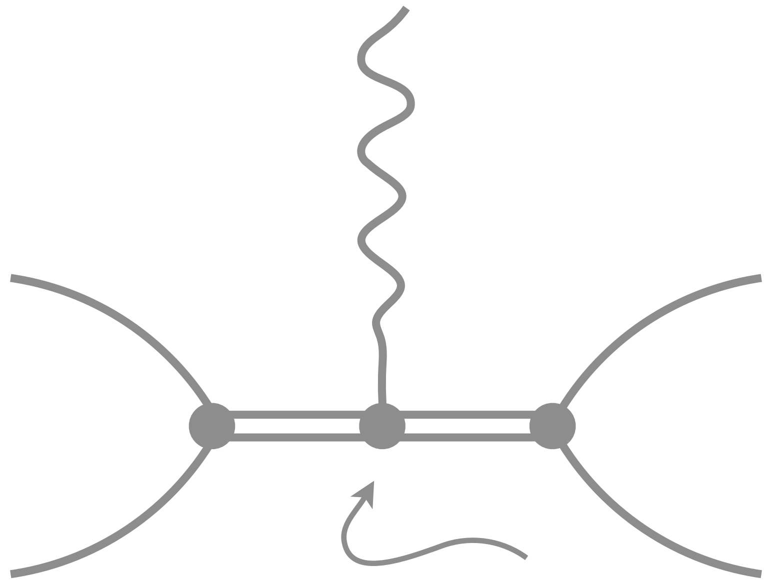
Production amplitude
with hadronic final state interactions

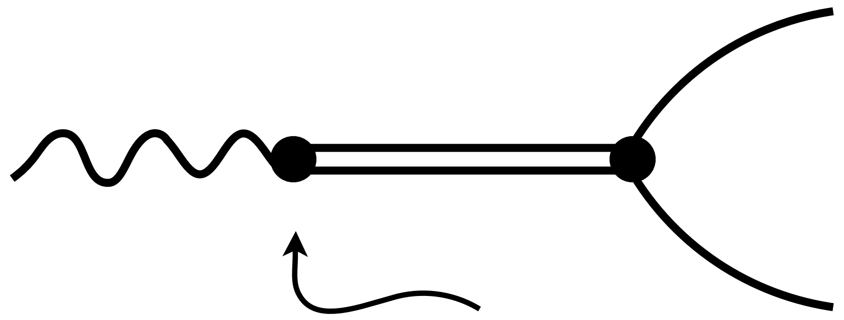
$\mathcal{M} =$ 

$\mathcal{W} =$ 

$f =$ 





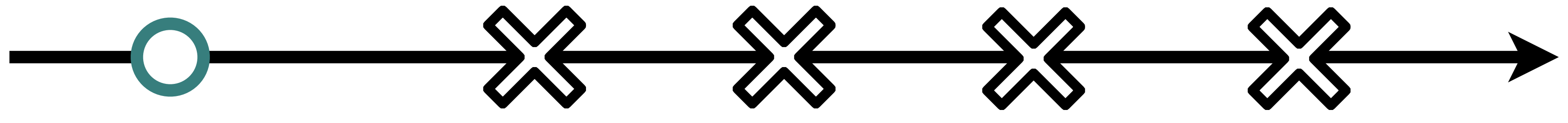


Lüscher formalism

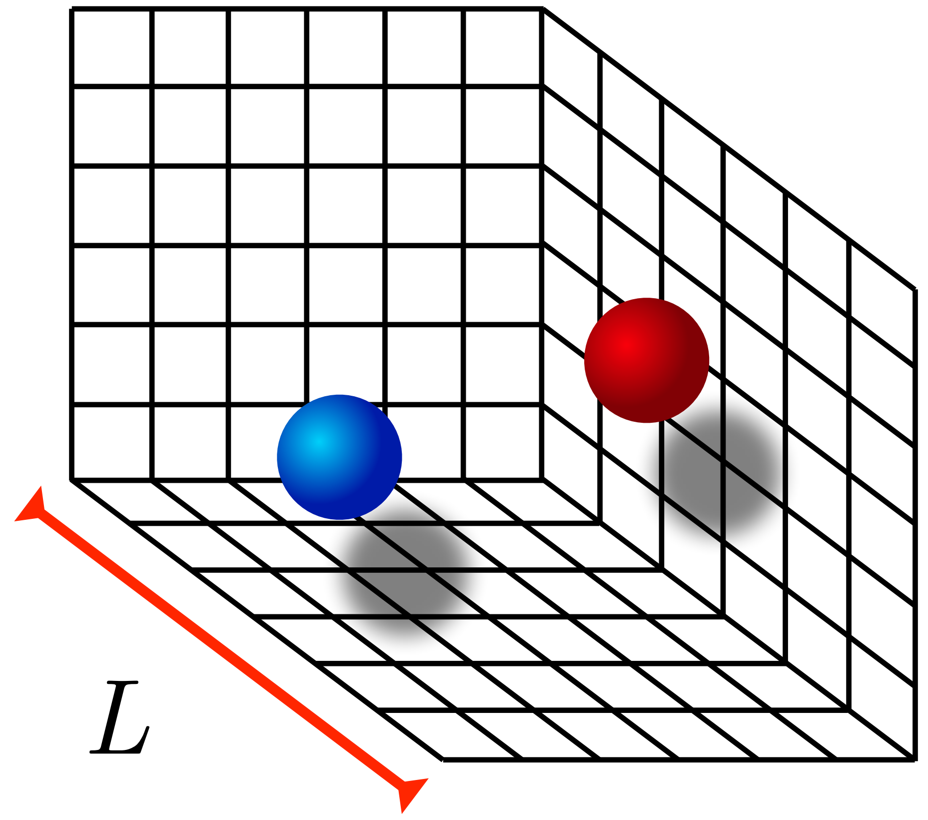
[*Commun.Math.Phys.* 105, 153 (1986) and extensions]
See Briceno et al. (2017) 1706.06223 for a review

$$\left\langle O_i(t)O_j^\dagger(0)\right\rangle = \sum_{n=0} Z_i^{n*} Z_j^{n} e^{-E_n t}$$

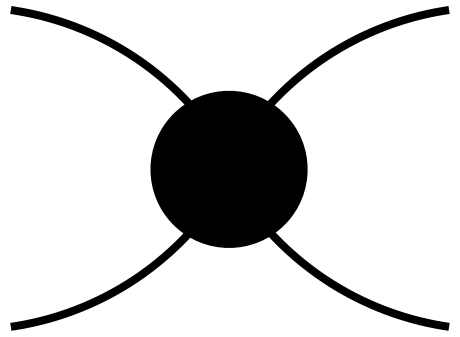
$C_L(P^2)$

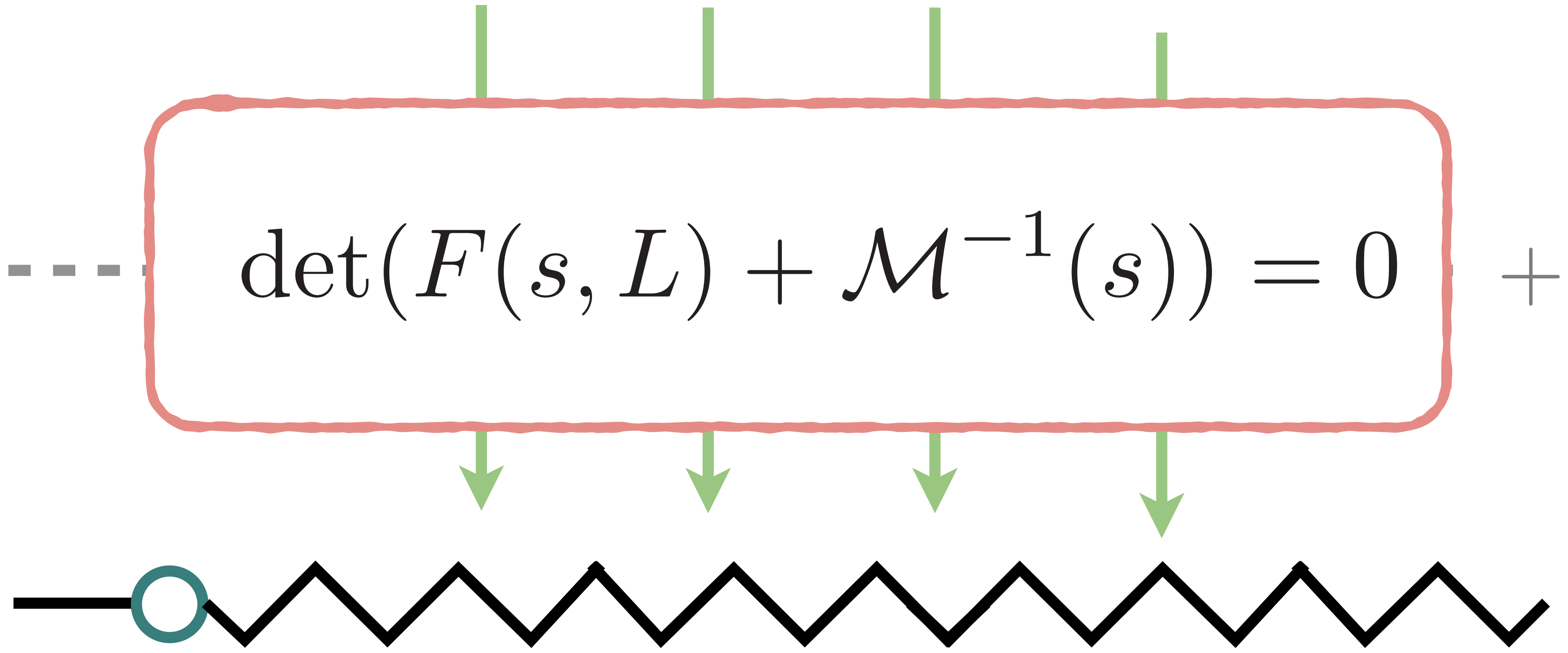


$E_n^{\star 2}(L)$

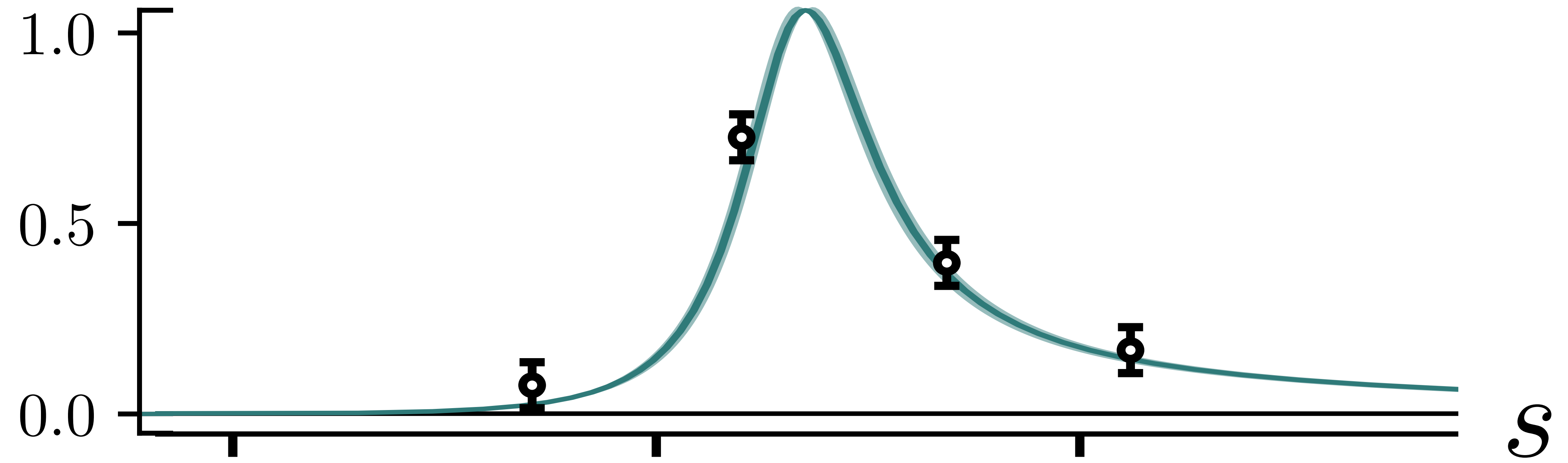


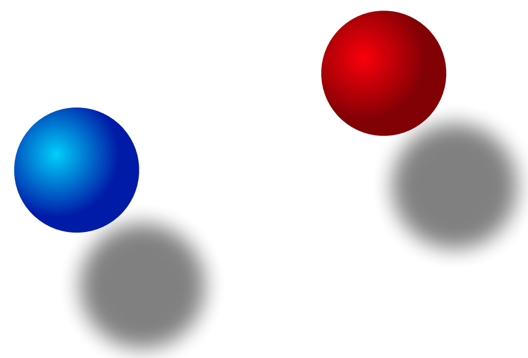
$$\det(F(s,L)+\mathcal{M}^{-1}(s)) = 0 \quad +\mathcal{O}(e^{-m_\pi L})$$

$\mathcal{M}(s) =$ 



$\rho^2|\mathcal{M}|^2$

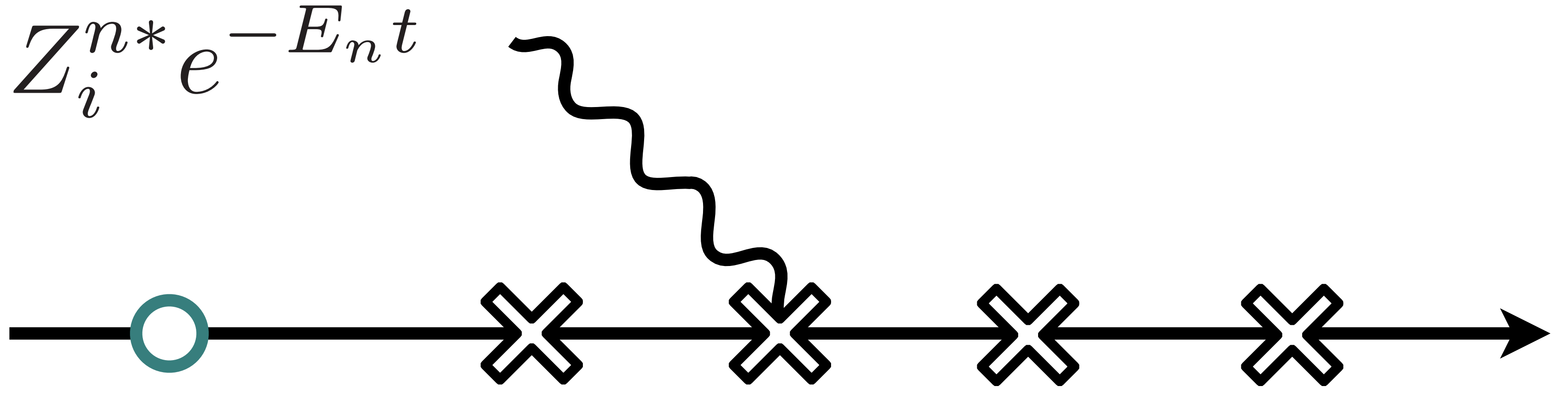




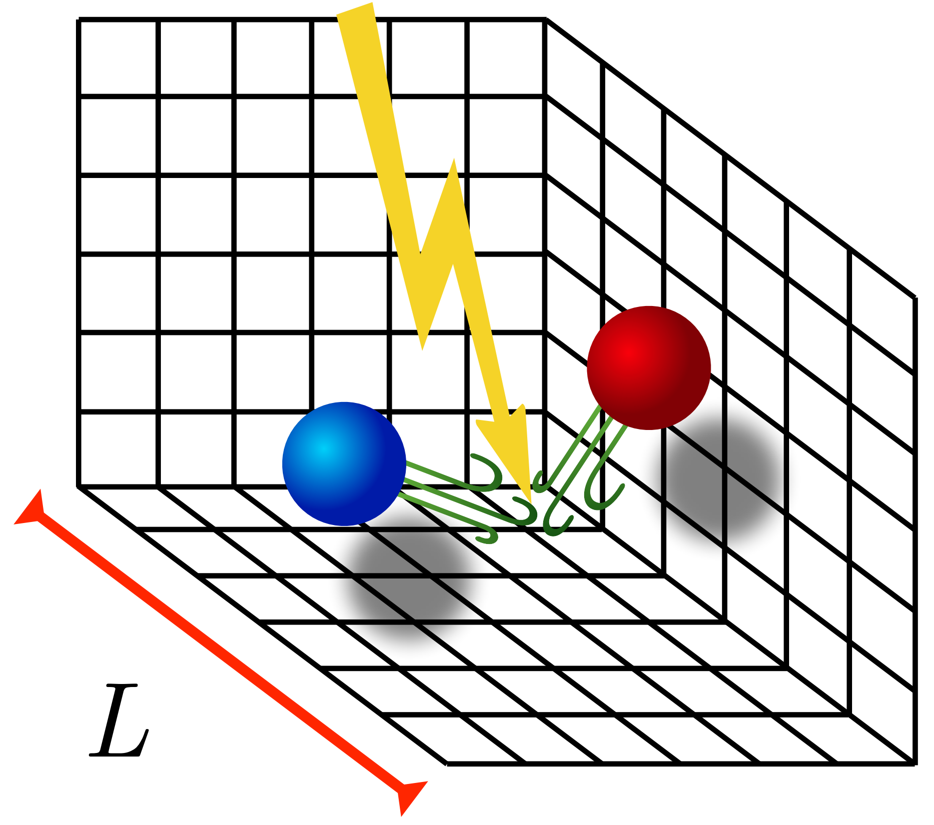
Two-hadron production

[Meyer (2011) 1105.1892]
[Lellouch, Luscher (2001) hep-lat/0003023]

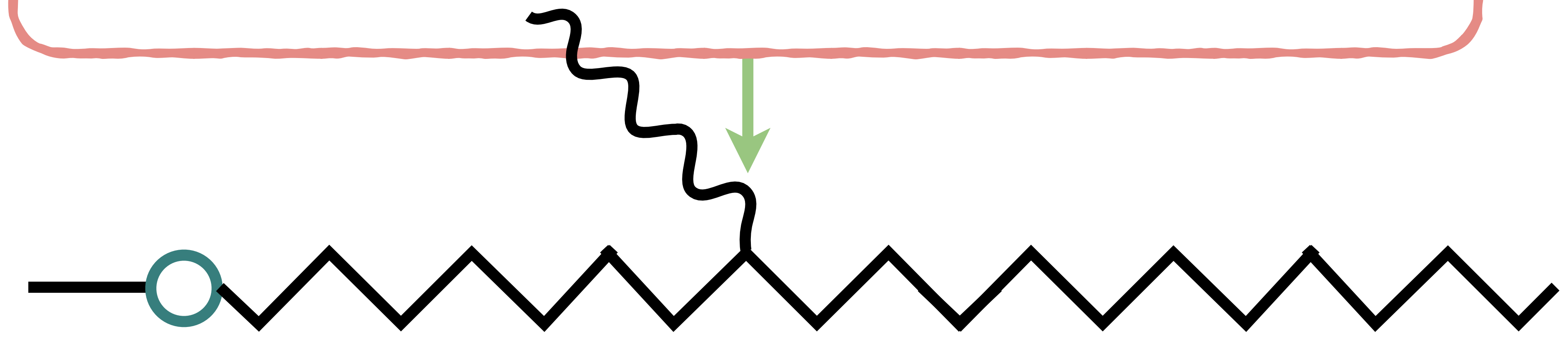
$$\langle O_i(t)\mathcal{J}^\mu(0)\rangle = \sum_{n=0} \langle n|\mathcal{J}^\mu(0)|0\rangle\, Z_i^{n*} e^{-E_n t}$$



$E_n^{\star 2}(L)$



$$|\langle n|\mathcal{J}(0)|0\rangle|^2 = \frac{1}{L^3} f(s) \cdot \mathcal{R}_n \cdot f(s)$$

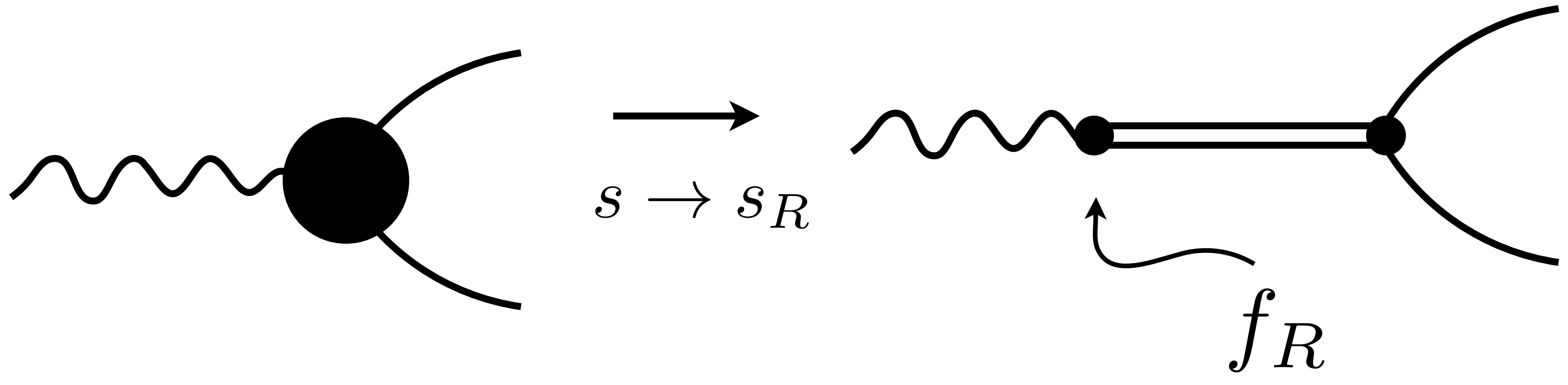


s

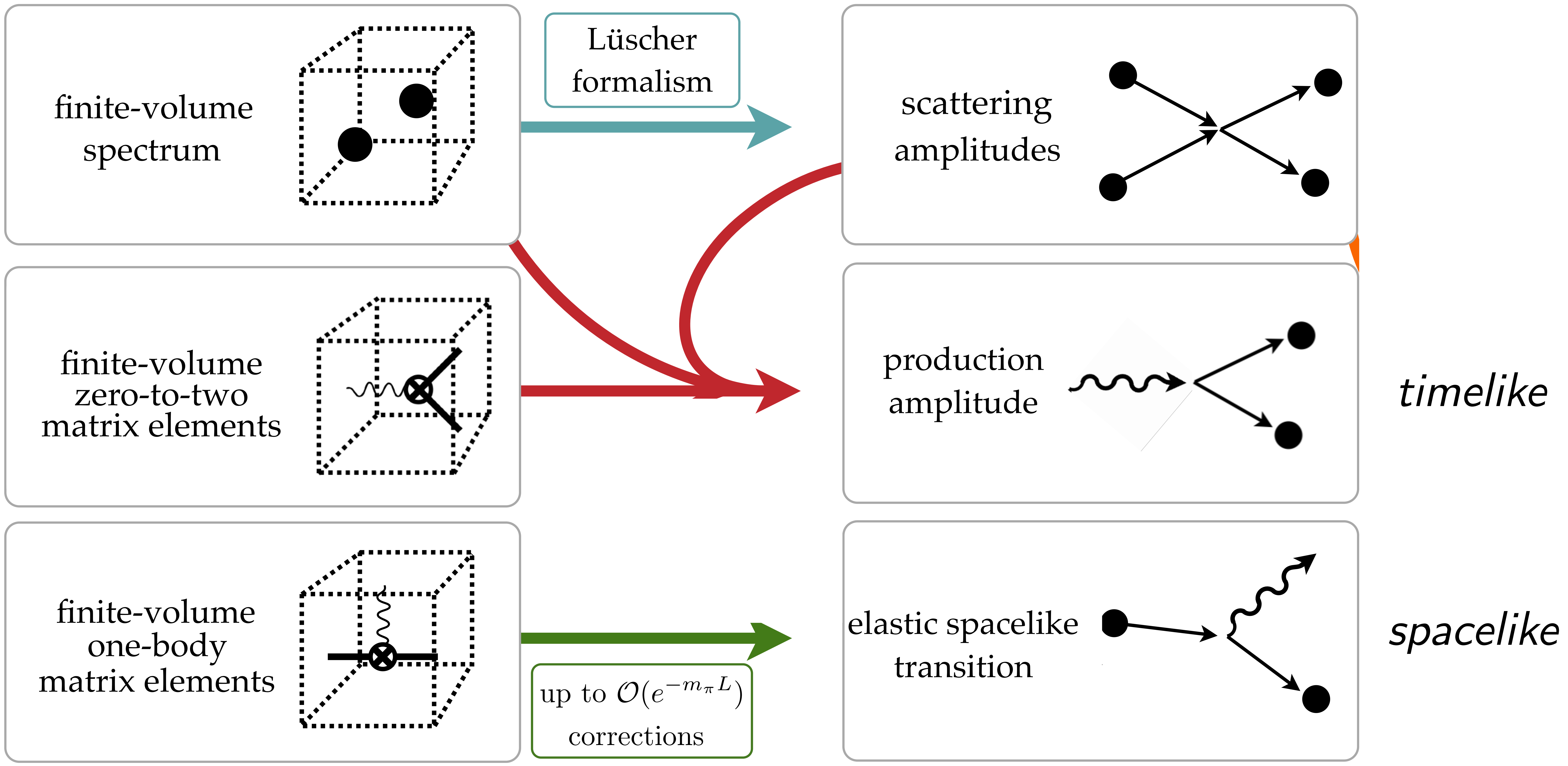
- Lellouch-Lüscher factor

$$\mathcal{R}_n \equiv \lim_{E\to E_n} \frac{E - E_n}{F^{-1} + \mathcal{M}}$$

Normalization of
dynamic FV state
& rescattering effects

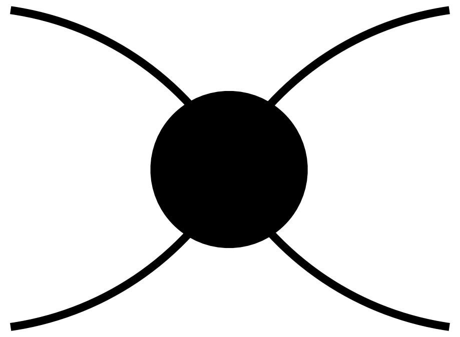
$f =$ 

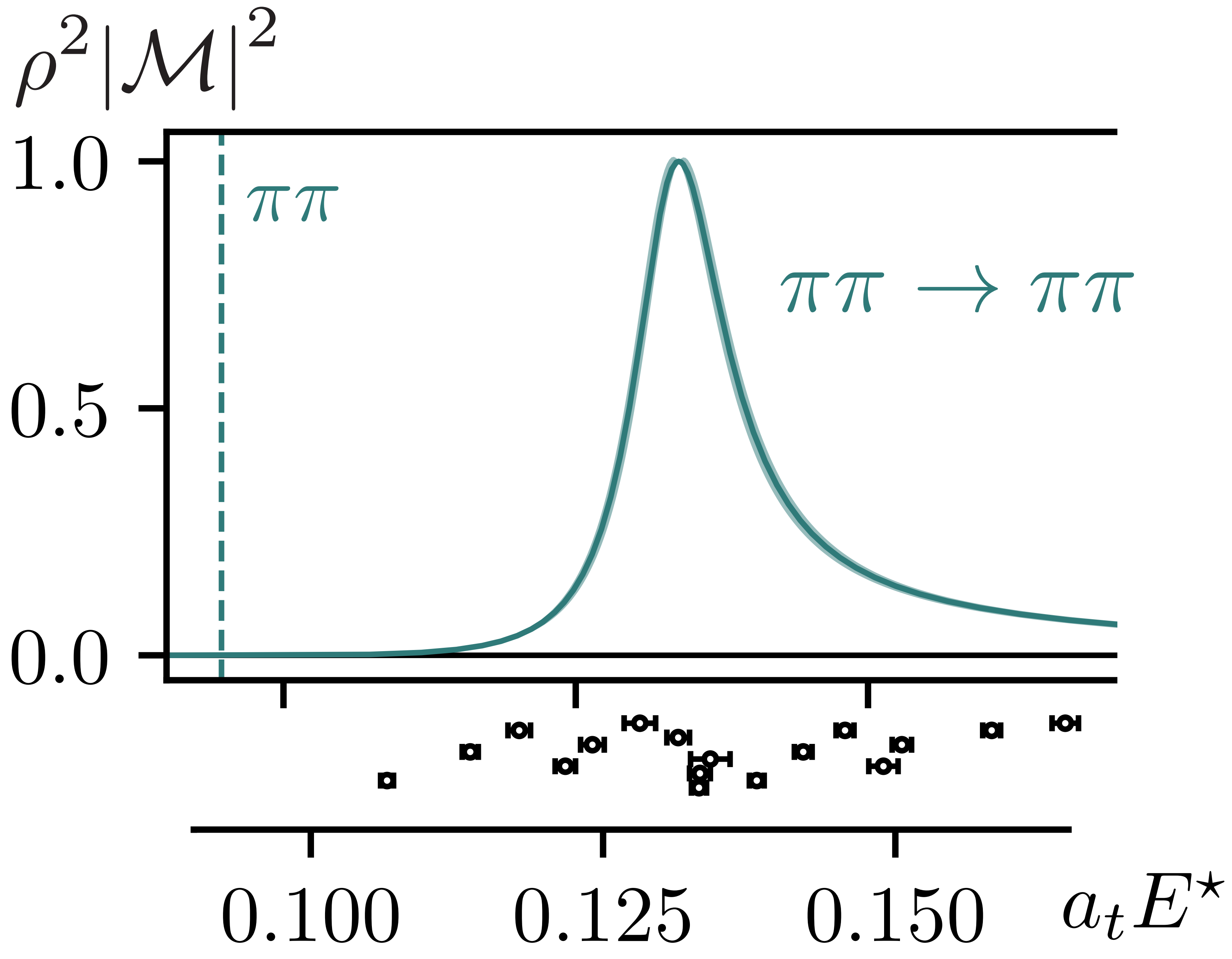
Lattice QCD form factors workflow



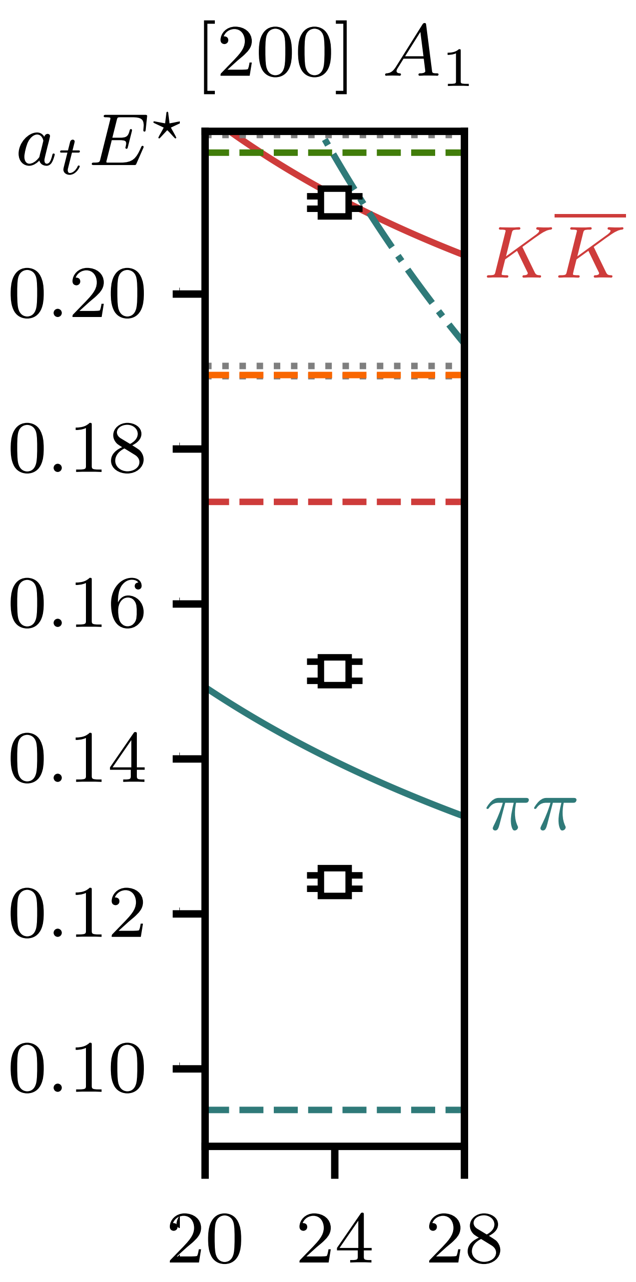
Matrix elements of excited states

[Ortega-Gama et al. (HadSpec) 2407.20617]

$\mathcal{M}(s) =$ 

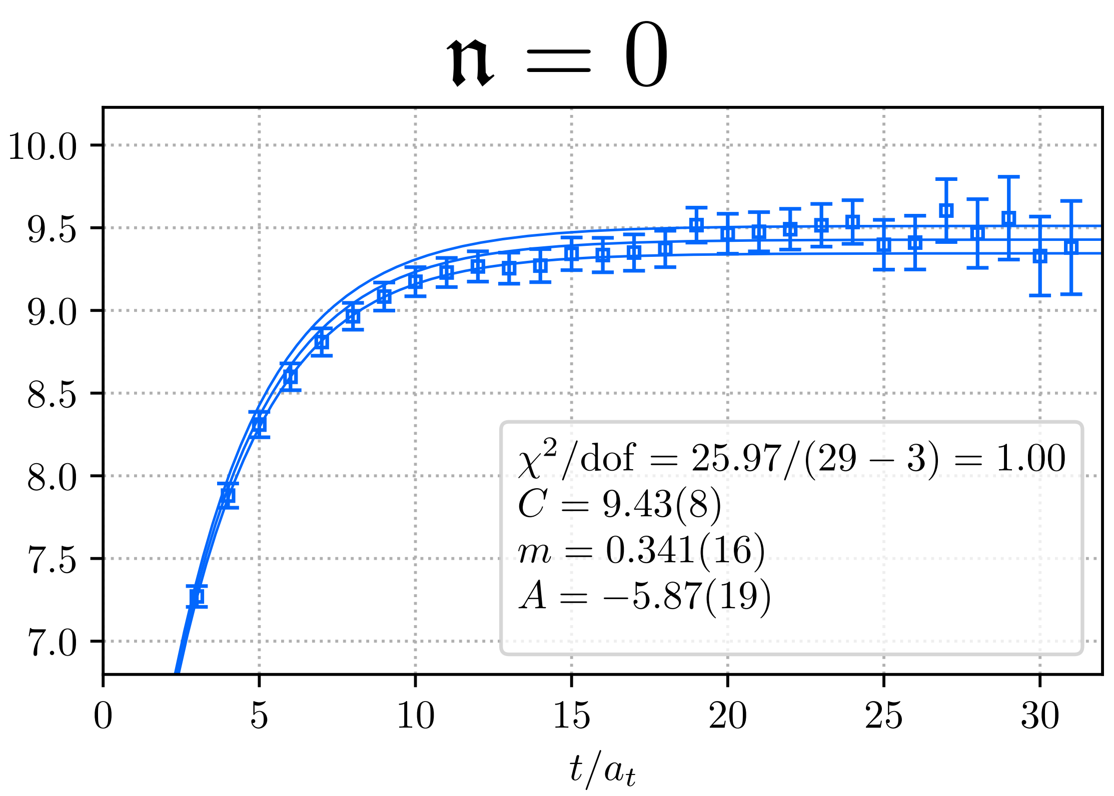


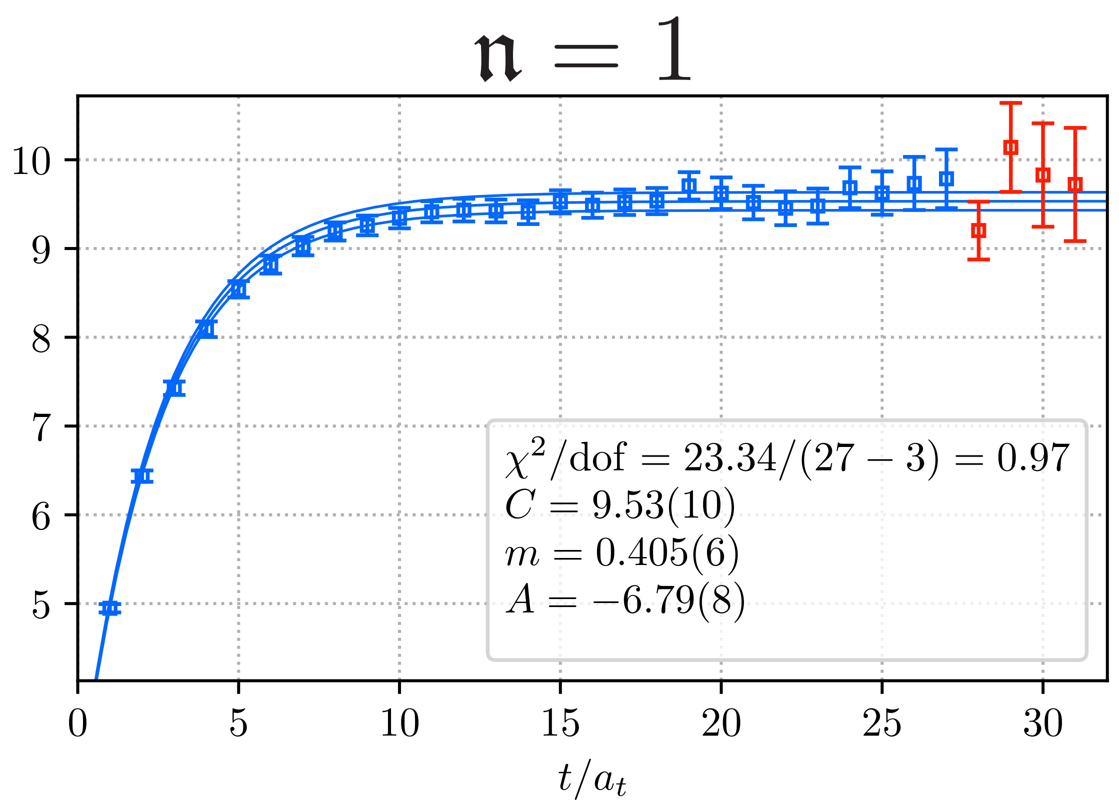
$$\frac{\langle \mathcal{J}(t)\Omega_{\mathfrak{n}}^\dagger(0)\rangle}{\langle \Omega_{\mathfrak{n}}(t)\Omega_{\mathfrak{n}}^\dagger(0)\rangle}$$

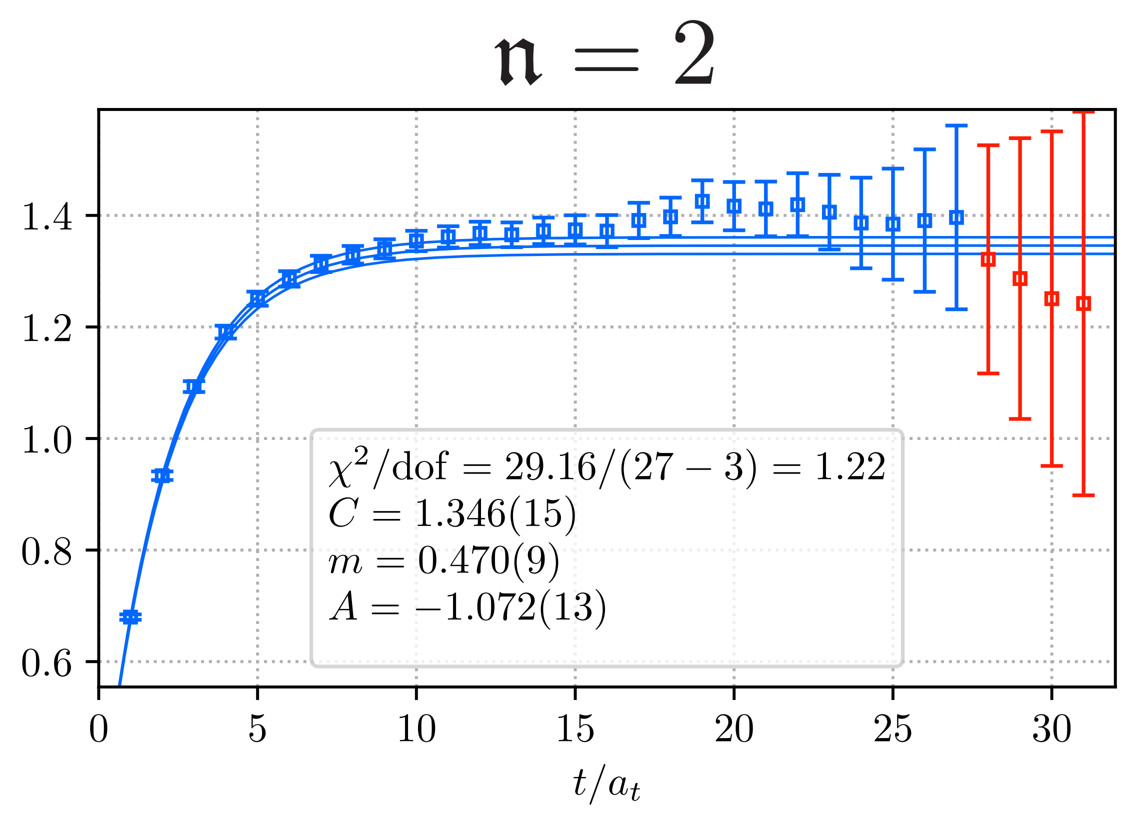


Matrix elements from optimized operators (solutions of GEVP)

	m/MeV
π	284
K	519







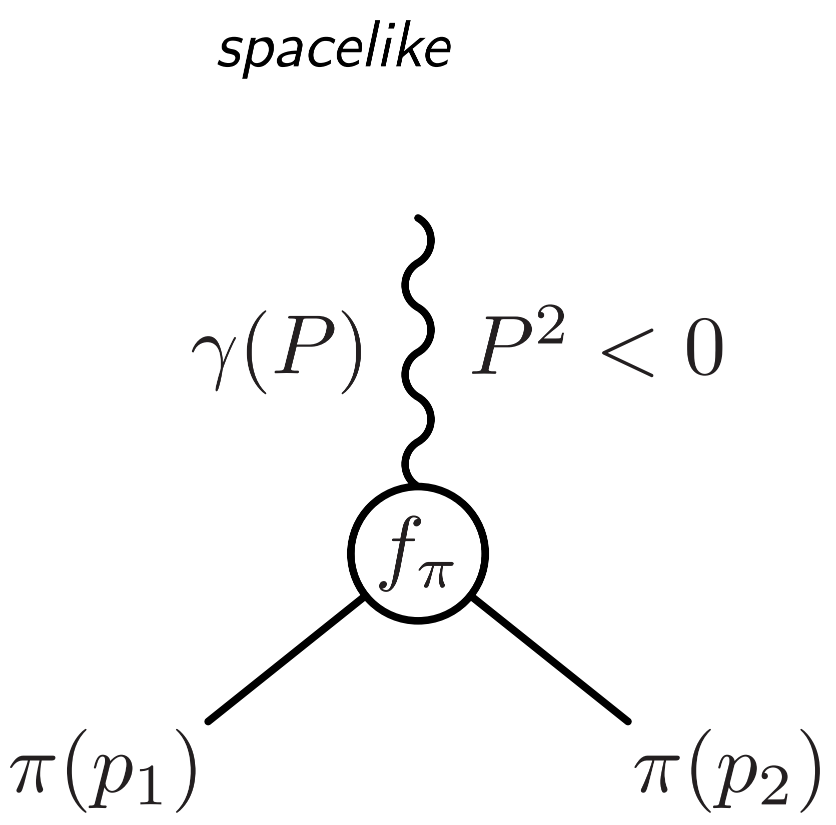
Energies from GEVP of $q\bar{q}$ and two-meson ops

K-matrix parameterization of scattering

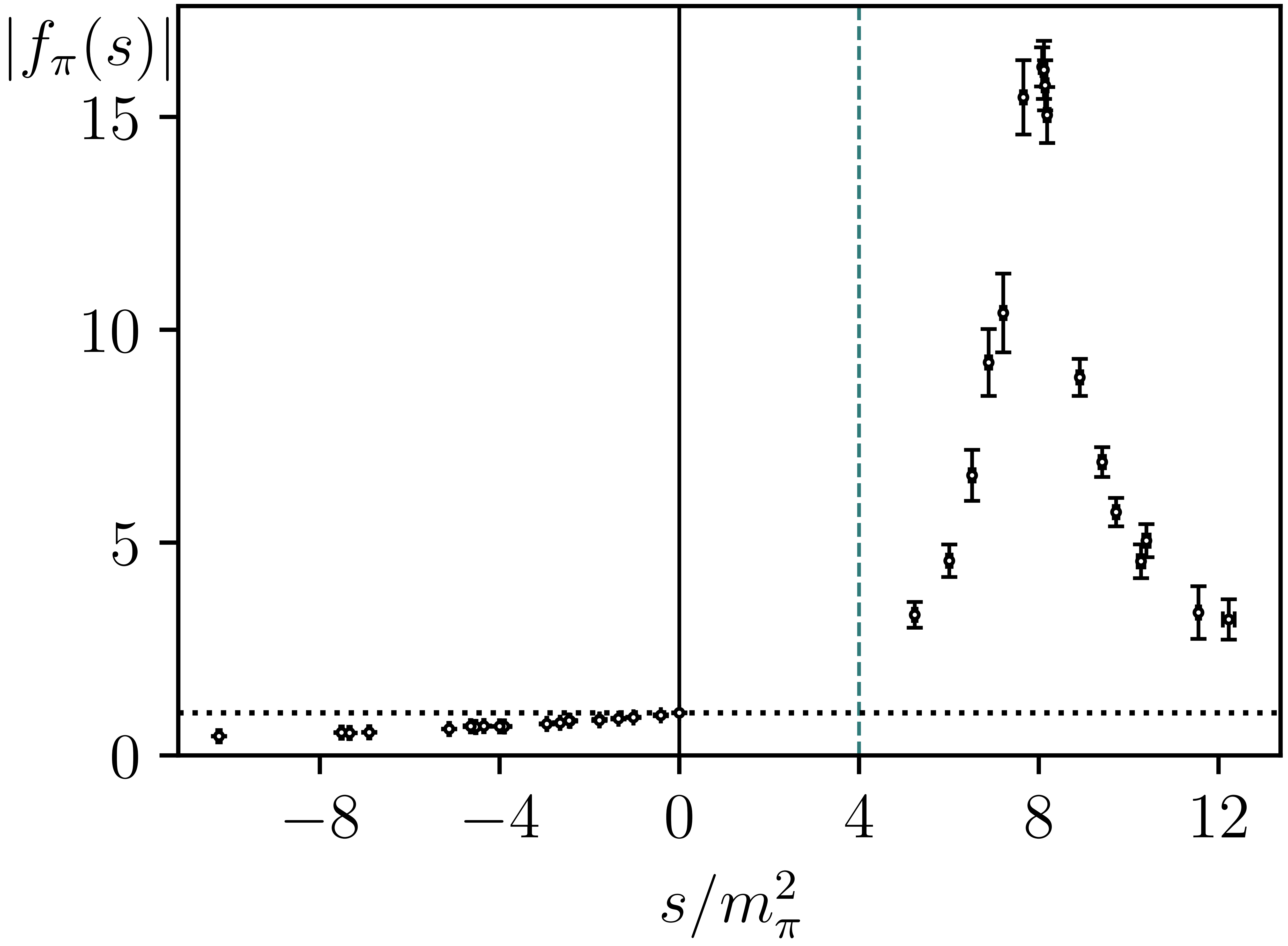
Fit form: $C + Ae^{-mt}$

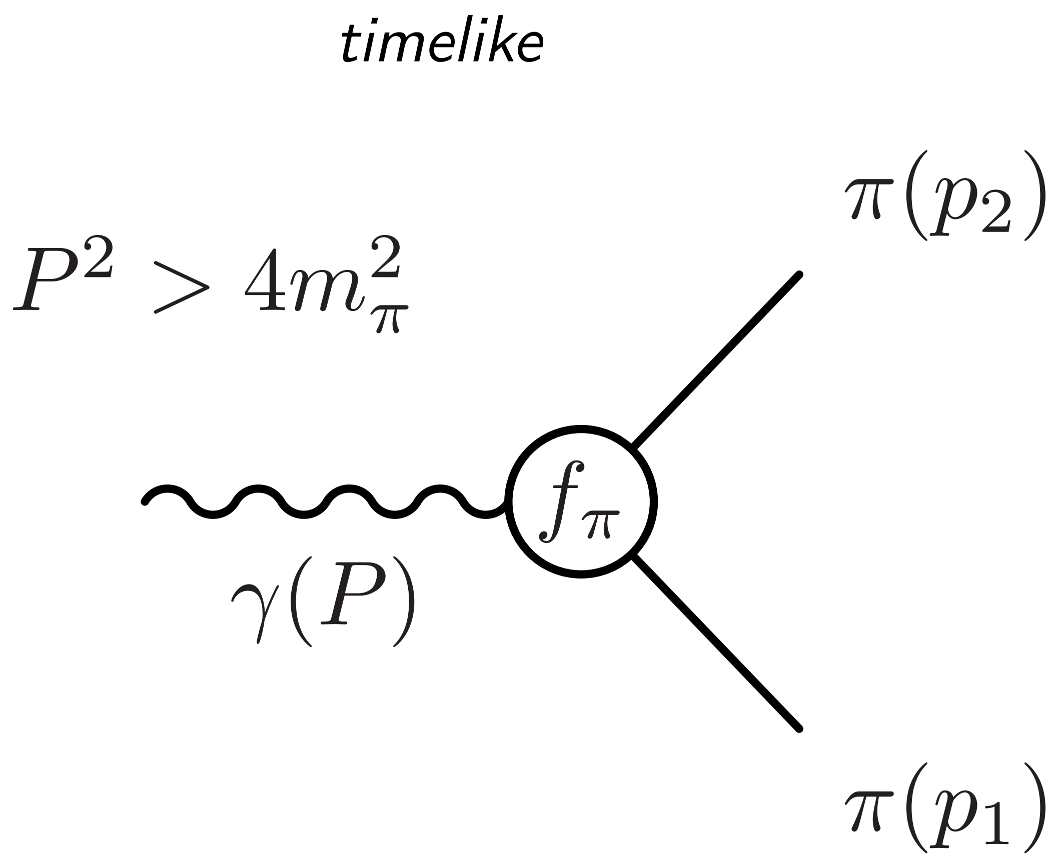
Pion form factor extraction

Previous pion timelike ff lattice calculations [Feng et al. 1412.6319] [Andersen et al. 1808.05007] [Erben et al. 1910.01083]



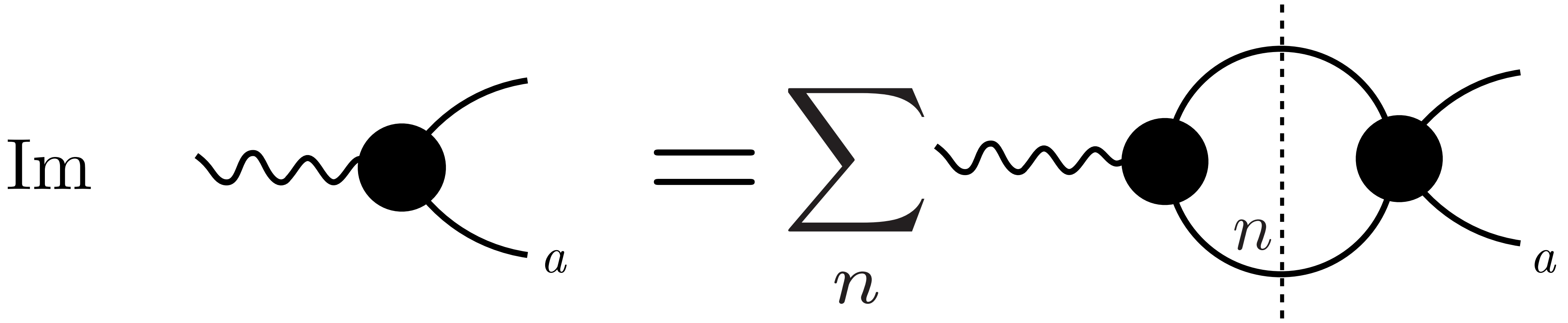
Directly from lattice matrix elements





Applying LL factor to lattice matrix elements

Unitarity and Watson's theorem



$$\mathrm{Im} f_a = \sum_n f_n \rho_n \mathcal{M}^*_{na}$$

Elastic solution:

K-matrix representation

$$f = \mathcal{M} \times \mathcal{A}$$

$$f = \frac{\mathcal{A}}{\mathcal{K}^{-1} - i\rho}$$

Satisfies ***unitarity***,

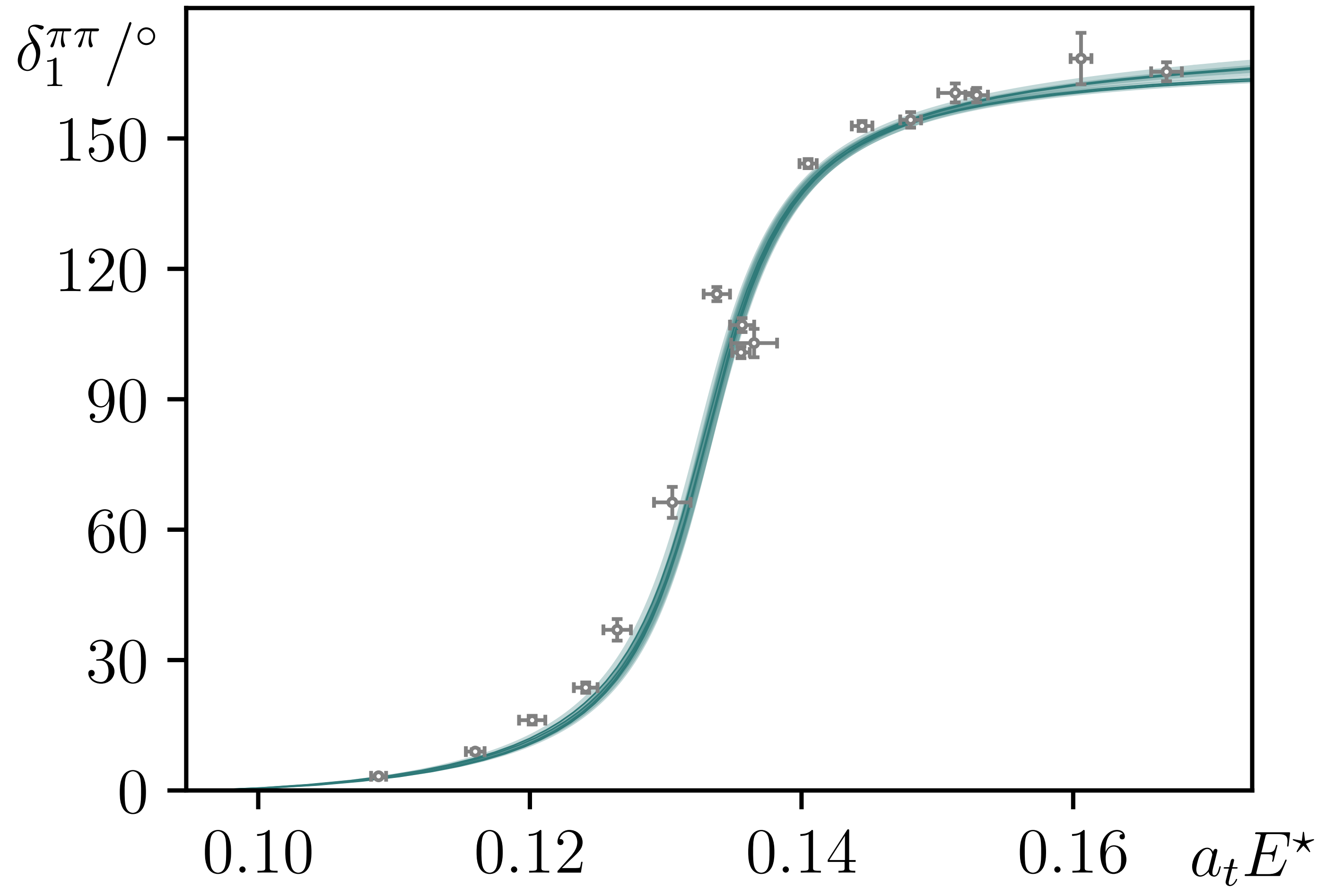
Analyticity is not guaranteed

Omnès representation

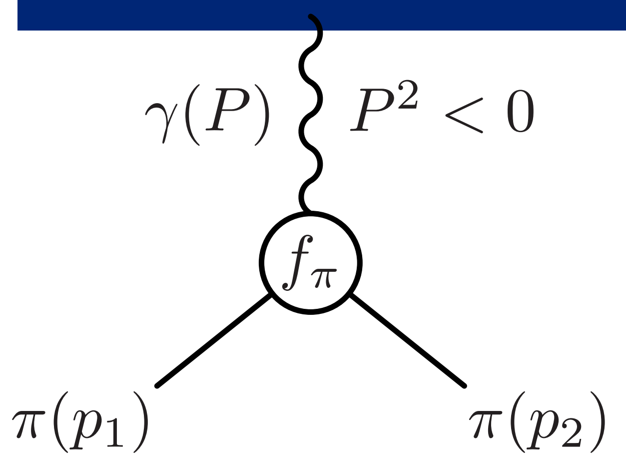
$$f = \Omega \times \mathcal{F}$$

$$\Omega(s) = \exp\left(\frac{s}{\pi} \int_{4m_\pi^2}^{\infty} s' \frac{\delta(s')}{s'(s'-s)} \right)$$

Dispersive solution satisfying ***unitarity*** and ***analyticity***



Pion vector form factor



	m/MeV
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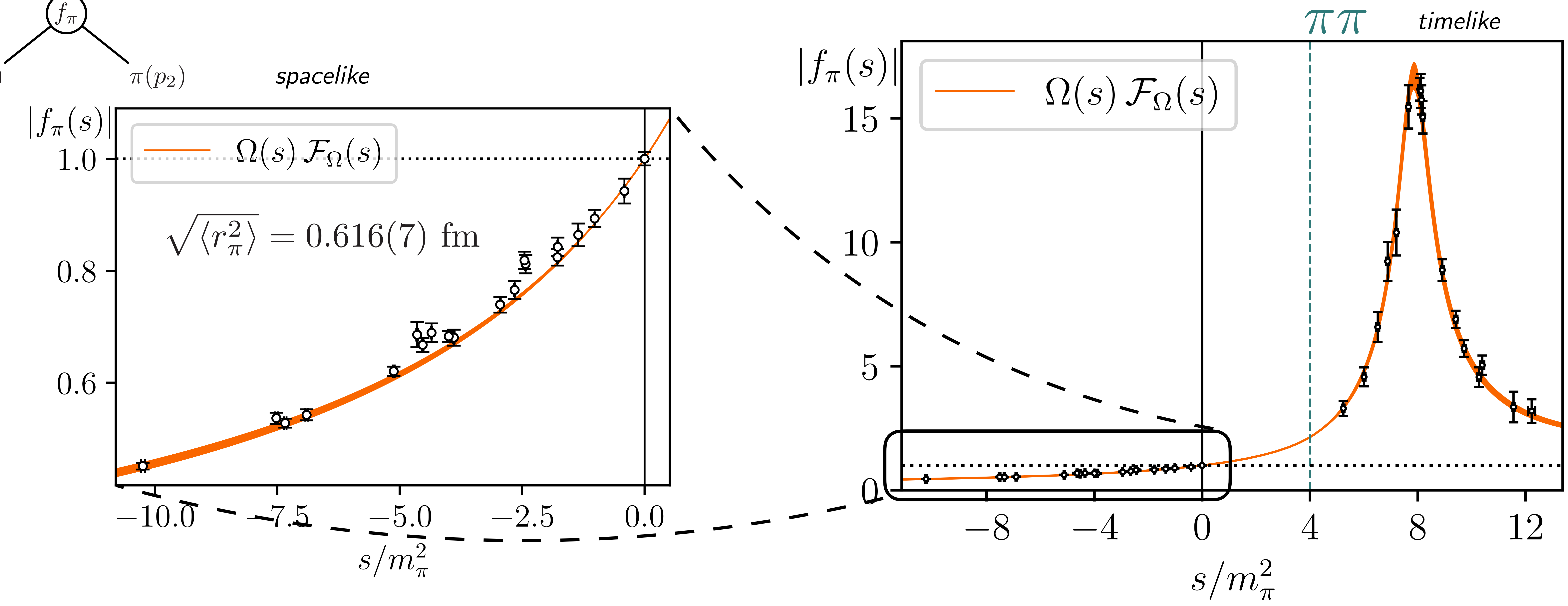
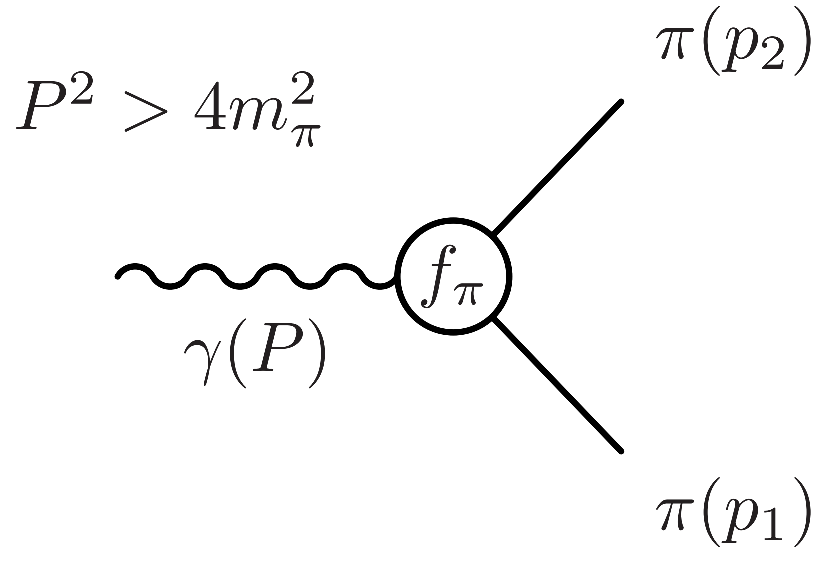




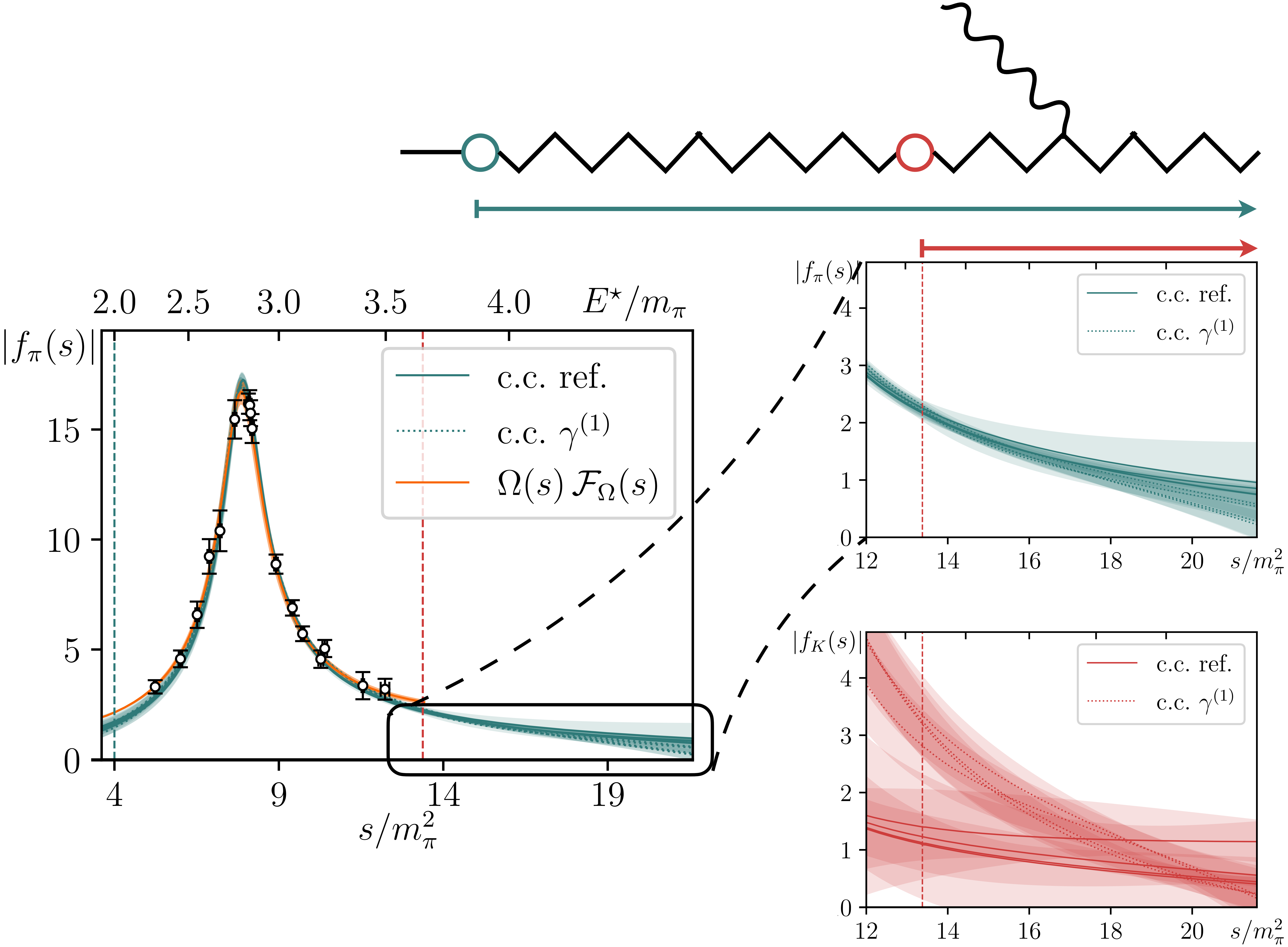
$$\Omega(s) = \exp\left(\frac{s}{\pi} \int_{4m_\pi^2}^{\infty} ds' \, \frac{\delta_1^{\pi\pi}(s')}{s'(s'-s)} \right)$$

$$\mathcal{F}_\Omega(s) = Q + \sum_{n=1}^{N} c_n \left(z_c(s)^n - z_c(0)^n\right)$$

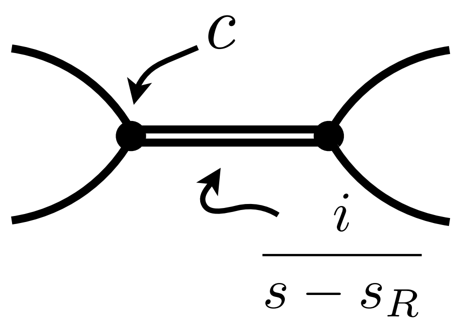
$$z_c(s) = \frac{\sqrt{s_c - s_0} - \sqrt{s_c - s}}{\sqrt{s_c - s_0} + \sqrt{s_c - s}}$$

Timelike form factor above KK threshold

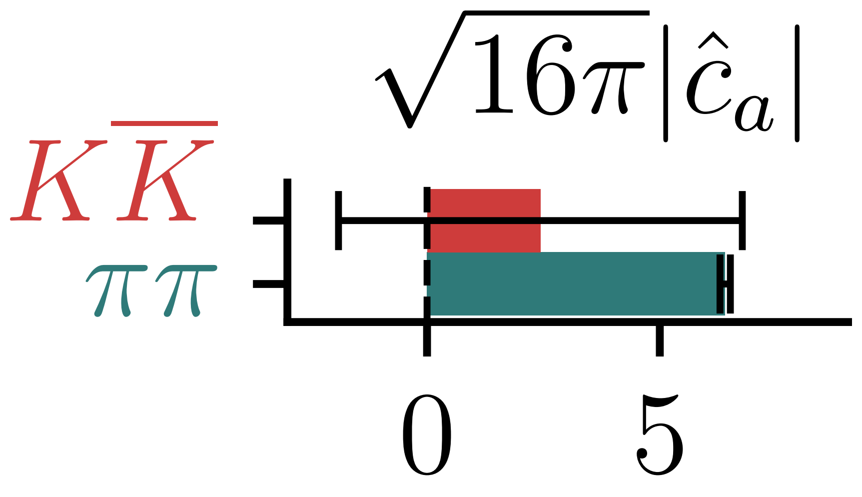
	m/MeV
π	284
K	519



The uncertainty near KK threshold is driven by the nearby resonance



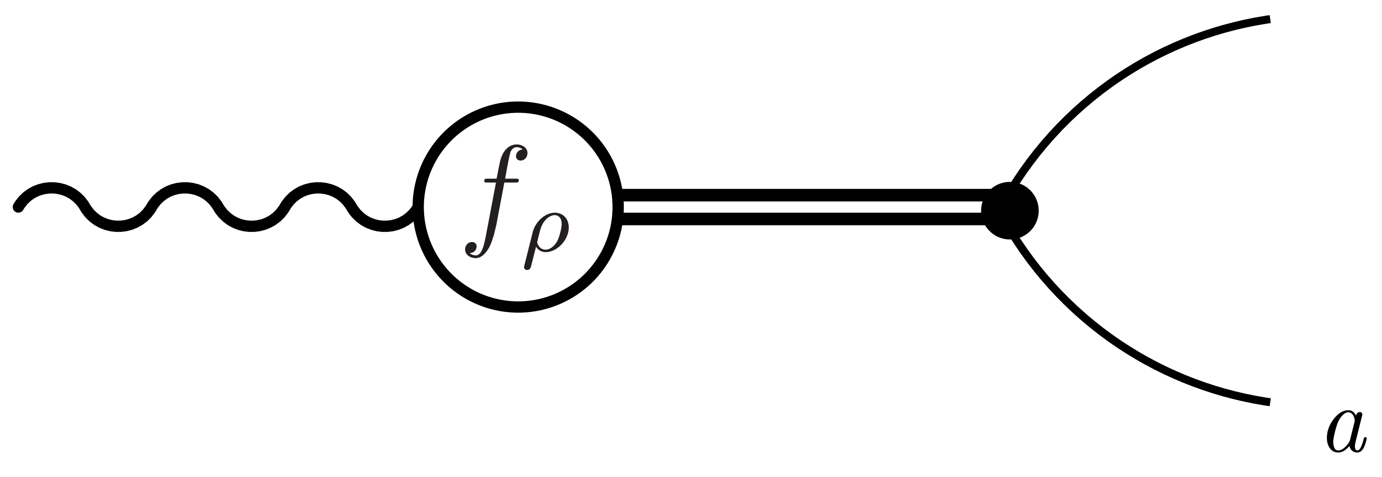
$$f_a(m_R^2) \propto \hat{c}_a + \mathcal{O}\left(\frac{\Gamma_R}{m_R}\right)$$



$$\hat{c}_a = \frac{c_a}{k_a^\star}$$

Relative momentum in cm frame

ρ meson decay constant



$$f_a(s \sim s_R) \sim \sqrt{16\pi}\,\hat{c}_a \frac{1}{s - s_R} f_\rho$$

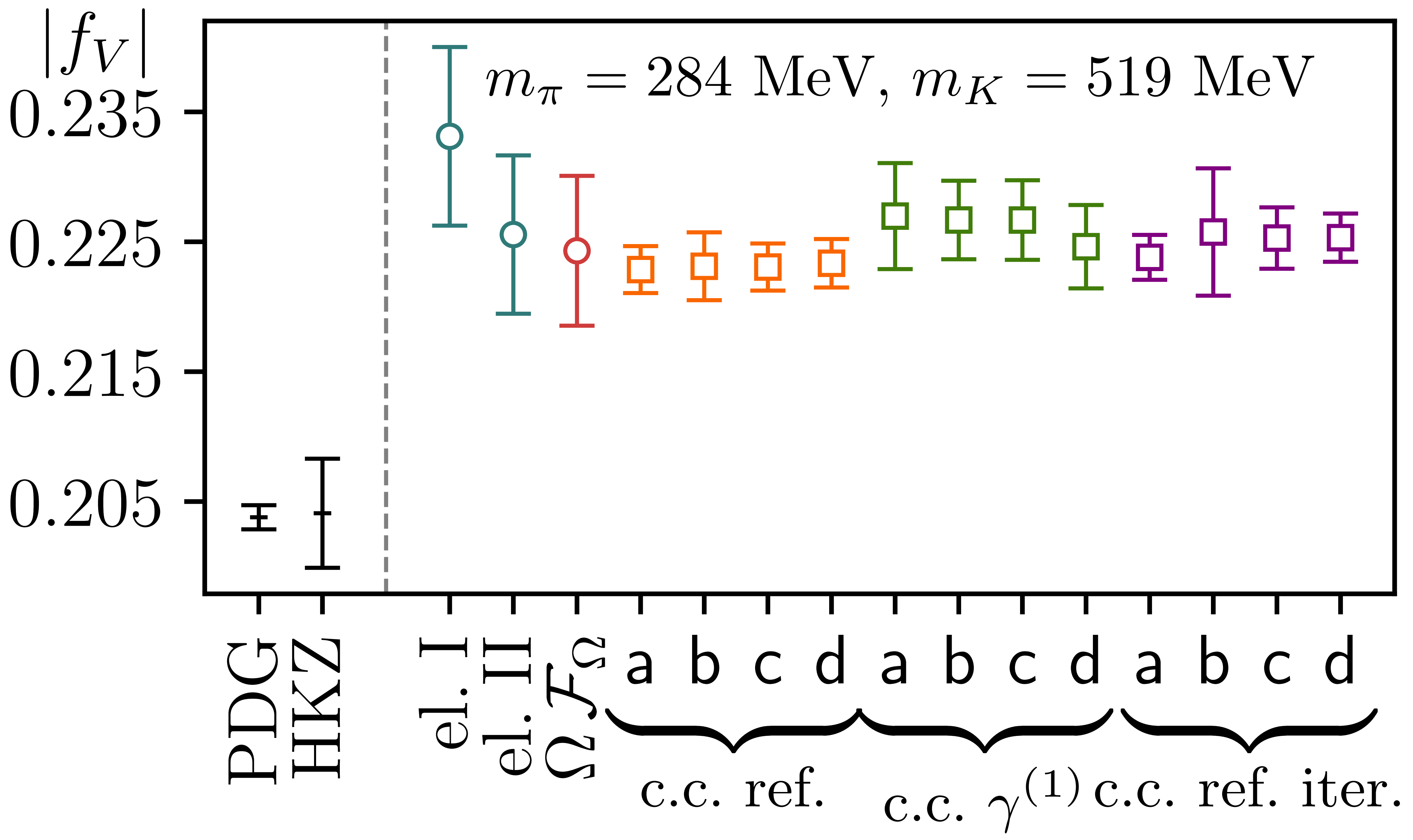
Coupled channel unitary parameterizations:

$$f_a = \sum_b \frac{1}{k_a^\star} \mathcal{M}_{ab} \frac{1}{k_b^\star} \mathcal{F}_b$$

P-wave barrier factor

Decay constant normalization

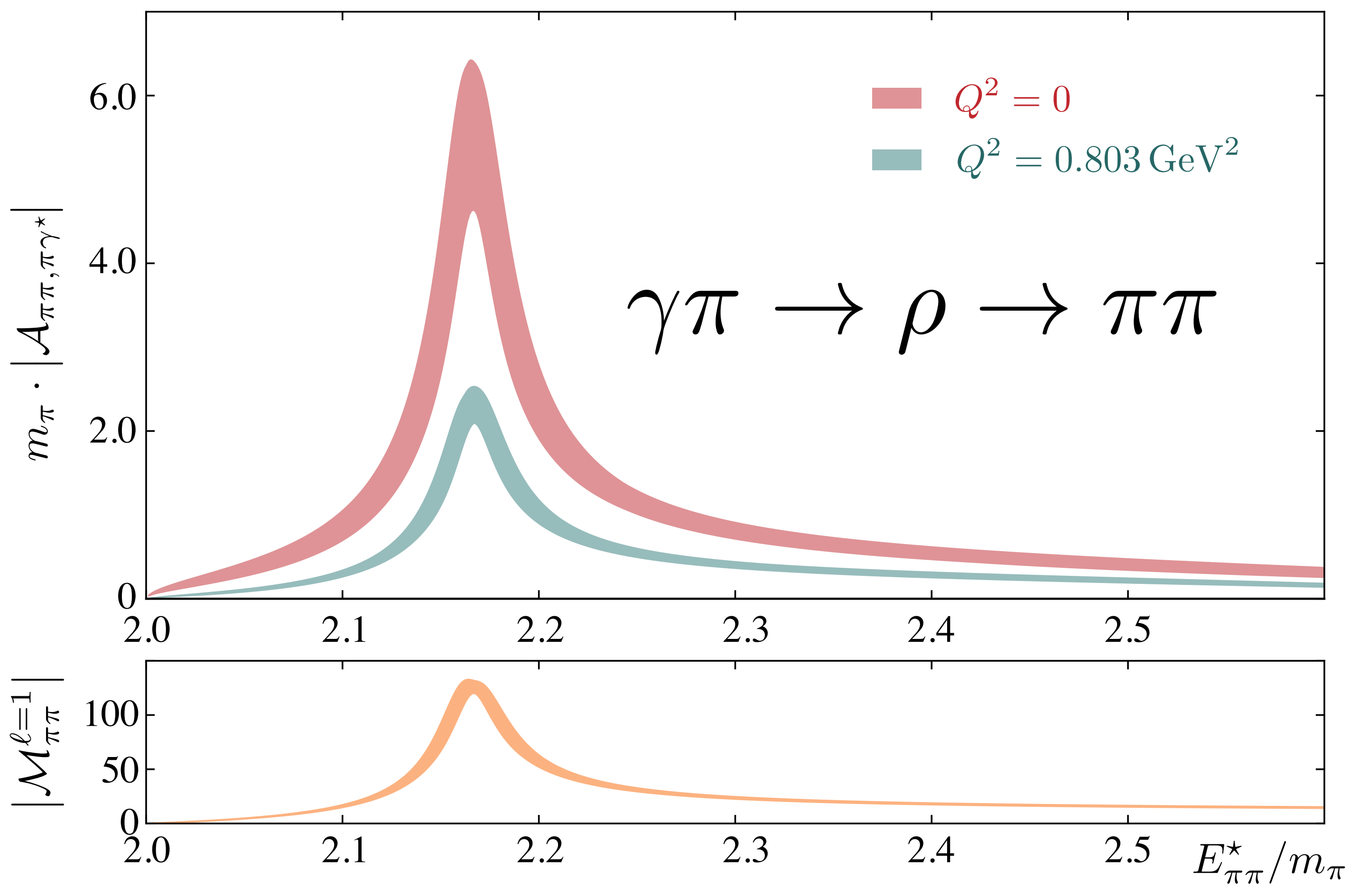
$$\langle \rho(\vec{P}, m) | \mathcal{J}^\mu | 0 \rangle = \epsilon^{\mu *}(P, m)\, f_\rho\, m_R^2$$



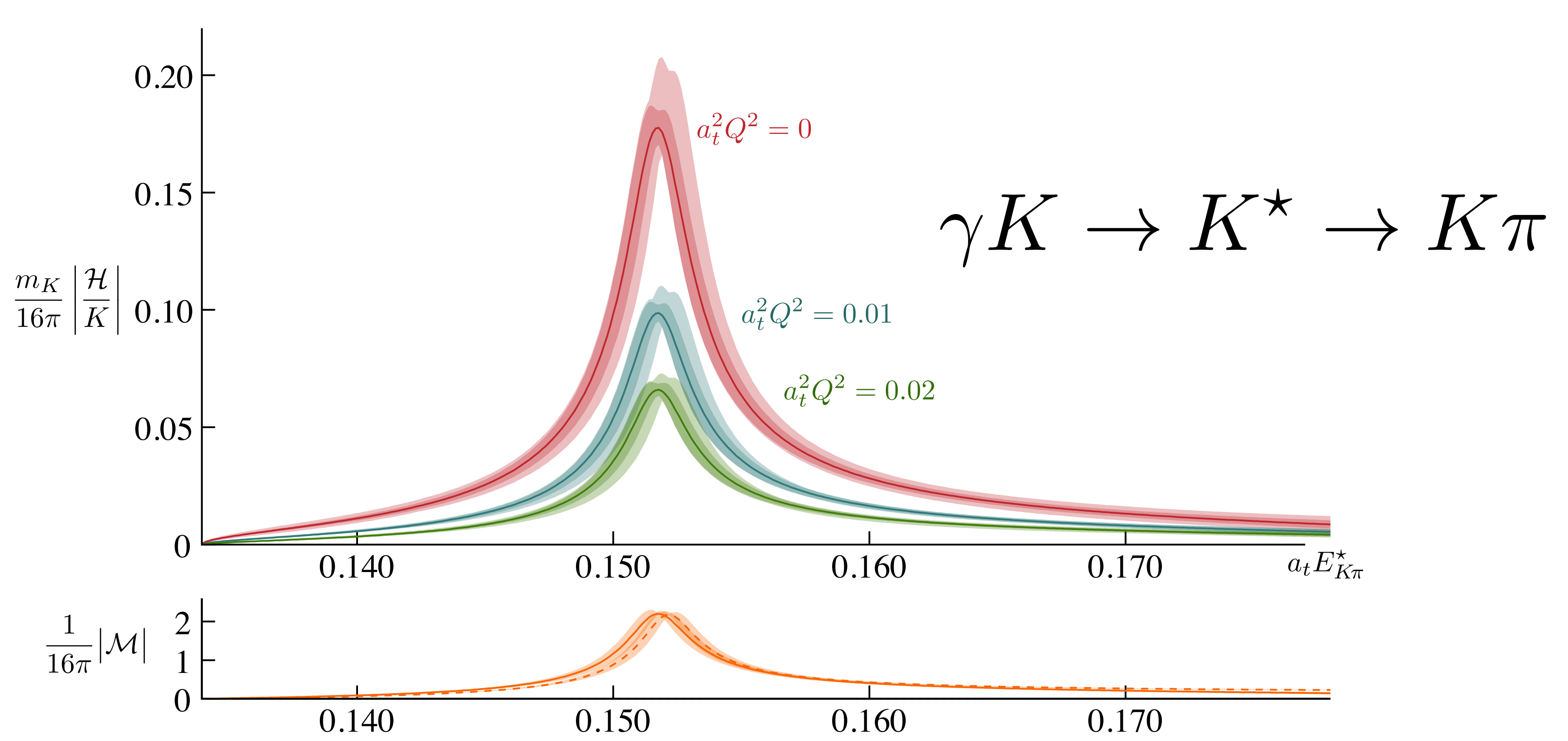
$$\mathcal{F}_a(s) = \sum_{n=0}^{N_a} h_{n,a}\, s^n$$

Model	$N_{\pi\pi}$	$N_{K\overline{K}}$
a	1	0
b	0	1
c	1	1
d	2	0

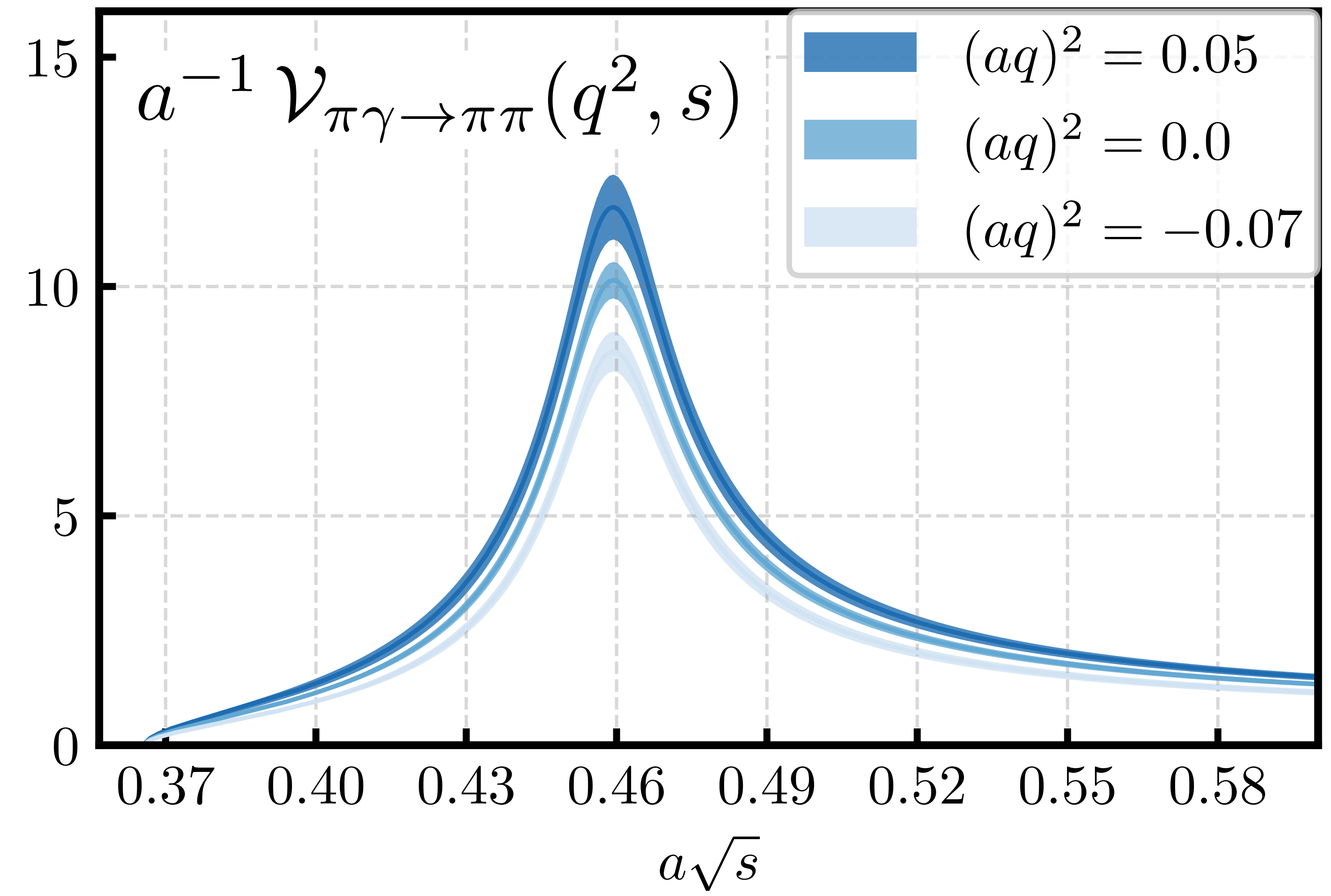
Transition form factors (resonances)



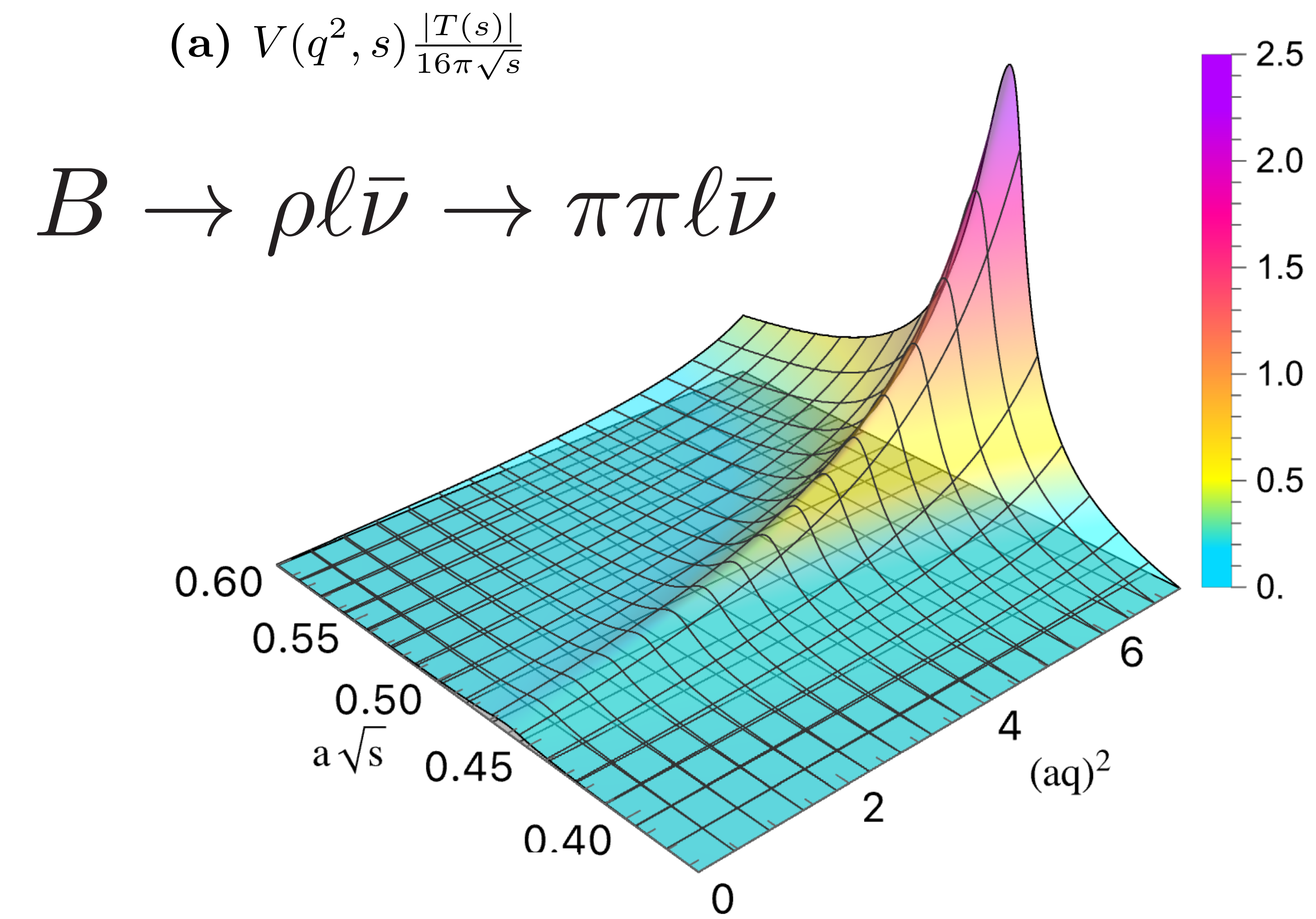
[Briceño et al. (HadSpec) 1604.03530]



[Radhakrishnan et al. (HadSpec) 2208.13755]



[Alexandru et al. 1807.08357]

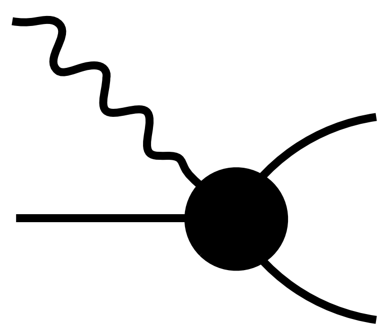


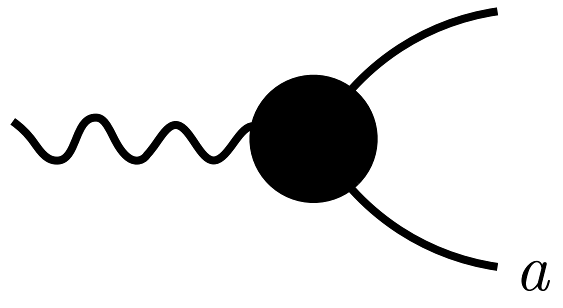
[Leskovec et al. 2501.00903]

Summary and outlook

The extraction of form factors of resonant states complements the determination of their masses and widths.

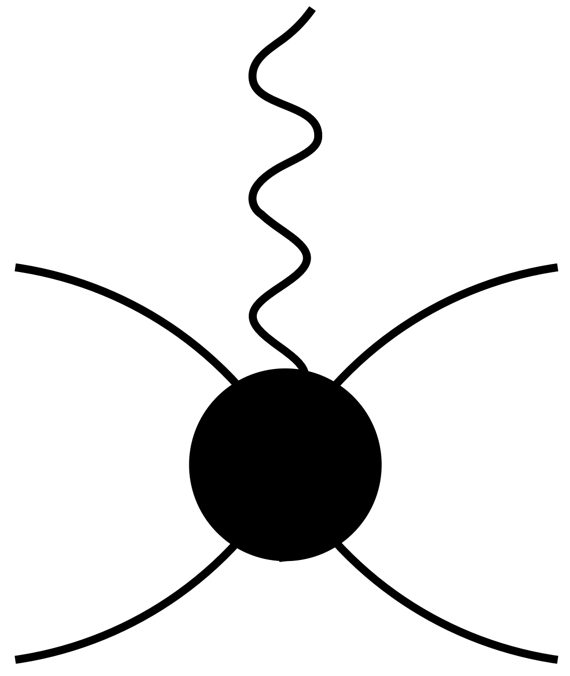
- [x] Transition amplitudes.
- [x] Timelike form factors in coupled channel region.





Future LQCD calculations of transition amplitudes:

- [ ] Non-resonant channels.
- [ ] Conventional resonances, eg. ρ decaying to $\pi\pi$.
- [ ] Exotic resonances, which typically decay to multiple channels.



Back up

Spectra

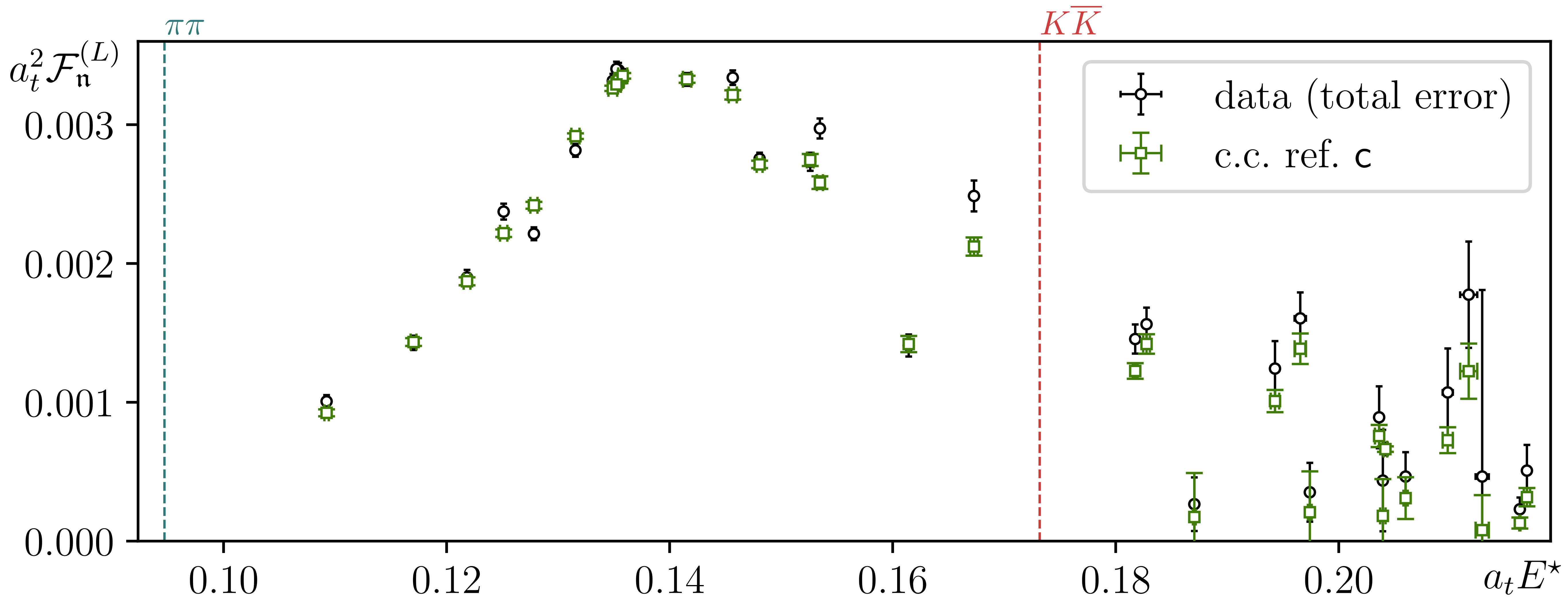
- 17 elastic levels
- 15 above KK threshold

Matrix elements

$$f_a = \frac{1}{k_a^\star}\mathcal{M}_{ab}\frac{1}{k_b^\star}\mathcal{F}_b$$

$$\mathcal{F}_a(s) = \sum_{n=0}^{N_a} h_{n,a}\, s^n$$

$$\mathcal{F}_n^{(L)} = \sum_a r_{n,a}\, f_a(s)$$



$$\widetilde{\mathcal{R}}(E_{\mathfrak{n}}, L) = 2E_{\mathfrak{n}} \lim_{E\to E_{\mathfrak{n}}} \frac{E - E_{\mathfrak{n}}}{\mathcal{M}(s) + F^{-1}(E, L)} = \frac{2E_{\mathfrak{n}}}{\lambda_0'}\,\mathbf{v}_0\,\mathbf{v}_0^\intercal$$

$$r_{\mathfrak{n},a}(L) = \sqrt{\frac{2E_{\mathfrak{n}}^\star}{\lambda_0'^\star}}\frac{(\mathbf{v}_0)_a}{k_a^\star}$$