Numerical exploration of Real-time Estimators for Scattering Observables

Marco A. Carrillo

Numerical exploration of RESOs
~~Real-time Estimators for Scattering Observables~~

Marco A. Carrillo

Outline

Motivation:

- **What is the order of volume sizes needed?**

Two types of explorations:

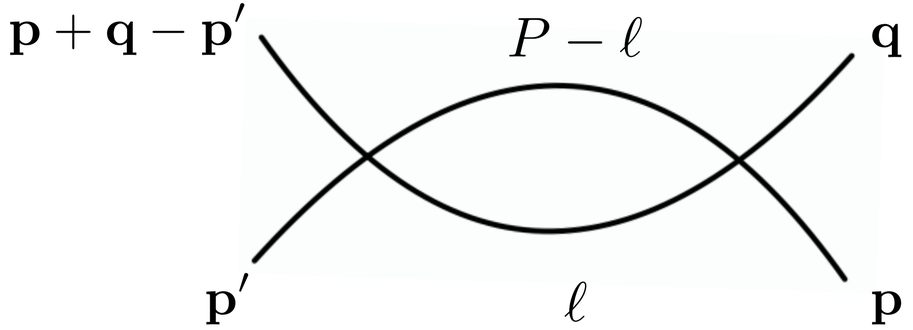
- **Non-interacting checks (1+1D & 3+1D)**
- **Strongly interacting checks (1+1D)**

I.Burbano, MC, R.Urek, et.al.
[2506.06511]

MC, R.Briceño, A.Sturzu
[PRD 110 054503 (2024)]

R.Briceño, J.Guerrero, M.Hansen, et.al.
[PRD 103 014506 (2021)]

Account of FV effects in 1+1D

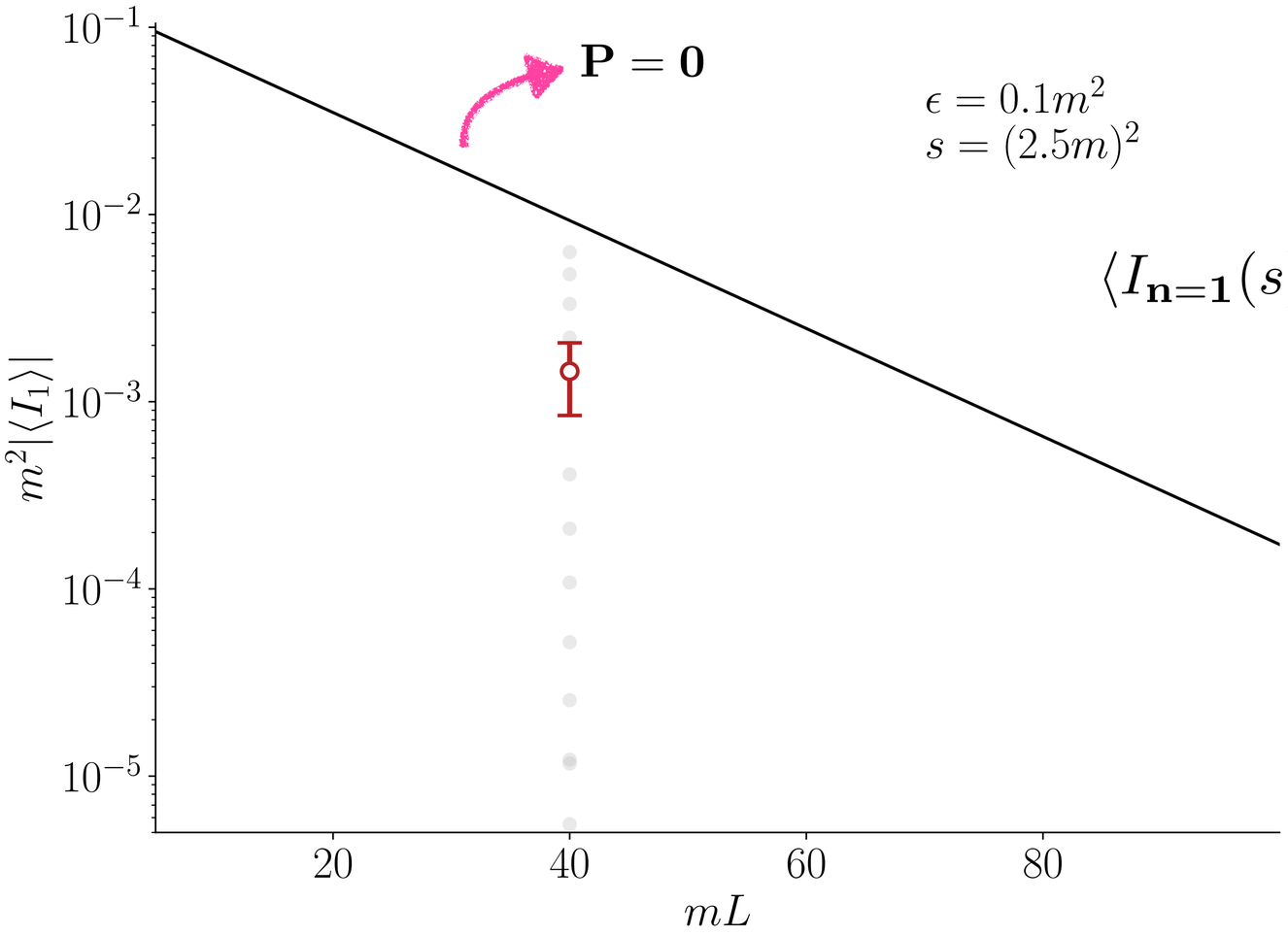


$$s = P^2$$

$$\langle I_{\mathbf{n}}(s)\rangle \sim L\int_0^1 du \left\langle e^{i(1-u)\mathbf{n}\cdot\mathbf{P}}\right\rangle \frac{\mathcal{K}_1(L|\mathbf{n}|\sqrt{\Delta})}{\sqrt{\Delta}}$$

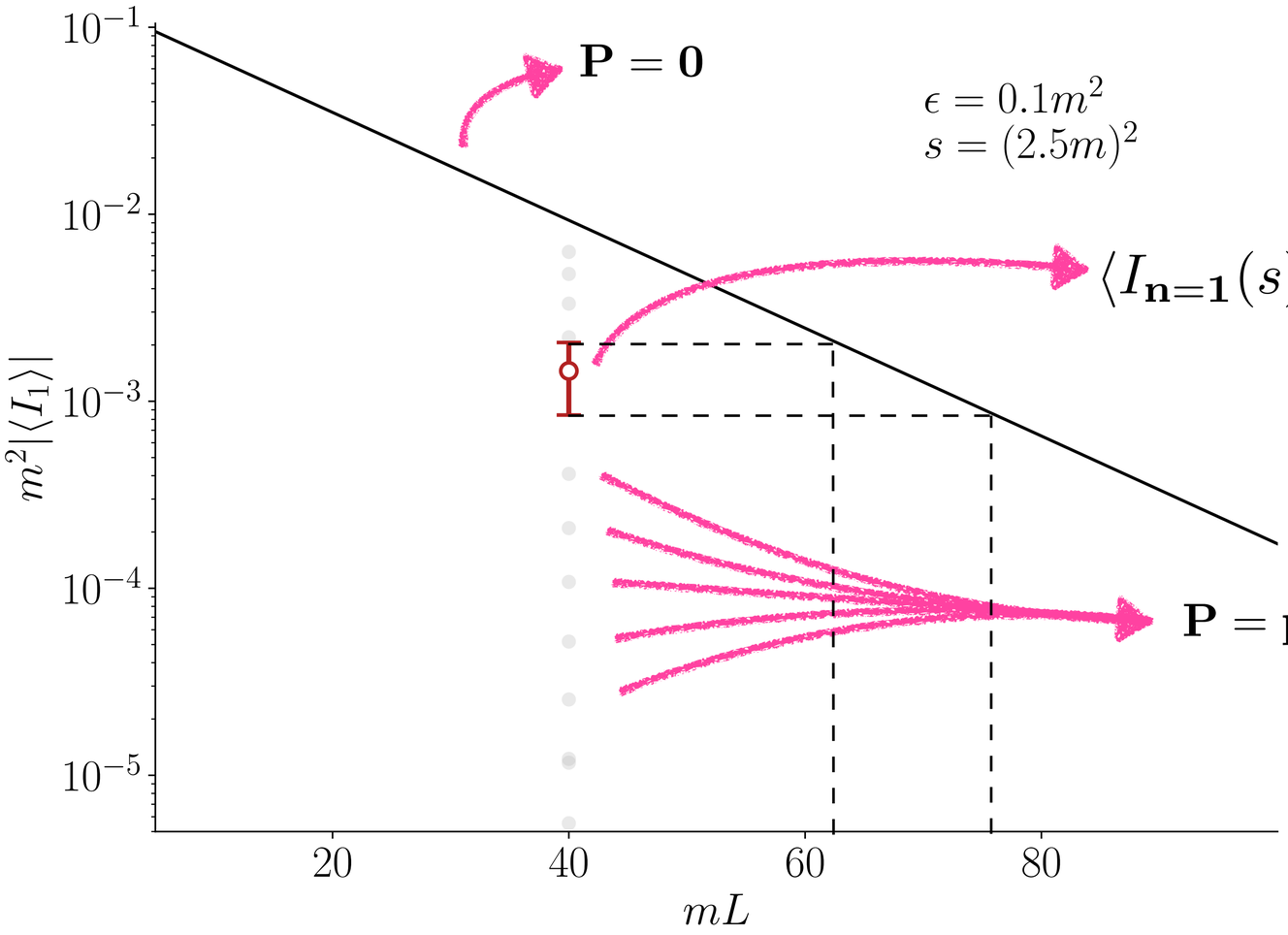
$$\Delta = m^2 + u(1-u)s - i\epsilon$$

Account of FV effects in 1+1D



$$\langle I_{\mathbf{n}=\mathbf{1}}(s) \rangle \sim L \int_0^1 du \left\langle e^{i(1-u)\mathbf{n}\cdot\mathbf{P}} \right\rangle \frac{\mathcal{K}_1(L|\mathbf{n}|\sqrt{\Delta})}{\sqrt{\Delta}}$$

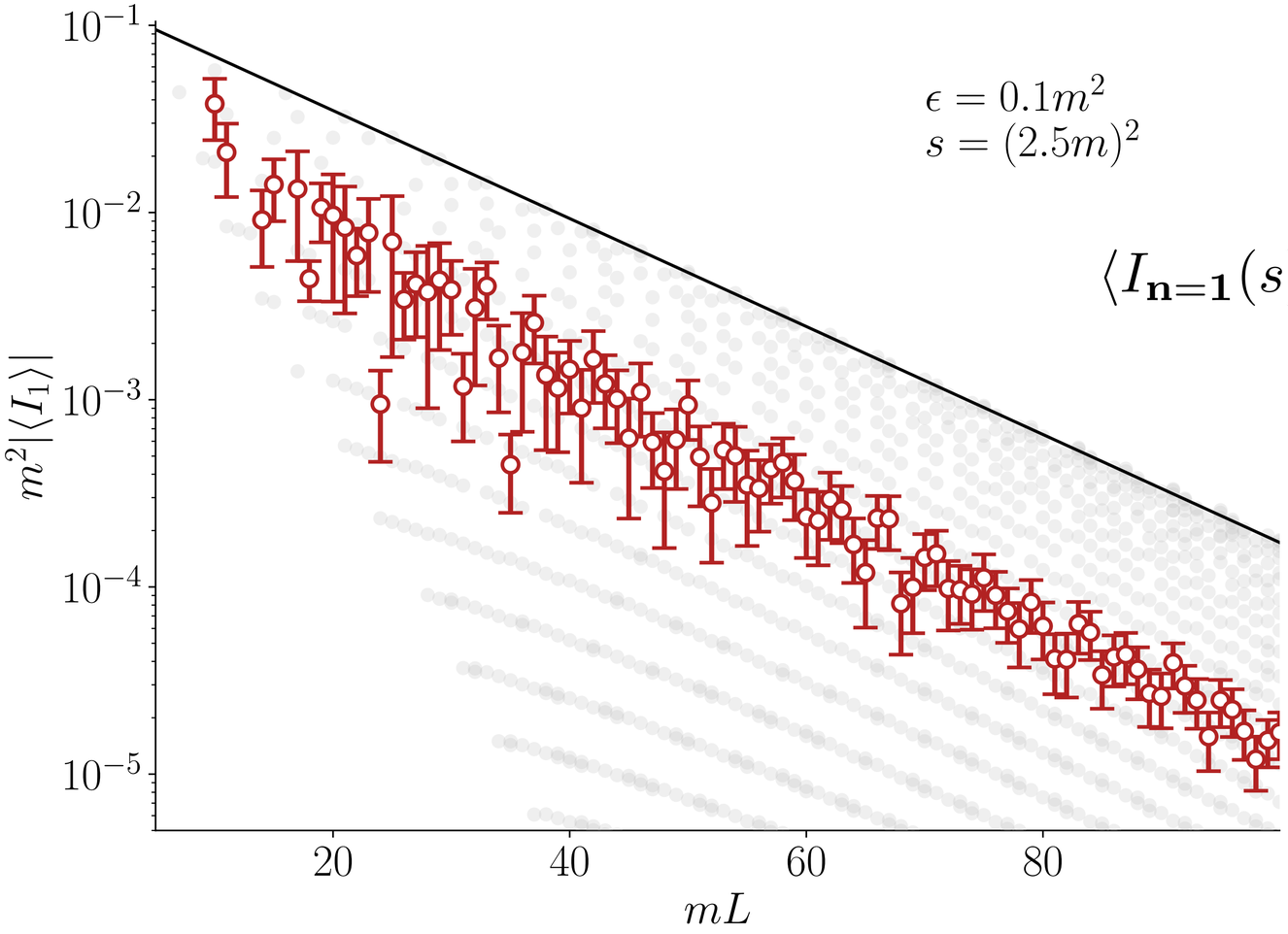
Account of FV effects in 1+1D



$$\langle I_{\mathbf{n}=\mathbf{1}}(s) \rangle \sim L \int_0^1 du \left\langle e^{i(1-u)\mathbf{n}\cdot\mathbf{P}} \right\rangle \frac{\mathcal{K}_1(L|\mathbf{n}|\sqrt{\Delta})}{\sqrt{\Delta}}$$

$$\left\{ (\mathbf{p}, \mathbf{q}) \;\middle|\; \frac{(p+q)^2 - s}{s} \leq 0.01 \right\}$$

Account of FV effects in 1+1D



$$\epsilon = 0.1m^2$$
$$s = (2.5m)^2$$

$$\langle I_{\mathbf{n}=\mathbf{1}}(s) \rangle \sim L \int_0^1 du \left\langle e^{i(1-u)\mathbf{n}\cdot\mathbf{P}} \right\rangle \frac{\mathcal{K}_1(L|\mathbf{n}|\sqrt{\Delta})}{\sqrt{\Delta}}$$

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- **Strongly interacting checks (1+1D)**

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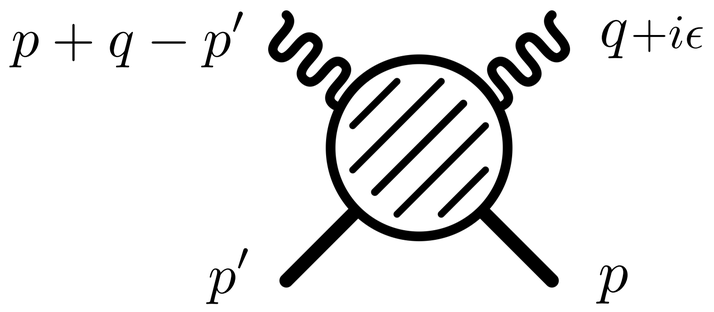
Consistency checks in 1+1D

R.Briceño, J.Guerrero, M.Hansen, et.al.
[PRD 103 014506 (2021)]

$$\overline{\mathcal{T}}(s,Q^2) = \frac{1}{\mathcal{N}} \sum_{\mathbf{p},\mathbf{q},\mathbf{p}'} \mathcal{T}_L$$



$$\mathcal{T}_L(\mathbf{p},\mathbf{q},\mathbf{p}') = \mathcal{A}(s,Q^2)\mathcal{M}_L(P)\mathcal{A}(s,Q^2)$$

$$\mathcal{M}_L(P) = \left[\mathcal{M}(s)^{-1} + F(P,L)\right]^{-1}$$

We get to chose

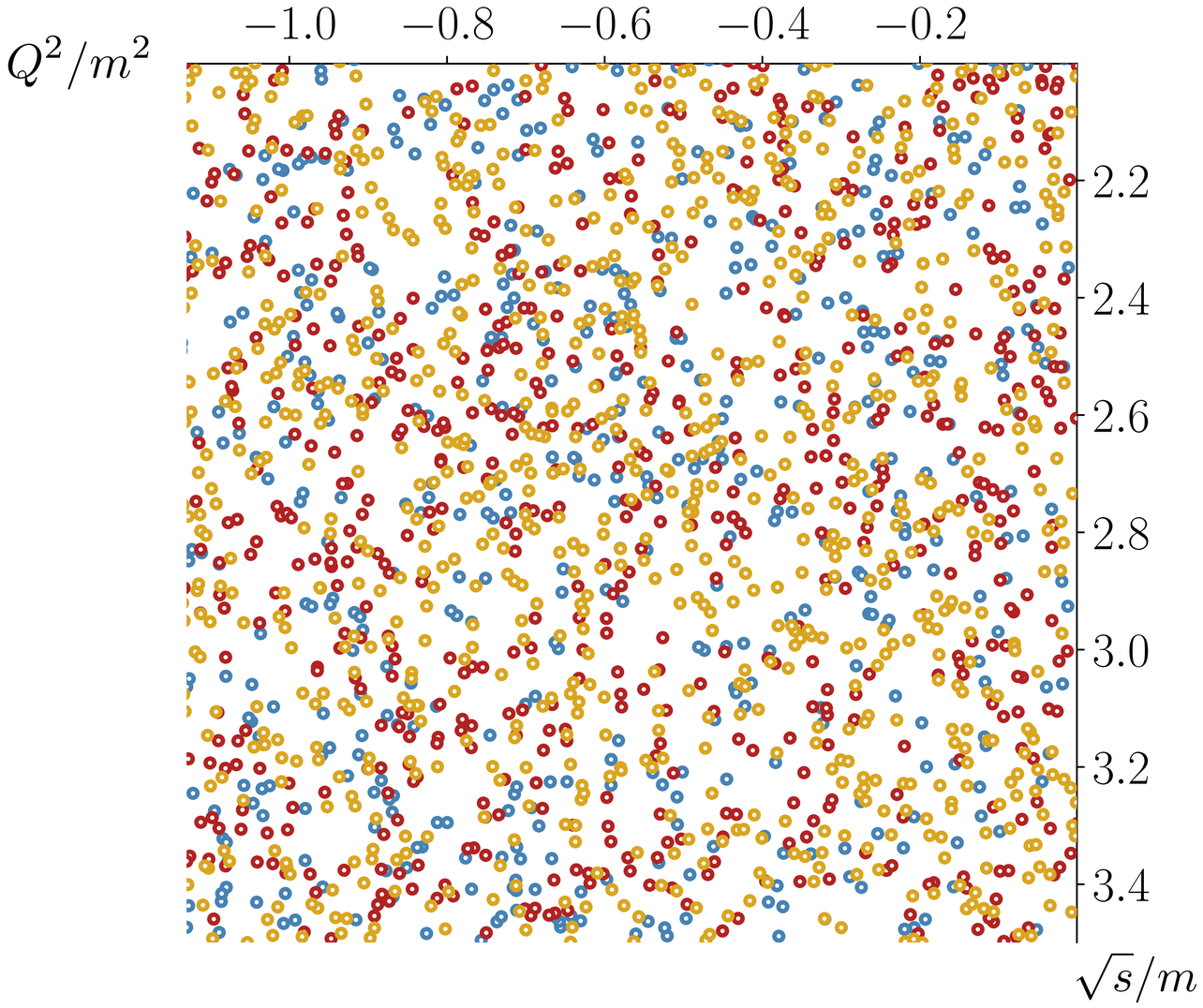
$$\mathcal{M}(s) = \left[K(s)^{-1} - i\rho(s)\right]^{-1}$$

$$F(P,L) = i\rho(s) + \rho(s)\cot\left(\frac{\gamma L k^\star - \mathbf{d}\pi}{2}\right)$$

$$\rho(s) = \frac{1}{8\sqrt{s}k^\star} \qquad k^\star = \sqrt{\frac{s}{4} - m^2}$$

$$(2m)^2 < s < s_{\text{thr}}$$

Consistency checks in 1+1D

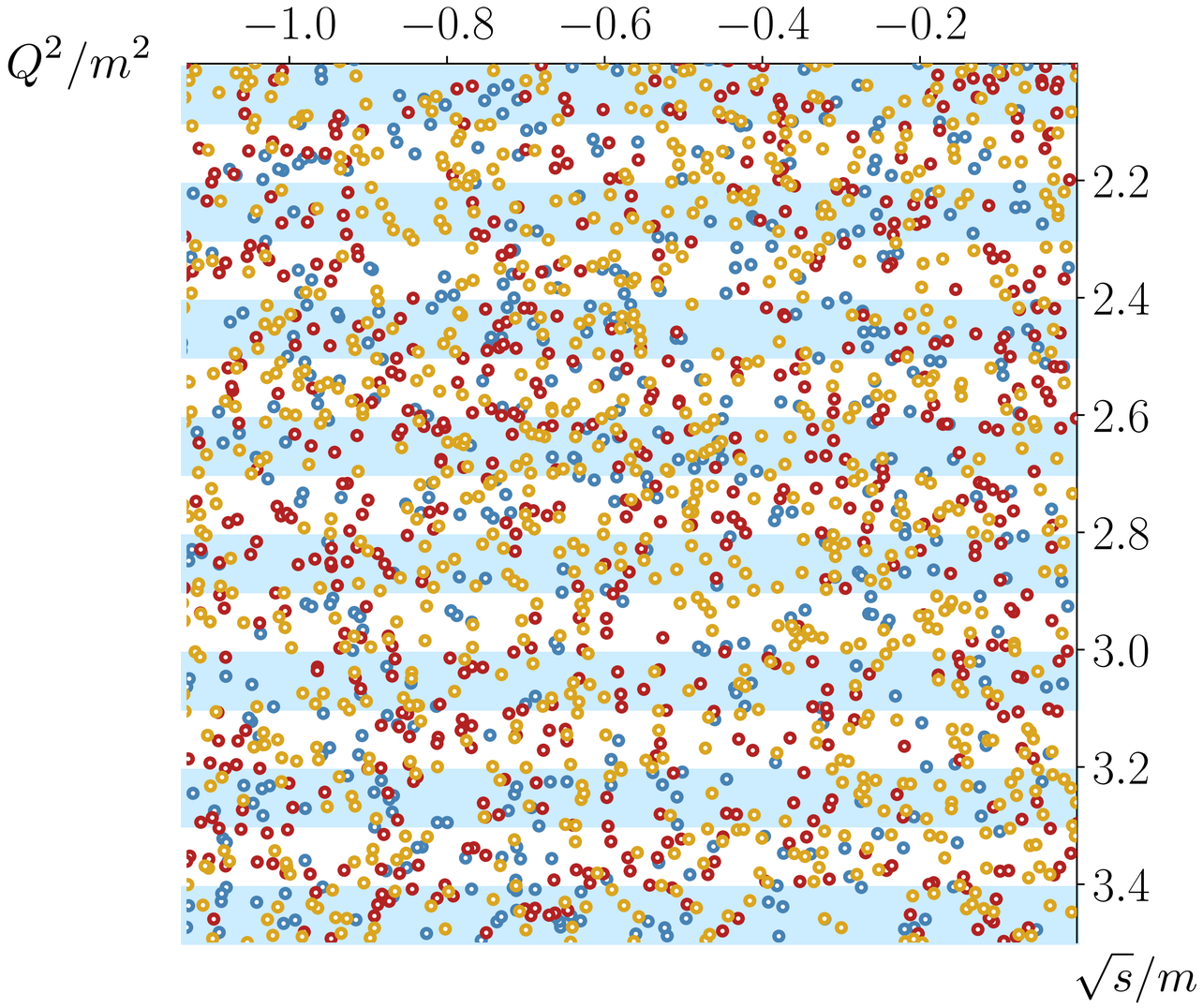


$$\epsilon = 1/\left(L\sqrt{mL}\right)$$

$$mL = 20, 25, 30$$

$$\overline{\mathcal{T}}(s, Q^2) = \frac{1}{\mathcal{N}} \sum_{\mathbf{p},\mathbf{q},\mathbf{p}'} \mathcal{T}_L$$

Consistency checks in 1+1D



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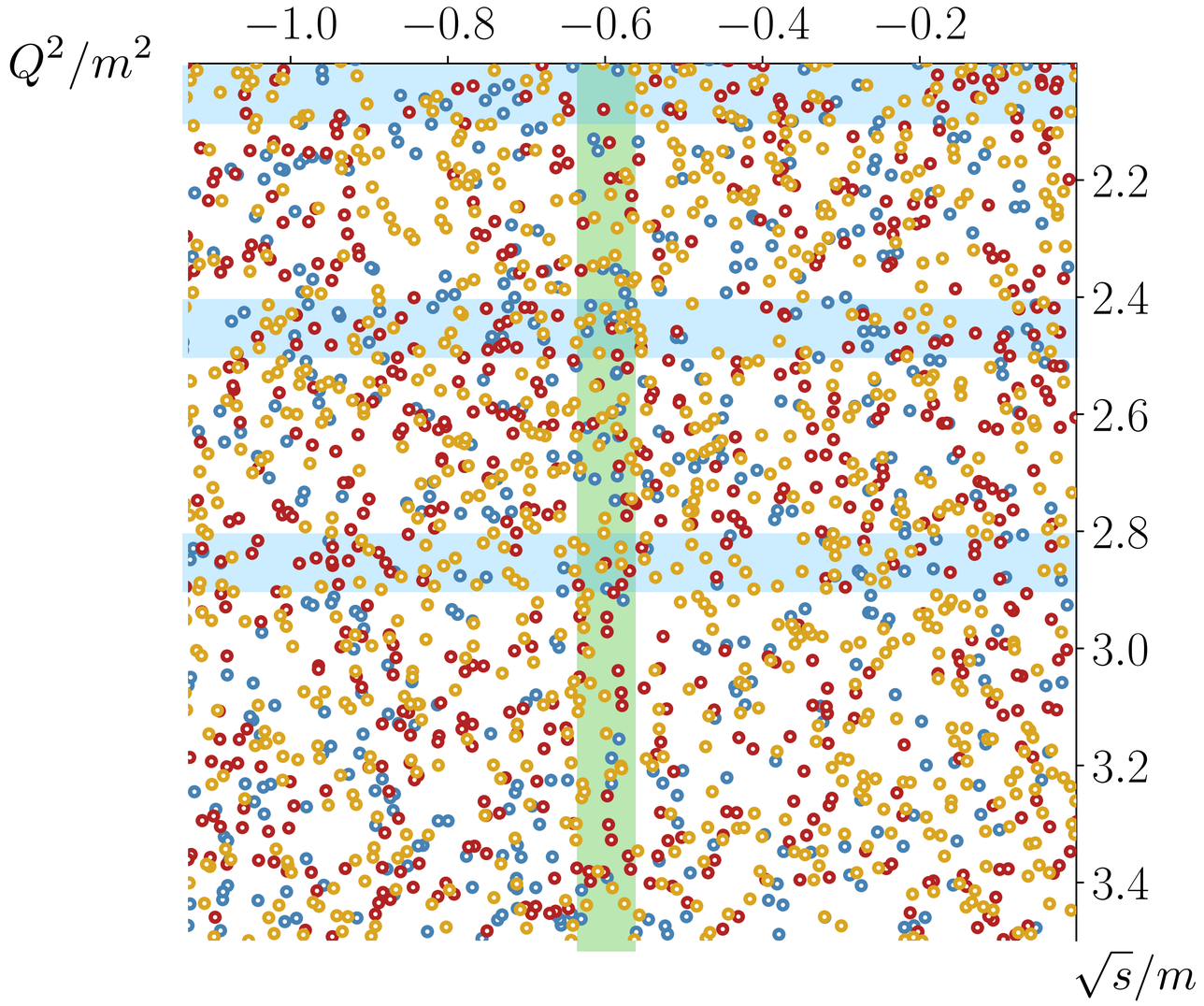

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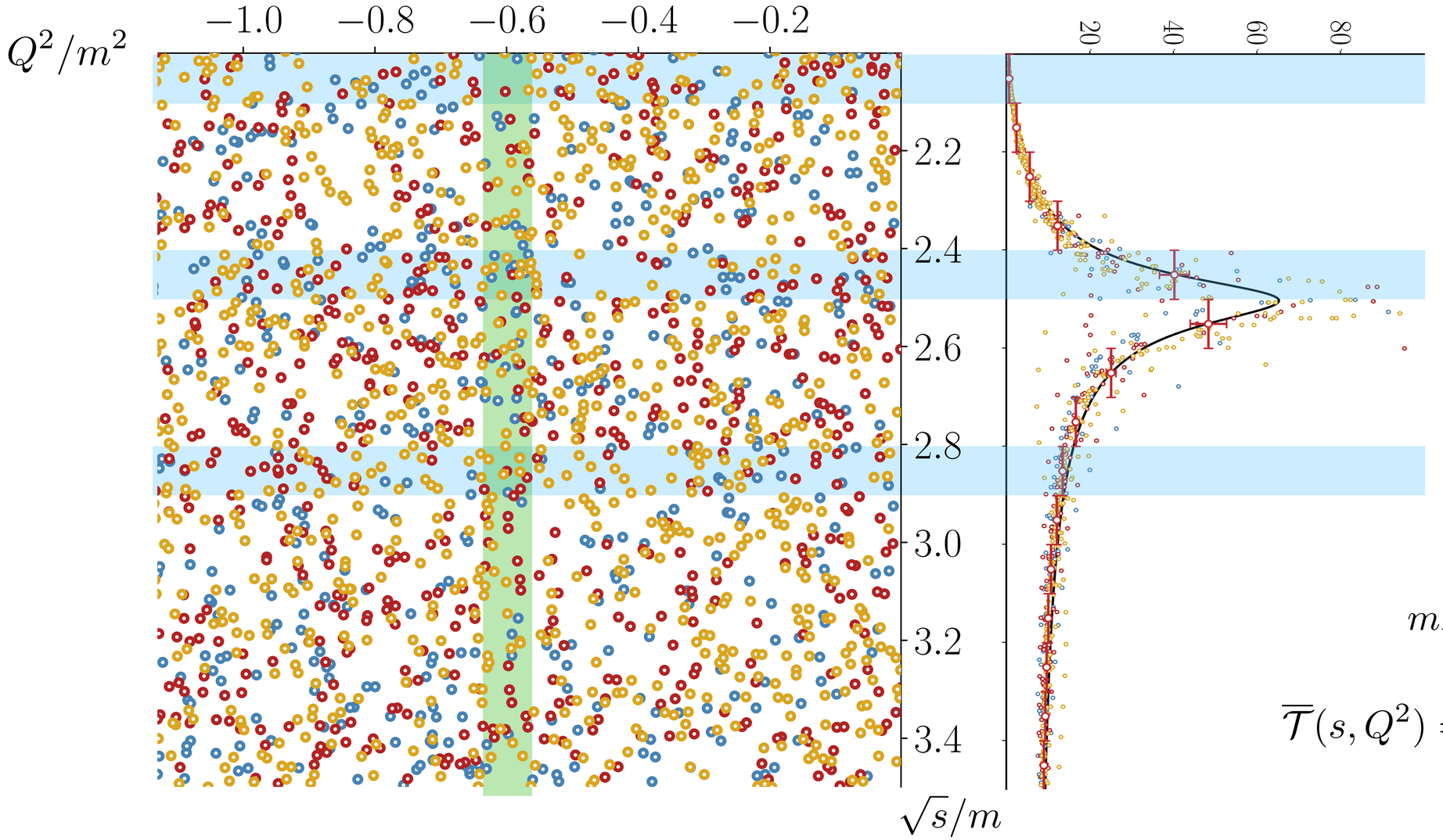
Consistency checks in 1+1D

$$\mathcal{A}(s,Q^2) \sim \frac{1}{Q^2+m^2} \qquad \mathcal{K}(s) \sim \frac{m^2(s^2/4-m^2)}{m_R^2-s}$$



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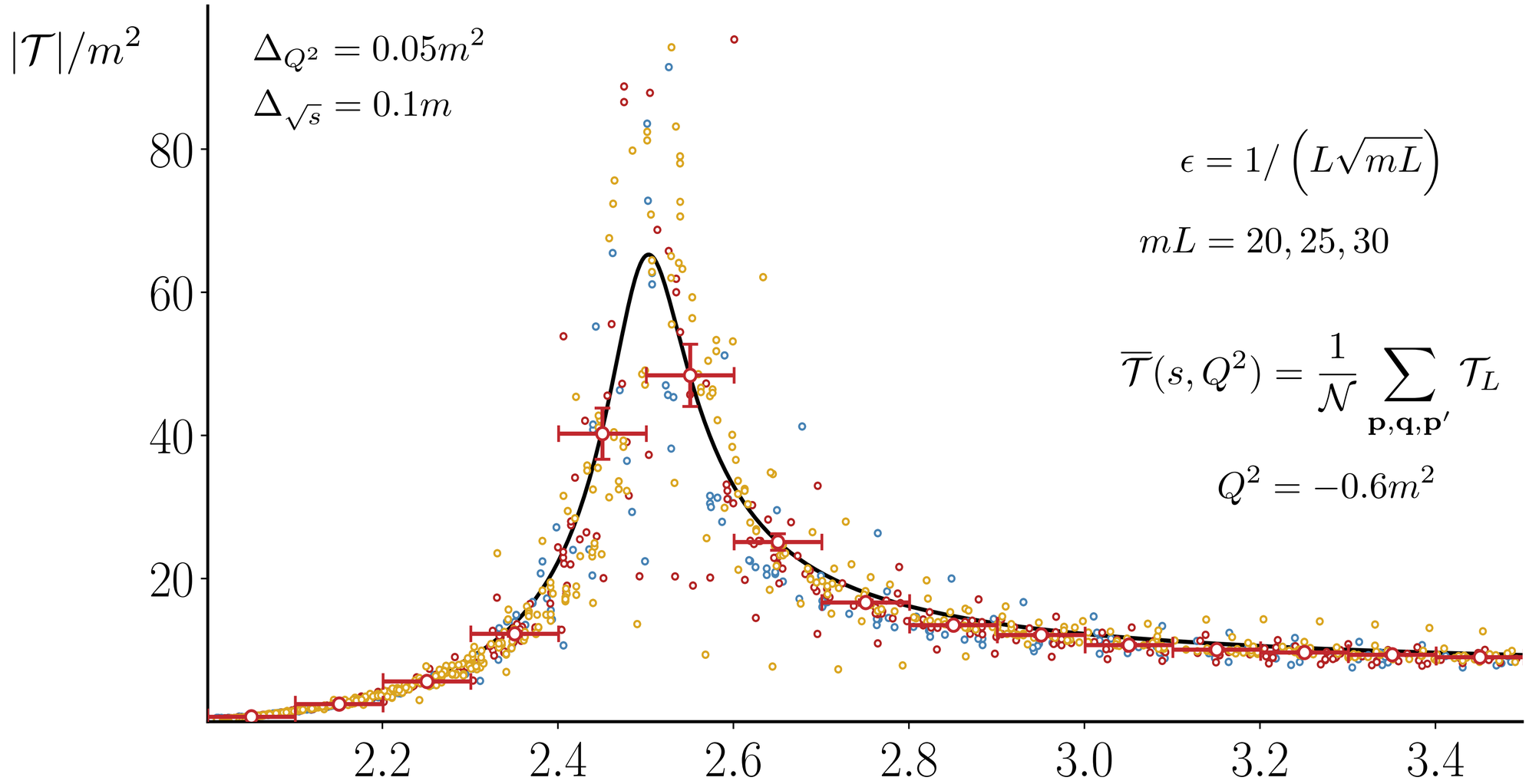

$$\epsilon = 1/\left(L\sqrt{mL}\right)$$

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$$\overline{\mathcal{T}}(s,Q^2) = \frac{1}{\mathcal{N}} \sum_{\mathbf{p},\mathbf{q},\mathbf{p}'} \mathcal{T}_L$$

$$Q^2 = -0.6m^2$$

Consistency checks in 1+1D

$$Q^2 \to -m^2$$

$$\overline{\mathcal{T}}(s,Q^2) = \frac{1}{\mathcal{N}} \sum_{\mathbf{p},\mathbf{q},\mathbf{p}'} \mathcal{T}_L \quad \Rightarrow \quad \overline{\mathcal{M}}(s) = \frac{1}{\mathcal{N}} \sum_{\mathbf{p},\mathbf{q},\mathbf{p}'} \frac{(Q^2+m^2)(Q^{\,2}+m^2)}{\langle 0|\mathcal{J}(0)|\mathbf{q}\rangle\langle \mathbf{p}+\mathbf{q}-\mathbf{p}'|\mathcal{J}(0)|0\rangle} \mathcal{T}_L$$

Consistency checks in 1+1D

$$\mathcal{A}_a(s,Q^2) \sim \frac{\delta_{a1}}{Q^2+m_1^2} \quad \mathcal{K}_{ab}(s) \sim \frac{m_a m_b (s^2/4-m_1^2)}{m_R^2 - s}$$

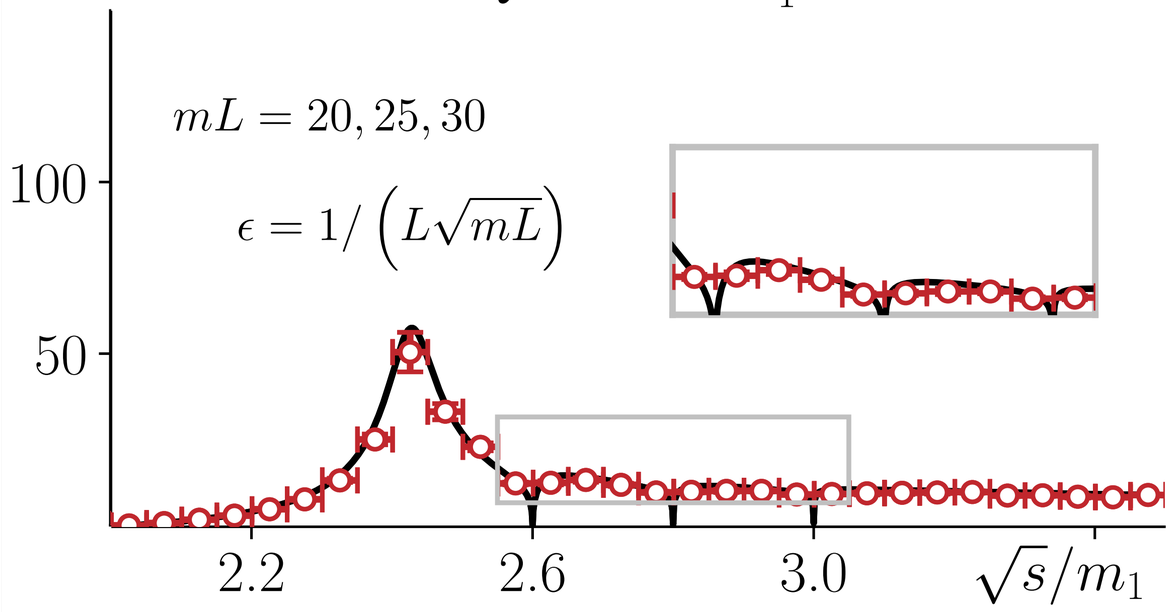
$|\mathcal{T}|/m_1^2$

$\Delta_{Q^2} = 0.05m^2$

$\Delta_{\sqrt{s}} = 0.05m$

$|\rho_{11}\mathcal{M}_{11}|$

$Q^2 = -0.6m_1^2$



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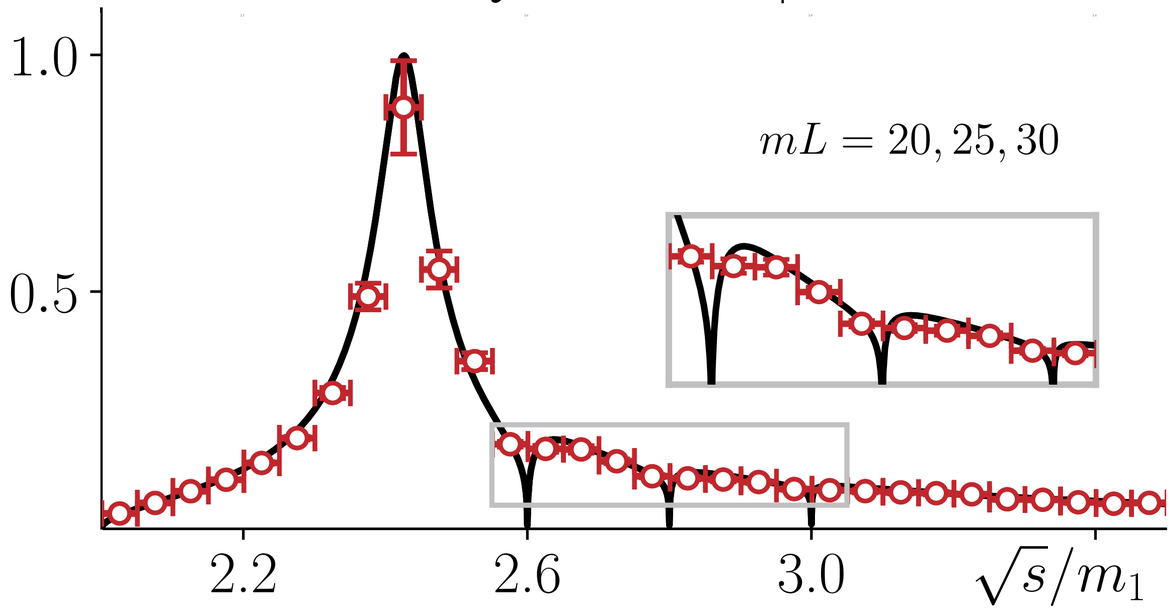


[PRD 110 054503 (2024)]

Consistency checks in 1+1D

R.Briceño, Z.Davoudi, M.Hansen, et.al. [PRD 101, 014509 (2020)]

$$\mathcal{T}_L \sim \int d^2x \; e^{iq\cdot x - \epsilon t} \langle p_f | \mathcal{J}(x)\mathcal{J}(0) | p_i \rangle_L$$

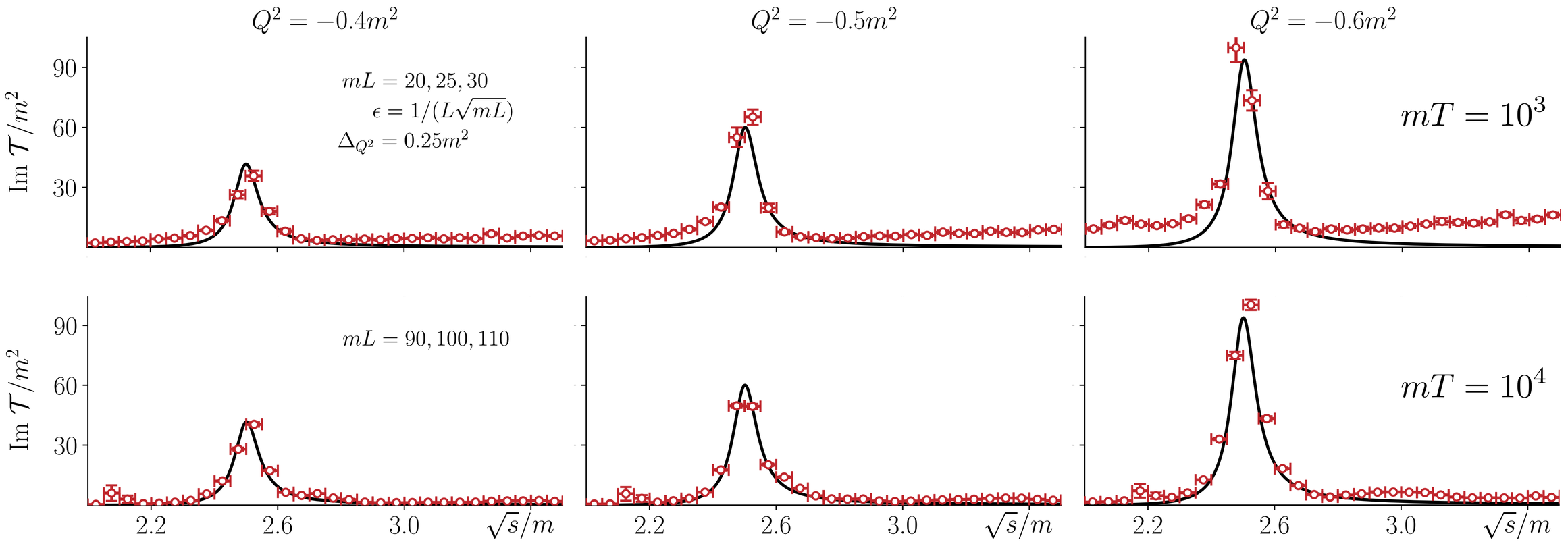
$$= \sum_{(E_n, \mathbf{P}_n)} \left\{ \int_0^T dt \; e^{i(q_0 + \omega_f - E_n + i\epsilon)t} \right\} \langle p_f | \mathcal{J}(0) | P_n \rangle_L \langle P_n | \mathcal{J}(0) | p_i \rangle_L \delta_{\mathbf{P}_n, \mathbf{q}+\mathbf{p}_f}$$

$$\lim_{\epsilon \to 0} \lim_{T \to \infty} \mathcal{T}_L \sim \sum_{(E_n, \mathbf{q}+\mathbf{p}_f)} \frac{\langle p_f | \mathcal{J}(0) | P_n \rangle_L \langle P_n | \mathcal{J}(0) | p_i \rangle_L}{q_0 + \omega_f - E_n}$$

$$\overline{\mathcal{T}}(s, Q^2) = \frac{1}{\mathcal{N}} \sum_{\mathbf{p}, \mathbf{q}, \mathbf{p}'} \mathcal{T}_L$$

Consistency checks in 1+1D

$$\mathcal{A}(s, Q^2) \sim \frac{1}{Q^2 + m^2} \qquad \mathcal{K}(s) \sim \frac{m^2(s^2/4 - m^2)}{m_R^2 - s}$$



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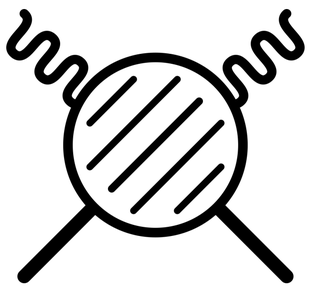
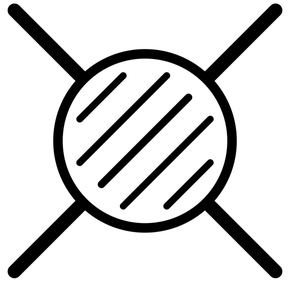
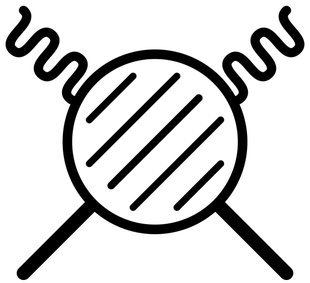
Order of volumes needed

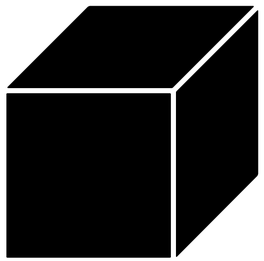



$$mL \sim \mathcal{O}(10)$$

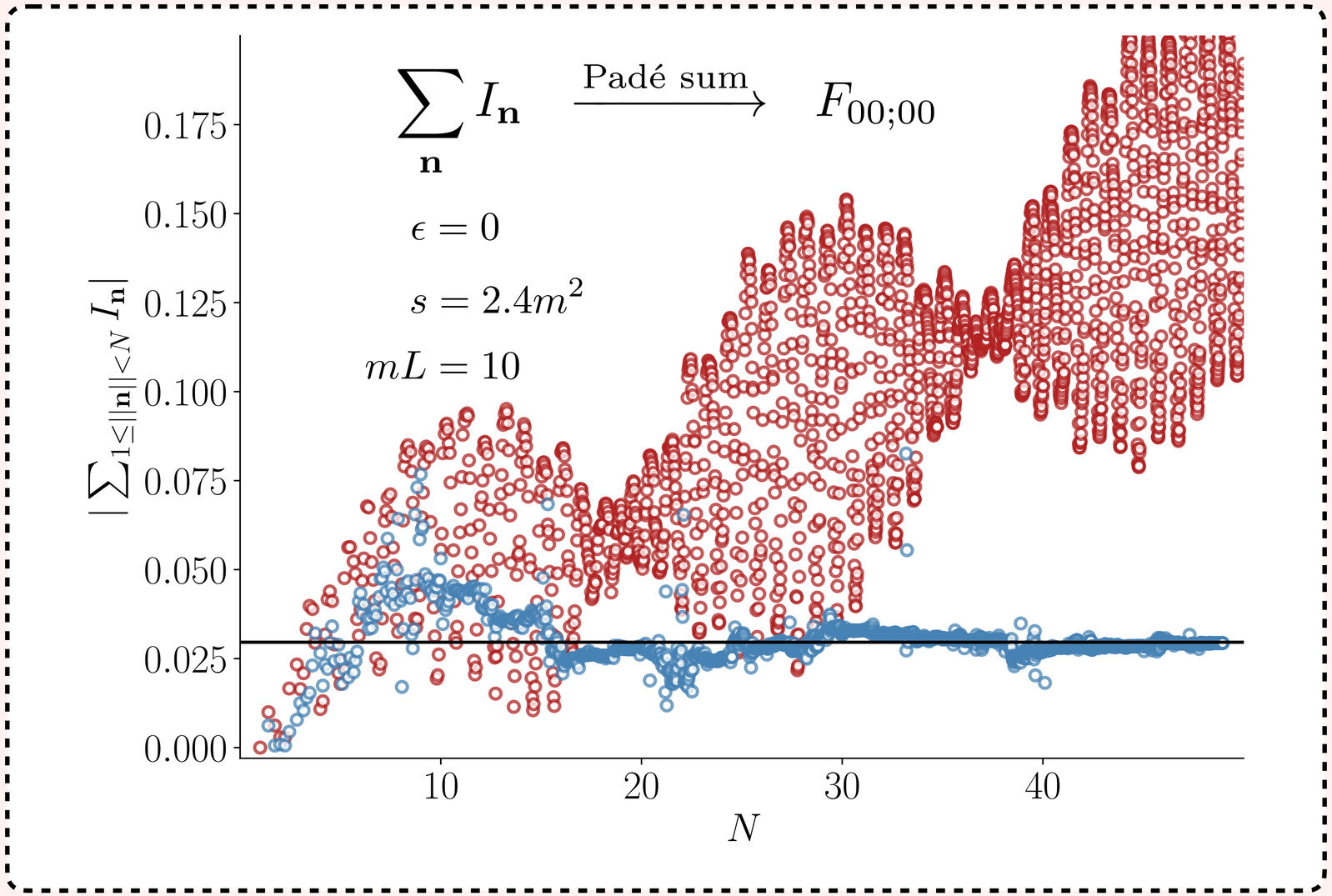



$$mL \sim \mathcal{O}(10^2)$$

$$mT \sim \mathcal{O}(10^3 - 10^4)$$



3+1D


$\sum_{\mathbf{n}} I_{\mathbf{n}}$
Padé sum
$F_{00;00}$
$\epsilon = 0$
$s = 2.4m^2$
$mL = 10$
$|\sum_{1\leq|\mathbf{n}|<N} I_{\mathbf{n}}|$
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
10
20
30
40
N

Next: Consistency checks for 3-body

Dimitra Pefkou

Hong Joo Ryoo

Raúl Briceño