A Renormalization Group Approach for Radiative Corrections in Nuclear Effective Field Theory

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Few Body Syst. 65, 79 (2024)

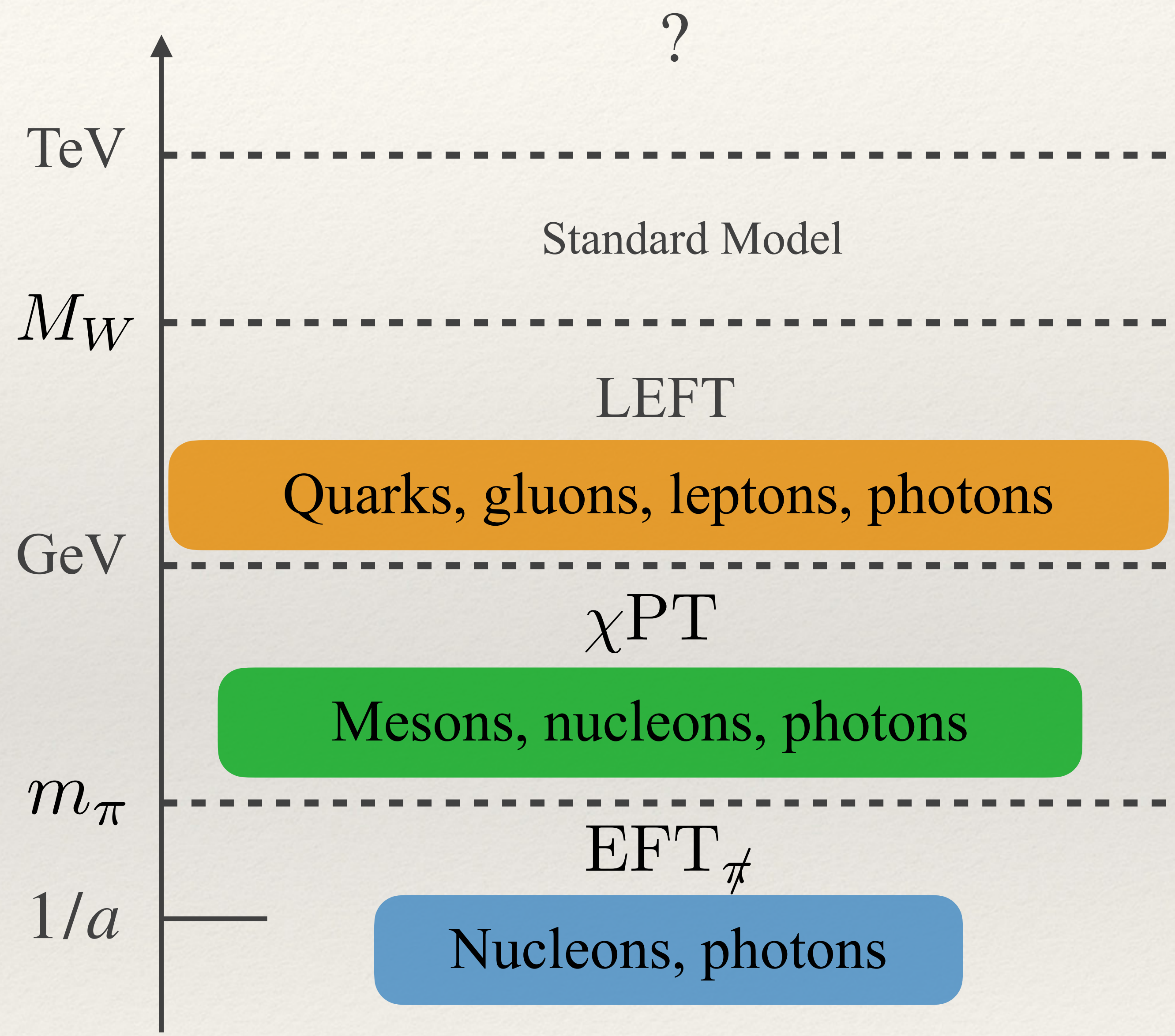
PRC 111, 064011 (2025)

Fluff #1: EFTs

Goal: Low energy, high precision, universal

Effective field theories should…

1. Have all particles that are near the mass-shell
2. Have a homogeneous power counting governed by a single ratio of scales
3. Be renormalization group invariance up to the order we are working
4. Preserve relevant symmetries (gauge, discrete, internal)
5. Make life easier



Fluff #2: Two Nucleon System

- Low-energy nucleons are "fine-tuned"

$$\mathcal{A}(p) = -\frac{4\pi}{M_N}\left[\frac{a}{1+iap} + \frac{a^2 r p^2/2}{(1+iap)^2} + \cdots\right]$$

$$a_t^{-1} \sim 36 \text{ MeV} \qquad a_s^{-1} \sim 8 \text{ MeV} \qquad r^{-1} \sim m_\pi$$

- Fine-tuned ↔ shallow bound state

$$B_d \sim \frac{1}{M_N a_t^2} \approx 1.4 \text{ MeV} \qquad B_d^{(\text{exp})} = 2.2245\ldots \text{ MeV}$$

- Can we reproduce this with an EFT and get electroweak matrix elements…

Kaplan, Savage, Wise NPB 478, PLB 424, NPB 534
van Kolck NPA 645

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Including radiative corrections and the renormalization group?

Kaplan, Savage, Wise NPB 478, PLB 424, NPB 534
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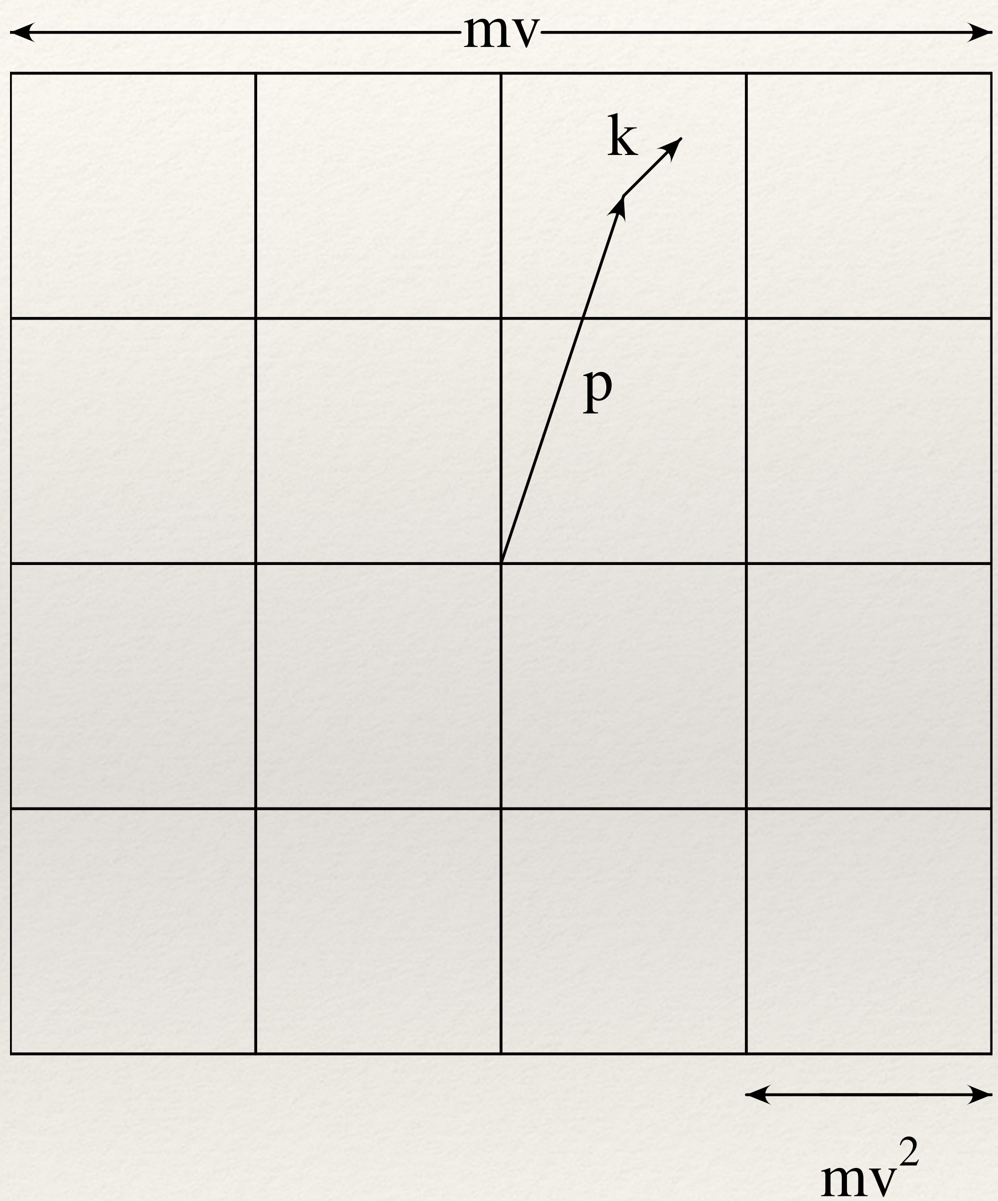
Nonrelativistic EFT with Virtual Photons*

- Bound state properties are easier with nonrelativistic theories[5]
- Energy and momenta are different but correlated scales

$$v \sim p/M = \sqrt{E/M}$$

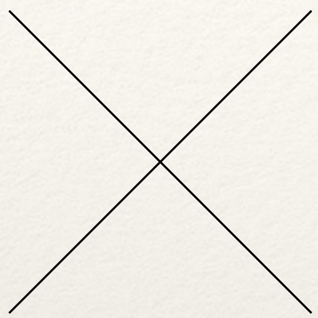
- Homogeneous power counting requires mode separation[1,2]

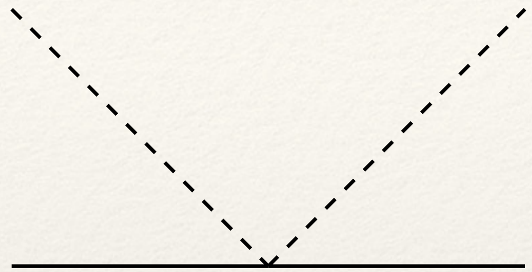
$$\text{potential} \sim (Mv^2, Mv) : N_{\mathbf{p}}$$
$$\text{soft} \sim (Mv, Mv) : A_p$$
$$\text{ultrasoft} \sim (Mv^2, Mv^2) : A$$

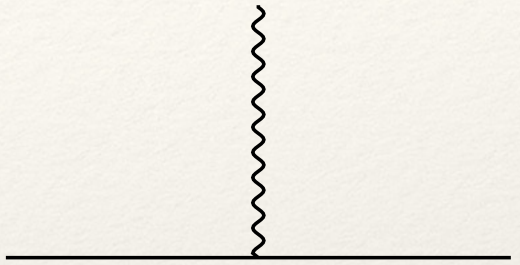


Caswell, Lepage, Labelle, Luke, Manohar, Rothstein, Savage, Grießhammer, Grinstein, Stewart, Hoang, Pineda, Soto, Brambilla, Vairo

Velocity EFT Lagrangian[4]



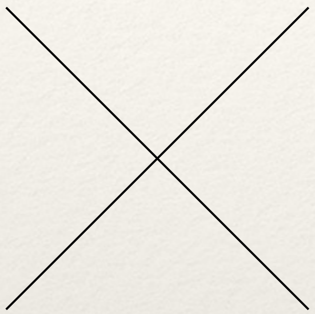




$$
\begin{aligned}
\mathcal{L} = & \sum_p N_p^\dagger \left(iD_0 - \frac{(p - iD)^2}{2M_N} \right) N_p - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
& + \sum_p \left| p^\mu A_p^\nu - p^\nu A_p^\mu \right|^2 - \sum_{p',p} V(p', p) \\
& - \frac{4\pi\alpha}{2M_N} \sum_{q,q',p,p'} A_{q'} \cdot A_q N_{p'}^\dagger Q N_p \\
& + \frac{e}{2M_N} \epsilon^{ijk} \left(\nabla^j A^k \right) \sum_p N_p^\dagger \sigma^i \left[\kappa_0 + \kappa_1 \tau^3 \right] N_p \,,
\end{aligned}
$$

TRR and Reis, FBS 65

Neutron-Proton Potential



$$V_{pn} = \sum_{v=-1} \sum_{p',p} V^{(v)}_{abcd}(p', p) p^{\dagger}_{p',a} p_{p,b} n^{\dagger}_{-p',c} n_{-p,d}$$

$$V^{(-1)}_{abcd} = C^{(S=1)}_{0,pn} P^{(1)}_{ab,cd} + C^{(S=0)}_{0,pn} P^{(0)}_{ab,cd}$$

$$V^{(0)}_{abcd} = \frac{1}{2}\left(p'^2 + p^2\right)\left[C^{(S=1)}_{2,pn} P^{(1)}_{ab,cd} + C^{(S=0)}_{2,pn} P^{(0)}_{ab,cd}\right]$$

$$V^{(1)}_{abcd} = \frac{1}{4}\left(p'^2 + p^2\right)^2\left[C^{(S=1)}_{4,pn} P^{(1)}_{ab,cd} + C^{(S=0)}_{4,pn} P^{(0)}_{ab,cd}\right]$$

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“Matching” to AV18

- The LECs need to be matched at $\mu = m_\pi$ at $O(\alpha^0)$

$$C_0^{(s)}(\mu_{\text{match}} = m_\pi) = \frac{4\pi a_s}{M_N}$$

$$C_2^{(s)}(\mu_{\text{match}} = m_\pi) = \frac{2\pi a_s^2 r_s}{M_N} \qquad C_4^{(s)}(\mu_{\text{match}} = m_\pi) = \frac{\pi a_s^3 r_s^2}{M_N}$$

- Use Argonne v18 as proxy for lattice QCD—“integrate out the pion”

$$a_0 = -23.084\,\text{fm} \qquad r_0 = 2.703\,\text{fm}$$

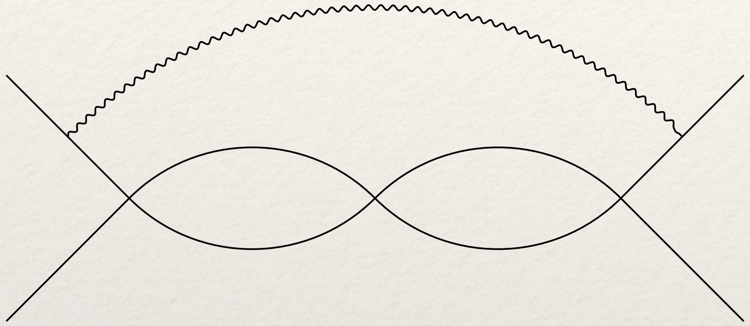
$$a_1 = 5.402\,\text{fm} \qquad r_1 = 1.752\,\text{fm}$$

- Next: Run μ from $m_\pi \rightarrow p$ at $O(\alpha)$ and calculate matrix elements

Wirigna et al. PRC 51

Loop Corrections and the Velocity Renormalization Group[3]

- Perturbation theory generates two (possibly) large logarithms



$$p^2 C_0^3 \left[\lambda_S \log \frac{\mu}{p} + \lambda_{US} \log \frac{\mu}{E} \right]$$

- One μ to rule them all? No!
- Introduce two correlated scales in dimensional regularization

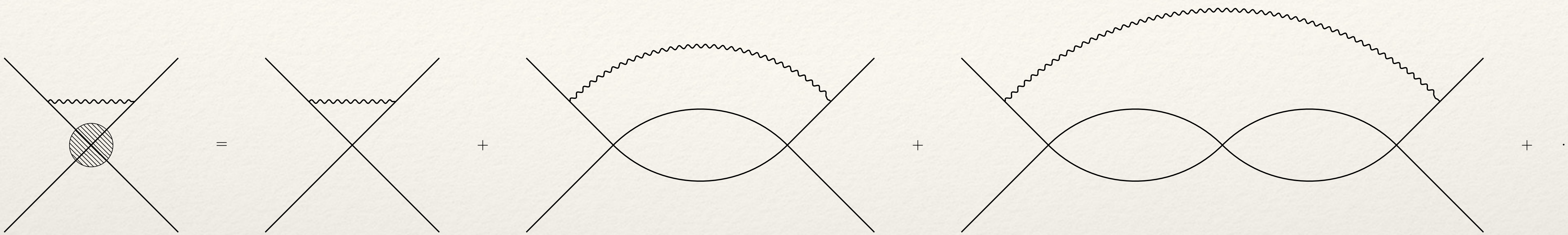
$$\mu_S = M_N \nu \qquad \mu_U = M_N \nu^2$$

- Run in ν—sum soft and ultrasoft logarithms simultaneously

$$\nu \frac{dV}{d\nu} = \gamma_S + 2\gamma_U \qquad \mu_U \frac{dV}{d\mu_U} = \gamma_U \qquad \mu_S \frac{dV}{d\mu_S} = \gamma_S$$

Luke et al. PRD 61

Velocity RG Solution



$$C_2(\nu) = C_2\left(\frac{m_\pi}{M_N}\right) - \frac{27}{8}\left(\frac{M_N}{4\pi}\right)^2 C_0^3 \log\left(\frac{\alpha(M_N\nu^2)}{\alpha(m_\pi^2/M_N)}\right),$$

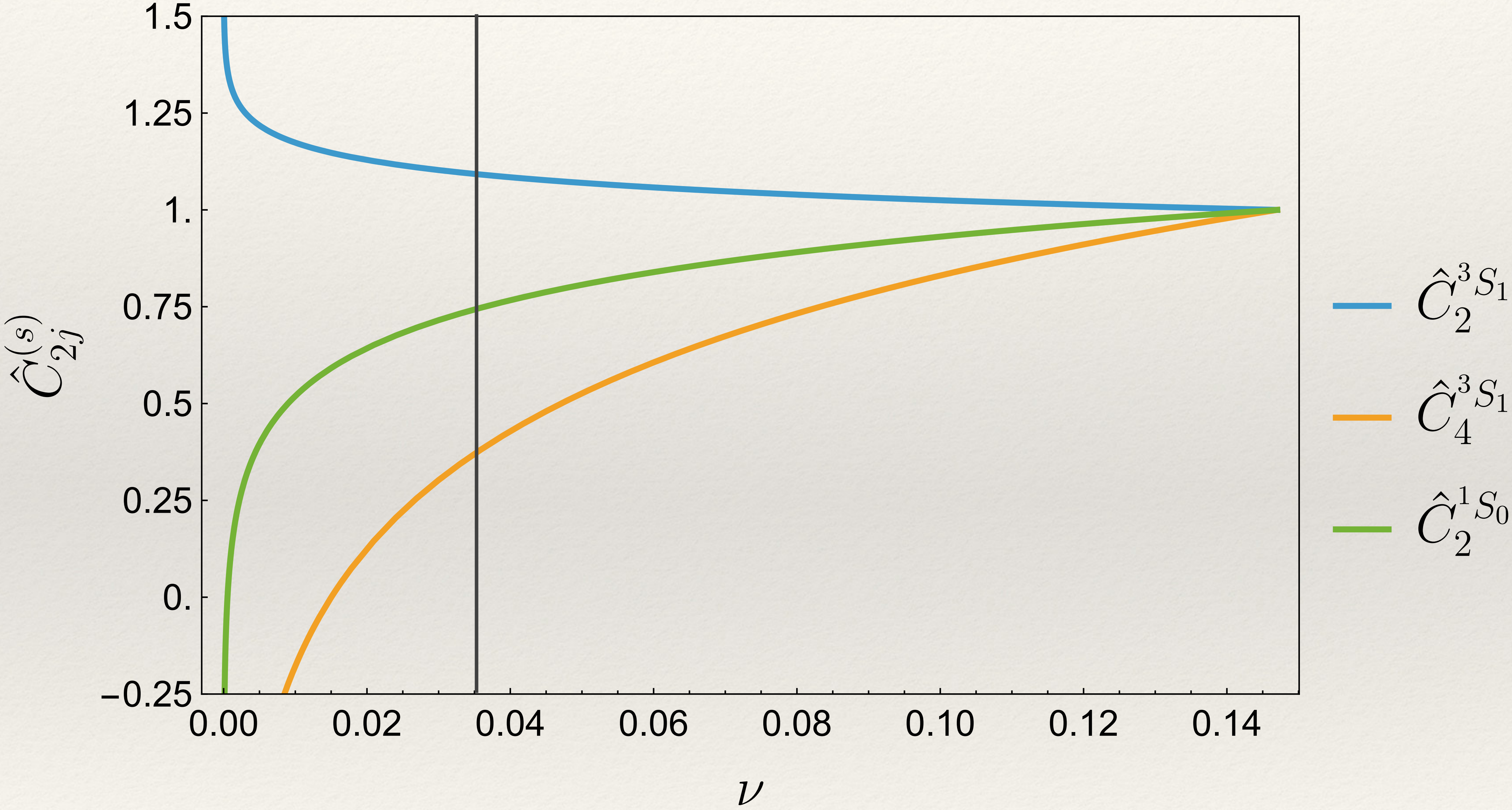
$$C_4(\nu) = C_4\left(\frac{m_\pi}{M_N}\right) + \frac{15}{4}\left(\frac{M_N}{4\pi}\right)^4 C_0^5 \log\left(\frac{\alpha(M_N\nu^2)}{\alpha(m_\pi^2/M_N)}\right)$$

Expanding logs gives complete sum $\sum_n \alpha^n \log^n(\nu)$

Varying ν probes higher-order logarithmic series

TRR and Reis, FBS 65

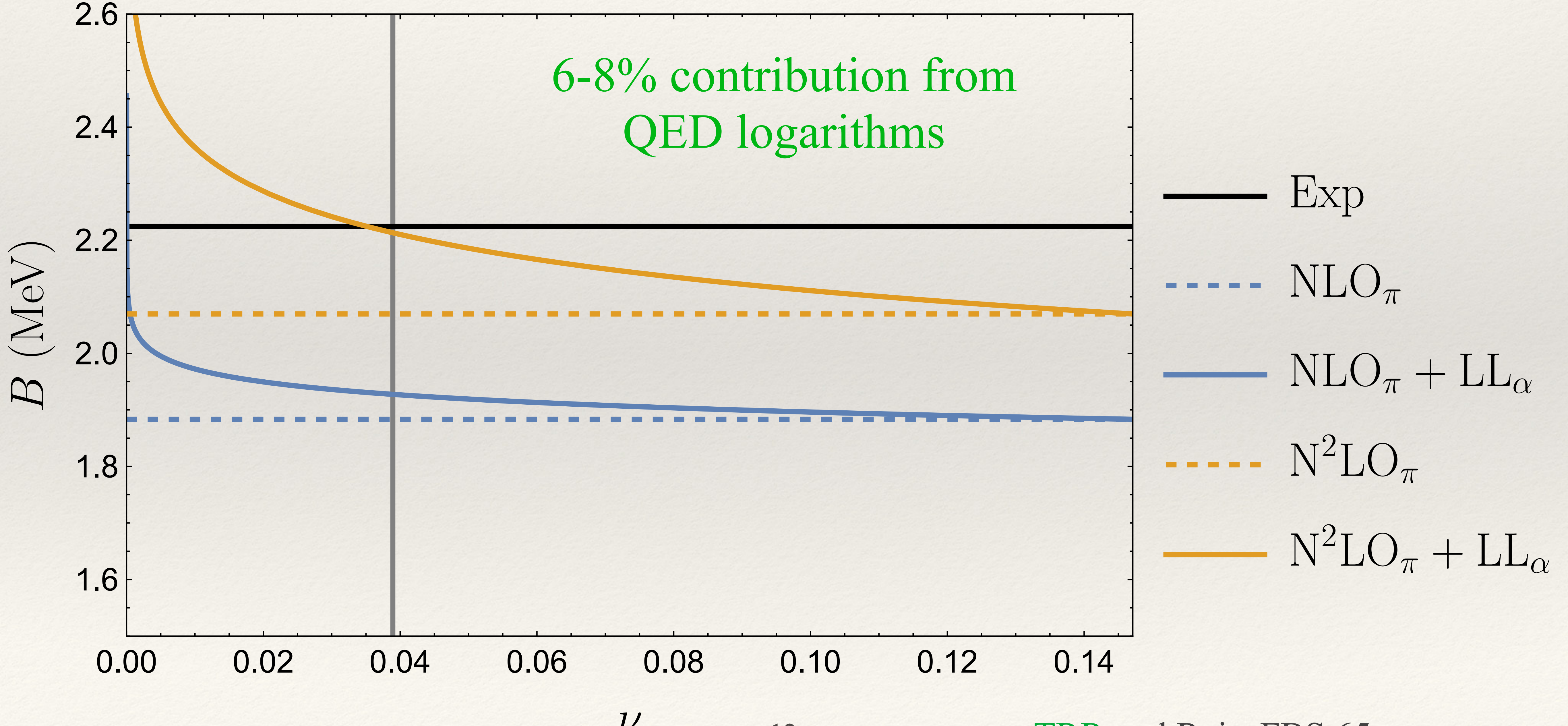
Running potentials



TRR and Reis, FBS 65

Deuteron Binding Energy

$$B = \frac{1}{M_N}\left(\frac{4\pi}{M_N C_0}\right)^2 + \frac{1}{2\pi}C_2\left(\frac{4\pi}{M_N C_0}\right)^5 + \frac{7}{16\pi^2}M_N C_2^2\left(\frac{4\pi}{M_N C_0}\right)^8 - \frac{1}{2\pi}C_4\left(\frac{4\pi}{M_N C_0}\right)^7$$



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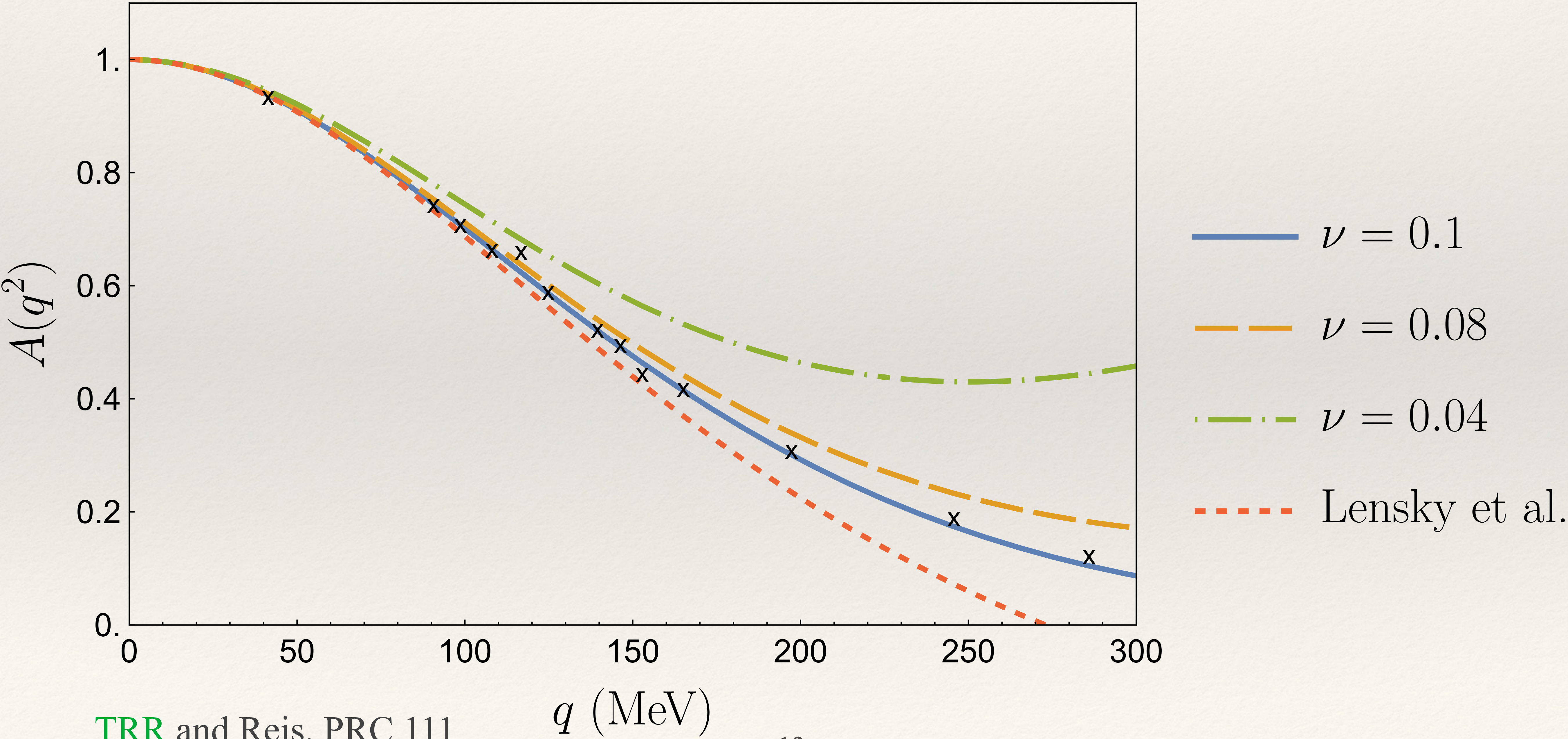
Electron Scattering Structure Function

$$A(q^2) = F_C^2(q^2) + \frac{2}{3}\eta F_M^2(q^2) + \frac{8}{9}\eta^2 F_Q^2(q^2)$$



TRR and Reis, PRC 111

Simon et al. NPA 364, KSW PRC 59, Lensky et al. PRC 104

Charge Radius

- Conventional definition of charge radius is not scale-independent at $O(\alpha)$ and higher

$$\langle r_d^2 \rangle_C = -6 \frac{dF_C(q^2)}{dq^2}\bigg|_{q^2=0}$$

$$r_{\text{CREMA}} = 2.12562(13)_{\text{exp}}(77)_{\text{th}}$$

CREMA Science 353

		$\nu = \frac{m_\pi}{M_N}$	$\nu = 0.1$	$\nu = 0.06$
r_d (fm)	no truncation error	2.154(6)	2.111(6)	2.049(7)
	with truncation error	2.15(6)	2.11(6)	2.05(6)

2% shift

Radiative Neutron Capture

- First step in Big Bang Nucleosynthesis network

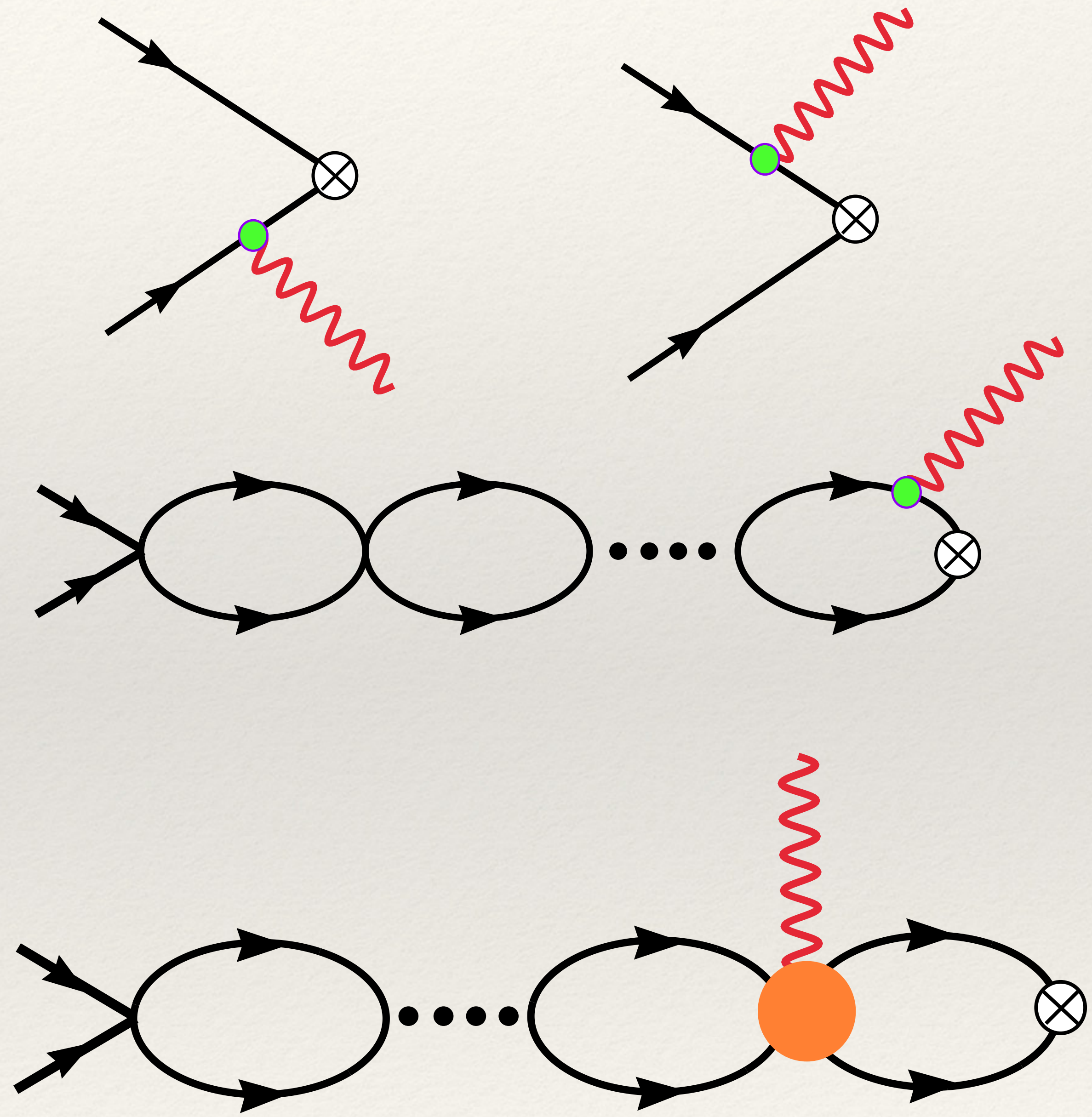
Wagoner, Fowler, Hoyle, Steigman, Iocco, Mangano, Miele, Pisanti, Serpico, Cyburt, Fields, Olive, Yeh

- Uncertainty in cross section sets scale for uncertainty in light element abundances

- Could be a probe of new physics with precise Standard Model predictions

- EFT analyses claim O(1%) uncertainty*

Chen et al. NPA, 653, Rupak NPA 678, Acharya and Bacca PLB 827



*with LECs fit to experimental data

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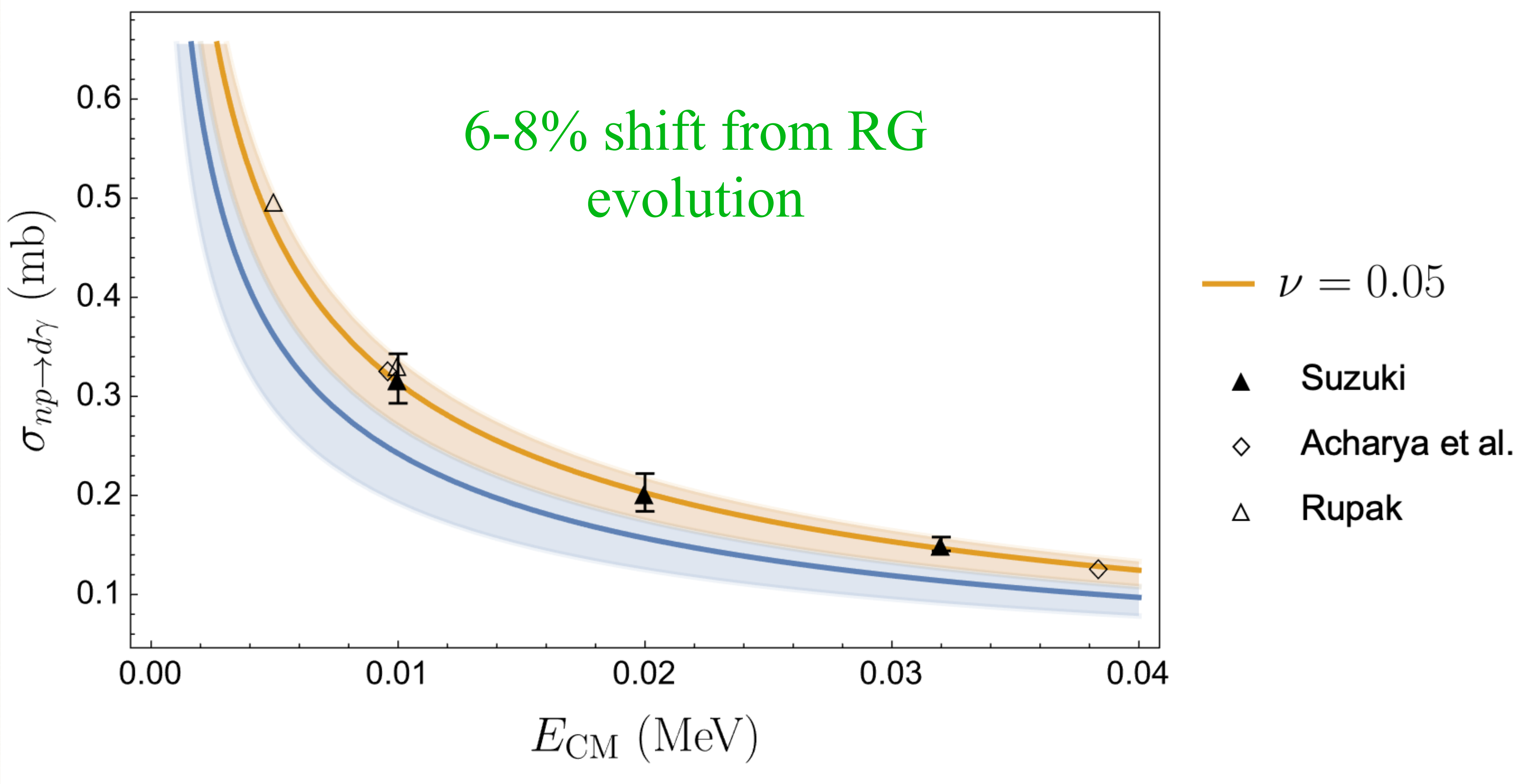

Cox et al. NP 74, Suzuki et al. 1995, Nagai et al. PRC 56, Tudoric-Ghemo, NPA 92, Bosman et al. PLB 82, Steiler et al. 1986, Michel et al. JPG 1989

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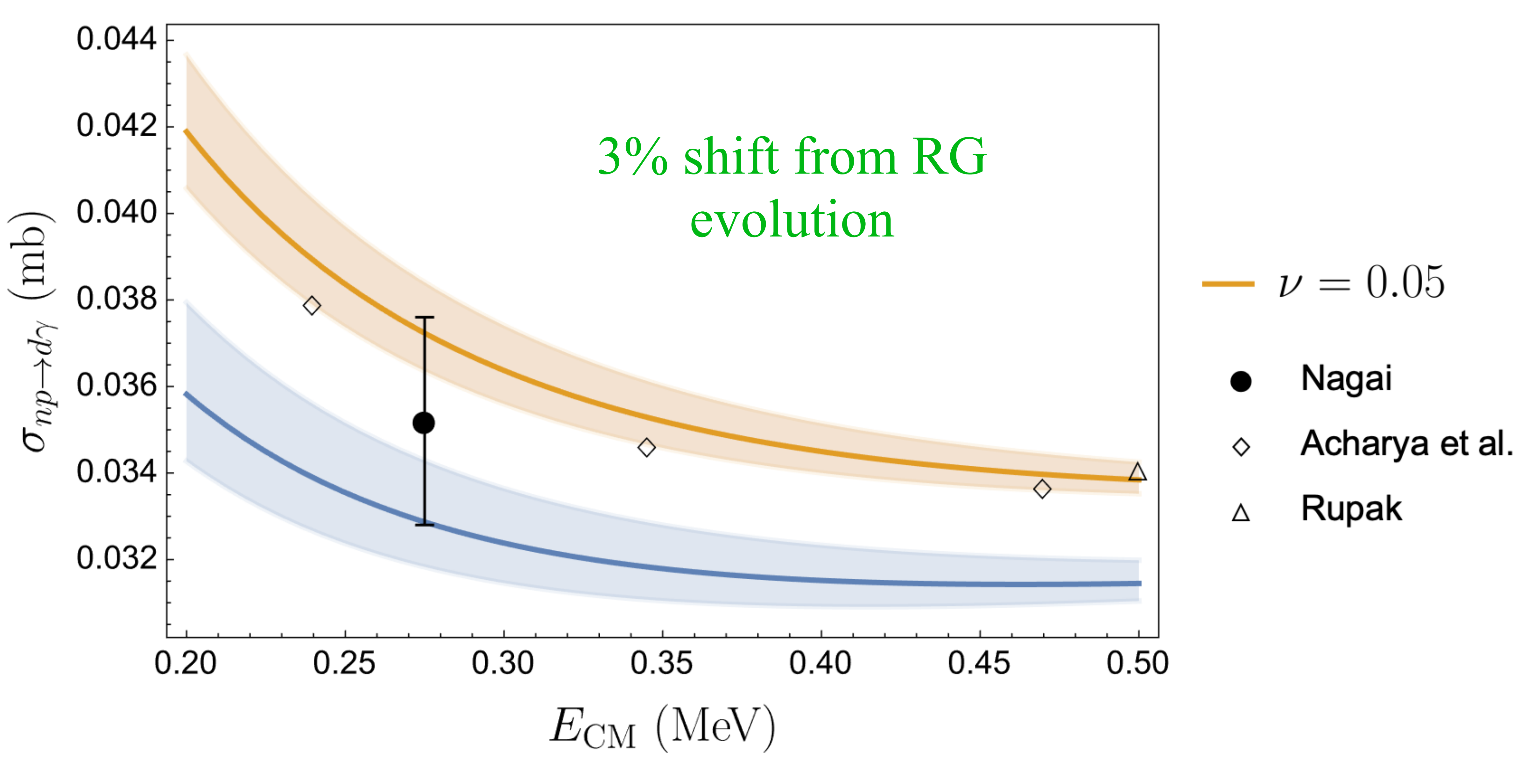

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Summary

- Nucleons + photons ≈ heavy quarkonium + gluons

- RG improvement leads to few percent corrections in
    1. Deuteron binding energy (7-8%)
    2. Charge form factor and radius (2%)
    3. $np \to d\gamma$ cross section relevant for BBN (3-8%)

➡ How robust is modern Bayesian uncertainty quantification without these corrections?

- Next steps: pp-fusion, pion-exchange (NN, T_{cc}, …), many-body?