Who is Steve Sharpe to me?

Jorge Baeza-Ballesteros

SteveFest - 1st August 2025

Deutsches Elektronen -Synchroton Zeuthen

Who is Steve Sharpe to me?

and *How can EFTs complement lattice computations?*

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Congratulations Steve!

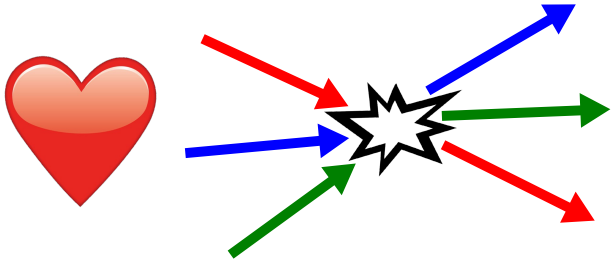
Overview

1. Three-pion K matrix at NLO in ChPT
2. Nucleon-nucleon interactions with plane-wave QC
3. Who is Steve Sharpe to me?

Overview

Once upon a time...

Steve has a **special relation** with three-particle systems



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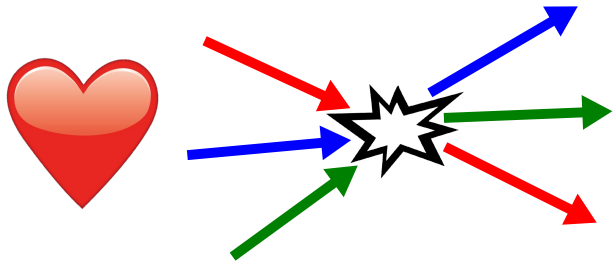
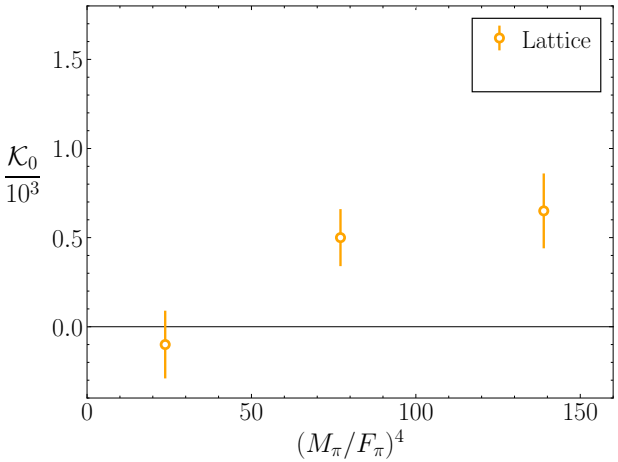
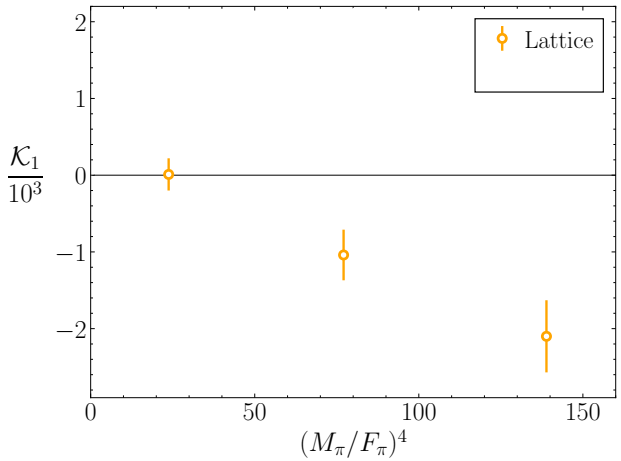


I **first met Steve** during Fernando's thesis defense:

$\pi^+\pi^+\pi^+$ K-matrix from lattice QCD

[Blanton et al. 2020, 2021]





$$\mathcal{K}_{\text{df},3}^{\text{LO}} = \mathcal{K}_0 + \Delta\mathcal{K}_1$$

$$\Delta = \frac{P^2 - 9M_\pi^2}{9M_\pi^2}$$

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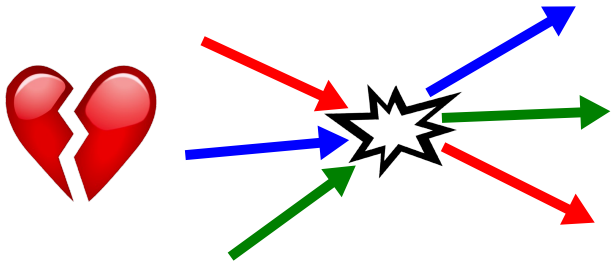

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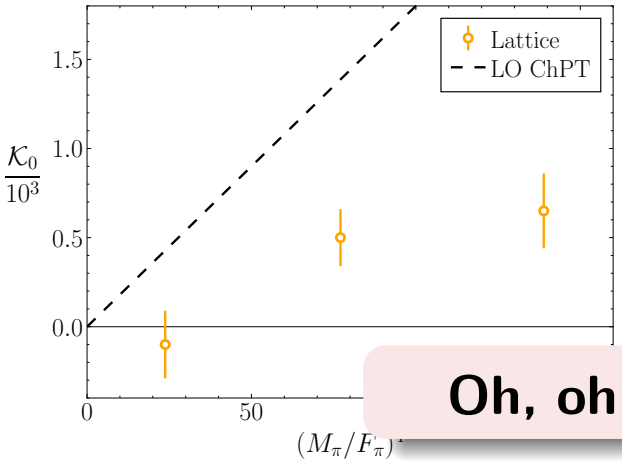

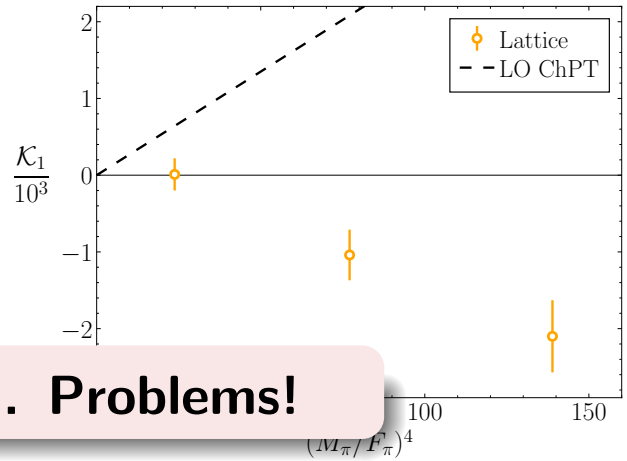

Oh, oh... Problems!

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$\pi^+\pi^+\pi^+$ from lattice QCD

Large tension between LO ChPT and lattice for $\pi^+\pi^+\pi^+$
[Blanton et al. 2020, 2021]

Origin of discrepancy
- Problems with the **quantization condition**
- Large **higher-order corrections**

Need to understand the origin of this discrepancy to **build confidence in the formalism**

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↓

Invitation: Determine three-pion K-matrix at NLO ChPT

Starting point: Three-pion amplitude at NLO in SU(2) ChPT
[Bijnens and Husek 2021]

My visit to U. of Washington

Visit to the University of Washington in fall 2022
(Fernando came too!)

→ Wonderful **hospitality** by Steve

My visit to U. of Washington

Visit to the University of Washington in fall 2022
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→ Wonderful **hospitality** by Steve

- Came to office every day in early post-covid times.
- Invitation to dinner at his place.
- Got special coffee for our stay
- Brought me as a guest to special event at U. Washington

Extremely grateful for his effort to make me feel comfortable both in and out the university!

An amazing team

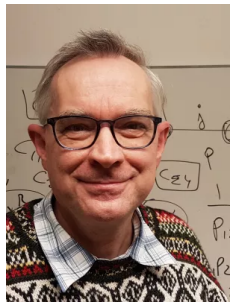
U. Washington team:

Stephen Sharpe
U. of Washington

Fernando Romero-López
U. of Bern

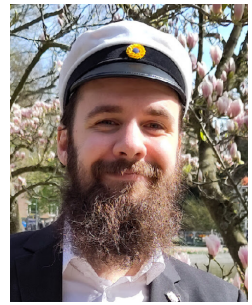
Jorge Baeza-Ballesteros
DESY

Lund U. team (The Swedes):



Hans Bijnens
Lund U.

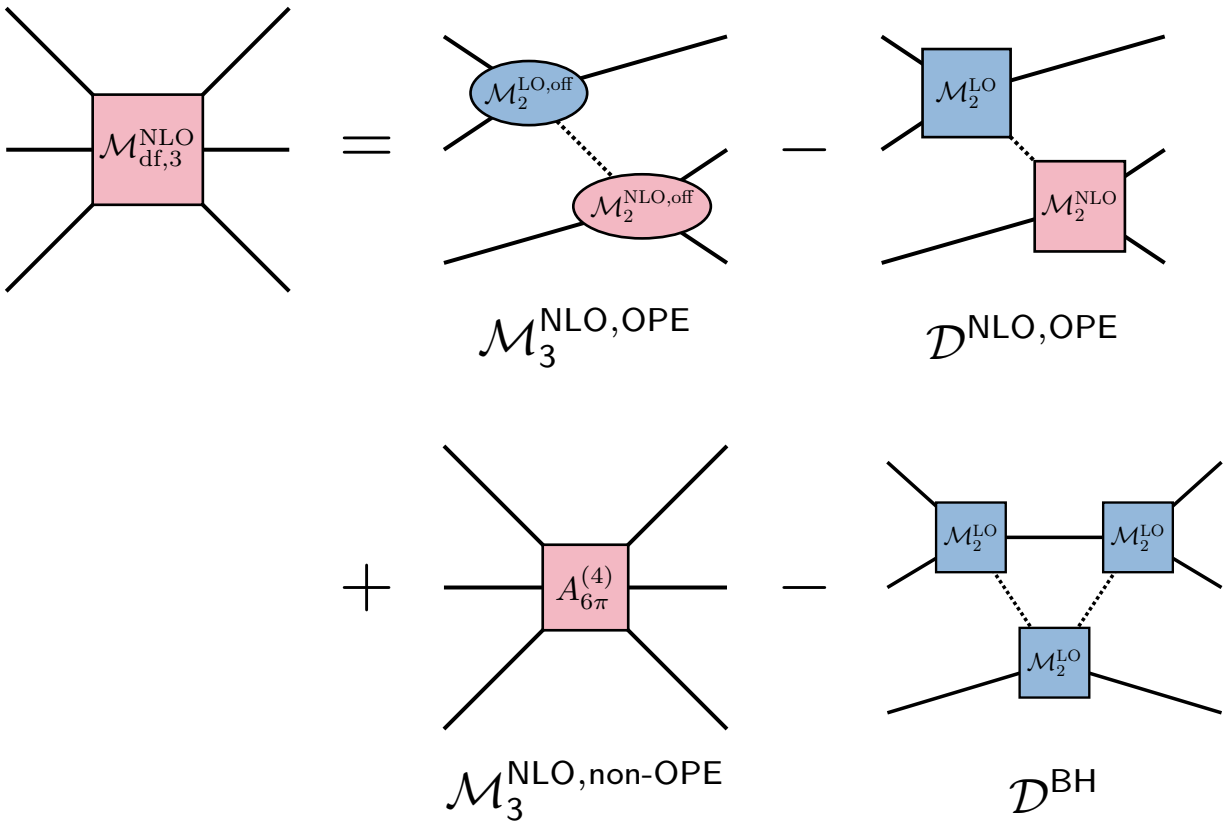
Tomáš Husek
Charles U.



Mattias Sjö
CPT Marseille

$\pi^+\pi^+\pi^+$ K-matrix at NLO in ChPT

Algebraic relation at NLO: $\mathcal{K}_{\text{df},3}^{\text{NLO}} = \text{Re}\,\mathcal{M}_{\text{df},3}^{\text{NLO}}$



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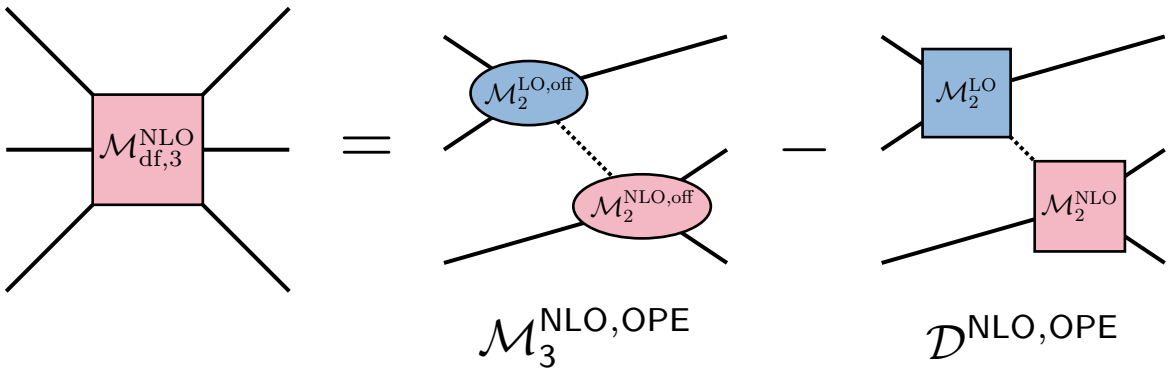

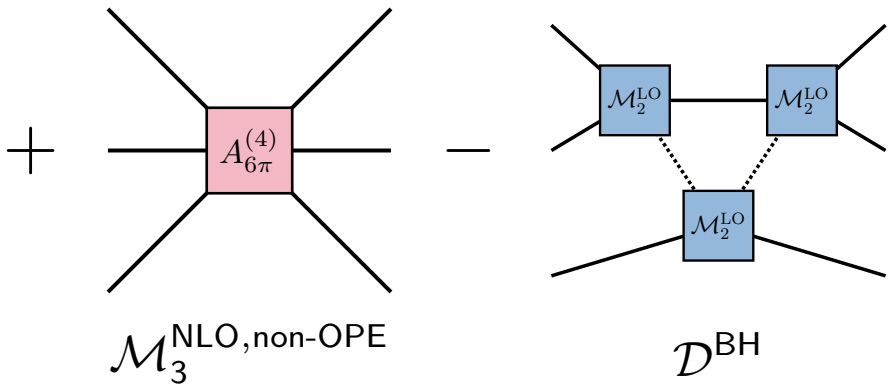

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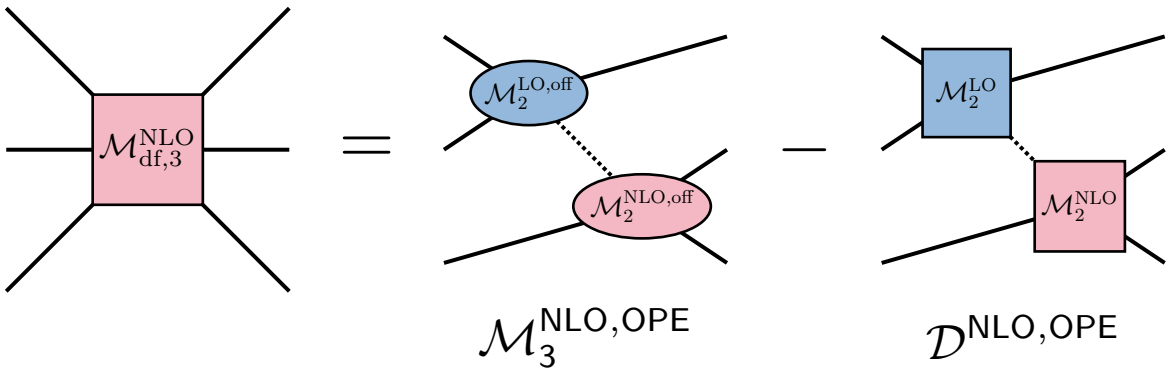

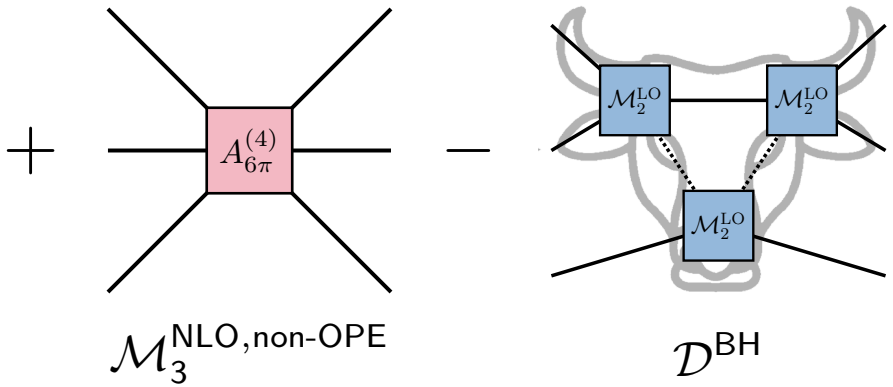

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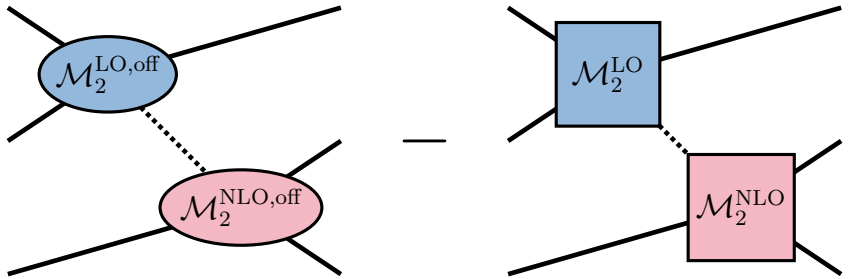
Anecdote: OPE contribution

Steve, Fernando and myself first focused on the **OPE contribution**



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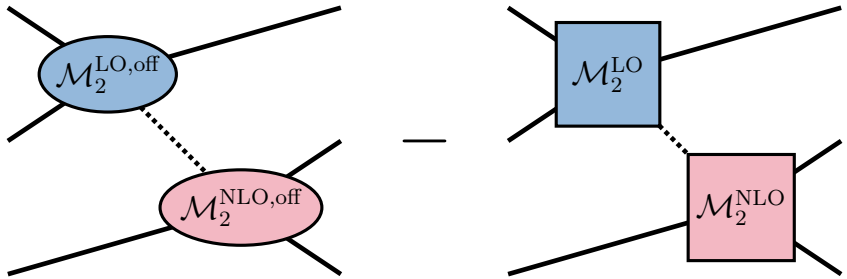

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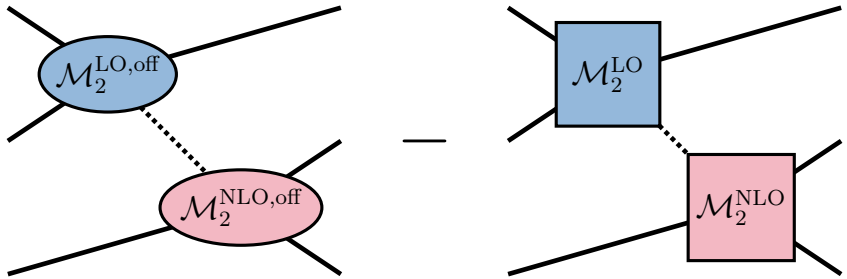

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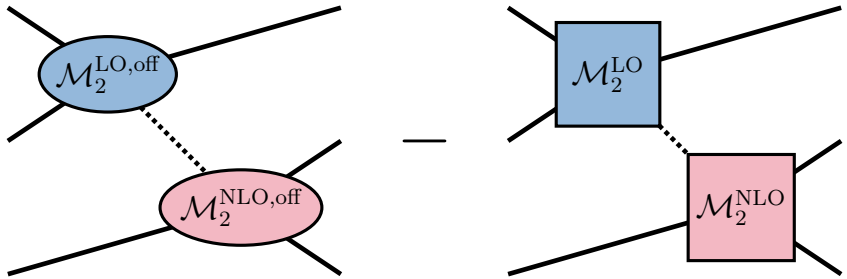

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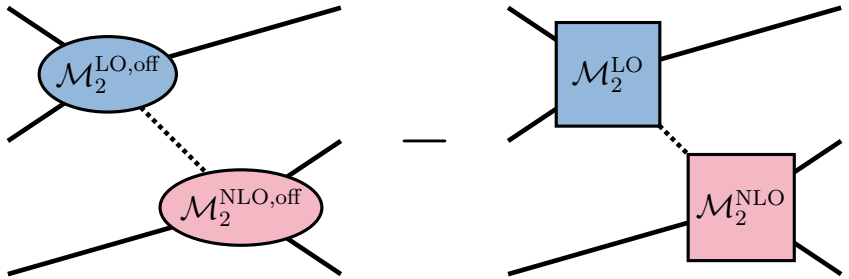

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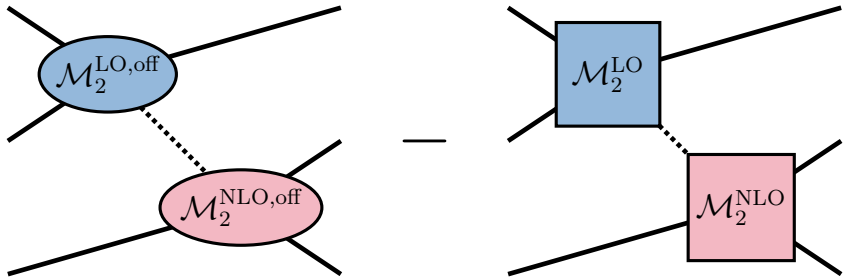

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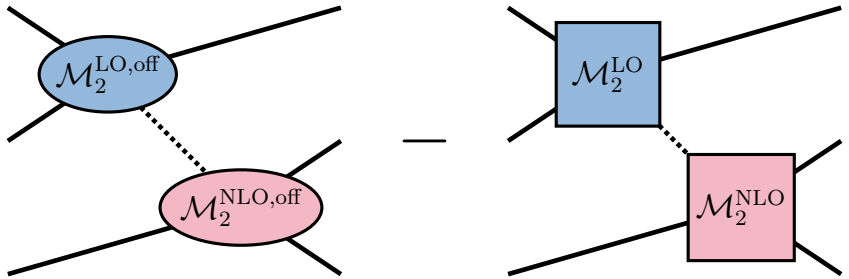

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- We all arrived to the same result! → **Incorrect**

"This [a factor of 81] is possibly the largest factor I have missed in my career" S. Sharpe

$\pi^+\pi^+\pi^+$ K-matrix at NLO in ChPT

Determined $\mathcal{K}_{\mathrm{df,3}}$ about threshold up to **quadratic order** [JBB et al. 2023]

$$M_\pi^2 \mathcal{K}_{\mathrm{df,3}} = \mathcal{K}_0 + \mathcal{K}_1 \Delta + \mathcal{K}_2 \Delta^2 + \mathcal{K}_{\mathrm{A}} \Delta_{\mathrm{A}} + \mathcal{K}_{\mathrm{B}} \Delta_{\mathrm{B}} + \mathcal{O}(\Delta^3)$$

Angular dependence

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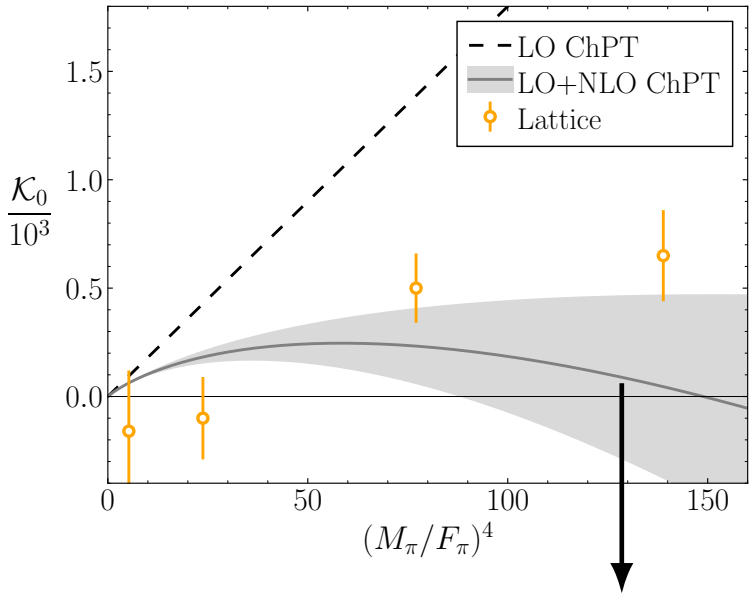
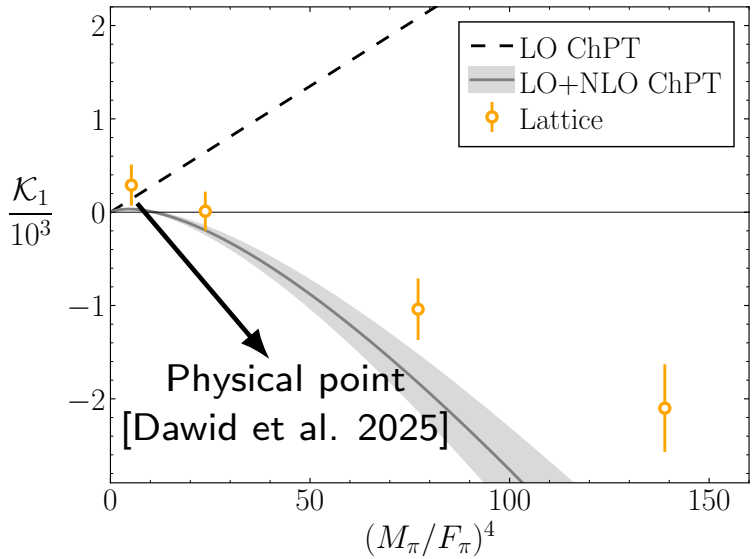
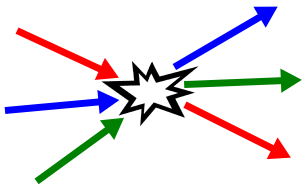


Phenomenological LECs



Much better agreement to lattice results

- **New features**: triangle diagrams, cutoff, angular dependence...



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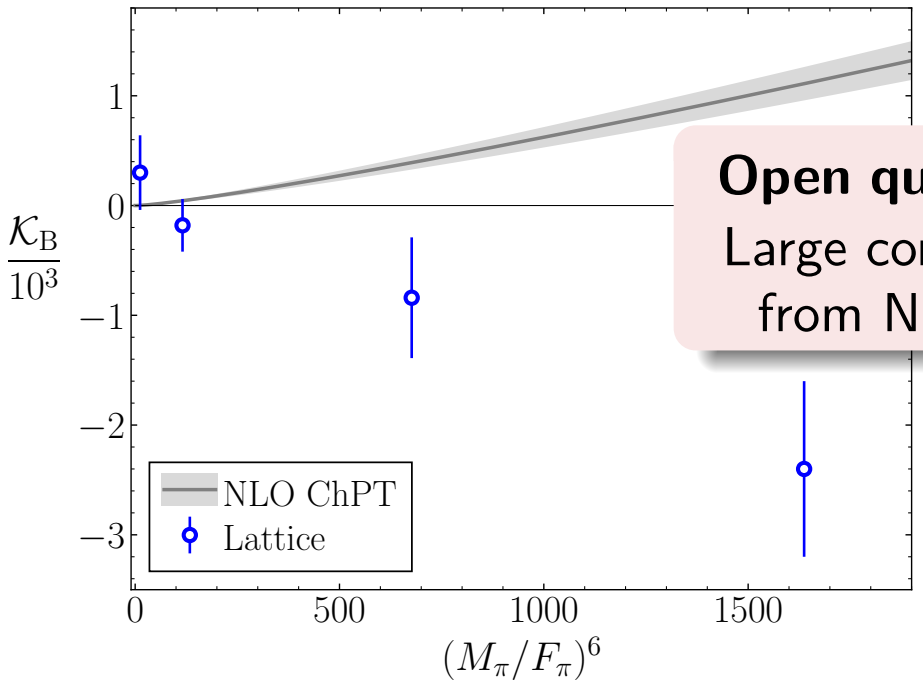

Open question:
Large correction from NNLO?

Follow-up: K-matrix for general isospin

Follow-up [JBB et al. 2024]:
LO and NLO computation in all isospin channels

Insight on **chiral expansion** for three-pion quantities

Useful for comparison to **future lattice results**

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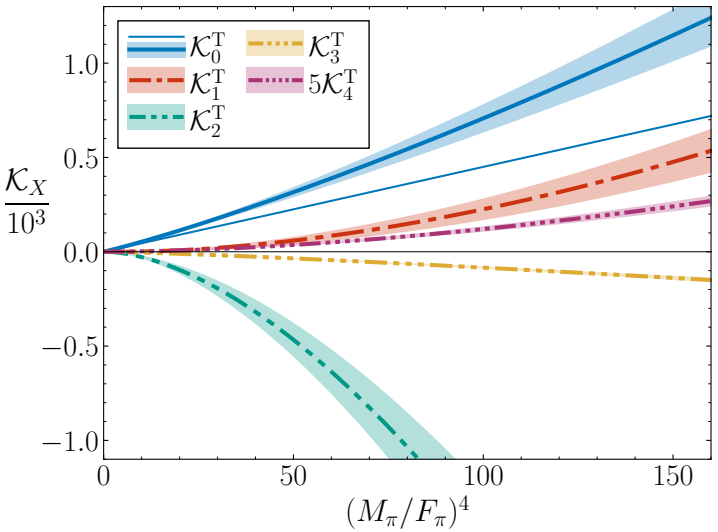

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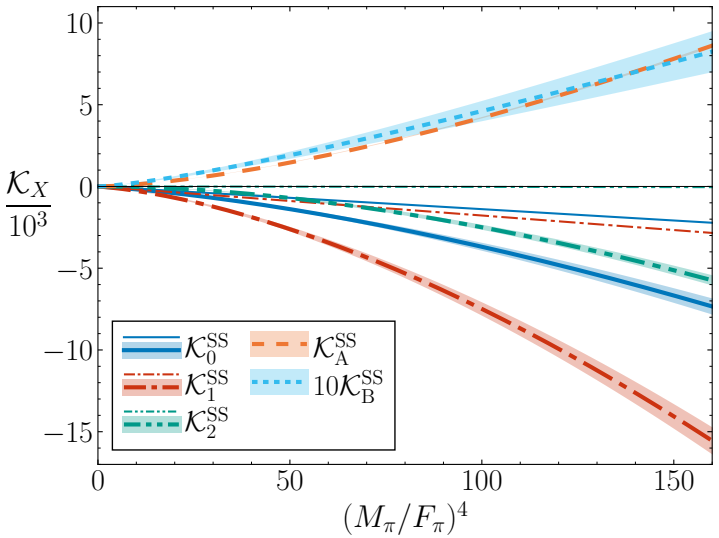

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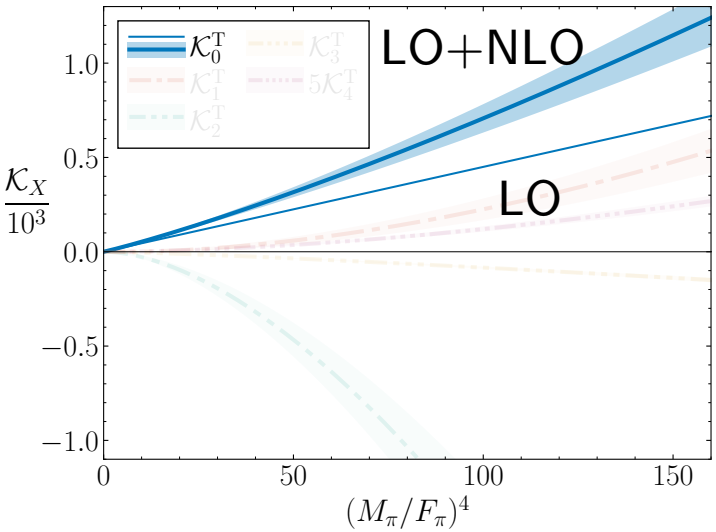

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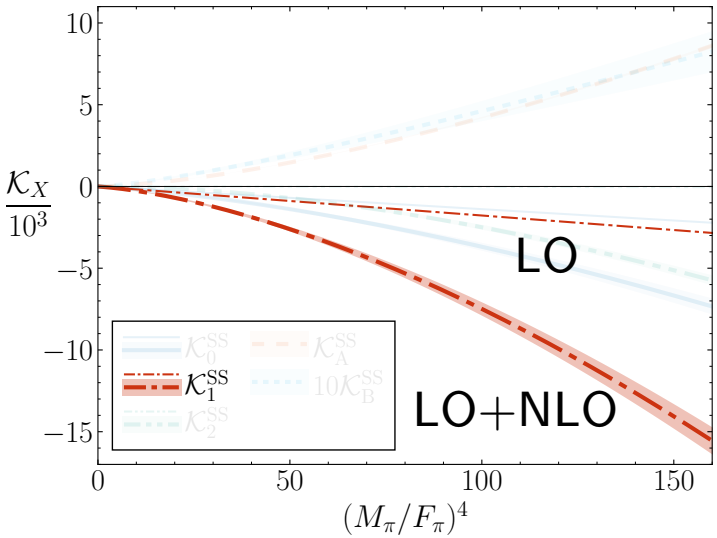


Large NLO correction across all isospin channels

The moral of this story

Getting to work with Steve was an **invaluable experience**:

- Very **supportive**, both academically and extra-academically.
- Keen on teaching and also learning new topics.

...and also **some physics**:

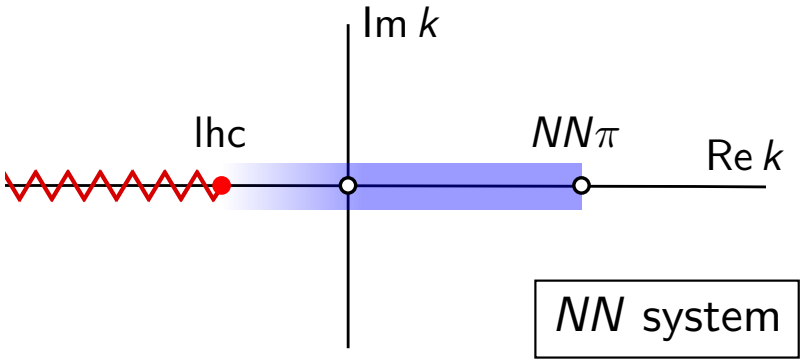
- NLO corrections to $\mathcal{K}_{\text{df},3}$ are, in general, large.
- NLO corrections lead to reasonable agreement with lattice results, **building confidence on the RFT formalism**.

Overview

Limitations of Lüscher's formalism

Lüscher's formalism breaks close to a **left-hand cut**

- **Large exponentially-suppressed effects** close to the lhc.
- Limits convergence of ERE and **partial-wave truncation**



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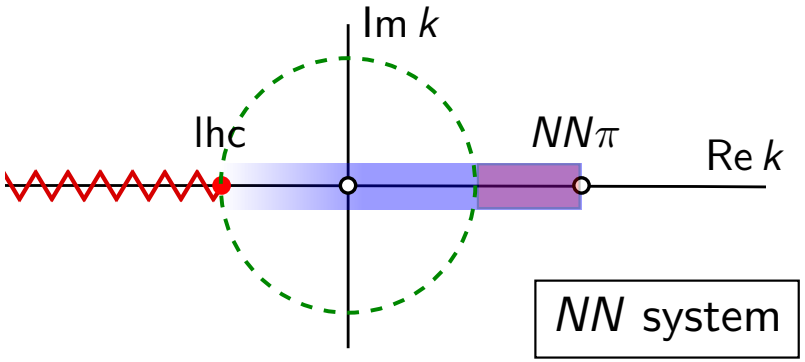

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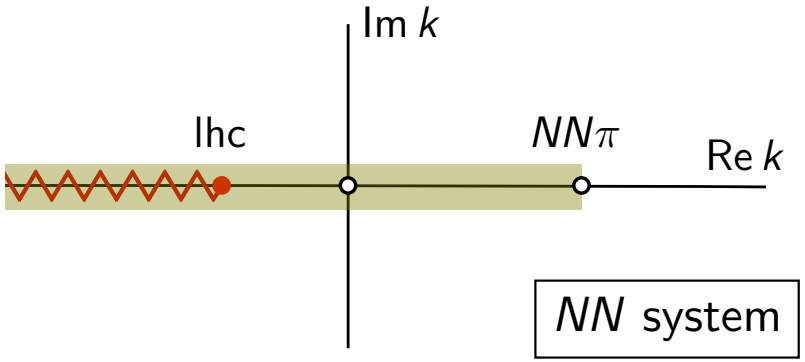

Multiple **alternatives and extensions** proposed in recent years
[Meng and Epelbaum 2021, Baião Raposo and Hansen 2024, **Steve** et al. 2024, Bubna et al. 2024, Dawid et al. 2025, ...]

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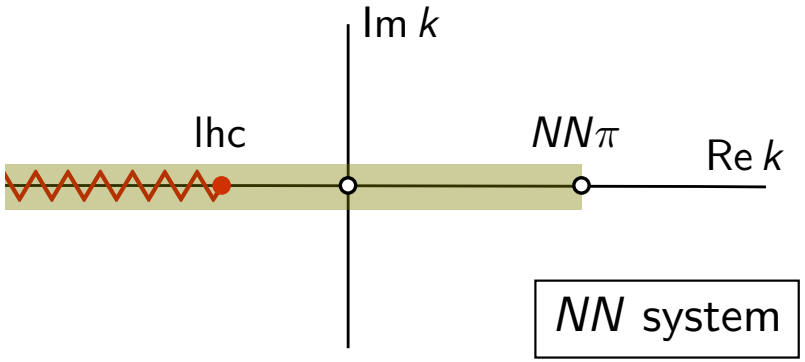

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Leave something for the youngsters!

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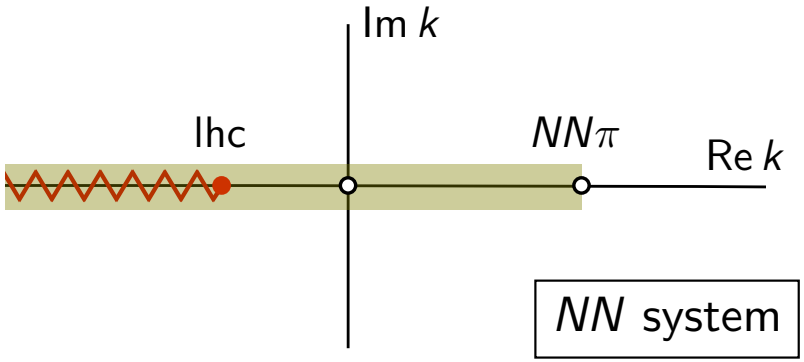

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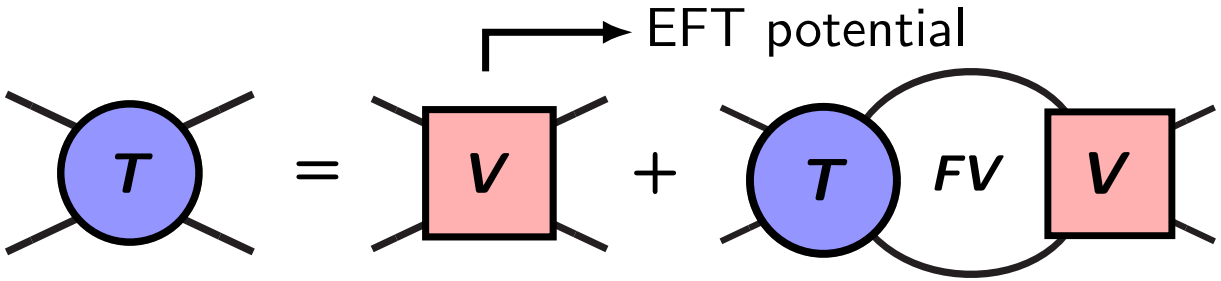
Plane-wave QC uses knowledge from **effective field theory**

Plane-wave quantization condition

Plane-wave QC for *NN*: Lippmann-Schwinger in finite volume in plane-wave basis



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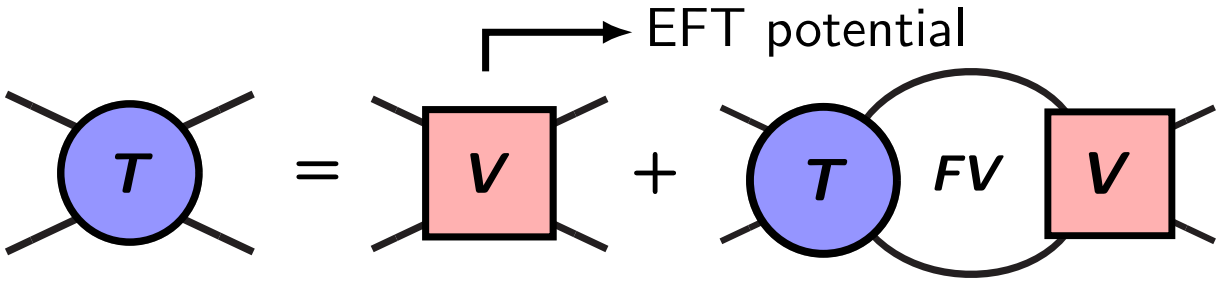


Pros:

- Avoids partial-wave projection by working on the **plane-wave basis**.
- Includes **knowledge from EFT**: lhc and less parameters
- **Numerically simple**: FV energies are solutions to eigenvalue problem

Cons:

- Need to regularize potential!

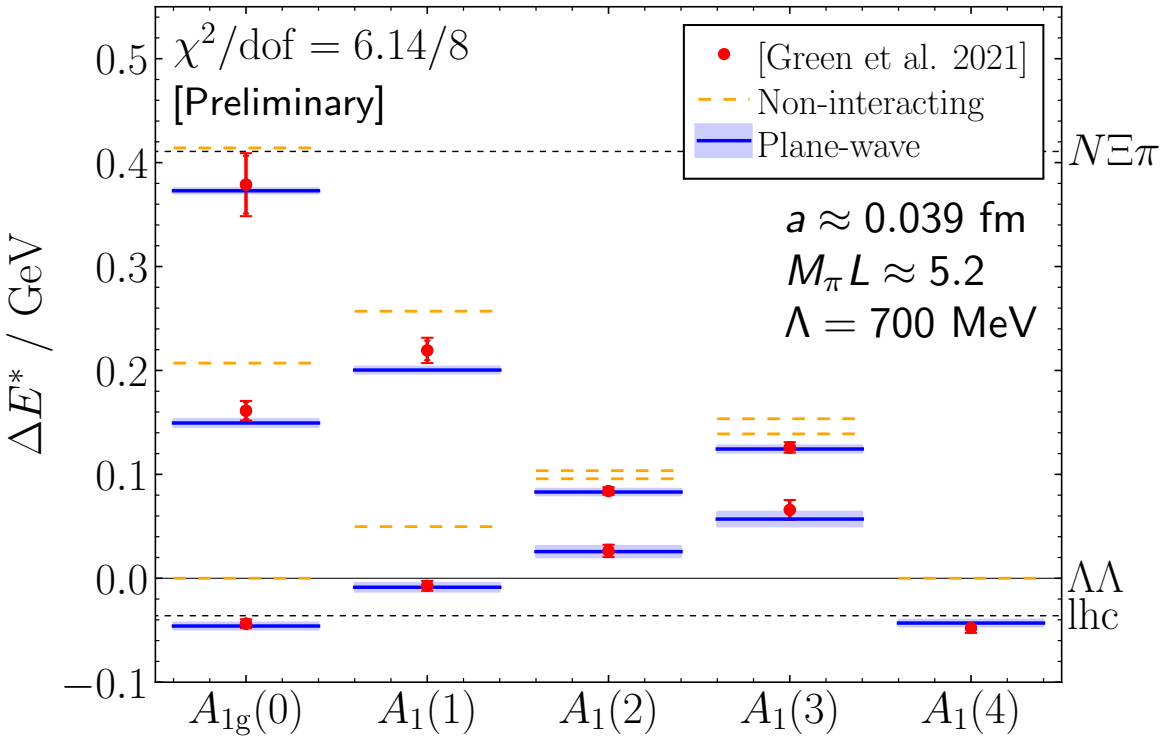
e.g. $$V(\boldsymbol{p}', \boldsymbol{p}) \longrightarrow V(\boldsymbol{p}', \boldsymbol{p}) \times \exp\left[-\frac{\boldsymbol{p}^2 + \boldsymbol{p}'^2}{\Lambda^2}\right]$$

Λ: Cutoff

Application: H dybarion

Application: NN data at $M_\pi \sim 420$ MeV and various a [Green et al. 2021]
(In collaboration with J. Green)

LO potential: $$V(\boldsymbol{p}', \boldsymbol{p}) = \frac{C_{\text{OPE}}}{4M_N^2} \frac{\boldsymbol{q}^2}{\boldsymbol{q}^2 + M_\pi^2} + C_{\text{cont}}$$



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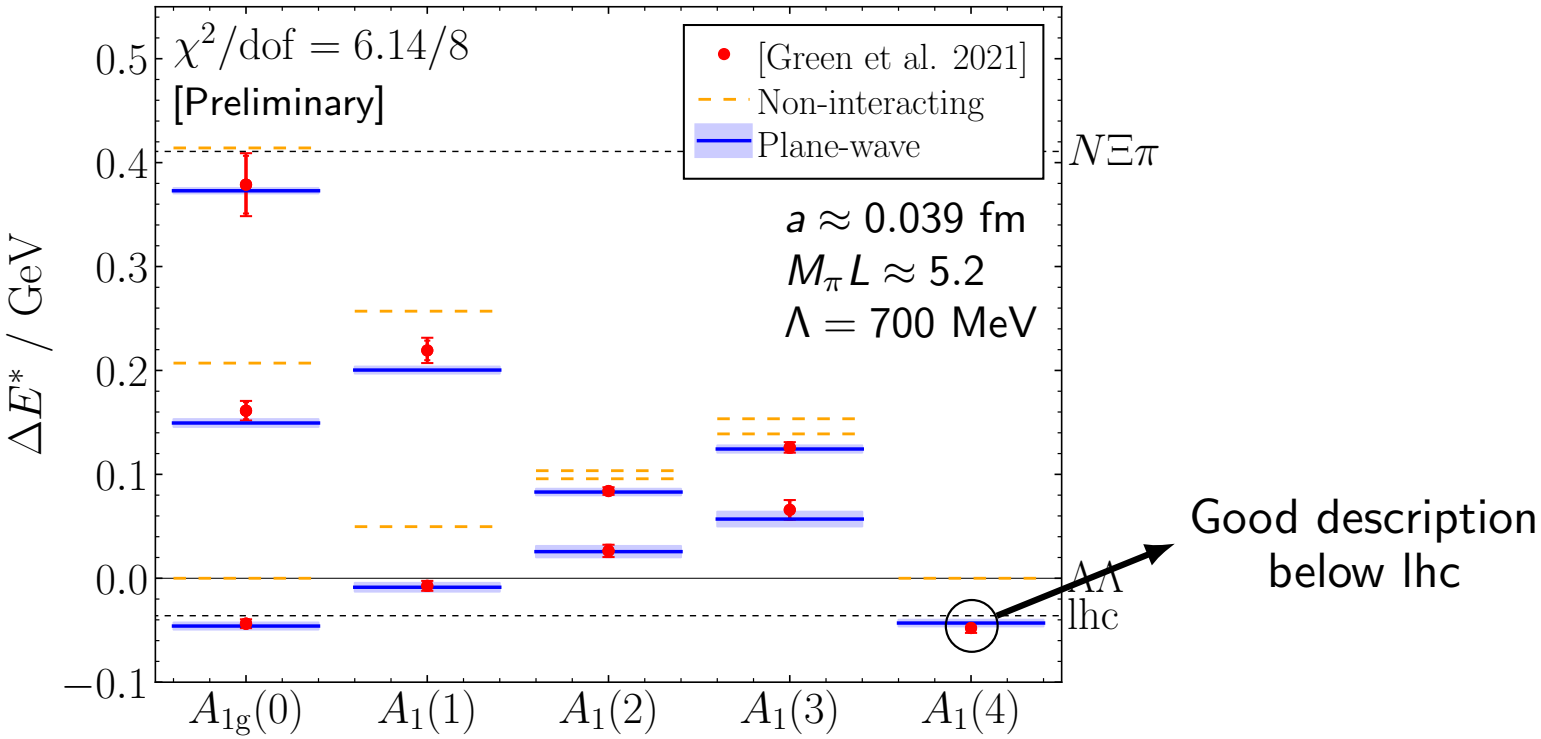

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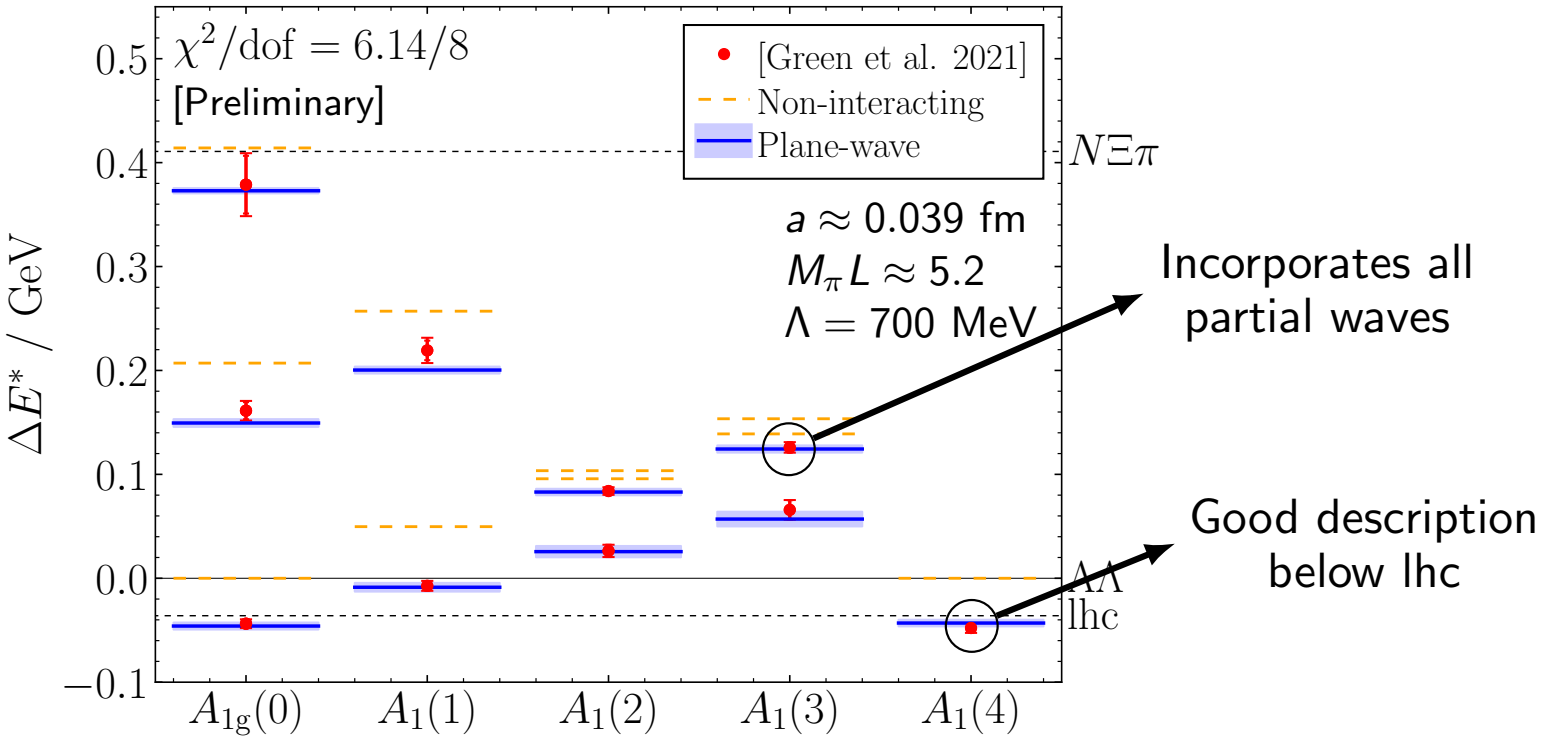

Application: H dybarion

Application: NN data at $M_\pi \sim 420$ MeV and various a [Green et al. 2021]
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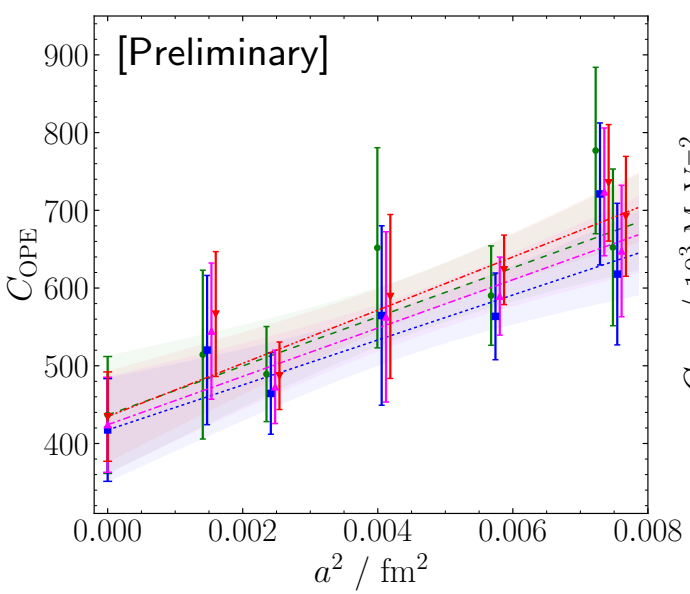
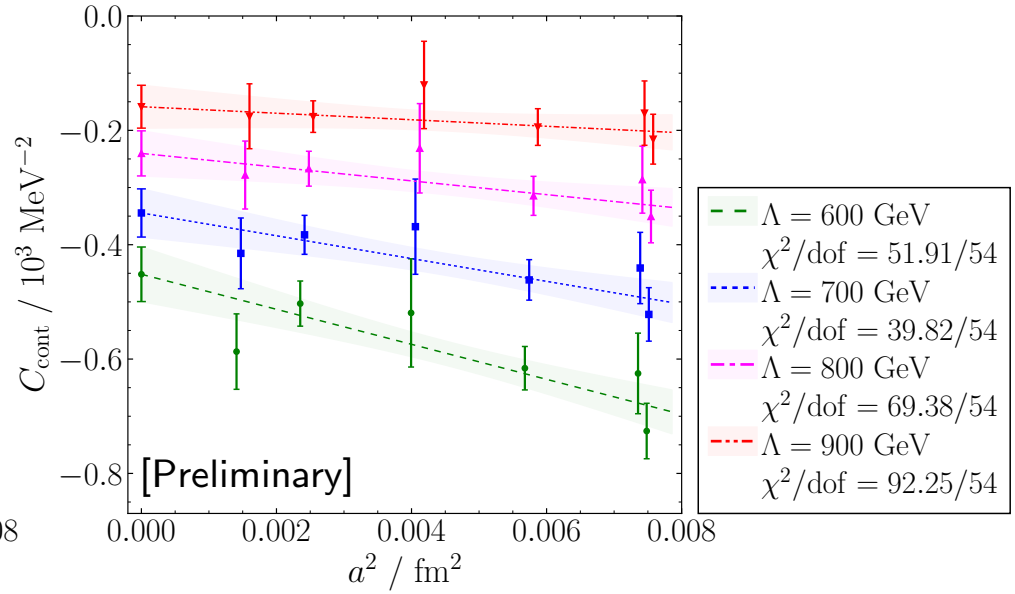
LO potential: $$V(\boldsymbol{p}', \boldsymbol{p}) = \frac{C_{\text{OPE}}^{(0)} + a^2 C_{\text{OPE}}^{(1)}}{4M_N^2} \frac{\boldsymbol{q}^2}{\boldsymbol{q}^2 + M_\pi^2} + C_{\text{cont}}^{(0)} + a^2 C_{\text{cont}}^{(1)}$$





- Physical observables are independent of Λ, $C_{\text{OPE}} \propto g_{NN\pi}^2$.
- Contact terms reabsorb UV effects

Overview

Some physics conclusions

EFTs complement lattice studies of multiparticle interactions

- We determined the $\pi^+\pi^+\pi^+$ K-matrix at NLO in ChPT, reconciling lattice and ChPT predictions
- We extended our computations to the remaining isospin channels, finding large NLO corrections in general
- EFTs can also be used to overcome limitation of the QC, as is the case of the **plane wave quantization condition**

Who is Steve Sharpe to me?

Who is Steve Sharpe to me?

Steve is a **role model for future researchers**

- His **passion and devotion** motivates others to make their best effort.
- He is **always on the front line of research**, eager to tackle new problems.
- He is open to listen, discuss and reconsider, because **even giants bleed**.

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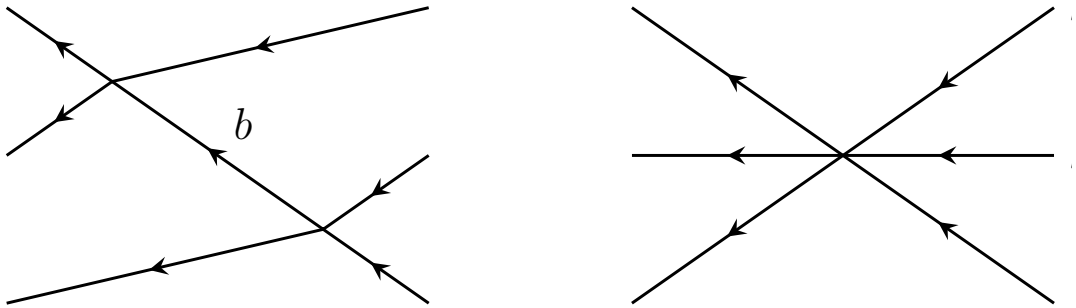
Congratulations Steve for an outstanding career!

Isospin-three K-matrix at LO in ChPT

ChPT allows the study of **three-pion interactions**. For $I = 3$

$$\mathcal{M}_3^{\text{LO}} = \mathcal{M}_3^{\text{LO,OPE}} + \mathcal{M}_3^{\text{LO,non-OPE}}$$



Divergent if $b^2 = M_\pi^2$

Relation between $\mathcal{M}_3$ and $\mathcal{K}_{\text{df},3}$ becomes **algebraic** in ChPT

$$\mathcal{K}_{\text{df},3}^{\text{LO}} = \mathcal{S}\left\{\mathcal{M}_3^{\text{LO,OPE}} - \mathcal{D}^{\text{LO,OPE}}\right\} + \mathcal{M}_3^{\text{LO,non-OPE}}$$

$\downarrow$

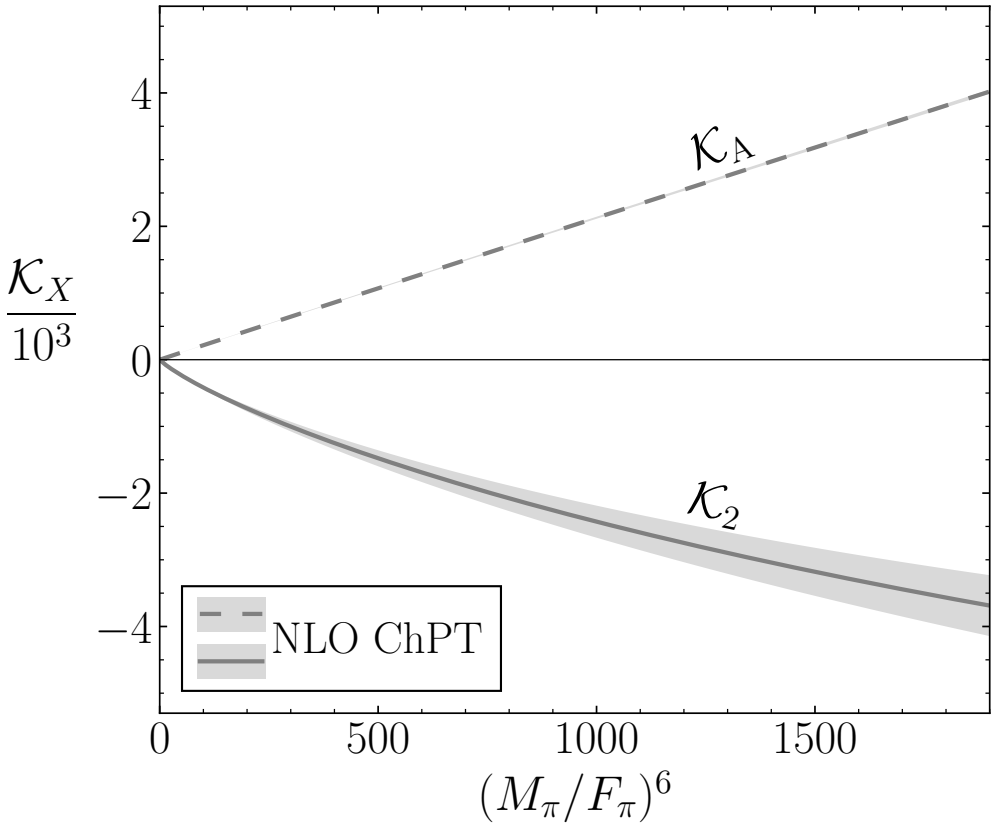
Removes divergencies from OPE

Isospin-three K-matrix at NLO in ChPT

We determined $\mathcal{K}_{\mathrm{df},3}$ about threshold up to **quadratic order**

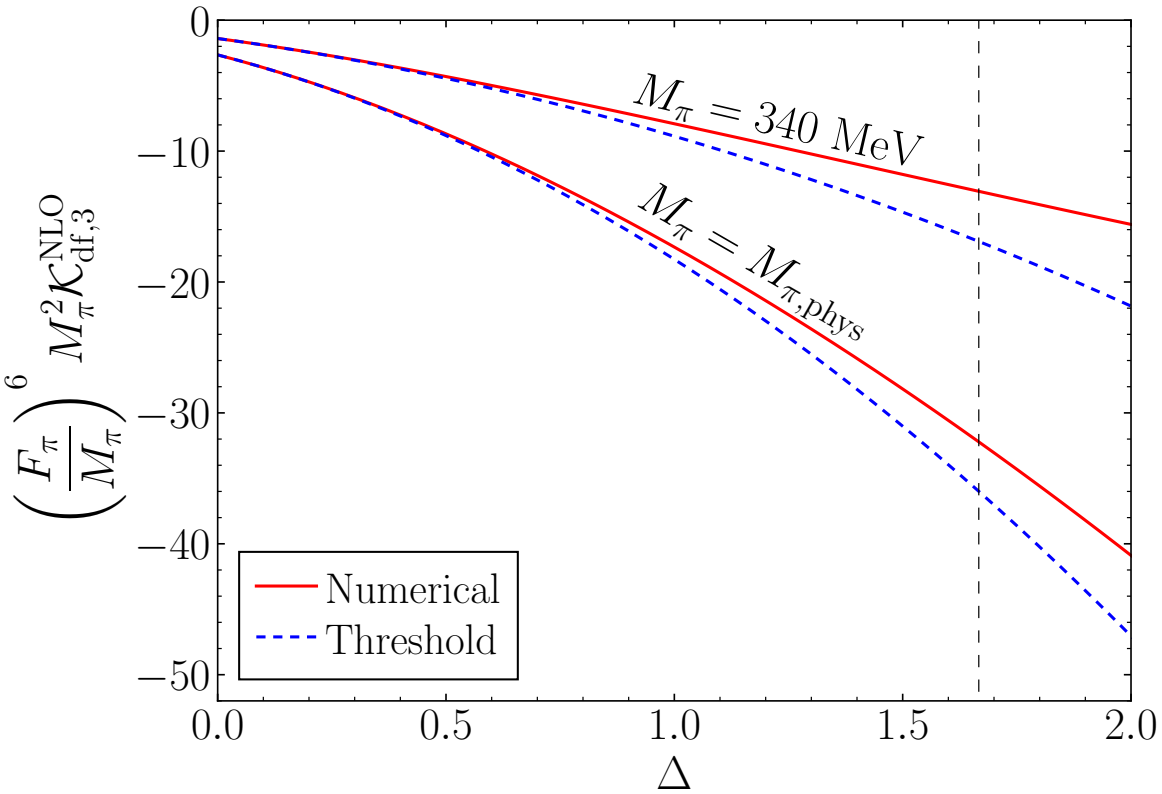
$$M_\pi^2 \mathcal{K}_{\mathrm{df},3} = \mathcal{K}_0 + \mathcal{K}_1 \Delta + \mathcal{K}_2 \Delta^2 + \mathcal{K}_A \Delta_A + \mathcal{K}_B \Delta_B + \mathcal{O}(\Delta^3)$$

Angular dependence



Isospin-three K-matrix at NLO in ChPT

We study the **convergence of the threshold expansion**



Isospin-three K-matrix at NLO in ChPT

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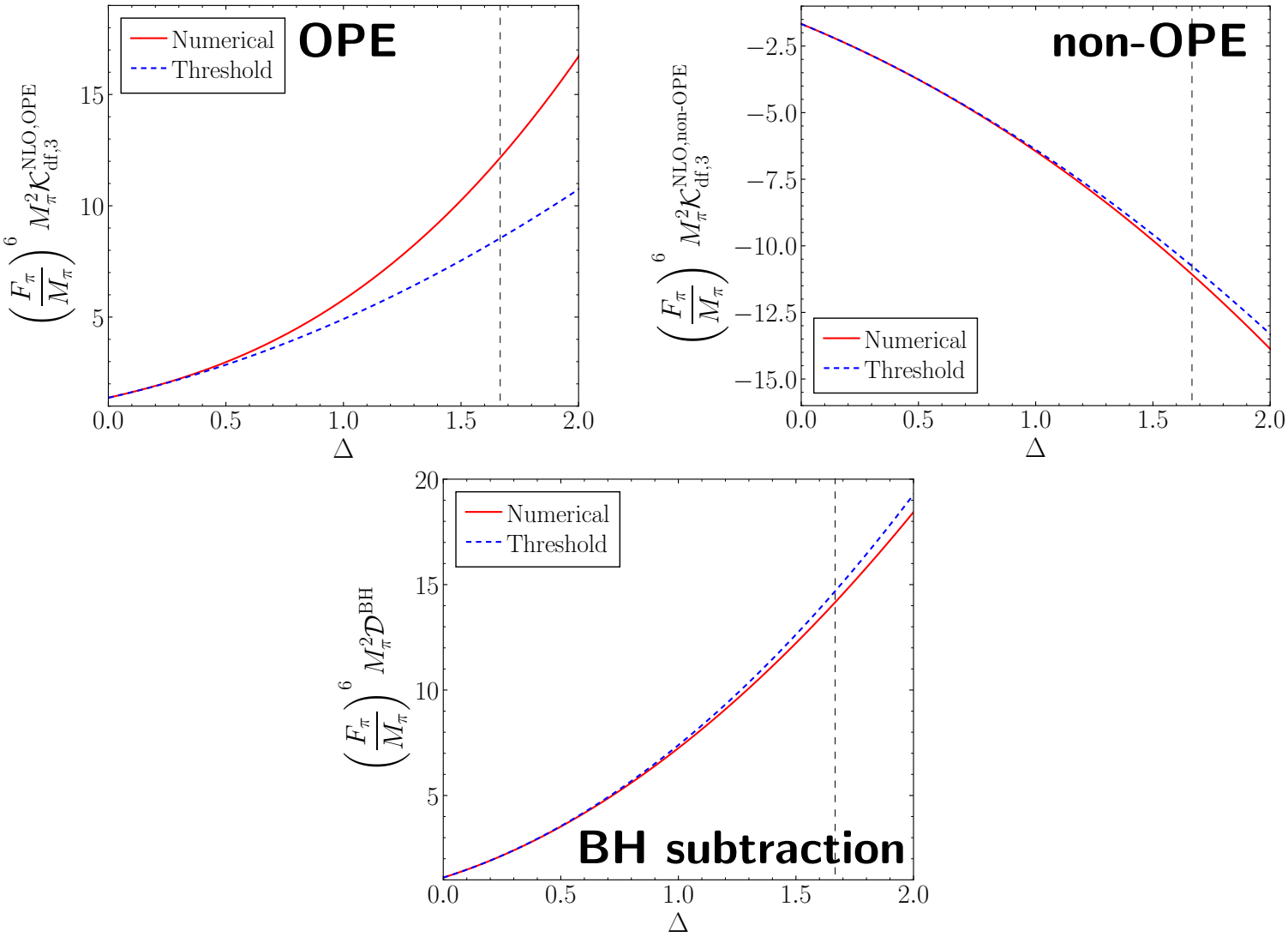
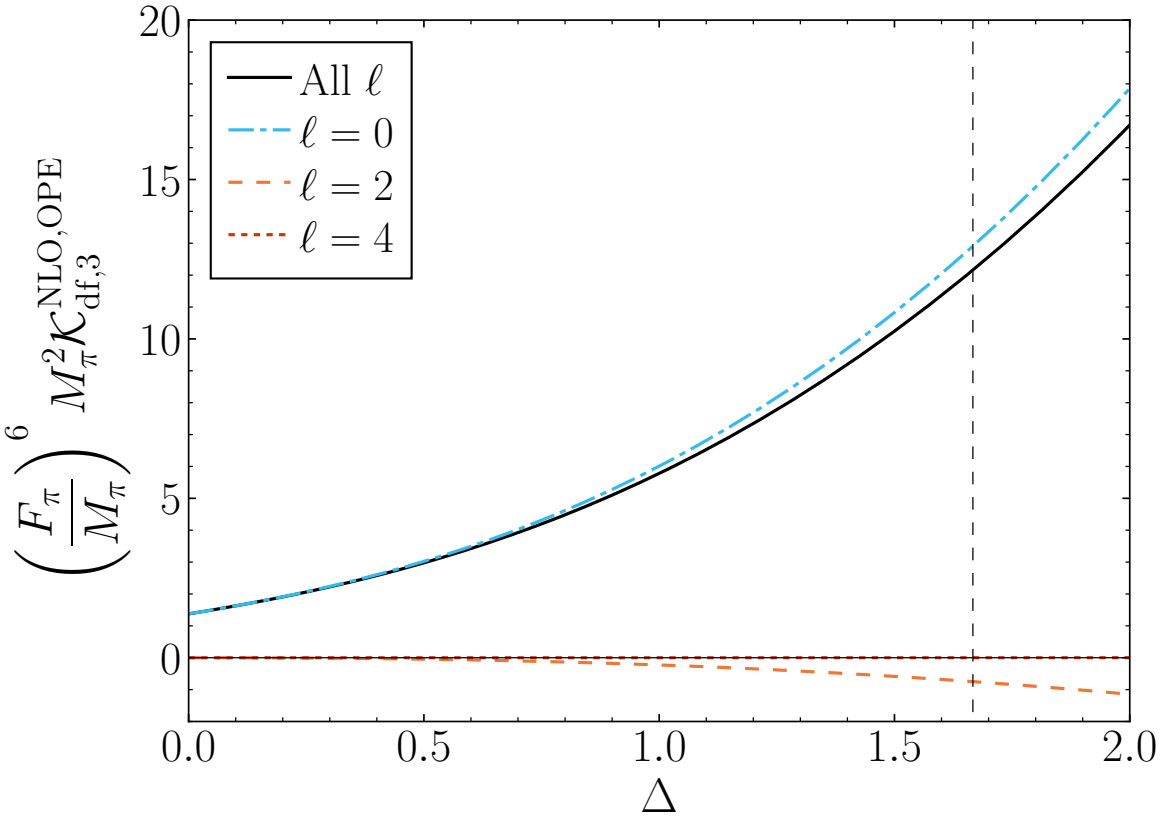


Isospin-three K-matrix at NLO in ChPT

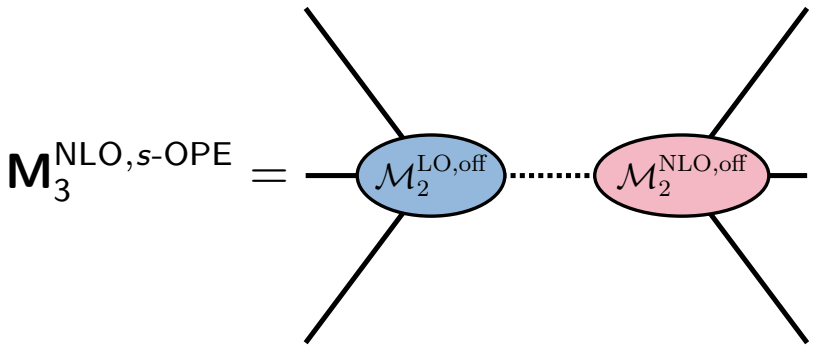
We study the **effect of two-particle partial waves** on OPE



Extension to general isospin

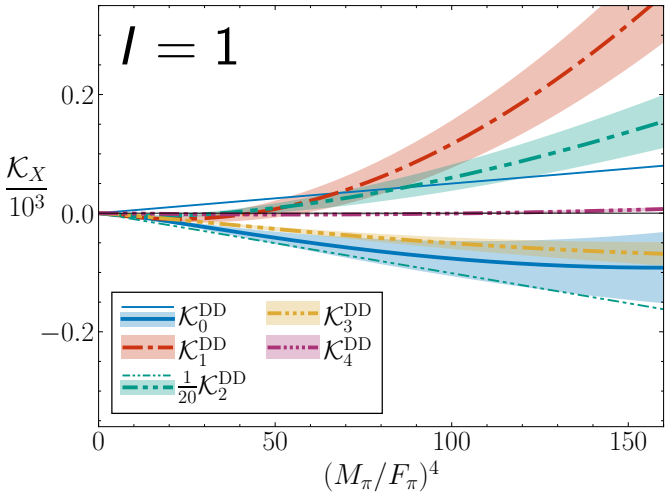
The computation for all isospin channels has **new complications**:

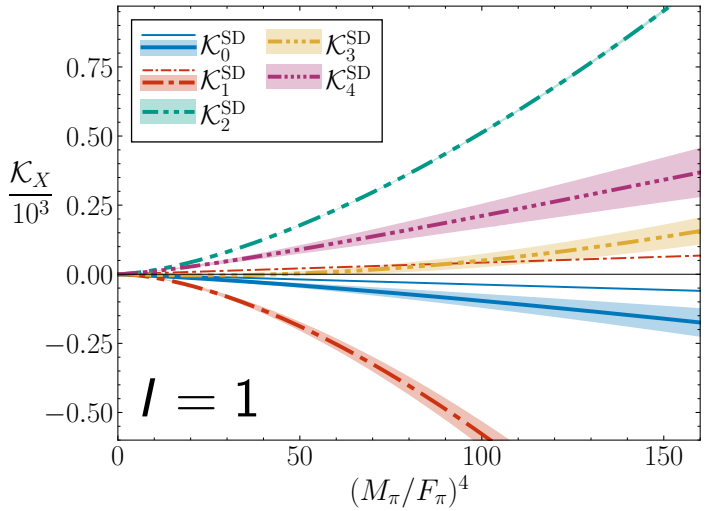
- Quantities are matrices in **flavor space** [Hansen et al, 2020]
- **New kinematic structures** (e.g., $I = 2$ transforms as a doublet,
  $I = 1$ as doublet + singlet,
  $I = 0$ as antisymmetric)
- Inclusion of ***p* and *f* waves**
- ***s*-channel OPE diagram** in $I = 1$

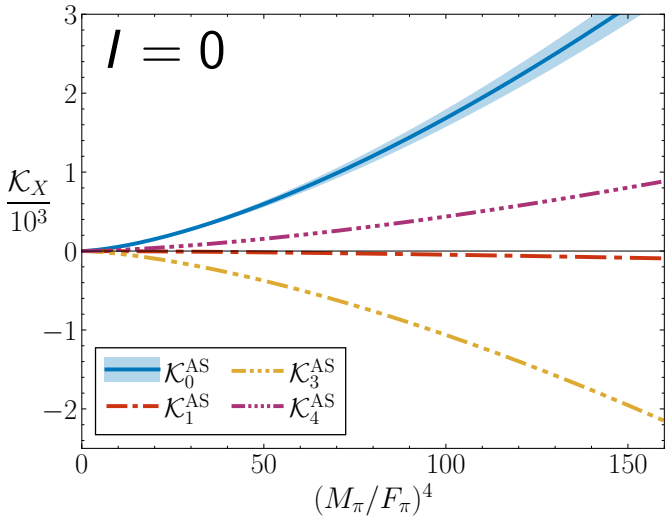


Three-pion K-matrix at NLO in ChPT

We computed $\mathbf{K}_{\mathrm{df},3}$ at LO and NLO for all isospin channels

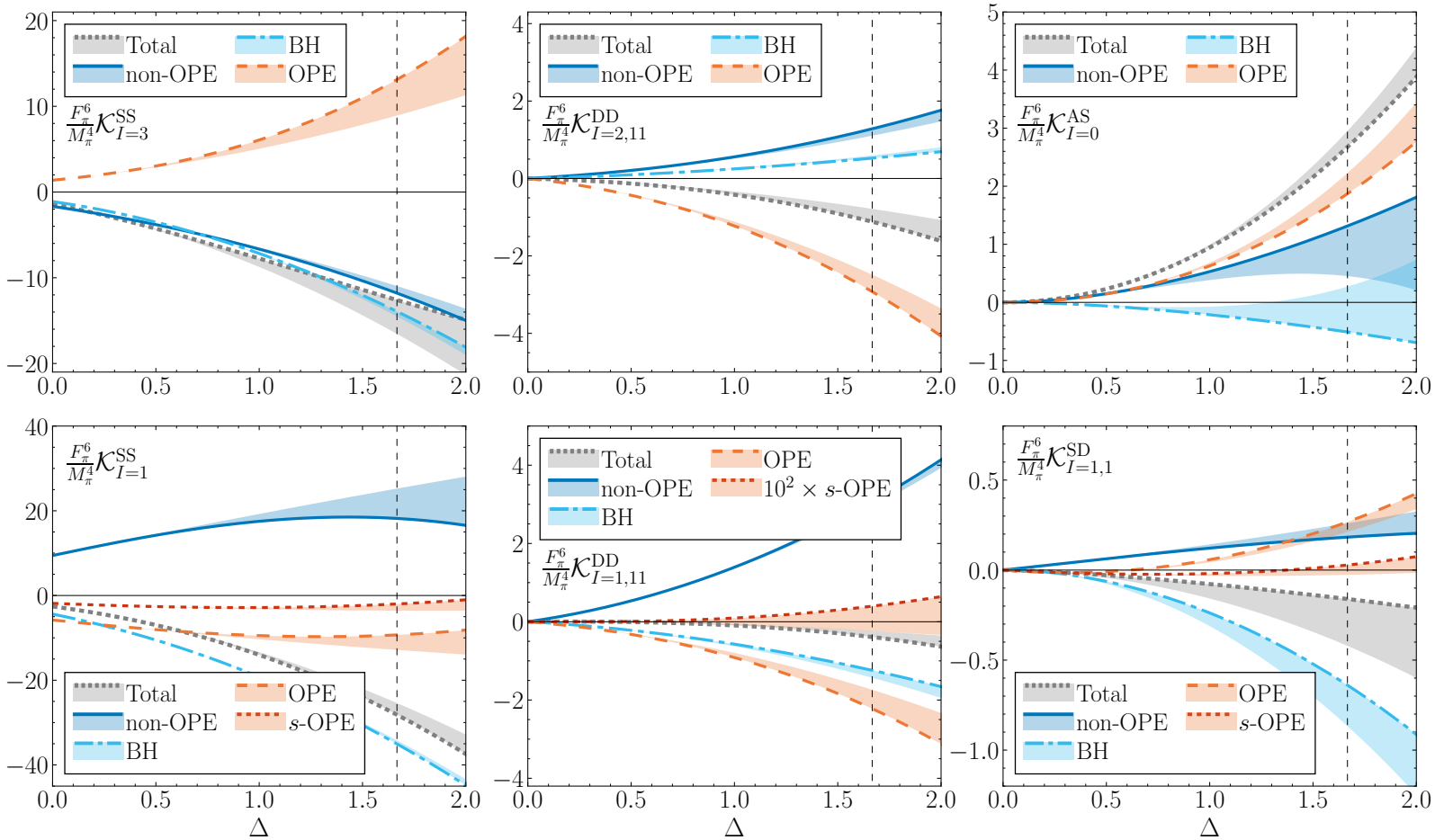






Three-pion K-matrix at NLO in ChPT

We study the **convergence of threshold expansion** (NLO only)



Three-pion K-matrix at NLO in ChPT

We study the **effect of two-particle partial waves** on OPE (NLO only)