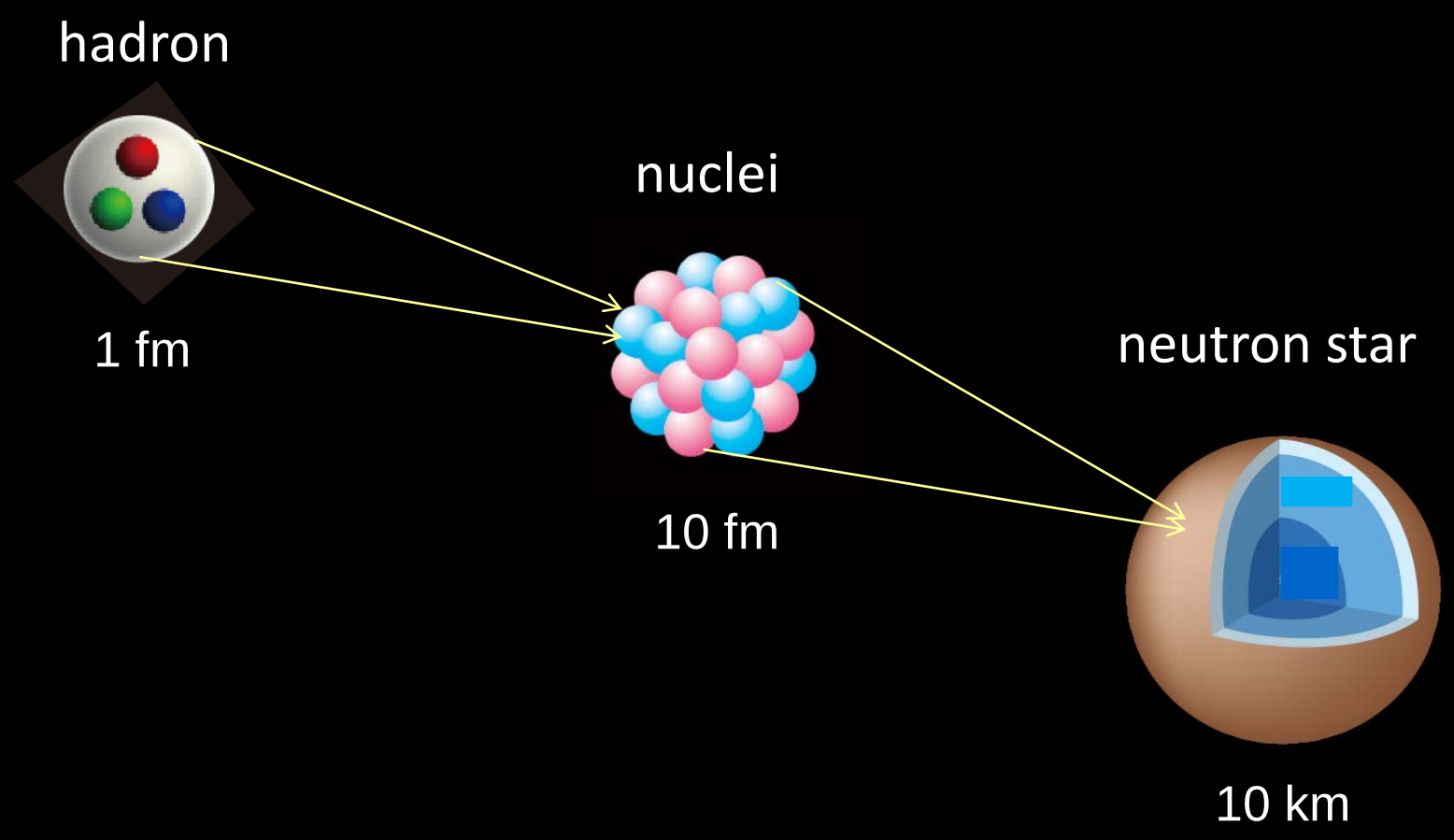
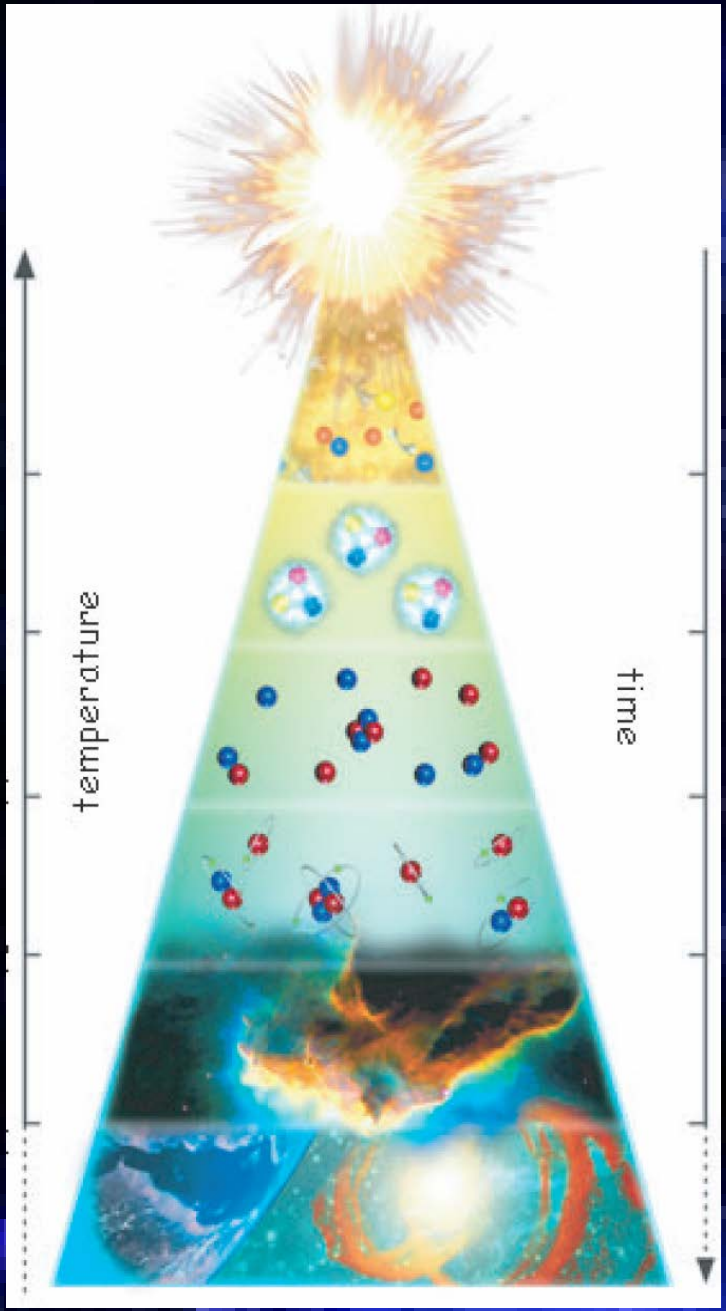
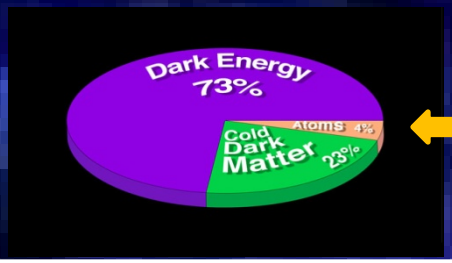
# *Lattice QCD in Nuclear Physics*




Tetsuo Hatsuda (RIKEN)
Berkeley School, May 16, 2012




# Major Challenges in Nuclear Physics




origin & evolution
of baryonic matter

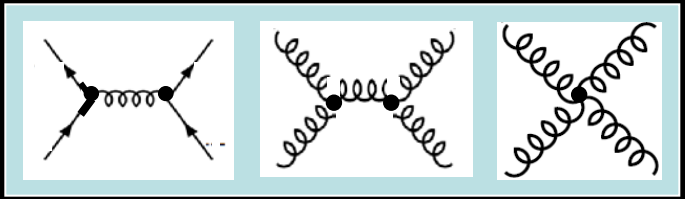
- **hot matter** ⇔ RHIC, LHC
  quark-gluon plasma in early universe
- **origin of elements** ⇔ RIBF
  nucleosynthesis
  in big-bang, stars, supernovae, …
- **dense matter** ⇔ J-PARC, FAIR
  neutron stars, exotic matter, …

LATTICE QCD inputs are crucial

# Contents

[1] Introduction
[2] Dense QCD
[3] Nuclear Force from QCD
[4] Dense QCD and Ultra-cold Atoms/Molecules
[5] Summary

# Quantum Chromo Dynamics



$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \bar{q}\gamma^\mu (i\partial_\mu - g t^a A^a_\mu) q - m\bar{q}q$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu$$

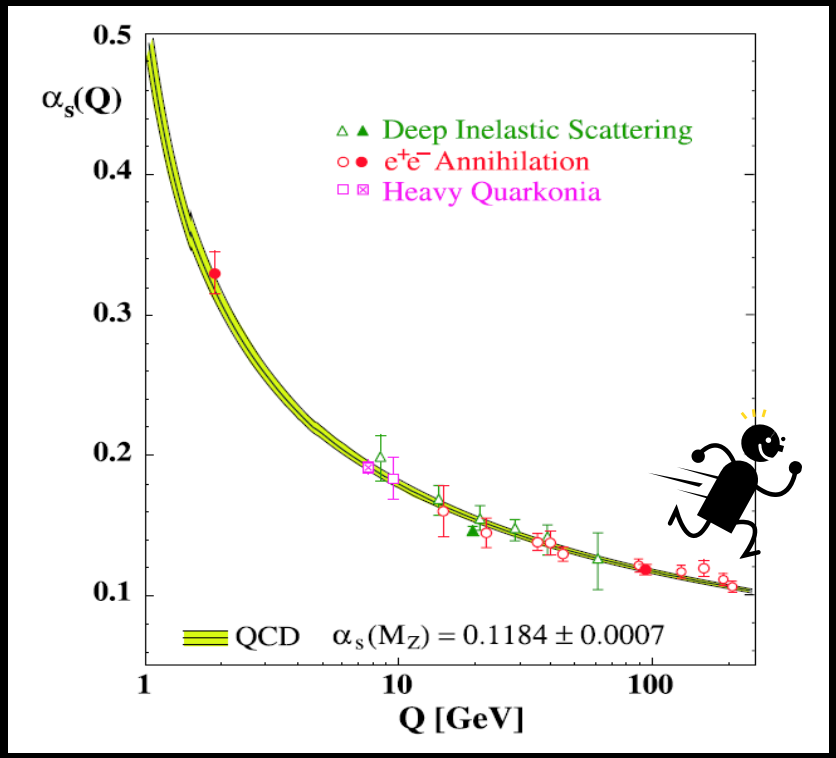
Nambu (1966)

## Running masses: $m_q(Q)$

| quark masses (from lattice QCD) | [MeV] (MS-bar @ 2GeV) |
|---|---|
| $m_u$ | 2.19(15) |
| $m_d$ | 4.67(20) |
| $m_s$ | 94(3) |

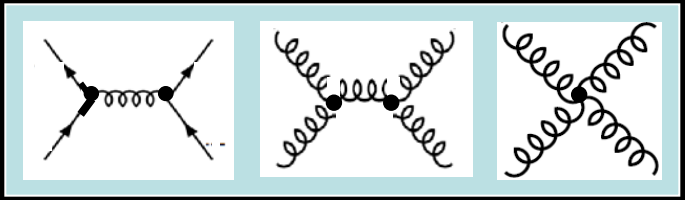
FLAG working group, arXiv:1011.4408 [hep-lat]

## Running coupling: $\alpha_s(Q)=g^2/4\pi$




Bethke, Eur. Phys. J C(2009)64:689 →

# Quantum Chromo Dynamics



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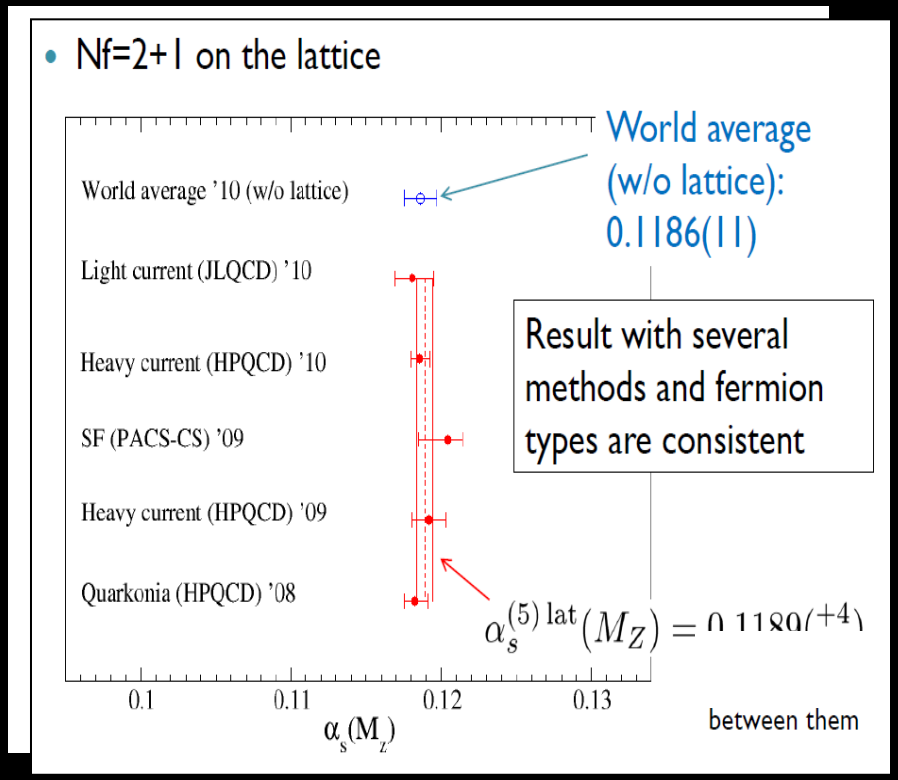
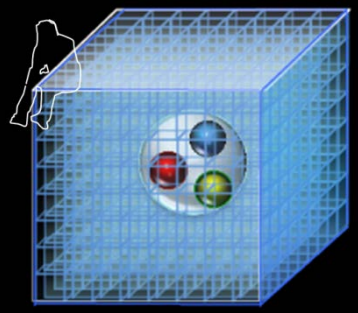


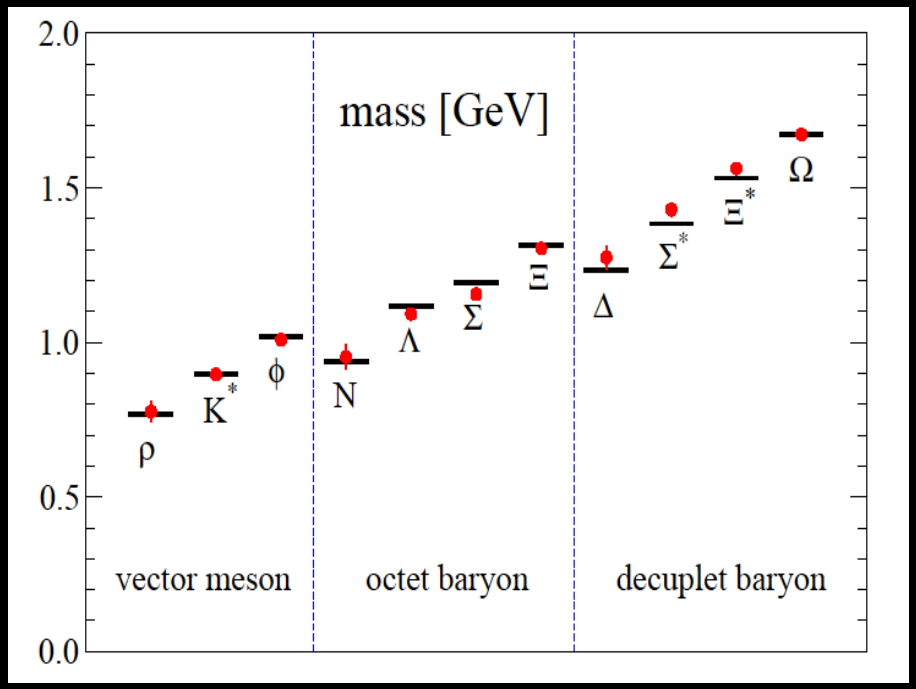

Bethke, Eur. Phys. J C(2009)64:689 →

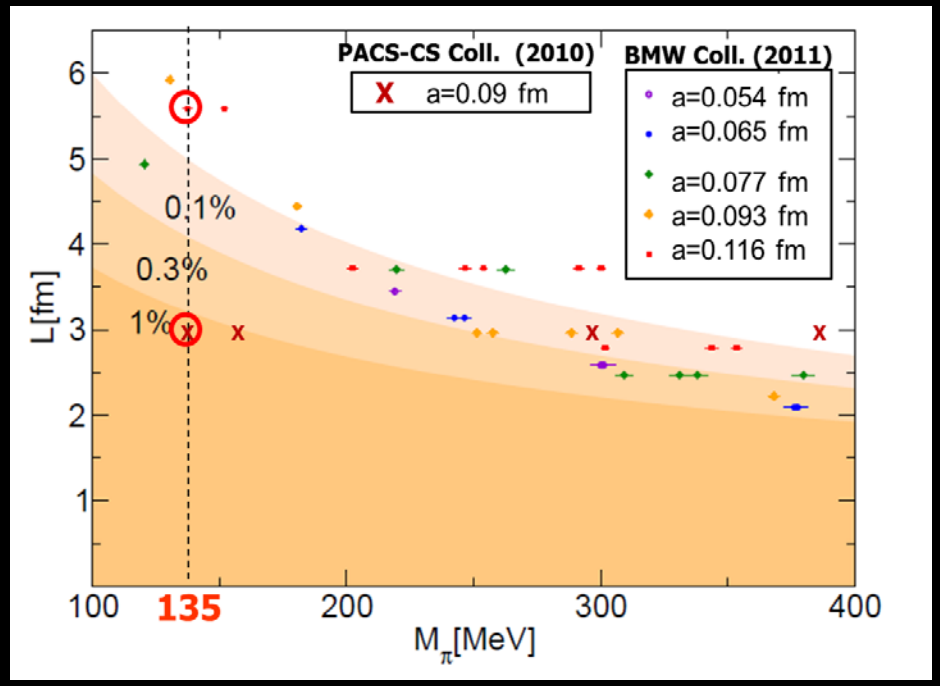
Shintani @Lattice2011 →

# Hadron masses @ 2009
# $m_\pi$ > 156 MeV

# Hadron masses @ 2011
# $m_\pi$ =135 MeV





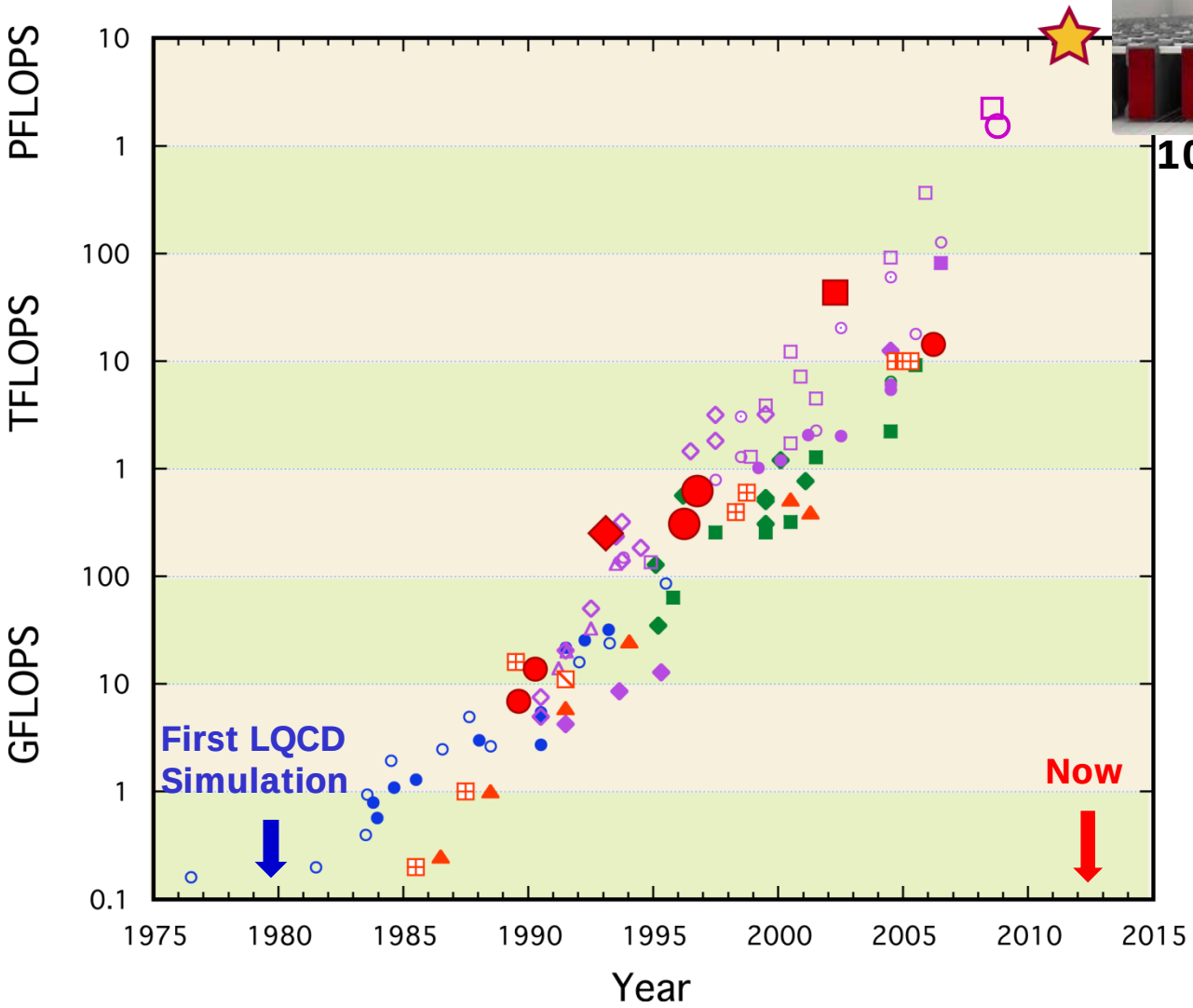

PACS-CS Collaboration,
Phys.Rev.D79 (2009)034503

PACS-CS Coll.: Phys. Rev.D81 (2010) 074503
BMW Coll.: Phys. Lett. B701 (2011) 265

# Toward large-scale Physical-point simulations

http://www.aics.riken.jp/en/

**10PFlops K computer (RIKEN)**




original plot by A. Ukawa

# Advanced Institute for Computational Science (AICS), RI KEN

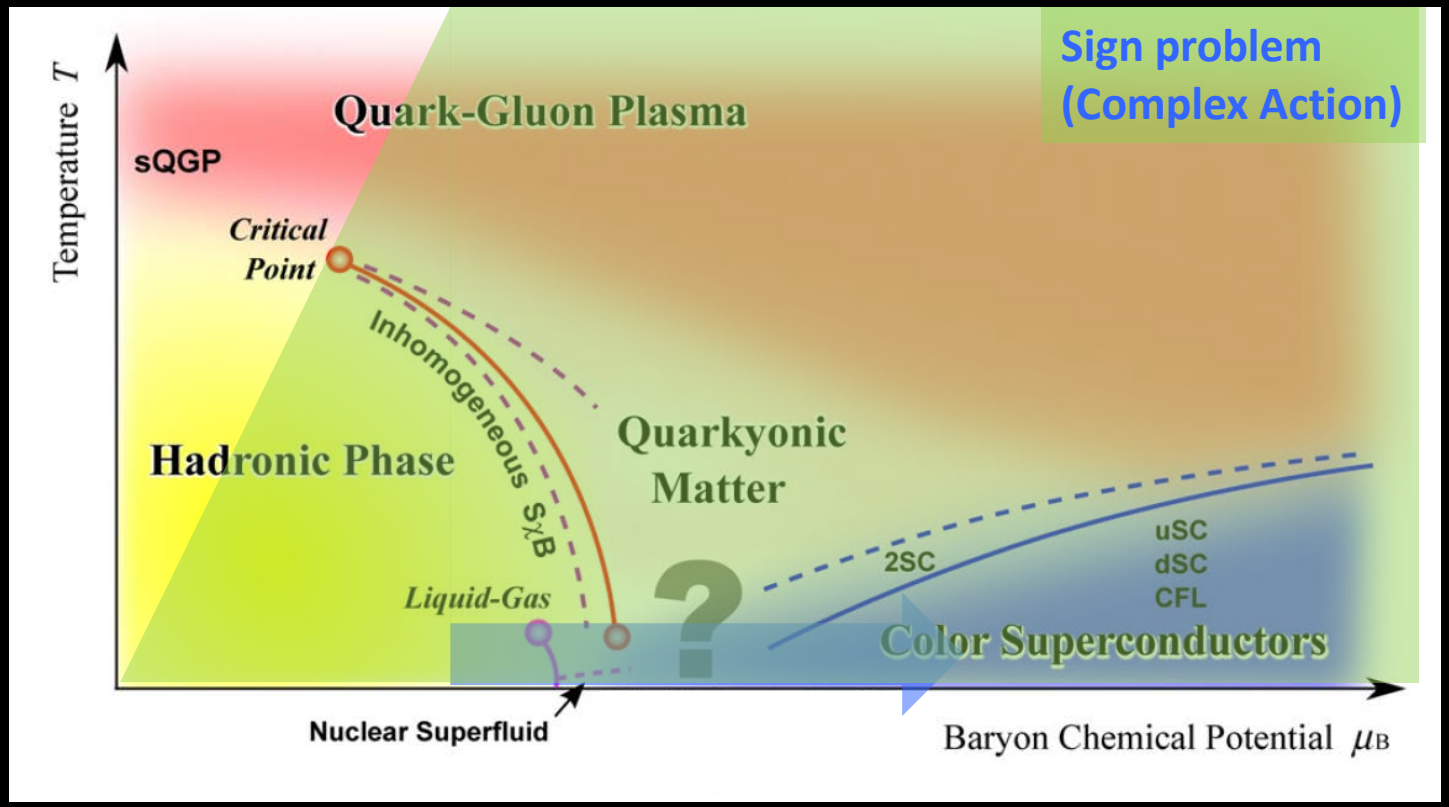
10 Pflops supercomputer KEI "京" (full operation in 2012)

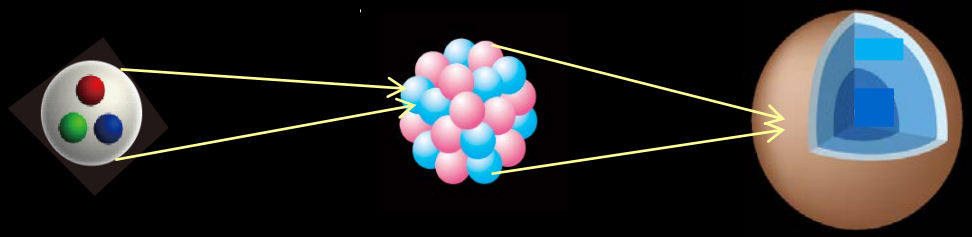
http://www.aics.riken.jp/en/

Five "strategic" programs (FY 2010-2015)

1. Life and Medicine
2. New Materials
3. Environment
4. Engineering
5. Particle, Nuclear and Astrophysics

# Dense QCD



K. Fukushima and T. Hatsuda,
Rep. Prog. Phys. 74 (2011) 014001

# sign problem

**Toy model:**

$$Z = \sum_{\{\phi(\mathrm{x})=\pm 1\}} \mathrm{sgn}(\phi)\ e^{-S(\phi)}$$

$$\left( Z_0 = \sum_{\{\phi(\mathrm{x})=\pm 1\}} e^{-S(\phi)} \right)$$

$$\langle \mathrm{sgn}(\phi) \rangle_0 = \frac{Z}{Z_0} = e^{-(f-f_0)V/T} \ll 1$$

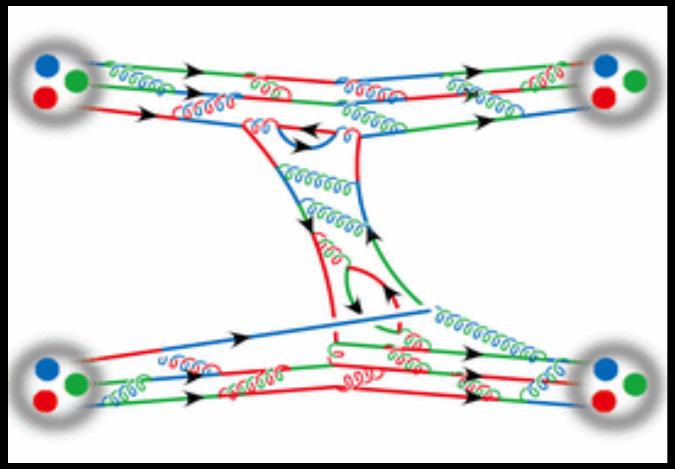
$$\frac{\Delta \mathrm{sgn}}{\langle \mathrm{sgn} \rangle_0} = \frac{\sqrt{\langle \mathrm{sgn}^2 \rangle_0 - \langle \mathrm{sgn} \rangle_0^2}}{\sqrt{N} \langle \mathrm{sgn} \rangle_0} \simeq \frac{e^{(f-f_0)V/T}}{\sqrt{N}} \ll 1 \quad \Rightarrow \quad N \gg \mathrm{e}^{2(f-f_0)V/T}$$

**QCD:**

$$Z = \mathrm{Tr}\left[ e^{-(H-\mu N)/T} \right] = \int [dA]\ \mathrm{Det}[\hat{D} + m + i\mu\gamma_4]\ e^{-S(A)}$$

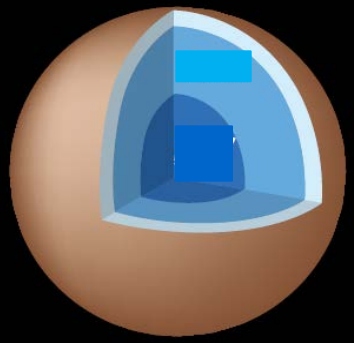
Complex

# Approach from low density



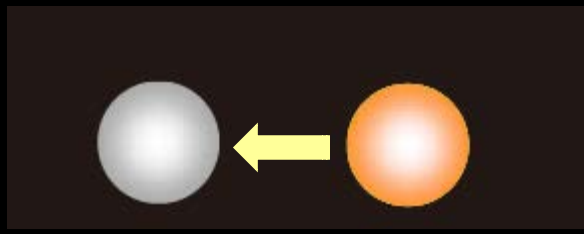
Lattice QCD

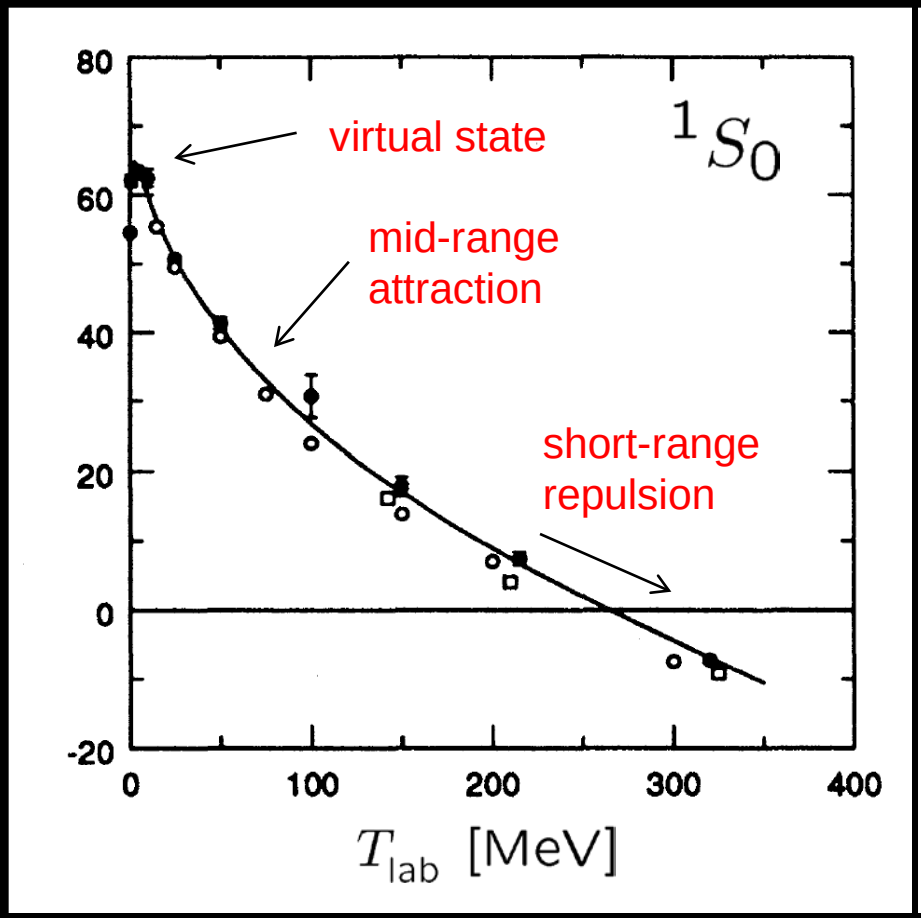
Nuclear many-body methods

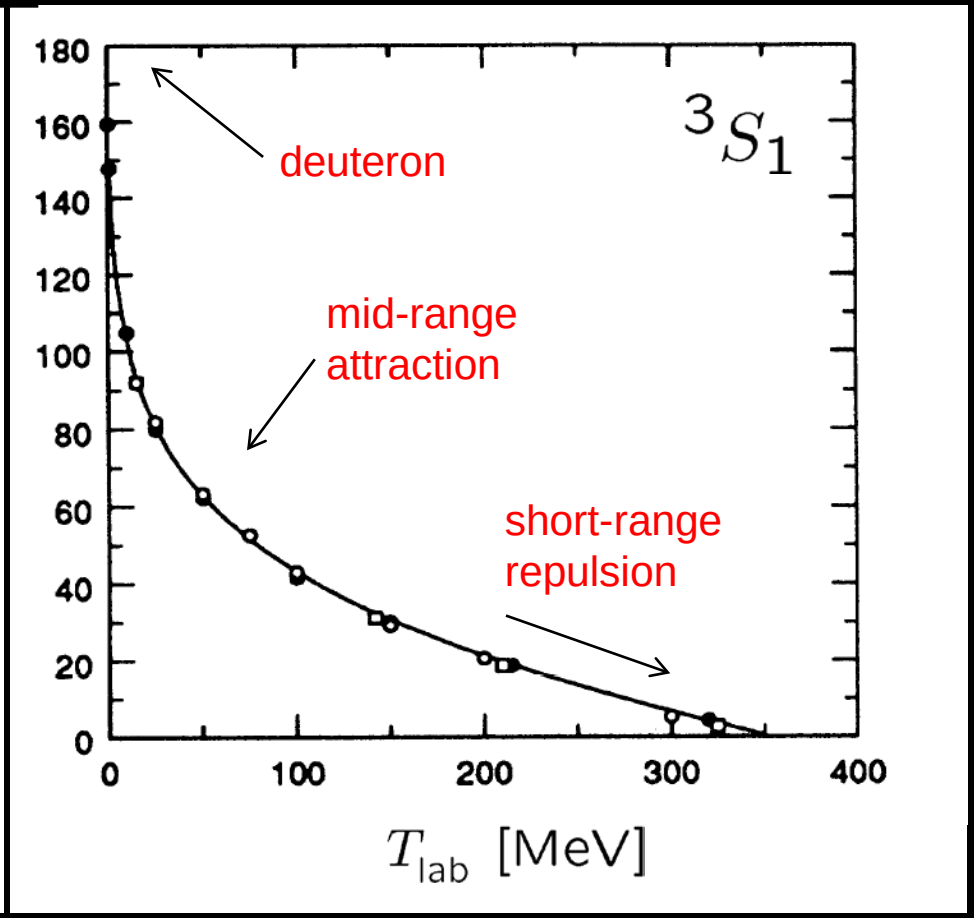


# NN interactions
# critical inputs in nuclear physics

$^{2S+1}L_J$

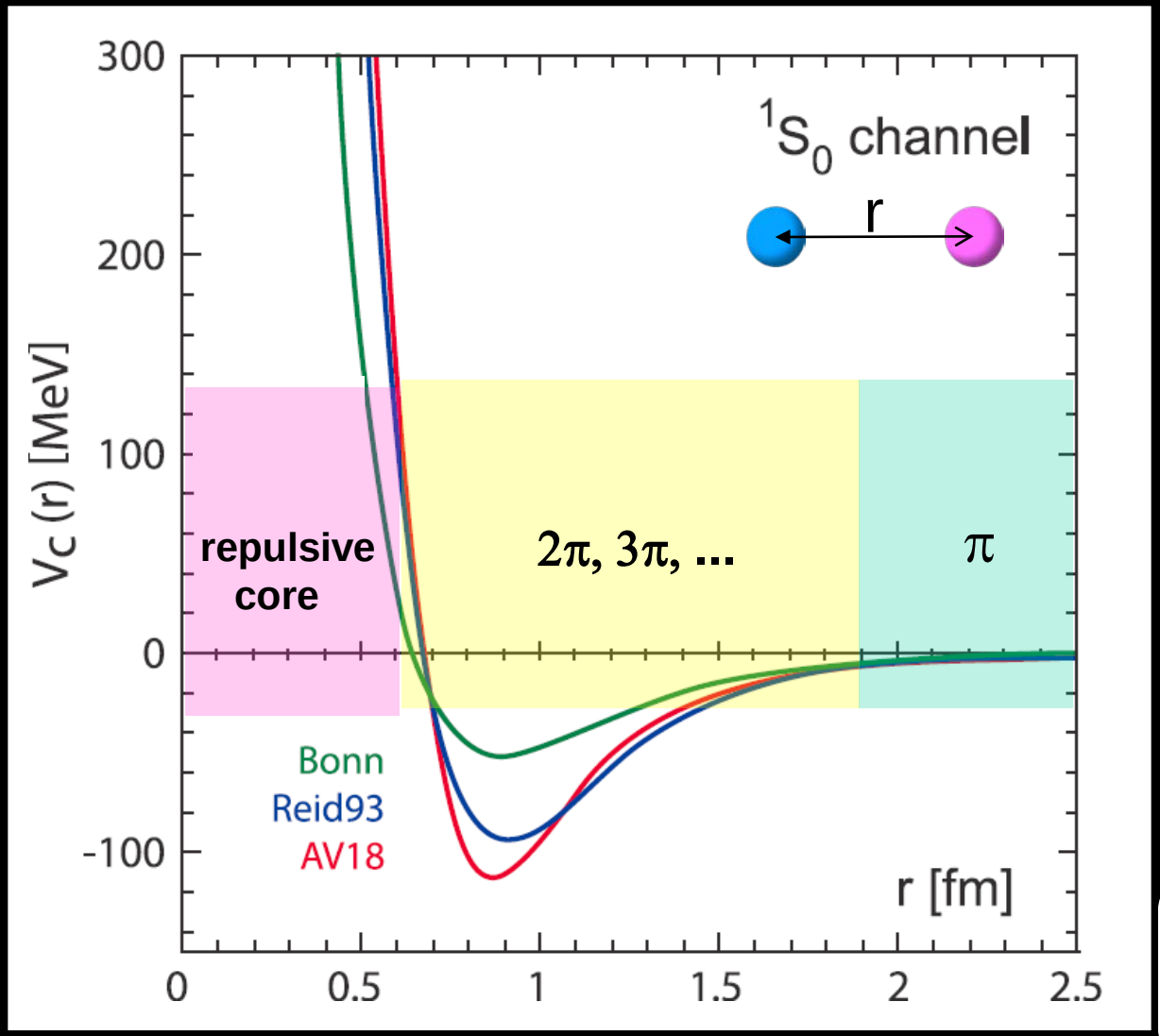





Nijmegen partial-wave analysis,
Stoks et al., Phys.Rev. C48 (1993) 792

# Nuclear force: a brief history




- One-pion exchange
  Yukawa (1935)

- Multi-pion
  Taketani et al. (1951)

- Repulsive core
  Jastrow (1951) Nambu (1957)

- EFT
  Weinberg (1990)

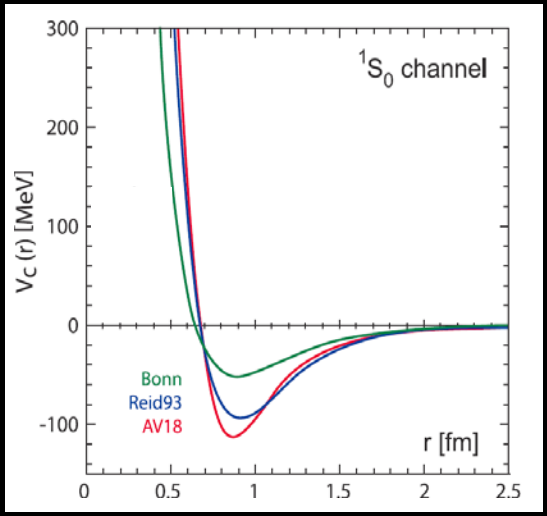
high precision NN force (90's-)
30-40 parameters
5000 phase shift data

# phenomenological nuclear forces

- NN int.: about 4500 np and pp scatt. data

| "high precision" NN interactions | | # of parameters |
|---|---|---|
| CD Bonn | (p space) | 38 |
| AV18 | (r space) | 40 |
| EFT in $N^3LO$ | ($n\pi$+contact) | 24 |

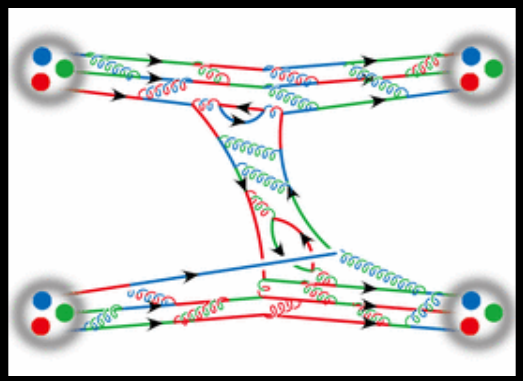
R. Machleidt, arXiv:0704.0807 [nucl-th]




- NNN, YN, YY : data very limited
- YYN, YNN, YYY : none

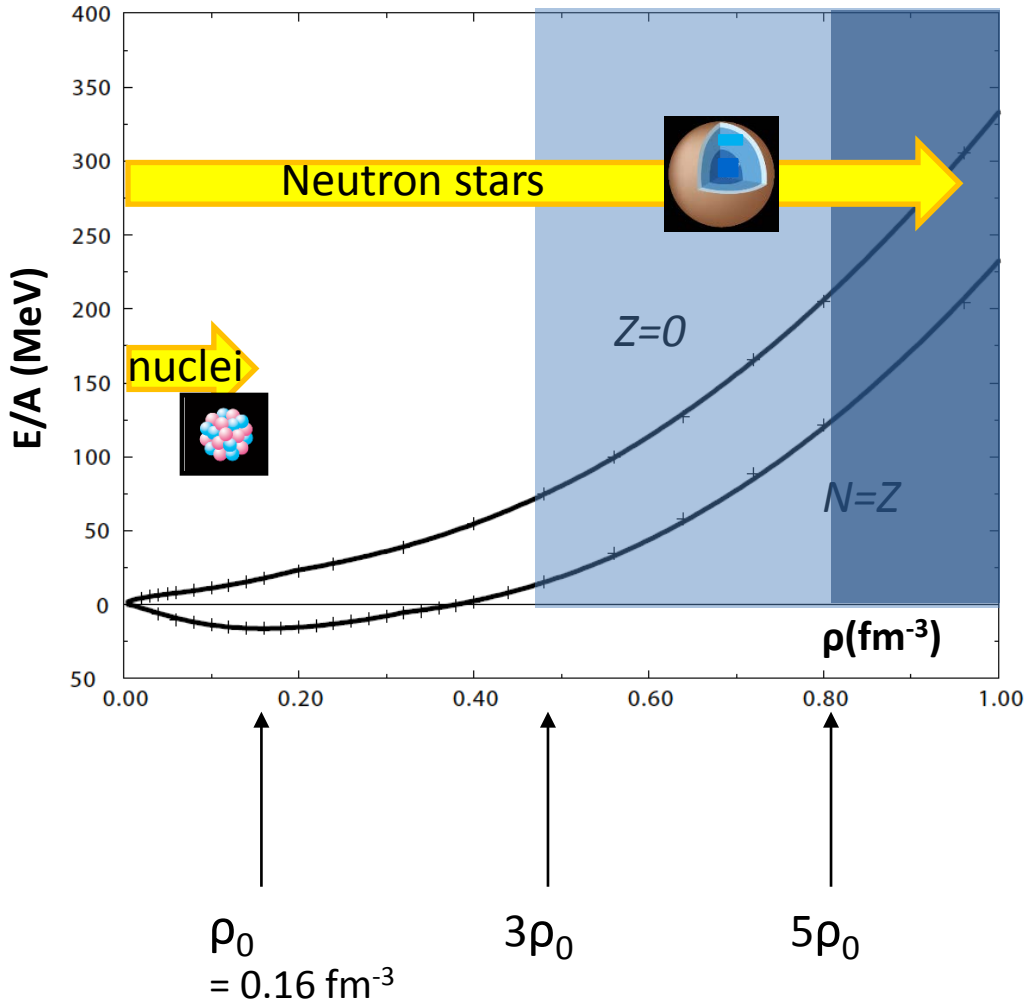
QCD has only four parameters :

$m_u$, $m_d$, $m_s$, $\Lambda_{QCD}$

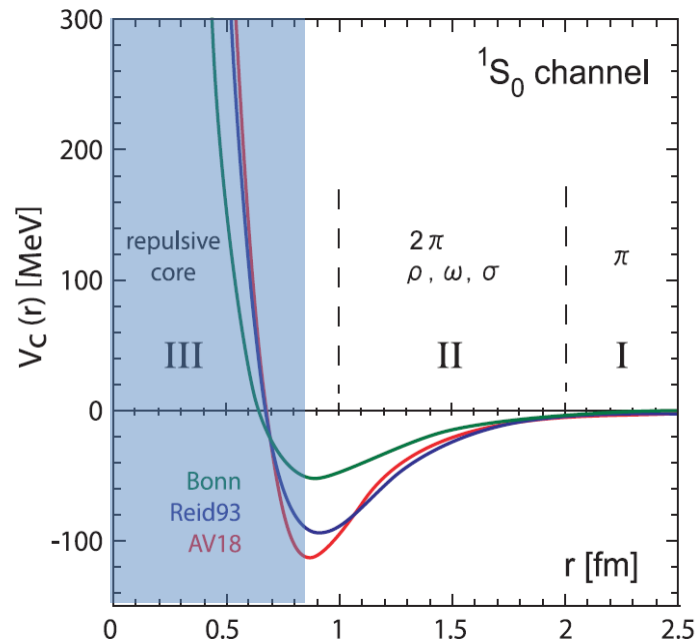


# Nuclear Force and EoS of Dense Matter

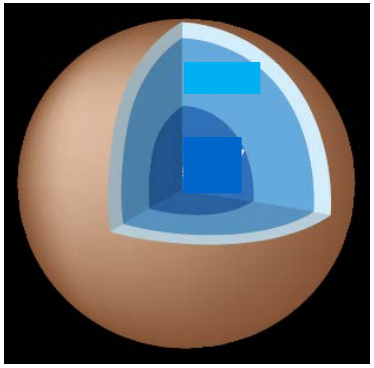
**Akmal, Pandharipande & Ravenhall, PRC58 ('98)**



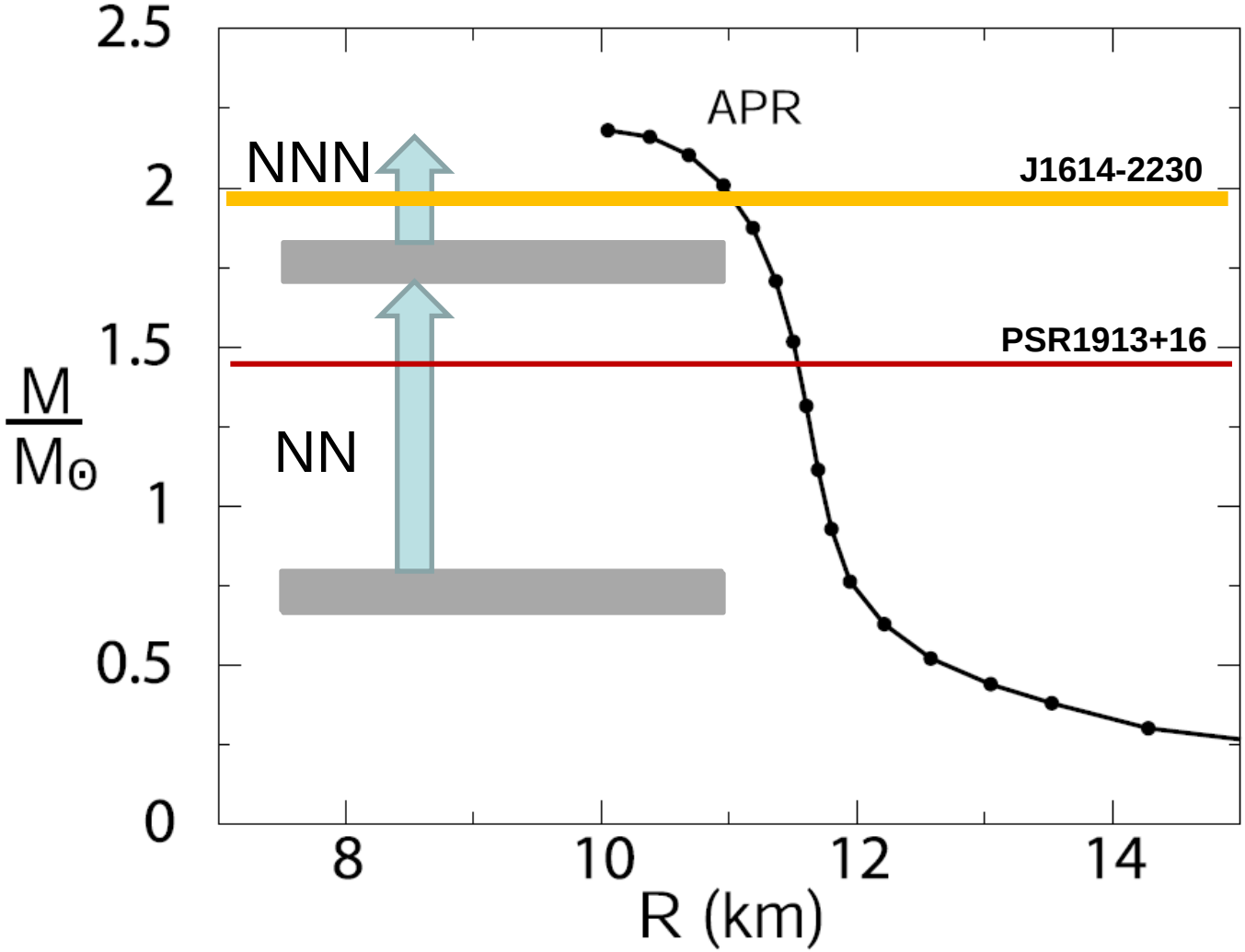

**Phenomenological NN forces**



# Mass-Radium Relation of Neutron Star

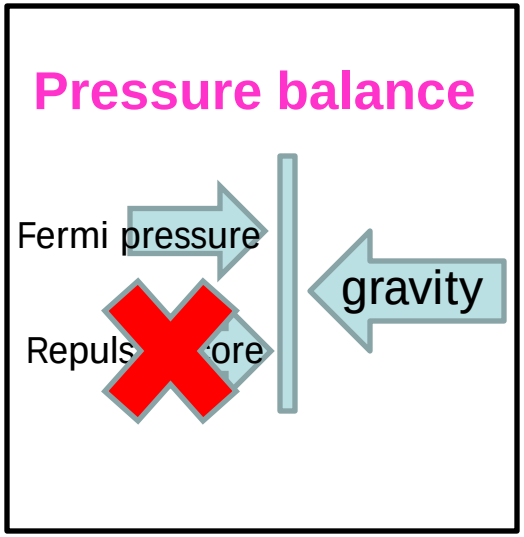


($\rho_{max} \sim 6\rho_0$)

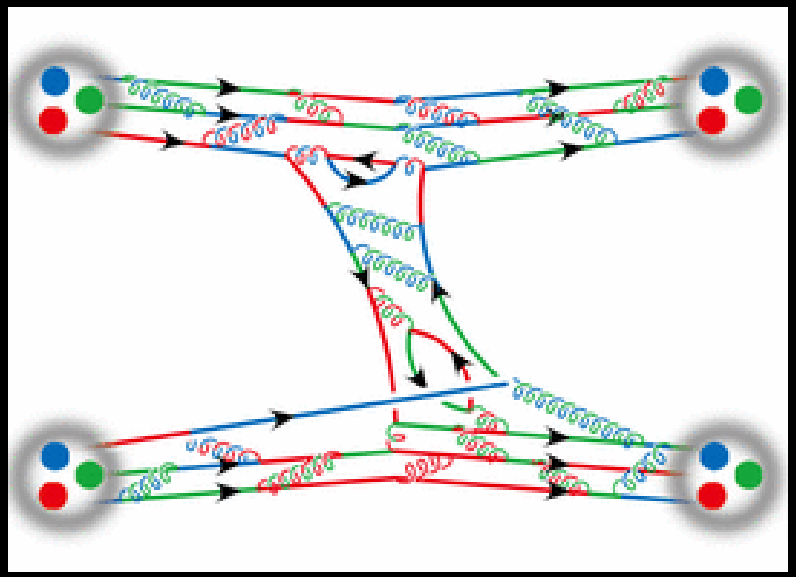



NS – WD binary

NS - NS binary

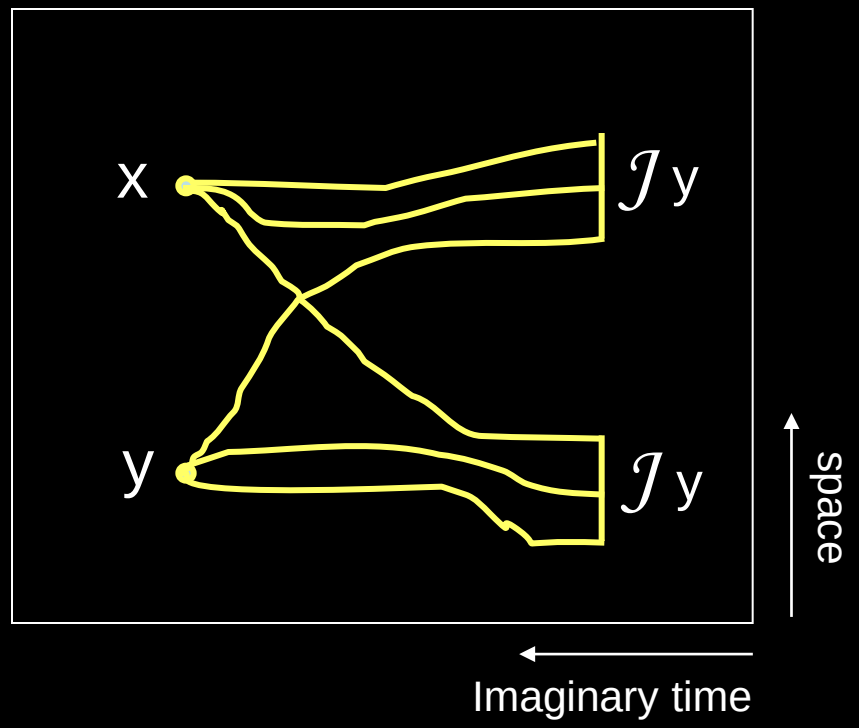


# Nuclear Force from Lattice QCD




Univ. Tsukuba S. Aoki, N. Ishii, H. Nemura, K. Sasaki
RIKEN K. Murano, T. Doi, T. Hatsuda
Nihon Univ. T. Inoue
Tokyo Inst. Tech. Y. Ikeda
Univ. Tokyo B. Charron

# Equal-time NBS amplitude $\phi(\mathbf{r})$ in lattice QCD



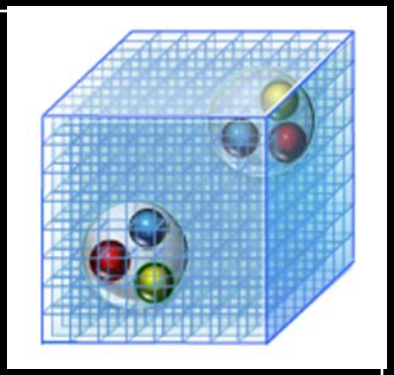

+ all possible combinations

$$
\begin{aligned}
C_4(\mathbf{r};t) &= \langle N_1(\mathbf{x},t) N_2(\mathbf{y},t) \mathcal{J}_1^\dagger(0) \mathcal{J}_2^\dagger(0) \rangle \\
&= \sum_n \langle 0 | N_1(\mathbf{x}) N_2(\mathbf{y}) | n \rangle A_n \; e^{-E_n t} \longrightarrow \phi(\mathbf{r}) A_0 e^{-E_0 t}
\end{aligned}
$$

$\phi(r > R)$ → phase shift : Luscher, Nucl. Phys. B354 (1991) 531

$\phi(r < R)$ → potential : Ishii, Aoki & Hatsuda, PRL 99 (2007) 022001

# How to define the NN potential from QCD ?



1. NN wave function from lattice QCD

$$\phi_n(\vec{r},t) = \langle 0|N(\vec{x}+\vec{r},t)N(\vec{x},t)|E_n\rangle$$

$$\phi(\vec{r},t) = \sum_n c_n \phi_n(\vec{r},t)$$

1. NN potential from the NN wave function

$$\left(-\frac{\partial}{\partial t} - H_0\right)\phi(\vec{r},t) = \int U(\vec{r},\vec{r}')\phi(\vec{r}',t)d^3r'$$

3. Derivative expansion

$$U(\vec{r},\vec{r}') = V(\vec{r},\nabla)\delta^3(\vec{r}-\vec{r}')$$

$$V(\vec{r},\nabla) = V_{\rm C}(r) + S_{12}V_{\rm T}(r) + \vec{\rm L}\cdot\vec{\rm S}\,V_{\rm LS}(r) + \{V_{\rm D}(r),\nabla^2\} + \cdots$$

LO LO NLO NNLO

Ishii, Aoki, Hatsuda, Phys.Rev.Lett. 99 (2007) 022001
HAL QCD Coll., Phys. Lett. B (2012)

- Potential is a nice <u>tool</u> to calculate observables
- Potential is <u>volume insensitive</u> (=Lattice Friendly)

# Key channels in NN scattering ($^{2s+1}L_J$)

$$V(\vec{r},\nabla) = V_{\rm C}(r) + S_{12}V_{\rm T}(r) + \vec{\rm L}\cdot\vec{\rm S}\, V_{\rm LS}(r) + \{V_{\rm D}(r), \nabla^2\} + \cdots$$

LO LO NLO NNLO

$^1S_0$ Central force ⟷ nuclear BCS pairing

Bohr, Mottelson & Pines, Phys. Rev. 110 (1958)

$^3S_1$-$^3D_1$ Tensor force ⟷ deuteron binding

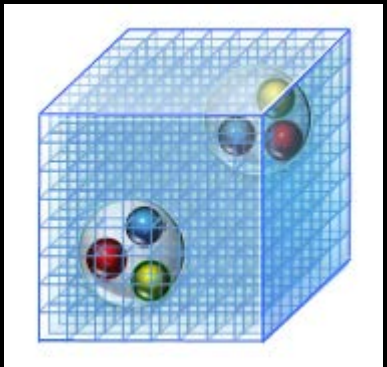
Schwinger, Phys. Rev. 55 (1939), Bethe, ibid.57 (1940)
Rarita & Schwinger, ibid. 59 (1941)

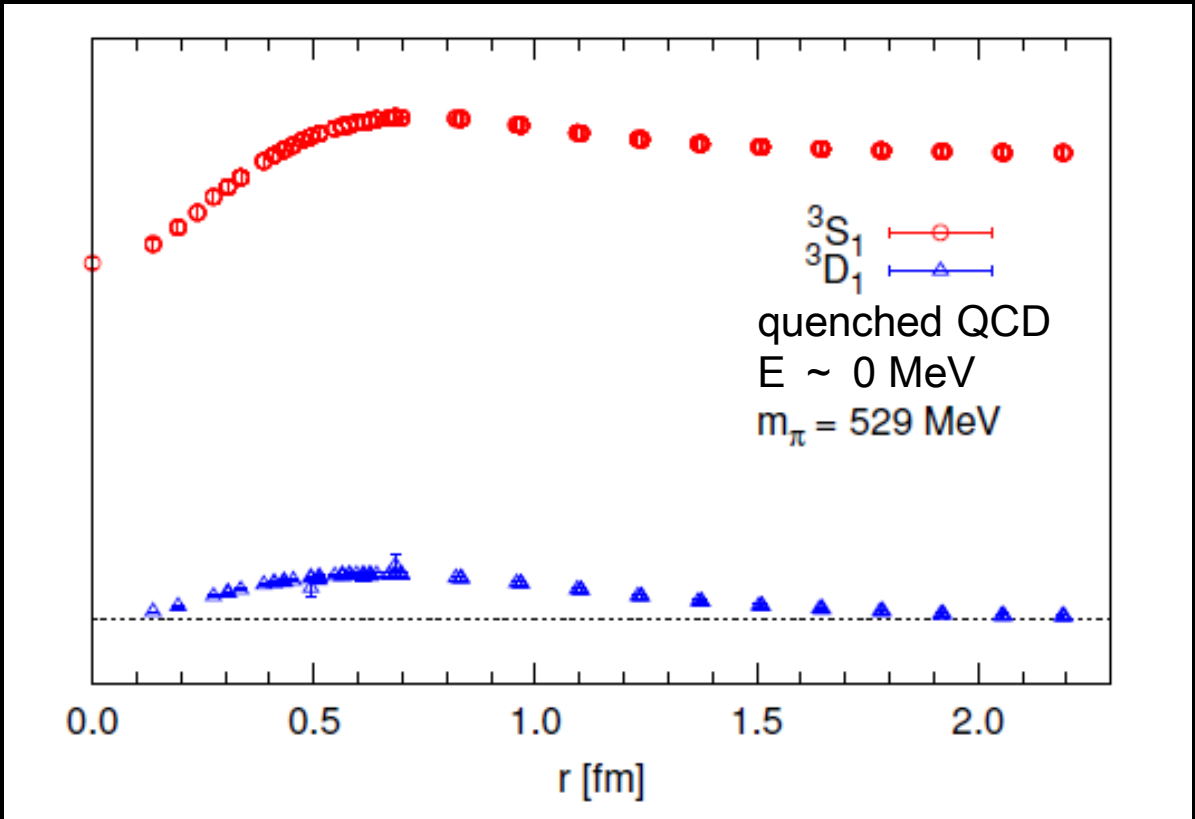
$^3P_2$-$^3F_2$ LS force ⟷ neutron superfluidity in neutron stars

Tamagaki, Prog. Theor. Phys. 44 (1970)
Hoffberg et al., Phys. Rev. Lett. 24 (1970)

Density profile of the deuteron with $S_z=\pm 1$
Pandharipande et al., (1998)

# [Exercise 1] LO potentials : $V_C(r)$ & $V_T(r)$






Aoki, Ishii & Hatsuda,
Prog. Theor. Phys. 123 (2010) 89

- Rapid quark-mass dependence of $V_T(r)$
- Evidence of the one-pion-exchange

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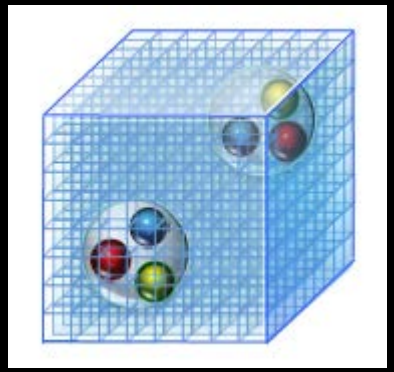

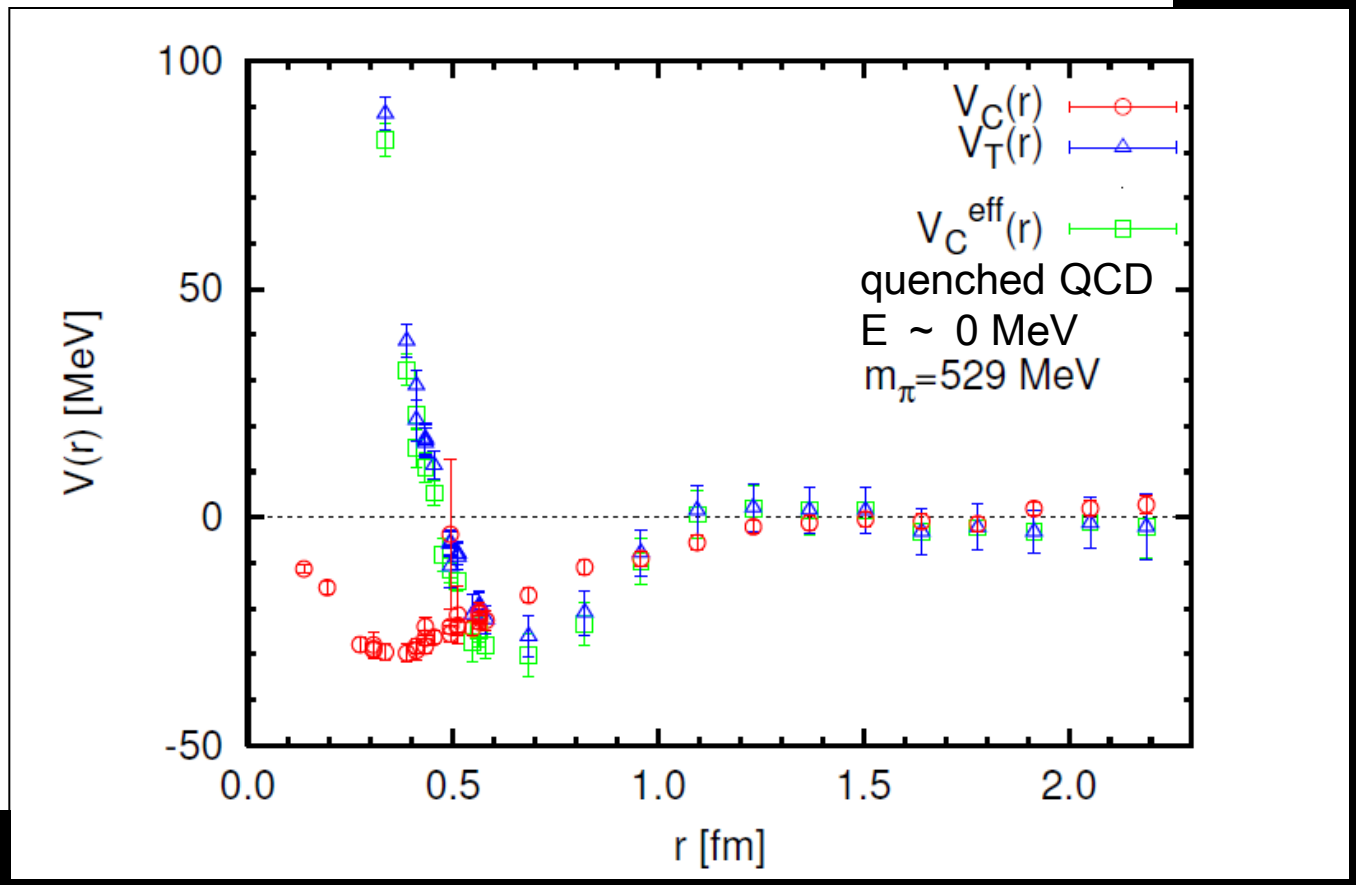


Aoki, Ishii & Hatsuda,
Prog. Theor. Phys. 123 (2010) 89

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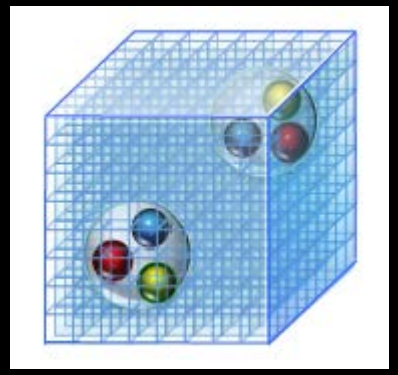
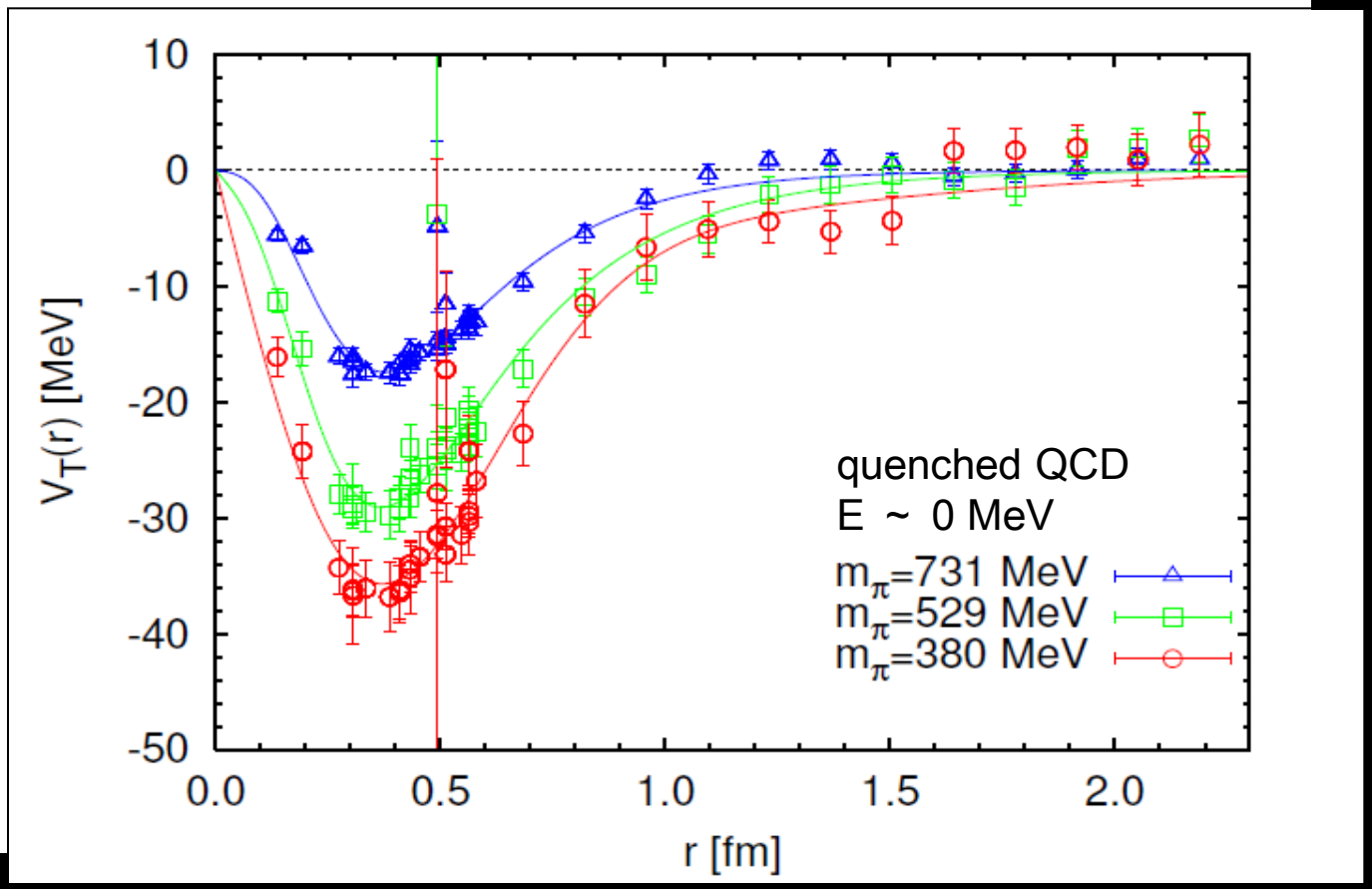




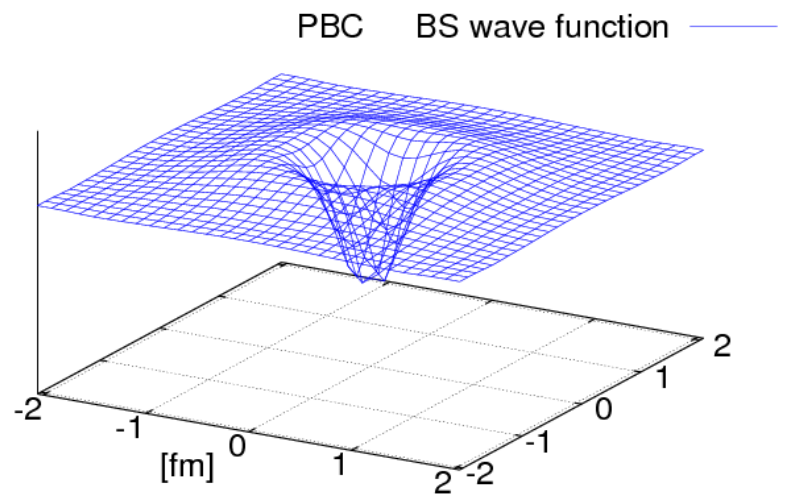

Aoki, Ishii & Hatsuda,
Prog. Theor. Phys. 123 (2010) 89

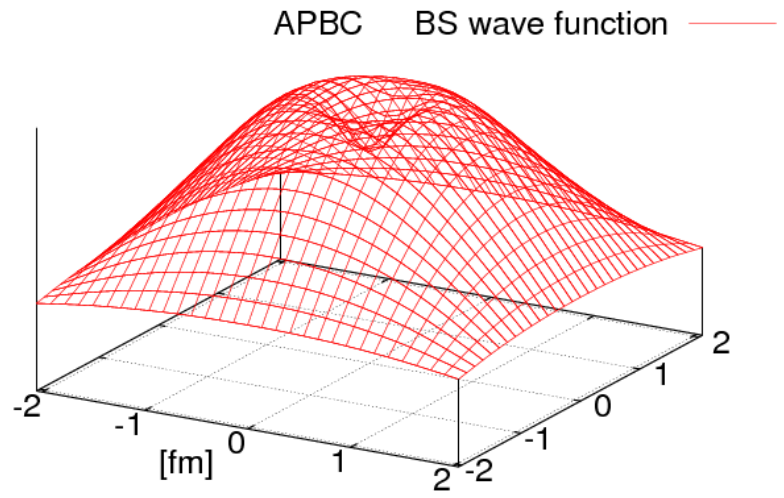
- Rapid quark-mass dependence of $V_T(r)$
- Evidence of the one-pion-exchange

# [Exercise 2] Magnitude of the non-locality

$$V(\vec{r},\nabla) = V_{\rm C}(r) + S_{12}V_{\rm T}(r) + \vec{\rm L}\cdot\vec{\rm S}\,V_{\rm LS}(r) + \{V_{\rm D}(r),\nabla^2\} + \cdots$$

- PBC ($T_{Lab}$~0 MeV)
- APBC ($T_{Lab}$~100 MeV)




Murano et al. (HAL QCD Coll.), Prog. Theor. Phys. 125 (2011) 1225

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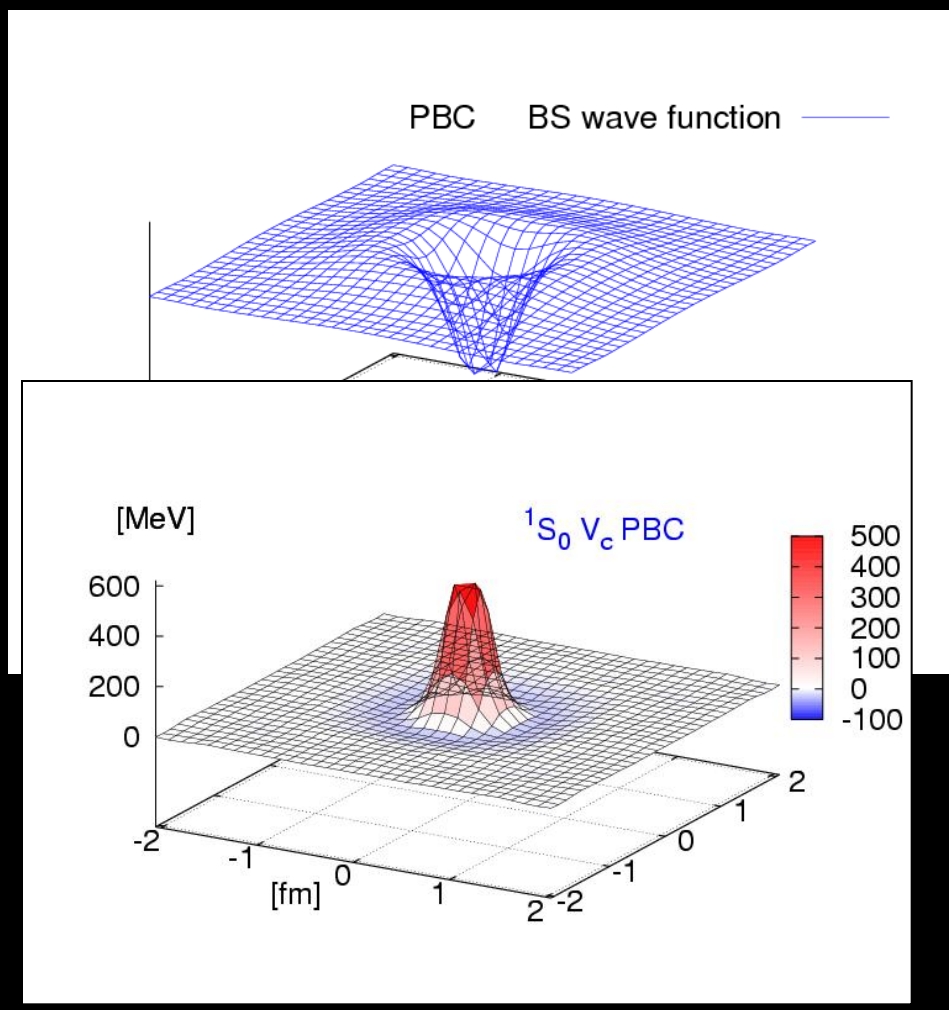
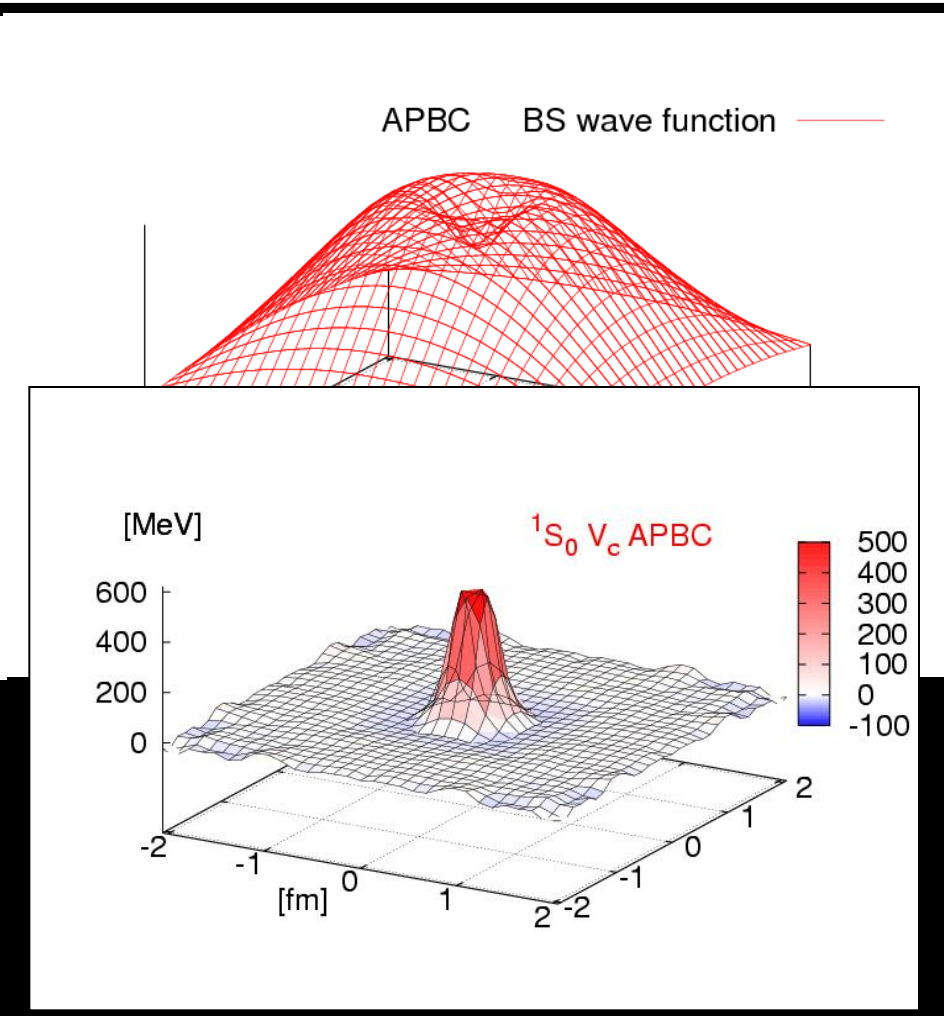
- APBC ($T_{Lab}$~100 MeV)



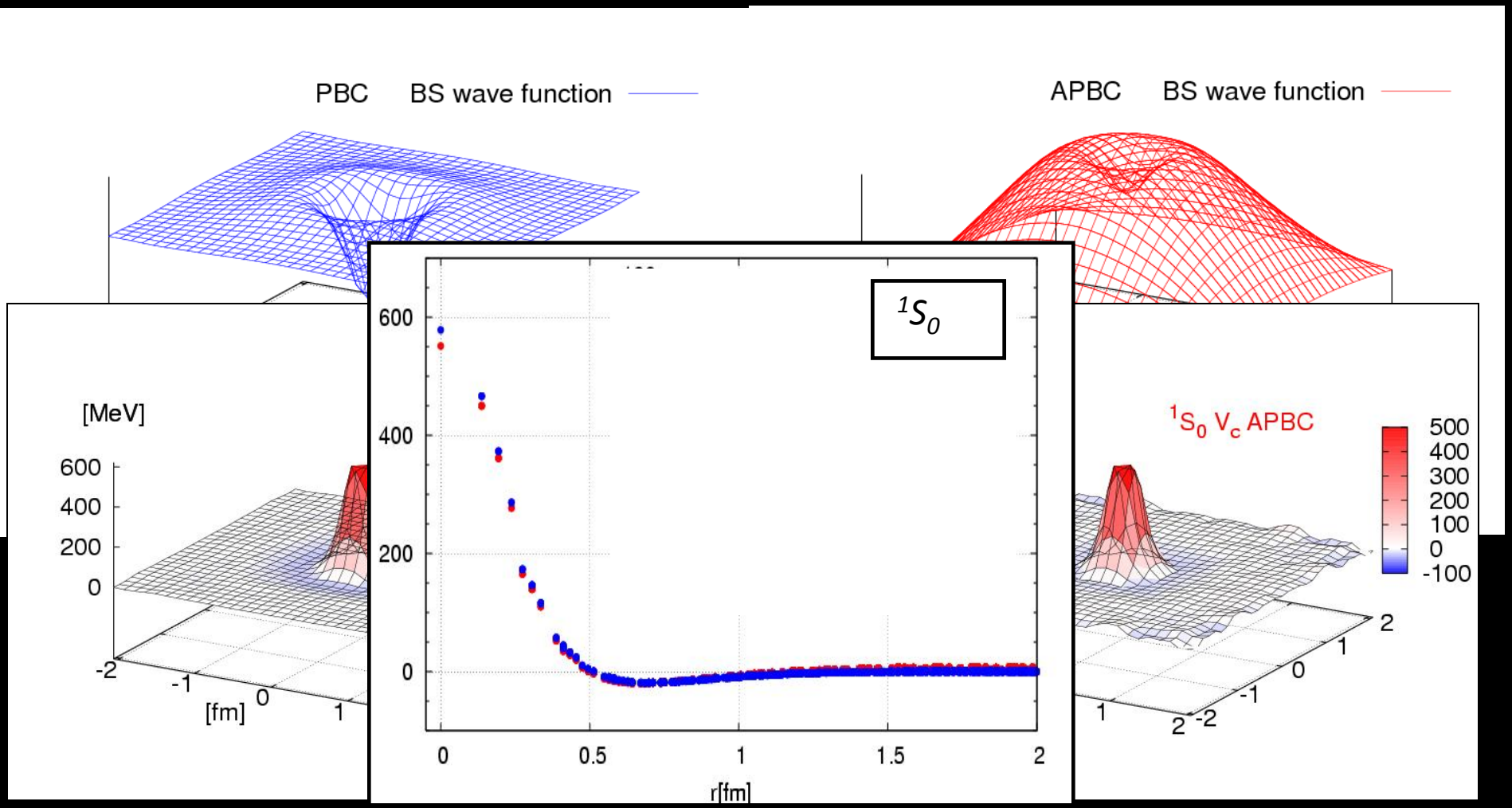




Murano et al. (HAL QCD Coll.), Prog. Theor. Phys. 125 (2011) 1225

# [Exercise 2] Magnitude of the non-locality

$$V(\vec{r},\nabla) = V_{\rm C}(r) + S_{12}V_{\rm T}(r) + \vec{\rm L}\cdot\vec{\rm S}\,V_{\rm LS}(r) + \{V_{\rm D}(r),\nabla^2\} + \cdots$$

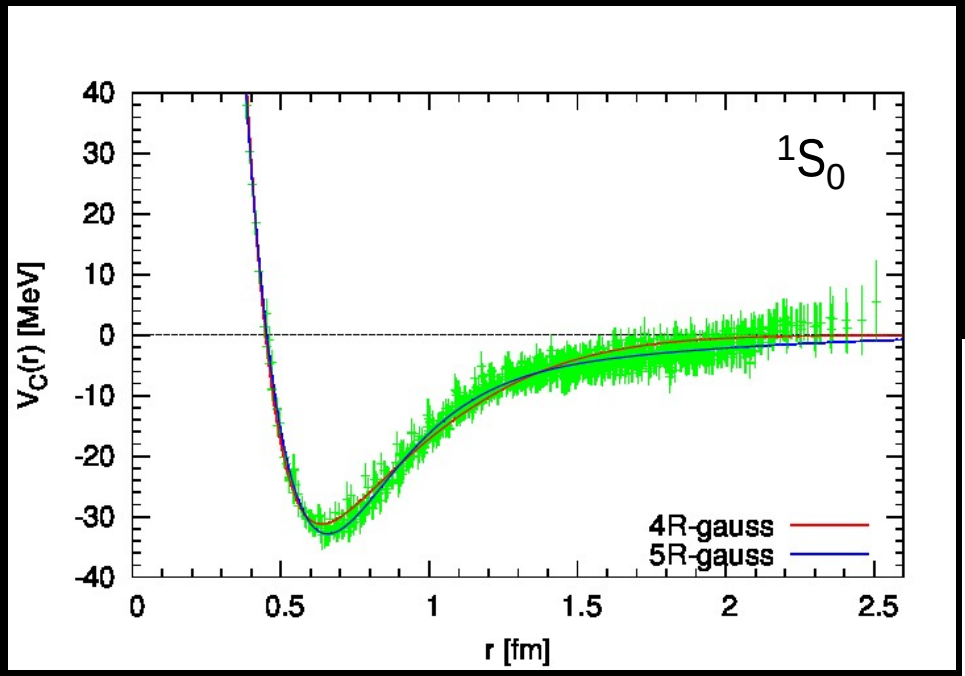
- PBC ($T_{Lab}$~0 MeV)
- APBC ($T_{Lab}$~100 MeV)



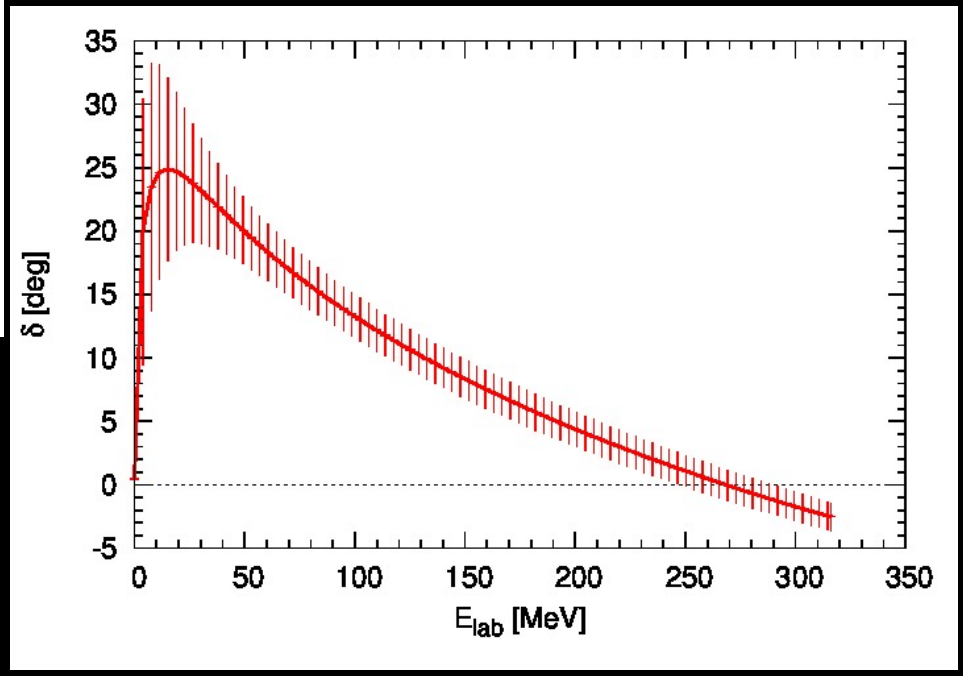

Murano et al. (HAL QCD Coll.), Prog. Theor. Phys. 125 (2011) 1225

# Central potential in (2+1)-flavor QCD

HAL QCD Coll., Phys. Lett. B (2012)



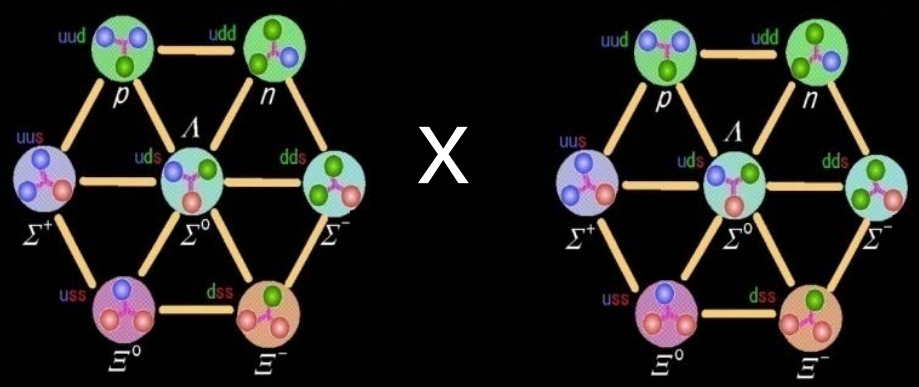

PACS-CS gauge config.
(Clover + Iwasaki)
a= 0.09 fm, L=2.9 fm
$m_\pi$= 700 MeV




Physical point simulations ($m_\pi$=135MeV with L=6fm & 9fm) will be carried out at KEI computer

Q1: Physical origin of the repulsive NN core ?
Q2: Hyperon forces : important for hyper-nuclei & neutron stars
Q3: Fate of H-dibaryon: exotic 6-quark state (uuddss)

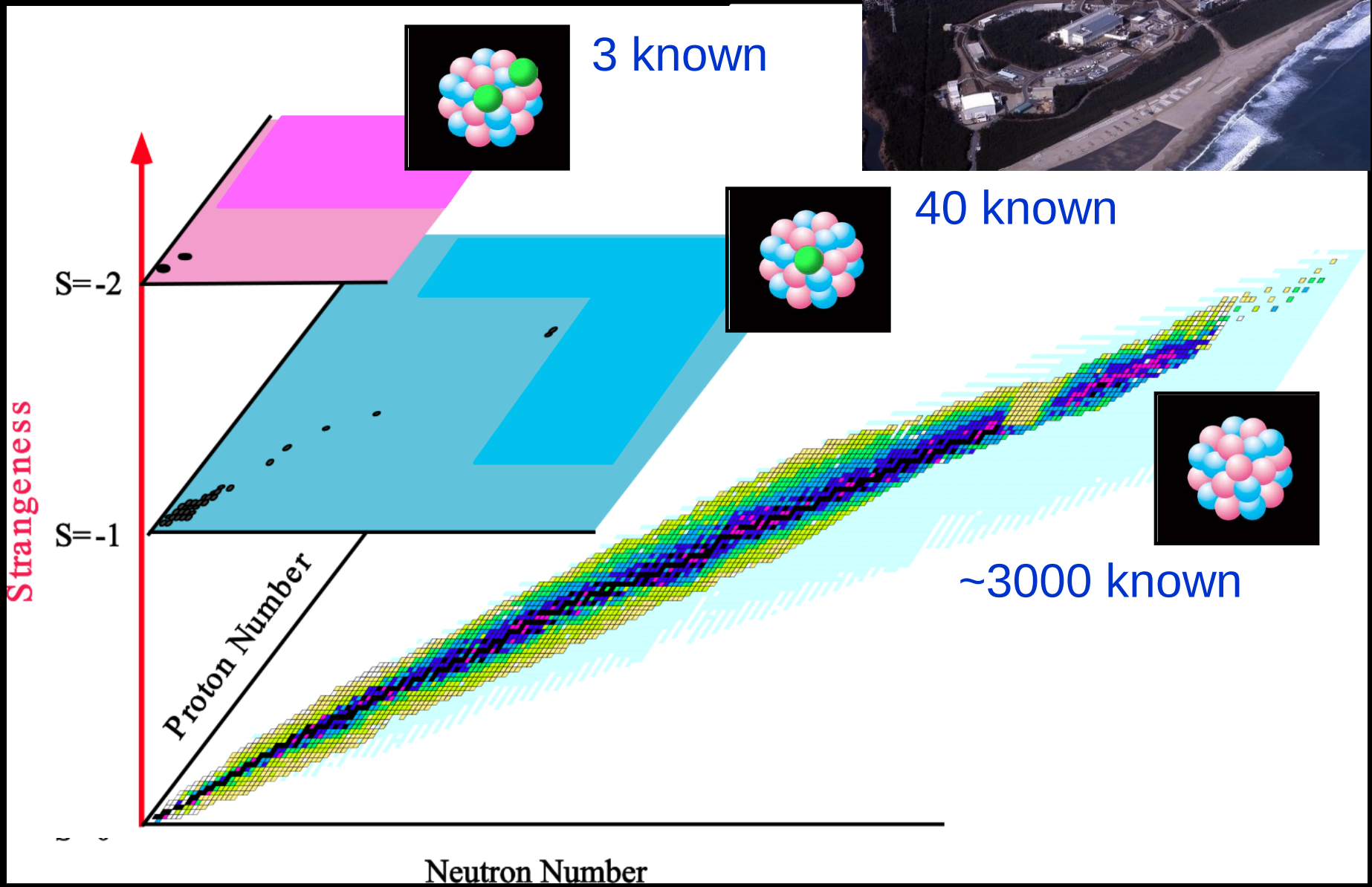
⇒ Baryon-baryon force in flavor SU(3)



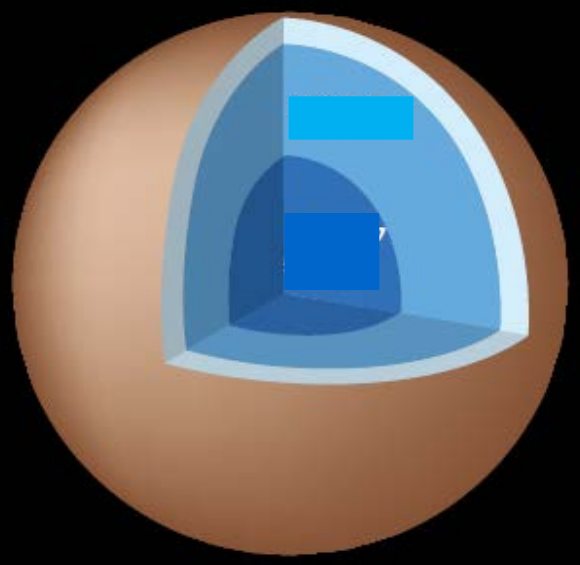

$$8 \times 8 = \underbrace{27 + 8s + 1}_{\text{Symmetric}} + \underbrace{10^* + 10 + 8a}_{\text{Anti-symmetric}}$$

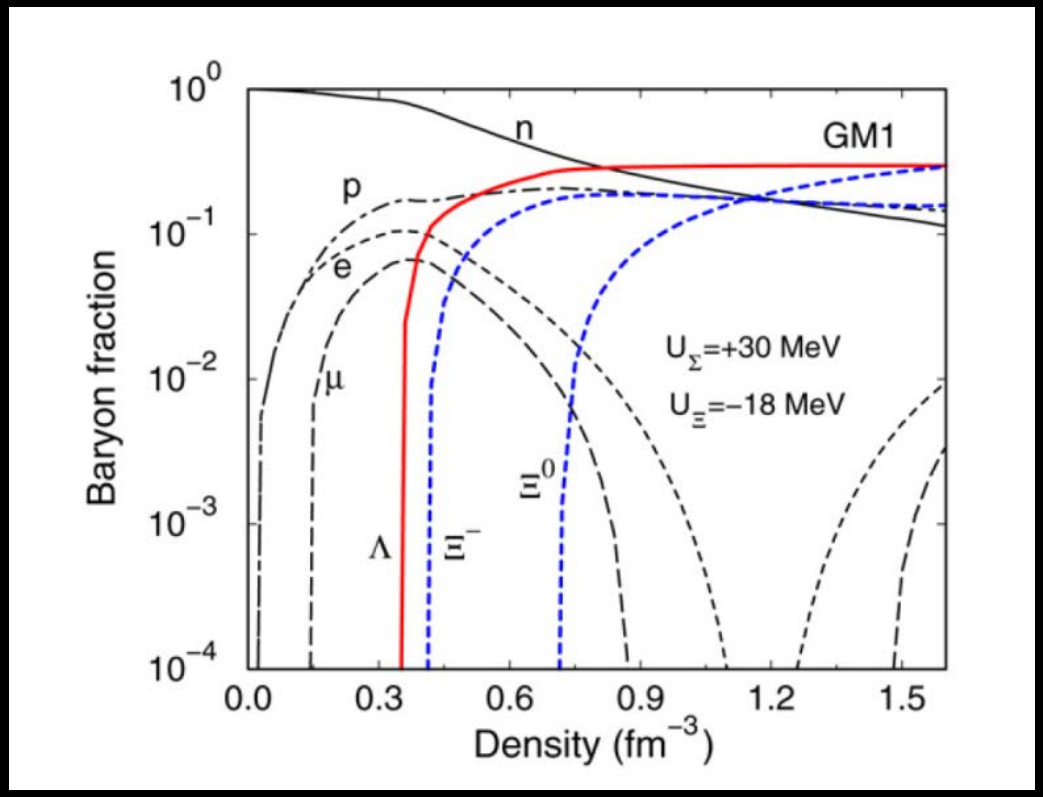
<u>Six</u> independent potentials in the flavor-basis

# Hypernuclei at J-PARC



# Inner core of neutron stars
## -- hyperon mixing --



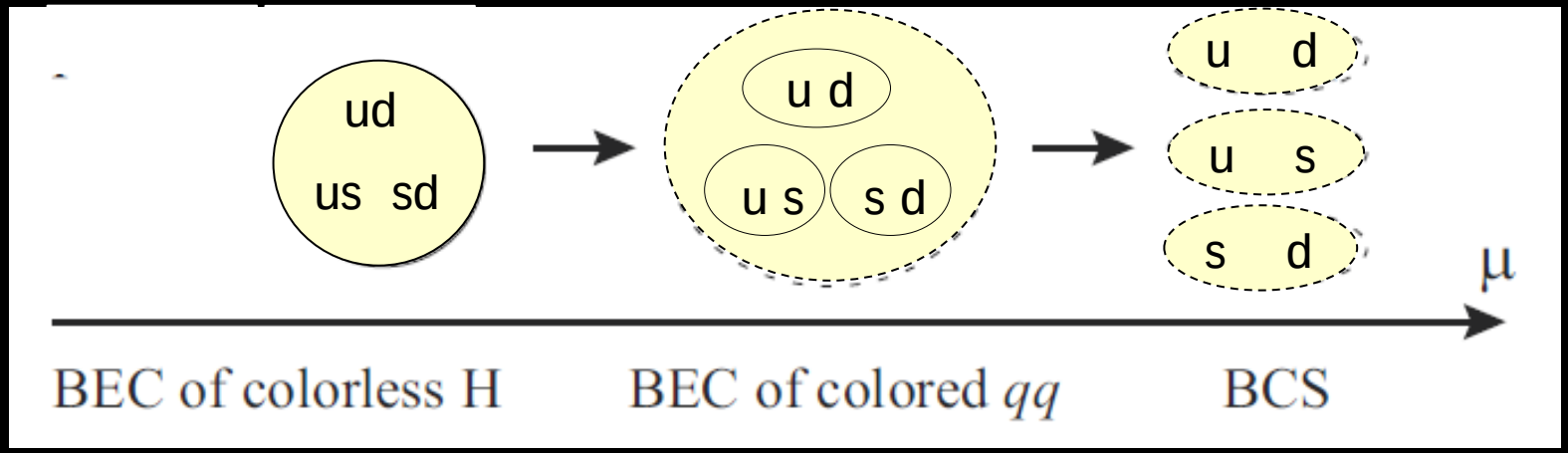



Schaffner-Bielich, Nucl. Phys.A 835, 279 (2010)

Onset of hyperon mixture ⇔ YN interactions

# H-dibaryon :
# doorway state to quark matter ?

Tamagaki, PTP 85 (1991) 321
Fukushima, PRD70 (2004) 094014



Q1: Physical origin of the repulsive NN core ?
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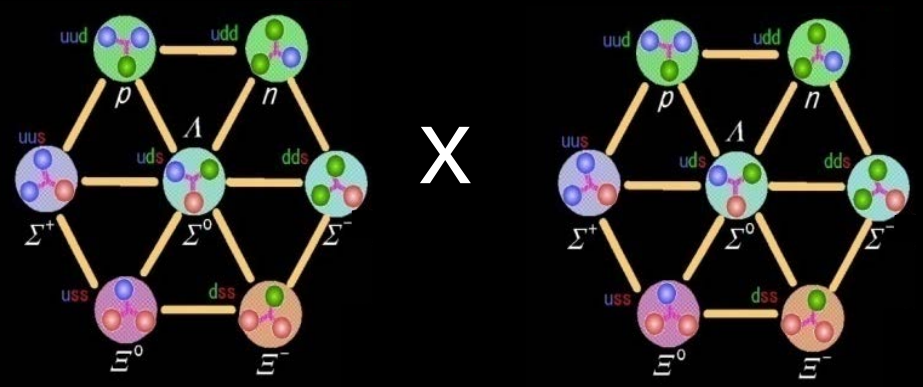
Q3: Fate of H-dibaryon: exotic 6-quark state (uuddss)

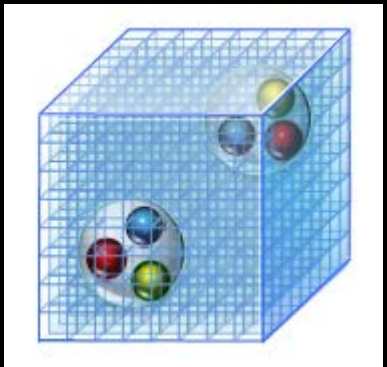
⇒ Baryon-baryon force in flavor SU(3)

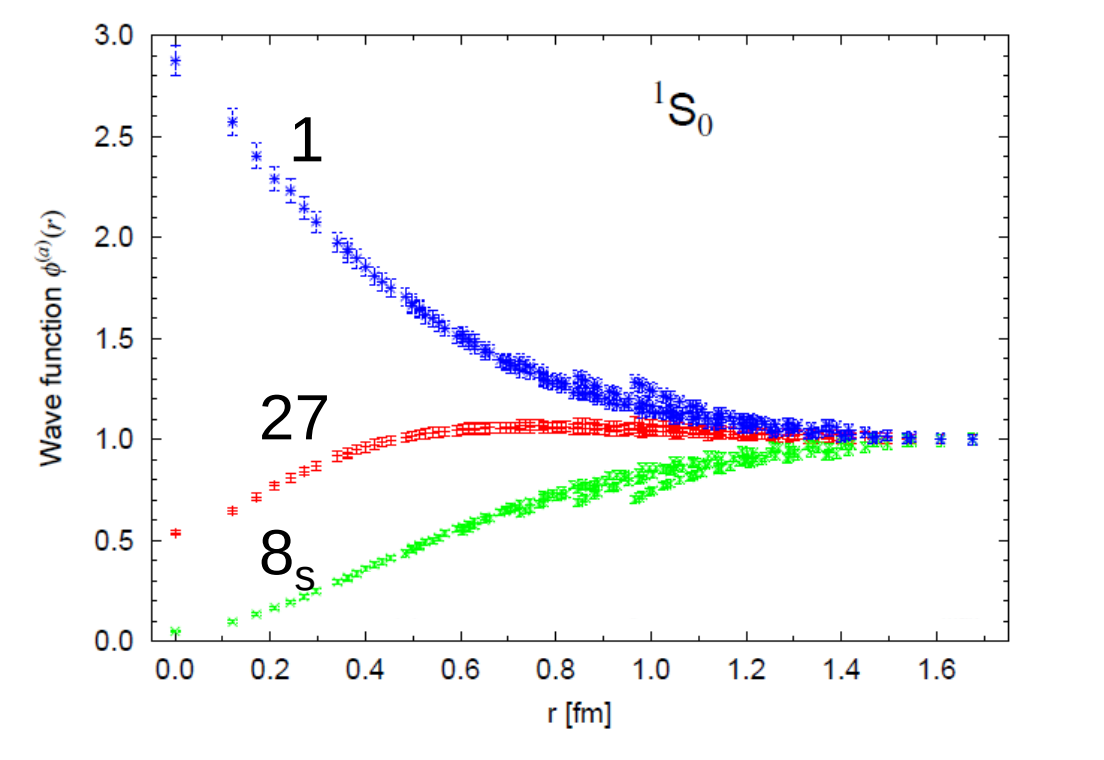



$$8 \times 8 = \underbrace{27 + 8s + 1}_{\text{Symmetric}} + \underbrace{10^* + 10 + 8a}_{\text{Anti-symmetric}}$$

<u>Six</u> independent potentials in the flavor-basis

# Lattice BB wave functions






Iwasaki + clover
(CP-PACS/JLQCD config.)
L=1.9 fm, a=0.12 fm, $16^3$x32
$m_\pi$=835 MeV, $m_B$=1752 MeV

HAL QCD Coll.,
Nucl. Phys. A881 (2012) 28

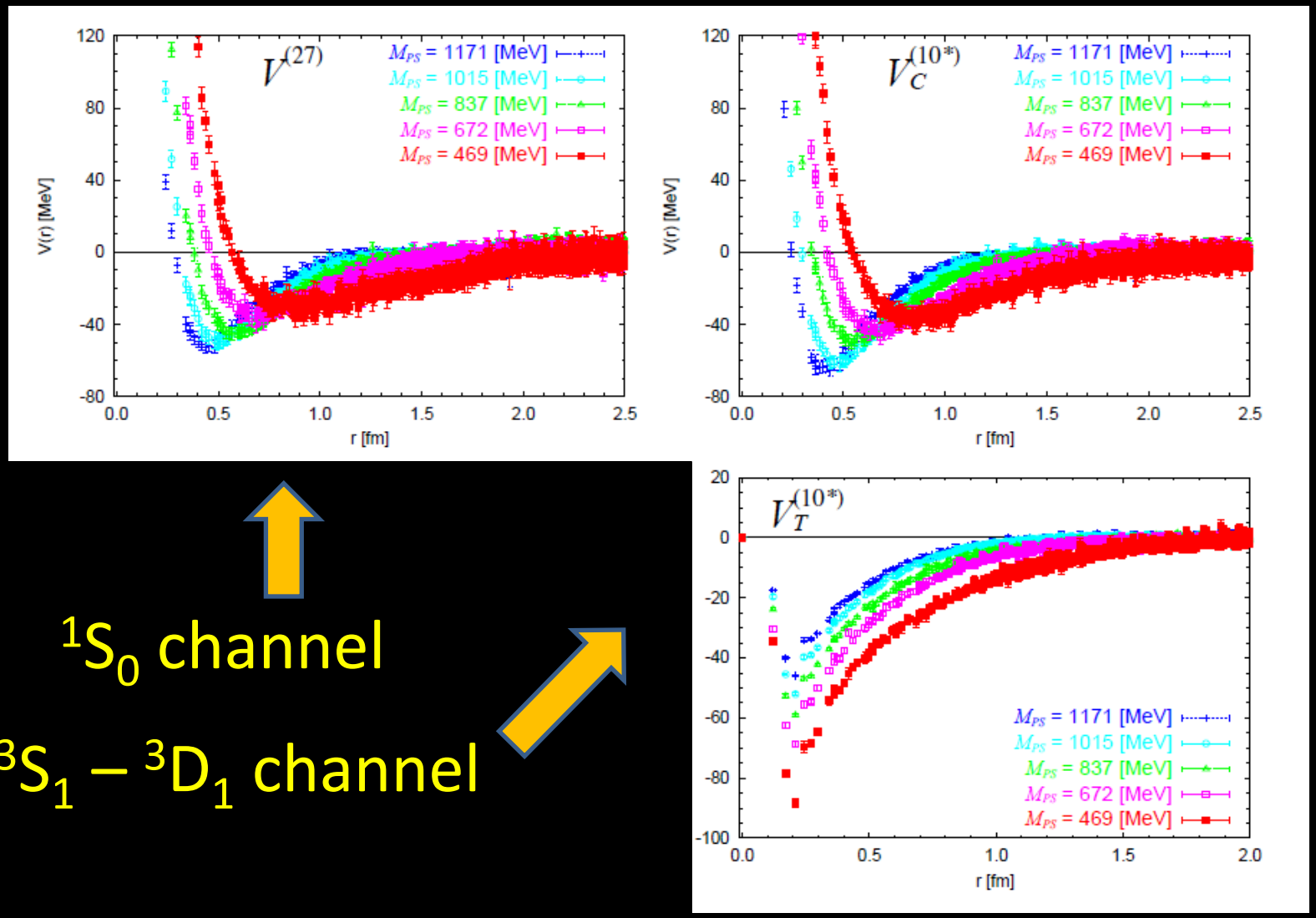
**Short range BB int. ⇔ Quark Pauli principle**

1 : allowed, 27 : partially blocked, $8_S$ : blocked

# BB potentials in flavor-basis

IHAL QCD Coll.
Phys. Rev. Lett. 106 (2011) 162002
Nucl. Phys. A881 (2012) 28

Repulsive core in NN channel

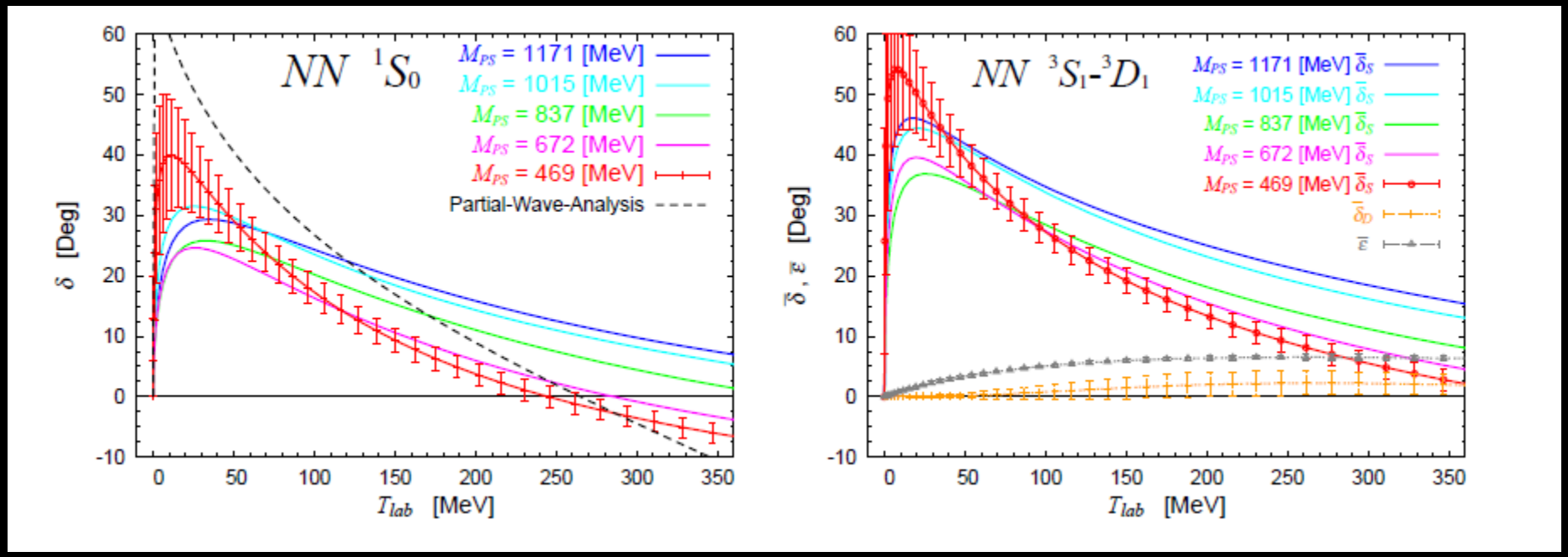



$^1S_0$ channel

$^3S_1 - {}^3D_1$ channel

Growing NN tensor force

# NN phase shifts in the SU(3) symmetric world

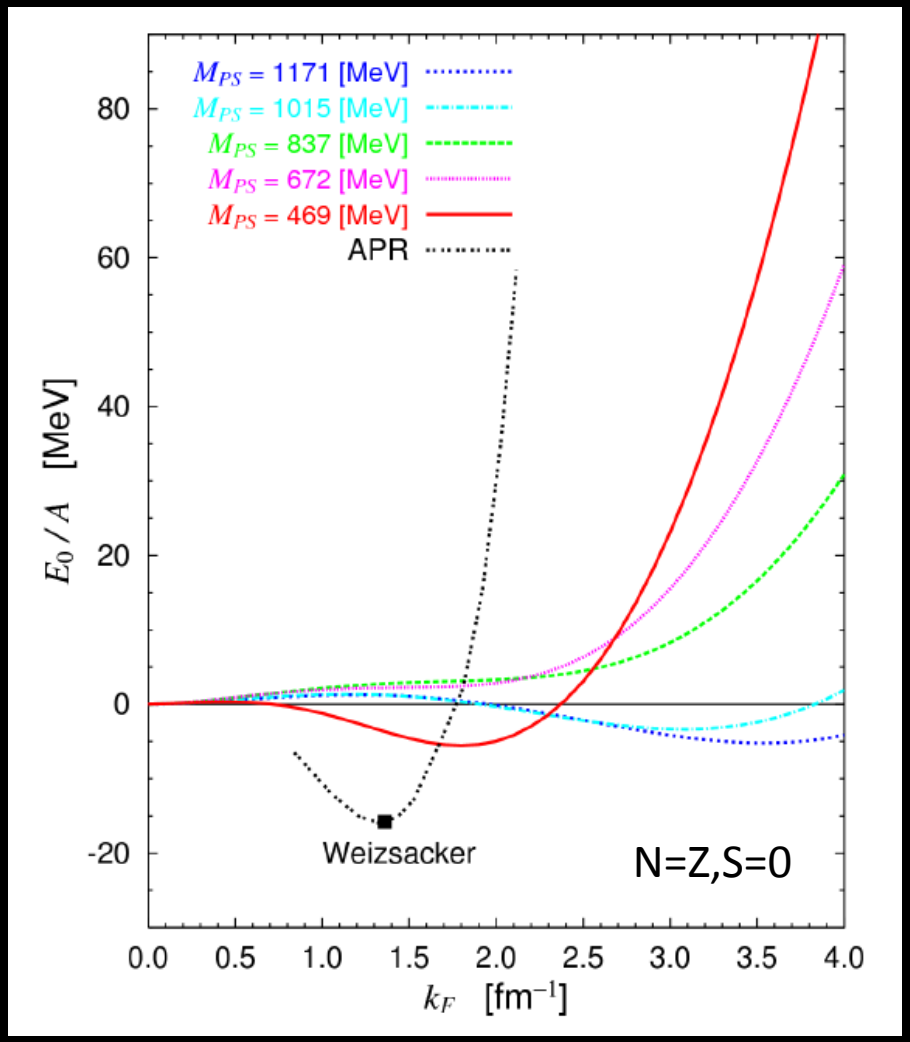


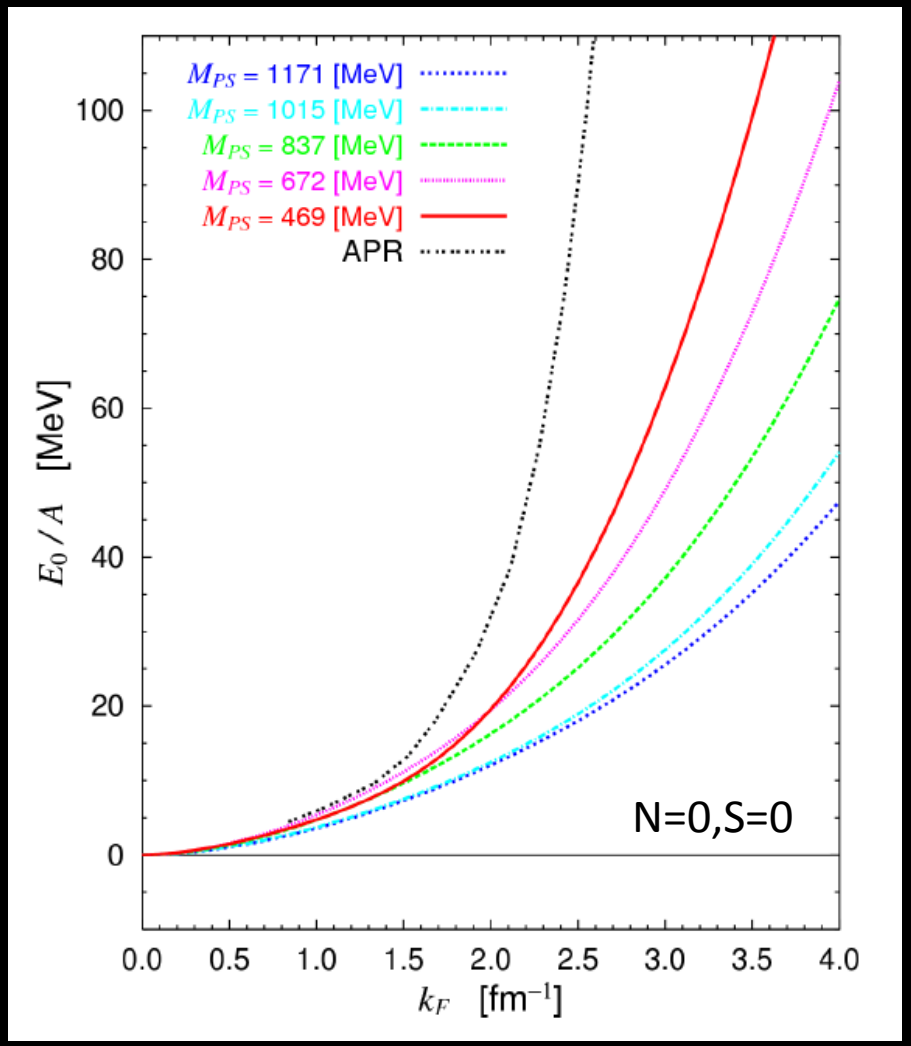

Stronger attraction in the deuteron channel

HAL QCD Coll.,
Phys. Rev. Lett. 106 (2011) 162002
Nucl. Phys. A881 (2012) 28

# Equation of state for nuclear/neutron matter from LQCD

## Brueckner-Hartree-Fock calculation with LQCD NN force




HAL QCD Coll, in preparation (2012)

Q1: Physical origin of the repulsive NN core ?
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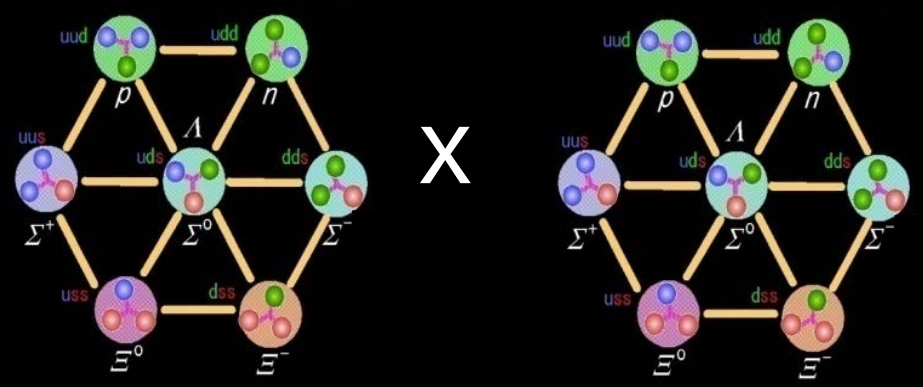
⇒ Baryon-baryon force in flavor SU(3)

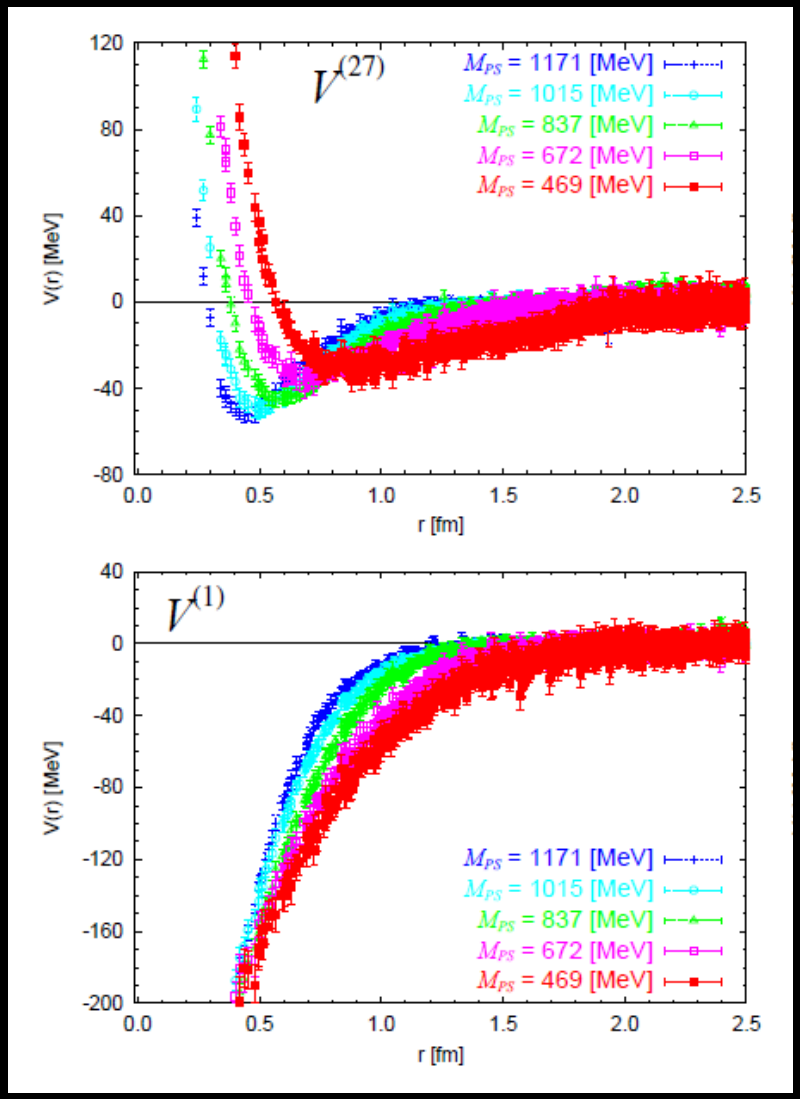



$$8 \times 8 = \underbrace{27 + 8s + 1}_{\text{Symmetric}} + \underbrace{10^* + 10 + 8a}_{\text{Anti-symmetric}}$$

<u>Six</u> independent potentials in the flavor-basis

# BB potentials in flavor-basis

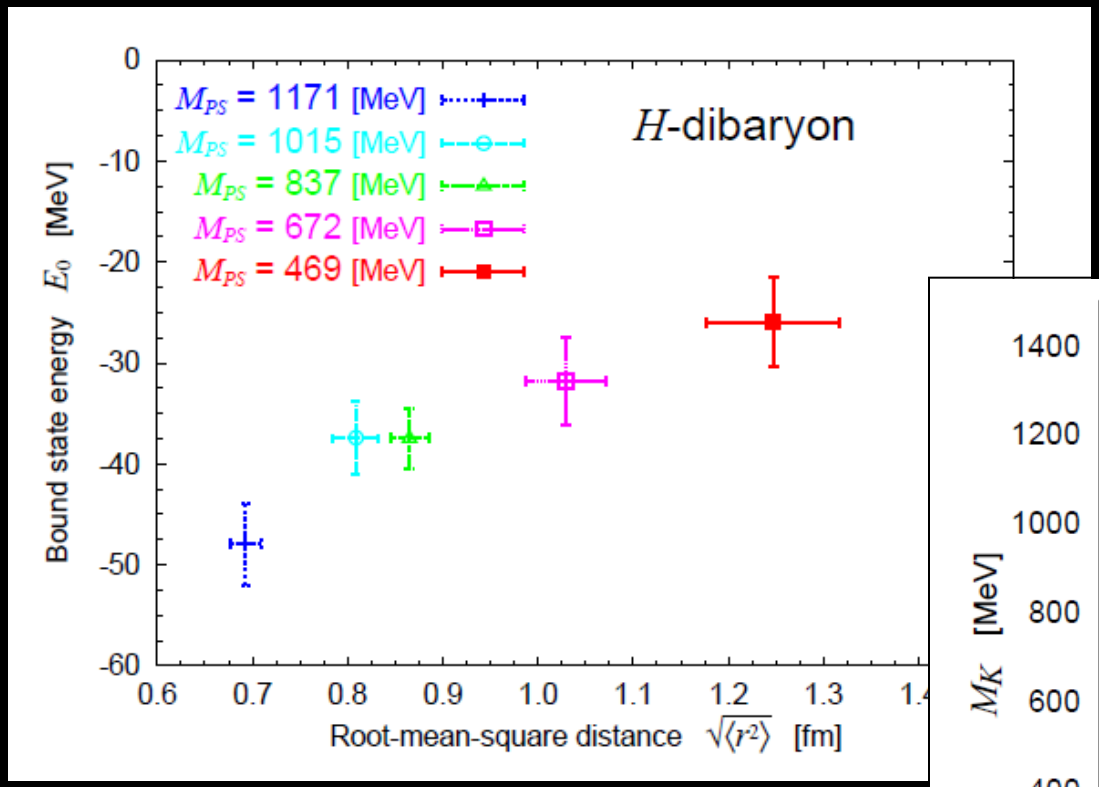
IHAL QCD Coll.
Phys. Rev. Lett. 106 (2011) 162002
Nucl. Phys. A881 (2012) 28

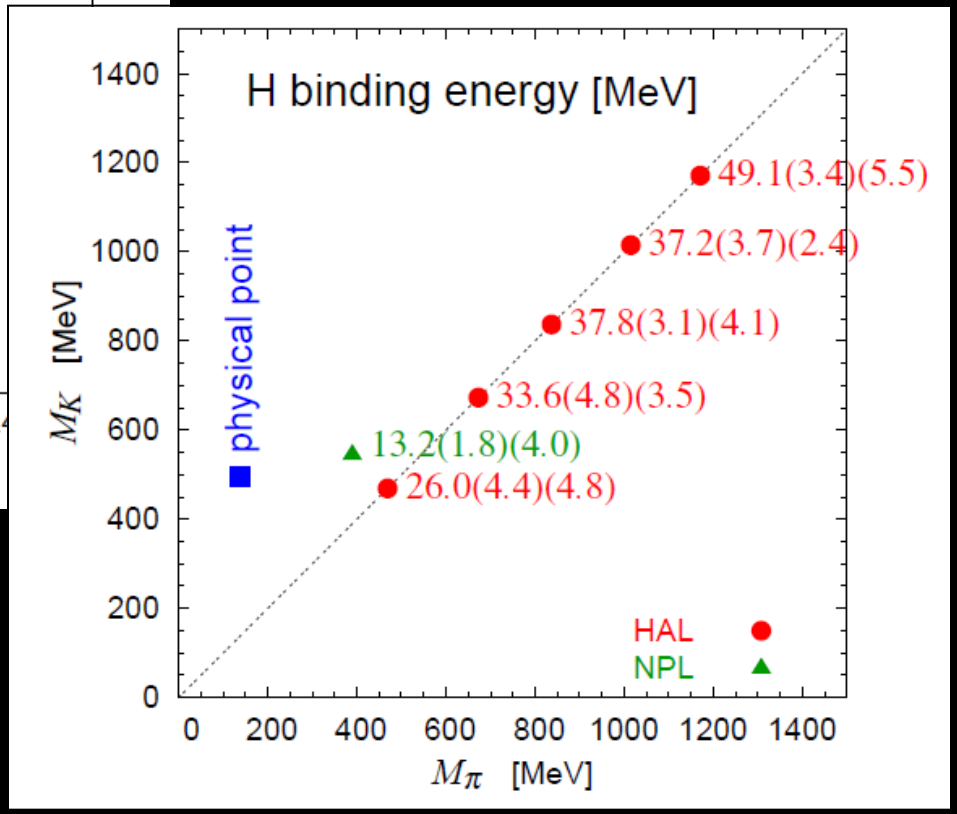



Repulsive core in NN channel

Attractive core in H channel

# H-dibaryon from LQCD




At physical point:

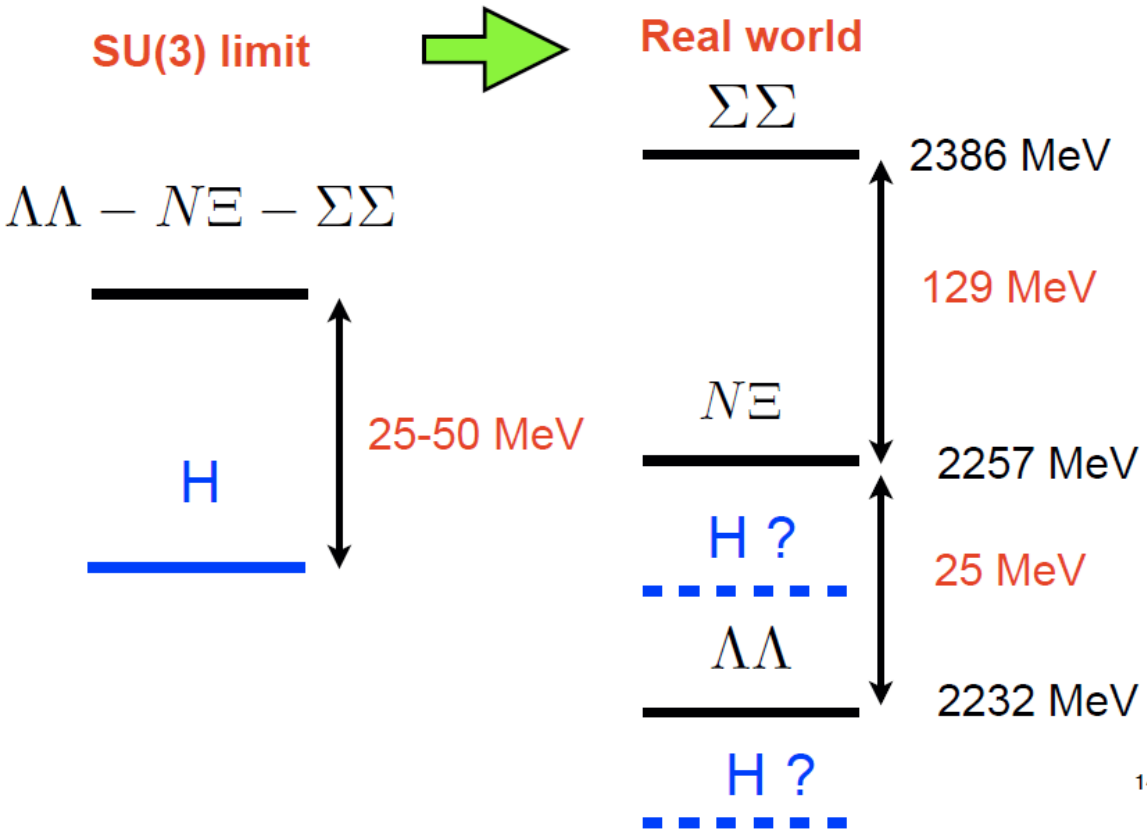
$M_{\Lambda\Lambda} < M_H < M_{\Xi N}$ ?

IHAL QCD Coll.
Phys. Rev. Lett. 106 (2011) 162002
Nucl. Phys. A881 (2012) 28

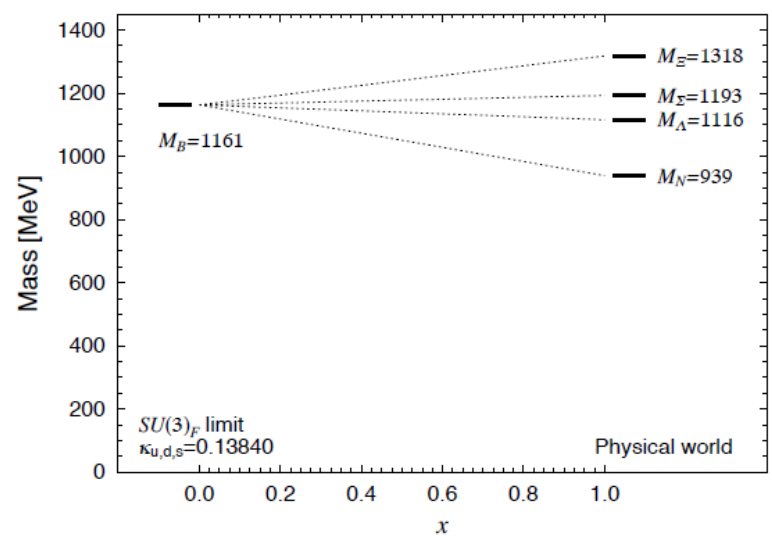
⇒ exp. search at RHIC & J-PARC

# H-dibaryon with the flavor SU(3) breaking

$m_u = m_d \neq m_s$




$m_\pi \simeq 470$ MeV $\quad$ $m_\pi \simeq 135$ MeV




## Our approximation for SU(3) breaking

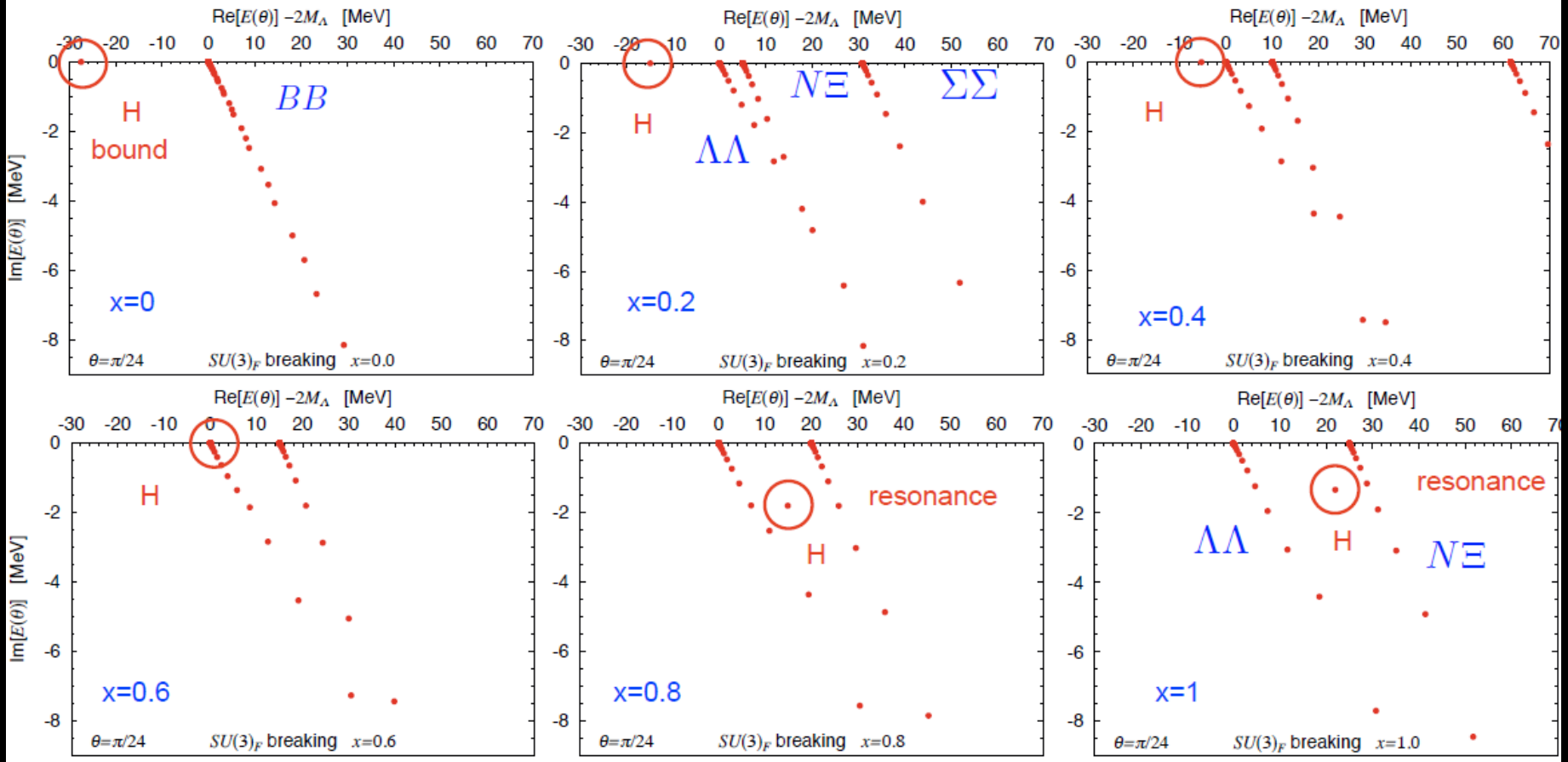
1. Linear interpolation of octet baryon masses

$$M_Y(x) = (1-x)M_Y^{\mathrm{SU}(3)} + xM_Y^{\mathrm{Phys}}$$

2. Potentials in particle basis in SU(3) limit

## Energy eigenvalues

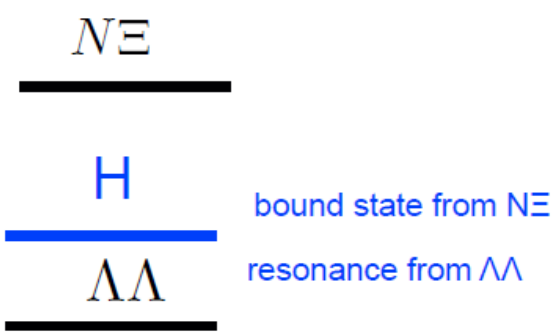
complex scaling method $\mathcal{H}(\theta)\Psi_\theta = E(\theta)\Psi_\theta \quad (r \text{ to } re^{i\theta})$




HAL QCD Coll., Nucl. Phys. A881 (2012) 28

H-dibaryon seems to become resonance at physical point.

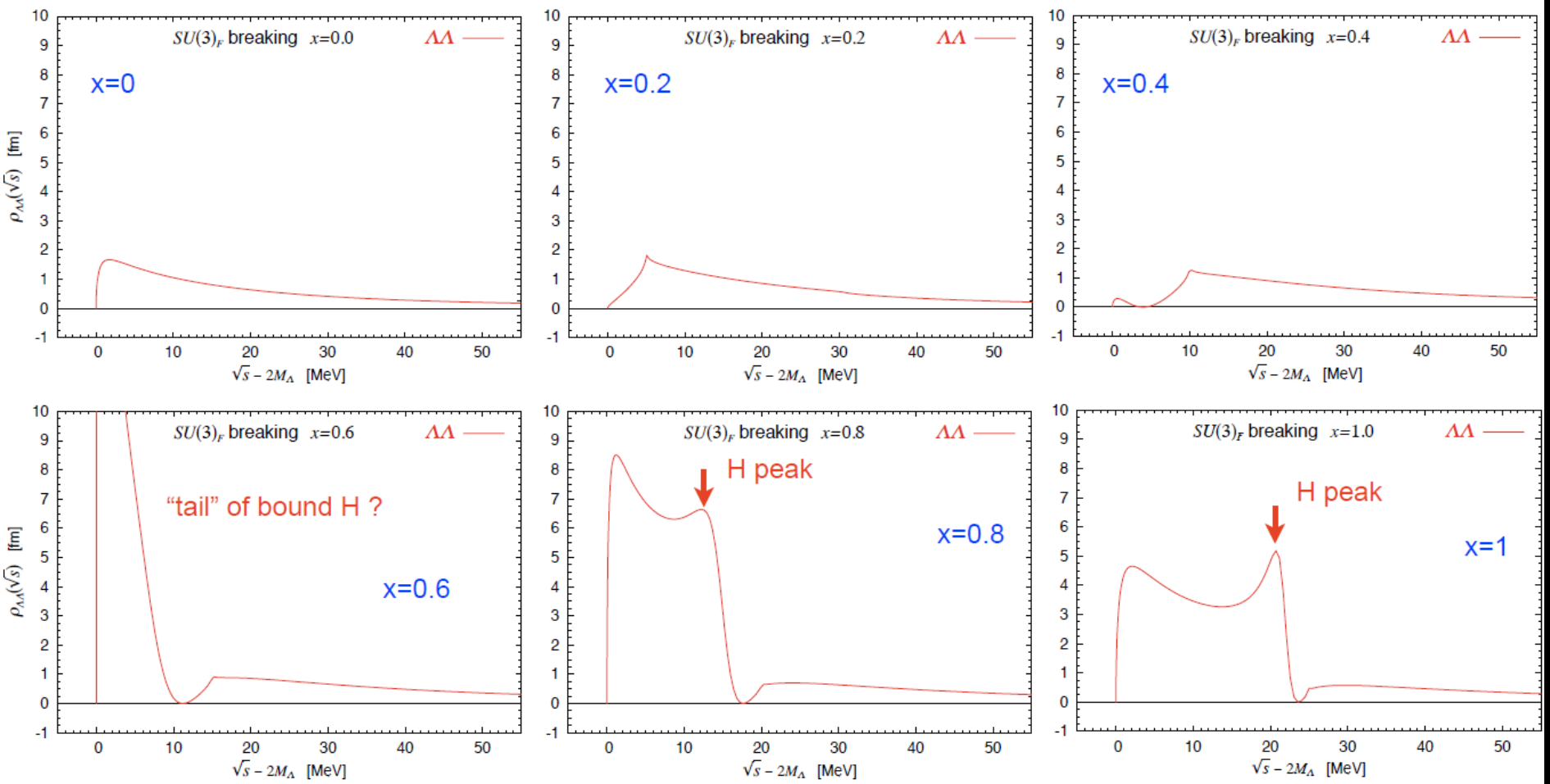
This needs a direct confirmation by 2+1 flavor QCD.



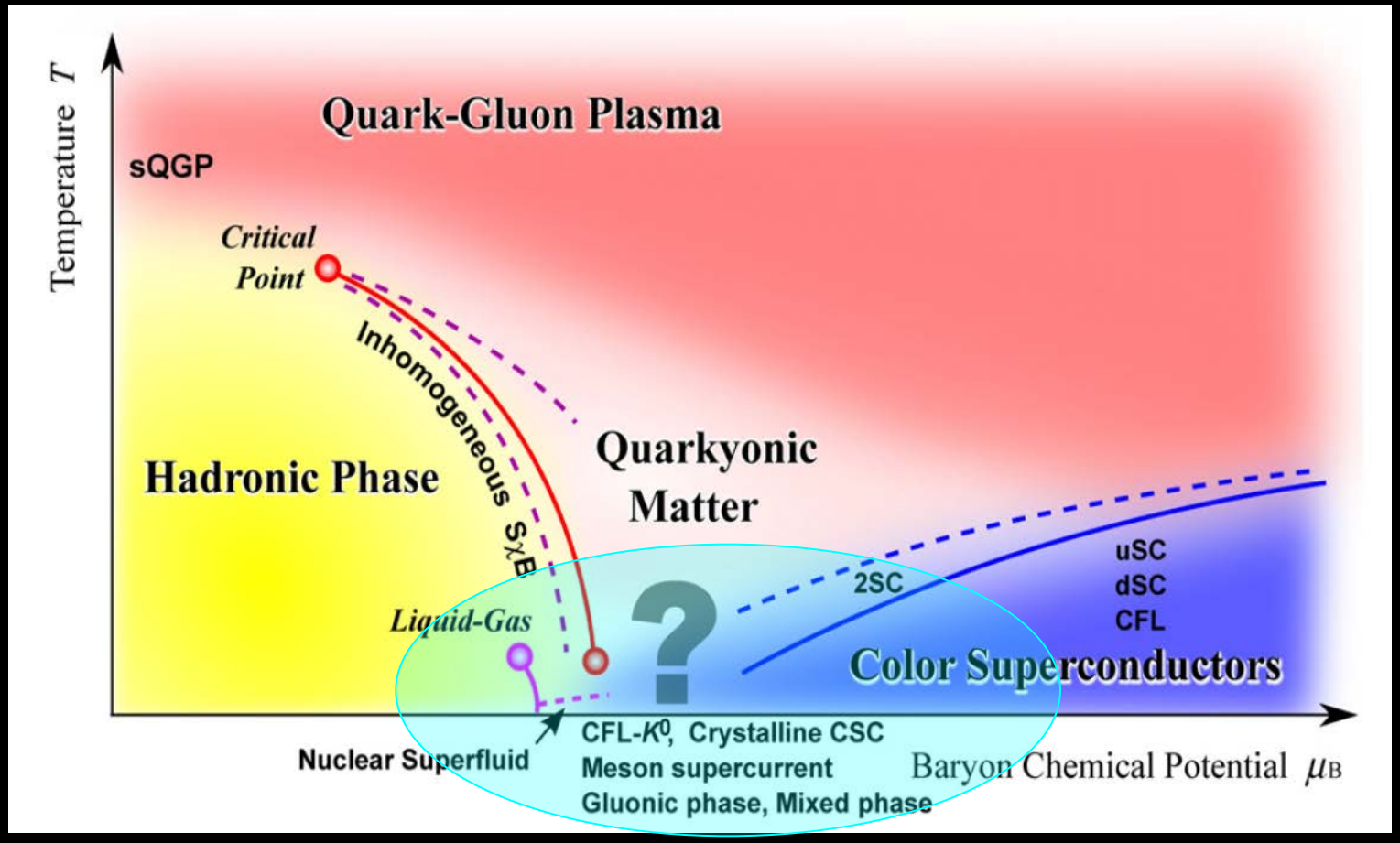
# Invariant mass spectrum

$\Lambda\Lambda \longrightarrow \Lambda\Lambda$

HAL QCD Coll., Nucl. Phys. A881 (2012) 28



# Simulating dense QCD by ultra-cold atoms/molecules



# Ultra-cold atomic Gasses

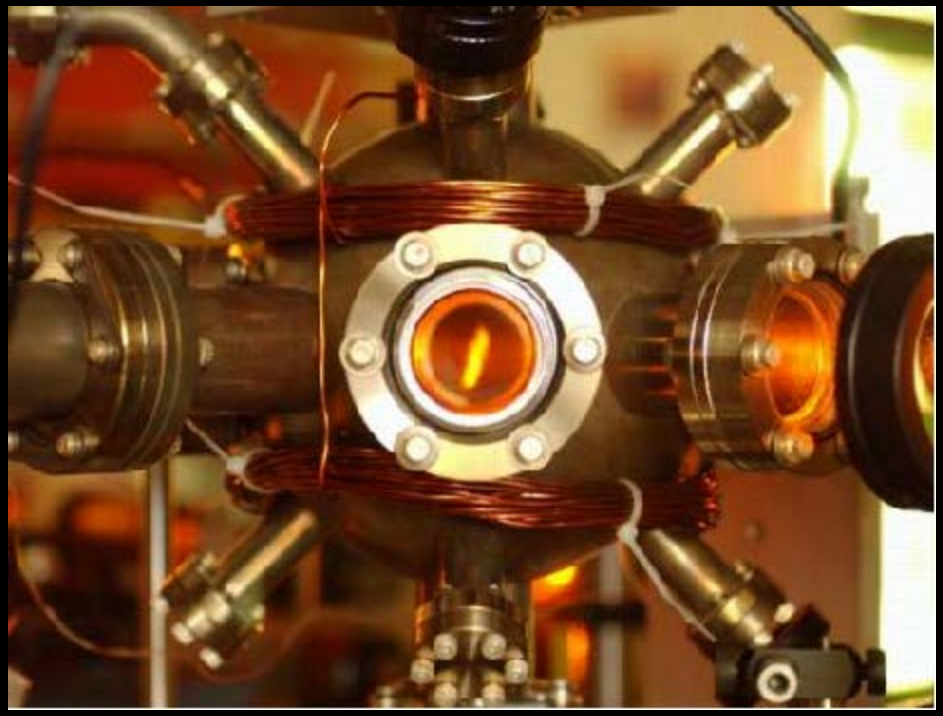
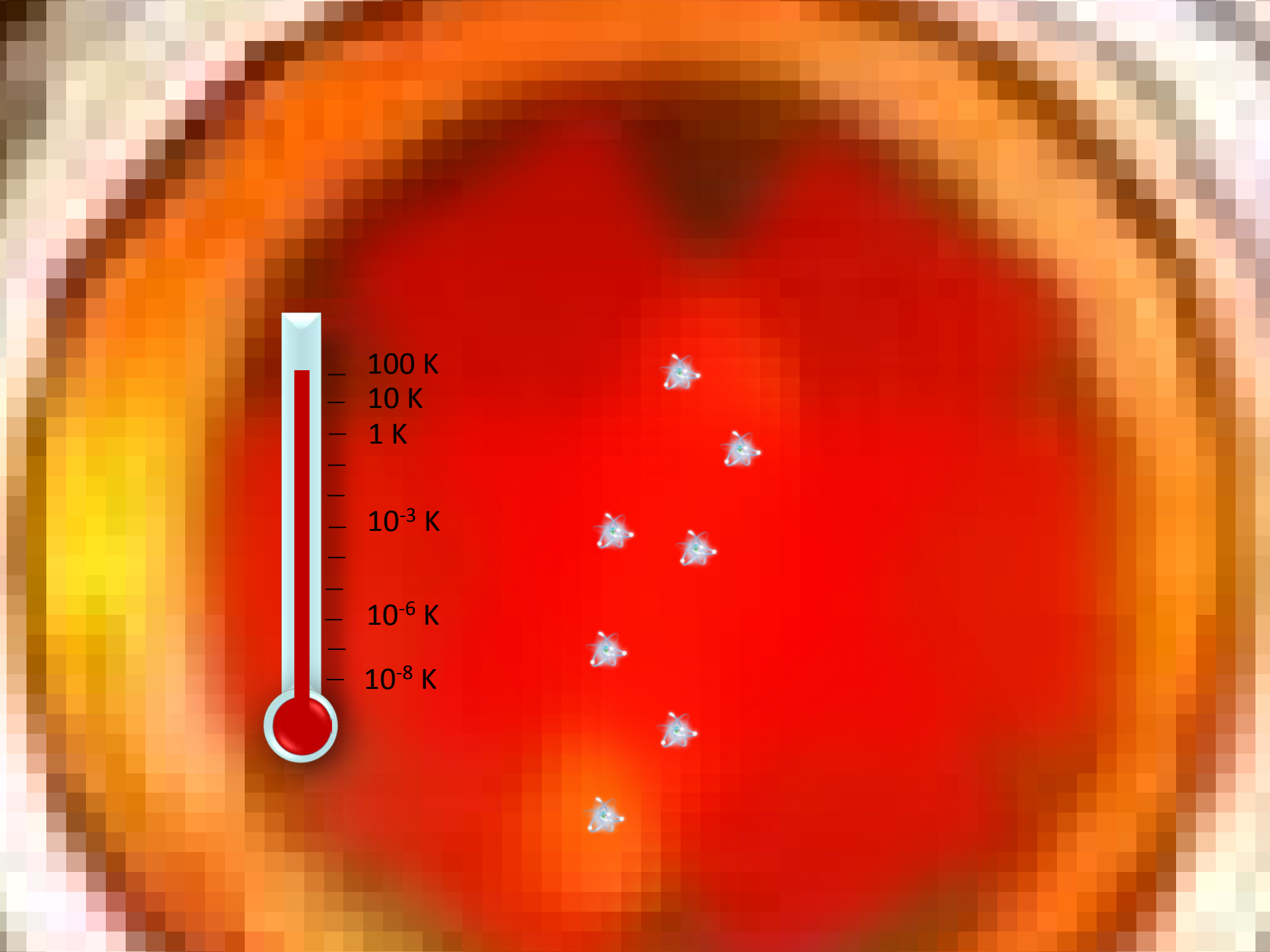
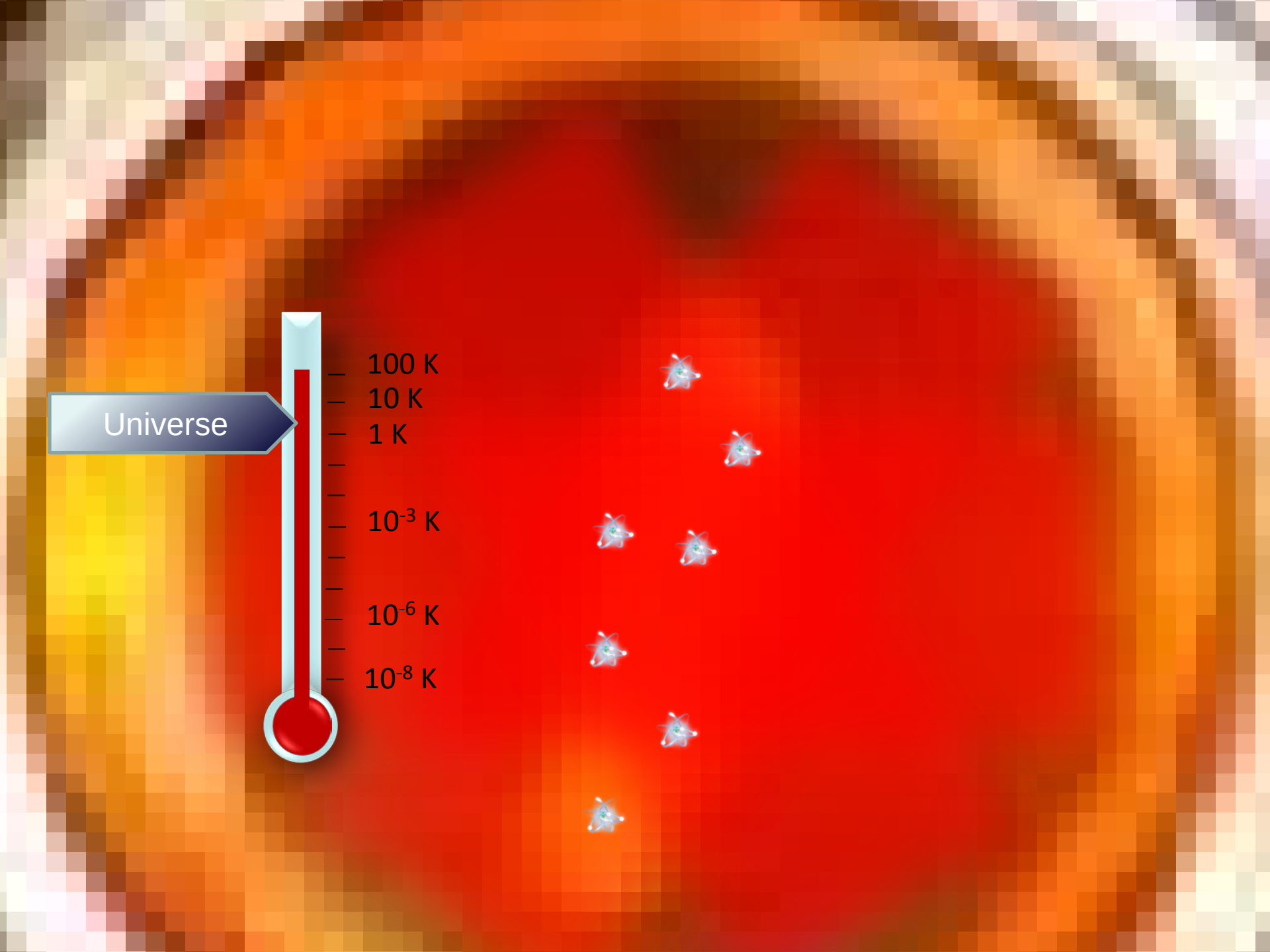
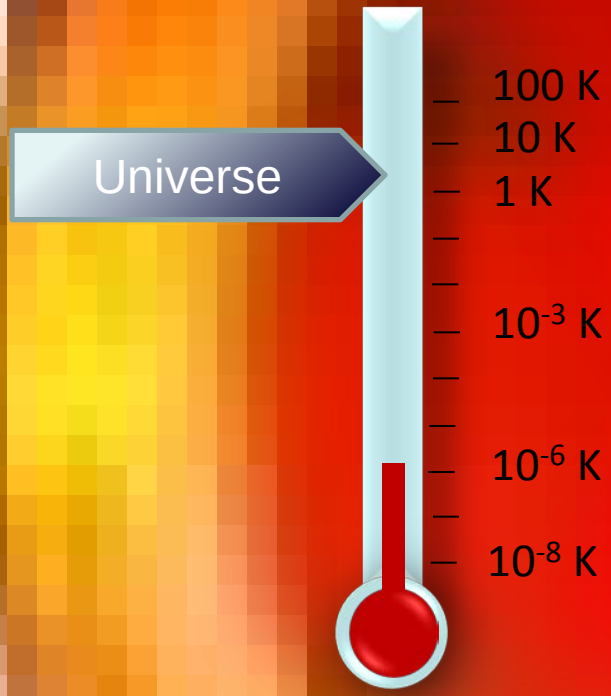


Figure from Pascal Naidon (RIKEN)


100 K
10 K
1 K
$10^{-3}$ K
$10^{-6}$ K
$10^{-8}$ K


Universe
100 K
10 K
1 K
$10^{-3}$ K
$10^{-6}$ K
$10^{-8}$ K

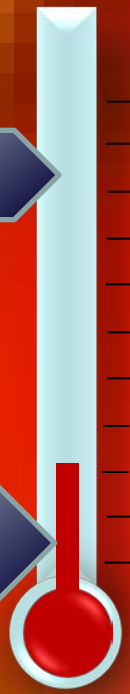


At these temperatures, *quantum effects* appear.

Quantum motion is described by waves

Universe

atomic Condensation



100

10

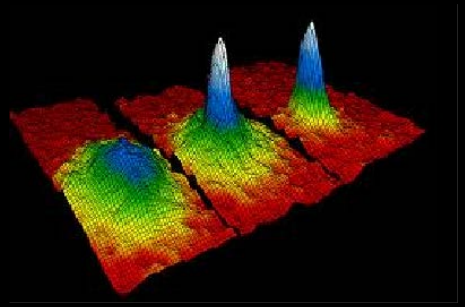
1 K

$10^{-1}$

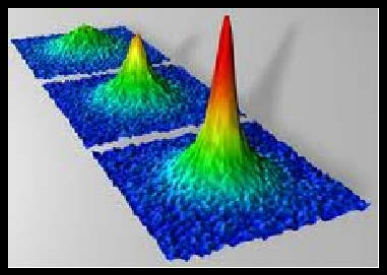
$10^{-6}$

$10^{-8}$

**Bose-Einstein Condensate 1995**



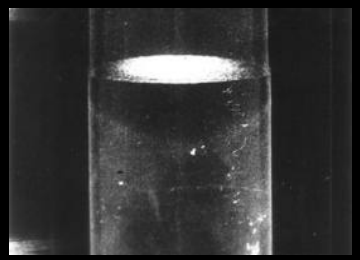
**Fermi superfluid 2003**



**10-7 K**

## QUANTUM FLUIDS (SUPERFLUIDS)

**Superfluid helium**



**$10^{-1}$ K**

**1 K**

**Superconducting electrons**

**10 K**

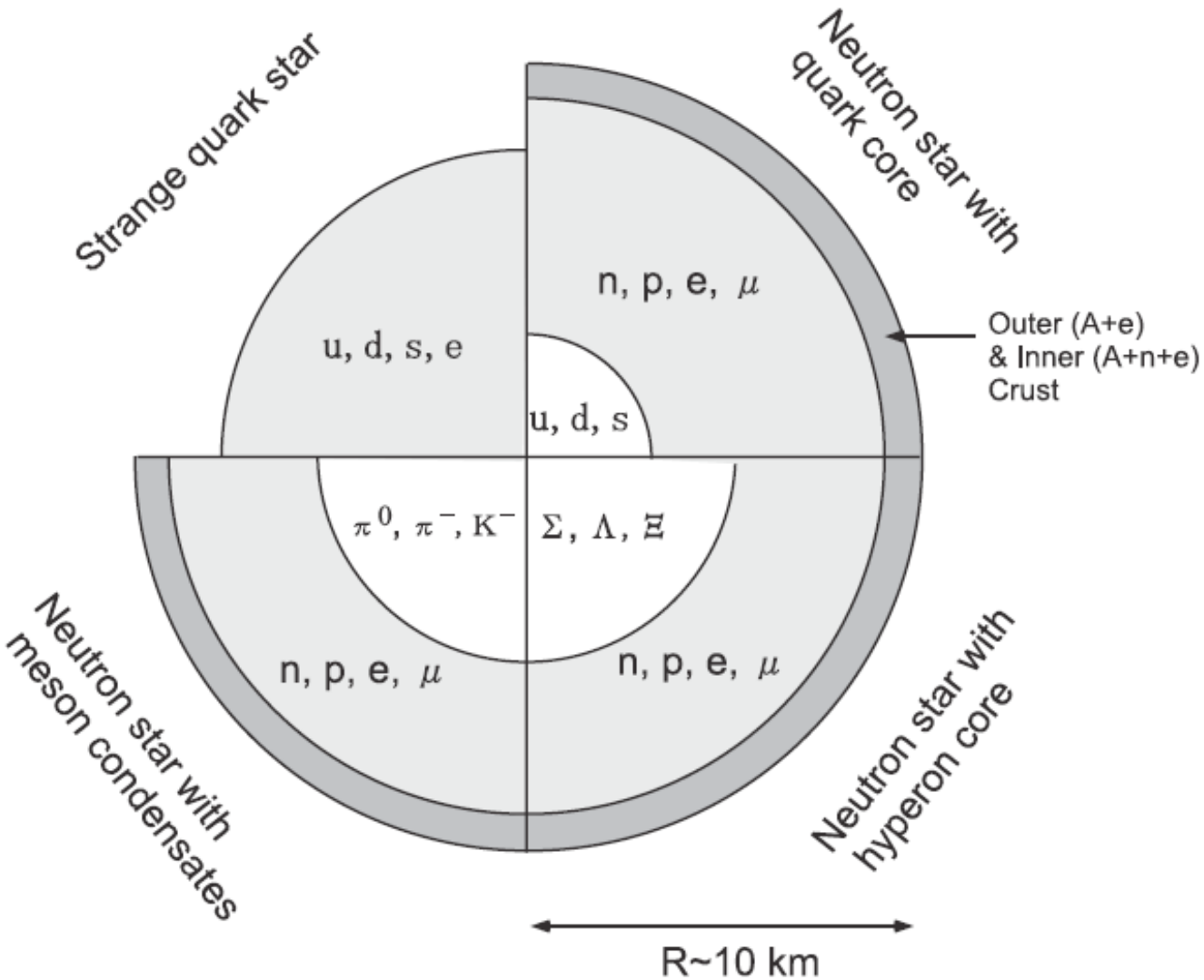
**Superfluid nucleons**

**Superconducting quarks**

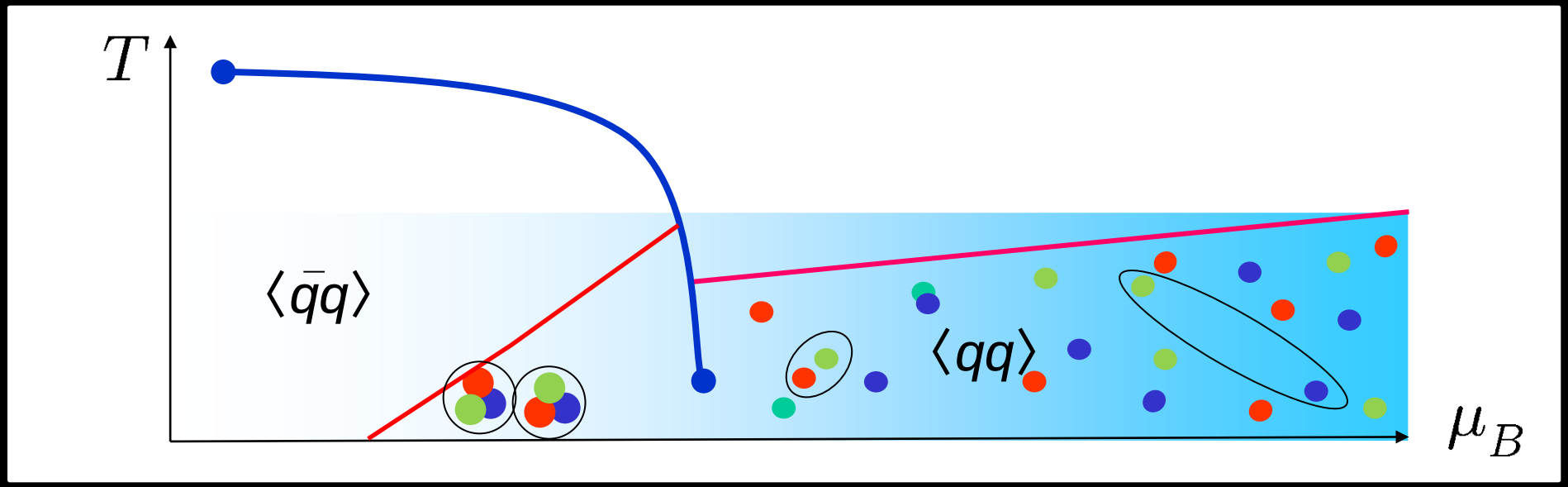
**$10^{9}$ K**

**$10^{10}$K**

# Neutron Star Structure

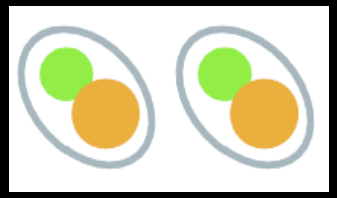


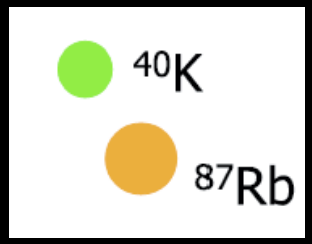
# "Quark-Hadron transition" in boson-fermion mixture of ultracold atoms




Nuclear superfluid ⇔ Fermion+Diquark ⇔ Quark superfluid

Induced superfluid ⇔ Fermi-Bose mixture

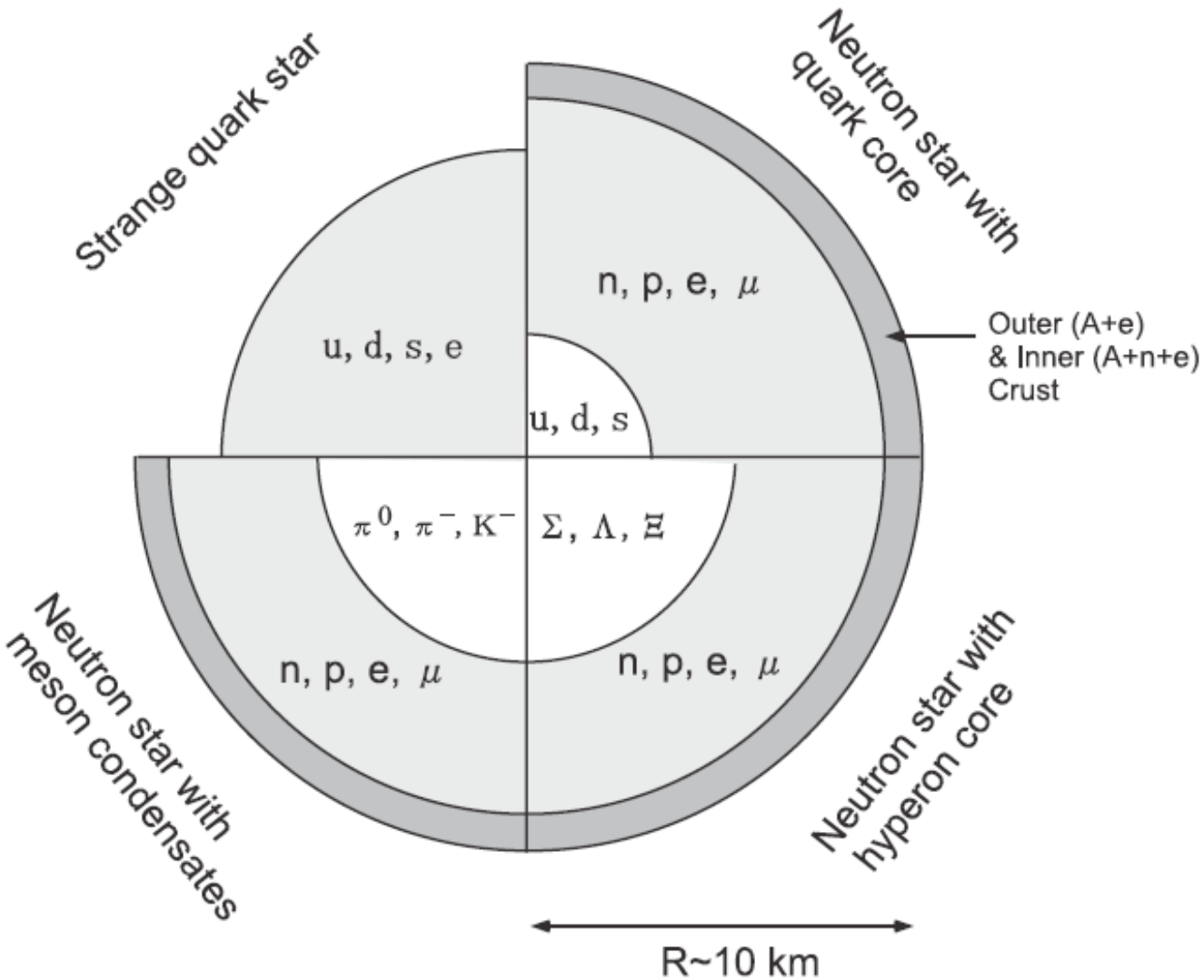





$$a_{\mathrm{NN}}^{\mathrm{Born}} = -\frac{m_{\mathrm{N}}}{2m_{\mathrm{R}}} a_{\mathrm{bf}}$$

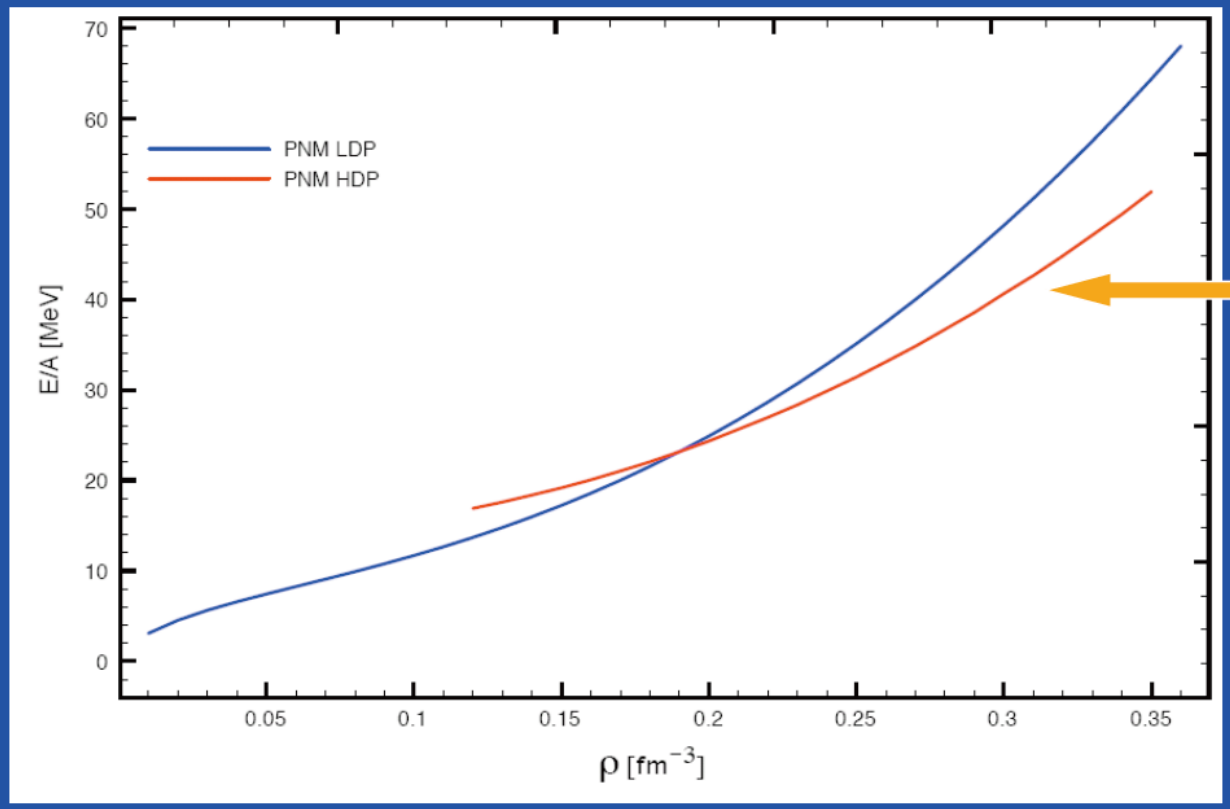
Maeda, Baym & Hatsuda, Phys. Rev. Lett. 103 ('09)

# Neutron Star Structure



# Energy per nucleon in pure neutron matter

*Morales, (Pandharipande) & Ravenhall, in progress*

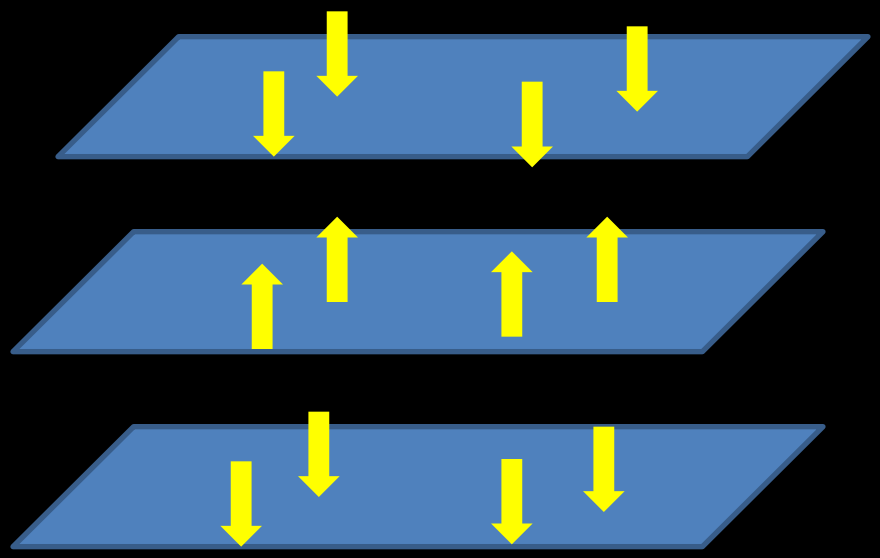


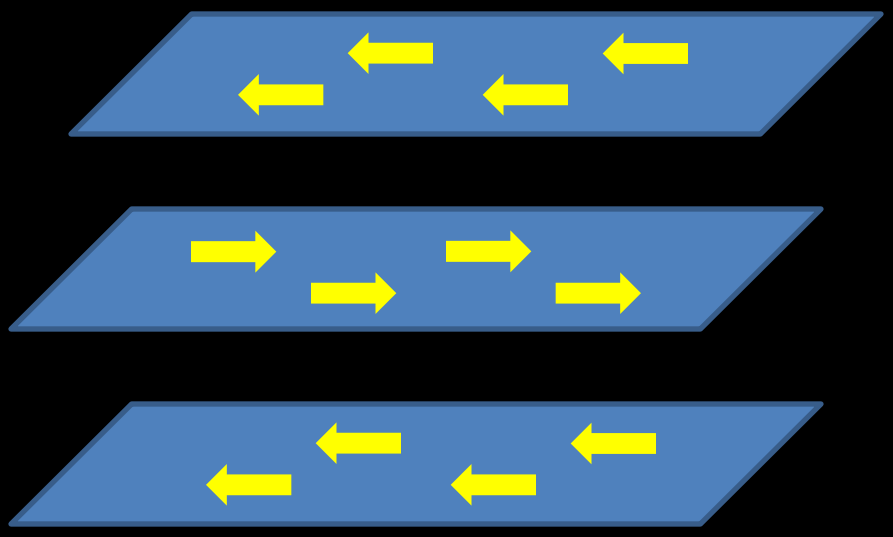

$\pi^0$ condensate

AV-18 + UIV 3-body (IL 3-body too attractive) Improved FHNC algorithms. Two minima!

E/A slightly higher than *Akmal, Pandharipande and Ravenhall, Phys. Rev. C58 (1998) 1804*

# $\pi^0$ and/or $\rho^0$ condensation in neutron matter





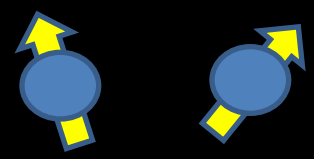
$$(-\nabla^2 + m_\pi{}^2)\,\varphi_c(\boldsymbol{r}) = (f/m_\pi)\,\boldsymbol{\nabla}\cdot\langle\psi^\dagger\boldsymbol{\sigma}\psi\rangle$$

$$(-\nabla^2 + m_\rho{}^2)\,\boldsymbol{\rho}_c(\boldsymbol{r}) = (f_\rho/m_\rho)\,\boldsymbol{\nabla}\times\langle\psi^\dagger\boldsymbol{\sigma}\psi\rangle$$

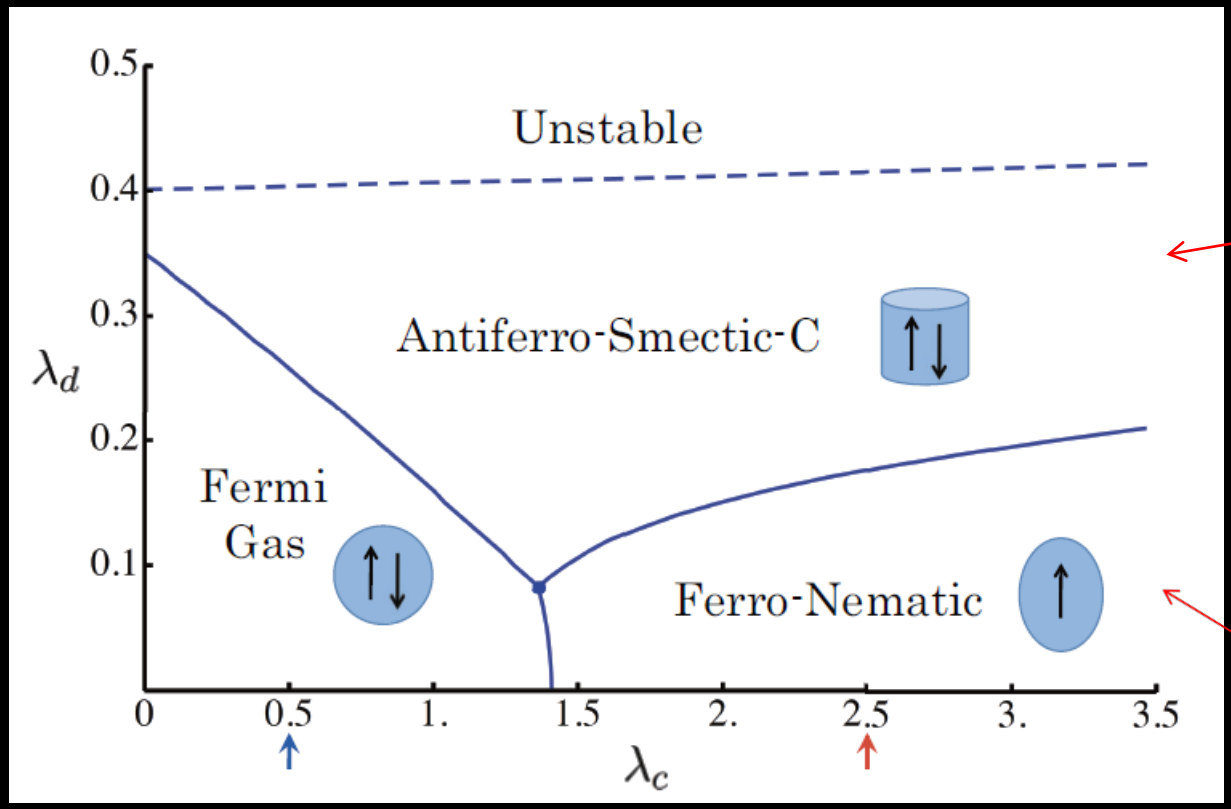
A. B. Migdal, NPA (1972)
Takatsuka, Tamagaki & Tatsumi,
Prog. Theor. Phys. Suppl. 112 ('93) 67

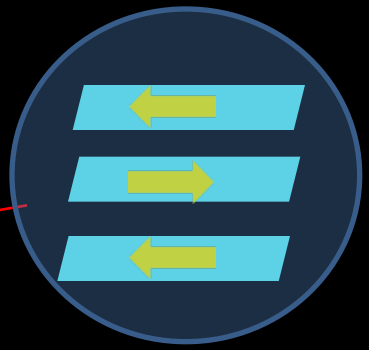
Kunihiro, Prog. Theor. Phys. 60 ('78) 1229

# “Meson condensation” in ultracold dipolar atoms/molecules



$$U = \frac{\mu^2}{r^3}\{\vec{\sigma}_1 \cdot \vec{\sigma}_2 - 3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r})\} + g\delta(\vec{r}),$$

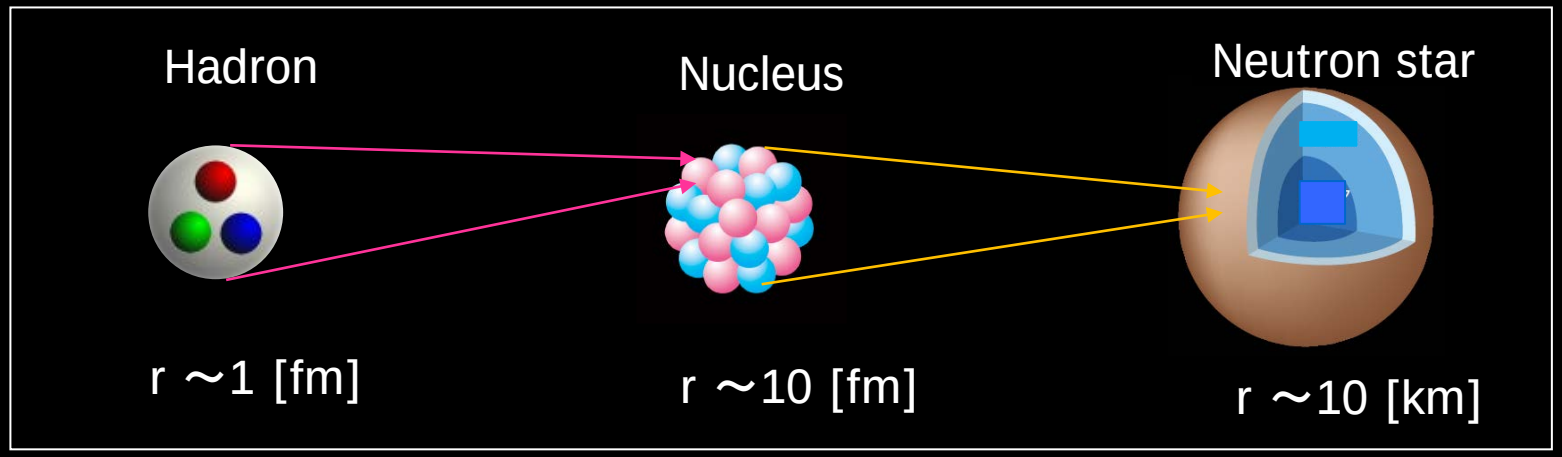


Maeda, Hatsuda & Baym, arXiv:1205.1086 [cond-mat.quant-gas]

Fregoso & Fradkin, PRL 103 (2009)

# “Summary”




1. New era of Latice QCD arrived
   → massive physical point simulations will start soon
   (u,d,s-flavor, L ~ 6fm, a ~ 0.05fm, $m_\pi$=135MeV)

2. LQCD started to provide 1st principle inputs for QGP physics
   → EoS (P(T), ε(T)), spectral functions, etc ⇔ RHIC, LHC

3. LQCD provides qualitative pictures on the NN, YN, YY & NNN forces
   → better constraints of the EoS of dense matter
   → physical point simulations will start soon

4. Ultracold atom/molecule experiments
   → better understanding of the hadron-quark transition in dense matter