# Charmonium properties in hot quenched lattice QCD

Heng-Tong Ding

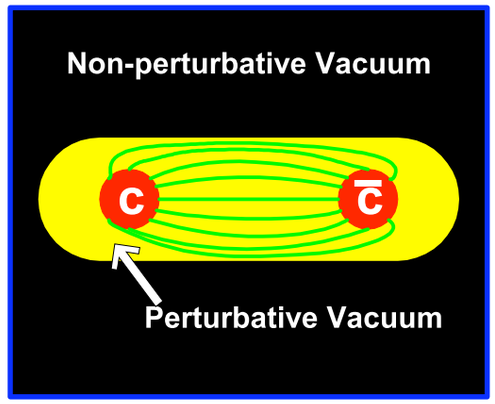
Brookhaven National Lab

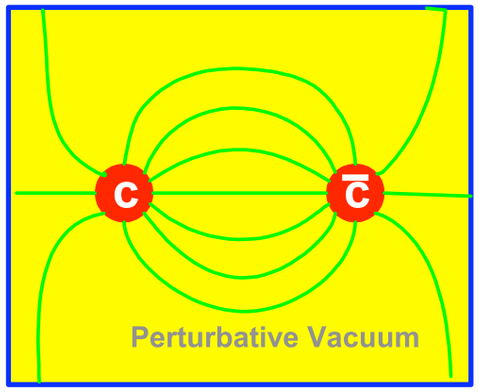
A. Francis, O. Kaczmarek, F. Karsch, W. Söldner, H. Satz

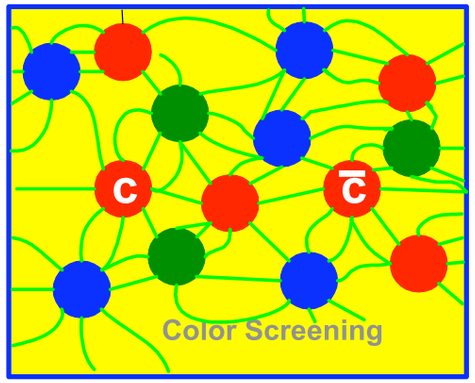
**arXiv:1204.4945**

# Charmonium at T>0

★ The suppression of the charmonium production can be a signature of the formation of the Quark Gluon Plasma [Mastui & Satz '86]




$$V(r) = -\frac{\alpha_{eff}}{r} + \sigma\, r$$

$$V(r,T) = -\frac{\alpha_{eff}}{r}\, e^{-r/\lambda_D(T)}$$

- Confining term: string tension becomes weaker with increasing T and vanishes at $T>T_c$
- Color screening due to the presence of the QGP

E(r) of the ccbar system should have a minimum wrt r if a bound state is possible

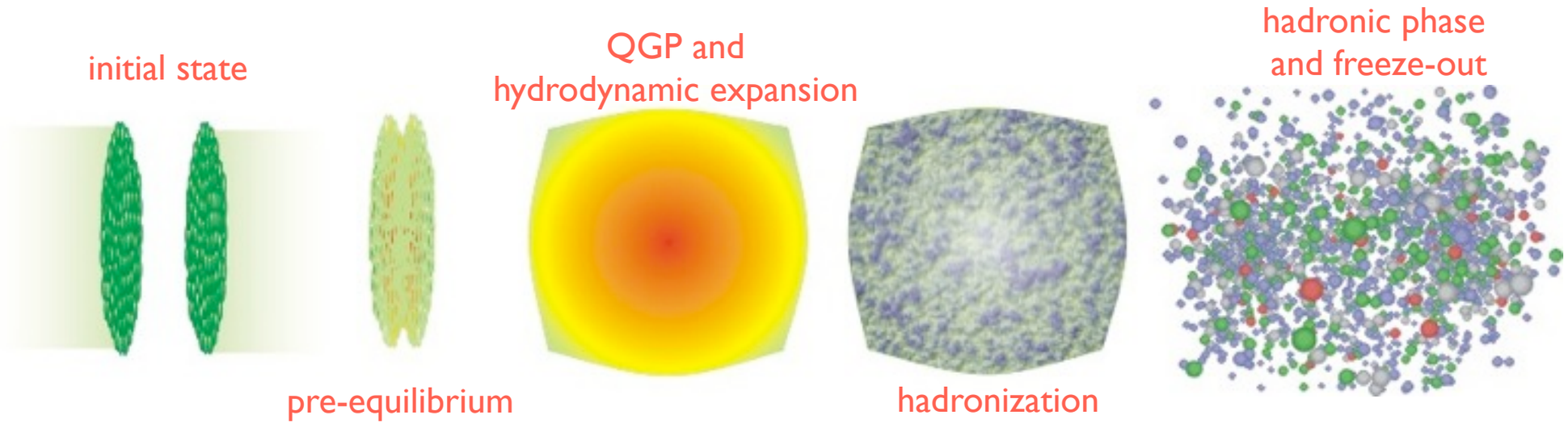
The condition for an extremum of E(r) is

$$x(1+x)\, e^{-x} = \frac{2}{m_c \alpha_{eff} \lambda_D} \quad \text{with} \quad x = r/\lambda_D$$

No solutions → the bound state will be "melted"

**QGP thermometers**

# The interpretation of experimental data




- Use nuclear modification factor $R_{AA}$ to define the charmonium modification

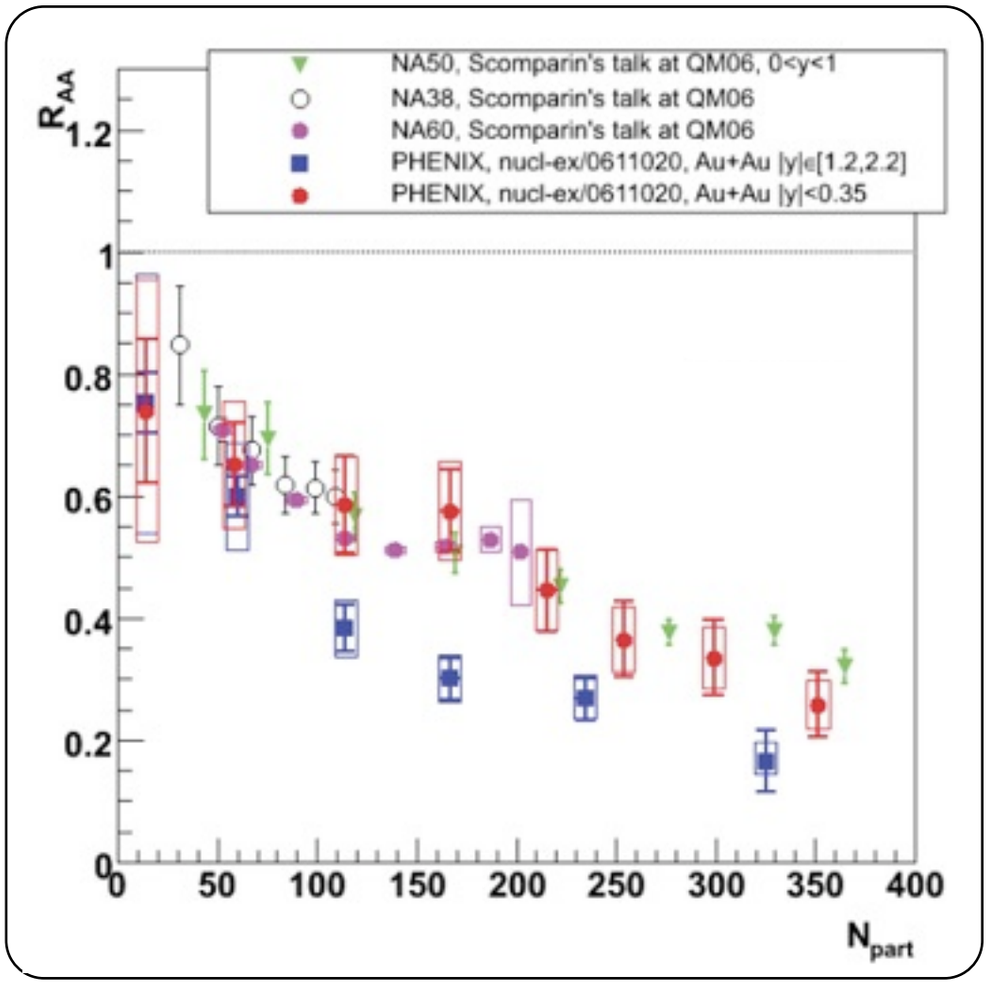
$$R_{AA}^{J/\psi} = \frac{\mathrm{d}N_{AA}^{J/\psi}/\mathrm{d}y}{N_{coll} \cdot \mathrm{d}N_{pp}^{J/\psi}/\mathrm{d}y}$$

- Two different classes of effects on the modification of the charmonium production

★ Cold nuclear effects: nuclear shadowing, comovers...

★ Hot medium effects: color screening effects...

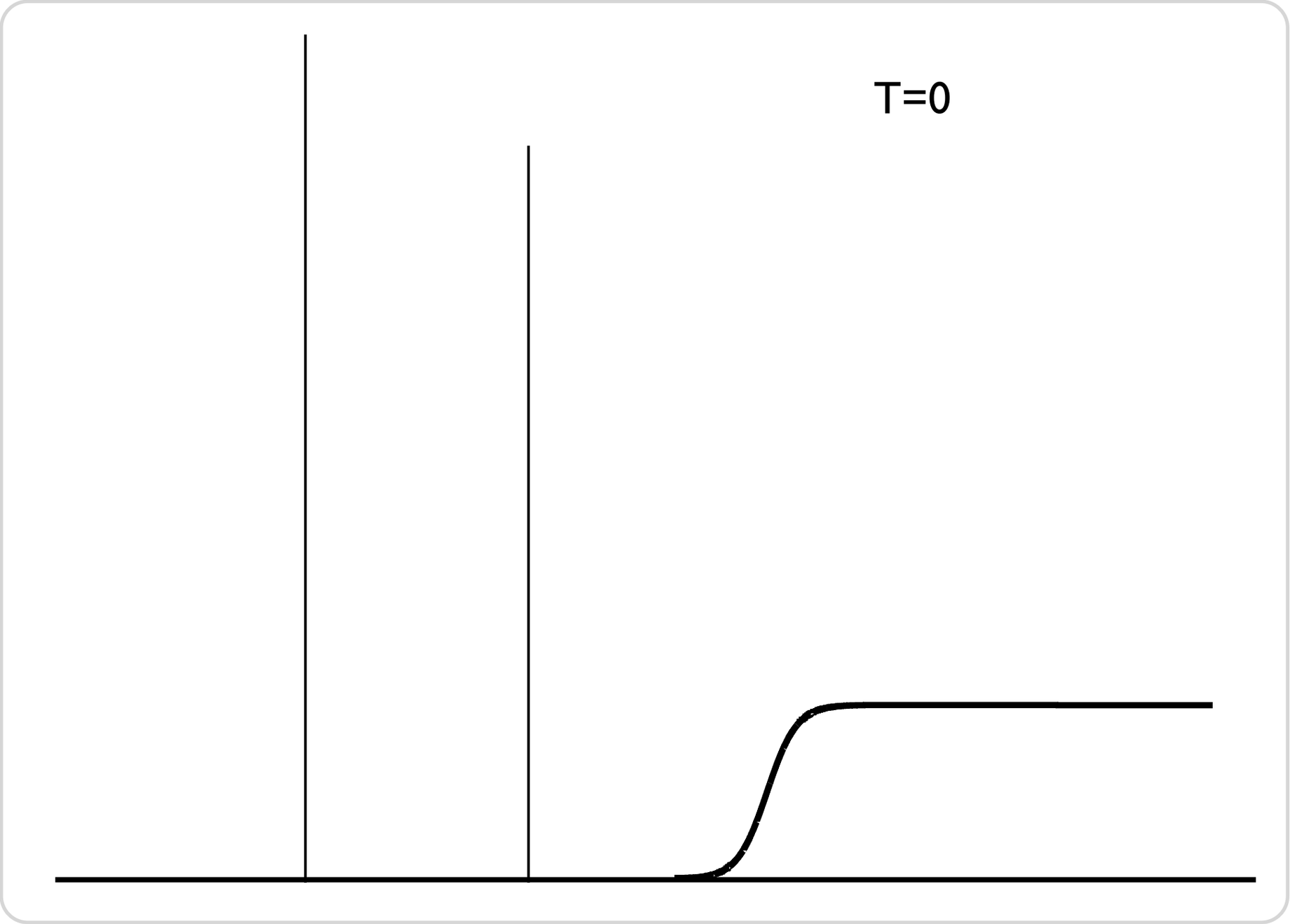
- Crucial to disentangle these two classes of effects to understand experimental pictures




comparison produced by R. Granier de Cassagnac

# Charmonium spectral functions

All the information about the properties of charmonium states are embedded in their spectral functions (spf)



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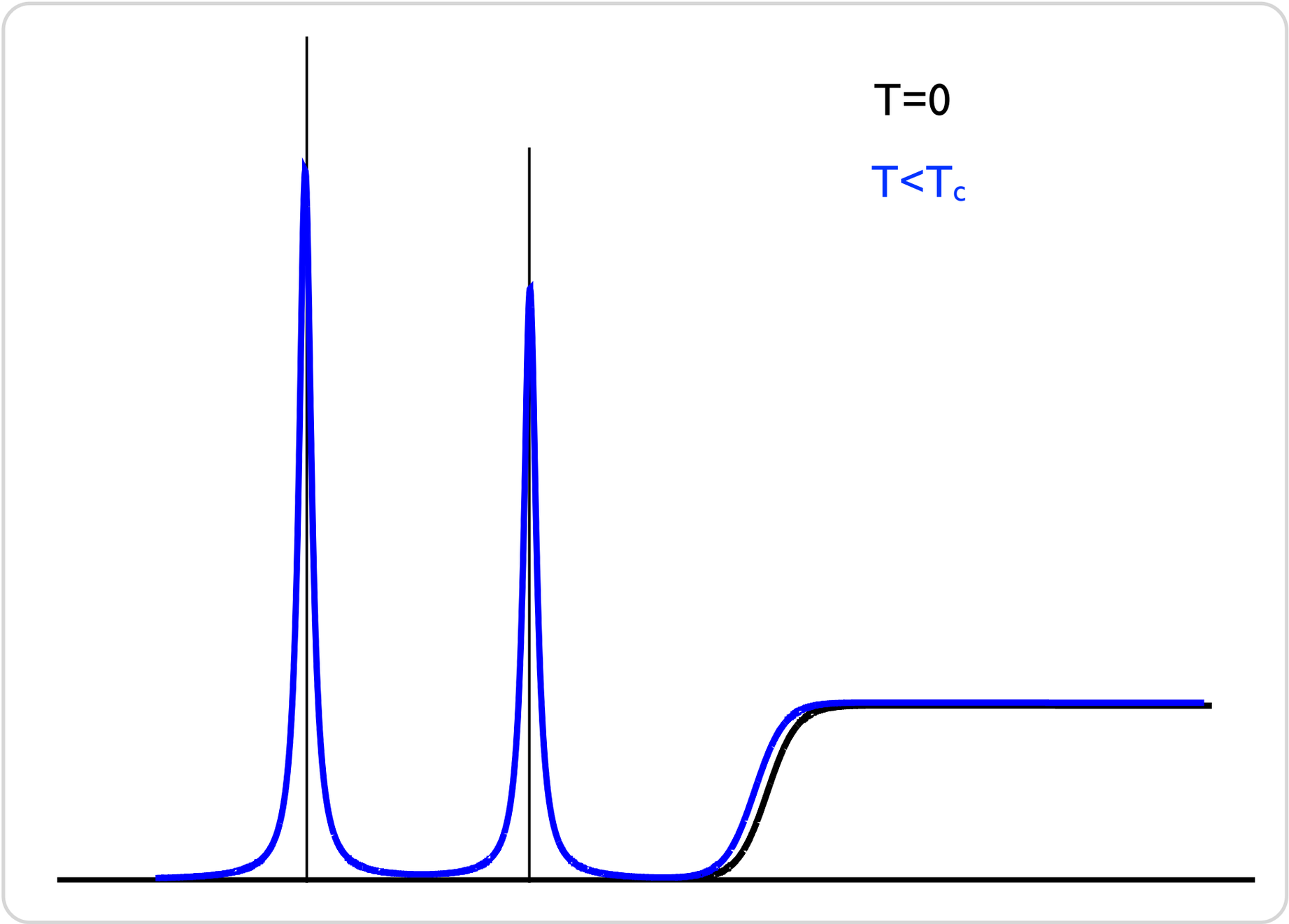

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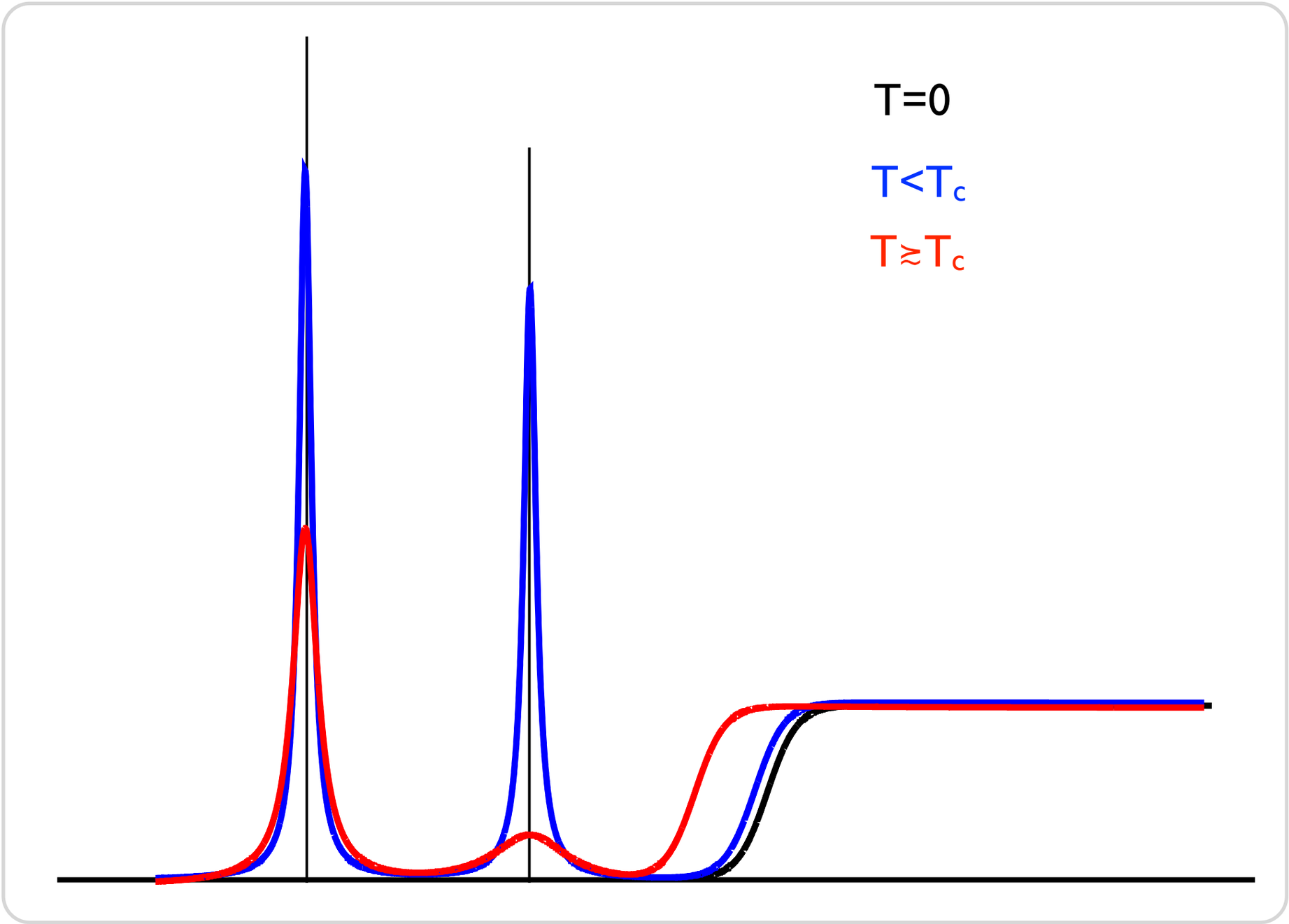

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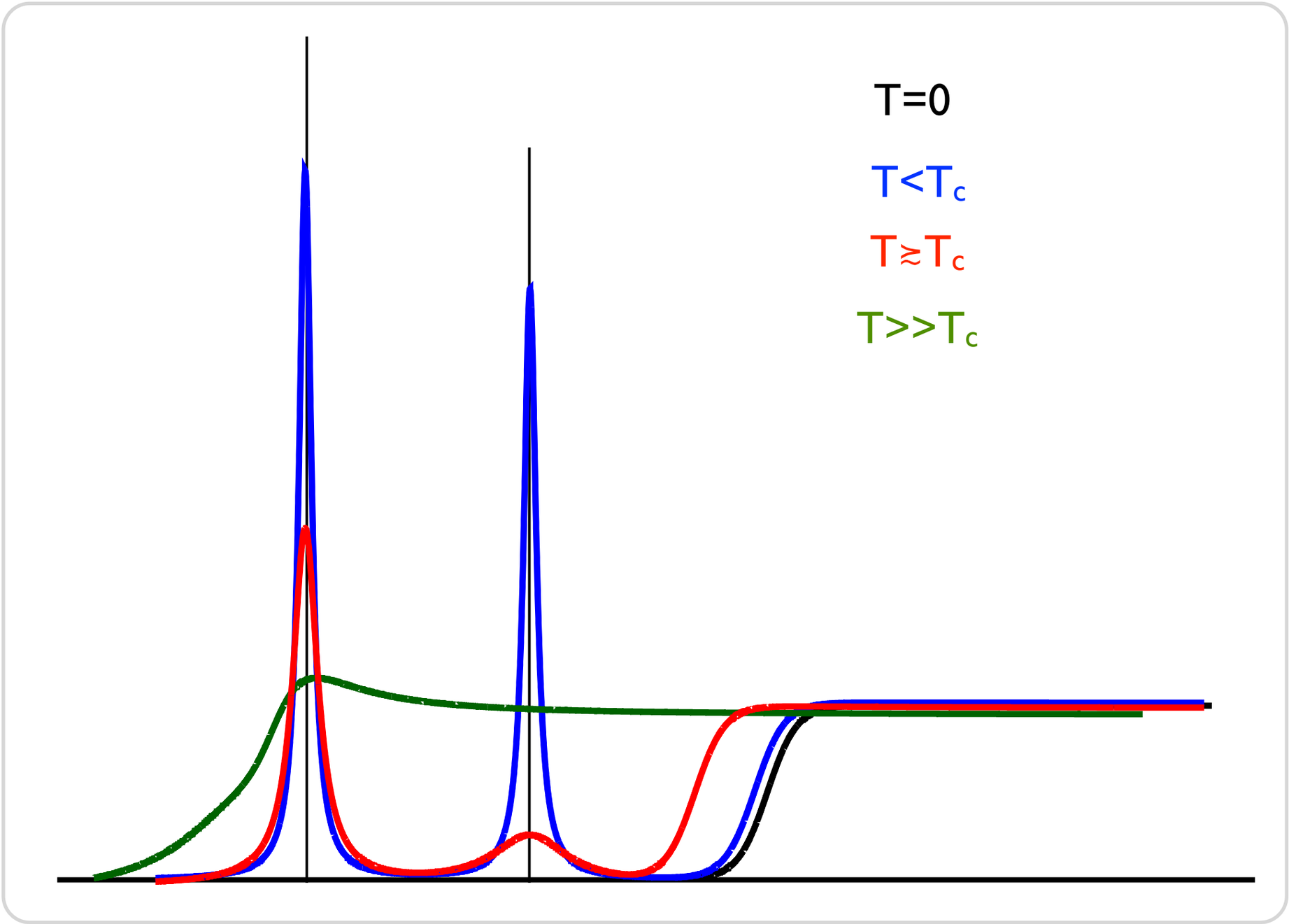

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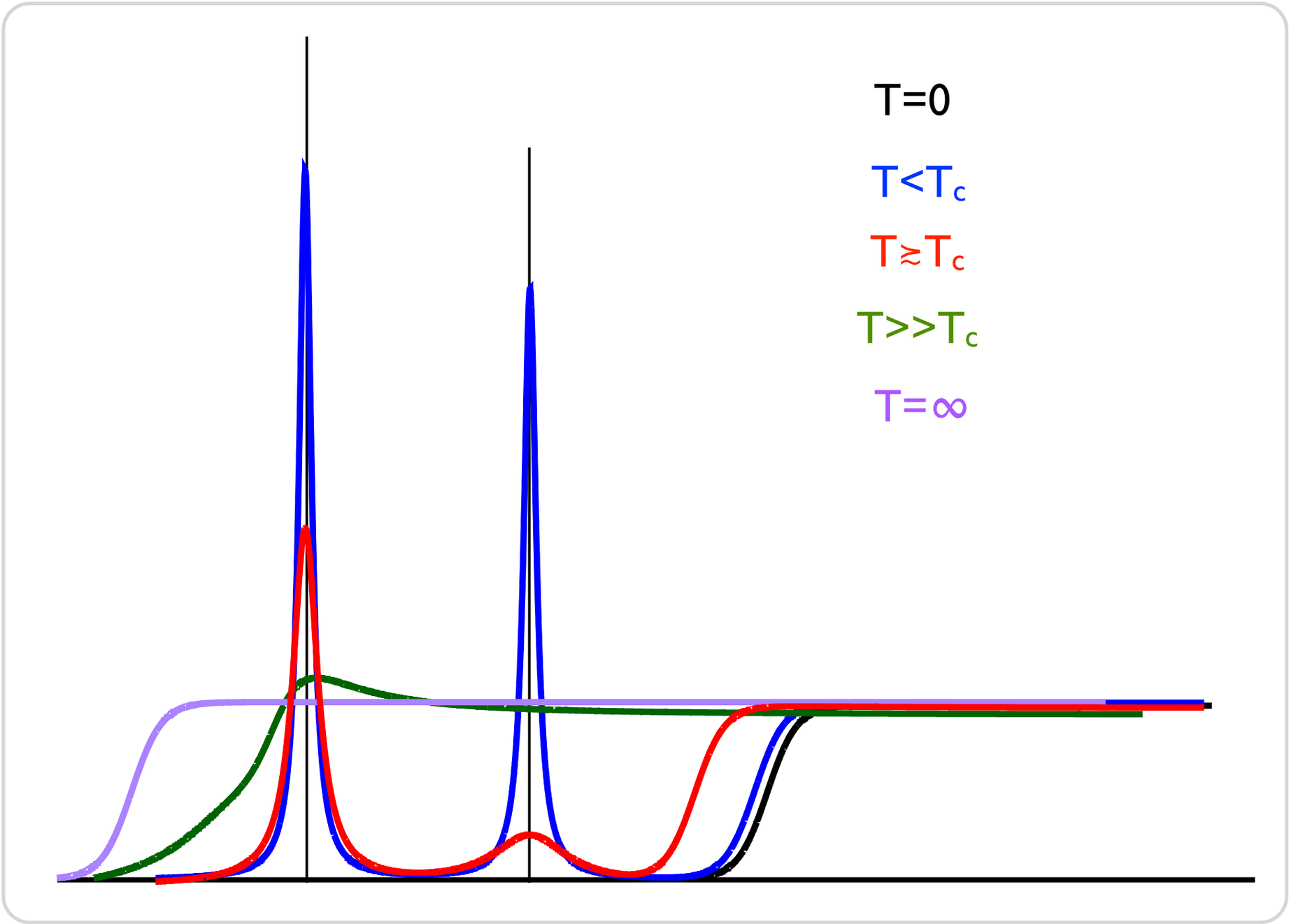
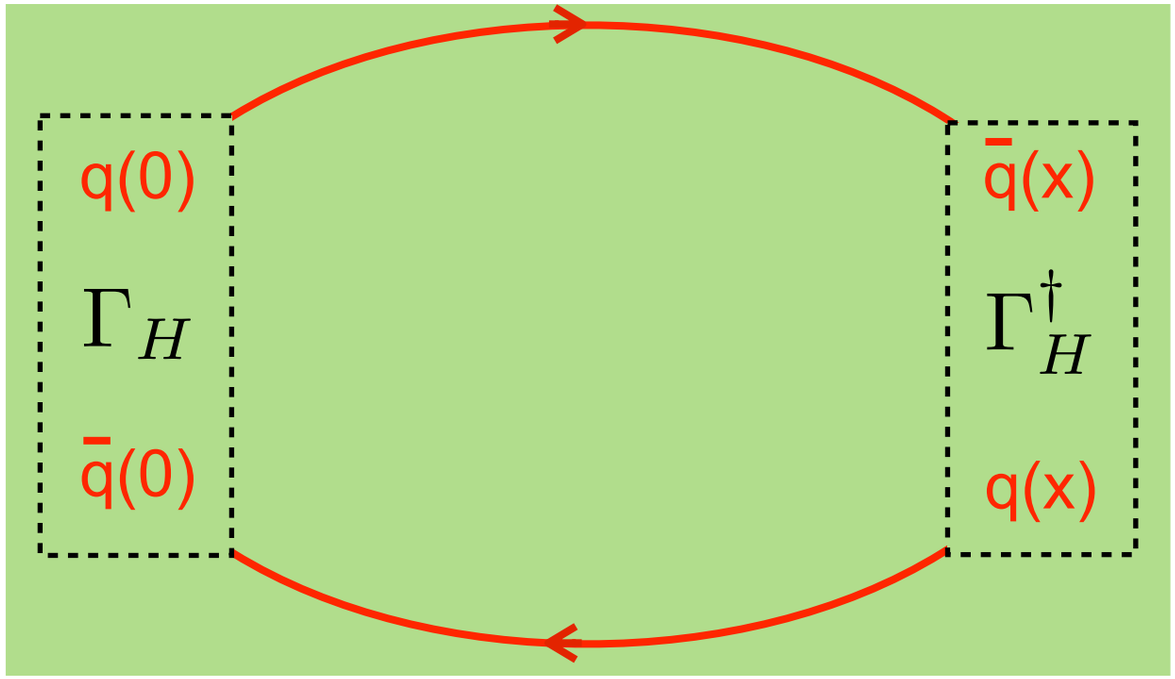


# charmonium correlation and spectral functions

$$G_H(\tau,T) = \sum_{\vec{x}} \left\langle J_H(0,\vec{0}) J_H^\dagger(\tau,\vec{x}) \right\rangle$$

$$J_H(\tau,\vec{x}) = \bar{q}(\tau,\vec{x})\,\Gamma_H\, q(\tau,\vec{x})$$

| Channel | $\Gamma_H$ | $^{2S+1}L_J$ | $J^{PC}$ | $c\bar{c}$ | M($c\bar{c}$)[GeV] |
|---|---|---|---|---|---|
| PS | $\gamma_5$ | $^1S_0$ | $0^{-+}$ | $\eta_c$ | 2.980(1) |
| VC | $\gamma_\mu$ | $^3S_1$ | $1^{--}$ | $J/\psi$ | 3.097(1) |
| SC | 1 | $^3P_0$ | $0^{++}$ | $\chi_{c0}$ | 3.415(1) |
| AV | $\gamma_5\gamma_\mu$ | $^3P_1$ | $1^{++}$ | $\chi_{c1}$ | 3.510(1) |




$$G(\tau,T) = D^+(-i\tau)\,, \qquad \rho(\omega) = 2\mathrm{Im}D^R(\omega) = D^+(\omega) - D^-(\omega)$$

## Spectral representation

$$G(\tau,T) = \int \frac{\mathrm{d}\omega}{2\pi}\, \rho(\omega,T)\; K(\omega,\tau,T); \qquad K(\tau,\omega,T) = \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}$$

# Lattice setup

- ★ non-perturbatively clover improved Wilson fermions
- ★ isotropic quenched lattice
- ★ very fine lattice close to continuum
- ★ simulation parameters tuned to reproduce nearly physical J/ψ mass
- ★ large $N_T$ makes the extraction of spf more reliable

| $\beta$ | $a$ [fm] | $a^{-1}$[GeV] | $L_\sigma$ [fm] | $c_{\rm SW}$ | $\kappa$ | $N_\sigma^3 \times N_\tau$ | T/T$_c$ | $N_{conf}$ |
|---|---|---|---|---|---|---|---|---|
| 6.872 | 0.031 | 6.432 | 3.93 | 1.412488 | 0.13035 | $128^3 \times 32$ | 0.74 | 126 |
| | | | | | | $128^3 \times 16$ | 1.49 | 198 |
| 7.457 | 0.015 | 12.864 | 1.96 | 1.338927 | 0.13179 | $128^3 \times 64$ | 0.74 | 179 |
| | | | | | | $128^3 \times 32$ | 1.49 | 250 |
| 7.793 | 0.010 | 18.974 | 1.33 | 1.310381 | 0.13200 | $128^3 \times 96$ | 0.73 | 234 |
| | | | | | | $128^3 \times 48$ | 1.46 | 461 |
| | | | | | | $128^3 \times 32$ | 2.20 | 105 |
| | | | | | | $128^3 \times 24$ | 2.93 | 81 |

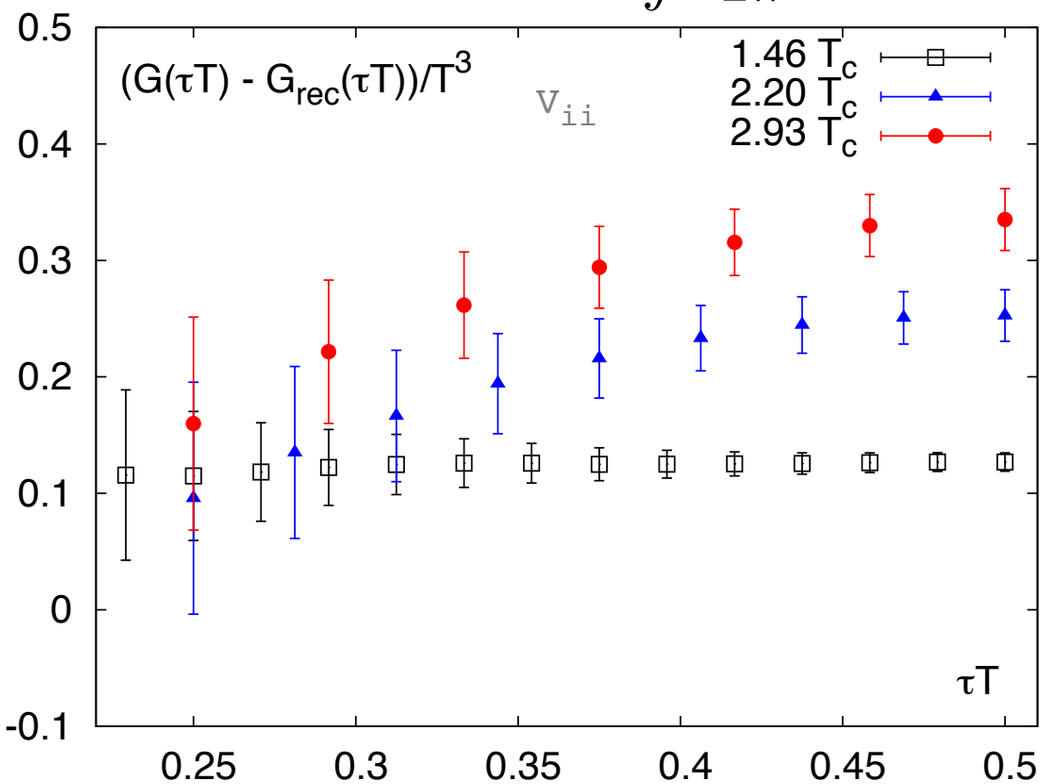
# differences of spf between above and below $T_c$

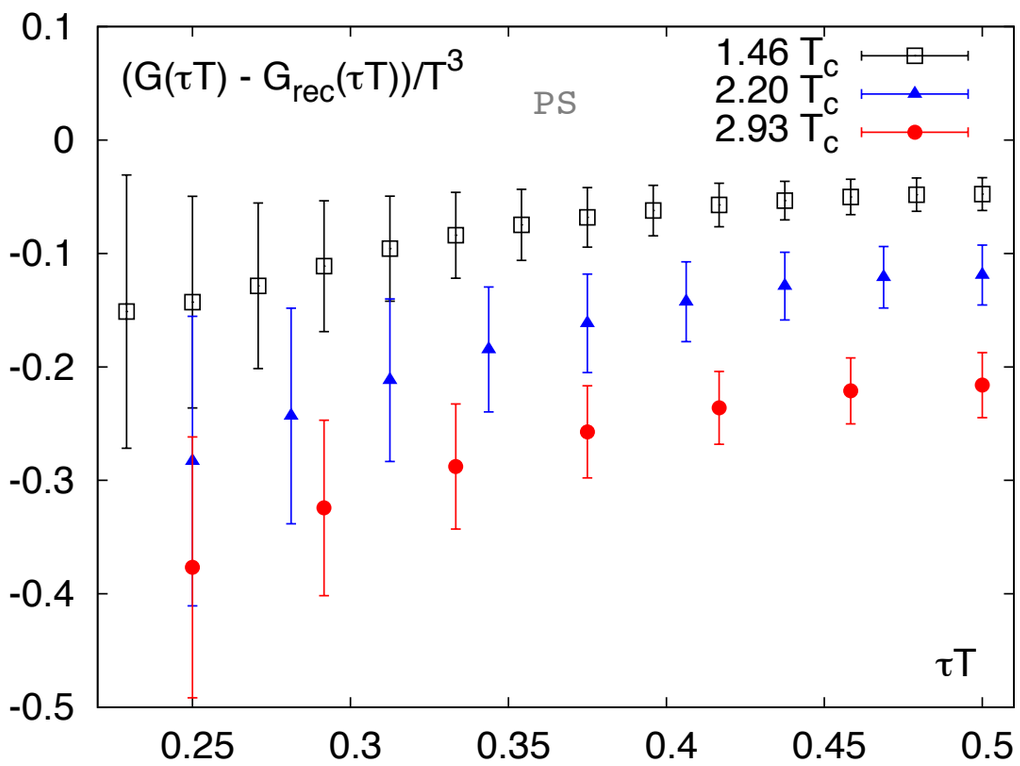
reconstructed correlator:

$$G_{rec}(\tau,T) = \int \frac{d\omega}{2\pi}\, \rho(\omega, 0.73T_c)\; K(\omega,\tau,T)$$ calculated from $G(\tau,0.73T_c)$ directly

differences of the correlation functions

$$G(\tau,T) - G_{rec}(\tau,T) = \int \frac{d\omega}{2\pi}\, \Delta\rho(\omega)\; K(\omega,\tau,T), \quad \Delta\rho(\omega) = \rho(\omega,T) - \rho(\omega,0.73T_c)$$




- differences of correlators increase with $\tau T$ in both channels
- $\Delta\rho(\omega)$ must be negative in some energy regions at $T>T_c$
- spf in PS channel suffers larger thermal modifications than in $V_{ii}$ channel

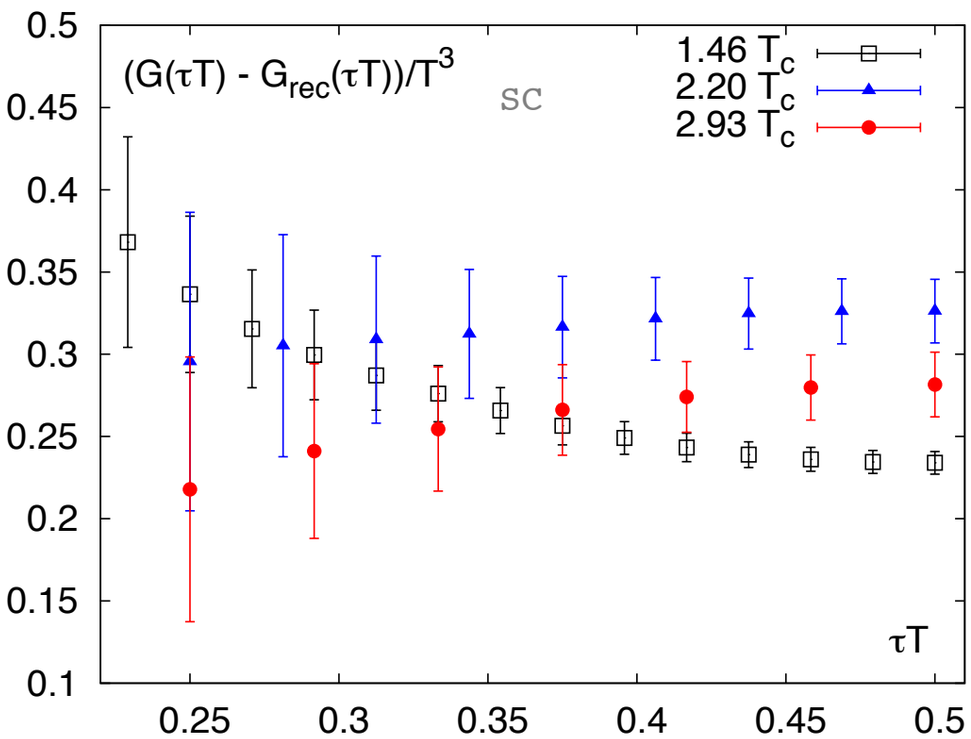
# differences of spf between above and below $T_c$

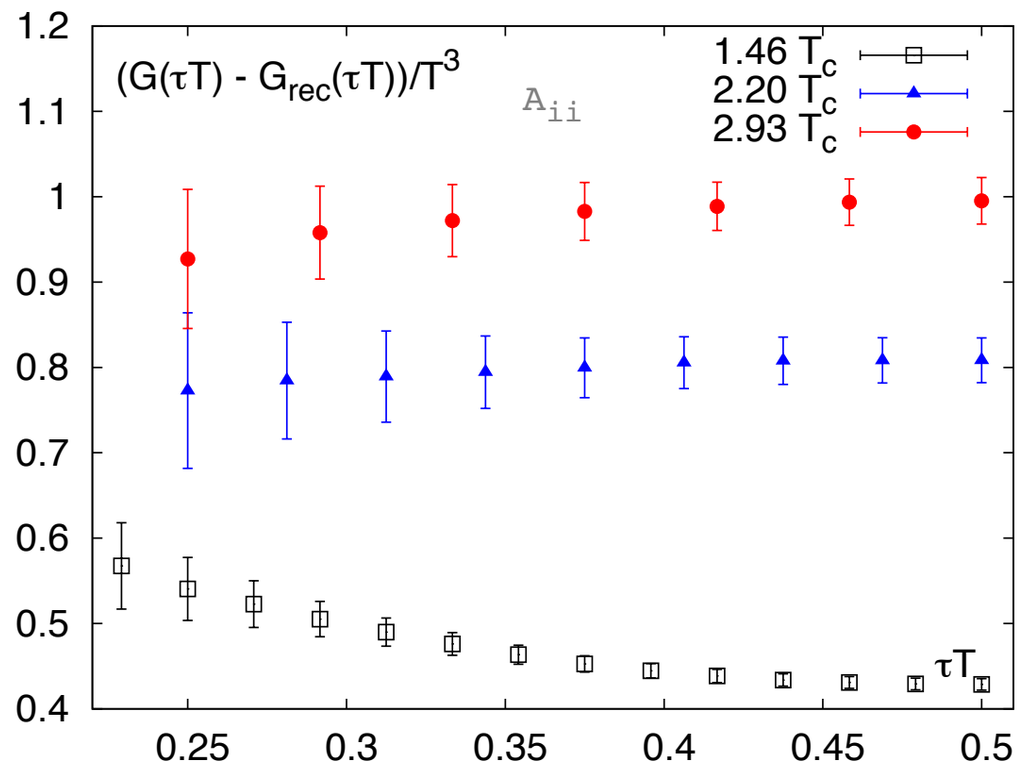
Taylor expansion of correlator differences about $\tau T=1/2$

$$(G(\tau,T)-G_{\rm rec}(\tau,T))/T^3 = \int \frac{d\omega}{2\pi}\,\Delta\rho(\omega,T)K(\omega,\tau T)/T^3 = \frac{\Delta G^{(0)}}{T^3}+\frac{\Delta G^{(2)}}{T^3}(\tau T-1/2)^2+\frac{\Delta G^{(4)}}{T^3}(\tau T-1/2)^4 + \cdots$$

thermal moments: $\Delta G^{(n)} = \frac{1}{n!}\int \frac{d\omega}{2\pi}\left(\frac{\omega}{T}\right)^n \frac{\Delta\rho(\omega)}{\sinh(\omega/2T)}$

If changes only happen in $\omega<T$ region, $\Delta G^{(0)}>2\Delta G^{(2)}>24\Delta G^{(4)}$ should hold




- $\Delta\rho(\omega)$ must be negative in some energy regions at $T=2.20, 2.93T_c$
- At $T=1.46\,T_c$ , $\Delta G^{(0)}>2\Delta G^{(2)}>24\Delta G^{(4)}$ does not hold in both SC and $A_{ii}$ channels
- At $T=1.46\,T_c$ , thermal modifications occurs to $\omega>T$ region

# Estimation of charm quark diffusion at 1.46 $T_c$

$$D = \frac{1}{6\chi_{00}} \lim_{\omega \to 0} \frac{\rho^V(\omega, T)}{\omega}$$

Assume: from T=0.73 $T_c$ to T=1.46 $T_c$, only the very low frequency part ($\omega$<T) of the vector spectral function changes

Fit to the value of $G(\tau T=1/2)-G_{rec}(\tau T=1/2)$ at 1.46 $T_c$

Ansatz of the very low frequency part of spectral function:

I $$\rho(\omega \ll T) = 2\chi_{00} \frac{T}{M} \frac{\omega\eta}{\omega^2 + \eta^2}, \quad \eta = \frac{T}{MD}$$

| | | |
|---|---|---|
| M=1.0GeV | ➜ | $2\pi TD \approx 0.6$ |
| M=1.8GeV | ➜ | $2\pi TD \approx 3.6$ |

II $$\rho(\omega \ll T) = b\,\omega$$

$$2\pi TD = \frac{\pi T}{3\chi_{00}} b \approx 2$$

# Maximum Entropy Method [Asakawa, Hatsuda & Nakahara, '01]

- Hard to extract spectral function (spf)

$$G(\tau,T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\, K(\tau,\omega,T)\, \rho(\omega,T)$$

Discretized $\mathcal{O}(10)$ — Continuous $\mathcal{O}(10^3)$

$\chi^2$ fitting inconclusive

- Maximum Entropy Method (MEM) ⟵ Bayesian theorem
  - A method to obtain the most probable image from insufficient data
  - Ingredients of MEM: $P[\sigma|GH] \propto P[G|\sigma H]\; P[\sigma|H]$

$P[G|\sigma H] \propto \exp(-\chi^2/2)$ : likelihood function

$P[\sigma|H] \propto \exp(\alpha S)$ : prior probability

ρ: spectral function
G: lattice data
H: prior information on ρ

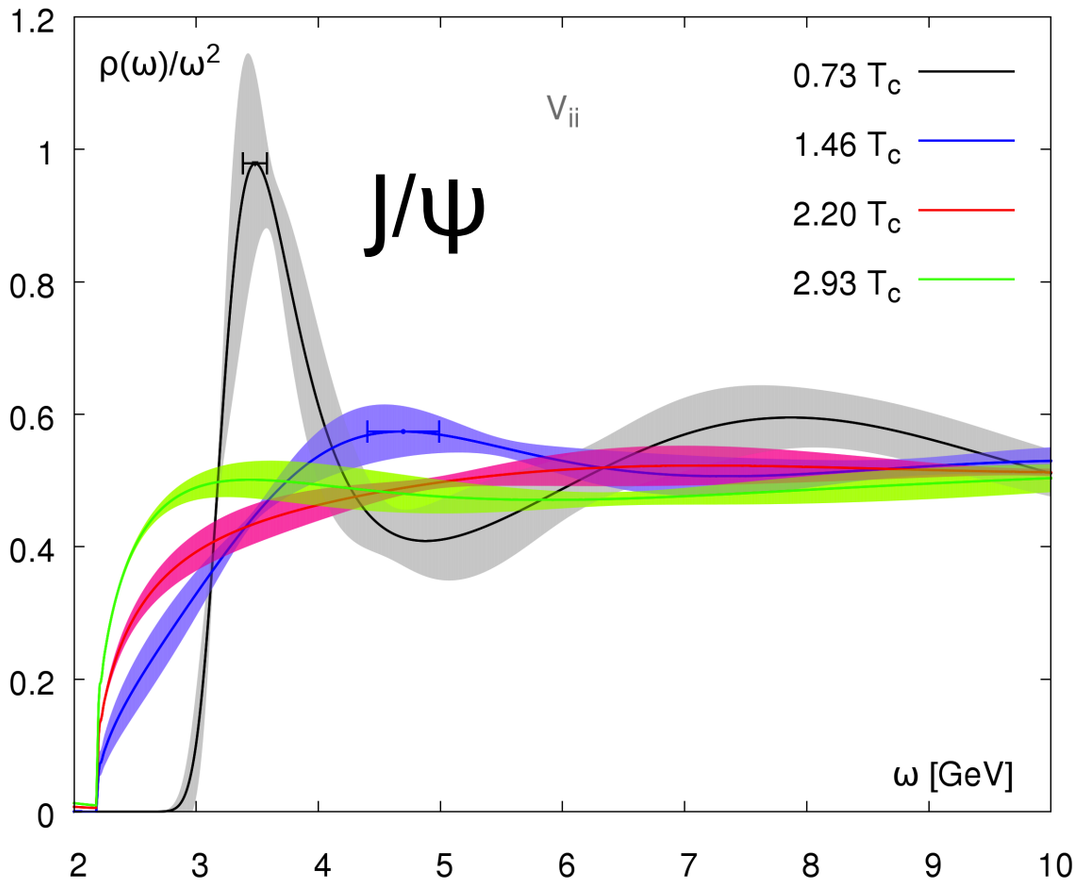
Information entropy: $S = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\left[\rho(\omega) - m(\omega) - \rho(\omega)\ln\left(\frac{\rho(\omega)}{m(\omega)}\right)\right]$

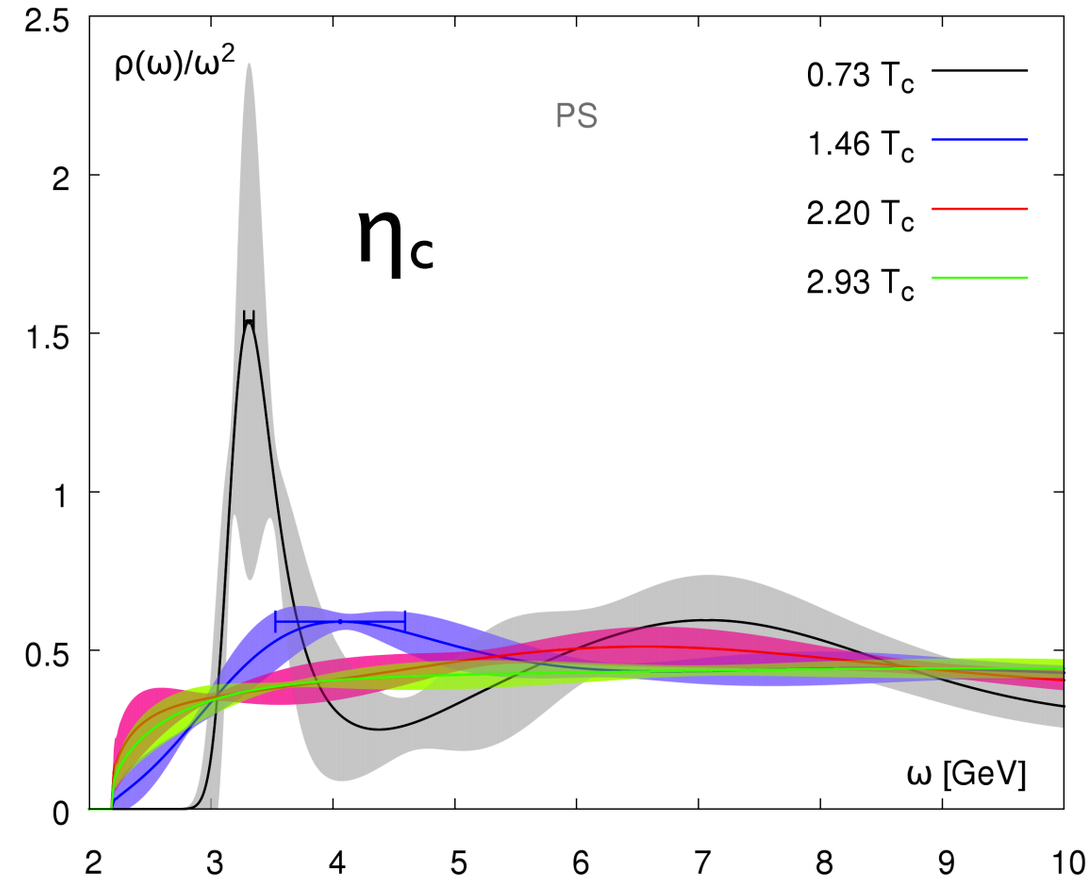
Default Model (DM): m(ω), includes the prior information on ρ, e.g. ρ is positive-definite

DM is the only input parameter in the MEM analysis

- Important to check the dependence of output spf on DMs

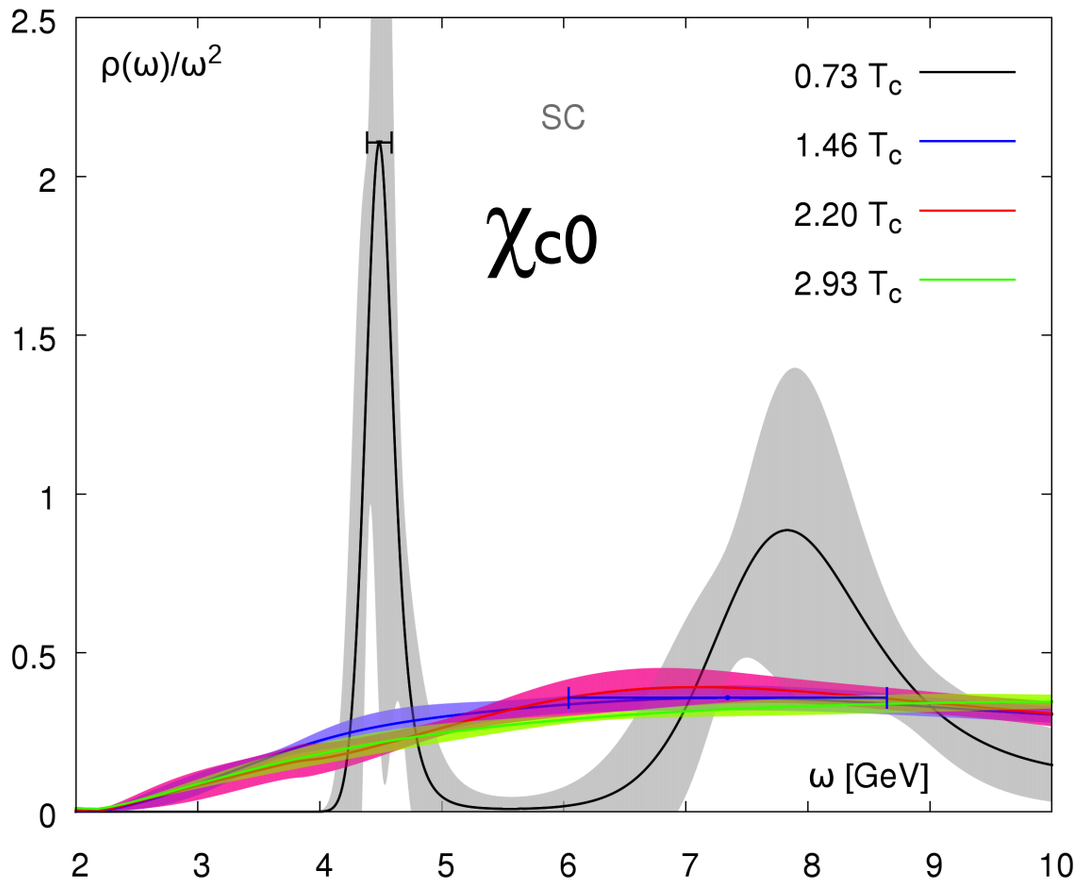
# charmonium spf in vector & pseudo-scalar channels

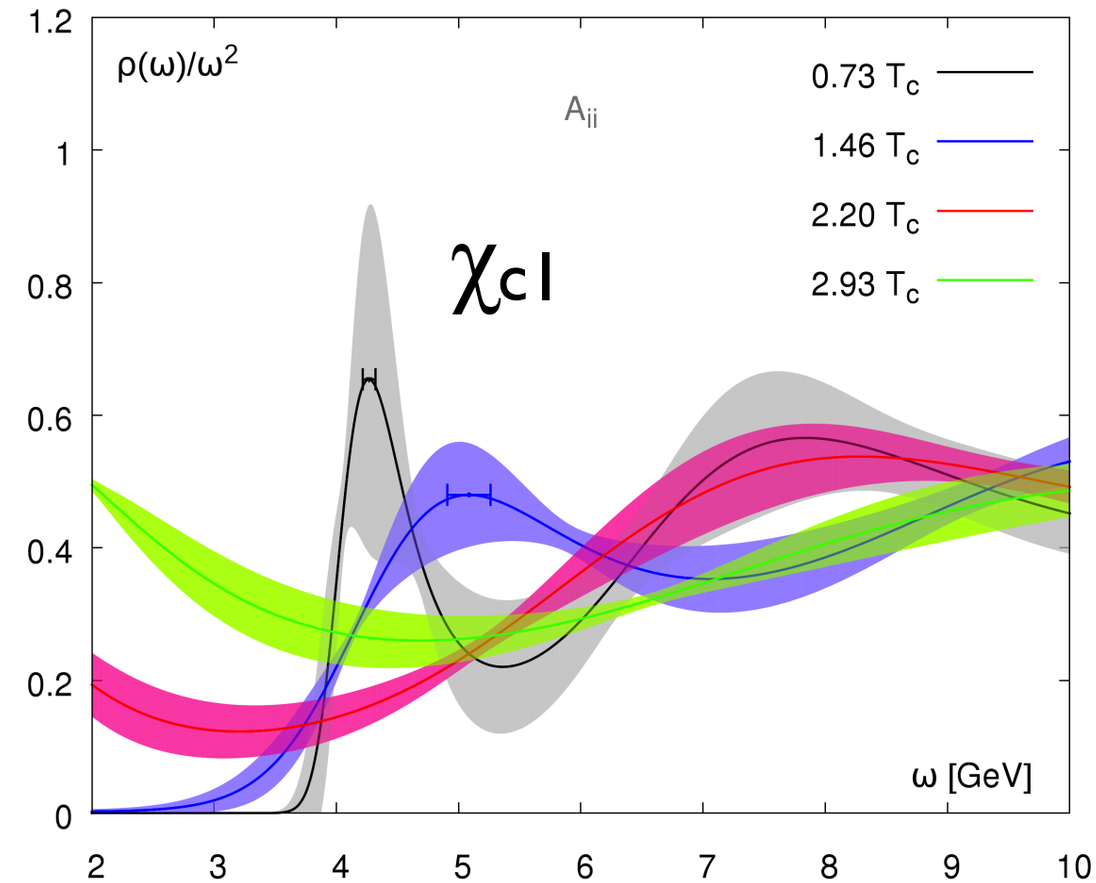



Both J/ψ and $\eta_c$ are dissociated already at 1.46 $T_c$

# charmonium spf in scalar & axial-vector channels

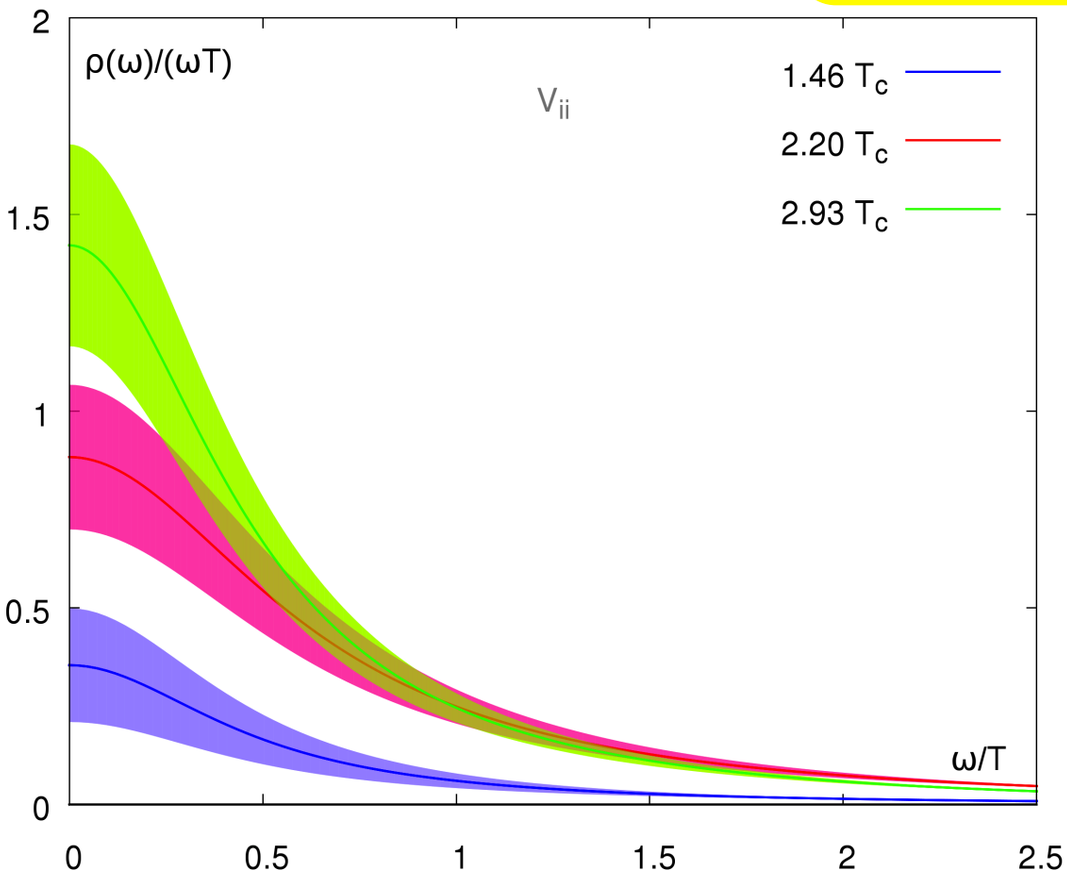


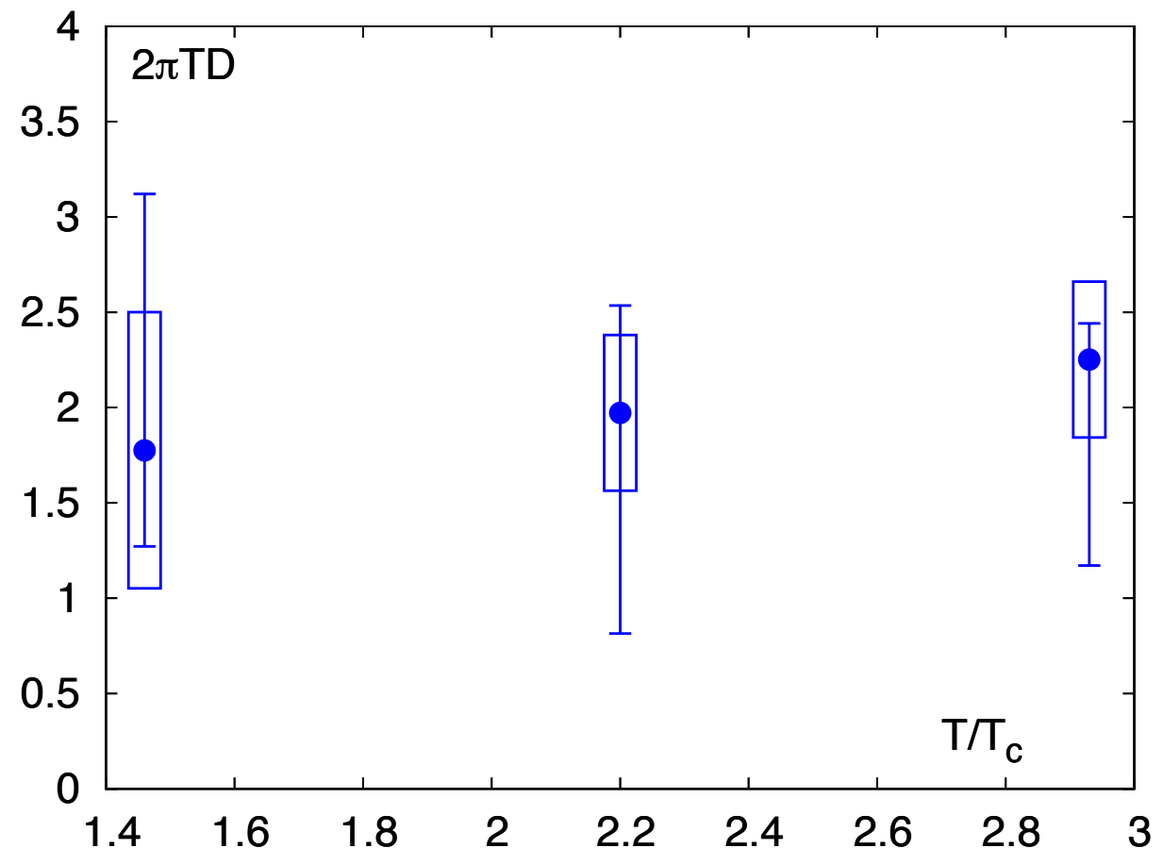




Both $\chi_{c0}$ and $\chi_{c1}$ are dissociated already at 1.46 $T_c$

# charm diffusion coefficient

$$D = \frac{1}{6\chi_{00}} \lim_{\omega \to 0} \frac{\rho^V(\omega, T)}{\omega}$$




- Strong coupling limit: $2\pi TD=1$ — Kovtun, Son & Starinets, JHEP 0310(2003)064
- pQCD: $\alpha_s \sim 0.2$, LO: $2\pi TD \approx 71$, NLO: $2\pi TD \approx 8$ — Moore & Teaney, PRD71(2005)064904; Caron-Huot & Moore, PRL 100(2008)052301
- T-Matrix (U-pot.): $T/T_c \approx 1.5$, $2\pi TD \approx 8$ — M. He, R. Rapp, arXiv: 1204.4442
- HQET(lQCD): $T_c < T < 2T_c$ $2\pi TD \approx 6$ — D. Banerjee, S. Datta, R. Gavai & P. Majumdar, PRD 85(2012)014510; A. Francis, O. Kaczmarek, M. Laine & J. Langelage, PoSLAT(2011)202

# Conclusion

- We have calculated charmonium correlation functions on large and fine isotropic lattices: $128^3 x N_T$, $N_T$=96,48,32,24 for 0.73, 1.46, 2.20 and 2.93 $T_c$

- We found that thermal modifications occur to the spectral function at $T>T_c$ by investigating differences of correlation functions from below to above $T_c$

- Through MEM analysis, all the ground states are melted already at $T=1.46\ T_c$

- Charm diffusion coefficients are estimated to be approximately $1/\pi T$ in the region of $1.46 T_c$ ... $2.93\ T_c$

# Current status of the lattice QCD study (MEM)

spf exists a strong peak at $0.9T_c$ as T=0, and at $1.1T_c$ the peak structure still persists almost the same place as below $T_c$

J/ψ and $\eta_c$ have little change up to $1.5T_c$ and then gradually weaken and disappear by $3T_c$, P wave states ($\chi_{c0}$ and $\chi_{c1}$) are possibly melted by $1.1T_c$

J/ψ and $\eta_c$ survive up to temperatures close to $2T_c$ while P waves states melt away below $1.2\,T_c$

| | $N_\sigma^3 \times N_\tau$ | $T/T_c$ | #conf | $a_\sigma[10^{-3}\mathrm{fm}]$ | $a_\sigma/a_\tau$ |
|---|---|---|---|---|---|
| Umeda02 | $20^3 \times 32$ | 0.88 | 1000 | | |
| EPJC 37S1(04)9 | $20^3 \times 26$ | 1.08 | 1000 | 96 | 4 |
| Asakawa03 | $32^3 \times 96$ | 0.78 | 194 | | |
| | $32^3 \times 54$ | 1.38 | 150 | 39 | 4 |
| PRL 92(04)012001 | $32^3 \times 46$ | 1.62 | 182 | | |
| | $32^3 \times 40$ | 1.87 | 181 | | |
| | $32^3 \times 32$ | 2.33 | 141 | | |
| Datta03 | $40^3 \times 40$ | 0.9 | 85 | | |
| | $64^3 \times 24$ | 1.5 | 80 | 20 | 1 |
| PRD 69(04)094507 | $48^3 \times 16$ | 2.25 | 100 | | |
| | $48^3 \times 12$ | 3 | 90 | | |
| Jakovác06 | $24^3 \times 44$ | 1.09 | 110 | | |
| | $24^3 \times 40$ | 1.20 | 1680 | 56 | 4 |
| PRD 75(2007)014506 | $24 \cdot 32 \times 32$ | 1.50 | 1000 | | |
| | $24^3 \times 24$ | 1.99 | 300 | | |
| | $24^3 \times 20$ | 2.39 | 640 | | |
| | $24^3 \times 16$ | 2.99 | 310 | | |
| Aarts07 | $12^3 \times 80$ | 0.42 | 250 | | |
| | $8^3 \times 32$ | 1.05 | 1000 | 162 | 6 |
| PRD 76(07)094513 | $8^3 \times 24$ | 1.40 | 1000 | | |
| | $8^3 \times 16$ | 2.09 | 1000 | | |

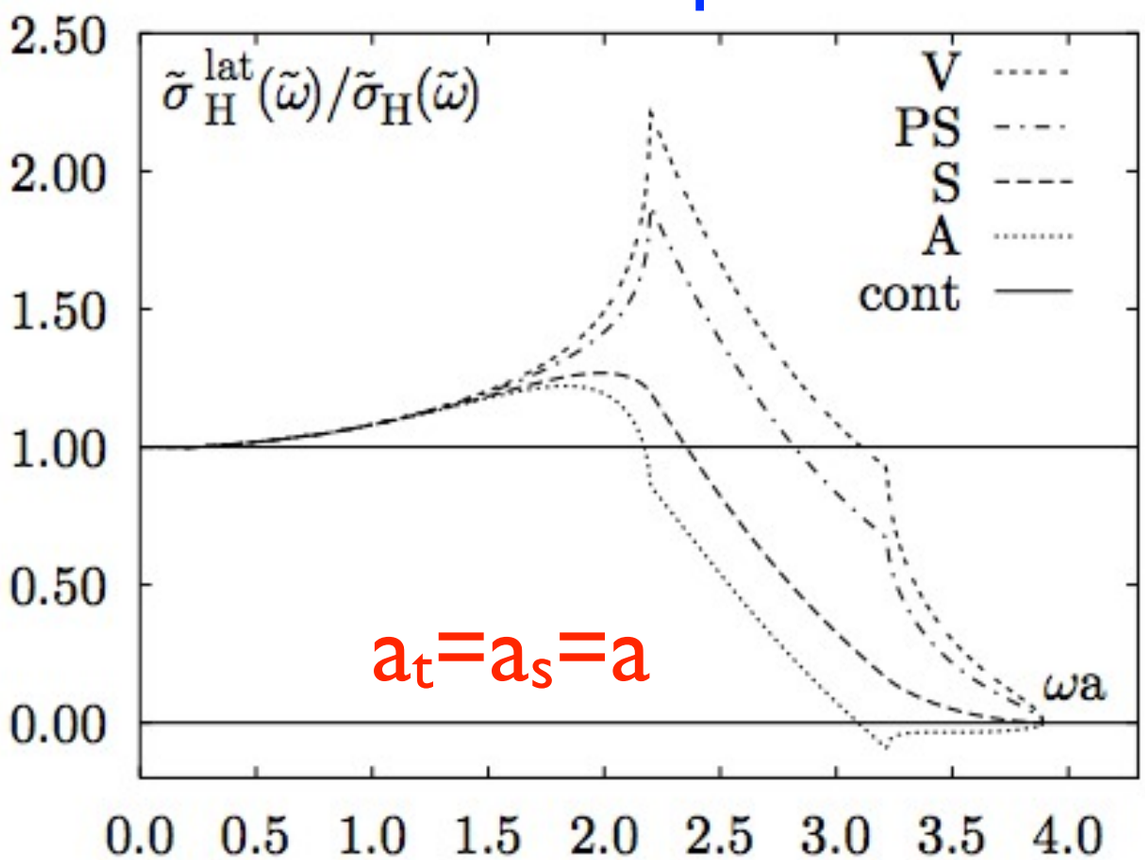
J/ψ and $\eta_c$ dissociate between $1.6T_c$ and $1.9\,T_c$

$\eta_c$ does not change up to $1.5\,T_c$, 1P states are melted at T=$1.1$-$1.2\,T_c$, the spectral functions in the vector channel at T=0 always differ from T≠0

- Lattice cutoff effects large on anisotropic lattices
- Redefinition of the kernel is needed
- Detailed default model dep. analysis as well as systematic uncertainty study are important

# isotropic v.s. anisotropic lattices

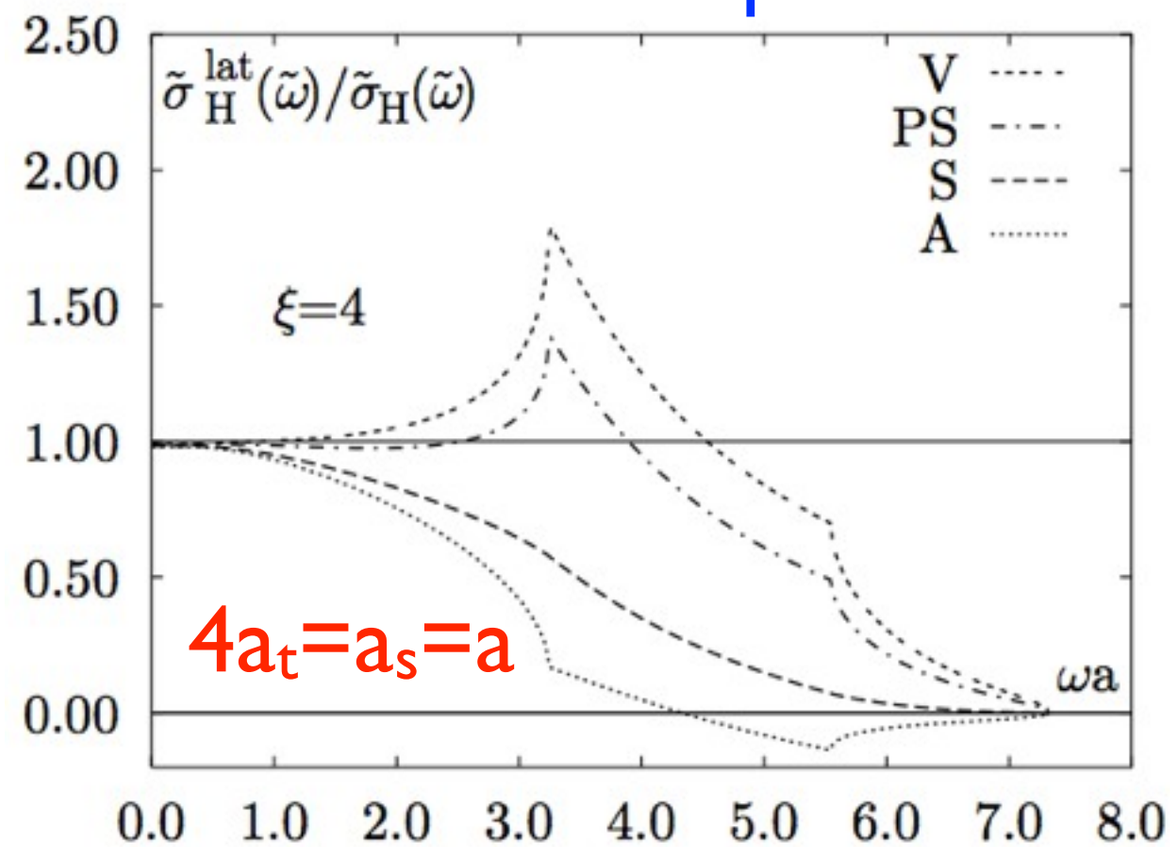
isotropic




$a_t=a_s=a$

$\omega/T=\omega a N_t$

anisotropic

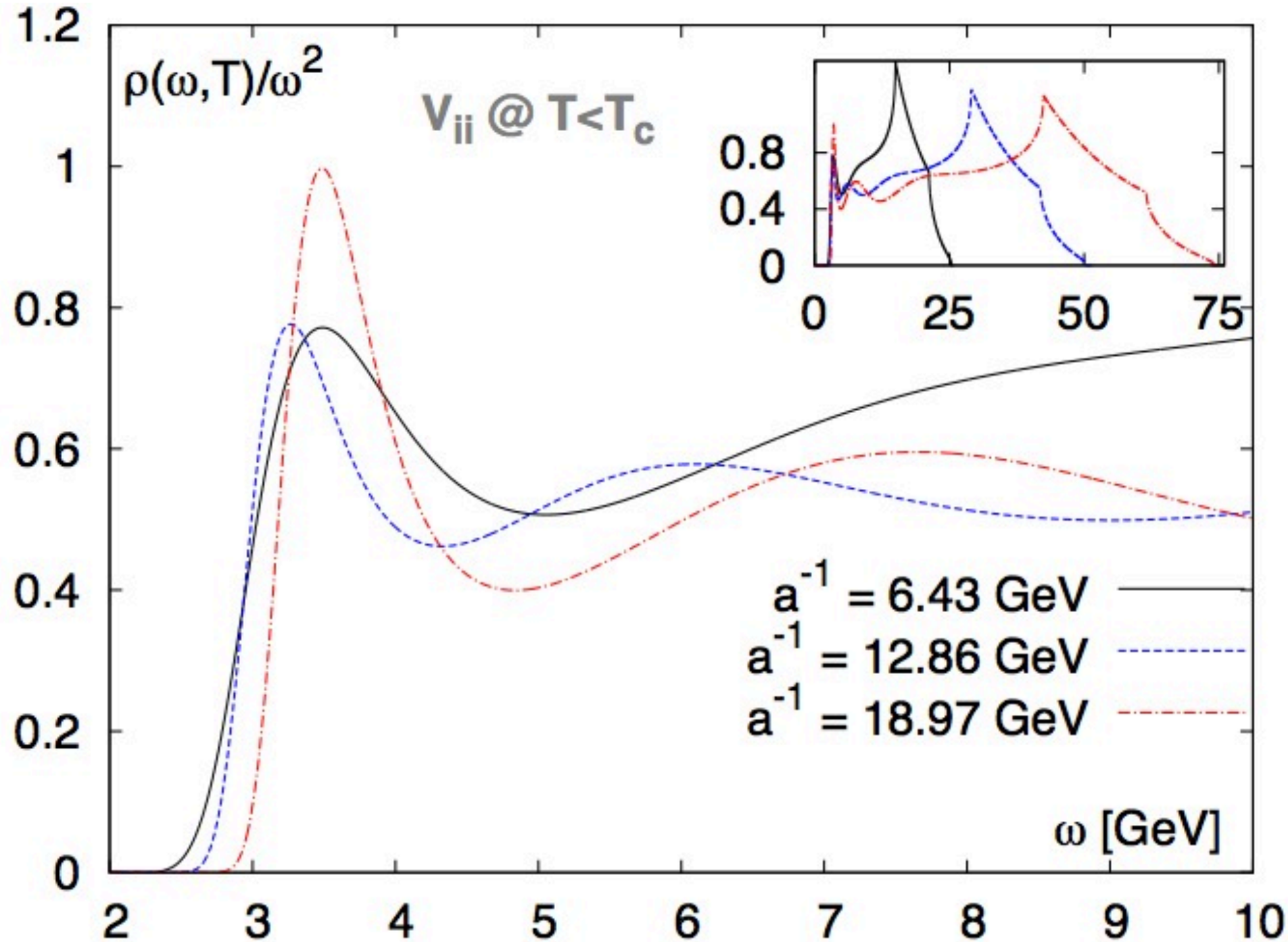



$4a_t=a_s=a$

$\omega/T=\omega a N_t/4$

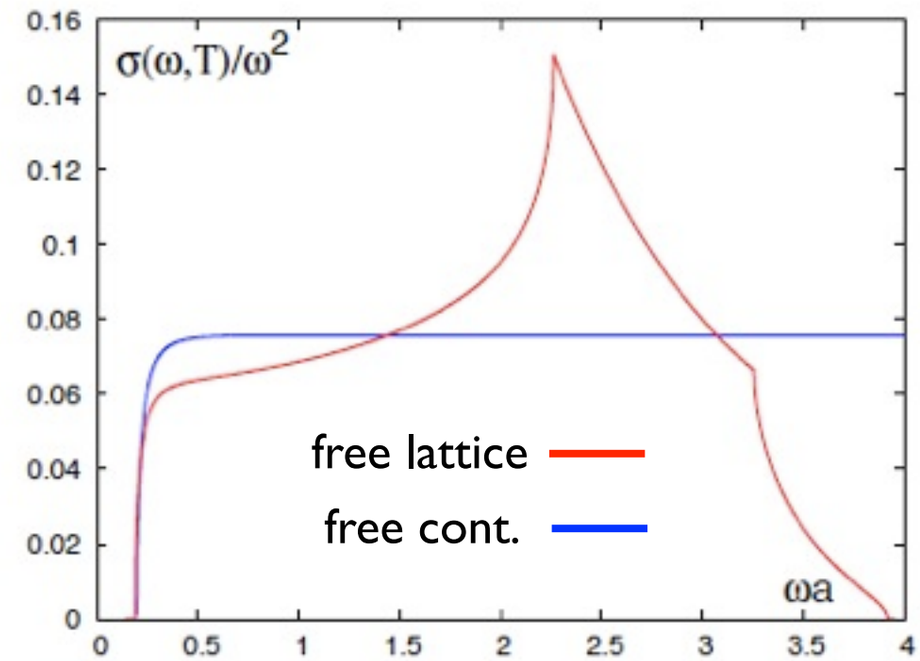
$1/T=a_t N_t$

# dependences on lattice spacing



# Prior information in the default model

- high frequency behavior of spf should resemble the spf in the free case
  - ★ free lattice spf rather than free continuum spf [HTD et al.,'09]




- low frequency behavior of spf

I: Non-interacting case: [Karsch et al., 03, Aarts et al., '05]

$$\sigma_H(\omega,0) = N_c\left[\left(a_H^{(1)}+a_H^{(3)}\right)I_1+\left(a_H^{(2)}-a_H^{(3)}\right)I_2\right]\omega\,\delta(\omega)$$

- ωδ(ω) term corresponds to a τ independent constant in the correlator, i.e. zero mode contribution [Umeda, '07]
- No zero mode contribution in the PS channel
- Zero mode contribution exists in the Vector, Scalar and Axial Vector channels

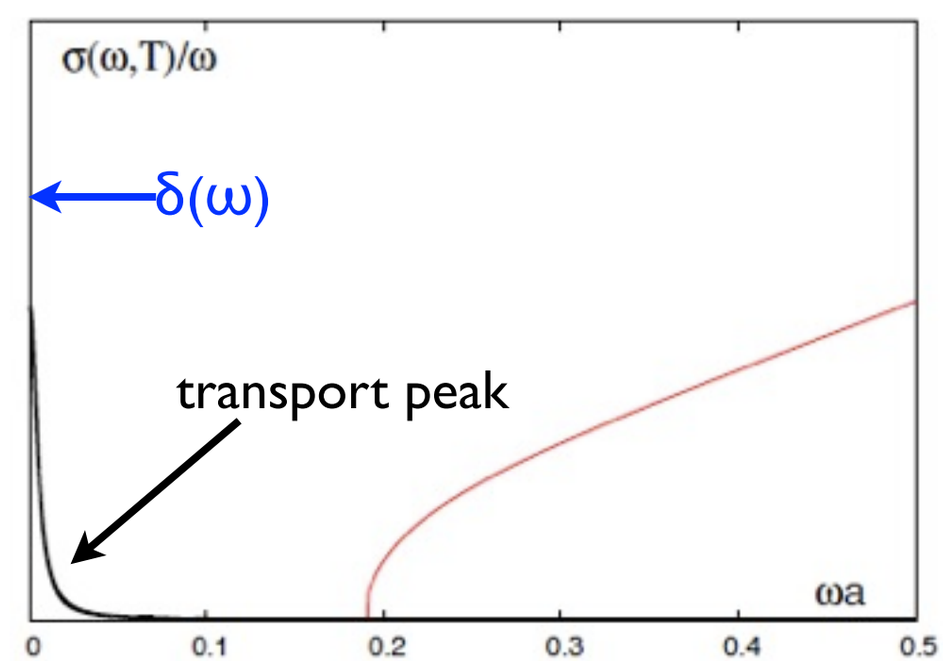
$$D \equiv \frac{\pi}{3\chi_{00}}\lim_{\omega\to 0}\sum_{i=1}^{3}\frac{\sigma_{ii}(\omega,0)}{\omega} = \infty$$

II: Interacting case: [Aarts & Martinez-Resco '02, Petreczky & Teaney '06]

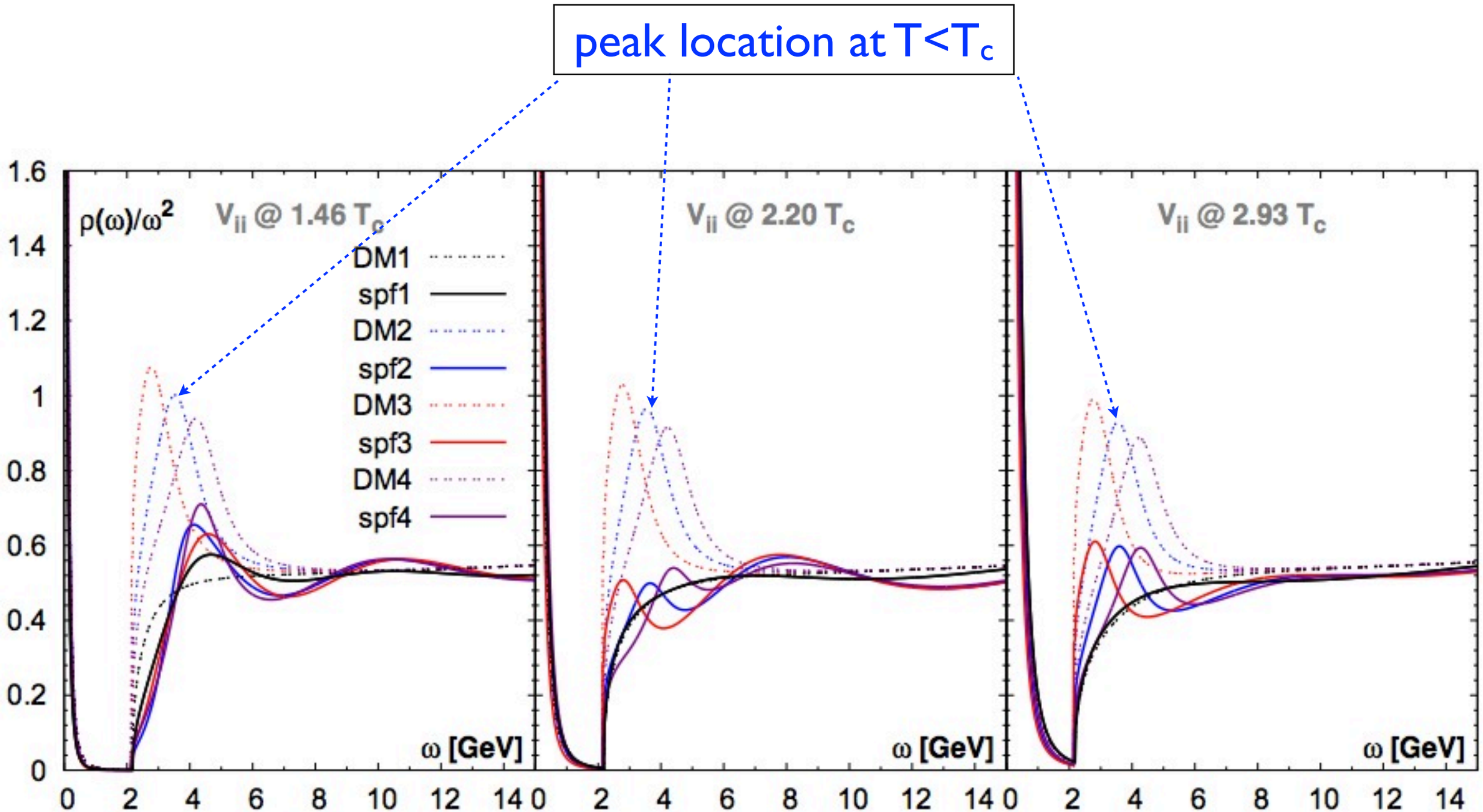
$$\delta(\omega) \to \frac{1}{\pi}\frac{\eta}{\omega^2+\eta^2} \longrightarrow D \propto 1/\eta$$

δ(ω) is smeared into a transport peak

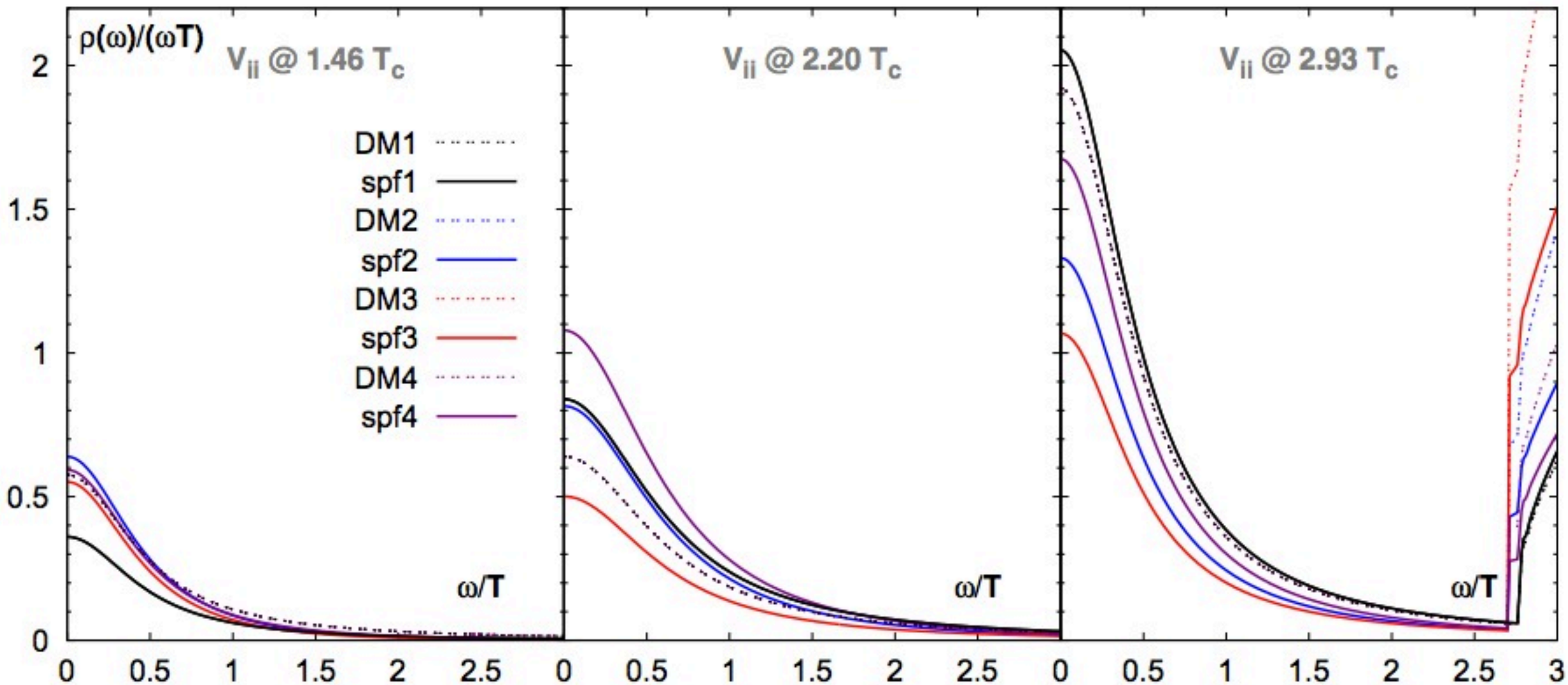
spf in vector channel should be linear in ω at ω~0



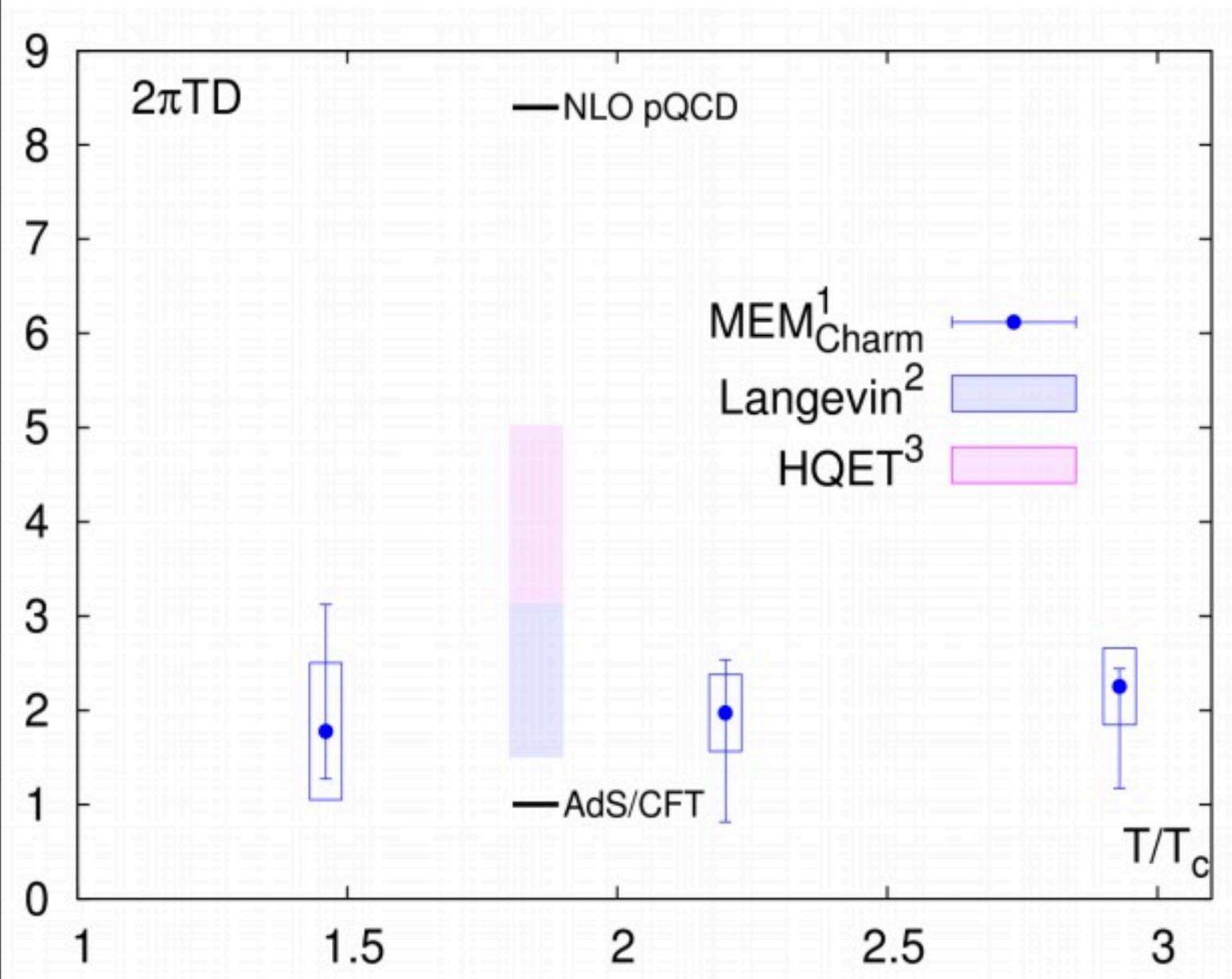
# default model dependences



# default model dependences



# heavy quark diffusion coefficients





1) HTD, A. Francis, O. Kaczmarek, F. Karsch, H. Satz, W. Söldner, arXiv: 1204.4945

2) G.D.Moore, D.Teaney; PRC71 (2005) 064904

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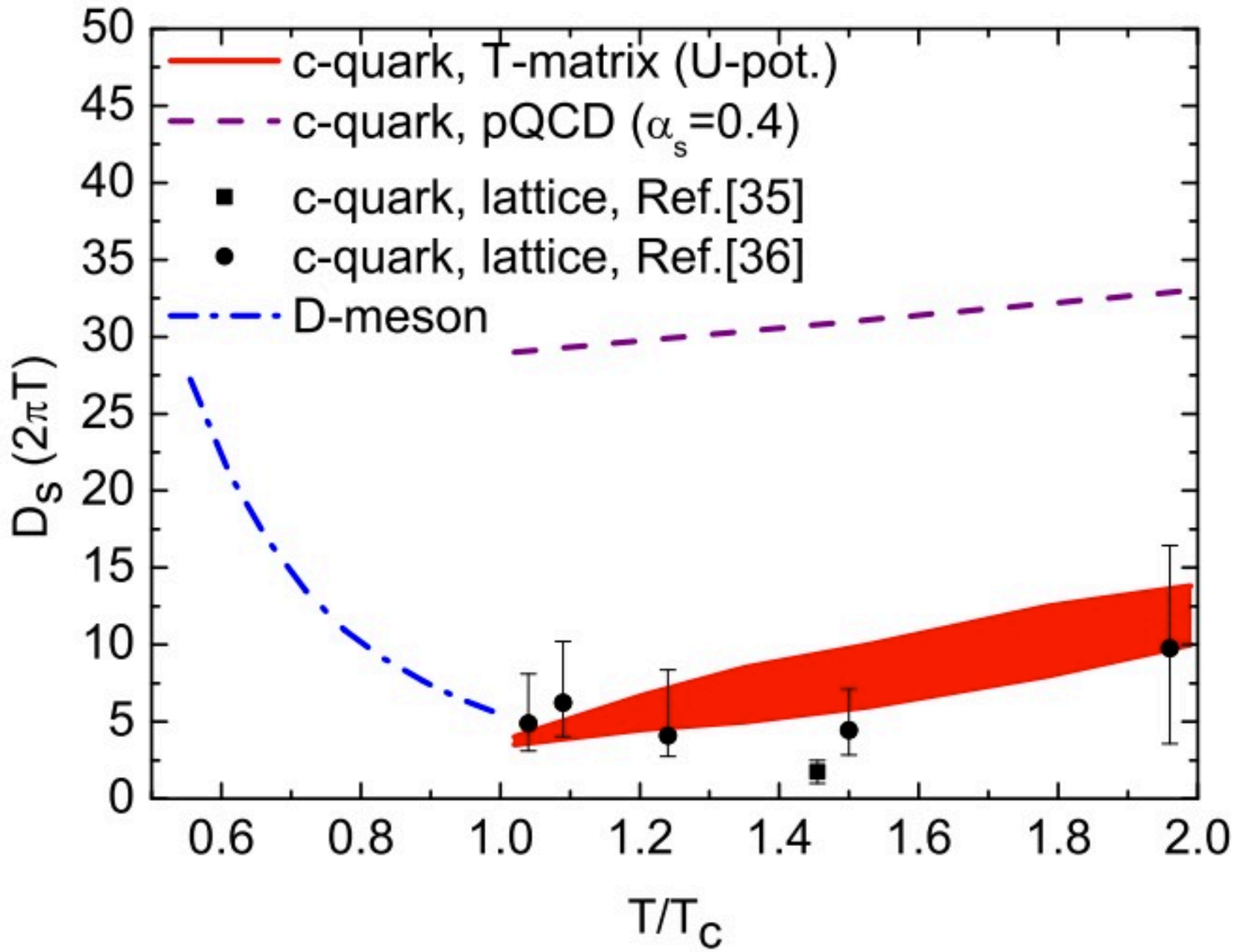
# heavy quark diffusion coefficients

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