CUORE BUQ Update

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Overview of Main Inference Problem

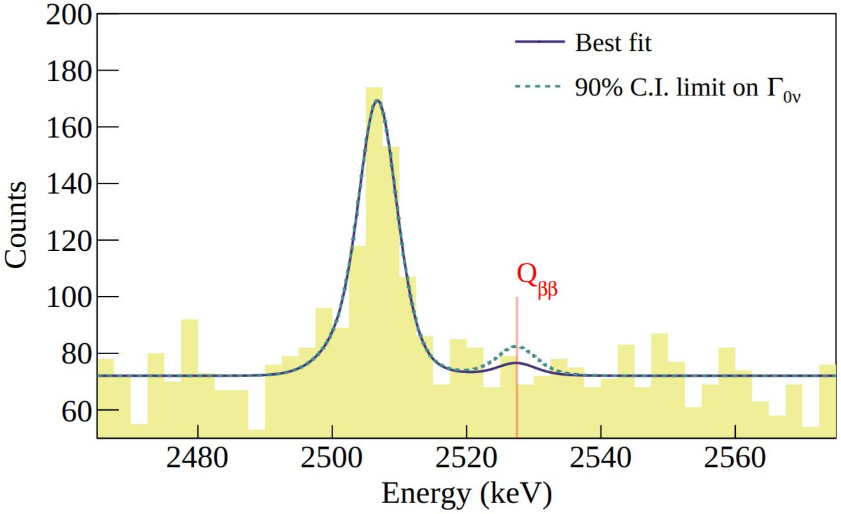
Looking for neutrinoless double-beta decay (0vββ): peak at known location (**$Q_{\beta\beta}$**) in detector energy spectrum.

Rare event search data is difficult to obtain. Detector lineshape function is complicated & typically described per-observation.

Unbinned (extended) likelihood: model region-of-interest (left) as mixture model of signal contribution, along with background contributions (nearby ^{60}Co peak & continuum background), plus nuisance parameters.

Promote statistical model to Bayesian inverse problem: priors are a combination of non-informative "default" priors (e.g. flat rate of 0vββ) & informative use from calibration measurements

Sample with MCMC to make inference on possible rate of 0vββ



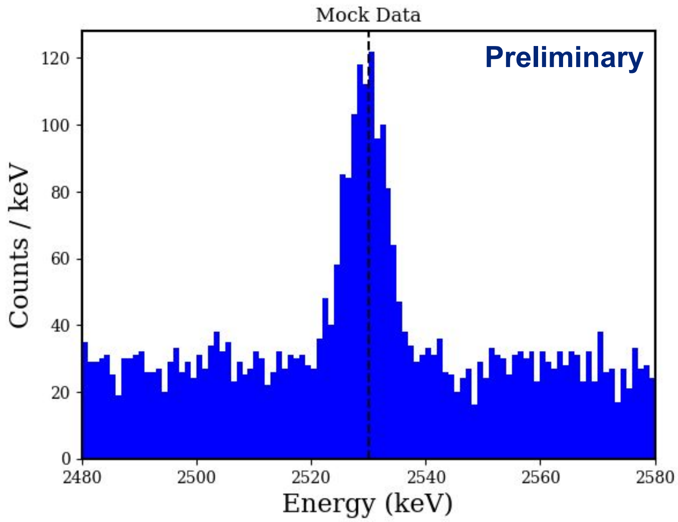
$$\mathcal{L} = \prod_{\text{DS, C}} \frac{e^{-\lambda}\lambda^n}{n!} \prod_i \left[\frac{s}{\lambda} f_{0\nu}(E_i|\vec{\theta}_{0\nu}) + \frac{c}{\lambda} f_{\text{Co}}(E_i|\vec{\theta}_{\text{Co}}) + \frac{b}{\lambda \Delta E} \right]$$

Problem Setting

- Likelihood evaluation is fast; posterior dimensionality is high
- CUORE data-taking is divided into 6-8 week periods: described with set of nuisance parameters
- Model uses 5 nuisance parameters / dataset, over 28 datasets: total posterior dimensionality of ~150, including “global” parameters such as $0\nu\beta\beta$ rate
  - This is still an approximation: including all lineshape nuisance parameters would push posterior-dimensionality to $\sim 10^4$
- Demands improved sampling techniques! Legacy Metropolis-Hastings Bayesian workflow no longer suitable to experiment needs…

Modernizing CUORE's Workflow

- Working with toy data/problem capturing key features & dimensionality of CUORE's $0\nu\beta\beta$ model to develop workflow & understand performance
  - Removing ^{60}Co bgd peak in current mock data - will reintroduce later
  - Some model modification required to support auto-diff. for all parameters
  - Variable-transformations needed for bounded parameters such as rates, efficiencies
  - Currently working unphysically deep in the "discovery regime": avoids complications due to parameter non-identifiability when limit-setting
- Using Blackjax sampler library: provides access to HMC & MCLMC implementations



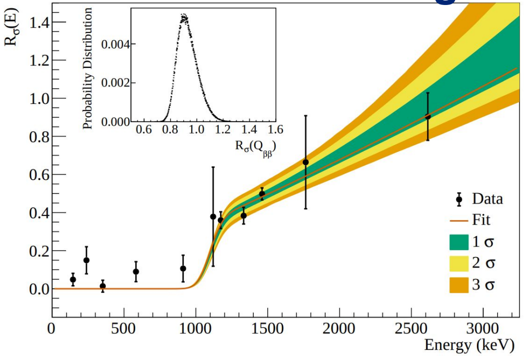
Posterior dimensionality of 86: equivalent to 2 t yr data release, without ^{60}Co parameters

Model input…

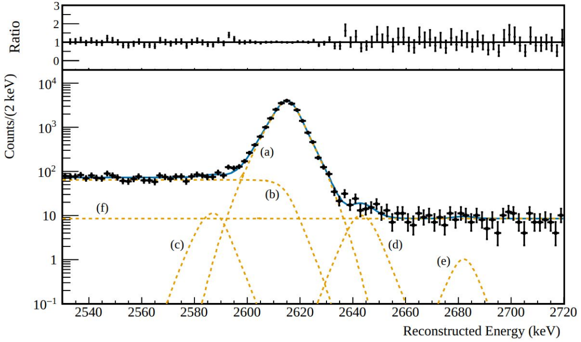
Model includes input from informative priors representing calibration input: efficiencies, lineshapes, etc.

Legacy workflow handles these priors numerically with histogram representations - need differentiable parameterization to support autodiff
E.g. efficiency histogram -> beta distribution prior

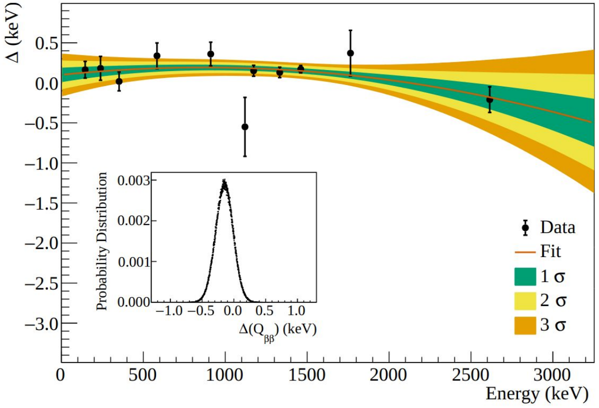
Resolution Scaling



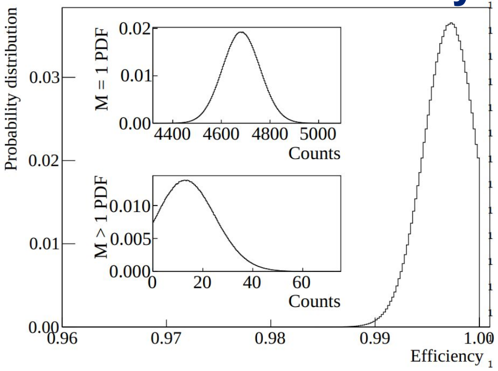
Lineshape Model



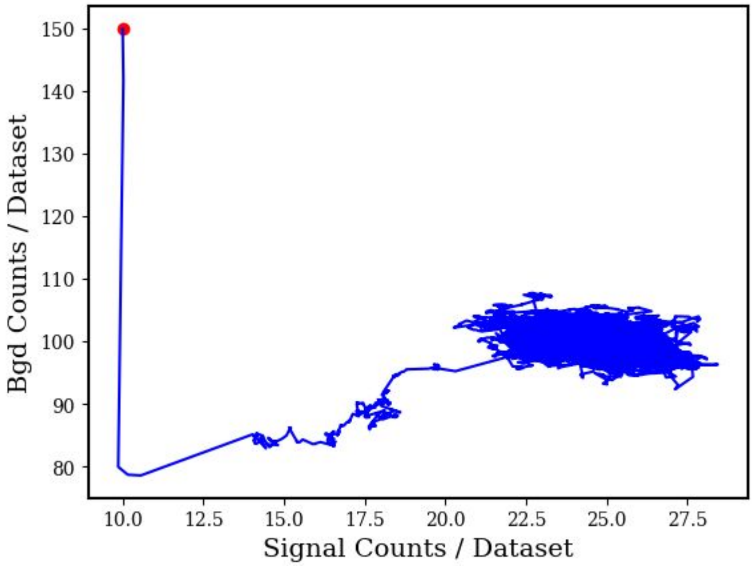
Calibration Bias

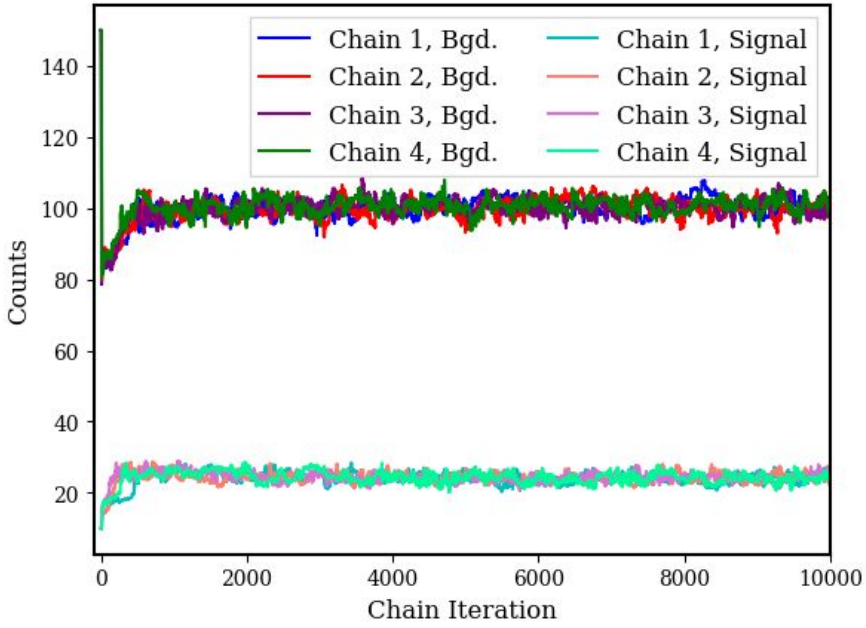


Selection Efficiency



HMC - Chain Convergence

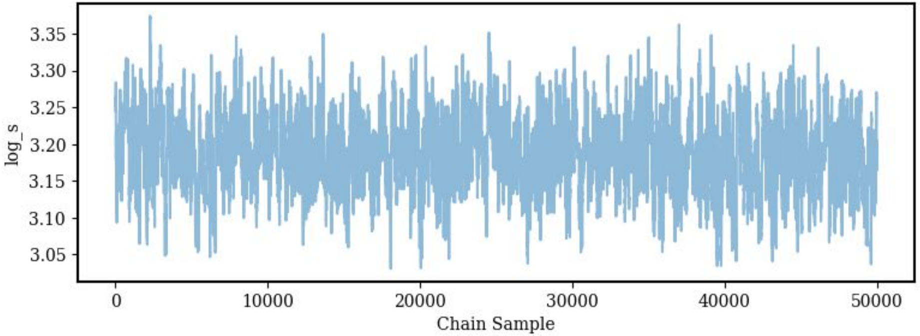


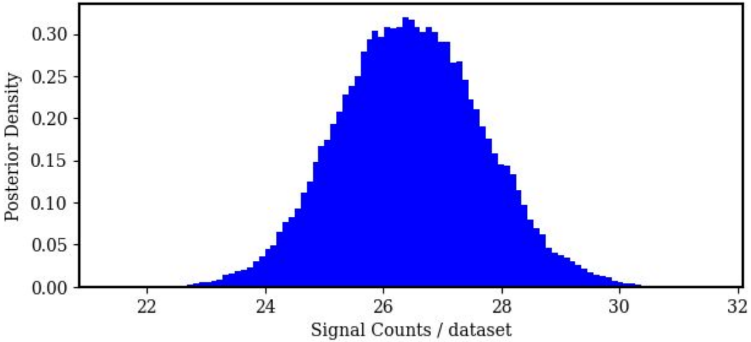


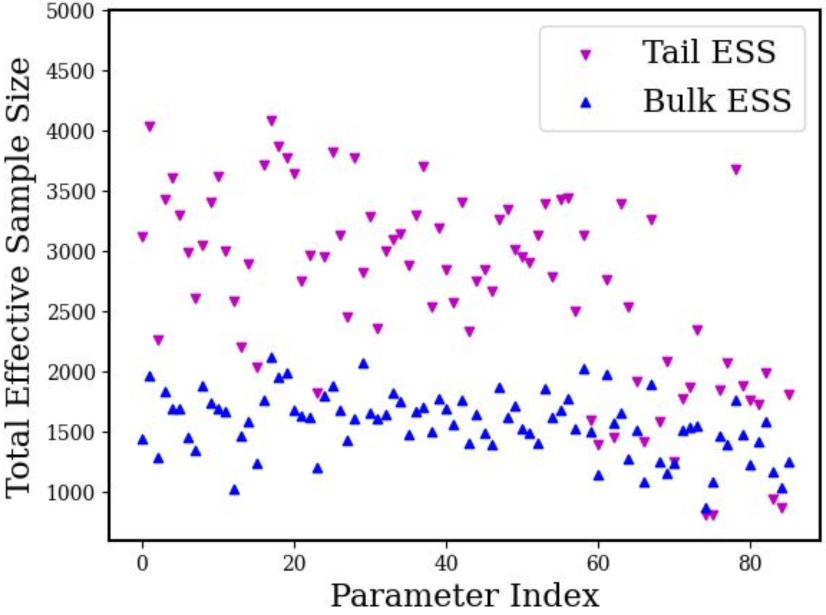
Running 4 chains, with 10000 iterations of warm-up/chain. 50000 iterations for inference.

Gelman-Rubin r-hat statistic 1.-1.01 for all parameters: good convergence

HMC - Inference



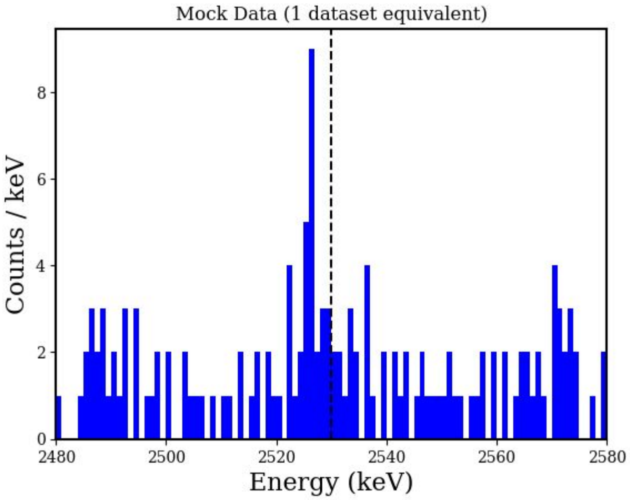


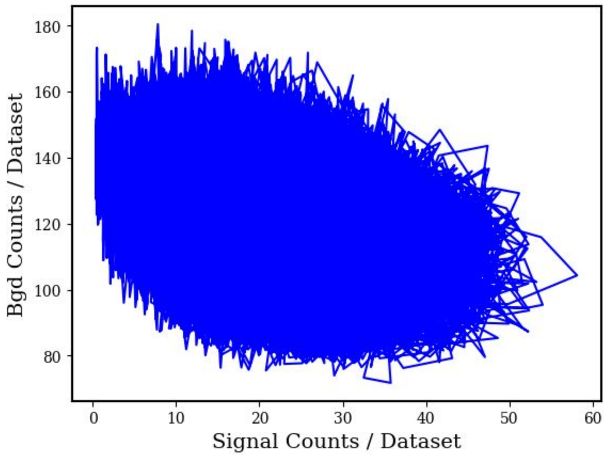


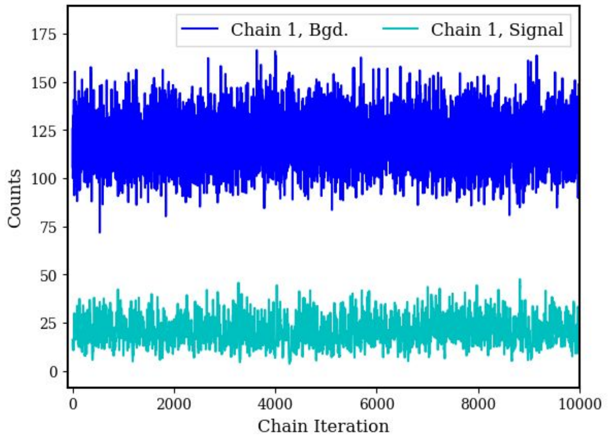
Effective Sample Size = equivalent number of samples from posterior, accounting for chain autocorrelation

Total ESS of 10^4 obtainable with 1-2 hours of computing time: ~10x faster compute than legacy workflow.

MCLMC - Growing Pains…

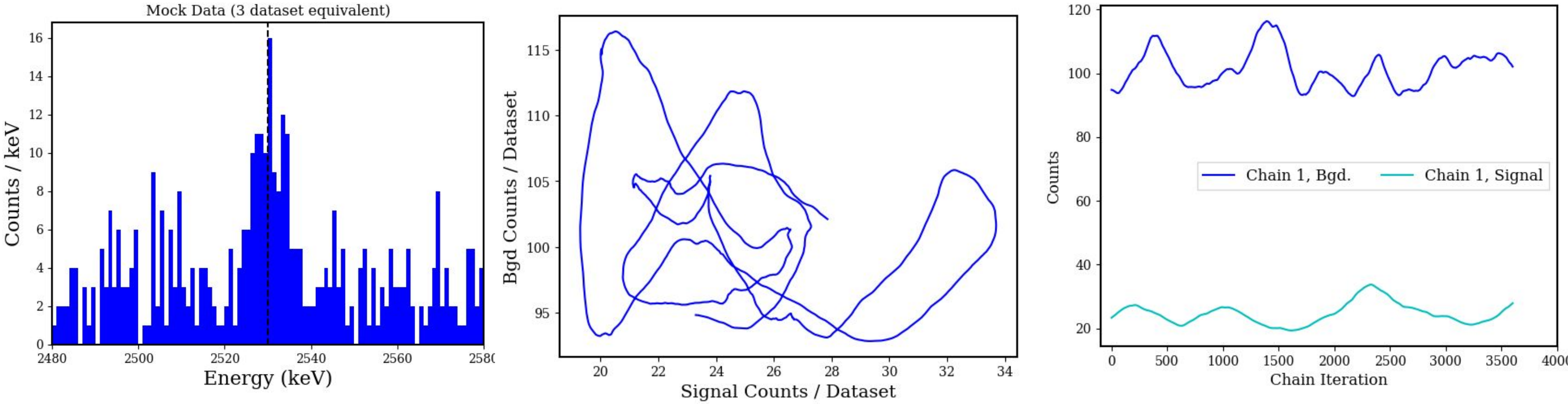






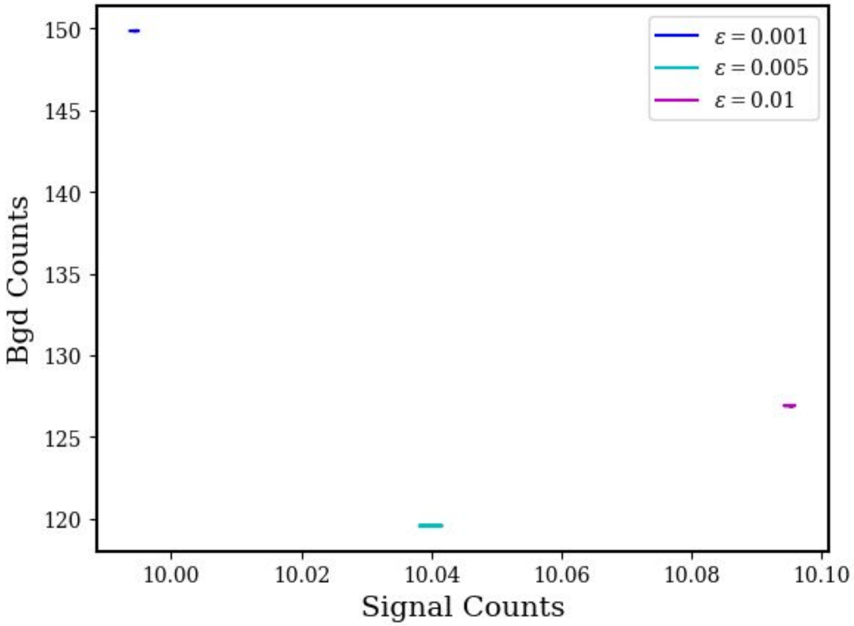
Microcanonical Langevin sampler seems to perform well on 1-dataset equivalent: only 5 parameters…

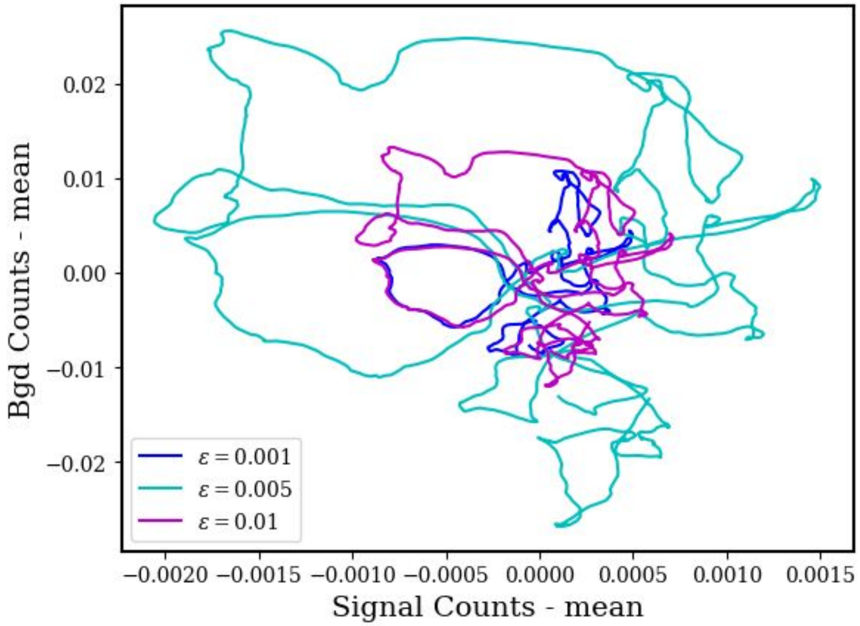
MCLMC - Growing Pains…



But not when scaled to > 1 dataset… Algorithm tuning leads to horrible chain mixing. No inference capability.

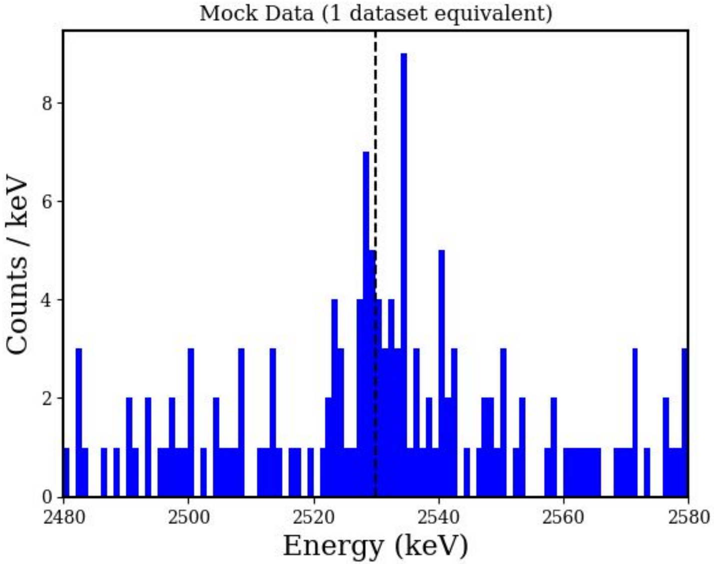
MCLMC - Growing Pains…





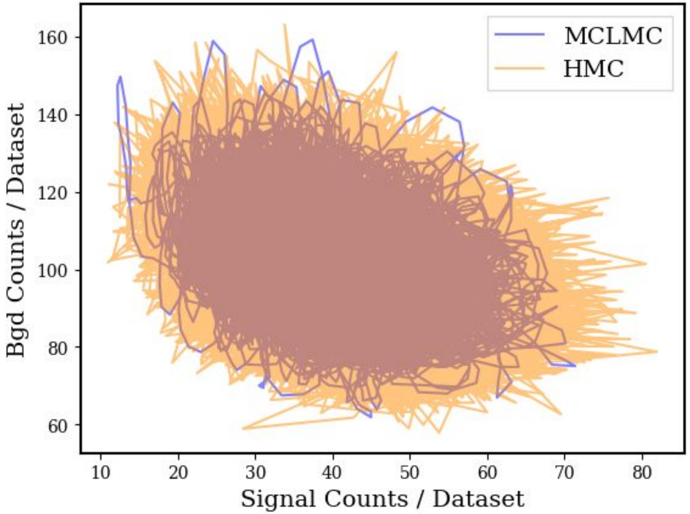
Tuning energy variance hyper-parameter just “rescales” chain walk - no effect at all from posterior landscape. Exact same statistical model which HMC handles well. Thoughts/diagnosis?

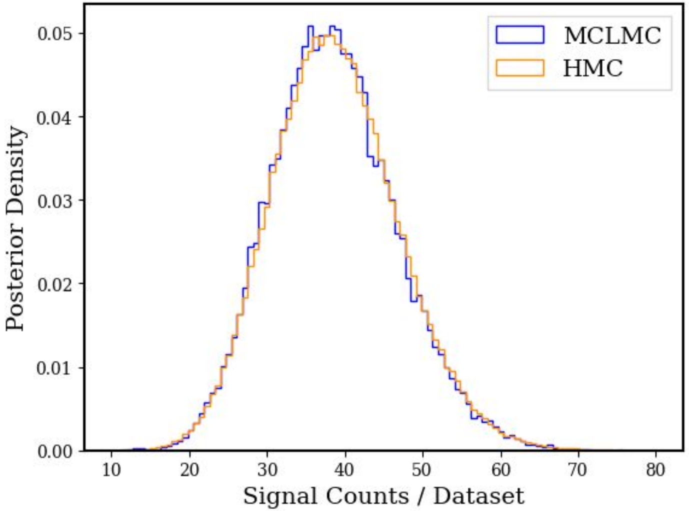
MCLMC vs HMC



When MCLMC works, it agrees well with HMC.

MCLMC compute time is ~10x faster than HMC on current workstation.





Ideas & Next Steps

- Attempt to diagnose cause of MCLMC failure for multi-dataset inference
  - Unclear what best plan of attack is for this: blackjax API leaves limited exposed options to user
- Can an HMC-based workflow be sufficient to meet experiment/project needs?
  - Still can provide large computational speed-up over legacy workflow
  - Interface with full detector data, efficiencies, calibration, etc.
- Limit-setting regime: issues with parameter non-identifiability?

Methods!

Last week: restarted discussions with Simon methodology opportunities for novel methods development for 0vββ:

- “Grouped” parameter tuning/structure: model contains many “copies” of similar structure from repetition across datasets
- Dimensional reduction vs full treatment of nuisance parameters
- Handling of non-sensitive parameters from informed priors