

HF Seminar - October 27, 2025

Heavy Quark Diffusion Coefficient from Lattice QCD

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HotQCD collaboration



Brookhaven
National Laboratory



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Motivation
Heavy Quark Diffusion
LATTICE ANALYSIS
Numerical setup
Double extrapolated correlators
PERTURBATIVE QCD ESTIMATES

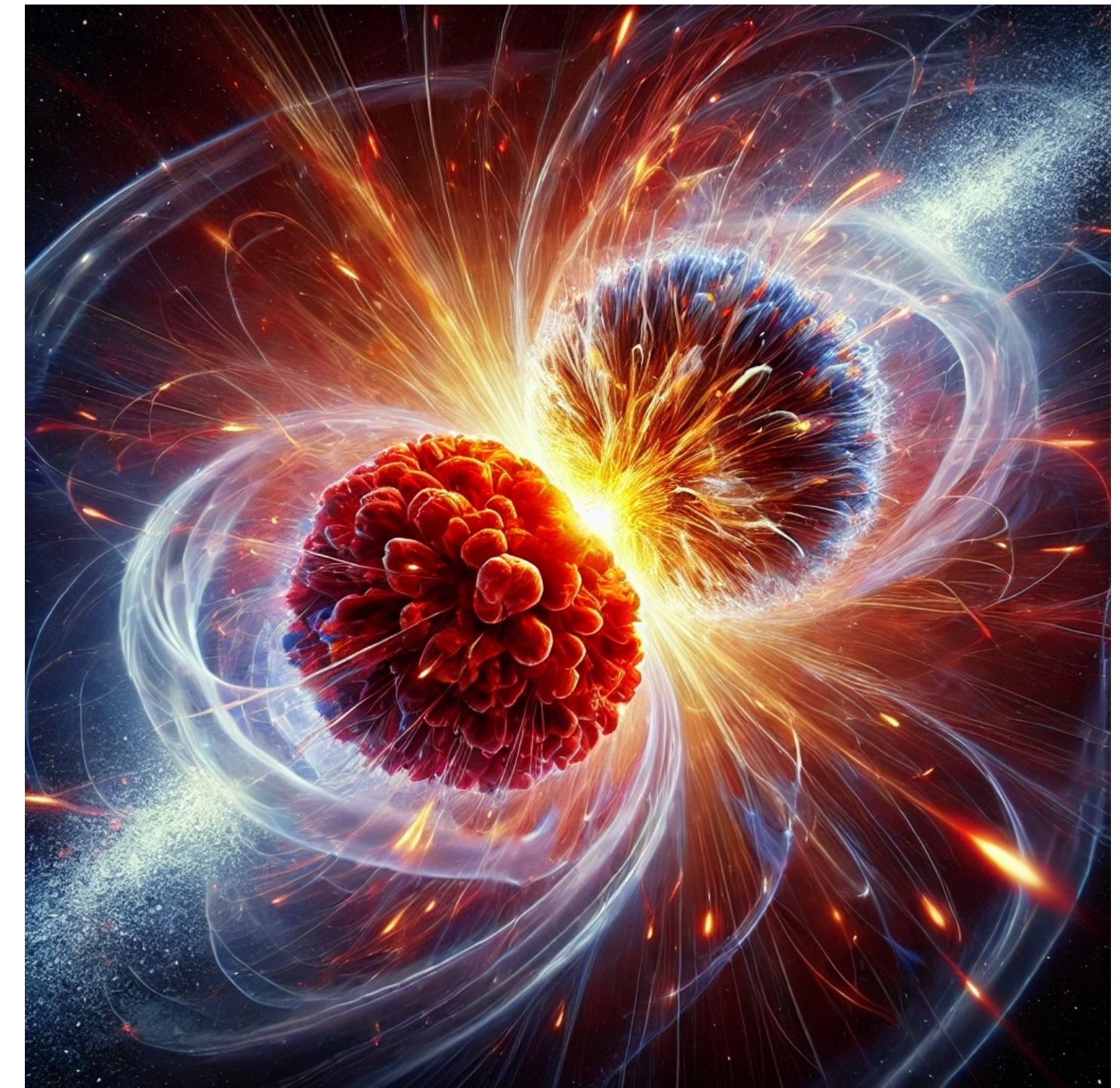
Perturbative results for the diffusion coefficient
Spectral reconstruction
HEAVY QUARK DIFFUSIONS RESULTS
Thermal squared velocity and momentum
Momentum Diffusion Coefficient
Spatial Diffusion Coefficient
CONCLUSIONS

- **HotQCD**, D. Bollweg et al. **Temperature Dependence of Heavy Quark Diffusion from (2+1)-flavor Lattice QCD**, *JHEP* 09 (2025) 180, [2506.11958].
- **TUMQCD**, N. Brambilla et al. **Heavy quark diffusion coefficient with gradient flow**, *Phys. Rev. D* 107 (2023) 054508, [2206.02861].
- **HotQCD**, L. Altenkort et al. **Heavy Quark Diffusion from 2+1 Flavor Lattice QCD with 320 MeV Pion Mass**, *Phys. Rev. Lett.* 130 (2023) 231902, [2302.08501].
- **HotQCD**, L. Altenkort et al. **Quark Mass Dependence of Heavy Quark Diffusion Coefficient from Lattice QCD**, *Phys. Rev. Lett.* 132 no. 5, (2024) 051902, [2311.01525].

PRELIMINARIES

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The quark gluon plasma (QGP) is a state of matter that exists under extreme conditions such as those found in the early universe.



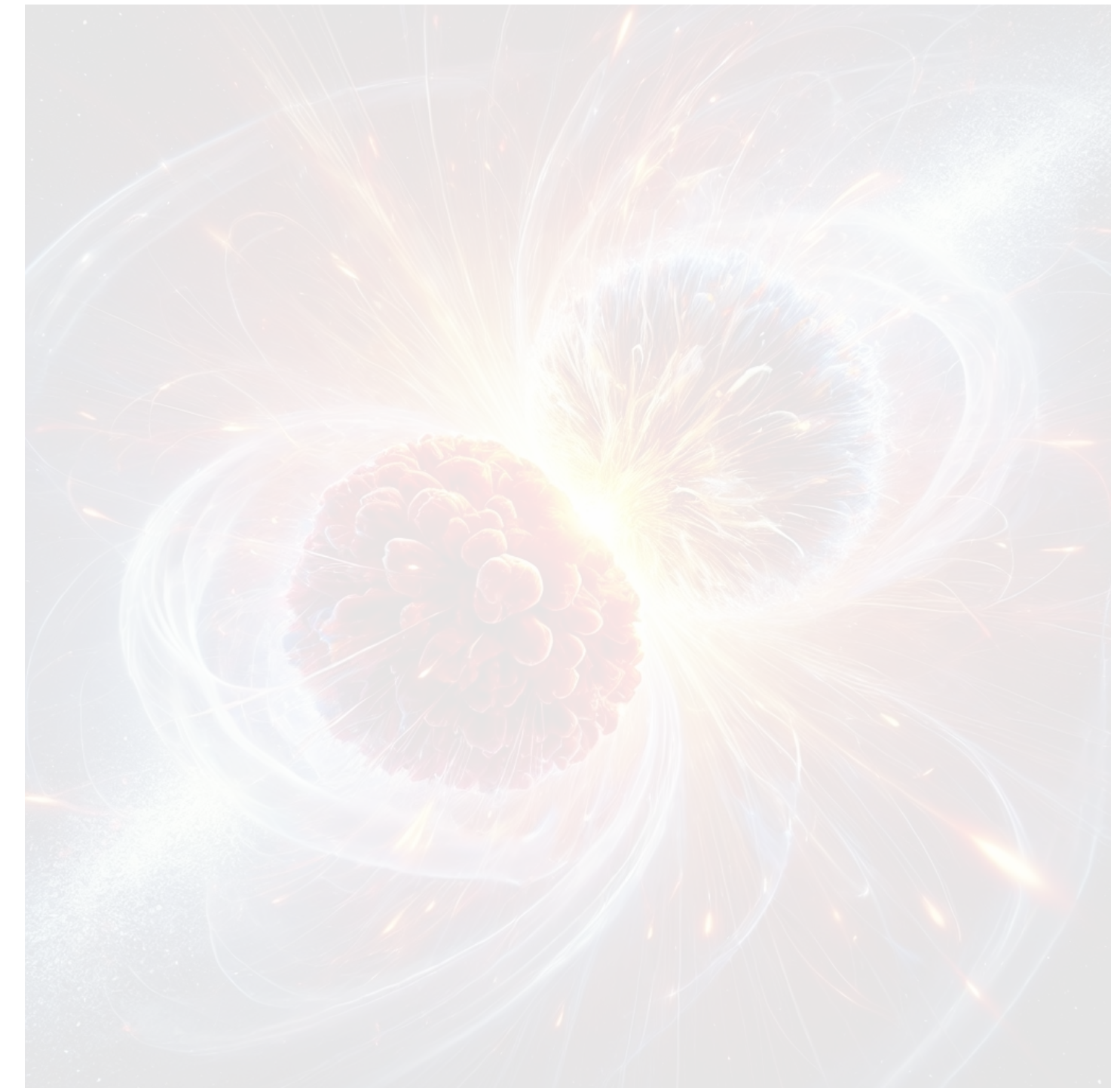
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Hot and dense assembly made of quarks and gluons



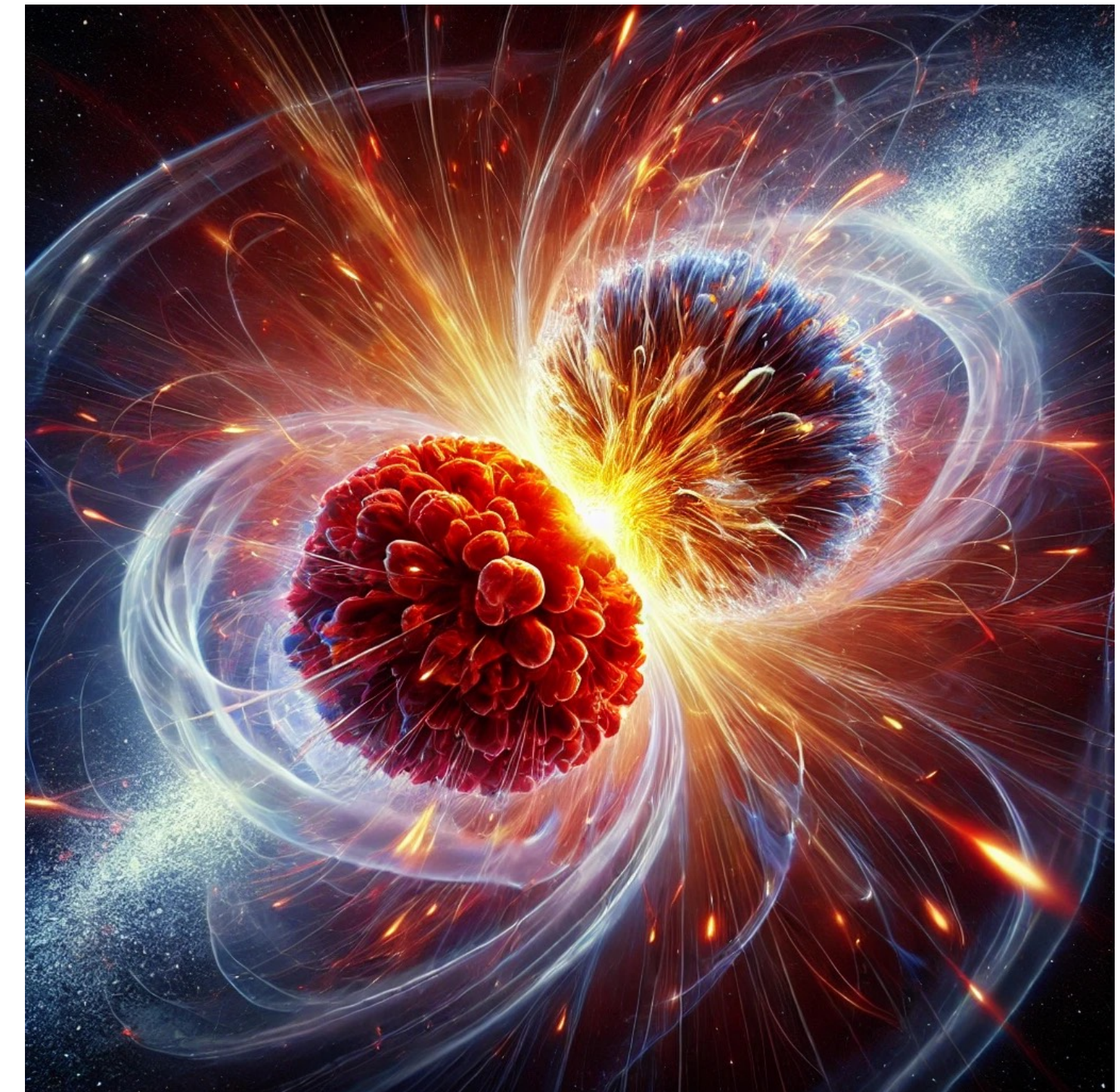
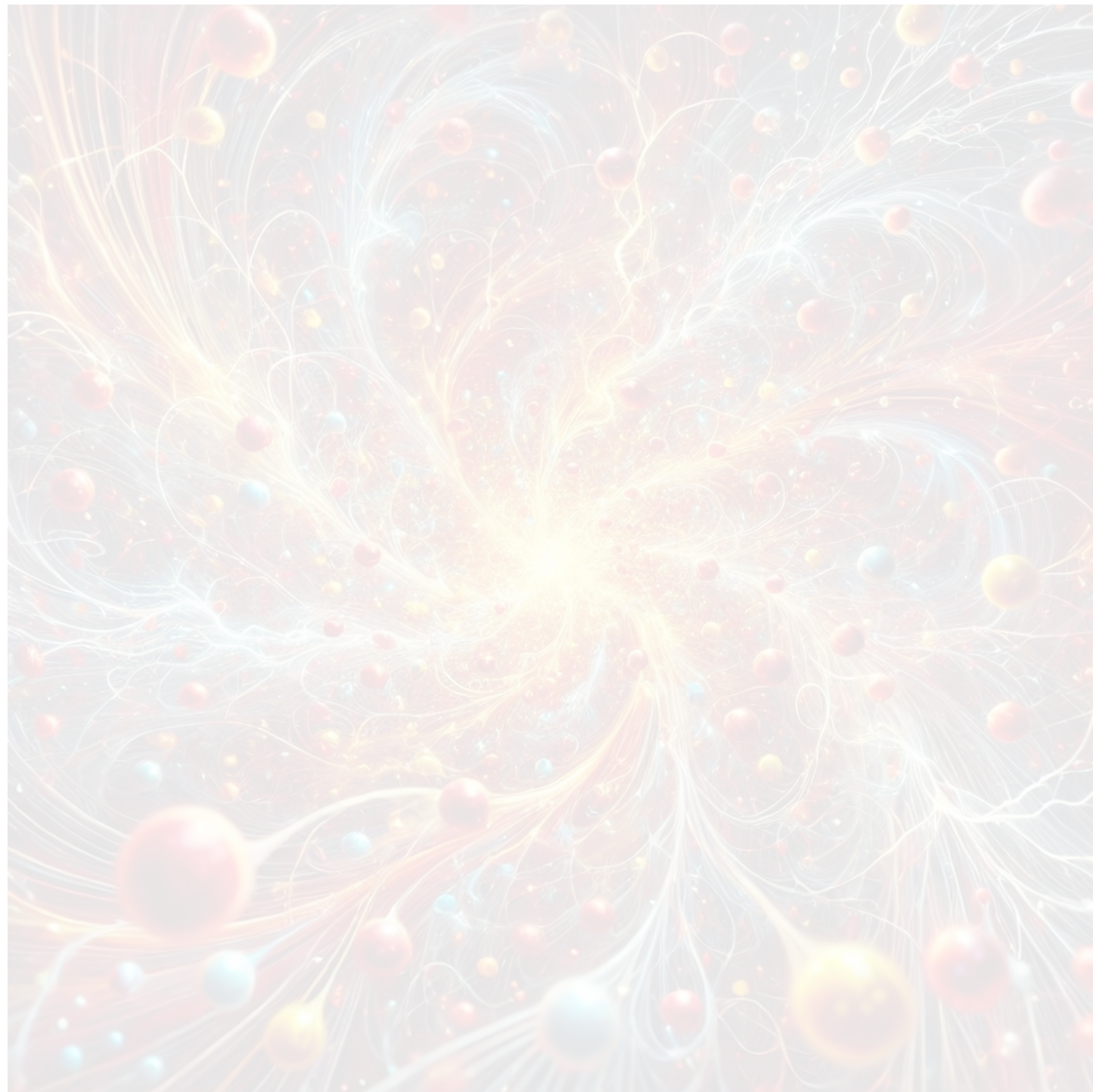
ChatGPT



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HEAVY-ION COLLISIONS (HICS)



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Collins, Perry (1978)

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We can study QGP at HICs using heavy flavor probes

HEAVY QUARKS: CHARM AND BOTTOM

+

BOUND STATES: QUARKONIA

MOTIVATION

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Transport coefficients

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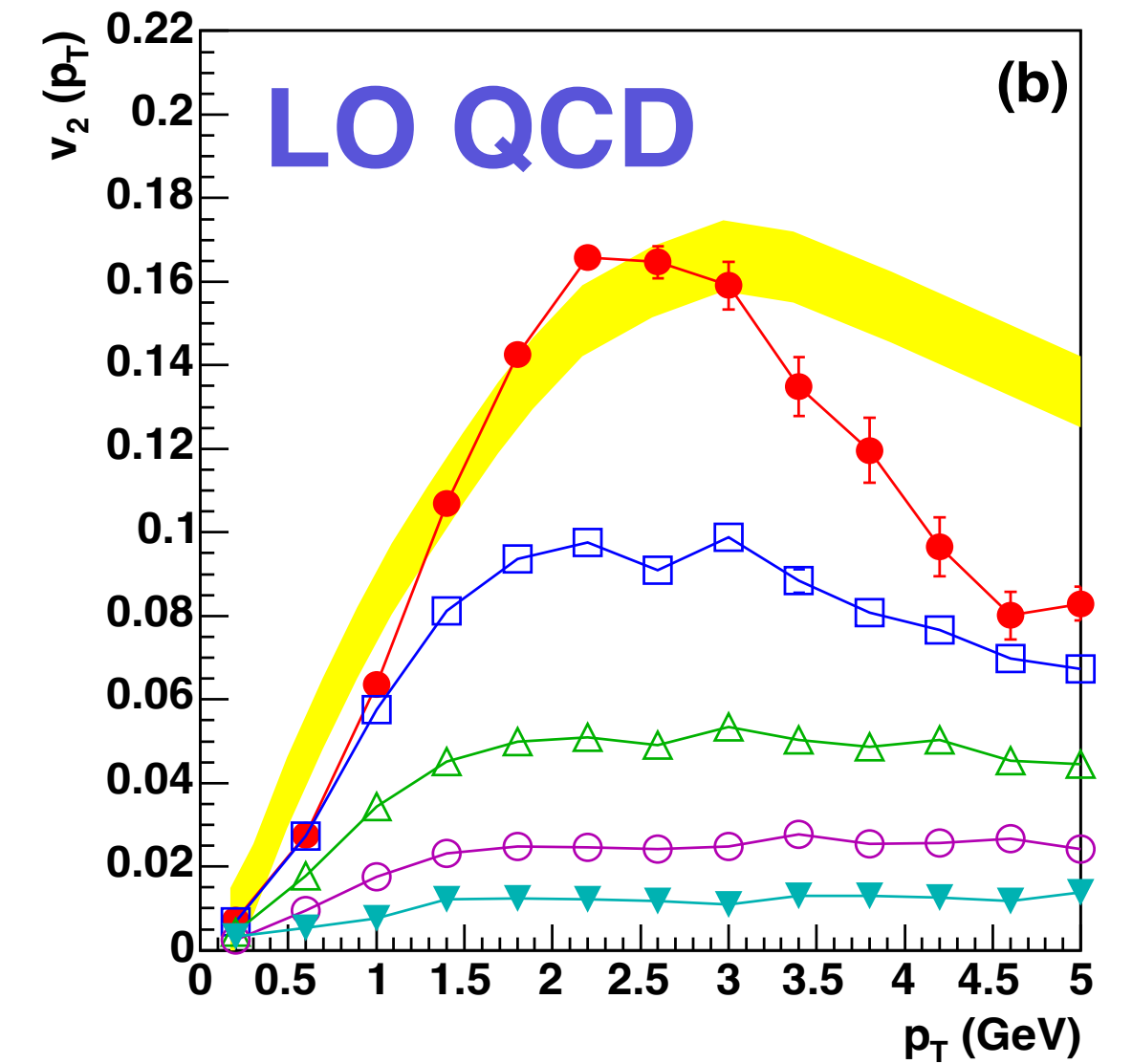
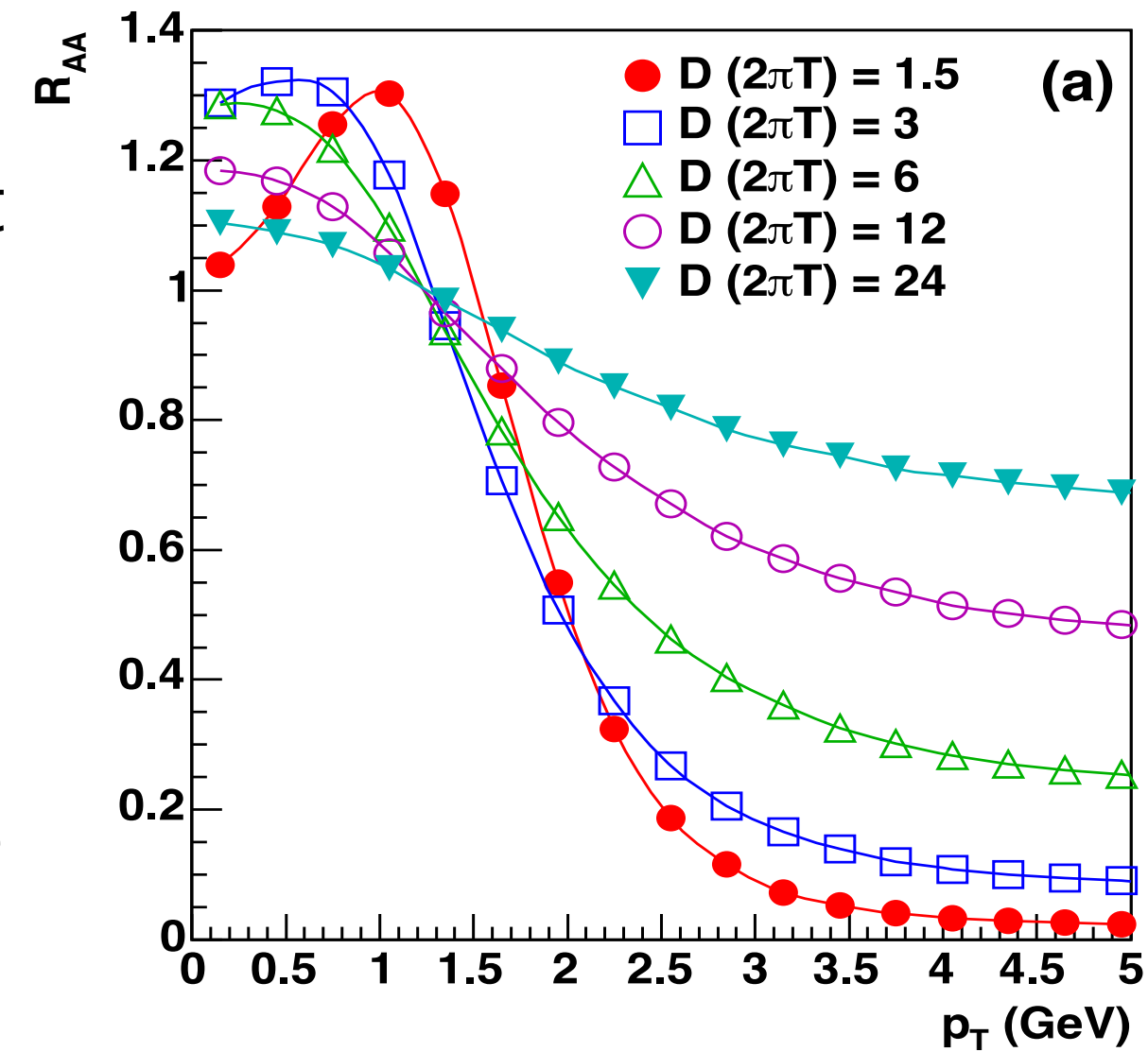
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The suppression factor R_{AA} and elliptic flow v_2 are highly dependent on the choice of D_s .



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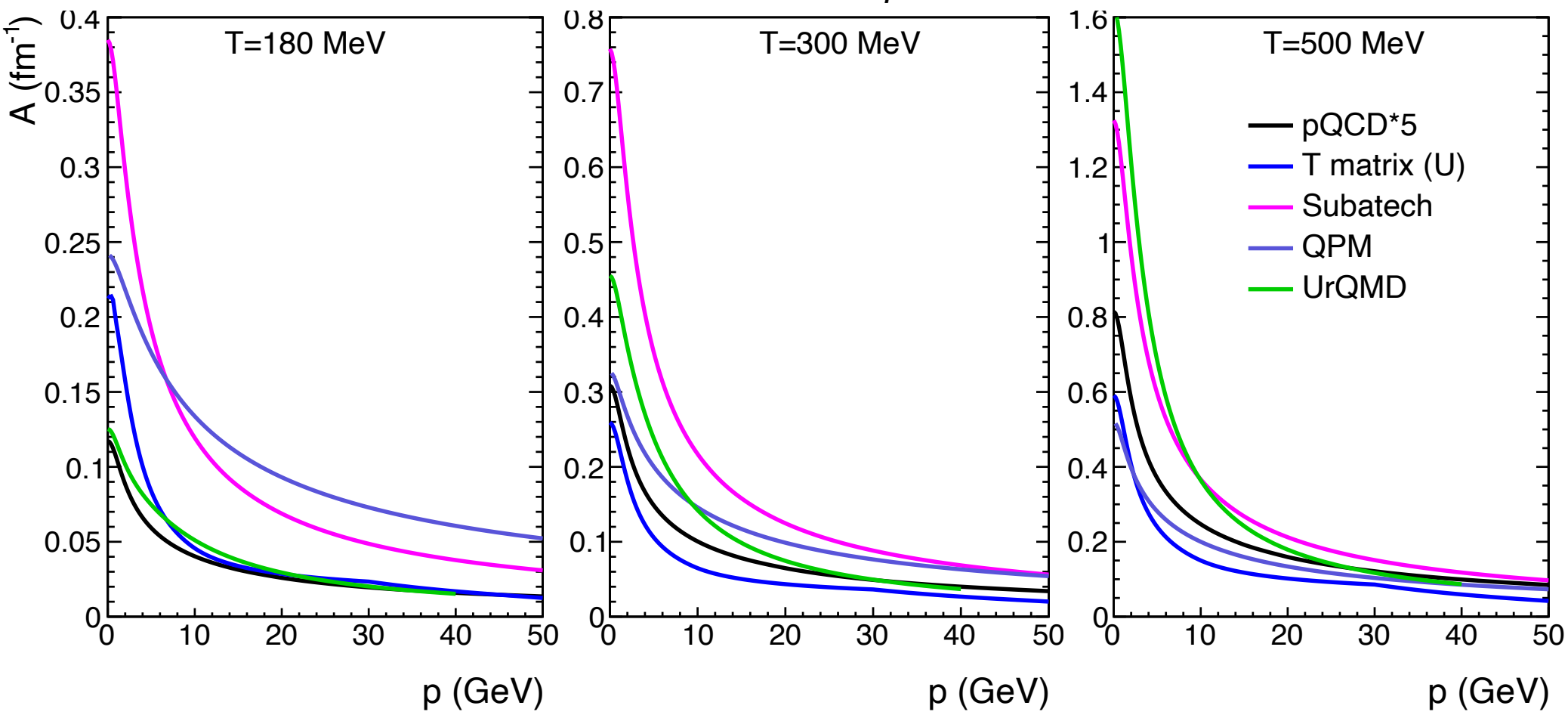
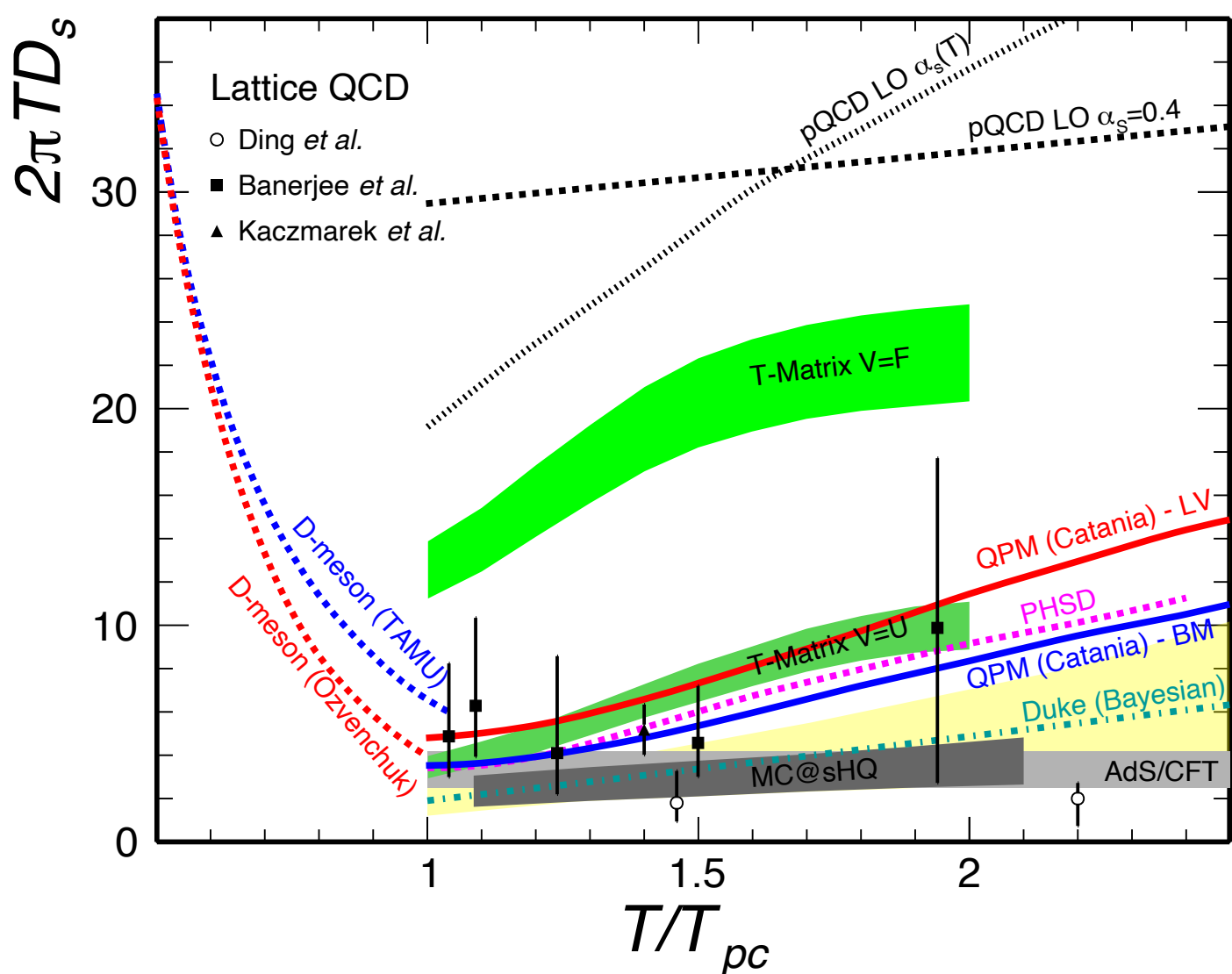
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Model predictions

Multiple models predict very different values for these transport coefficients.

NEED FOR NON-PERTURBATIVE DETERMINATIONS



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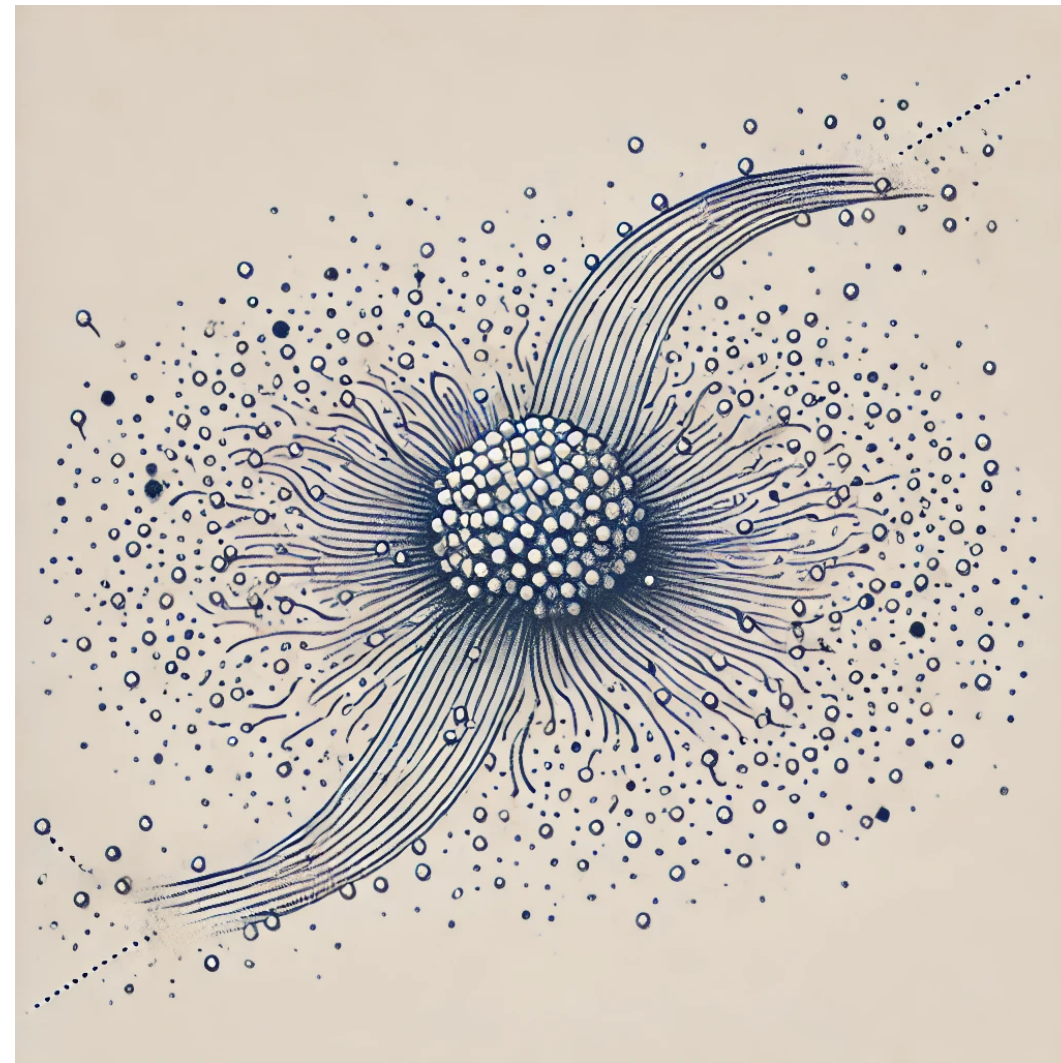
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CHATGPT, DRAW ME QGP
BROWNIAN MOTION

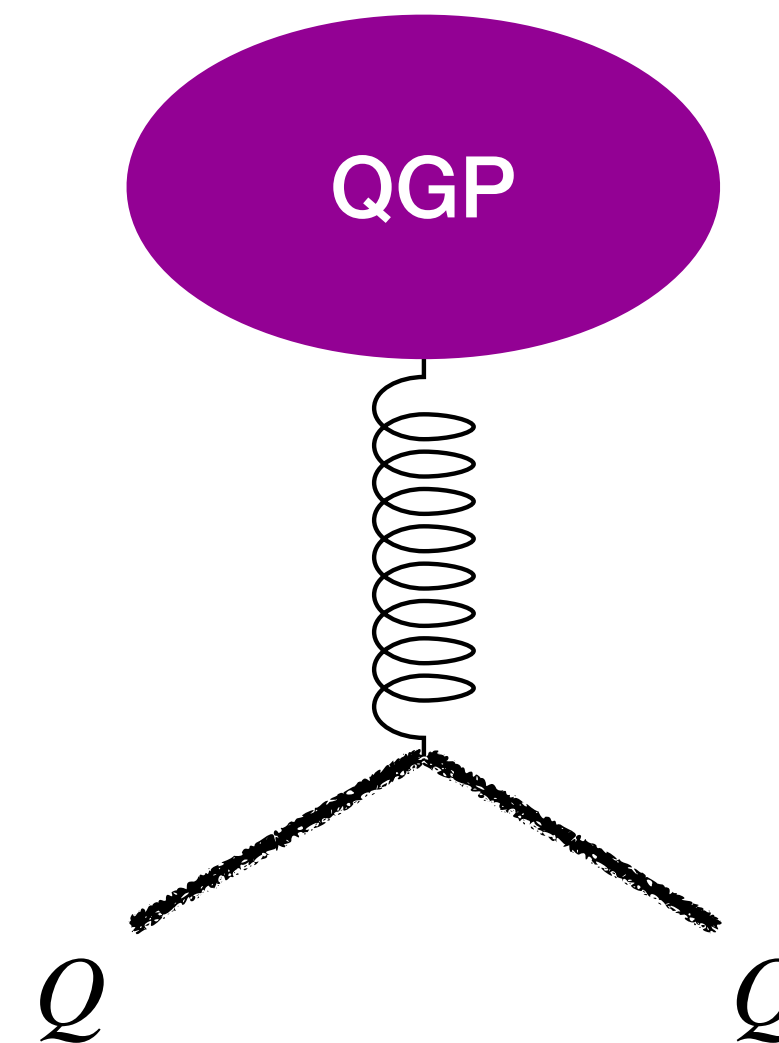
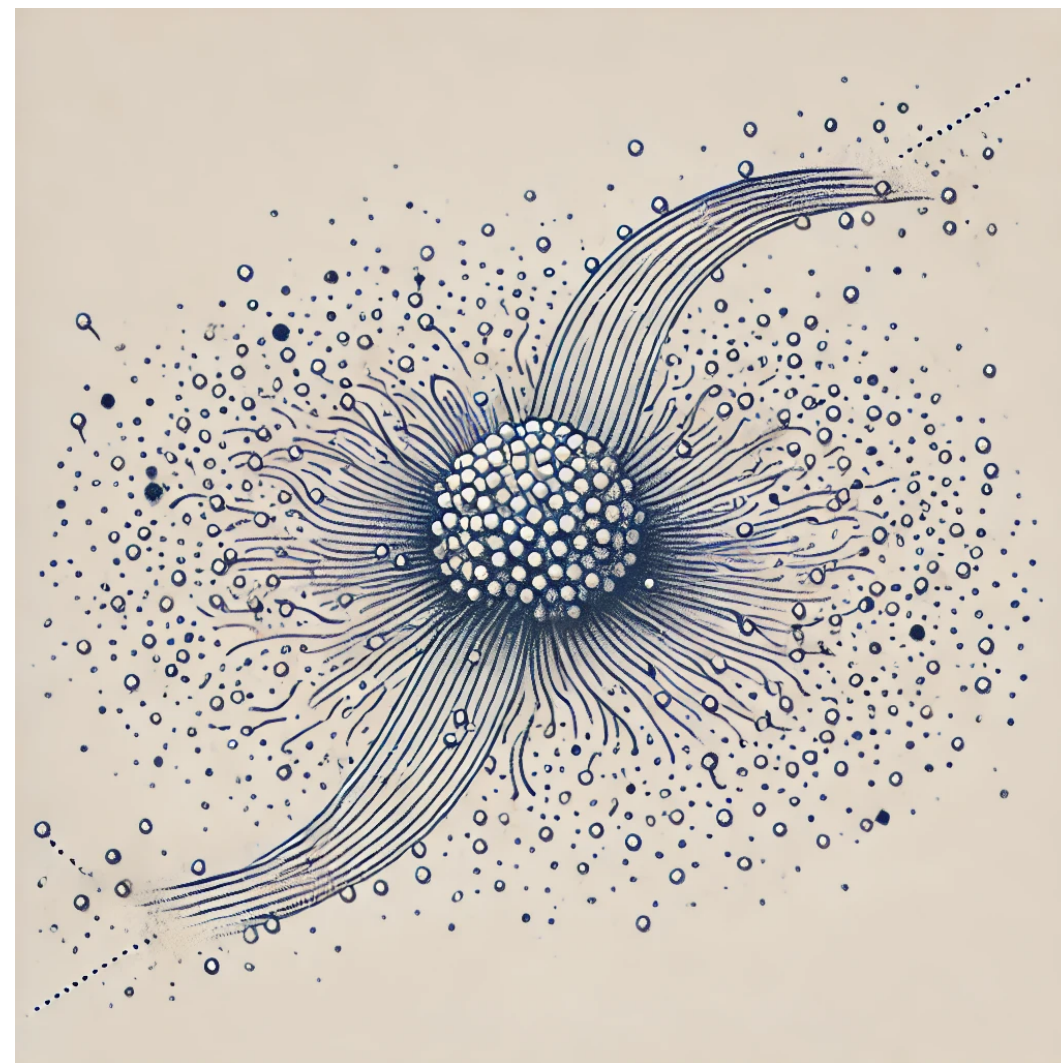


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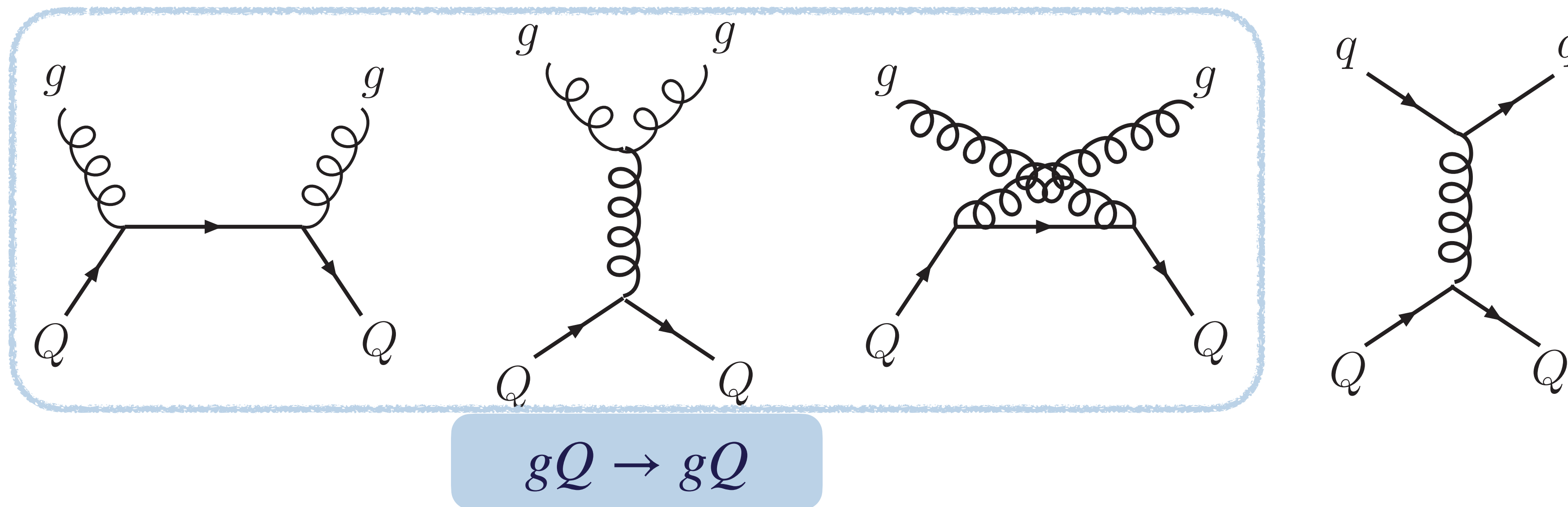


HEAVY QUARK INTERACTIONS

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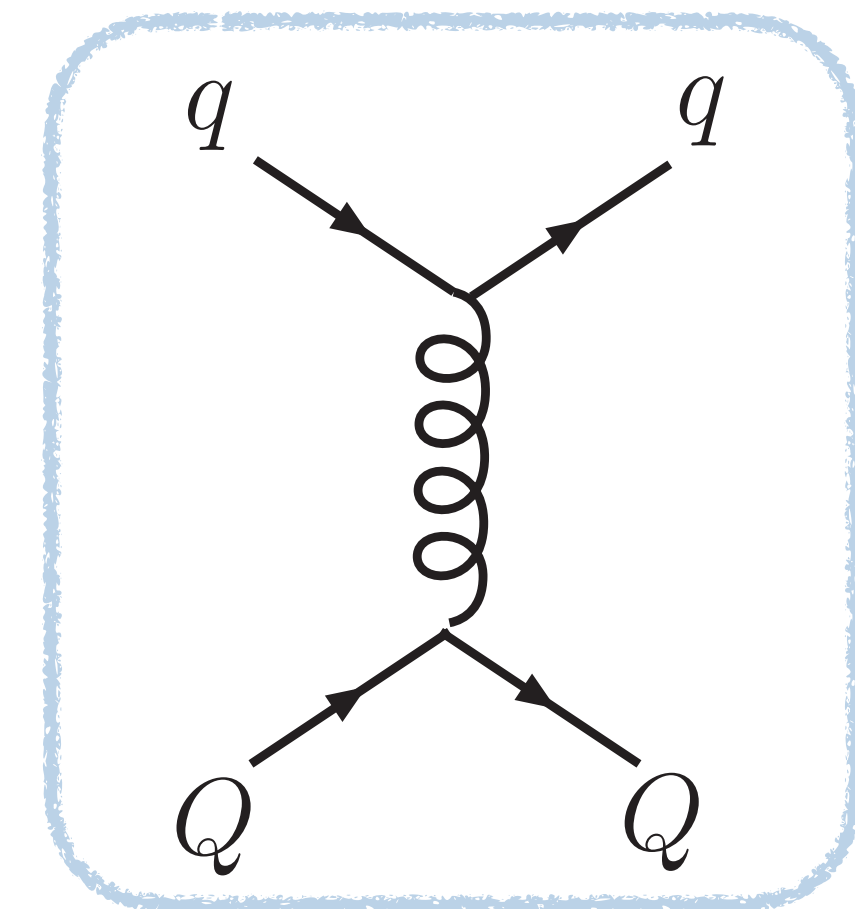
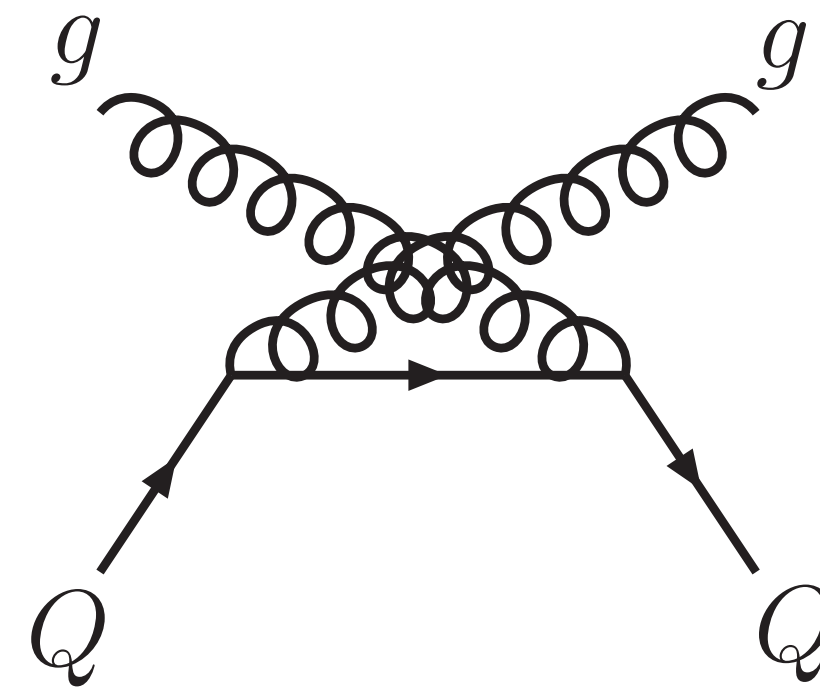
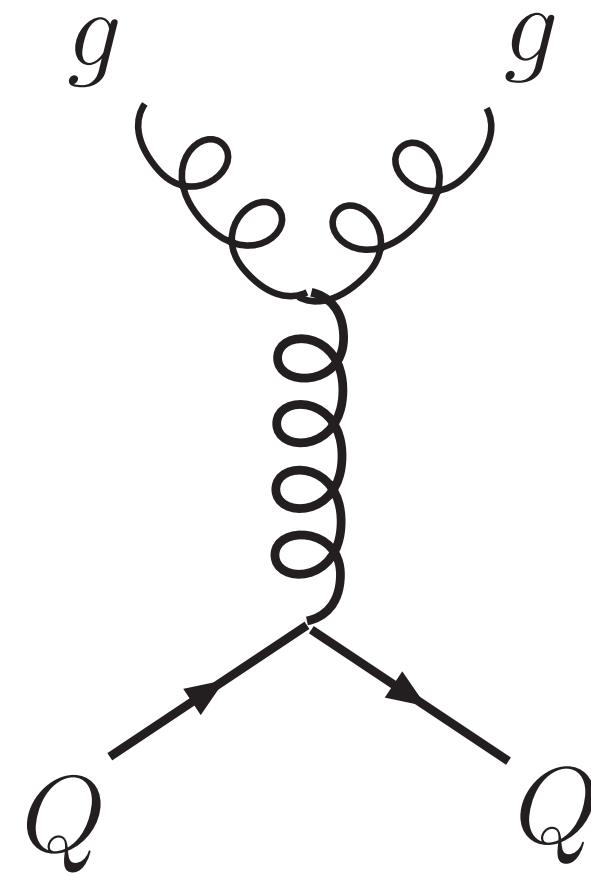
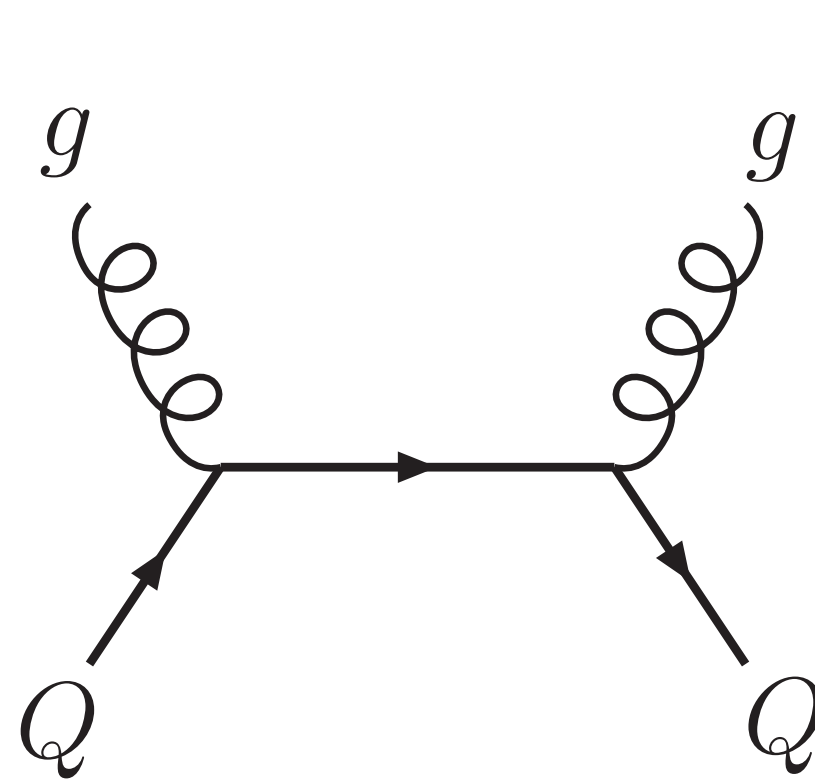
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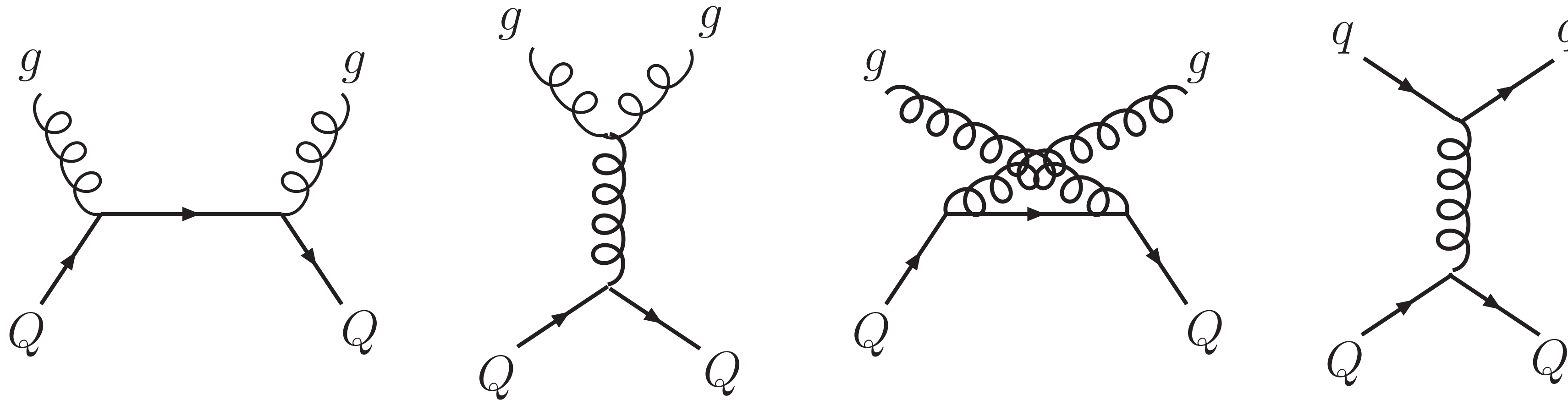


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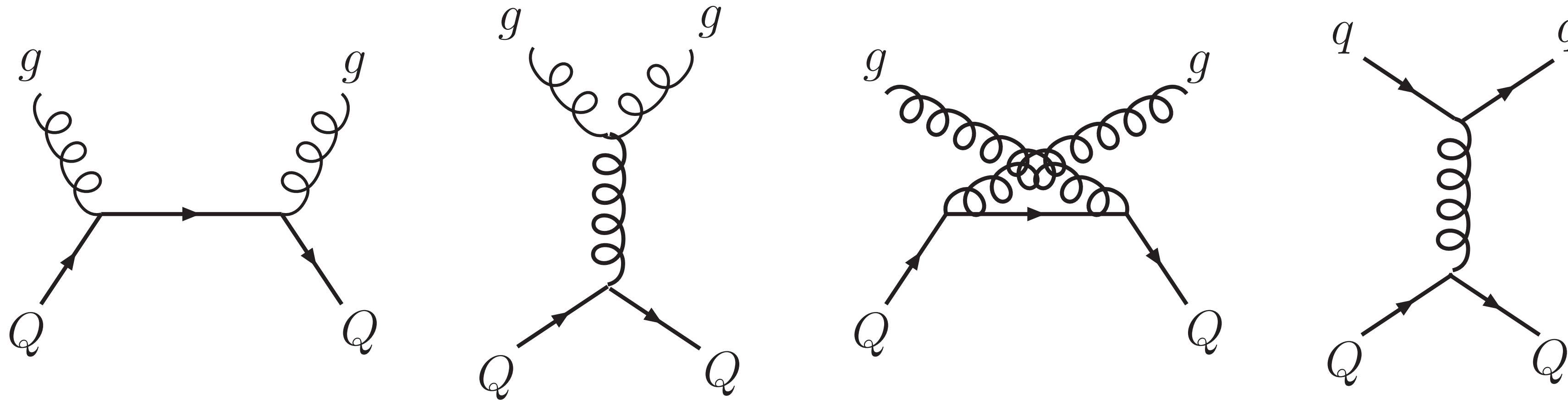


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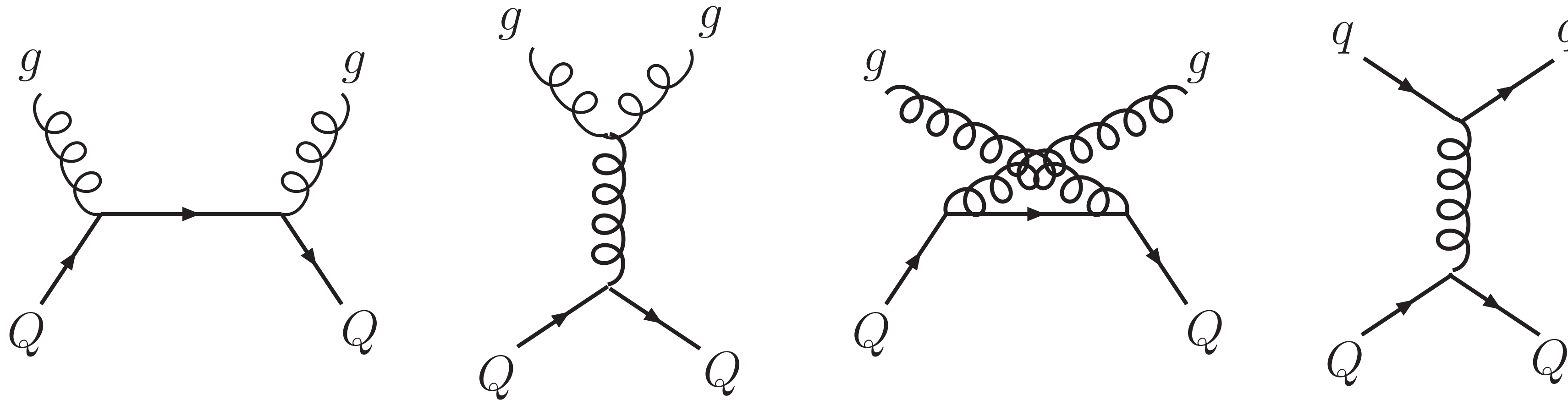
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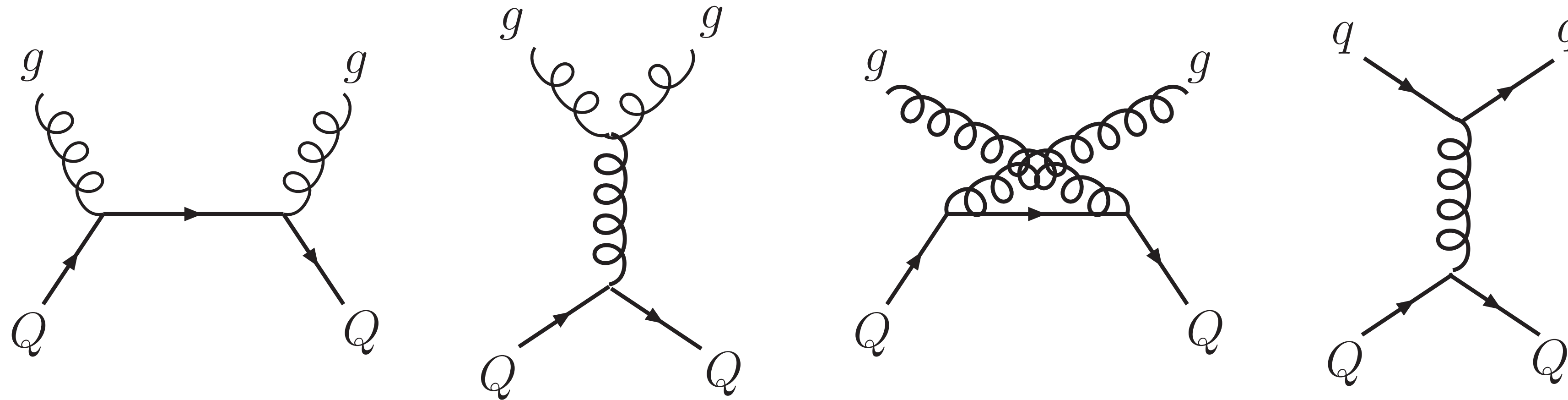
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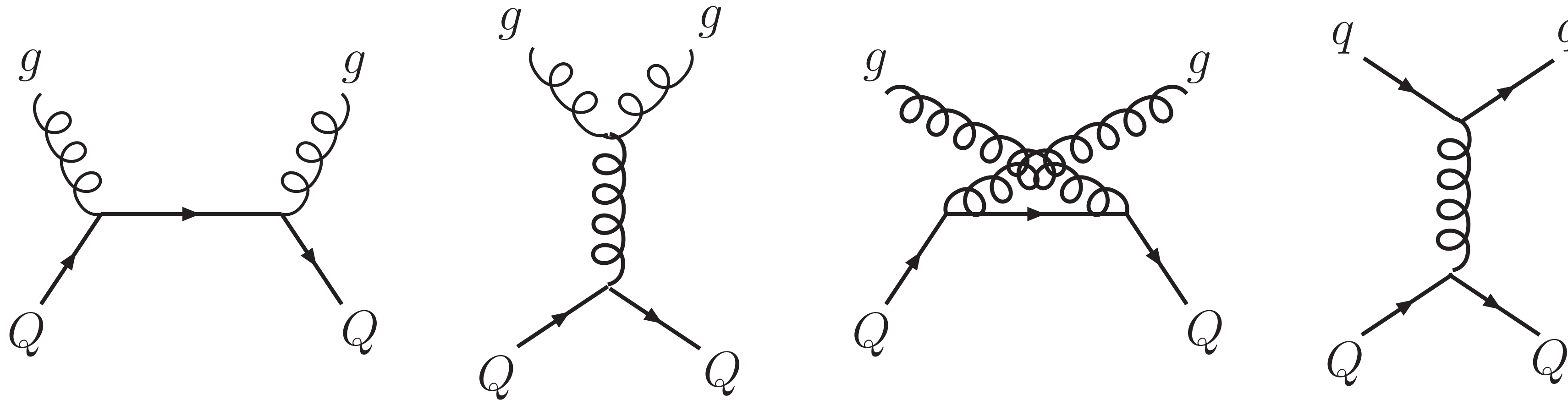
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Weakly coupled medium

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The coefficient should go to zero

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The motion is described by the Langevin equation:

$$\frac{dp_i}{dt} = -\eta p_i + f_i, \text{ with } \langle f_i(t')f_i(t) \rangle = \kappa \delta(t' - t) \text{ and } \kappa = 2MT\eta = \frac{2T^2}{D_s}$$

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LORENTZ FORCE

Matching both descriptions:

$$\langle f_i(t')f_j(t) \rangle = \langle E_i(t')E_j(t) \rangle + \frac{1}{3}\langle \mathbf{v}^2 \rangle \langle \delta_{ij}B_k(t')B_k(t) - B_j(t')B_i(t) \rangle, \text{ with } \langle \mathbf{v}^2 \rangle = \frac{3T}{M}$$

TOTAL DIFFUSION COEFFICIENT:
 $\kappa = \kappa_\infty + 2/3\langle \mathbf{v}^2 \rangle \kappa_{1/M}$

$\kappa_\infty \equiv \kappa_E$ and $\kappa_{1/M} \equiv \kappa_B$

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Casadelrrey-Solana, Teaney, Caron-Huot, Laine, Moore

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$$G_{E,B}(\tau, T) \equiv \int_0^\infty \frac{d\omega}{\pi} \rho_{E,B}(\omega, T) K(\omega, \tau, T) \text{ with } K(\omega, \tau, T) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$

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PHYSICALLY MOTIVATED ANSATZ FOR THE SPECTRAL FUNCTION

LATTICE ANALYSIS

NUMERICAL SETUP

We have generated lattice data ensembles in 2+1 flavor QCD with physical strange quark masses m_s over a temperature range from $T = 133$ to 10000 MeV on a volume of $N_\sigma^3 \times N_\tau$ with $N_\sigma = 64$.

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HIGHLY IMPROVED STAGGERED QUARKS (HISQ)

HPQCD, UKQCD (2007)

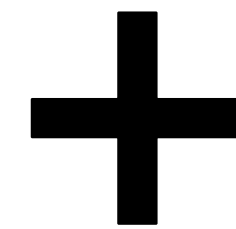
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TREE-LEVEL IMPROVED LÜSCHER-WEISZ

M. Lüscher, P. Weisz (1985)

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- For **high temperatures** we set $m_l = m_s/5$ (with $T_c \simeq 180$ MeV and $m_\pi \simeq 320$ MeV).

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At *sufficiently high* temperatures, the effect of the light quark masses is expected to be negligible. At lower temperatures, and especially near the pseudo-critical temperature T_c , these masses can have significant effects.

- For **high temperatures** we set $m_l = m_s/5$ (with $T_c \simeq 180$ MeV and $m_\pi \simeq 320$ MeV).
- For **low temperatures** we set $m_l = m_s/20$ (with $T_c \simeq 160$ MeV and $m_\pi \simeq 160$ MeV).

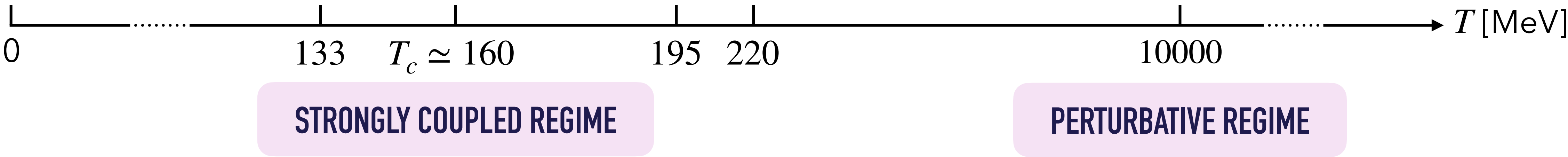
NUMERICAL SETUP

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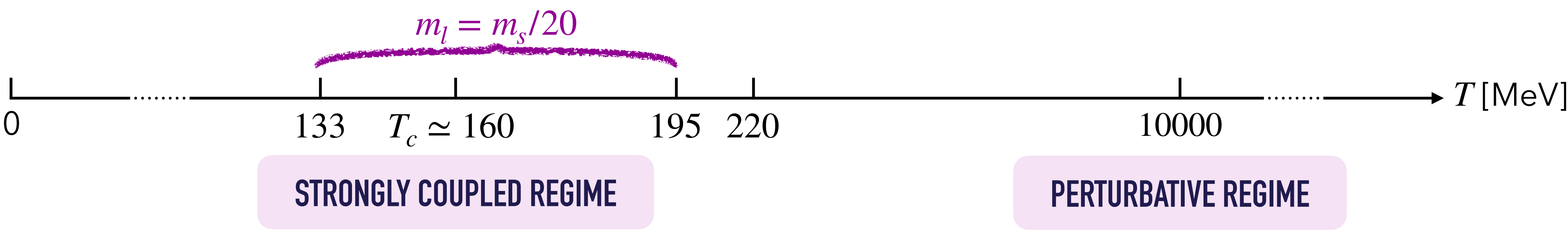
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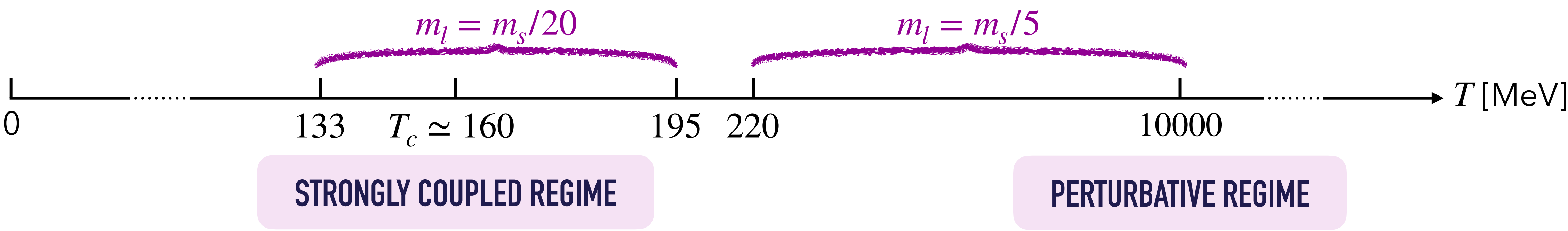
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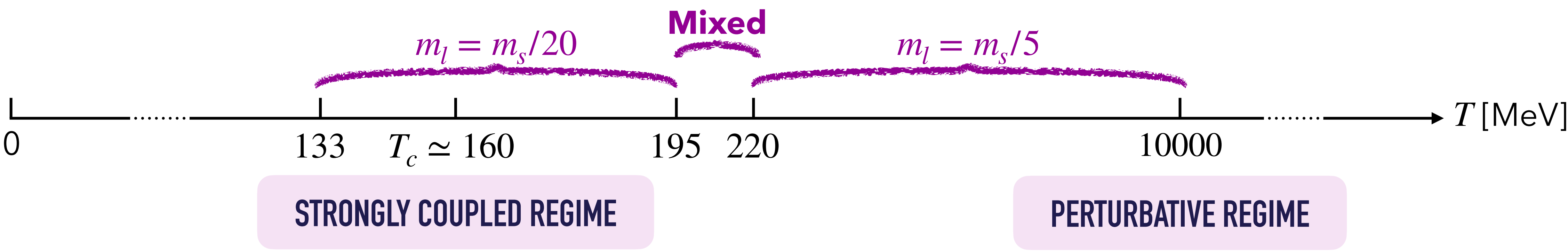
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Based on previous perturbative studies, we expect that the slopes of the extrapolations will approach zero for very large τT , with a leading term proportional to $1/(\tau T)^2$:

TUMQCD, HotQCD

CONTINUUM LIMIT

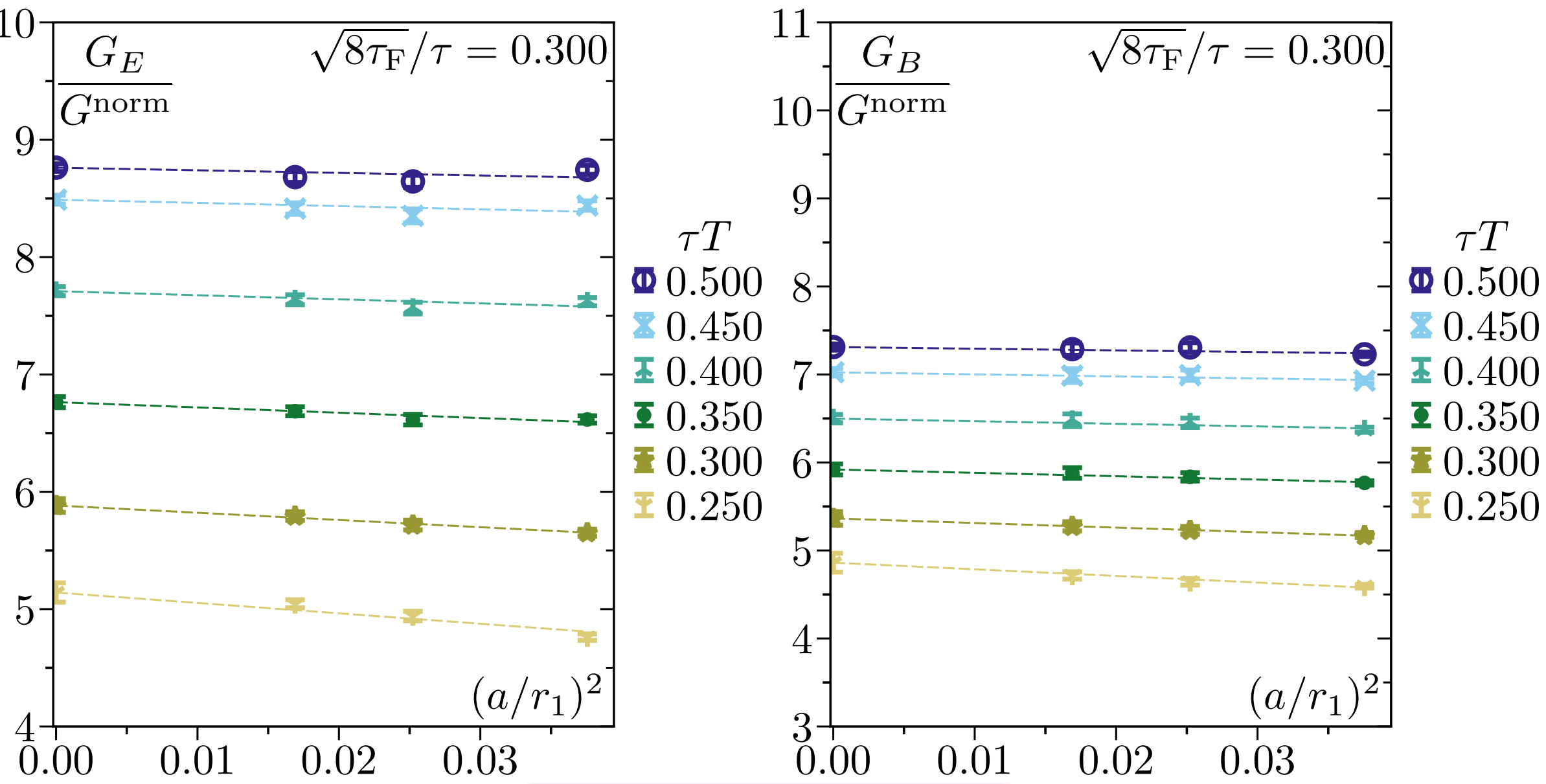
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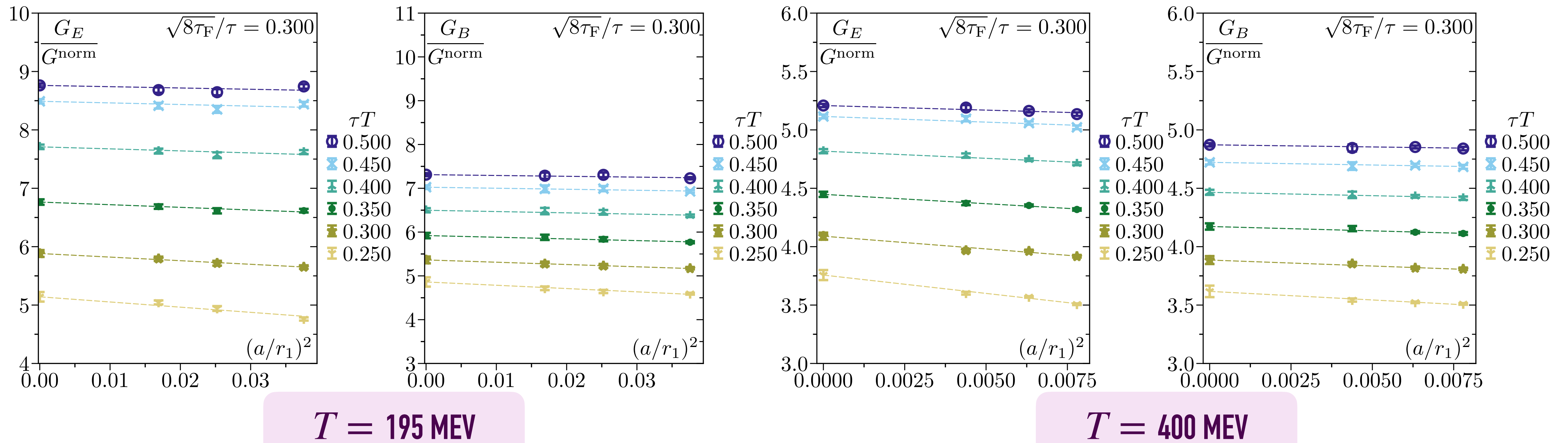


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ELECTRIC MATCHING

The heavy quark is so “heavy” that the quark number is conserved

$$M \rightarrow \infty$$

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First $1/M$ correction, so the current is approximately conserved

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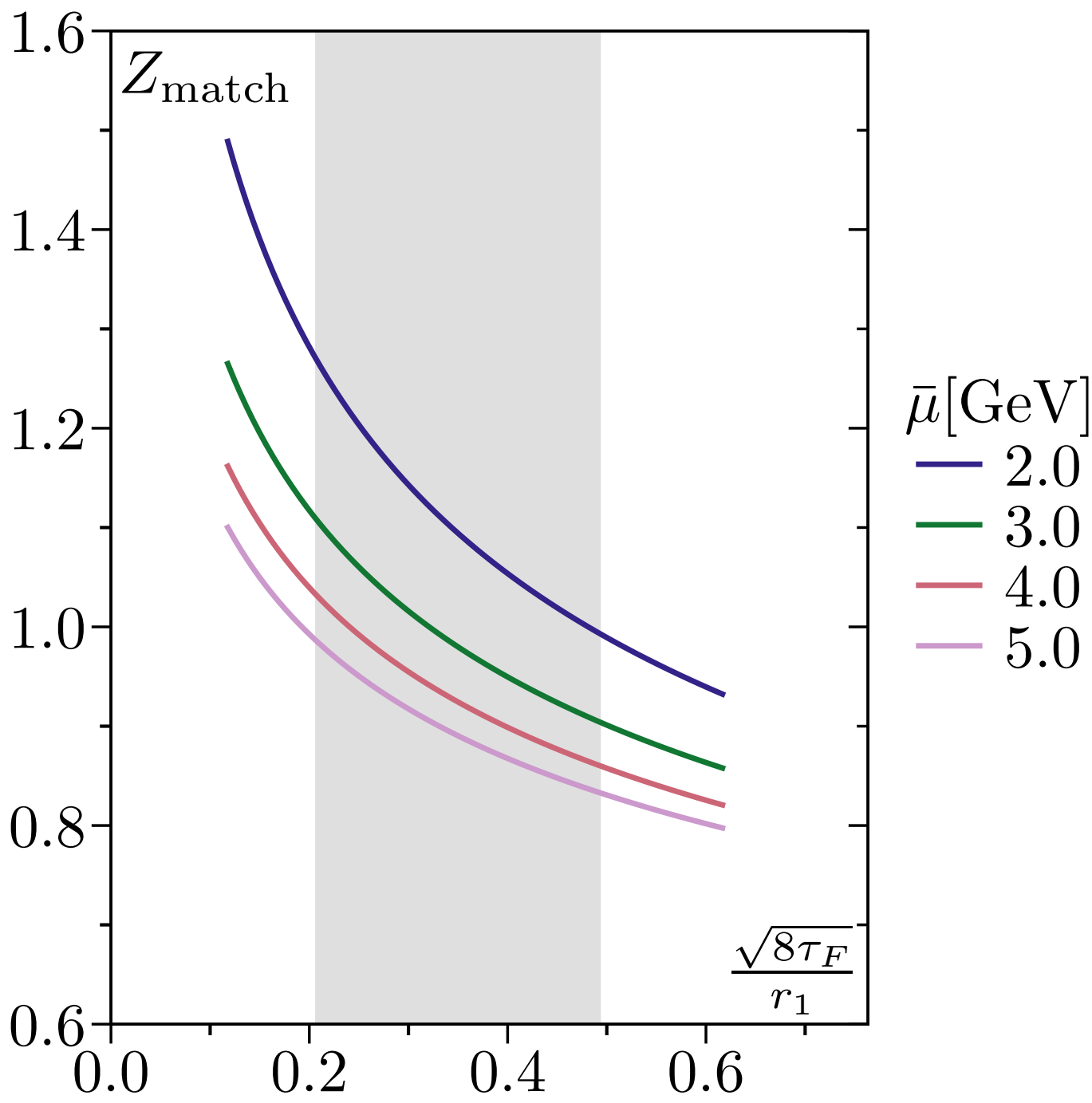
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All τT together

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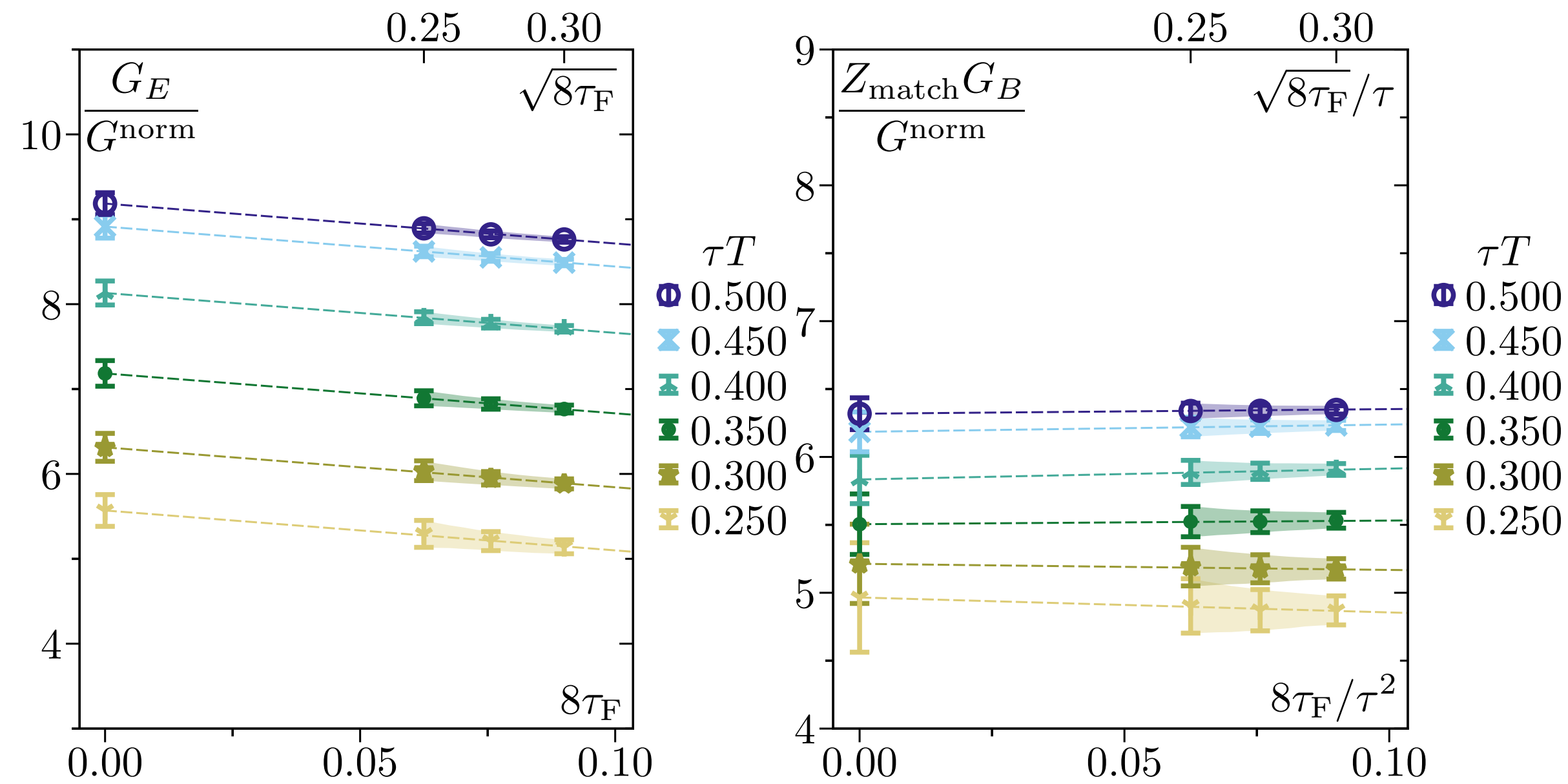
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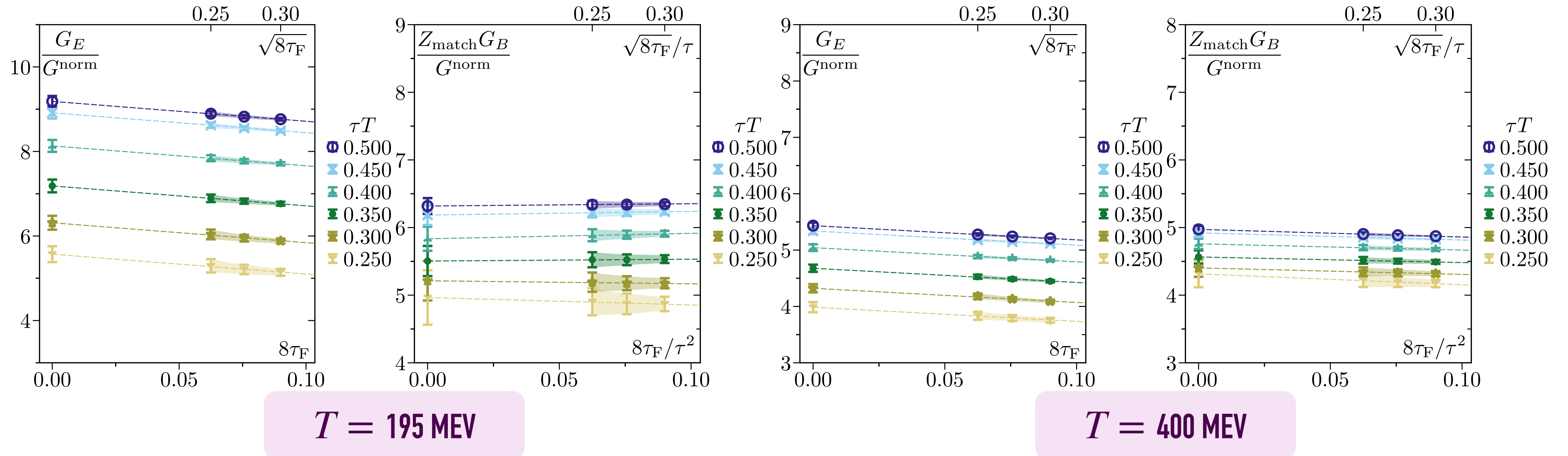
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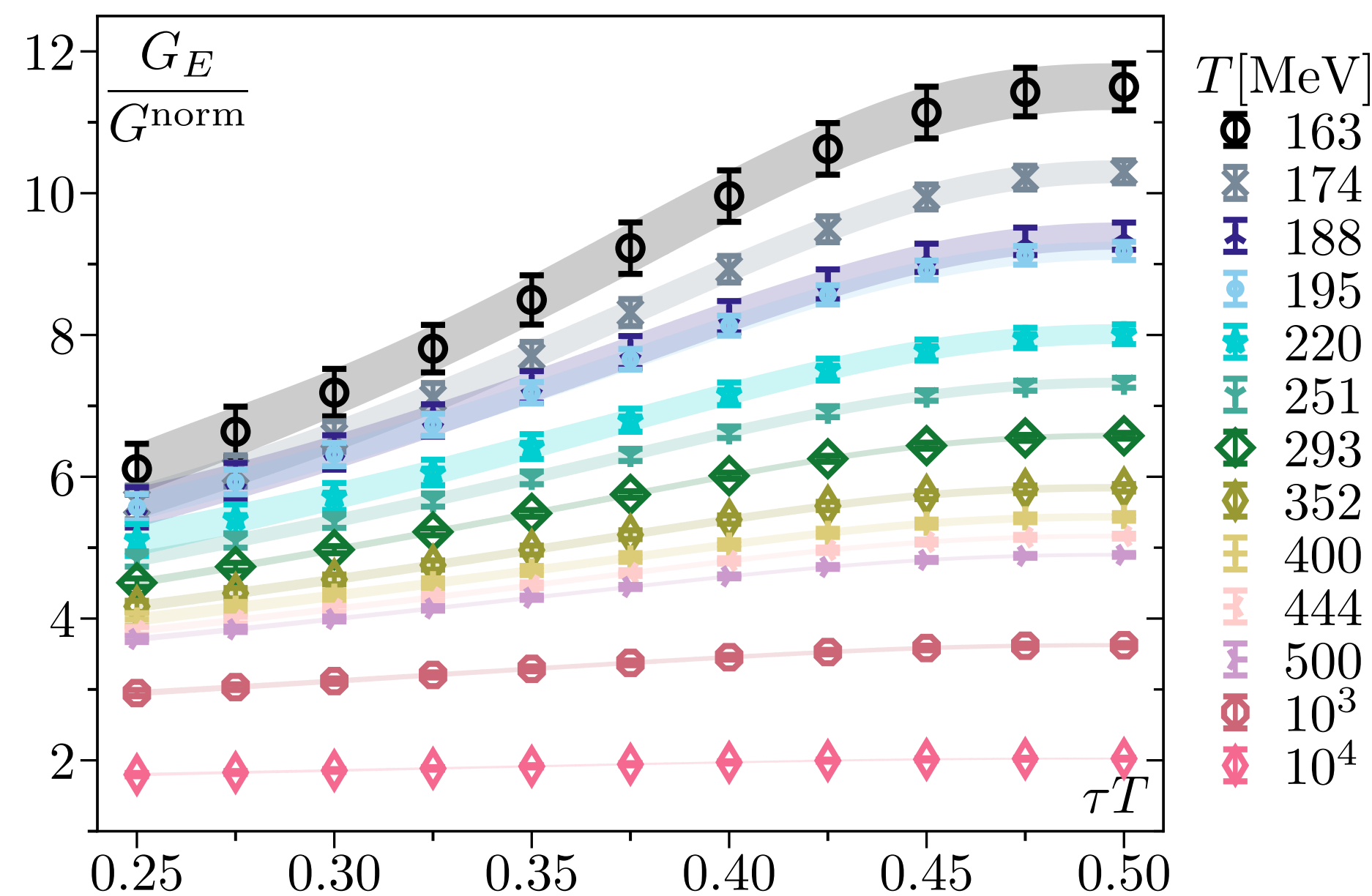
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DOUBLE EXTRAPOLATED CORRELATORS

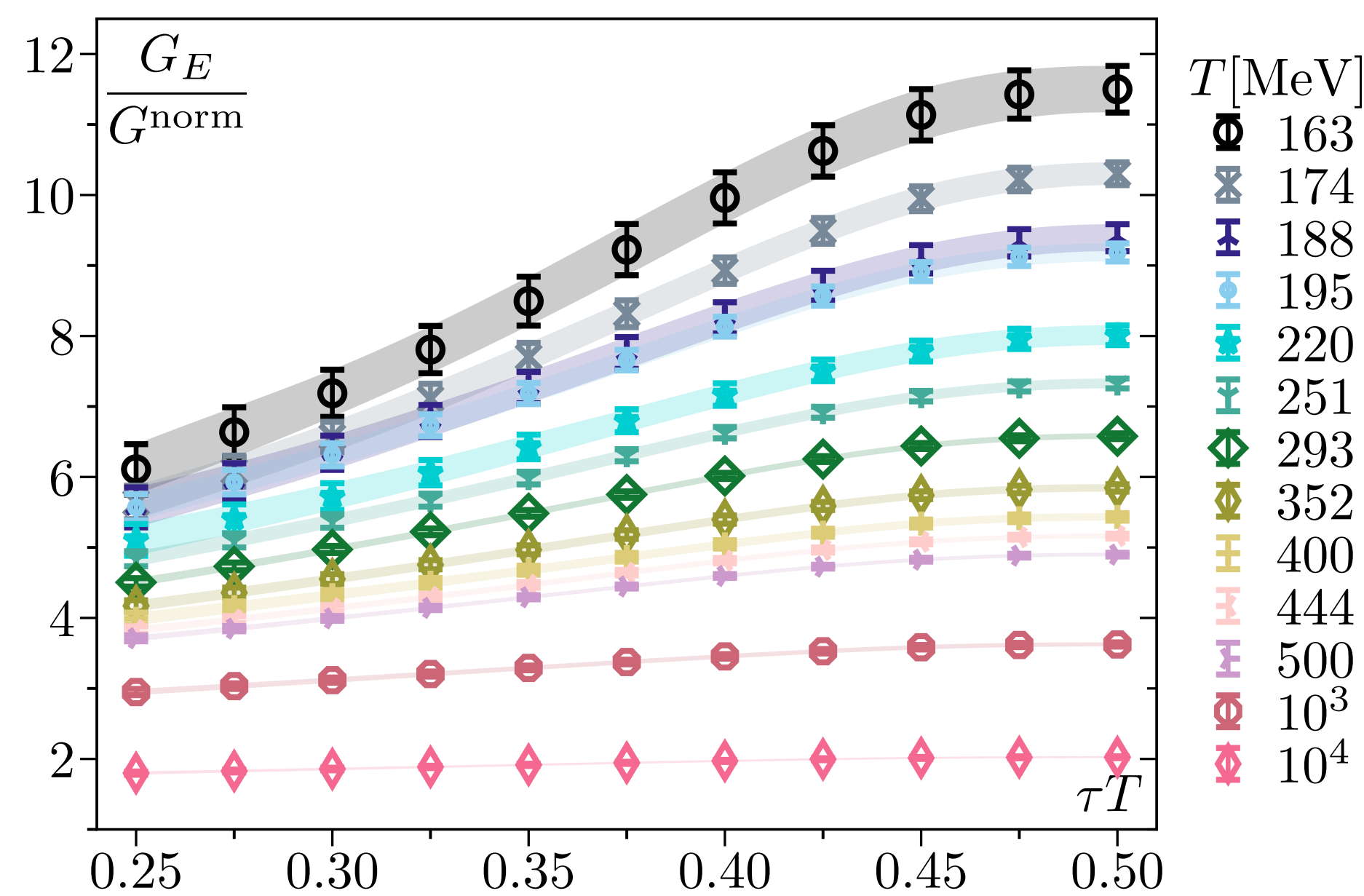
Let us give a look to the continuum and zero flow extrapolated results for both correlators:



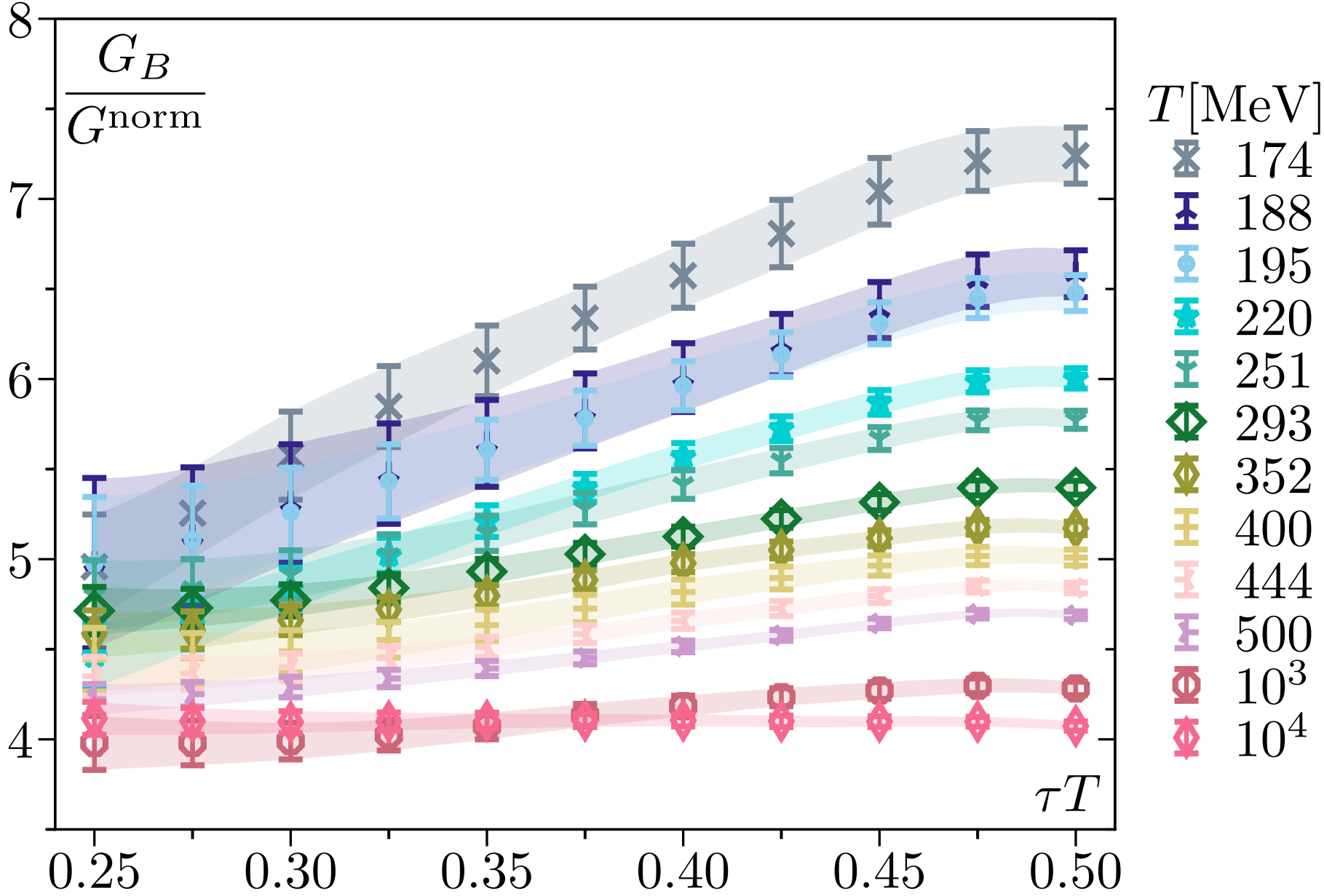
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Smaller curvature $\rightarrow$ Smaller κ

PERTURBATIVE QCD ESTIMATES

PERTURBATIVE RESULTS FOR THE DIFFUSION COEFFICIENT

The diffusion coefficient is encoded in the IR region of the spectral function, so we need to reconstruct the spectral function in some physically motivated way.

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PERTURBATION THEORY

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Caron-Huot, Laine, Moore (2009)

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Burnier, Laine, Langelage, Mether (2010)

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Banerjee, Datta, Laine (2022)

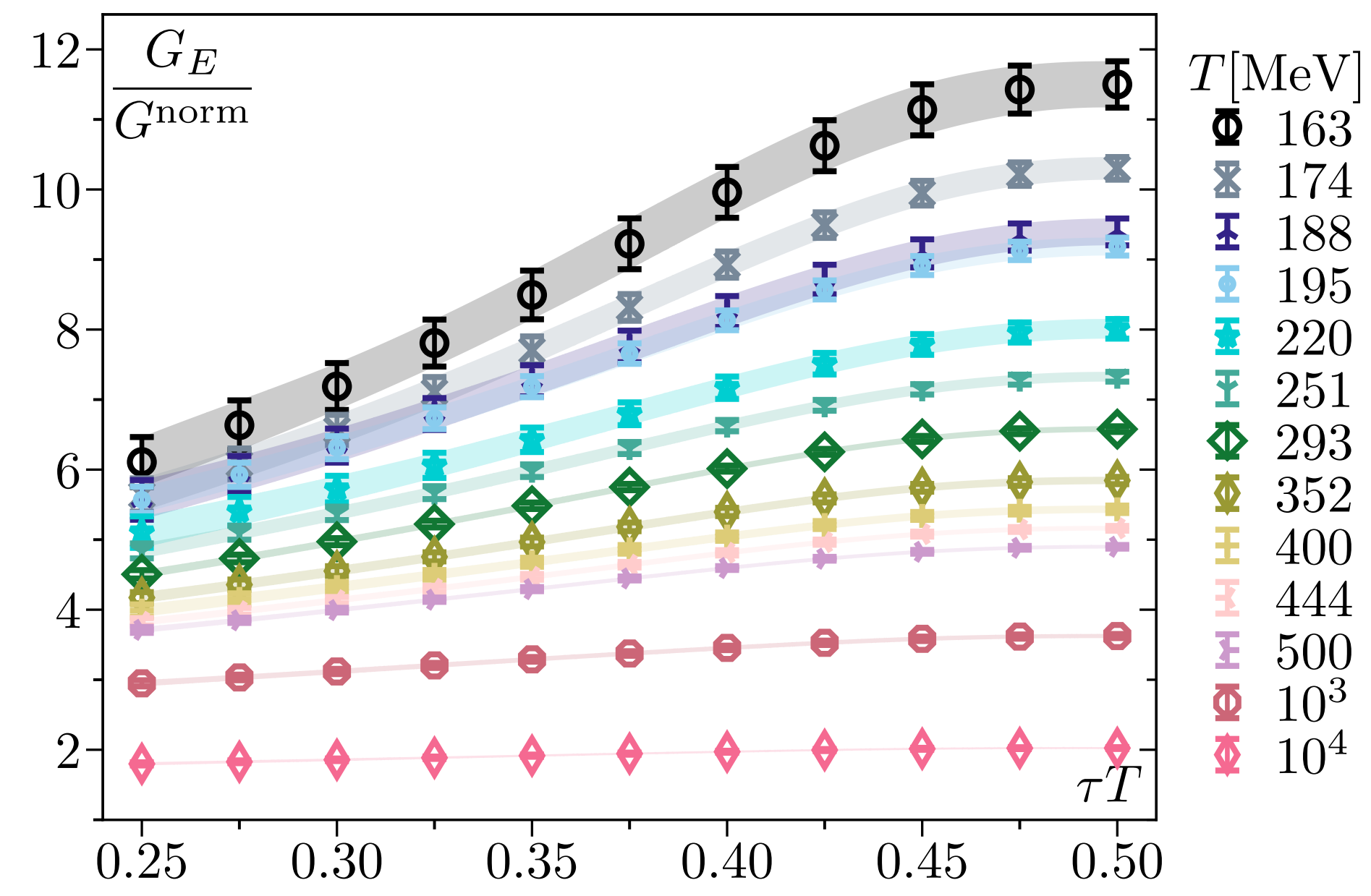
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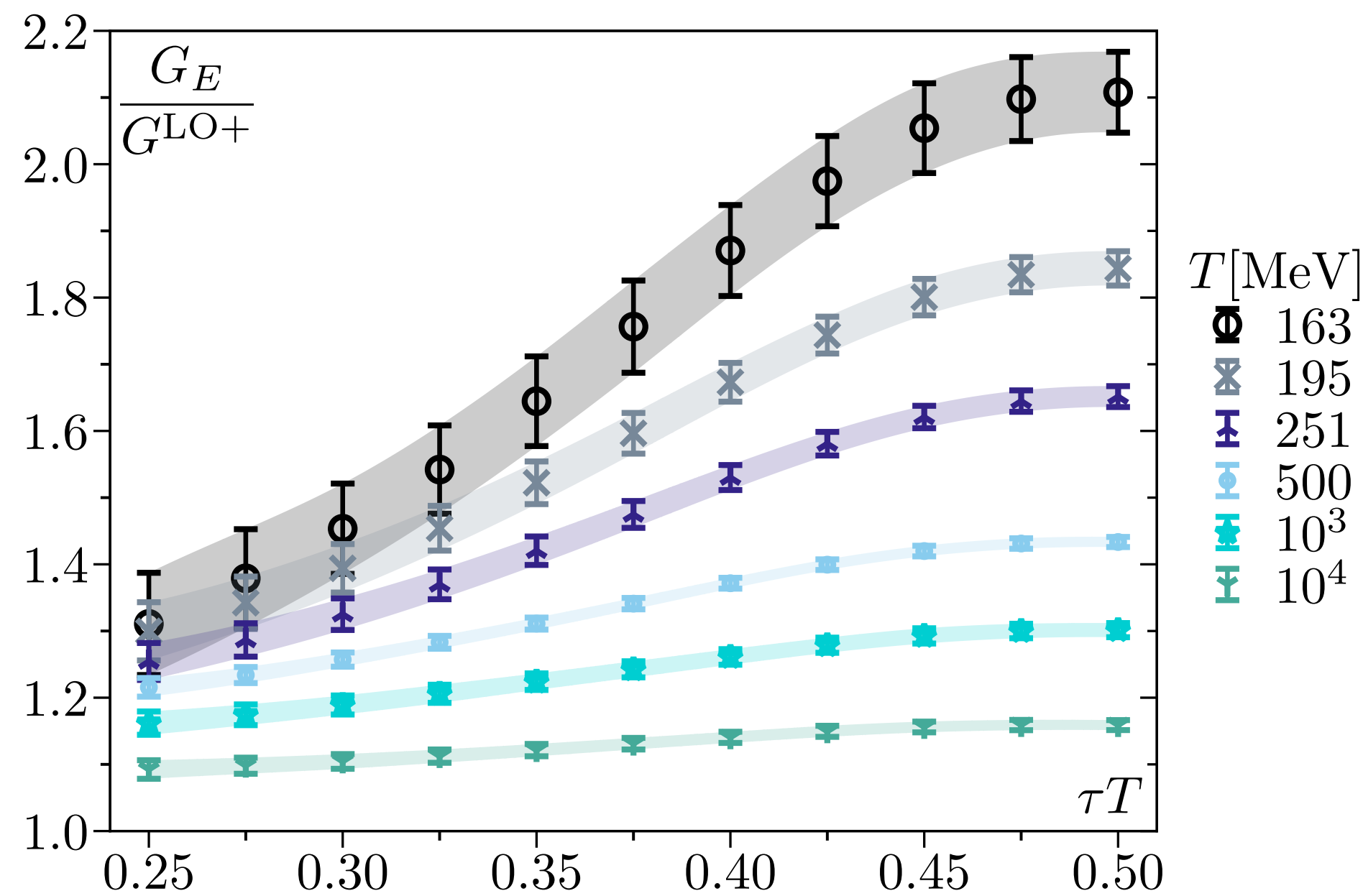
CORRELATORS NORMALIZE BY G^{norm}

Computed with $\rho_E^{\text{LO}}(\omega, \tau)$ but factorizing out the running coupling

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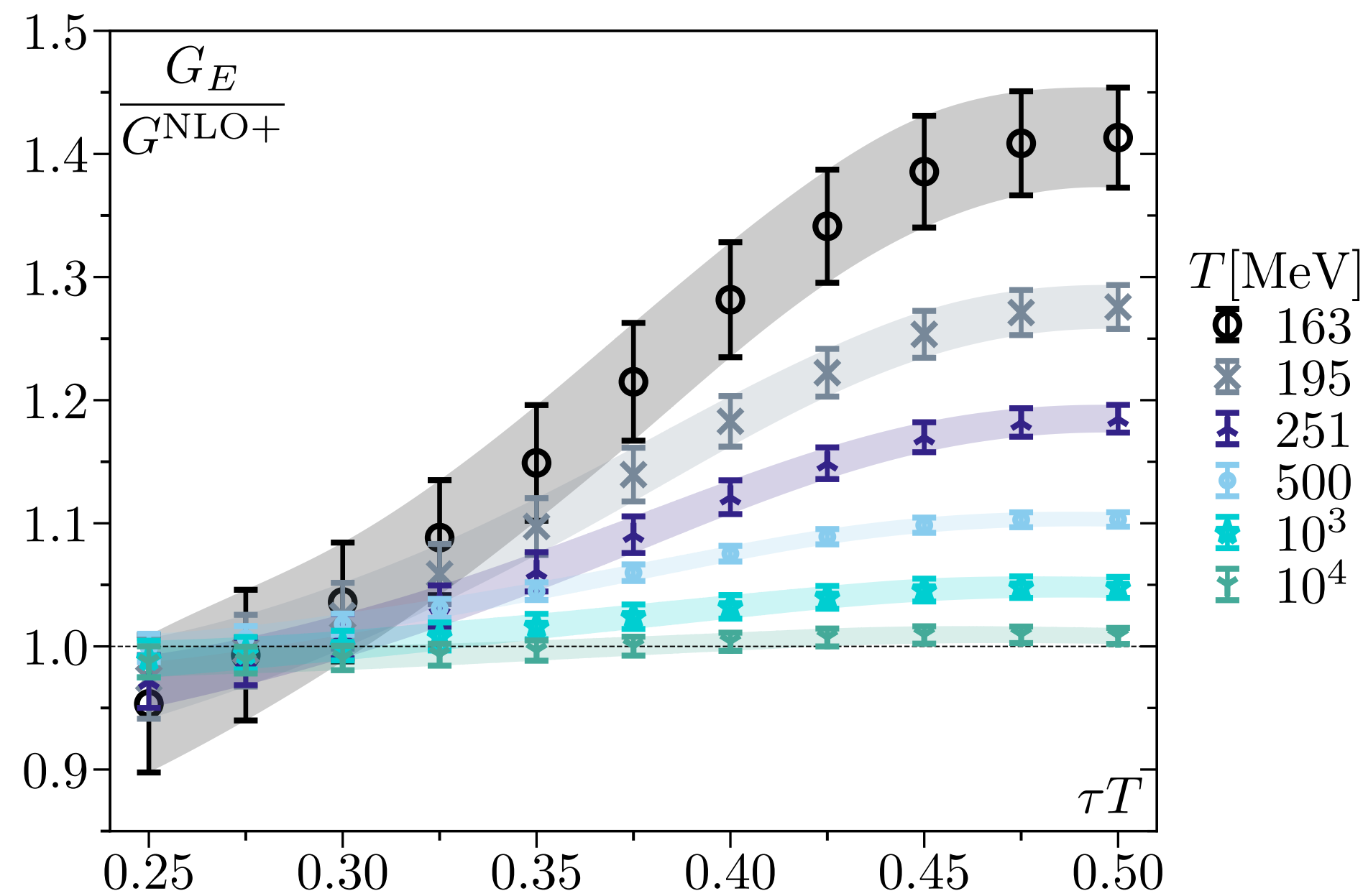
CORRELATORS NORMALIZE BY G^{LO+}

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CORRELATORS NORMALIZE BY $G^{\text{NLO}+}$

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THE RATIO IS ALMOST FLAT FOR LARGE T , SO $\kappa \rightarrow 0$

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There exist a LO of and NLO direct estimations of the diffusion coefficient:

$$\frac{\kappa_E^{\text{LO}}}{T^3} = \frac{g^4 C_F}{18\pi} \left(\left[N_c + \frac{N_f}{2} \right] \left[\ln \frac{2T}{m_D} + \xi \right] + \frac{N_f \ln 2}{2} \right)$$

Burnier, Laine, Langelage, Mether, Caron-Huot, Moore

$$\xi = 1/2 - \gamma_E + \zeta'(2)/\zeta(2) \simeq -0.64718$$

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The “convergence” of the expansion, however, is problematic unless the temperature is asymptotically high

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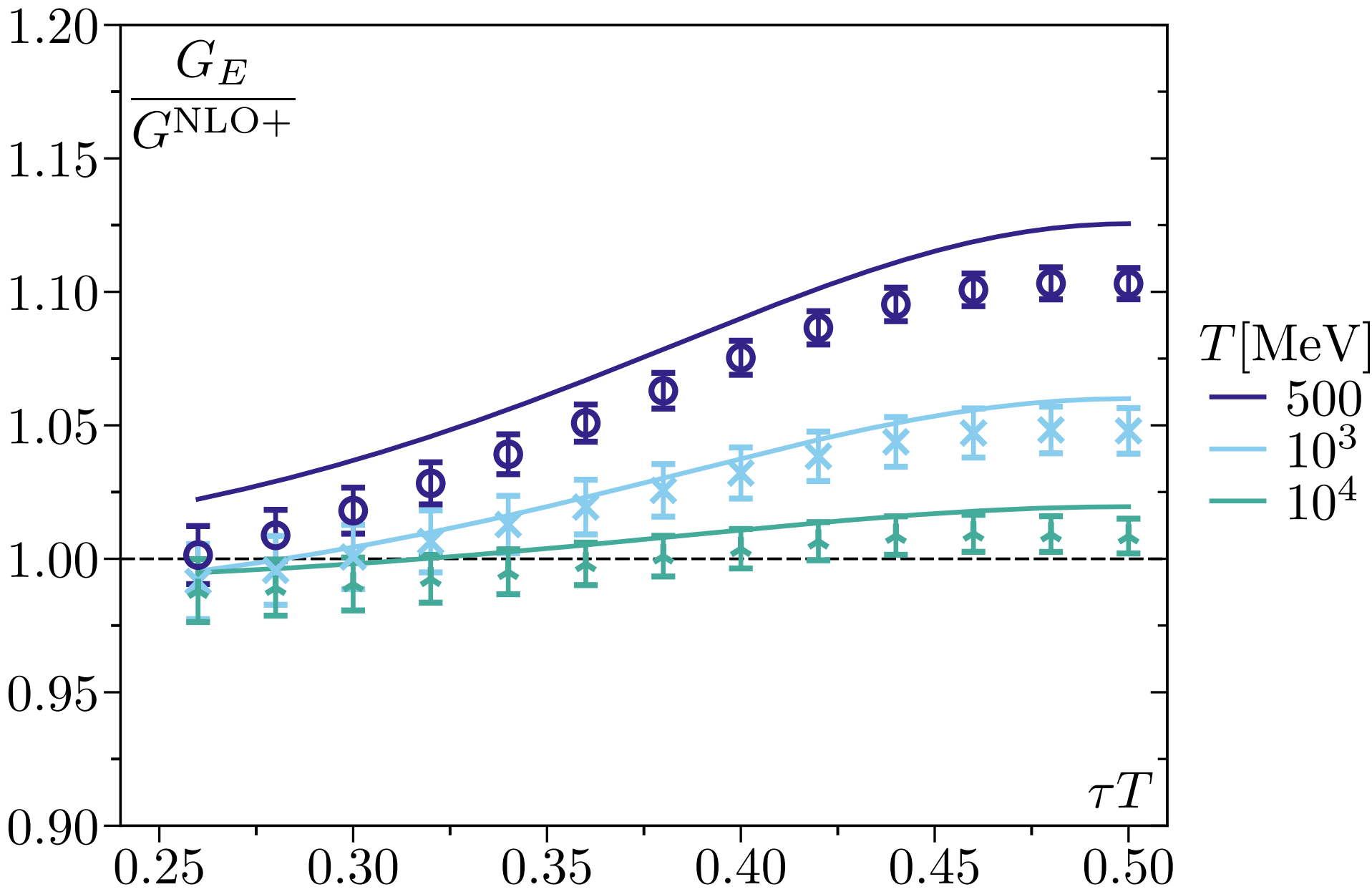
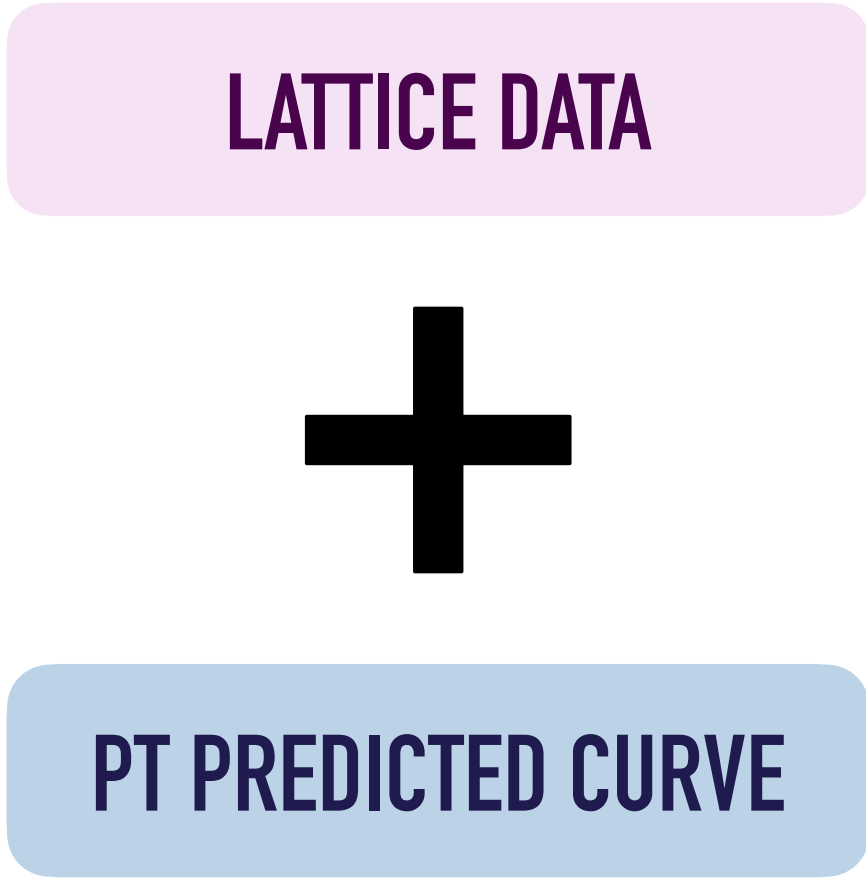
On the other hand, the NLO correction is $O(g^5)$ and very large numerically, and therefore κ_E^{NLO} is positive.

PERTURBATIVE RESULTS FOR THE DIFFUSION COEFFICIENT

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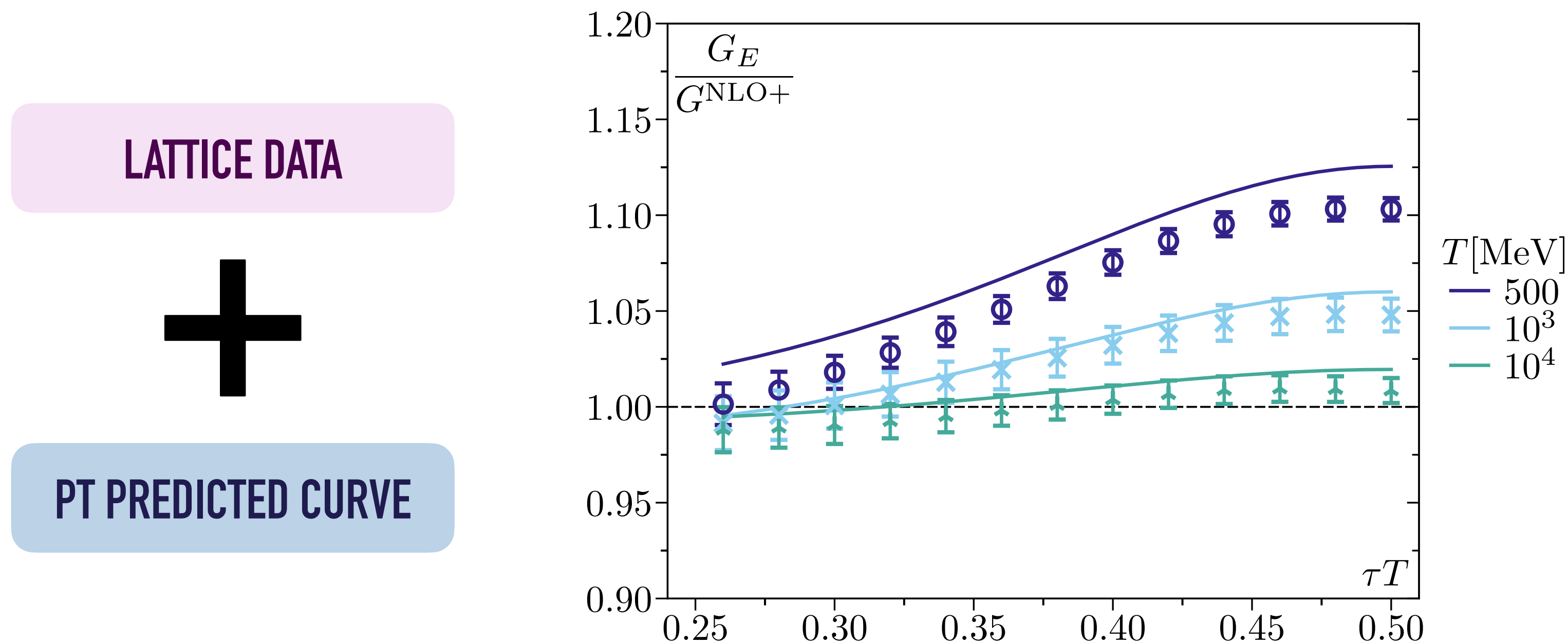
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GREAT AGREEMENT BETWEEN LATTICE DATA AND PT PREDICTIONS

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$$\left\{ \begin{array}{ll} \rho^{\text{IR}}(\omega, T) & \text{for } 10^{-6} \leq \omega/T \leq \omega_{\text{IR}} \\ c\omega^p & \text{for } \omega_{\text{IR}} \leq \omega/T \leq \omega_{\text{UV}} \\ \rho^{\text{UV}}(\omega) & \text{for } \omega_{\text{UV}} \leq \omega/T \leq 10^3 \end{array} \right.$$

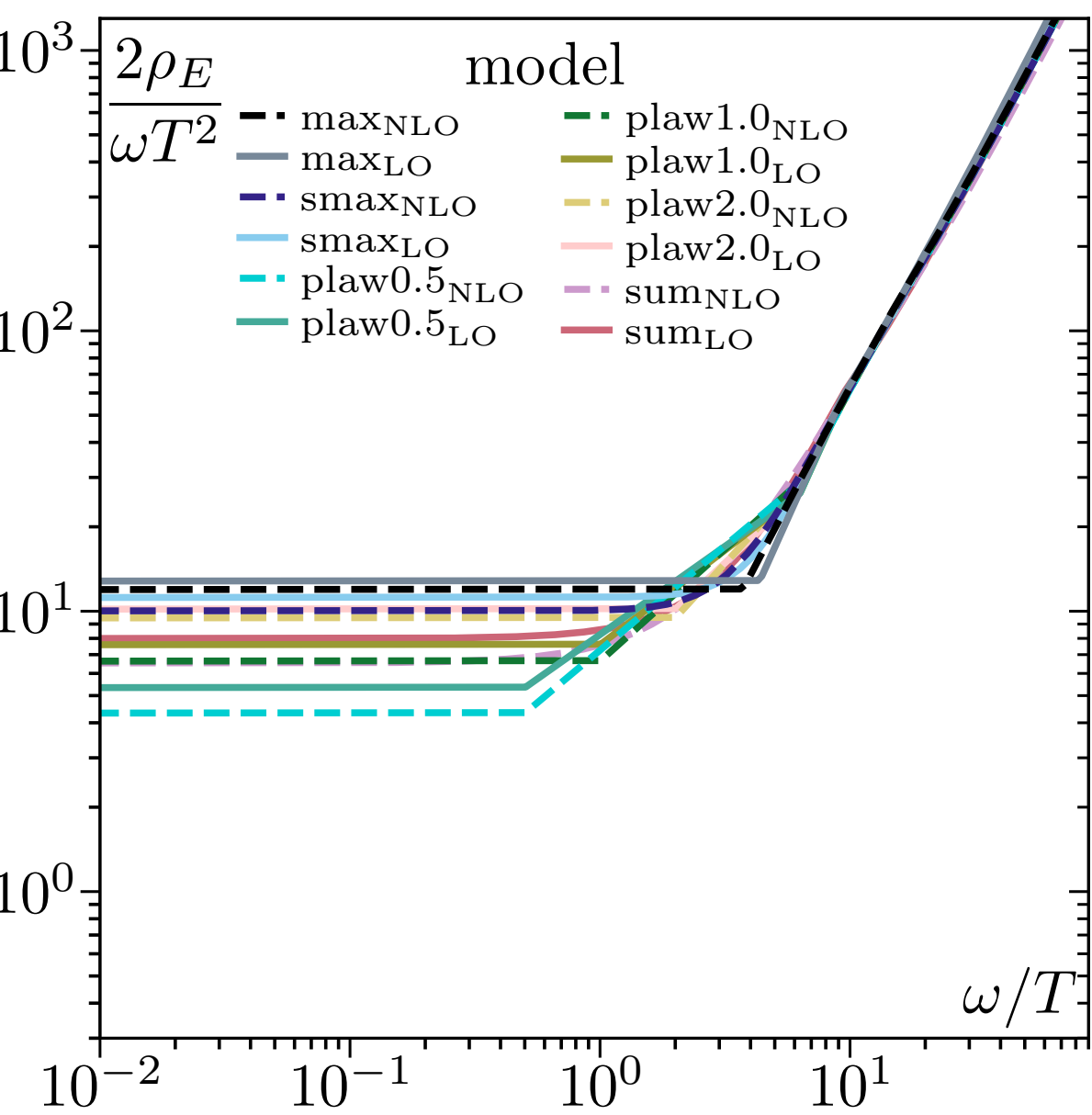
$$\omega_{\text{IR}}/T = \mathbf{0.5, 1.0, 2.0}$$
$$+$$
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We now present the results of using the previously discussed fitting techniques to extract the diffusion coefficients $\kappa_{E,B}/T^3$.

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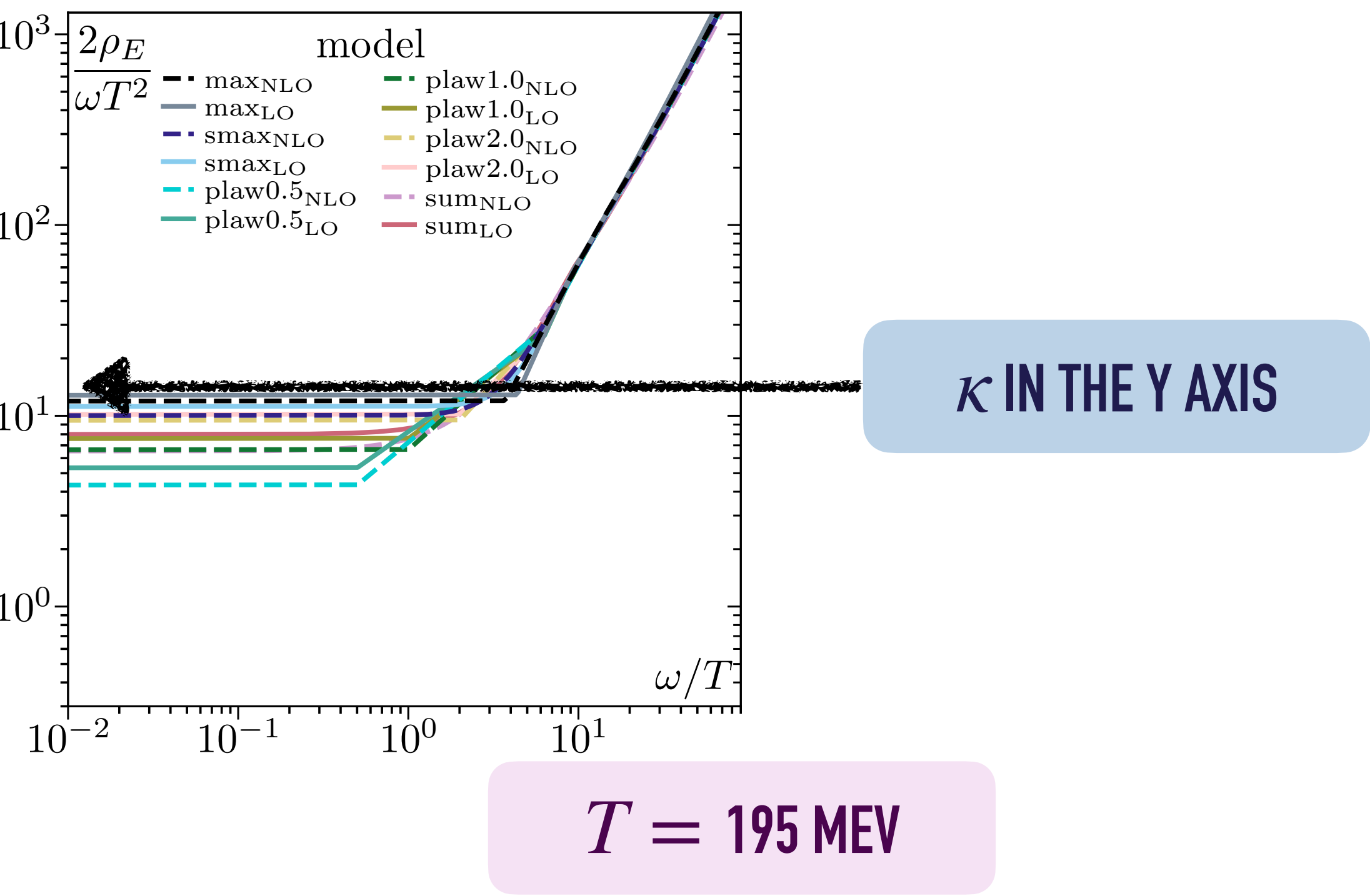
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$T = 195 \text{ MeV}$

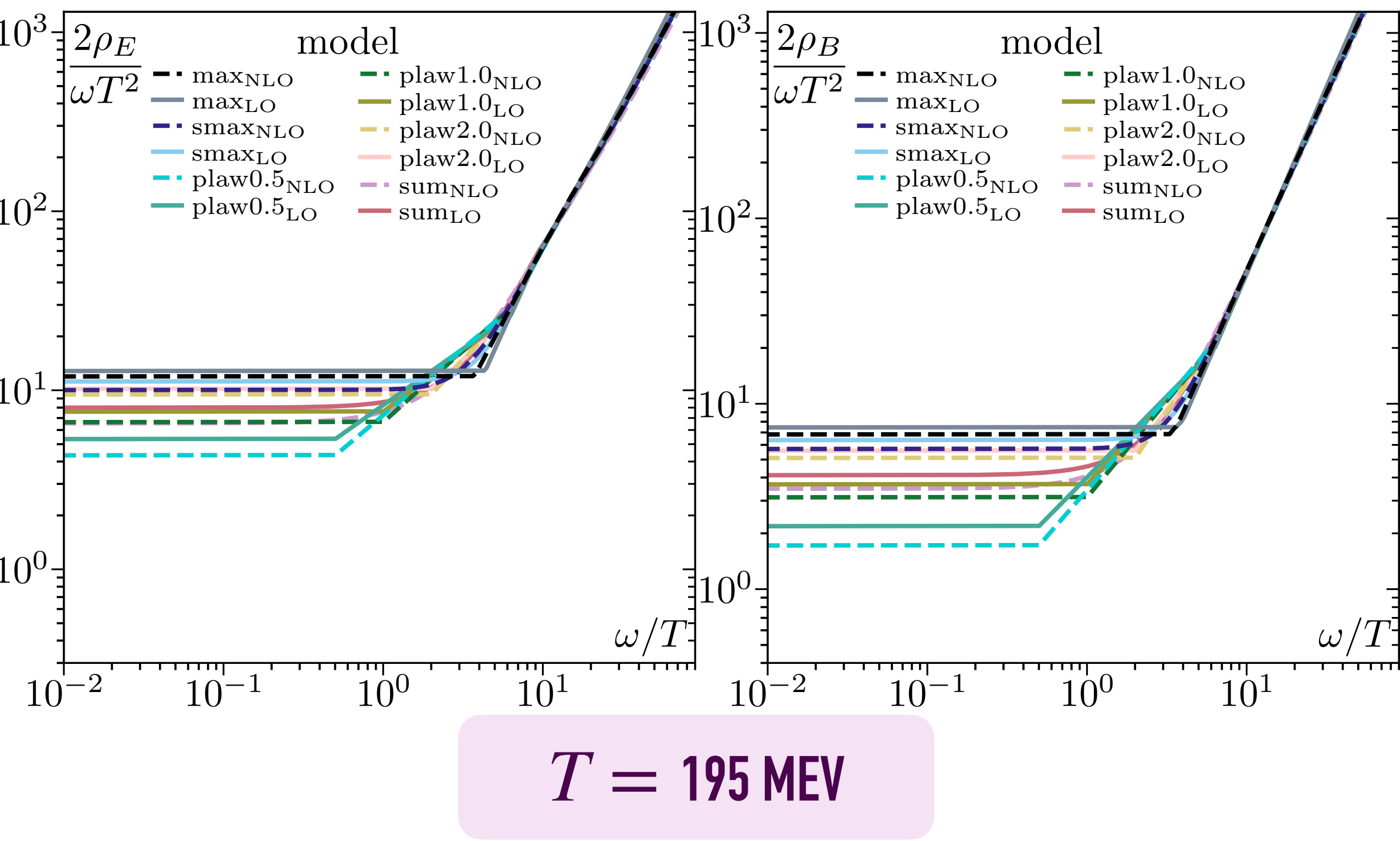
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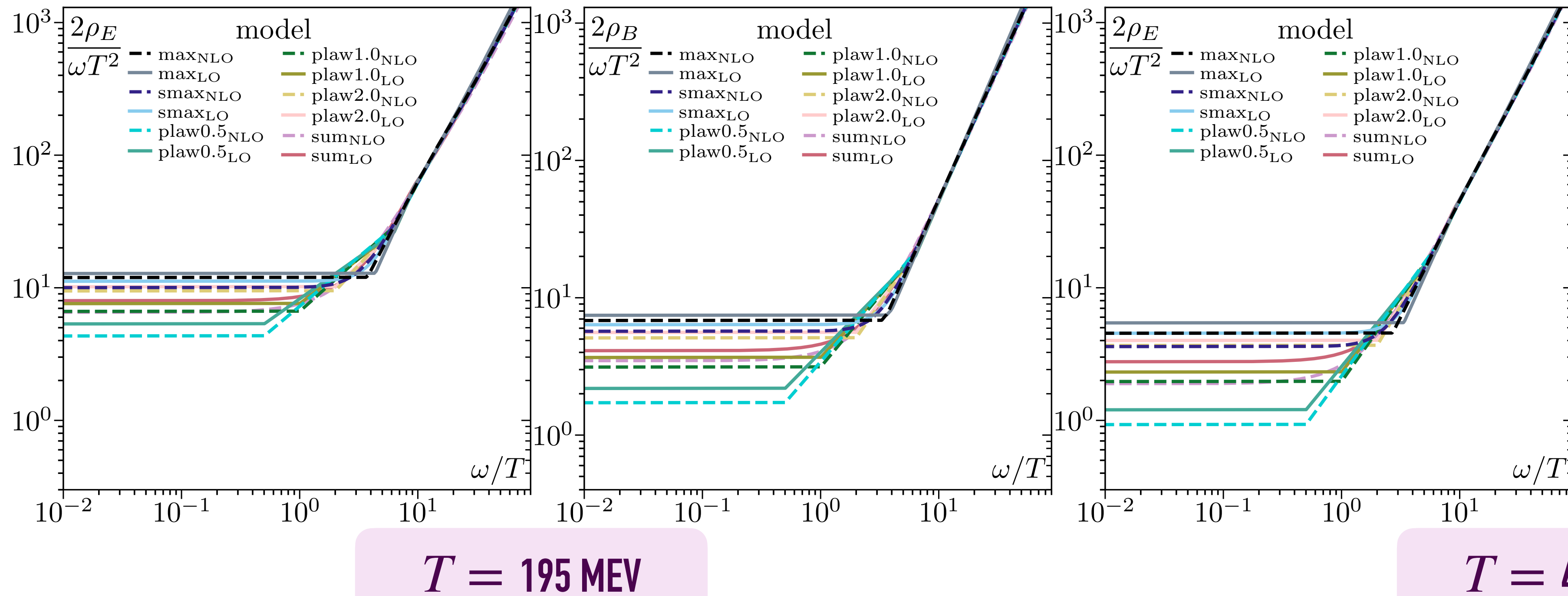
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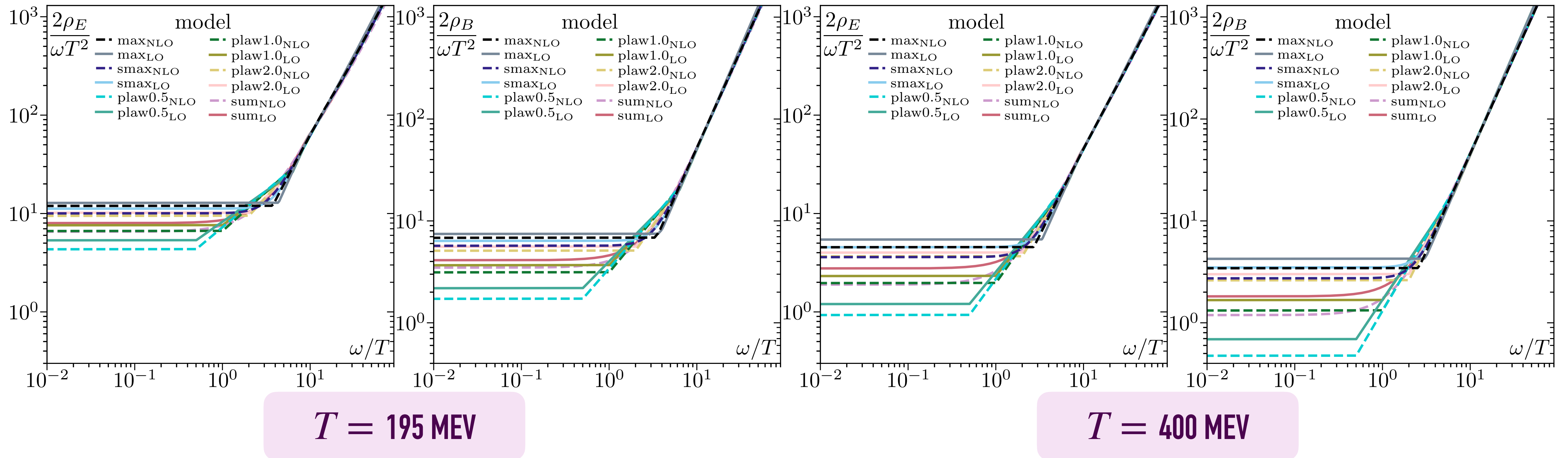
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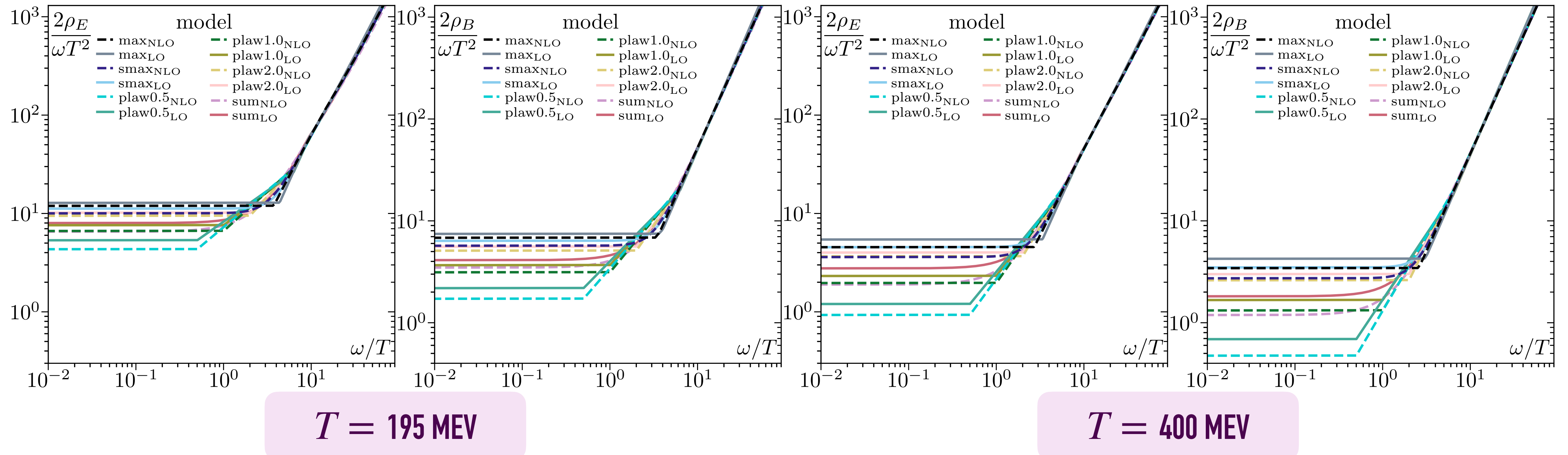
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The max model consistently yields the largest values of κ , while the power-law fit with $\omega_{IR}/T = 0.5$ systematically yields the smallest one

HEAVY QUARK DIFFUSIONS RESULTS

MOMENTUM DIFFUSION COEFFICIENT

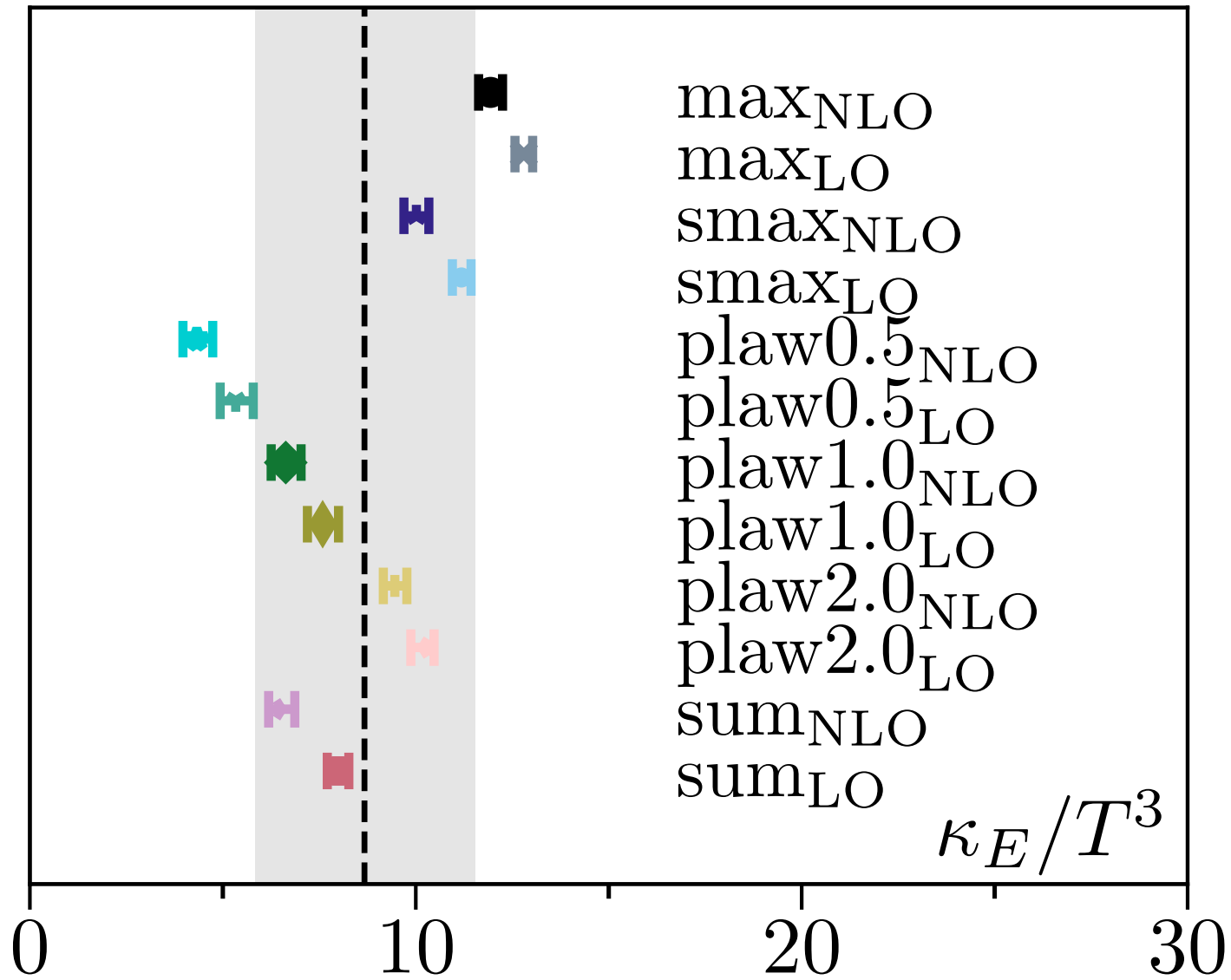
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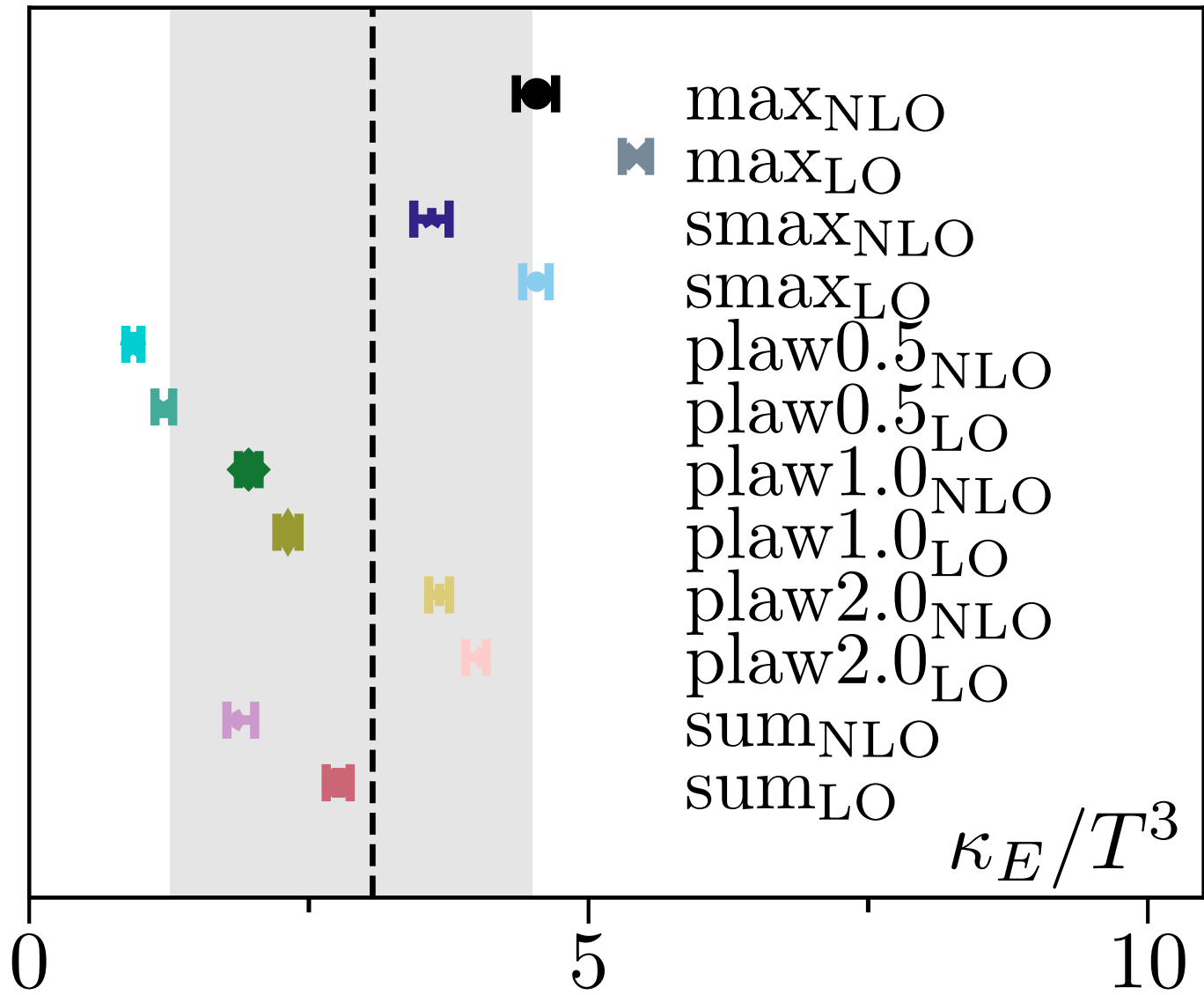
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ELECTRIC MOMENTUM COEFFICIENT

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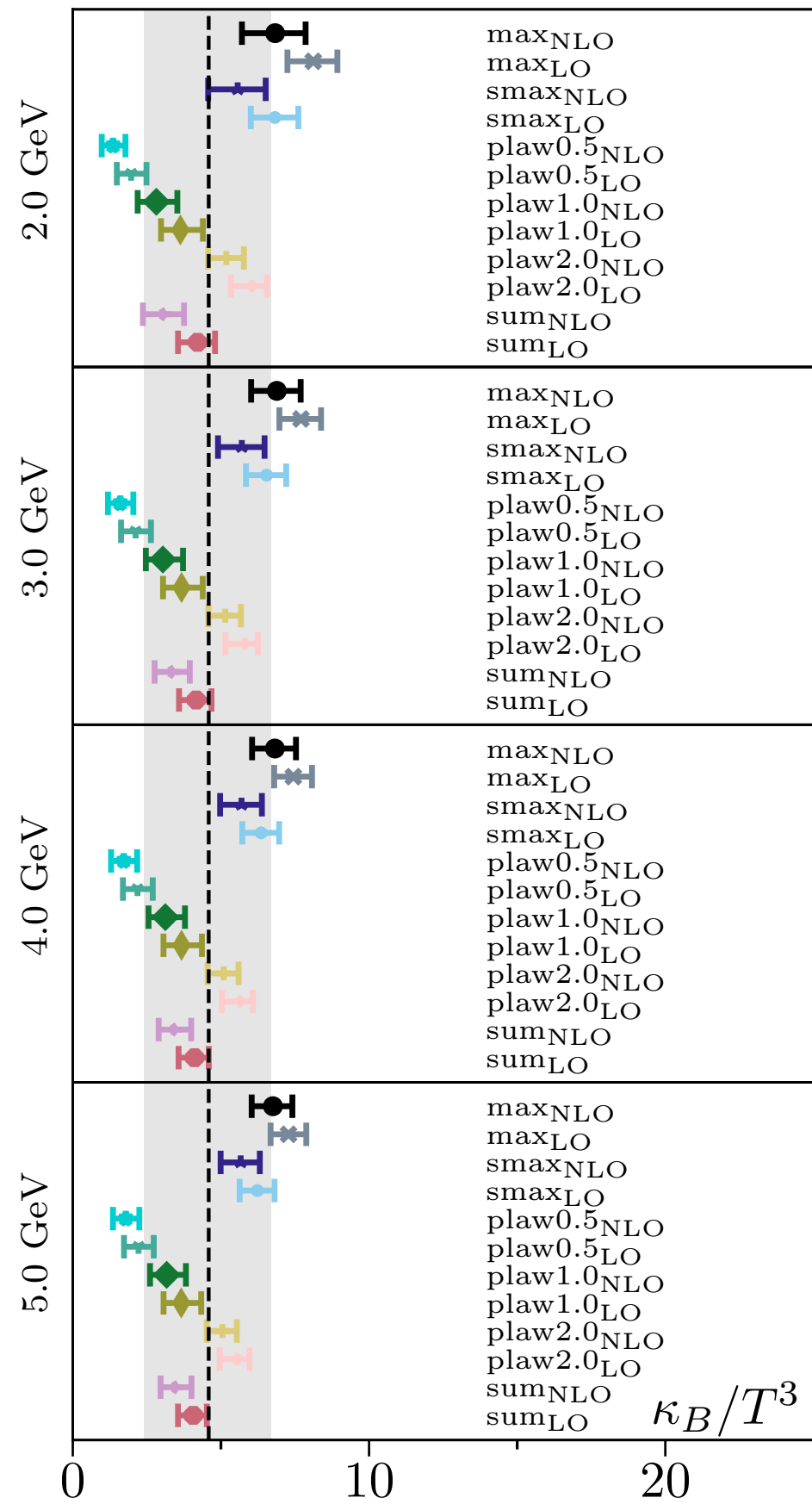


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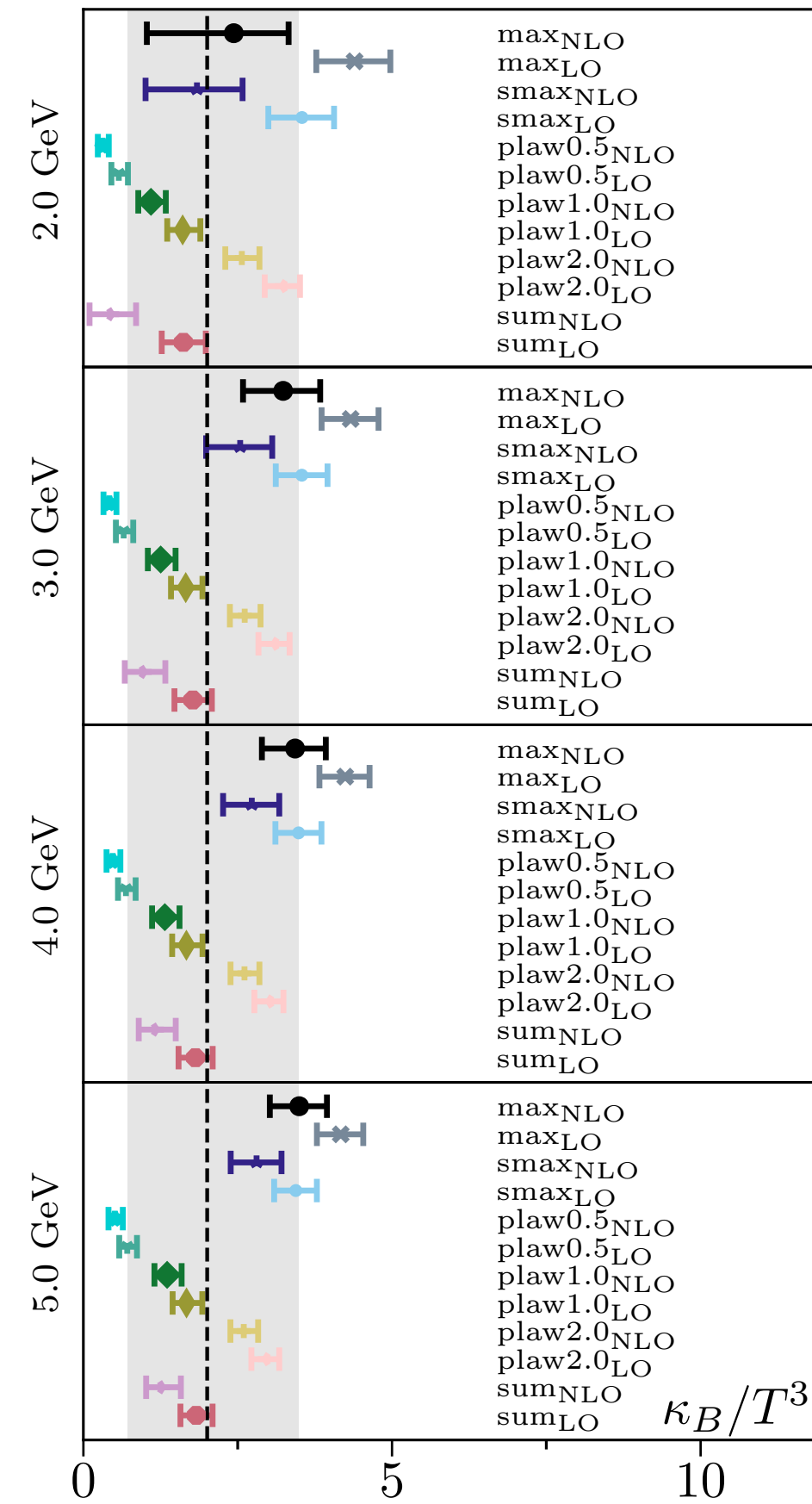
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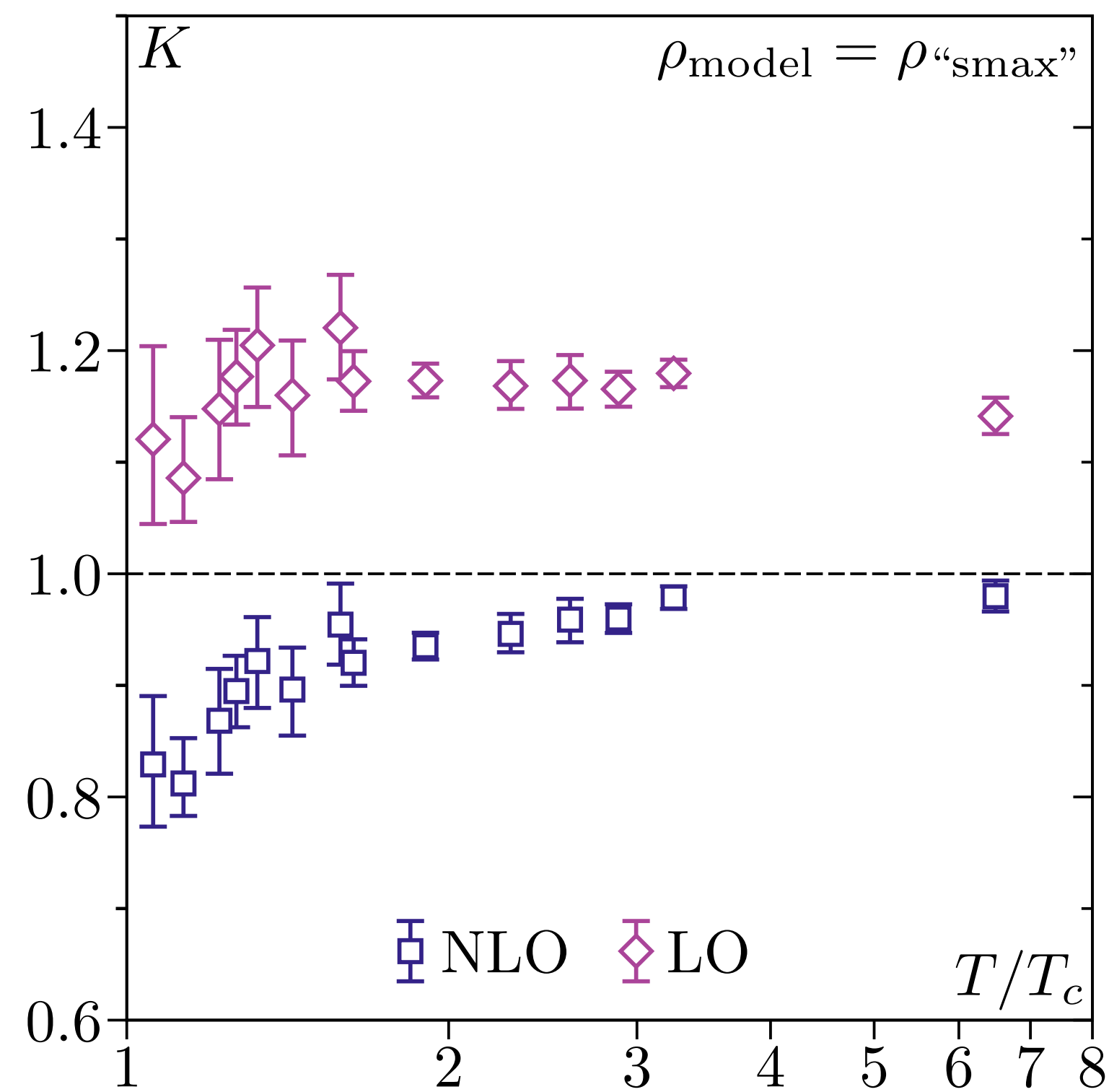
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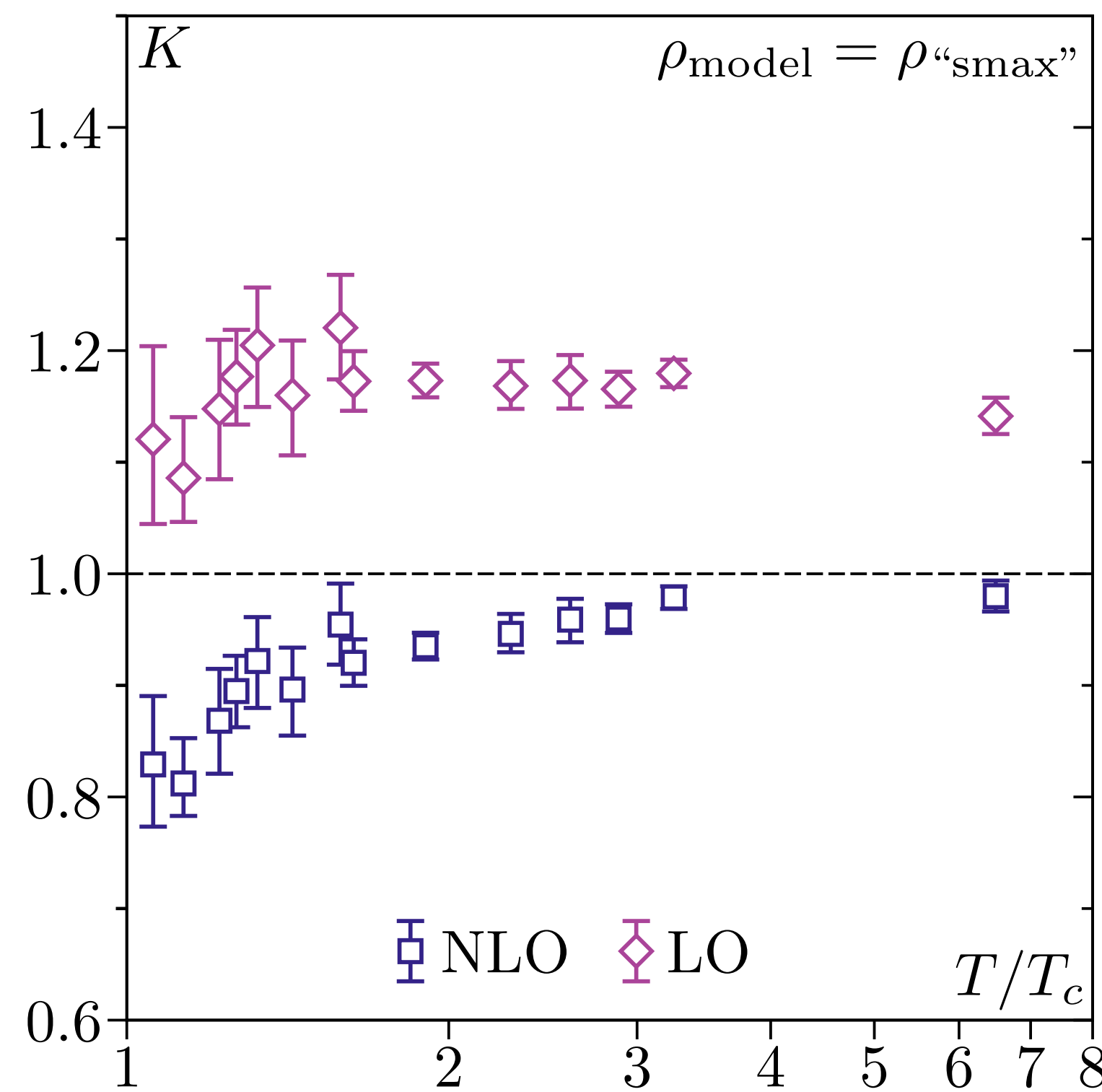
ELECTRIC K FACTOR



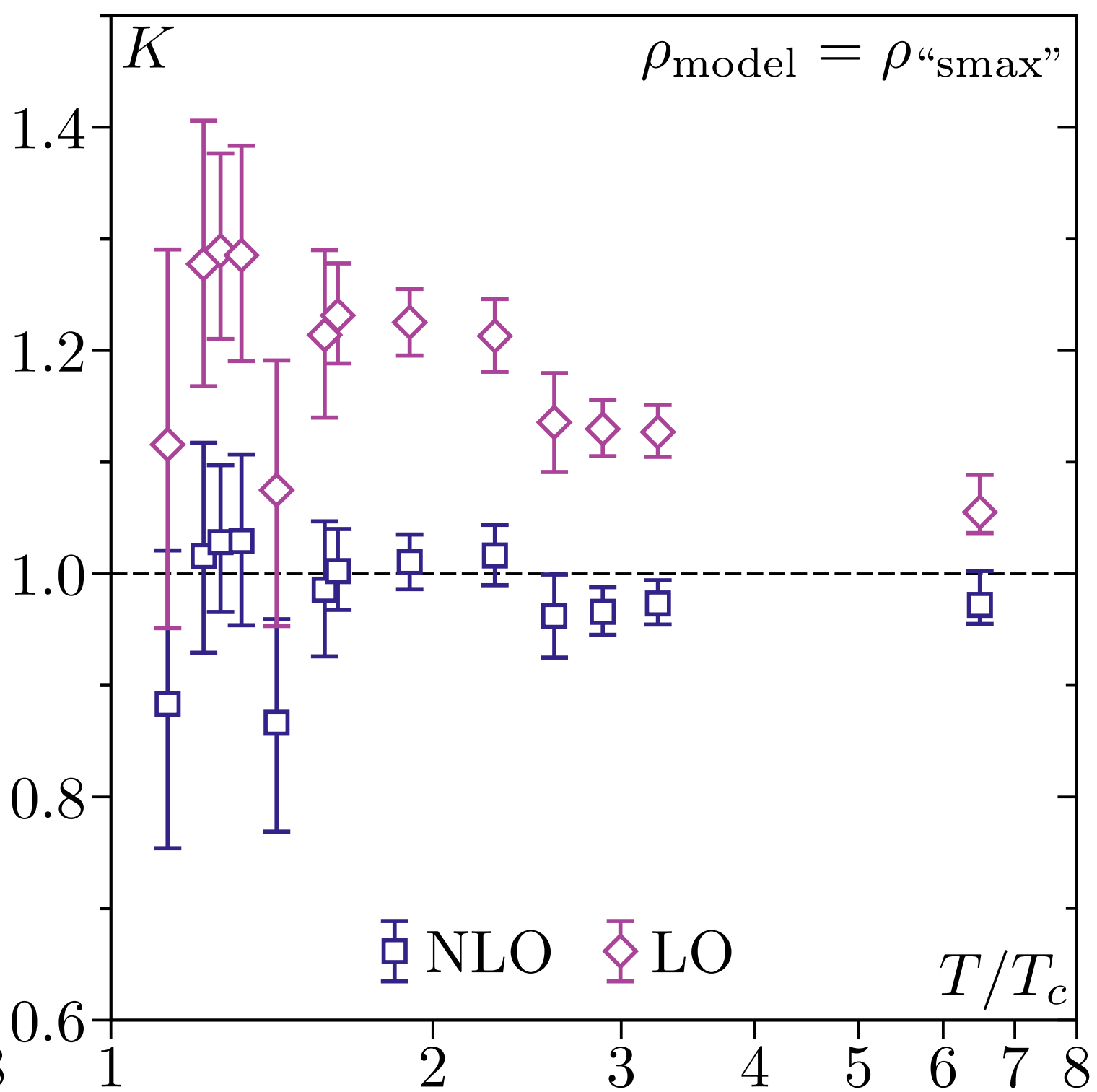
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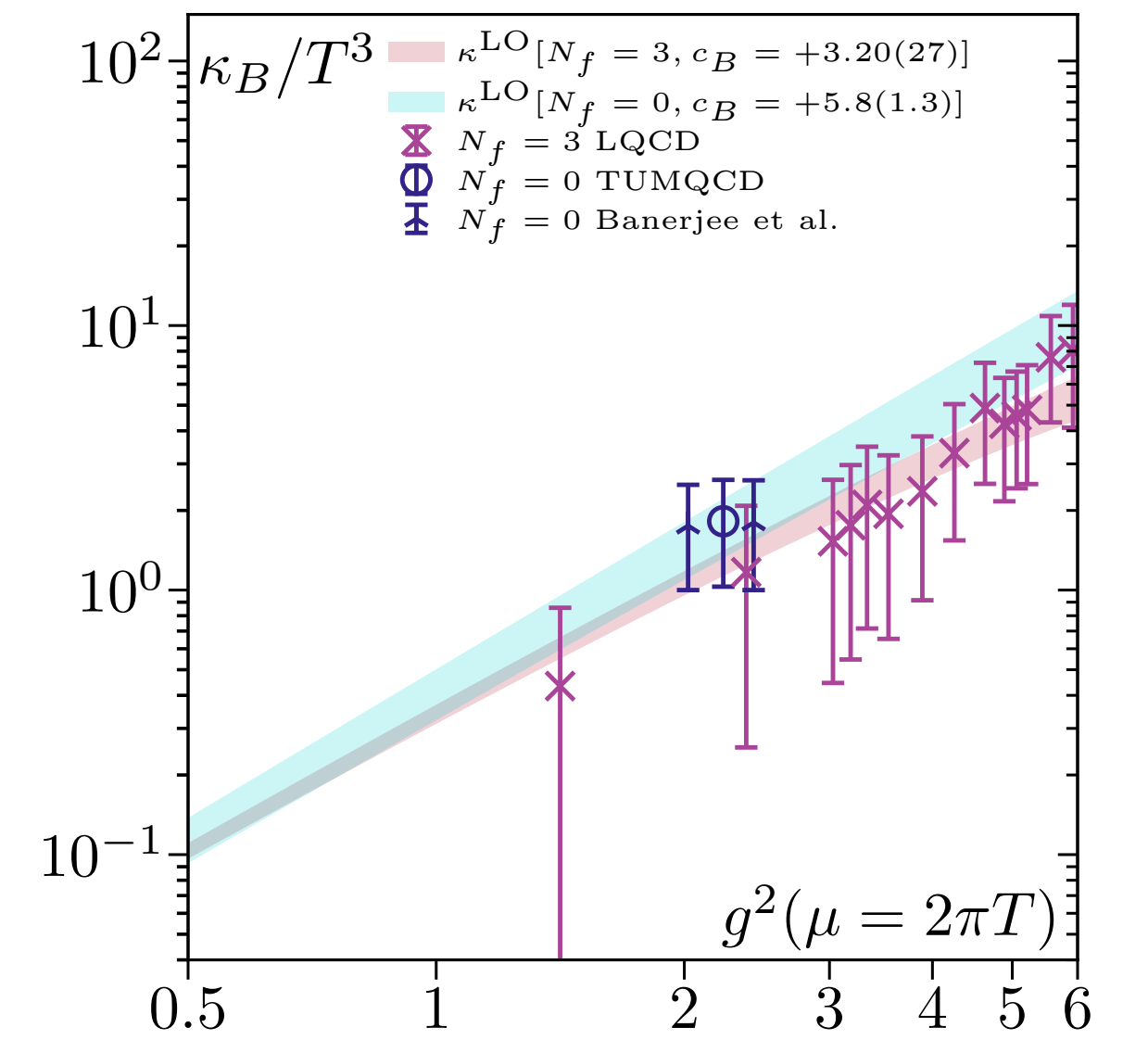
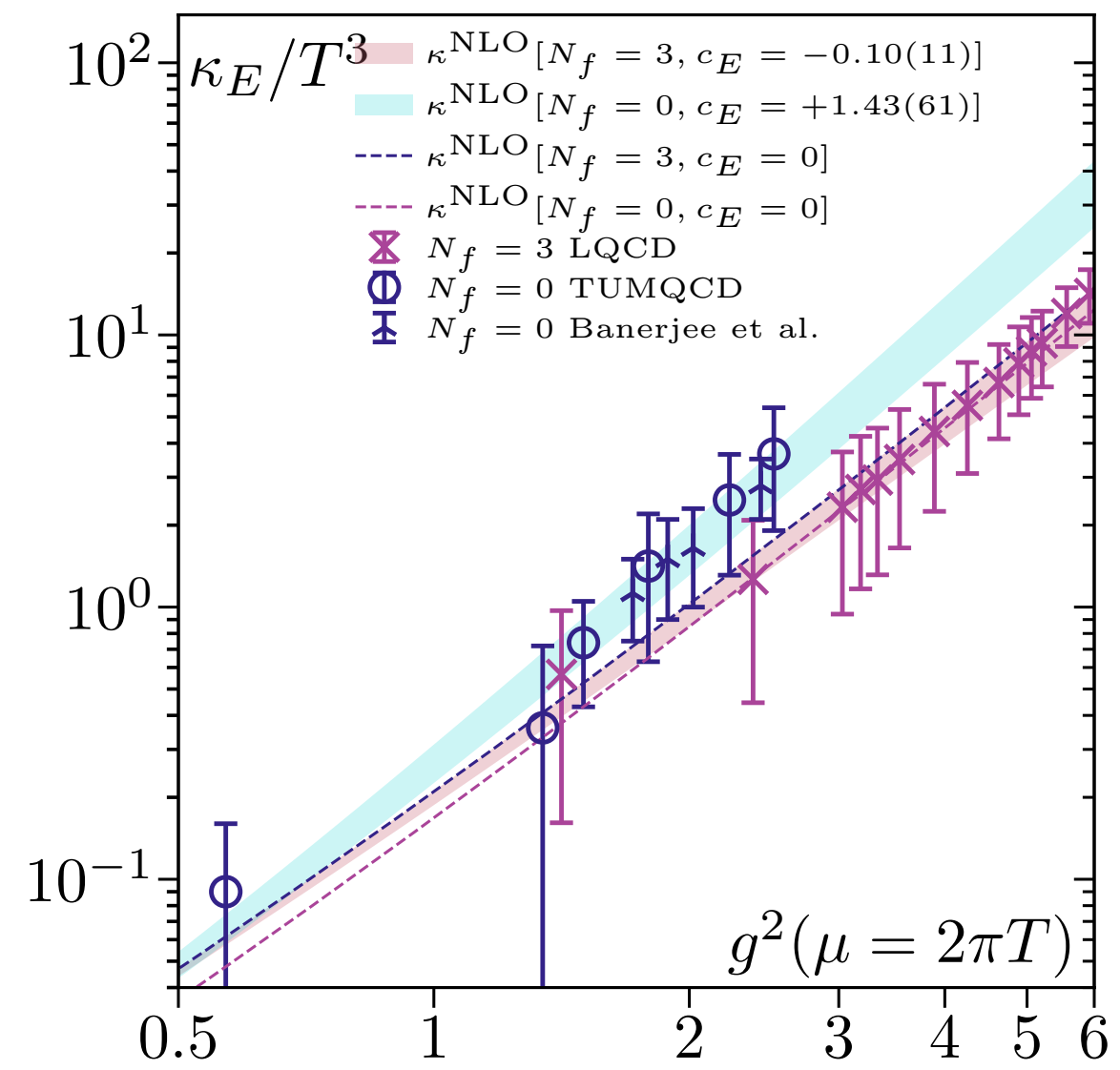
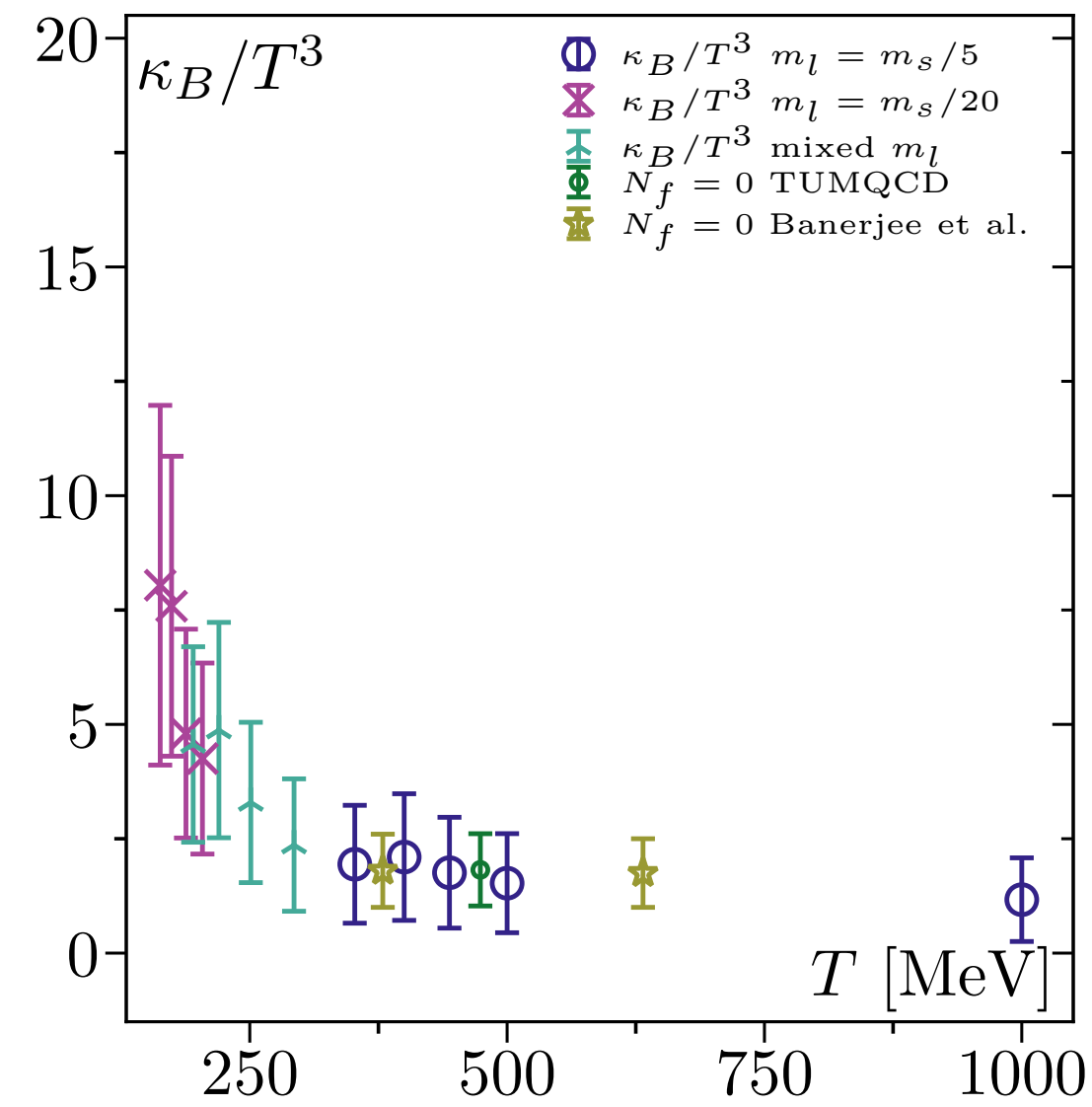
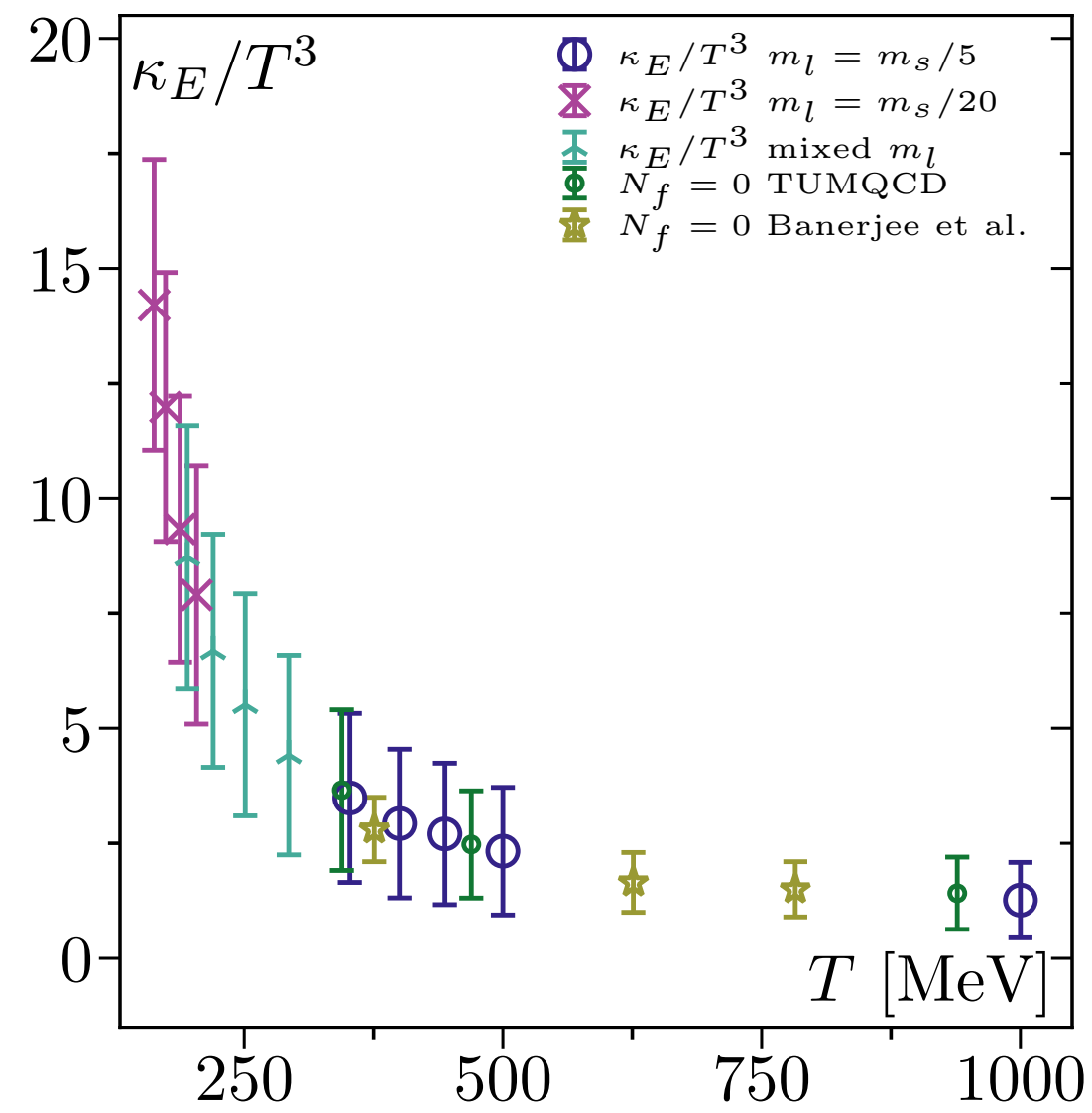


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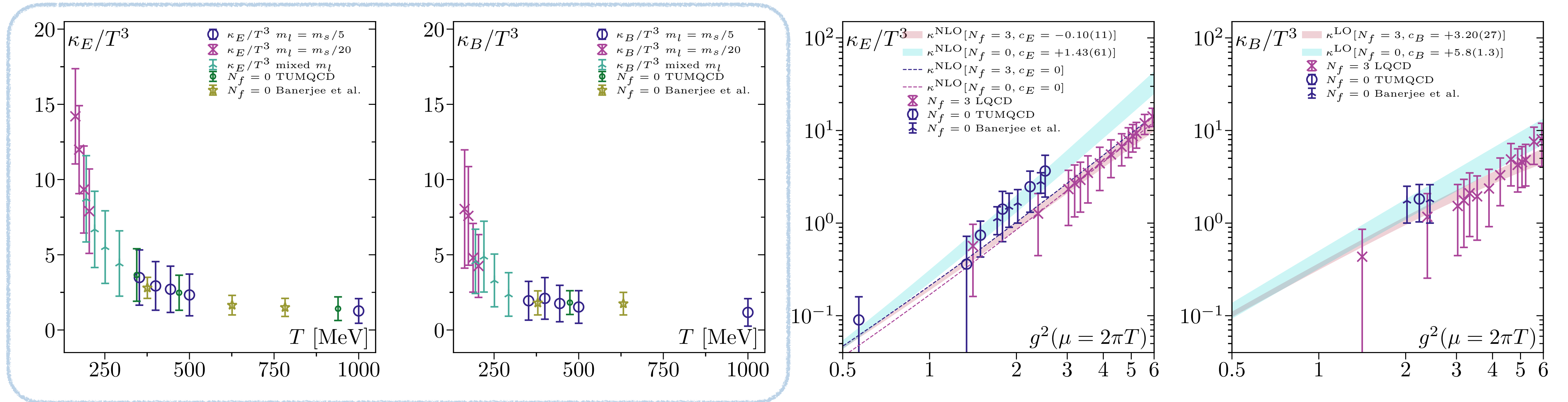
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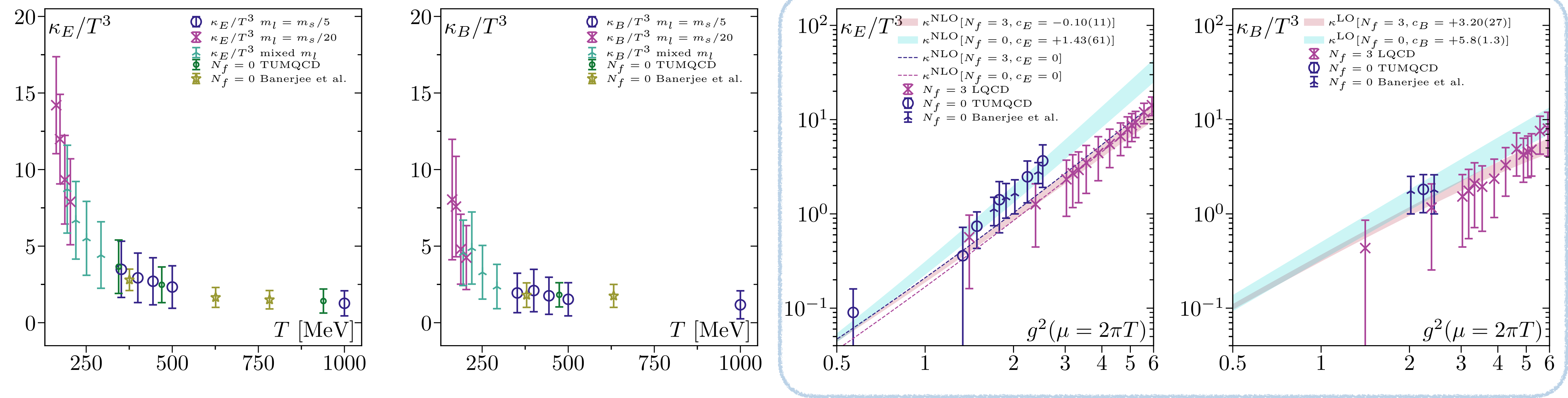


LARGE TEMPERATURE DEPENDENCE

COMPATIBLE RESULTS WITH QUENCHED THEORY

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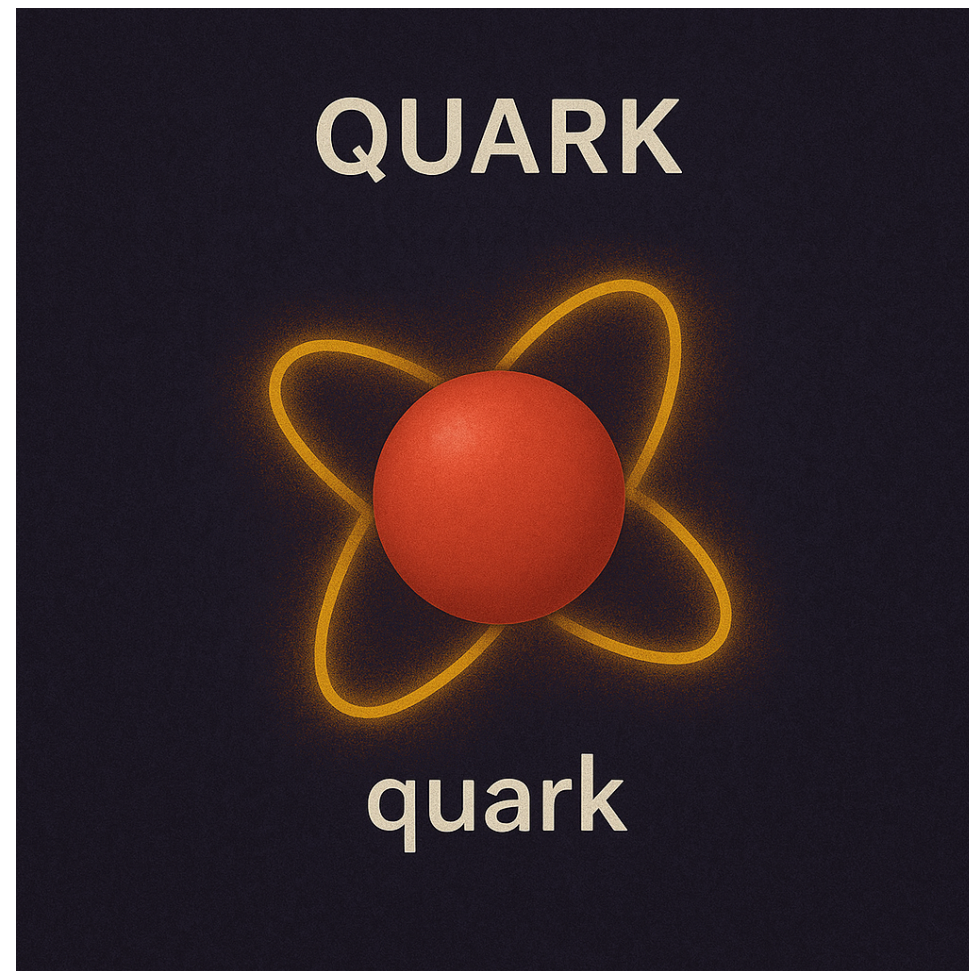


INCOMPATIBLE WITH g^2

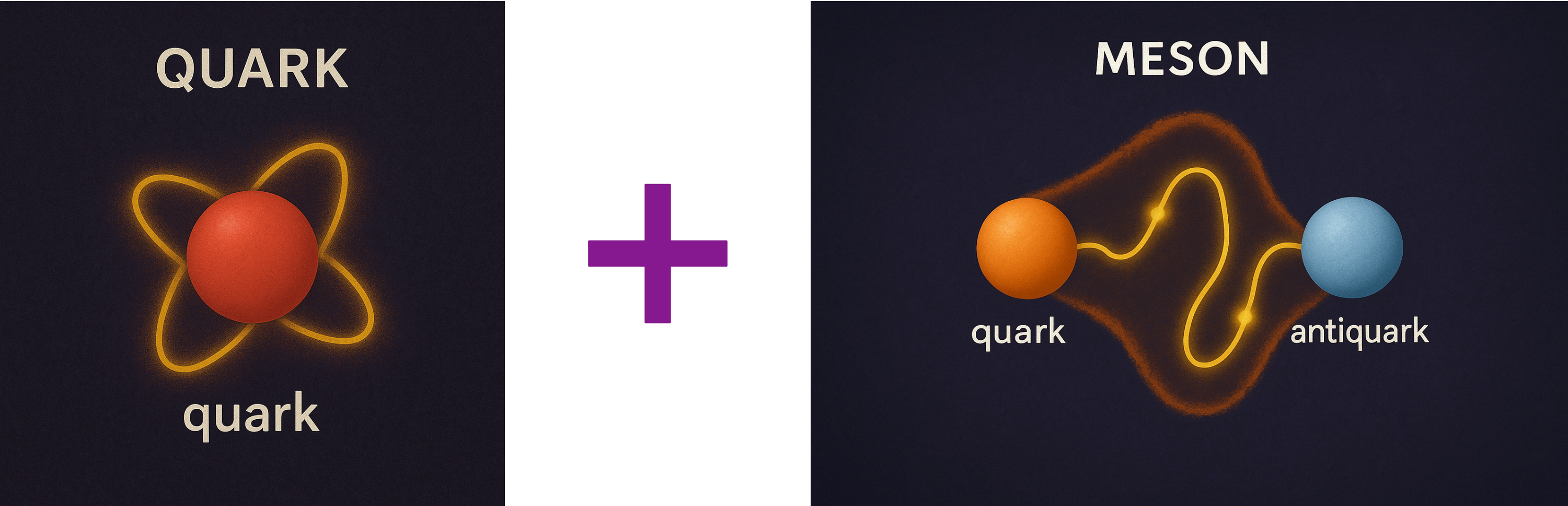
QCD IS MUCH CLOSER TO PERTURBATION THEORY

QUASI-PARTICLE MODEL

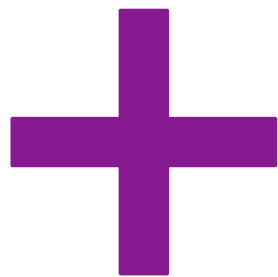
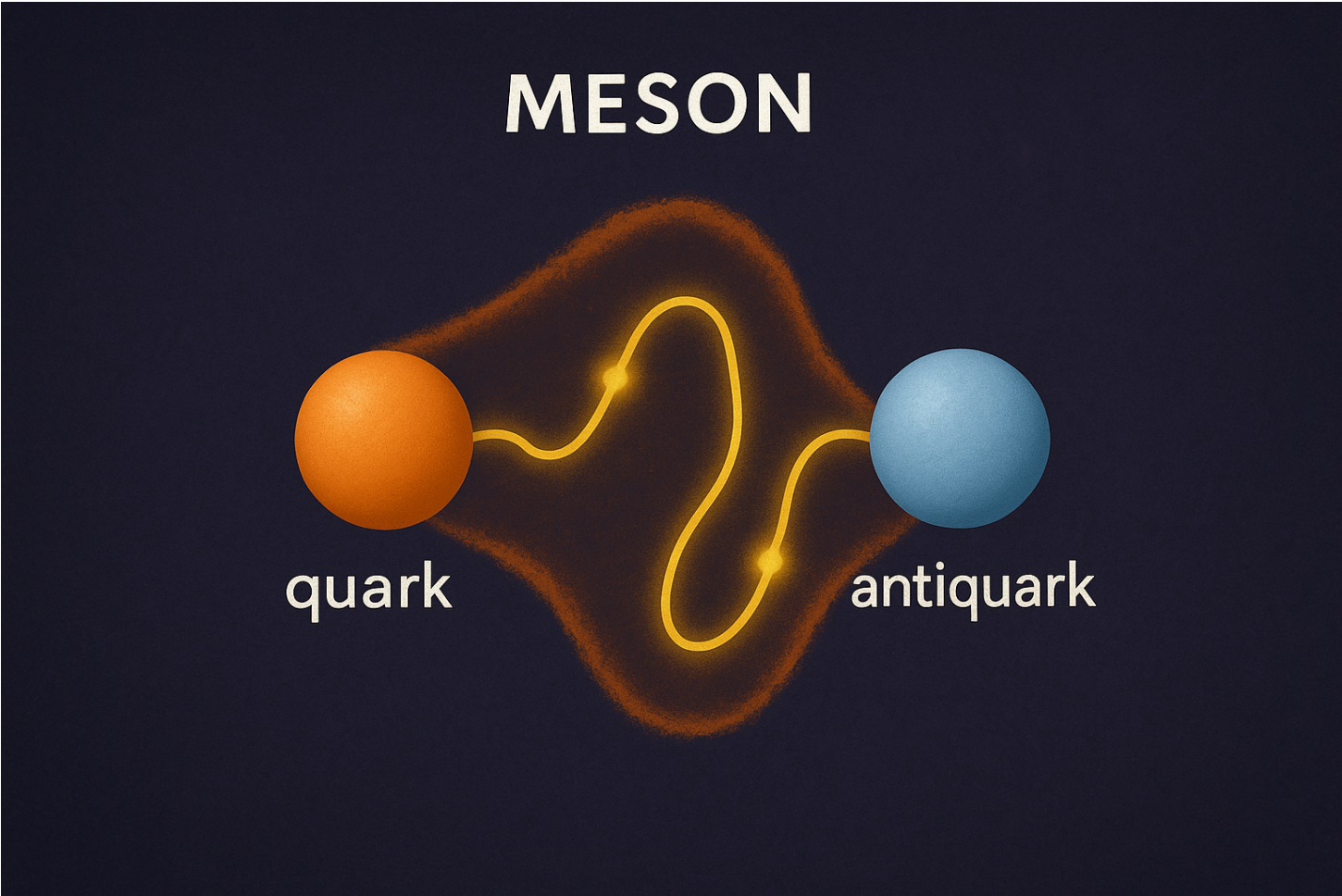
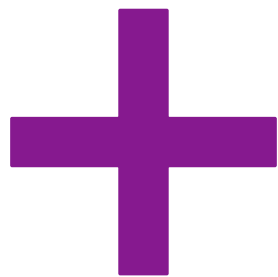
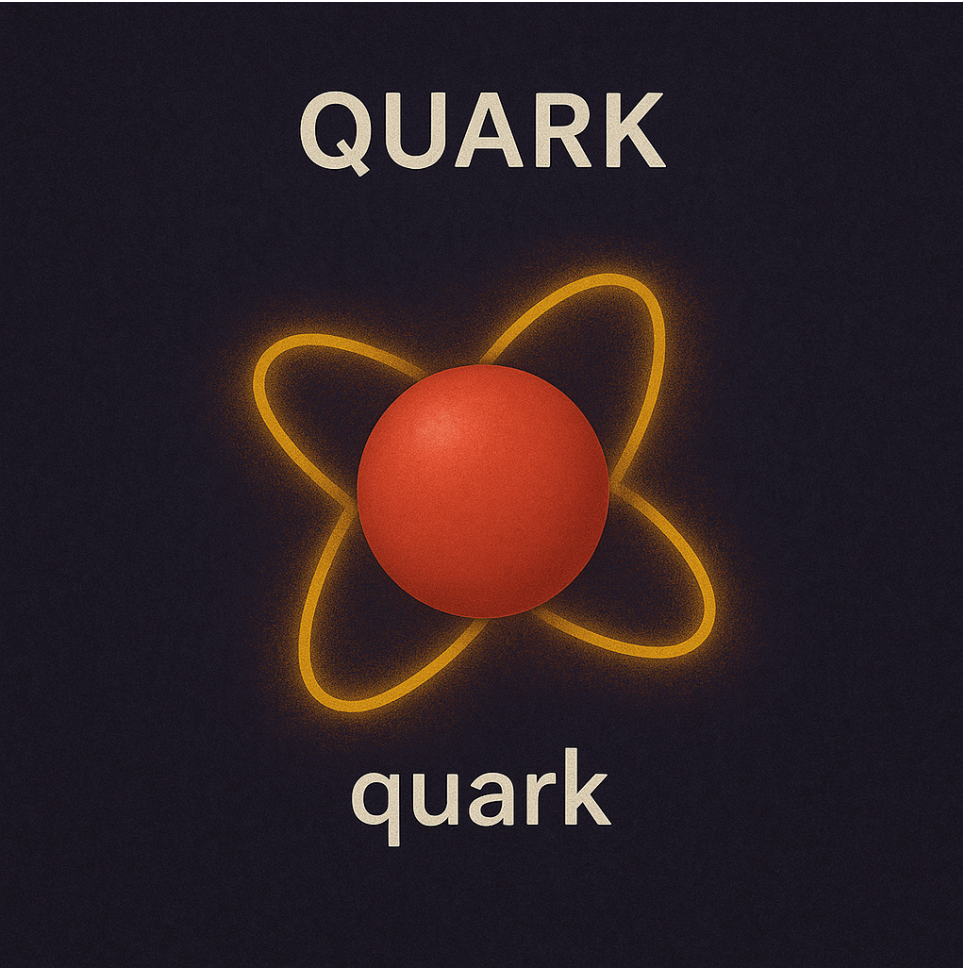
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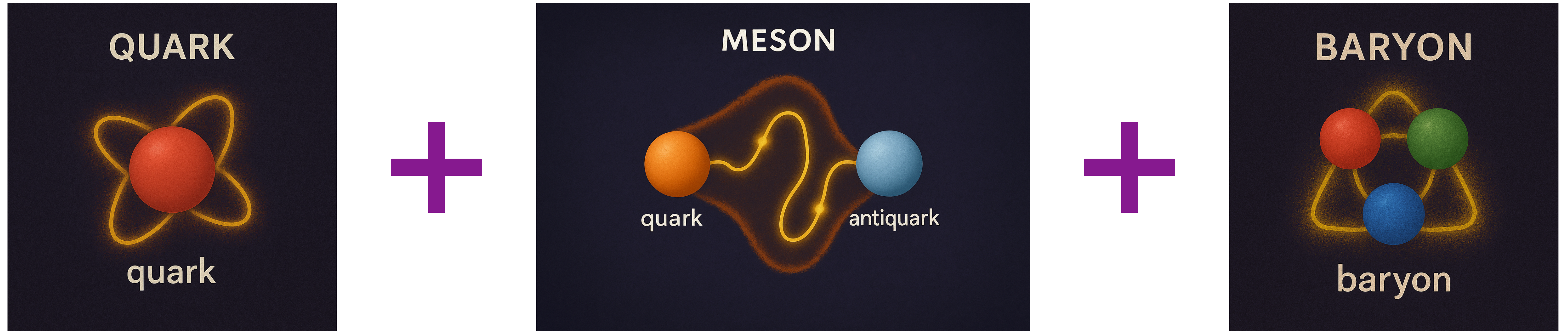
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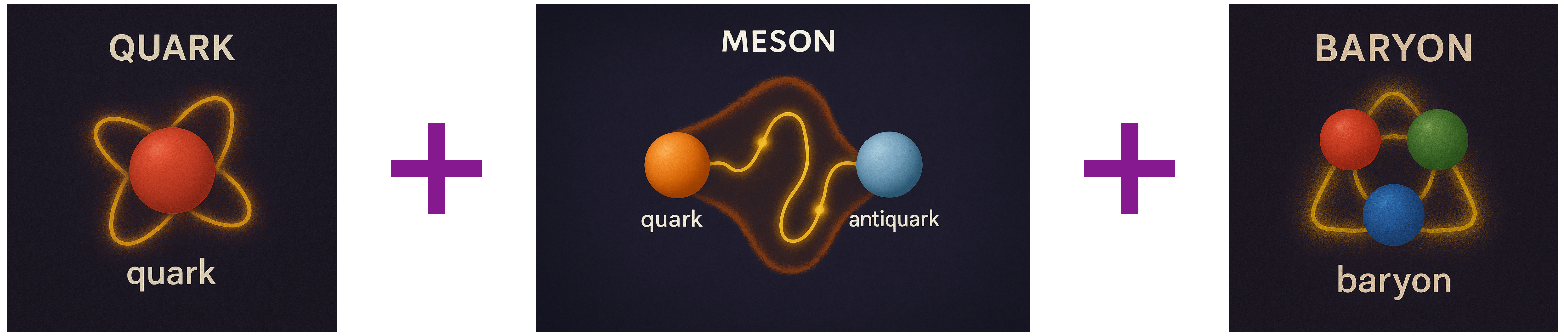
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In the QPM, the quark susceptibility is written as:

$$\chi_q(T) = \frac{1}{\pi} \sum_i g_i \int \frac{d^3p}{(2\pi)^3} e^{-E_p^i/T} \text{ with } E_p^i = \sqrt{p^2 + M_i^2}$$

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The mean thermal squared-velocity and square momentum are:

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We use the charm susceptibilities from the lattice.

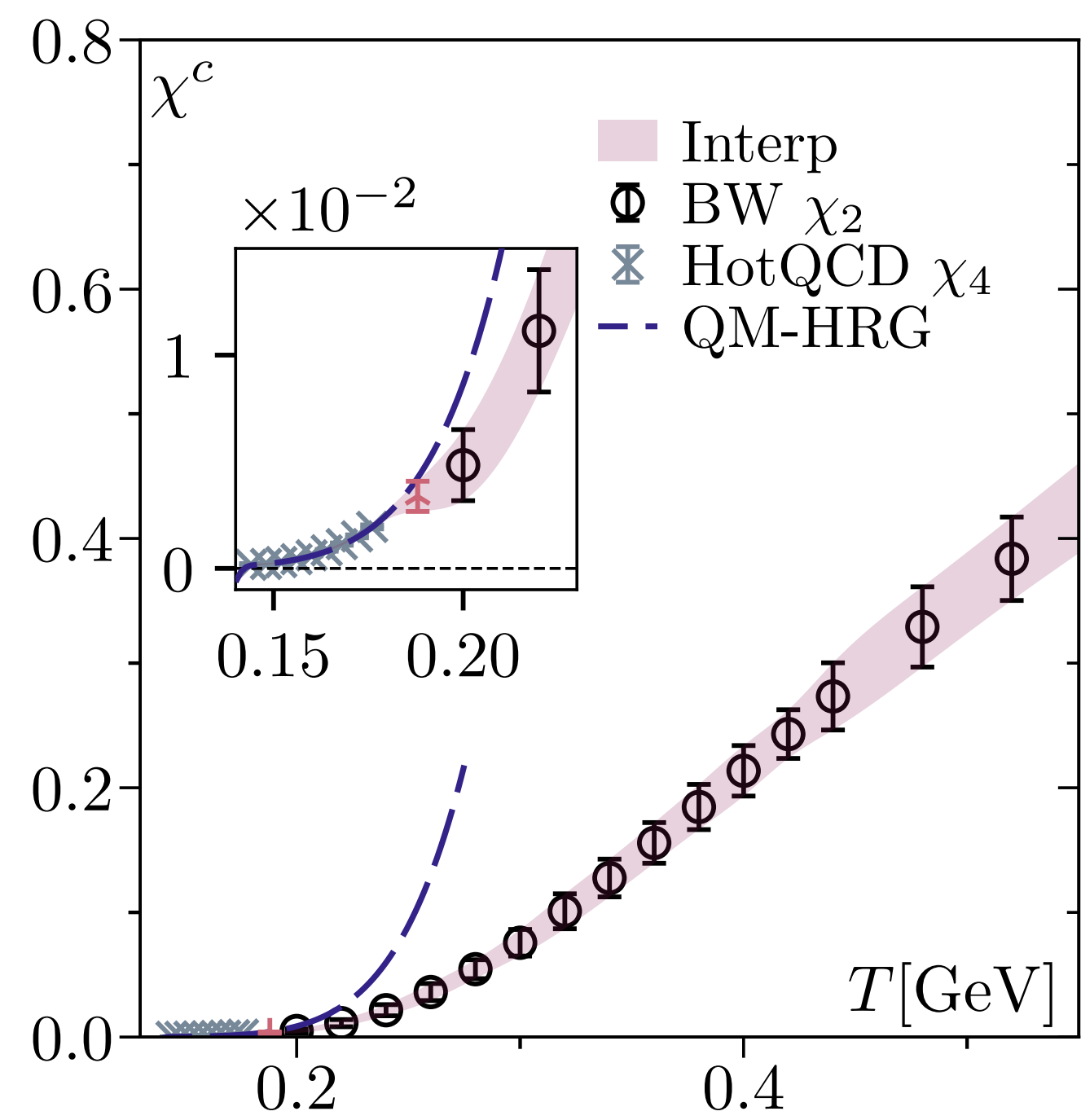
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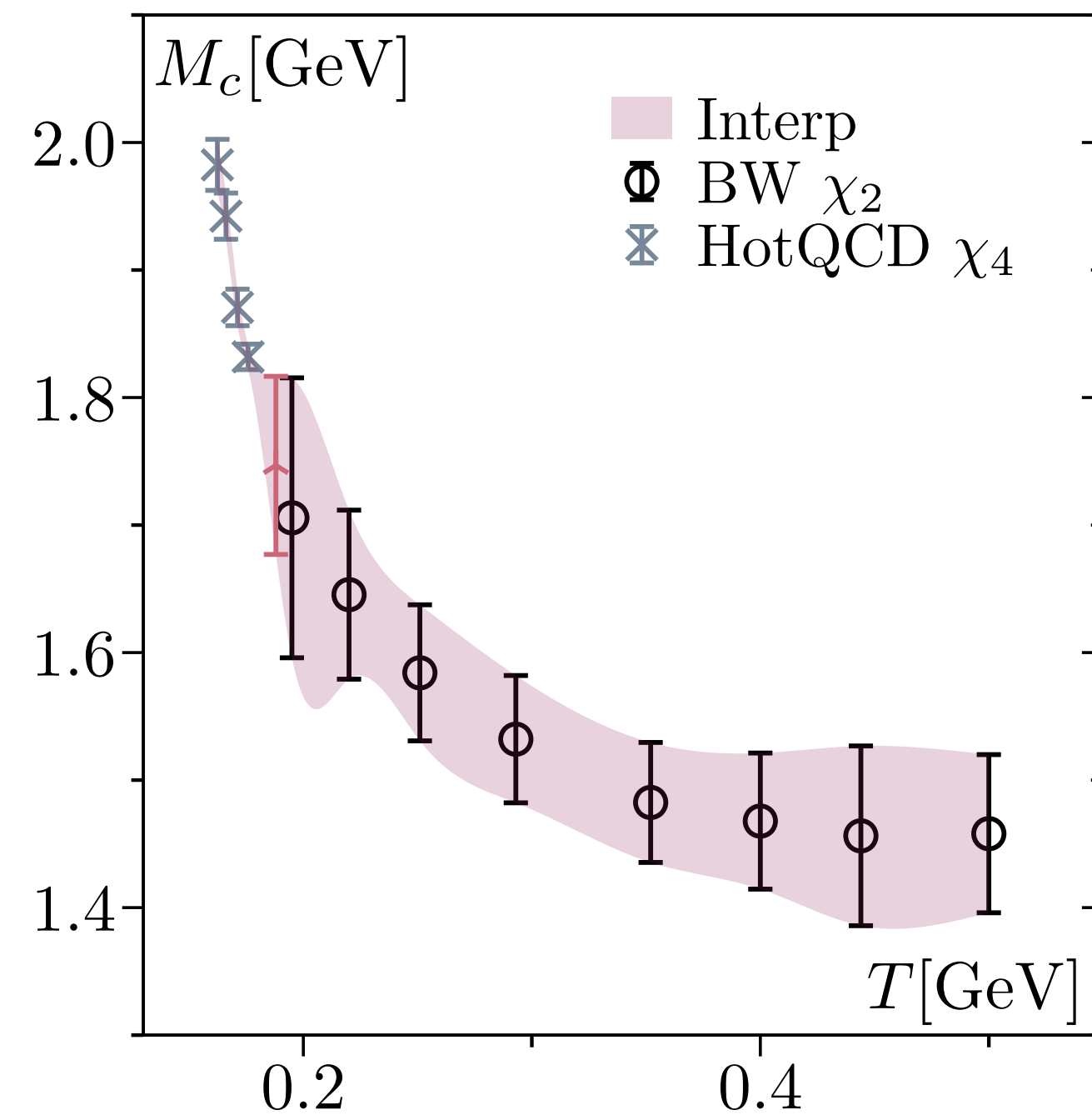


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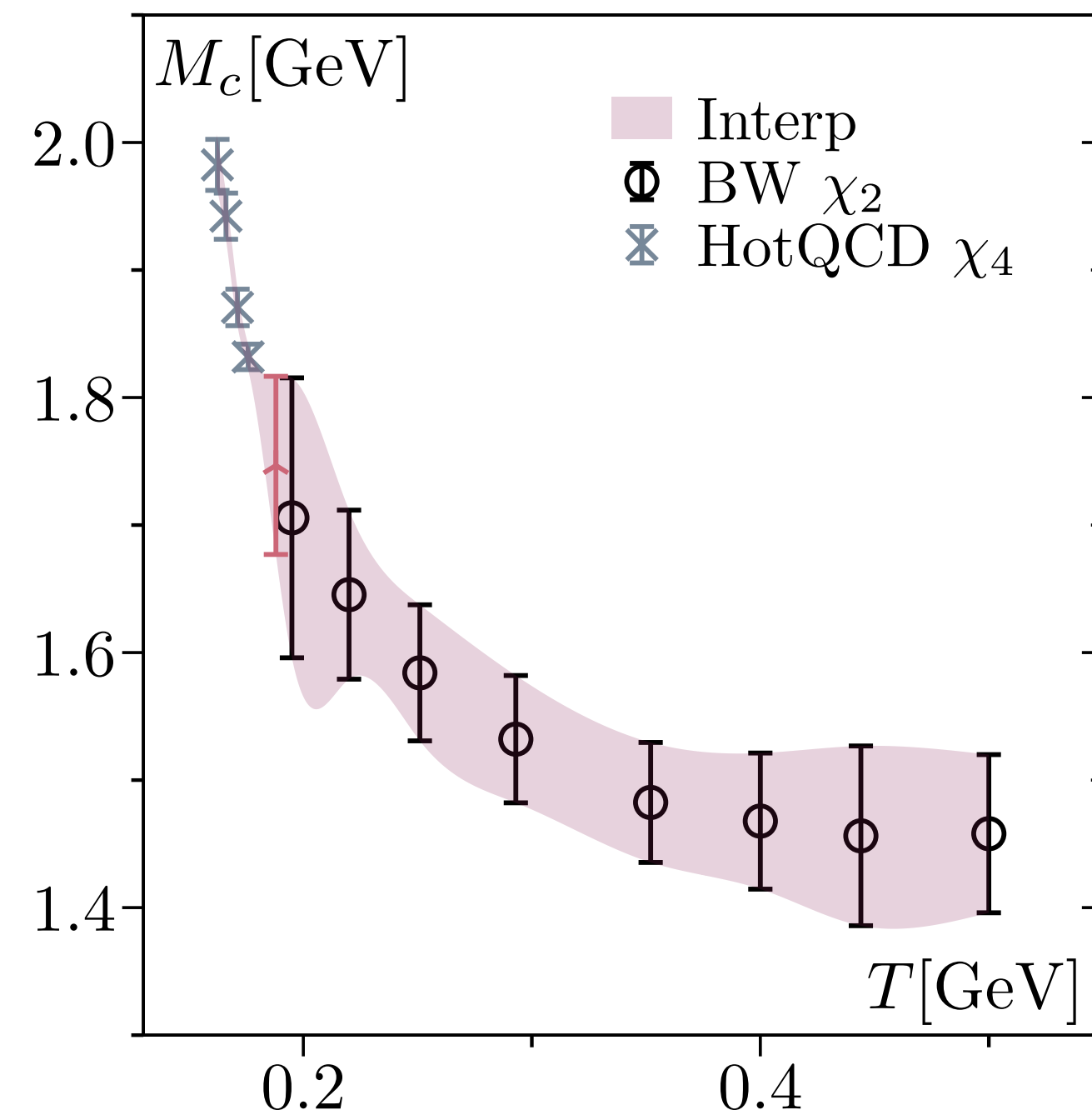
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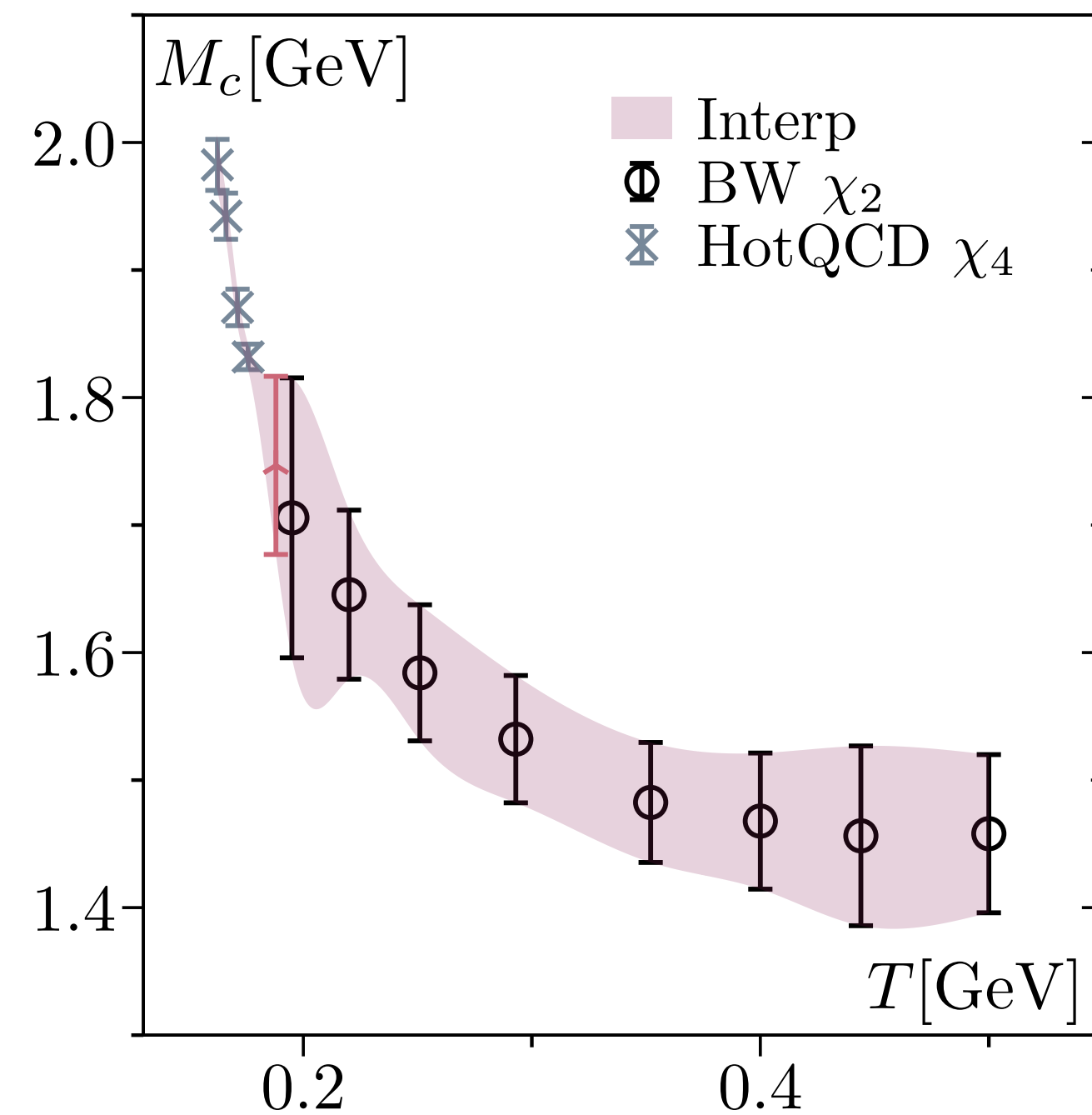
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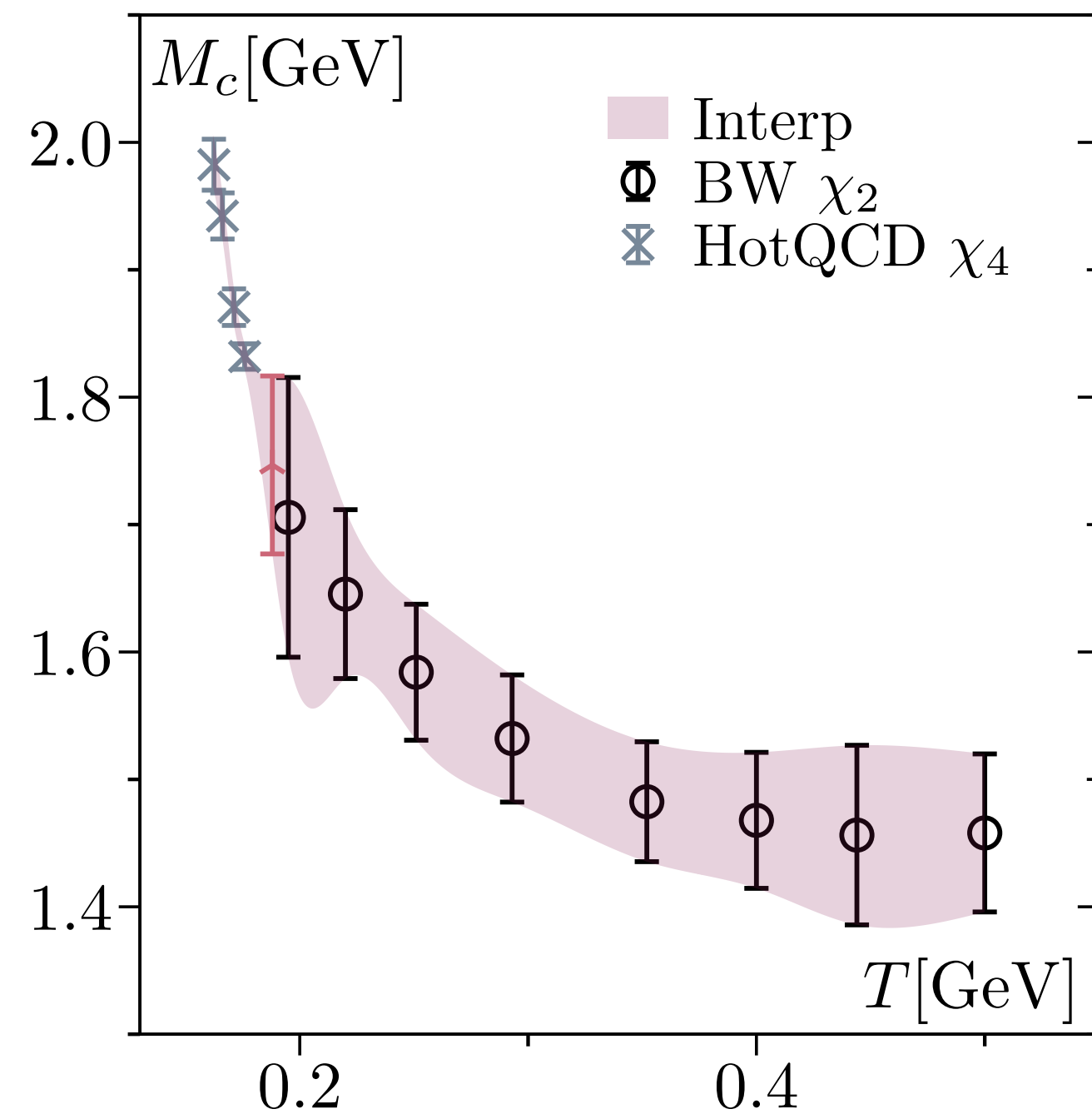
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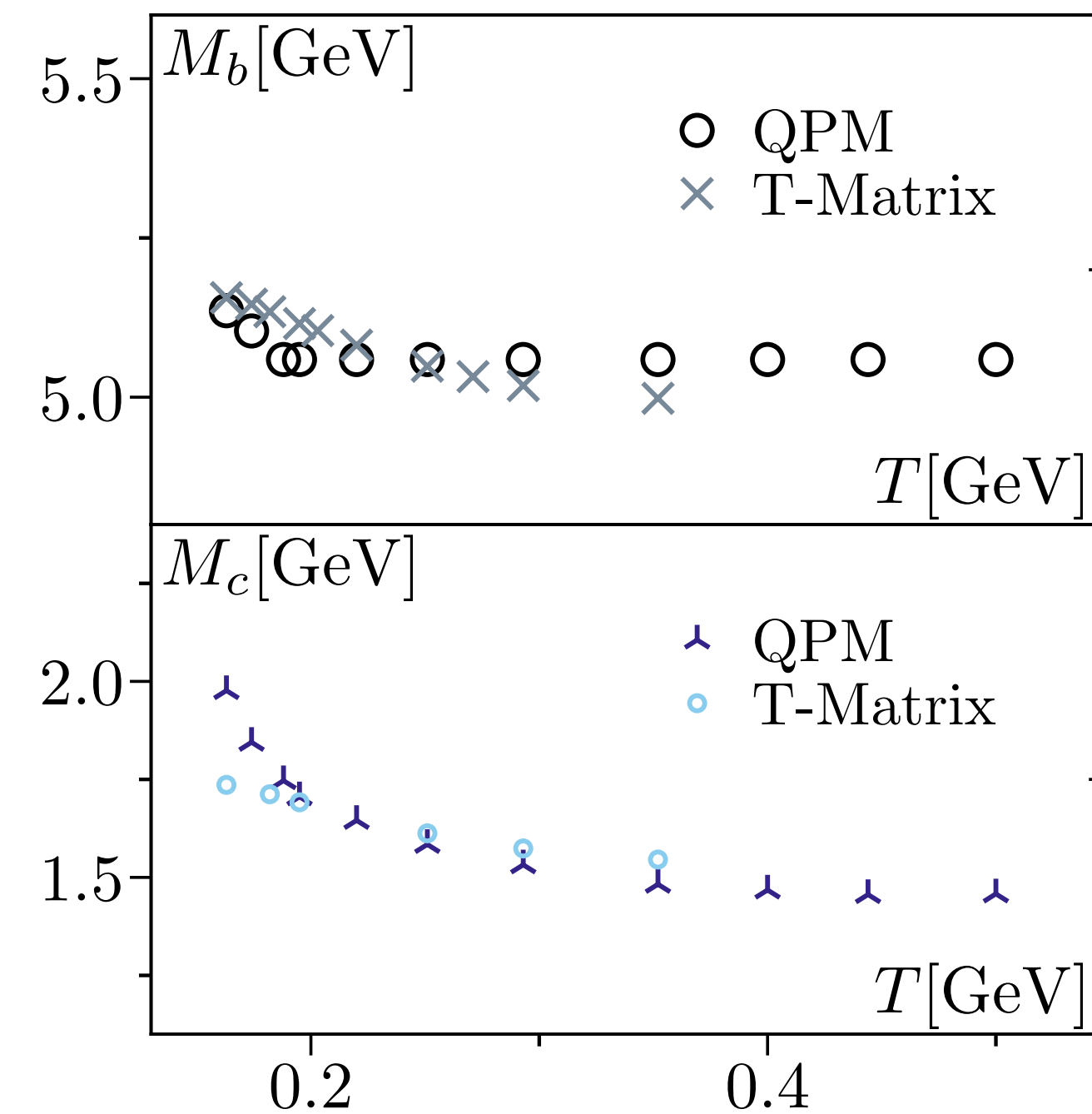


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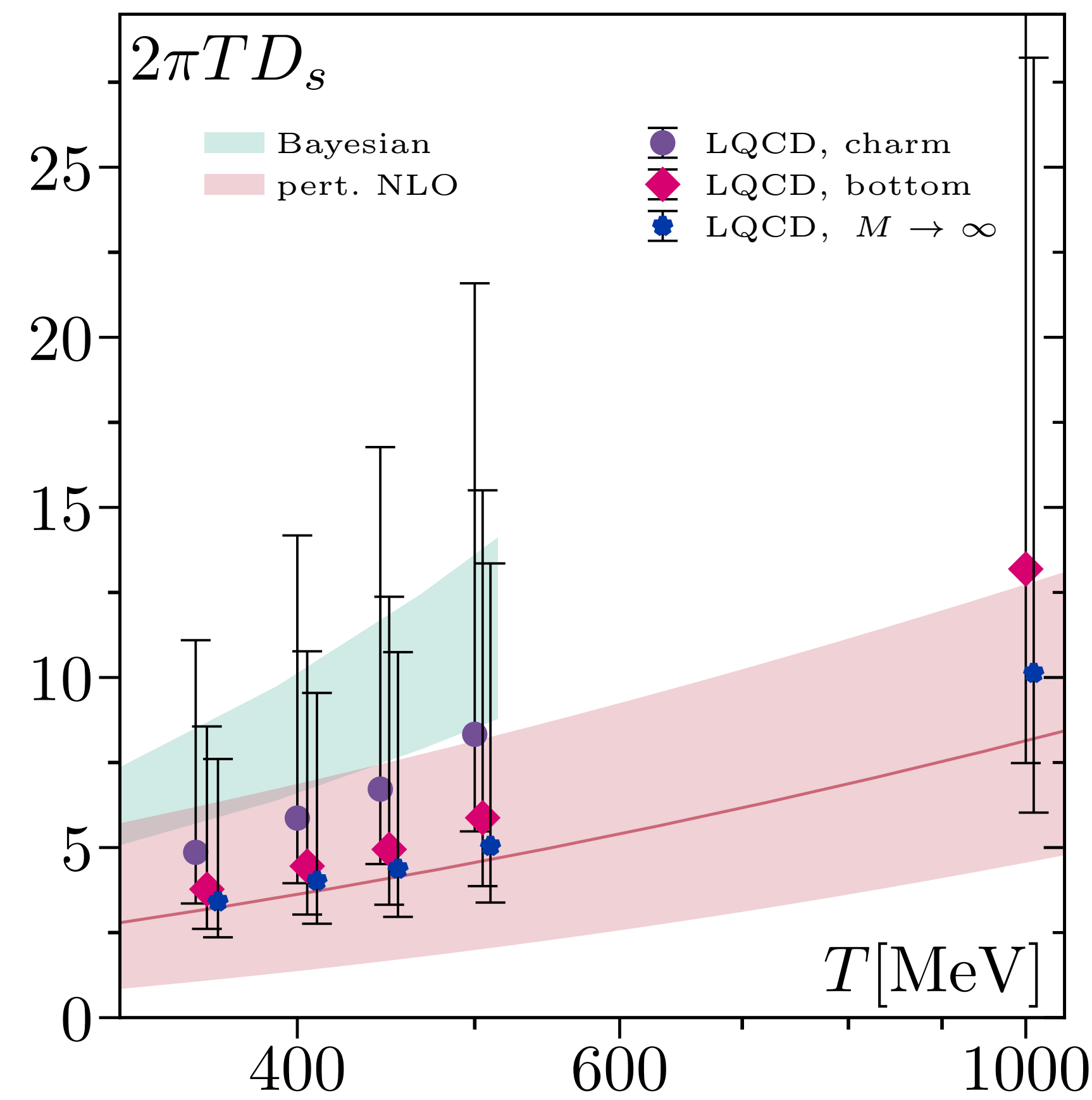
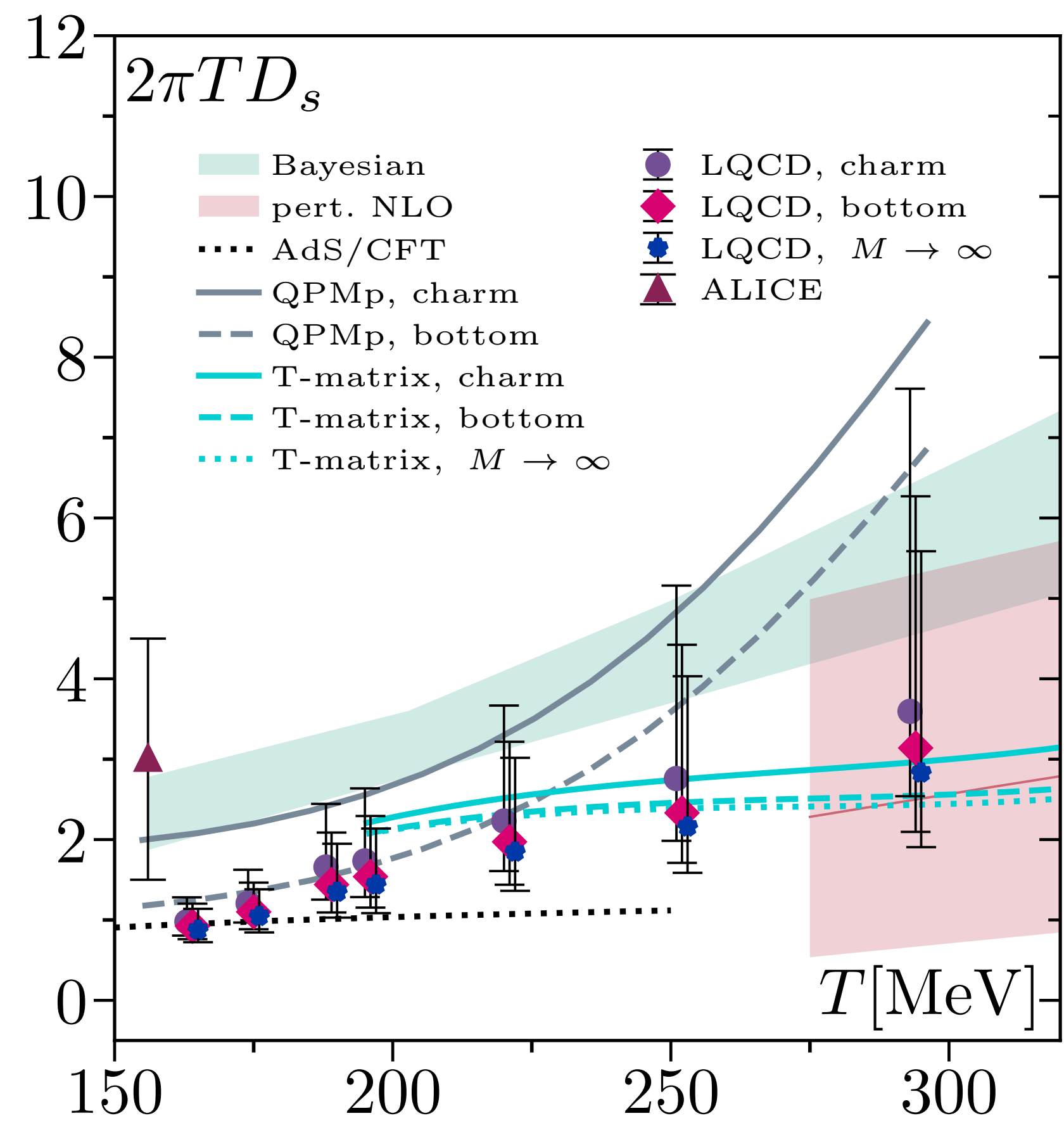
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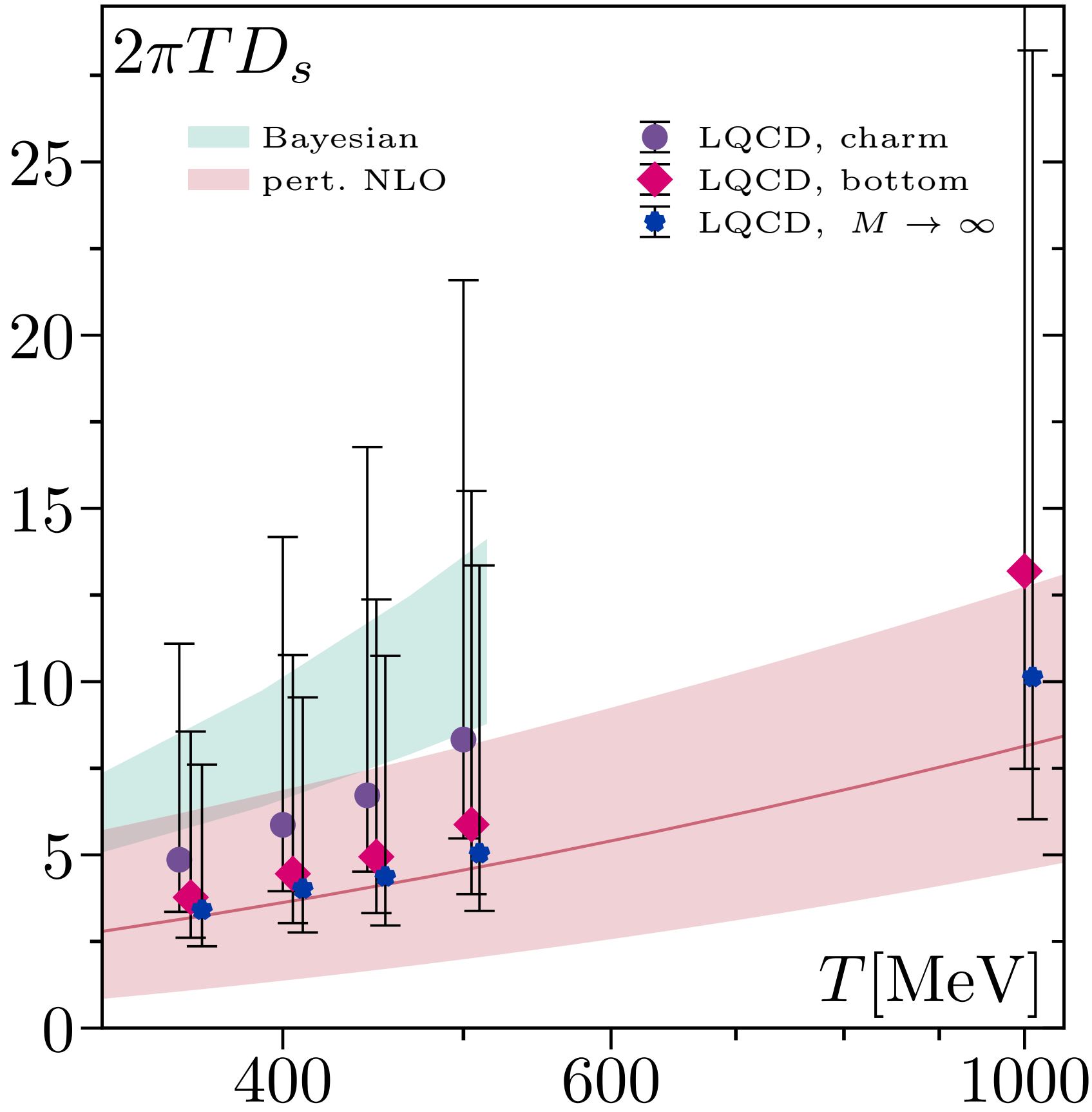
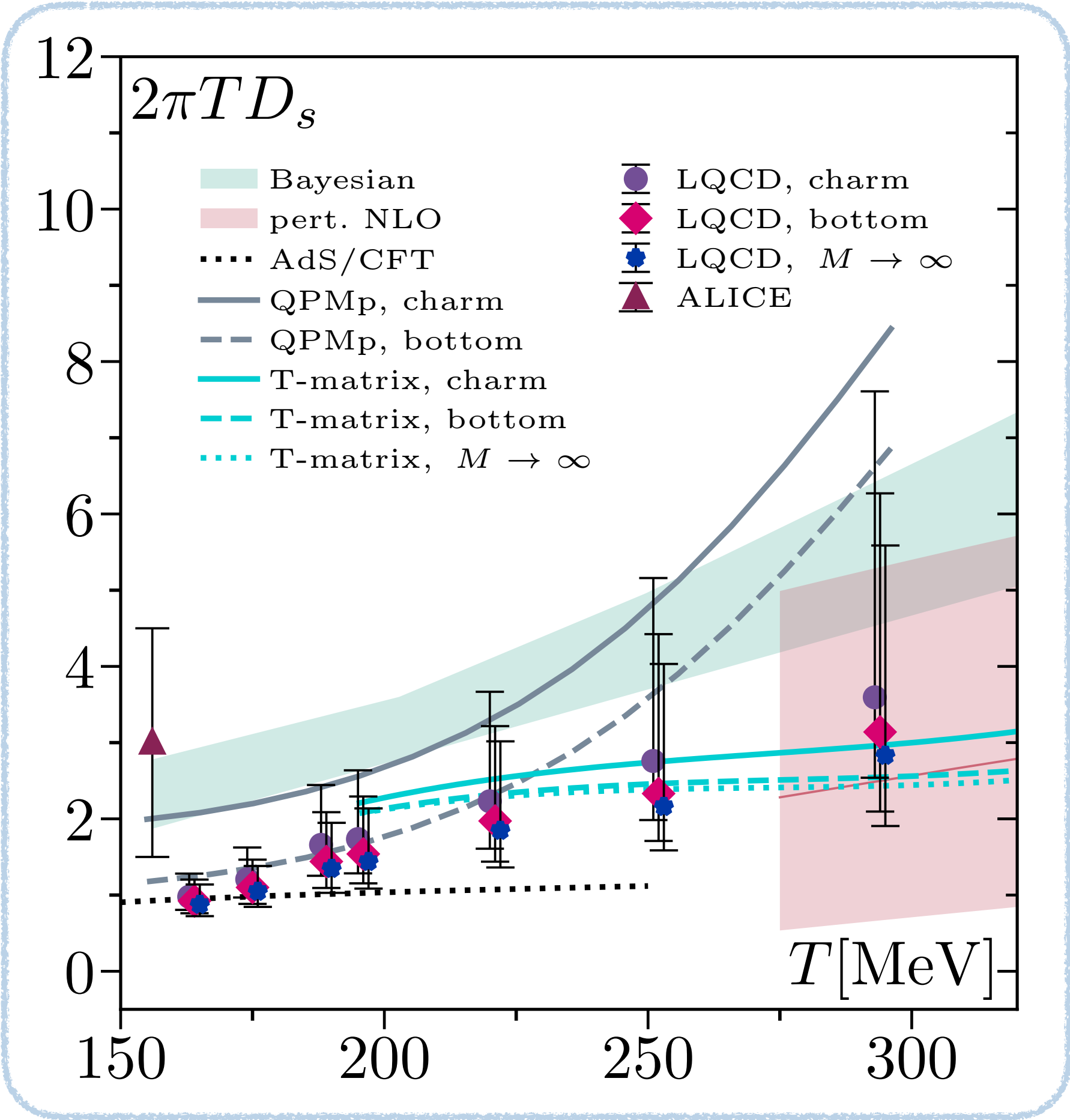
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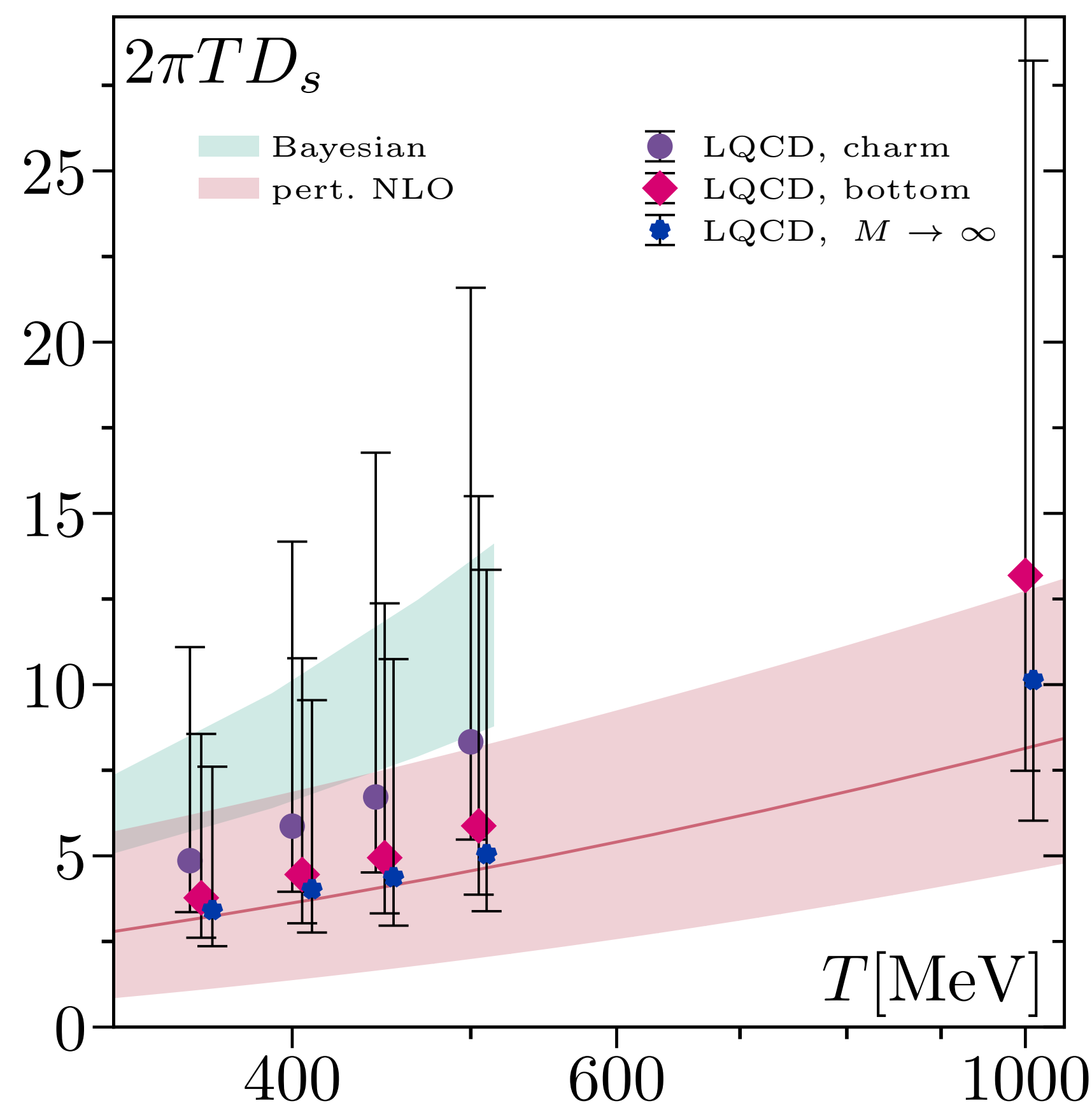
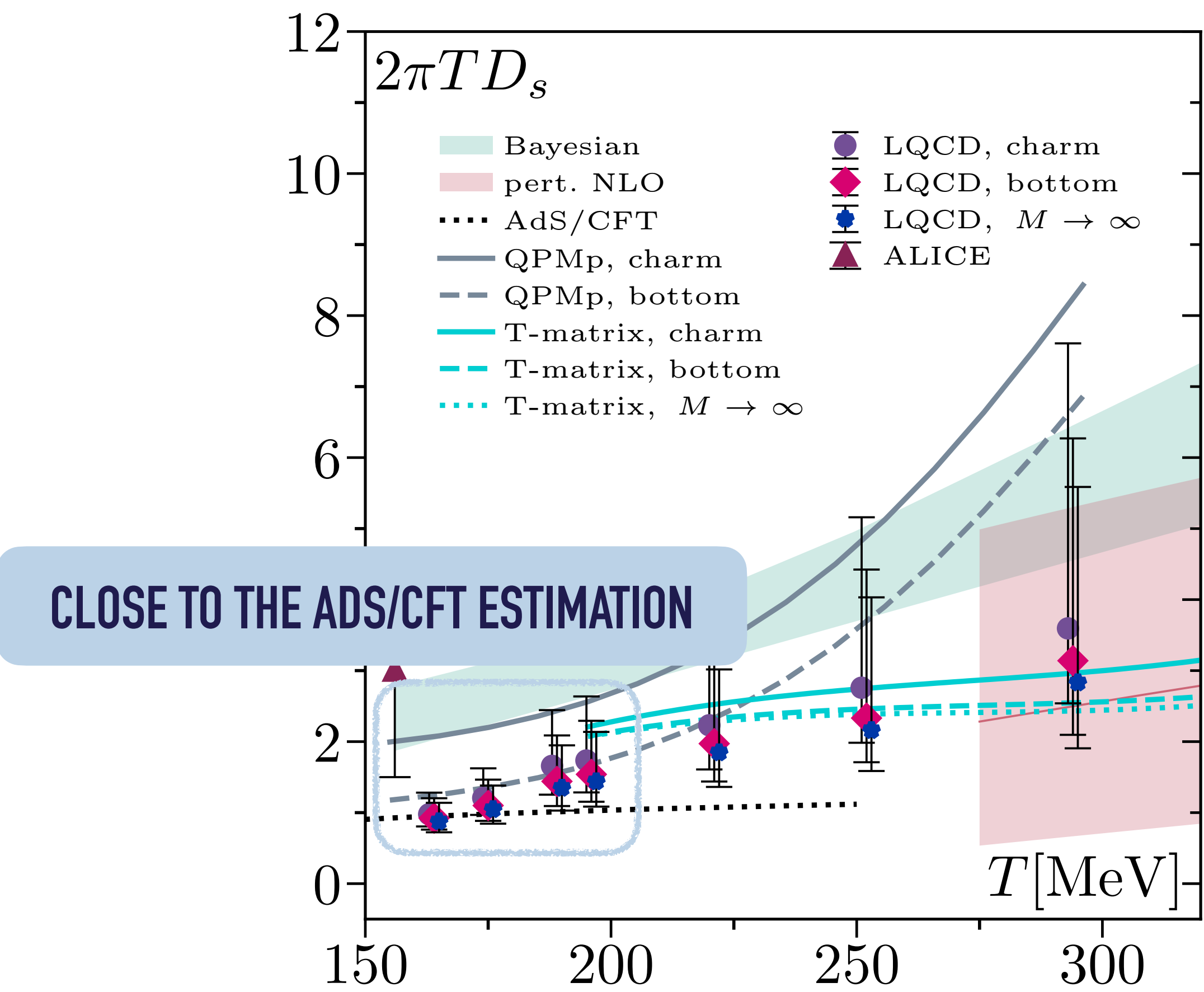
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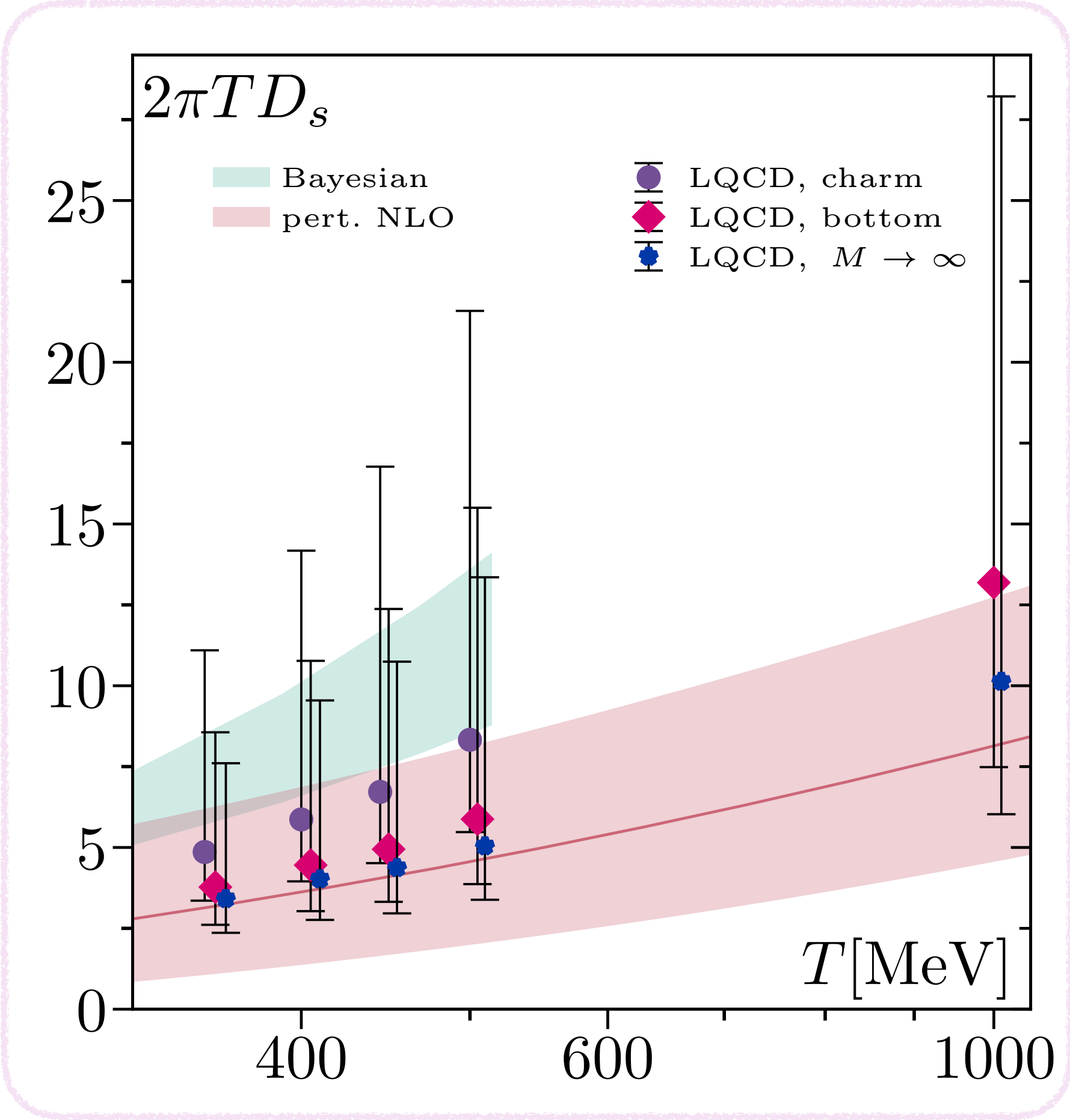
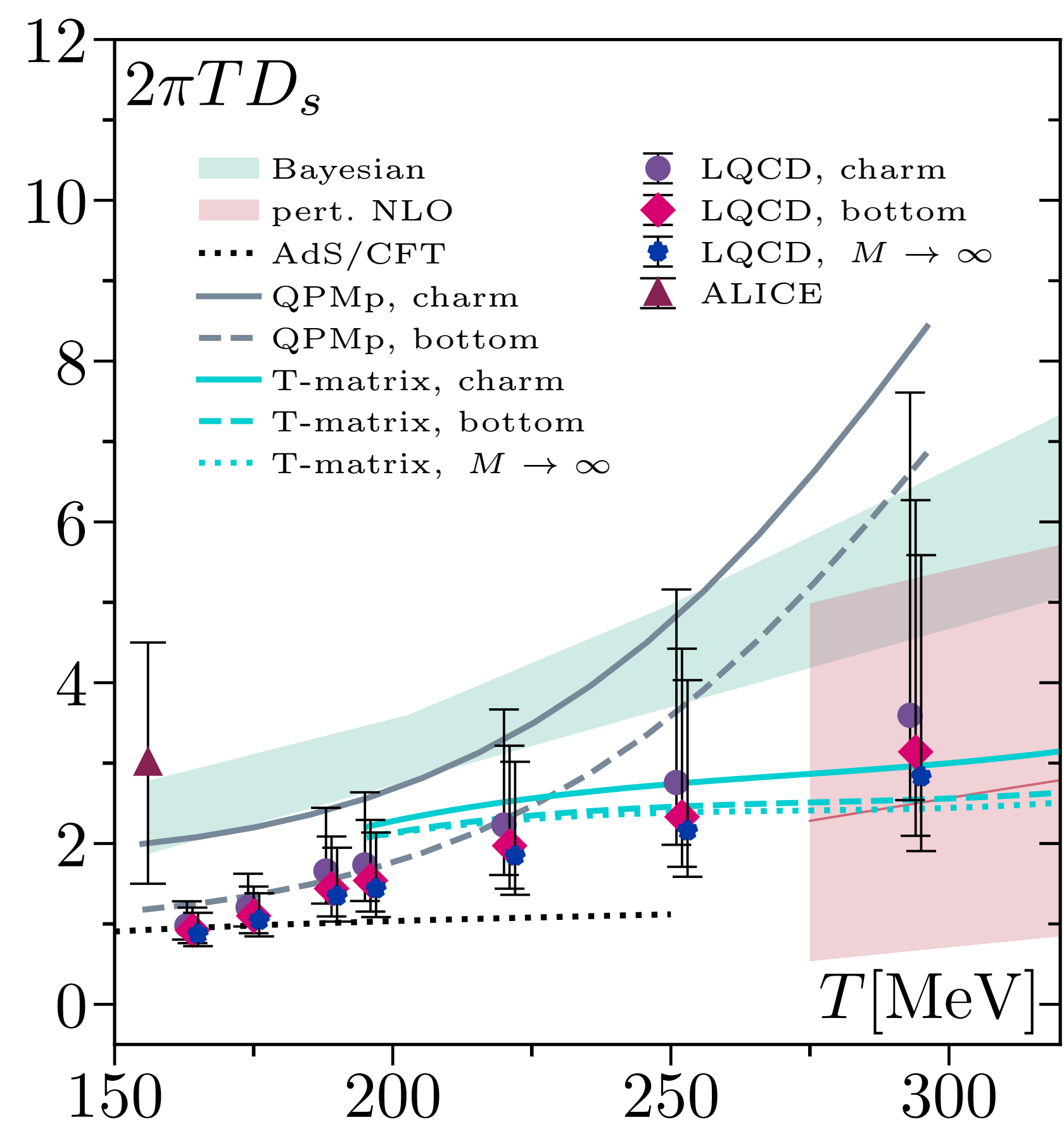
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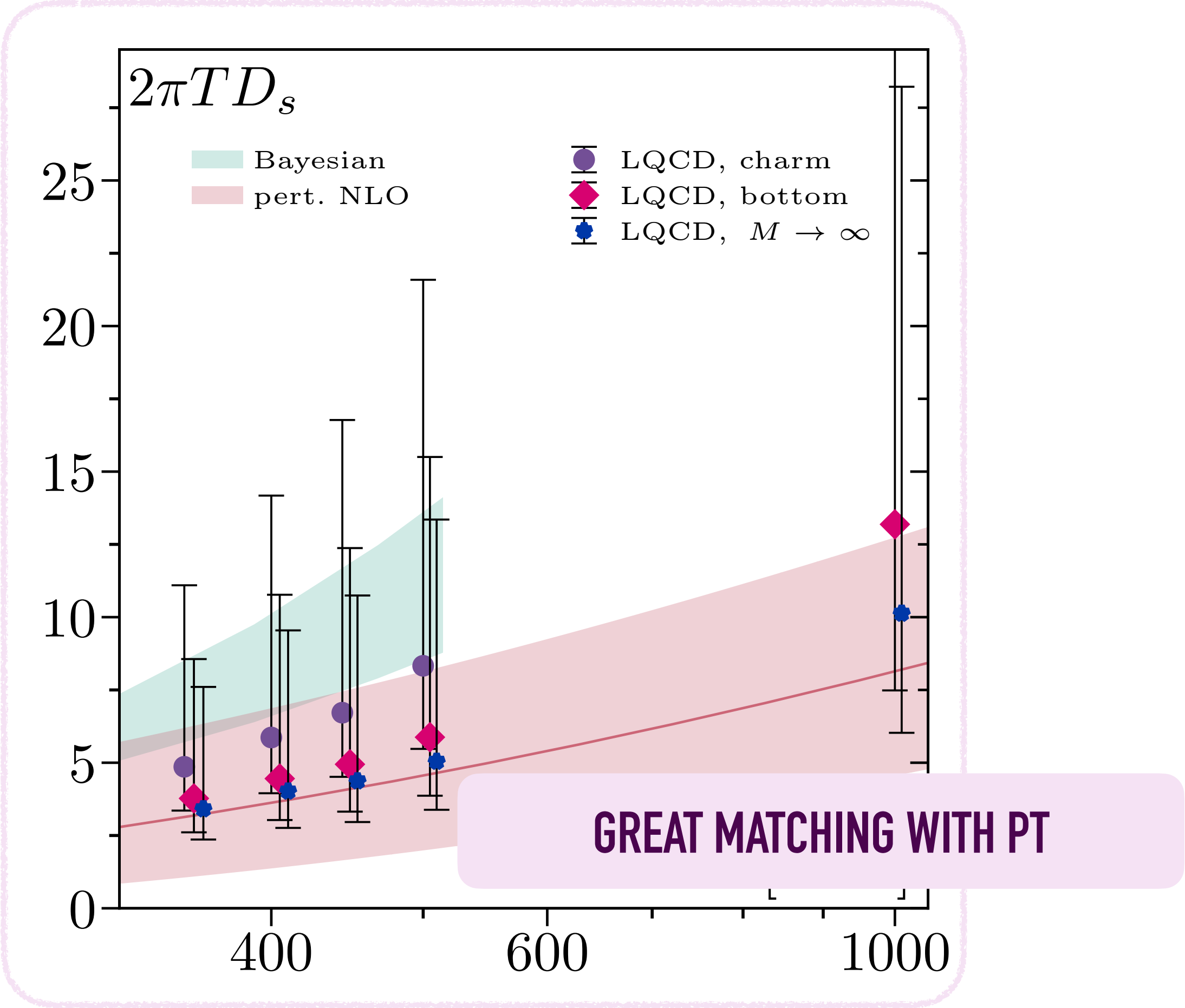
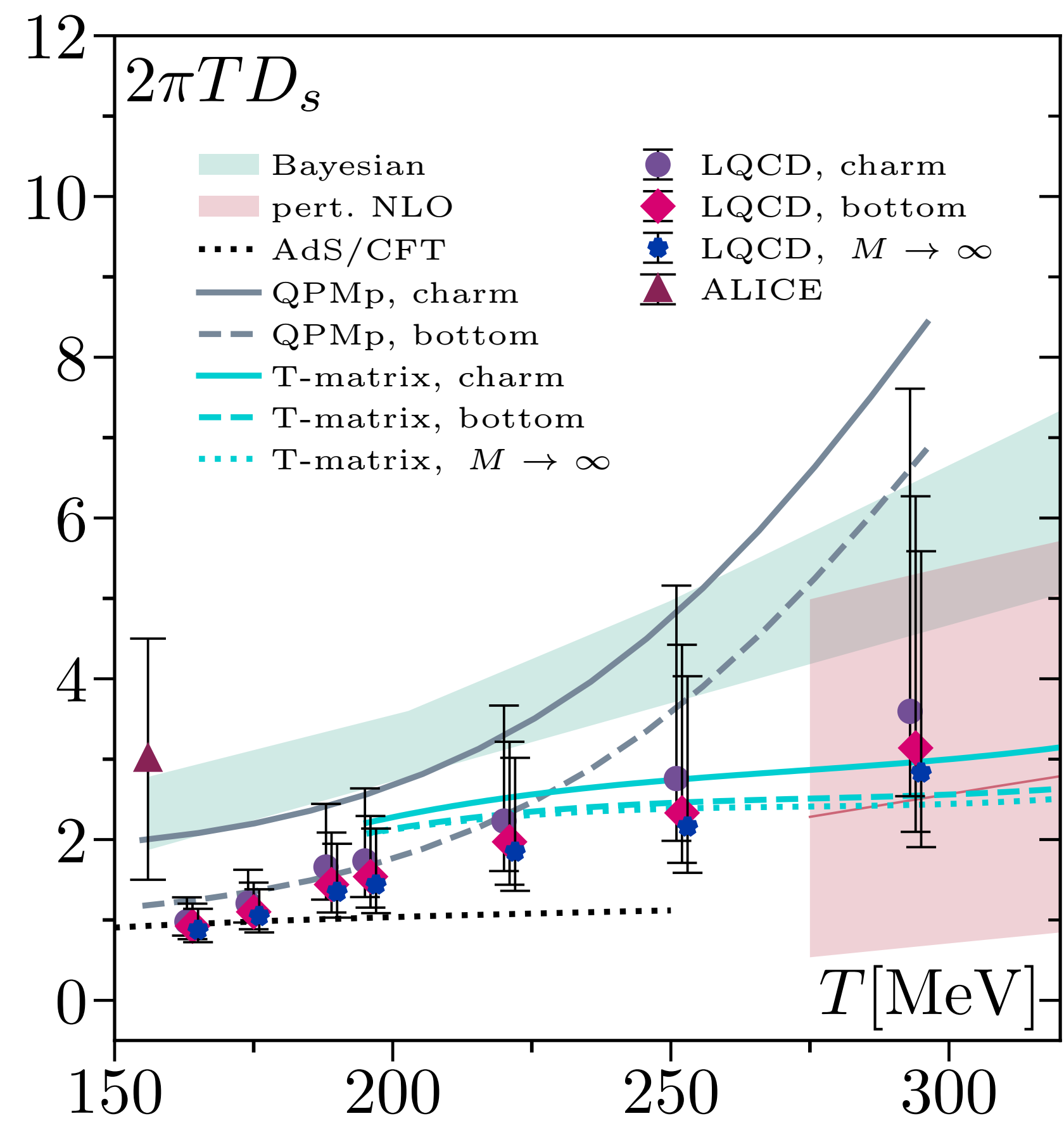
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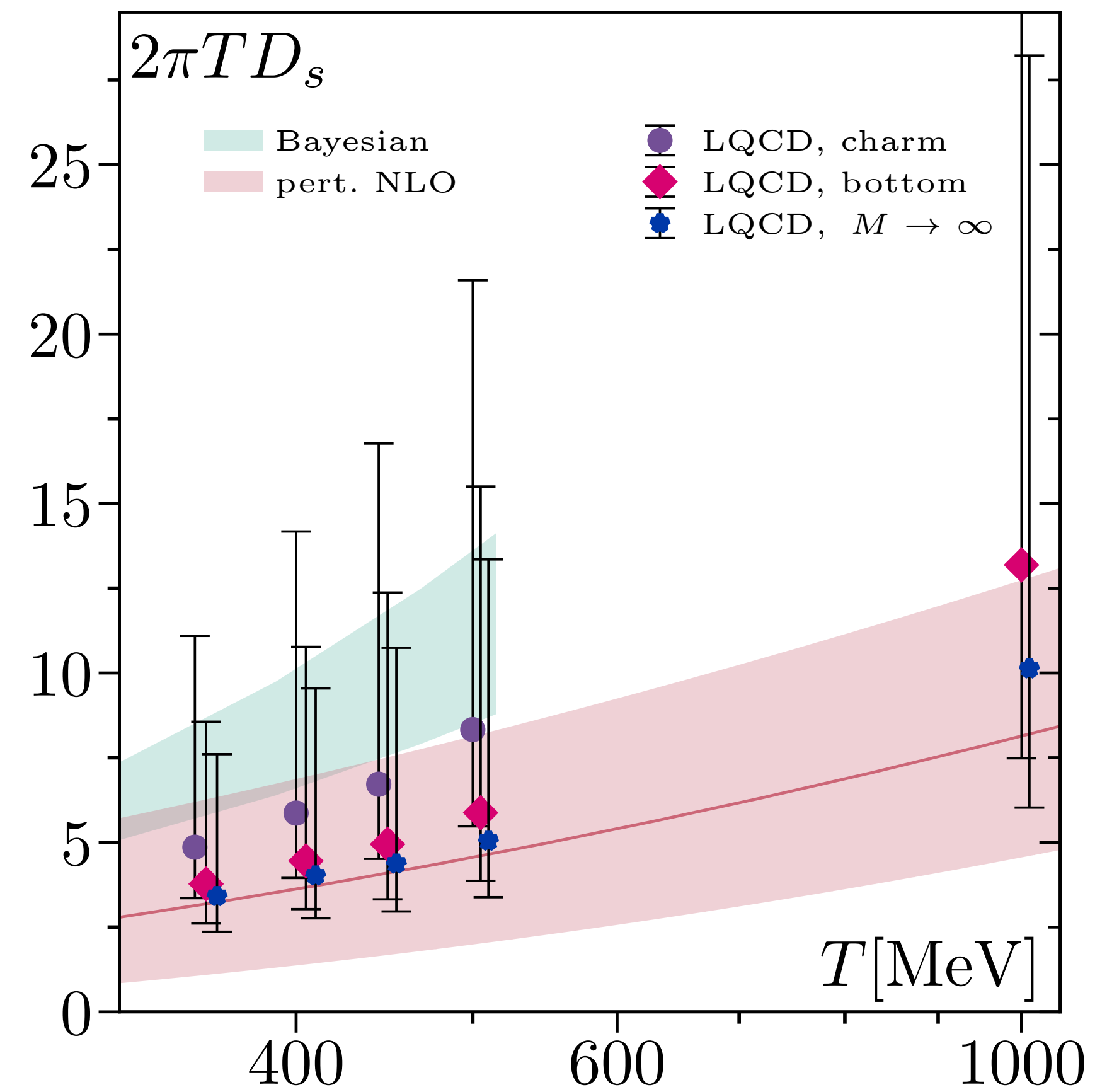
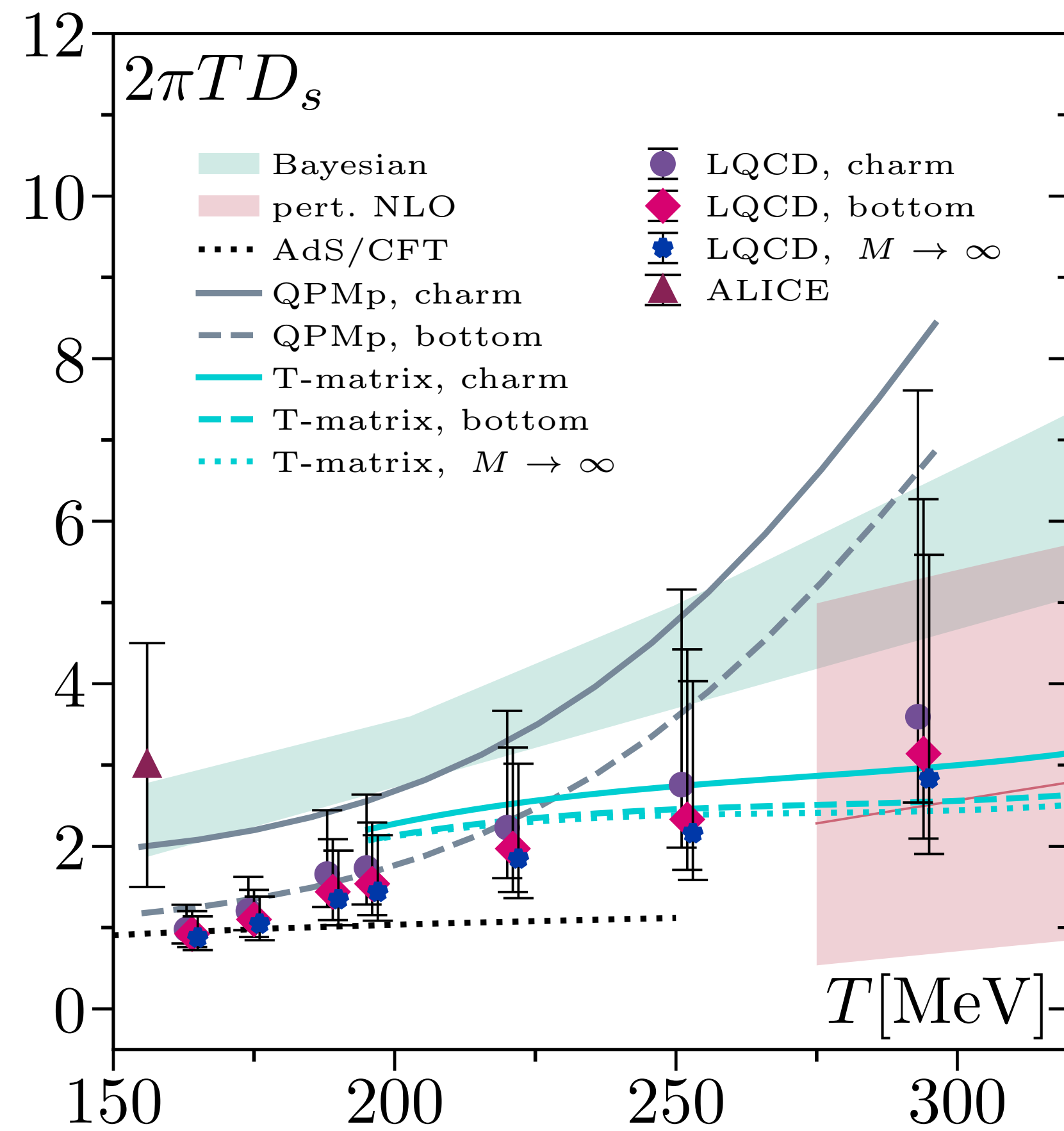
Near $T_c \simeq 150$ MeV, the diffusion coefficient approaches the strongly coupled **AdS/CFT** bound.

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Our results remain systematically lower than estimates **Bayesian** reconstructions and **data-driven** determinations.

LOWER THAN THE DATA DRIVEN ESTIMATION

THANKS FOR YOUR ATTENTION



BACKUP SLIDES

LATTICE ANALYSIS

NUMERICAL SETUP

CONFIGURATIONS WITH $m_l = m_l/5$

NUMERICAL SETUP

T [Mev]	β	a [fm]	$a \times m_s$	$a \times m_l$	N_σ	N_τ	configs	stream
286	8.4000	0.0247	0.008870	0.0017740	64	28	4234	16
308	8.4000	0.0247	0.008870	0.0017740	64	26	4841	16
333	8.4000	0.0247	0.008870	0.0017740	64	24	4728	4
364	8.4000	0.0247	0.008870	0.0017740	64	22	5272	4
400	8.4000	0.0247	0.008870	0.0017740	64	20	5664	4
444	8.4000	0.0247	0.008870	0.0017740	64	18	6188	4
500	8.4000	0.0247	0.008870	0.0017740	64	16	5971	4
352	8.1260	0.0311	0.011380	0.0022760	64	18	4214	12
352	8.3620	0.0255	0.009095	0.0018190	64	22	3609	16
400	8.2763	0.0274	0.009861	0.0019722	64	18	3441	16
400	8.6165	0.0205	0.007174	0.0014348	64	24	4097	24
444	8.2612	0.0278	0.010004	0.0020008	64	16	4462	16
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10000	12.1034	0.00110	0.0003221	0.00006442	64	18	1373	8
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286	8.4000	$r_1 = 0.3160 \text{ fm}$ $r_2 \left. \frac{dV(r)}{dr} \right _{r=r_1}$	0.008870	0.0017740	64	28	4234	16
308	8.4000		0.008870	0.0017740	64	26	4841	16
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NUMERICAL SETUP

CONFIGURATIONS WITH $m_l = m_l/20$

NUMERICAL SETUP

T [Mev]	β	a [fm]	$a \times m_s$	$a \times m_l$	N_σ	N_τ	configs	stream
137	7.3730	0.0602	0.02500	0.00125	64	24	2273	4
149	7.3730	0.0602	0.02500	0.00125	64	22	4663	35
164	7.3730	0.0602	0.02500	0.00125	64	20	7424	36
182	7.3730	0.0602	0.02500	0.00125	64	18	6245	37
205	7.3730	0.0602	0.02500	0.00125	64	16	4785	4
133	7.5960	0.0493	0.02020	0.00101	64	30	1683	4
143	7.5960	0.0493	0.02020	0.00101	64	28	2036	4
154	7.5960	0.0493	0.02020	0.00101	64	26	9162	47
167	7.5960	0.0493	0.02020	0.00101	64	24	6669	37
182	7.5960	0.0493	0.02020	0.00101	64	22	7115	37
200	7.5960	0.0493	0.02020	0.00101	64	20	3017	4
222	7.5960	0.0493	0.02020	0.00101	64	18	4952	8
250	7.5960	0.0493	0.02020	0.00101	64	16	7130	9
153	7.8250	0.0404	0.01640	0.00082	64	32	2574	8
163	7.8250	0.0404	0.01640	0.00082	64	30	4757	24
174	7.8250	0.0404	0.01640	0.00082	64	28	14128	49
188	7.8250	0.0404	0.01640	0.00082	64	26	13911	48
204	7.8250	0.0404	0.01640	0.00082	64	24	4555	7
222	7.8250	0.0404	0.01640	0.00082	64	22	5109	7
244	7.8250	0.0404	0.01640	0.00082	64	20	4433	4
271	7.8250	0.0404	0.01640	0.00082	64	18	5340	4
305	7.8250	0.0404	0.01640	0.00082	64	16	6238	4

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T [Mev]	β	a [fm]	$a \times m_s$	$a \times m_l$	N_σ	N_τ	configs	stream
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NUMERICAL SETUP

T [Mev]	β	a [fm]	$a \times m_s$	$a \times m_l$	N_σ	N_τ	configs	stream
195	7.5700	0.0505	0.019730	0.0039460	64	20	5911	12
195	7.7770	0.0421	0.016010	0.0032020	64	24	5480	4
220	7.7040	0.0449	0.017230	0.0034460	64	20	7933	12
220	7.9130	0.0374	0.014000	0.0028000	64	24	5754	4
251	7.8570	0.0393	0.014790	0.0029580	64	20	9443	4
251	8.0680	0.0327	0.012040	0.0024080	64	24	5336	12
293	8.0360	0.0336	0.012410	0.0024820	64	20	9287	4
293	8.1470	0.0306	0.011150	0.0022300	64	22	9105	12
352	8.2490	0.0280	0.010110	0.0020220	96	20	6167	4

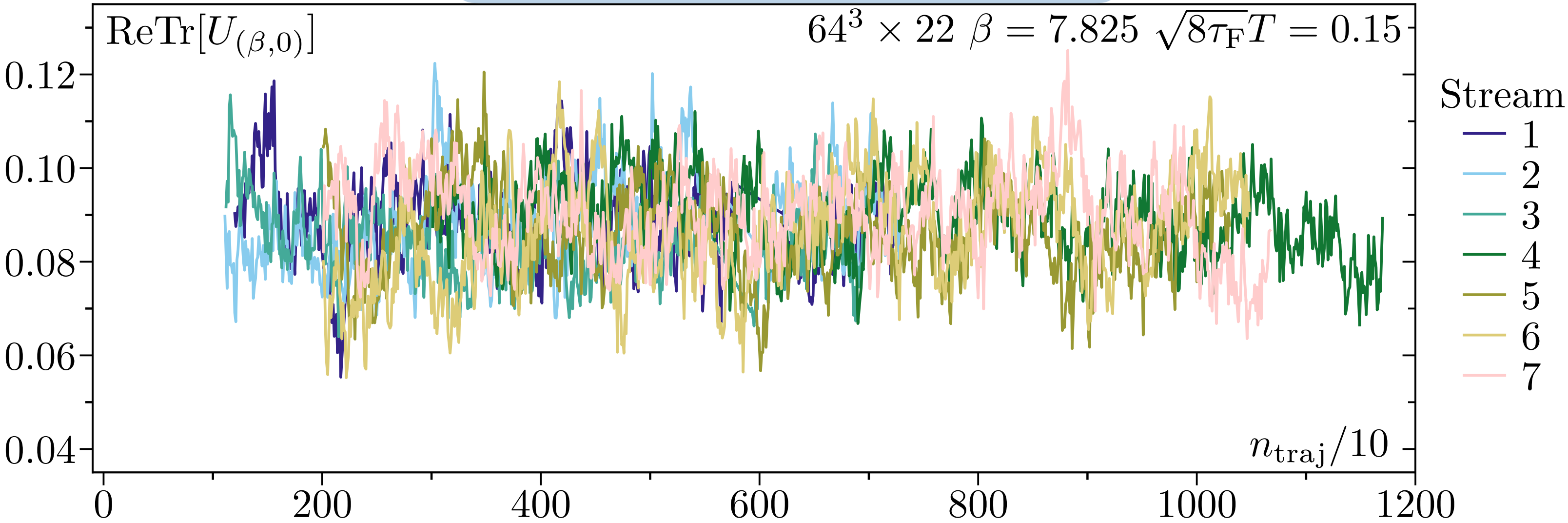
PERVIOUSLY GENERATED BY HOTQCD COLLABORATION

HotQCD

NUMERICAL SETUP

T [Mev]	β	a [fm]	$a \times m_s$	$a \times m_l$	N_σ	N_τ	configs	stream
195	7.5700	0.0505	0.019730	0.0039460	64	20	5911	12
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$\beta = 7.8250$ AND $N_\tau = 22$

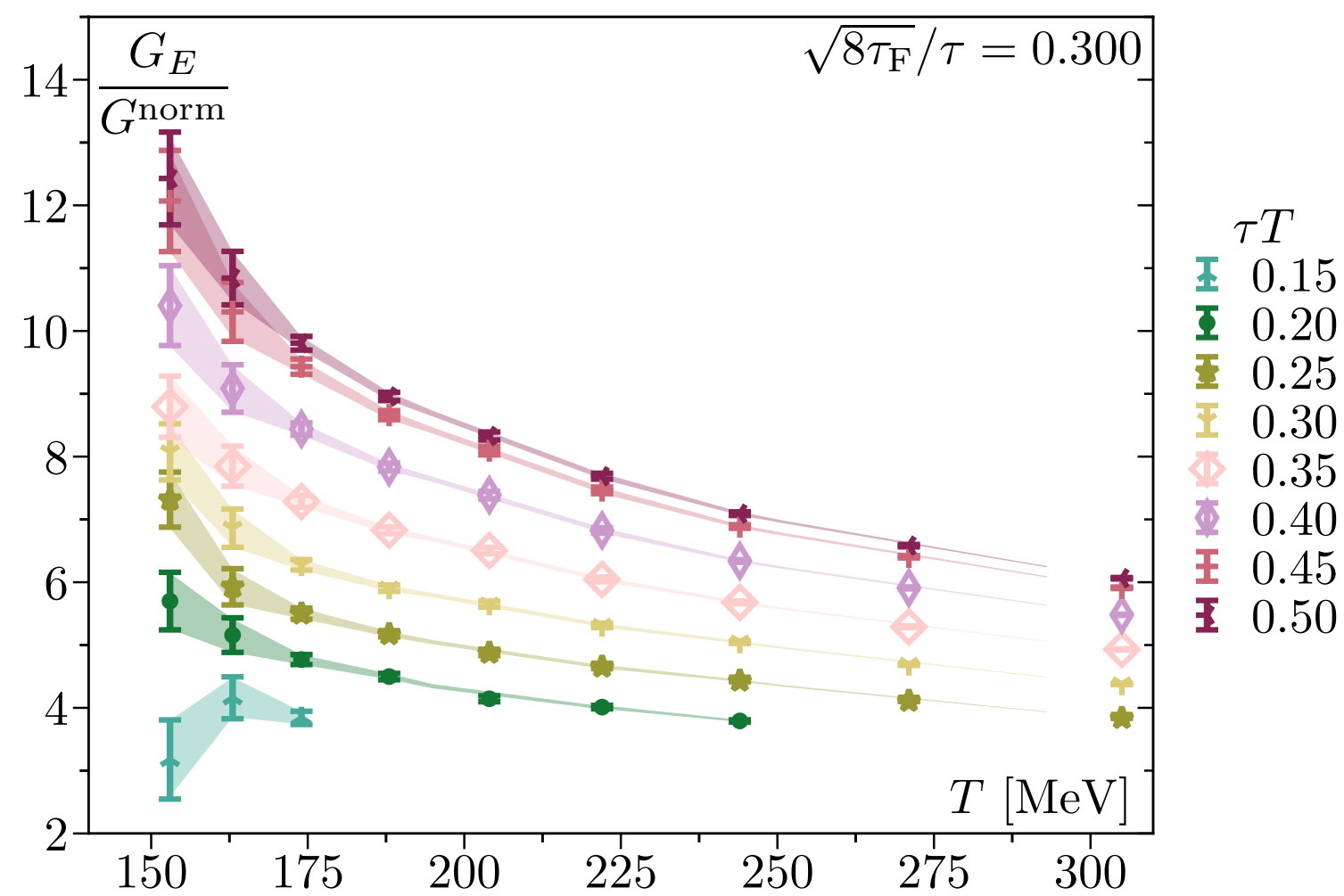


NUMERICAL SETUP

Our data is, **in general**, not obtained at a fixed temperature, so we need to interpolate the correlators also in T .

NUMERICAL SETUP

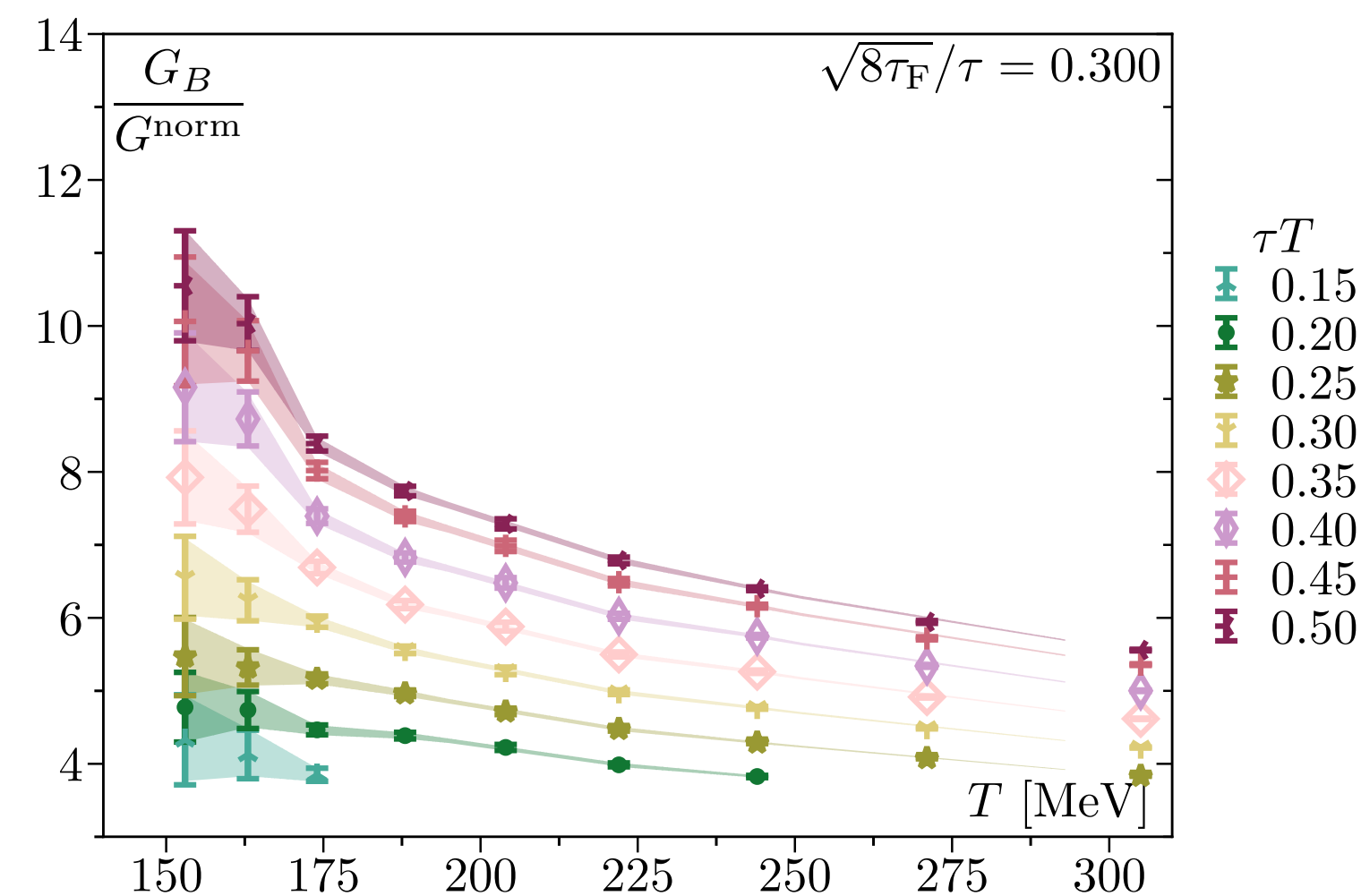
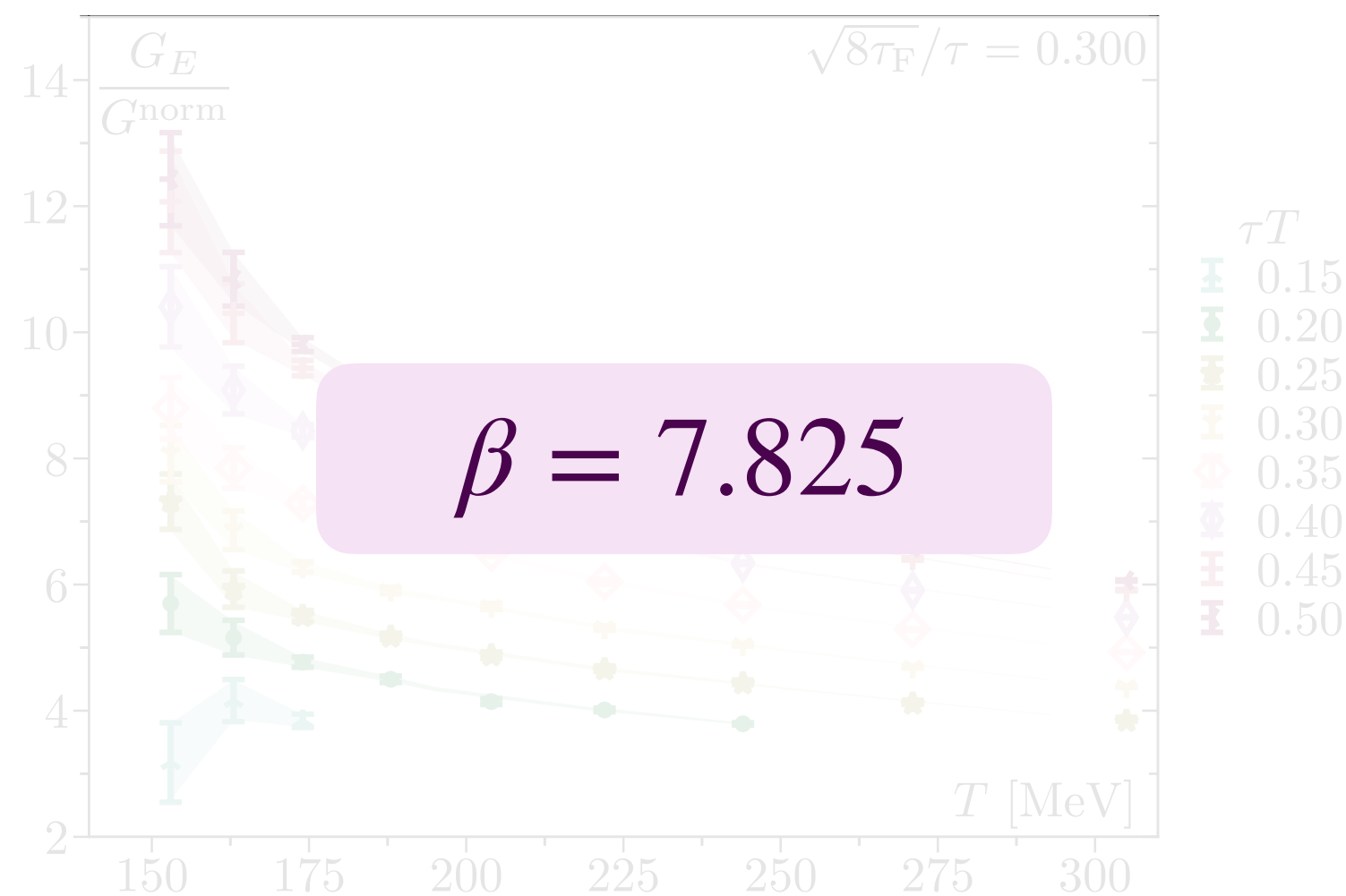
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$\beta = 7.825$

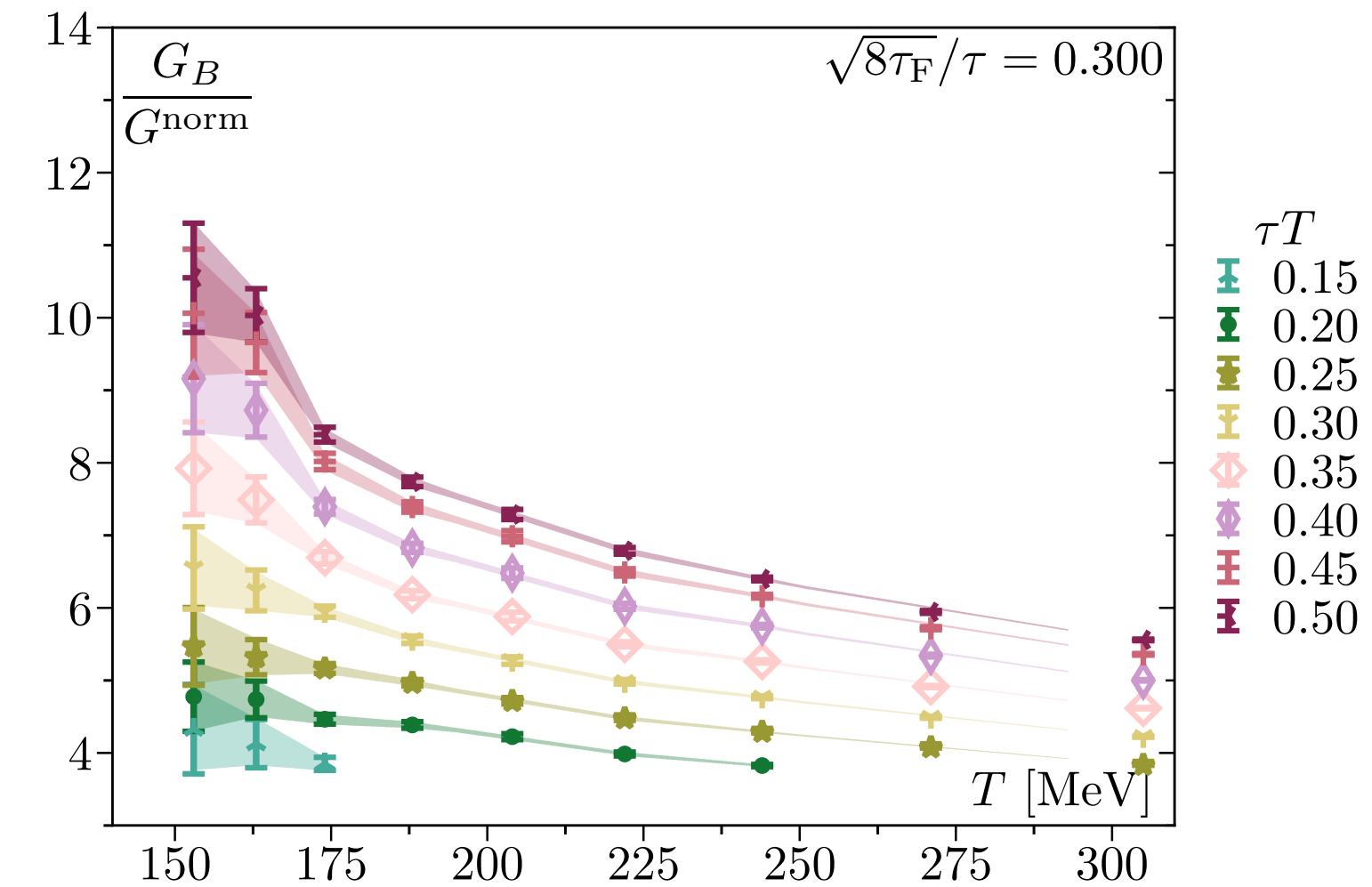
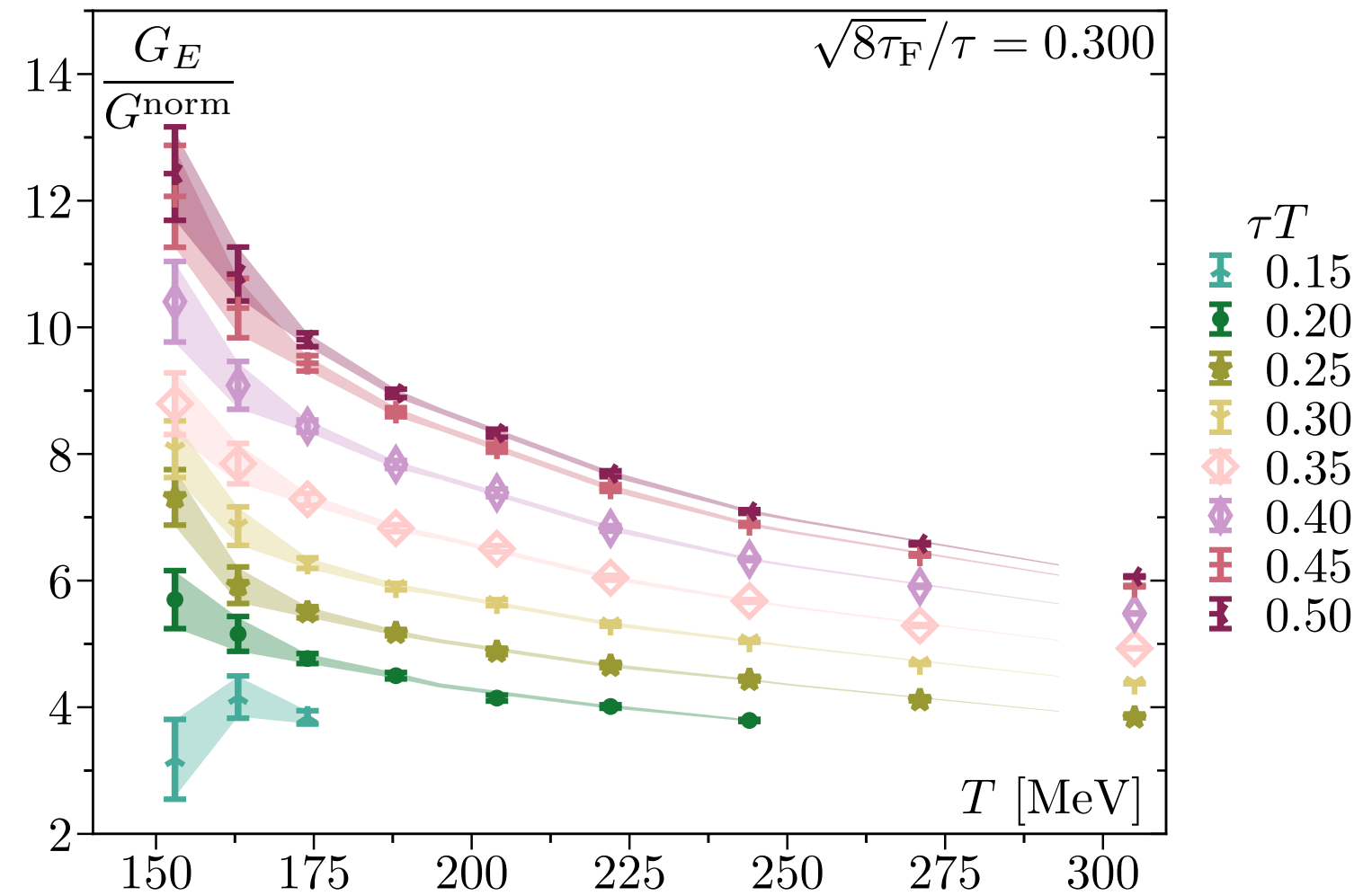
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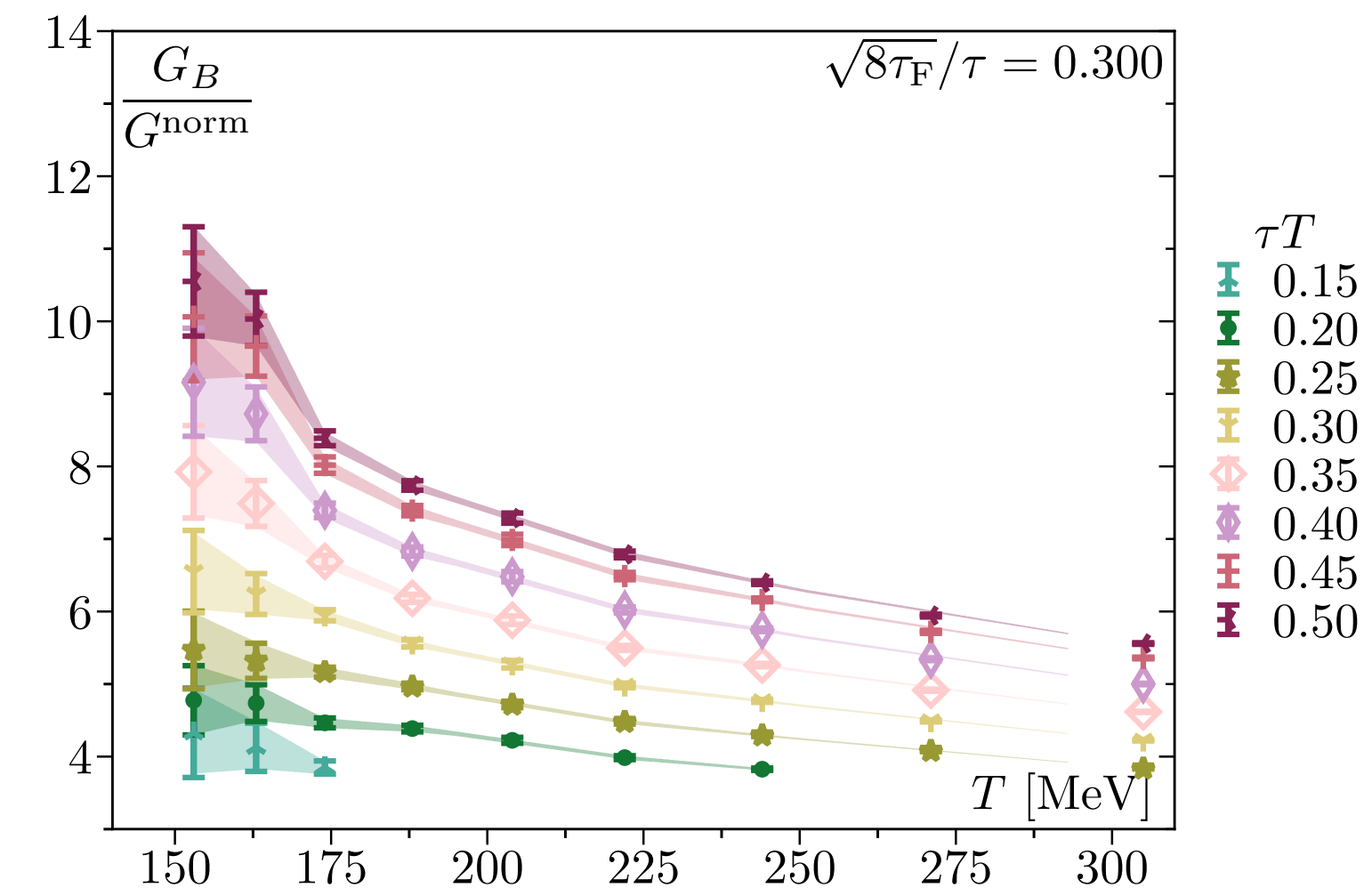
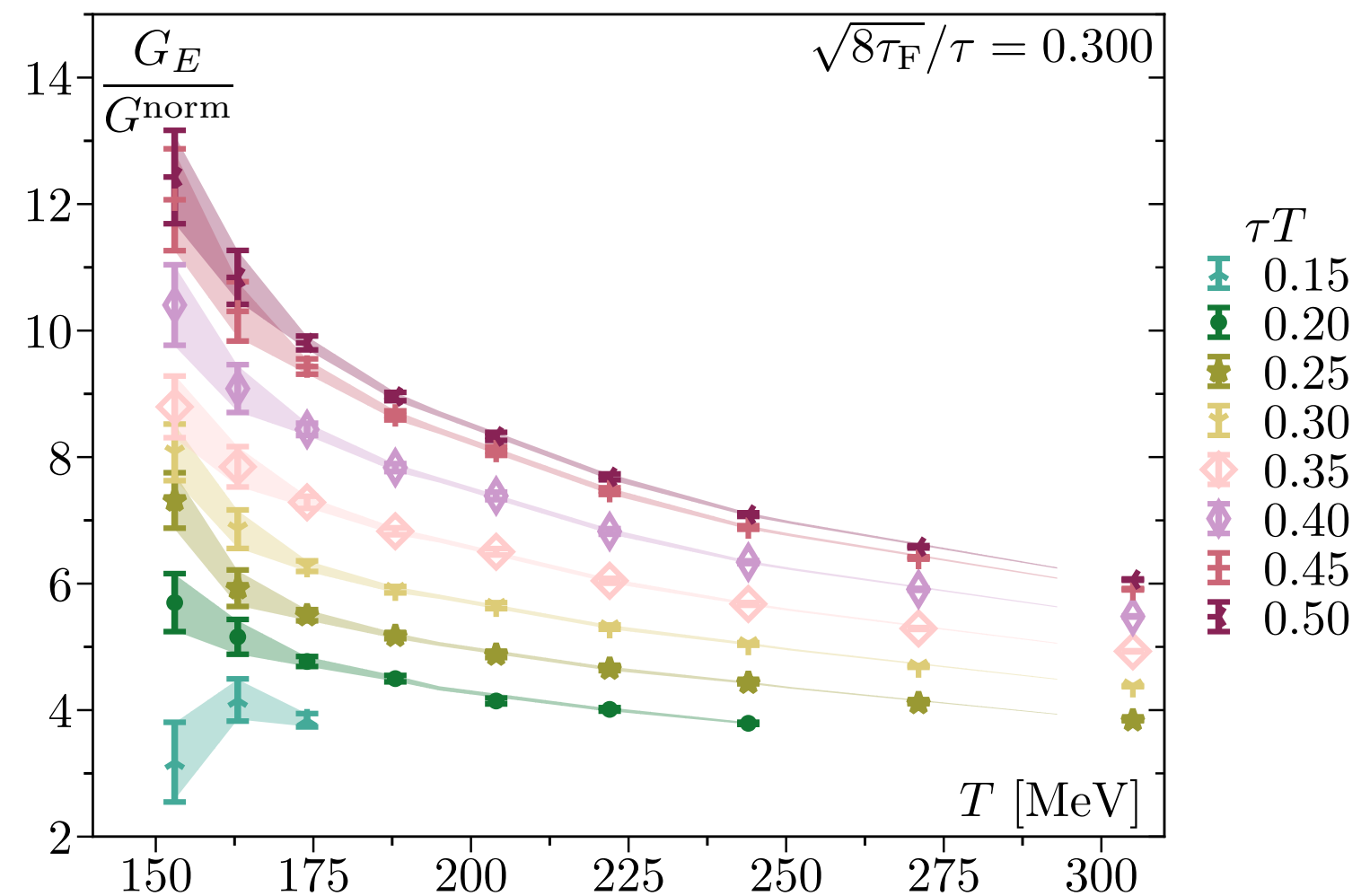


We target the following temperatures:

$T = 153, 163, 174, 188, 195, 204, 220, 244, 251, 293, 352, 400, 444, 500, 1000, \text{ and } 10000 \text{ MeV}.$

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Our data is, **in general**, not obtained at a fixed temperature, so we need to interpolate the correlators also in T .



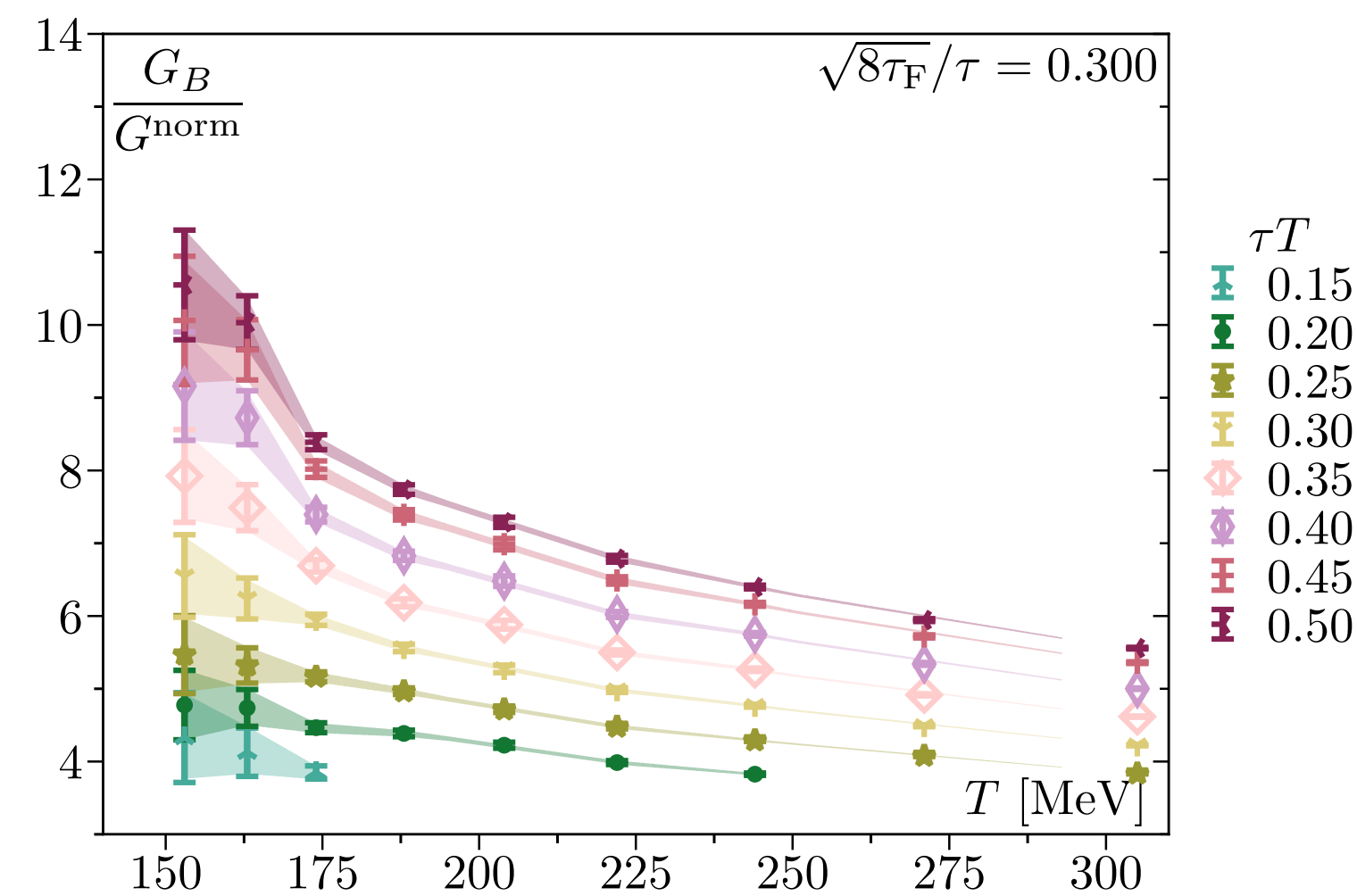
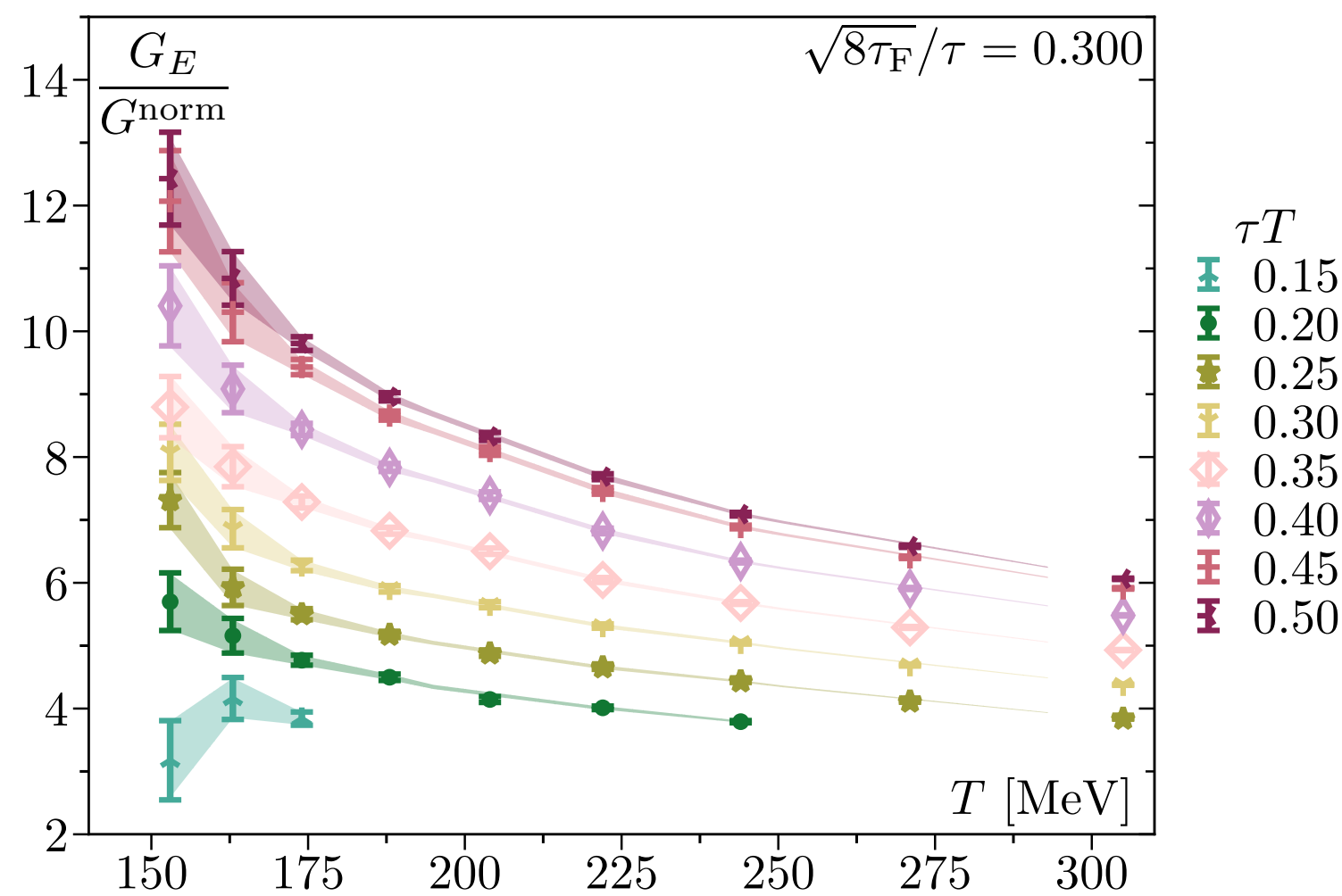
We target the following temperatures:

$T = 153, 163, 174, 188, 195, 204, 220, 244, 251, 293, 352, 400, 444, 500, 1000, \text{ and } 10000$ MeV.

Interpolation needed

NUMERICAL SETUP

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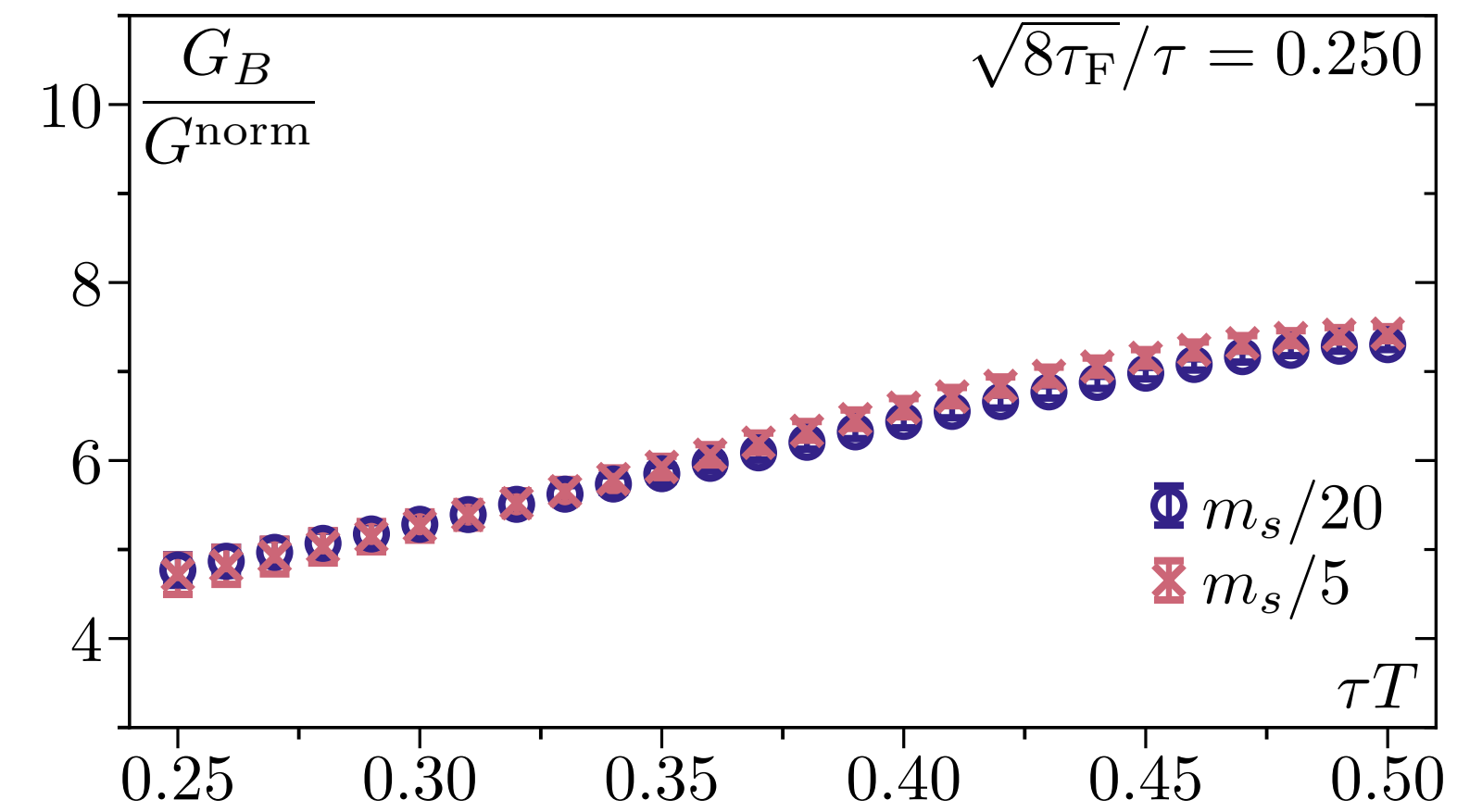
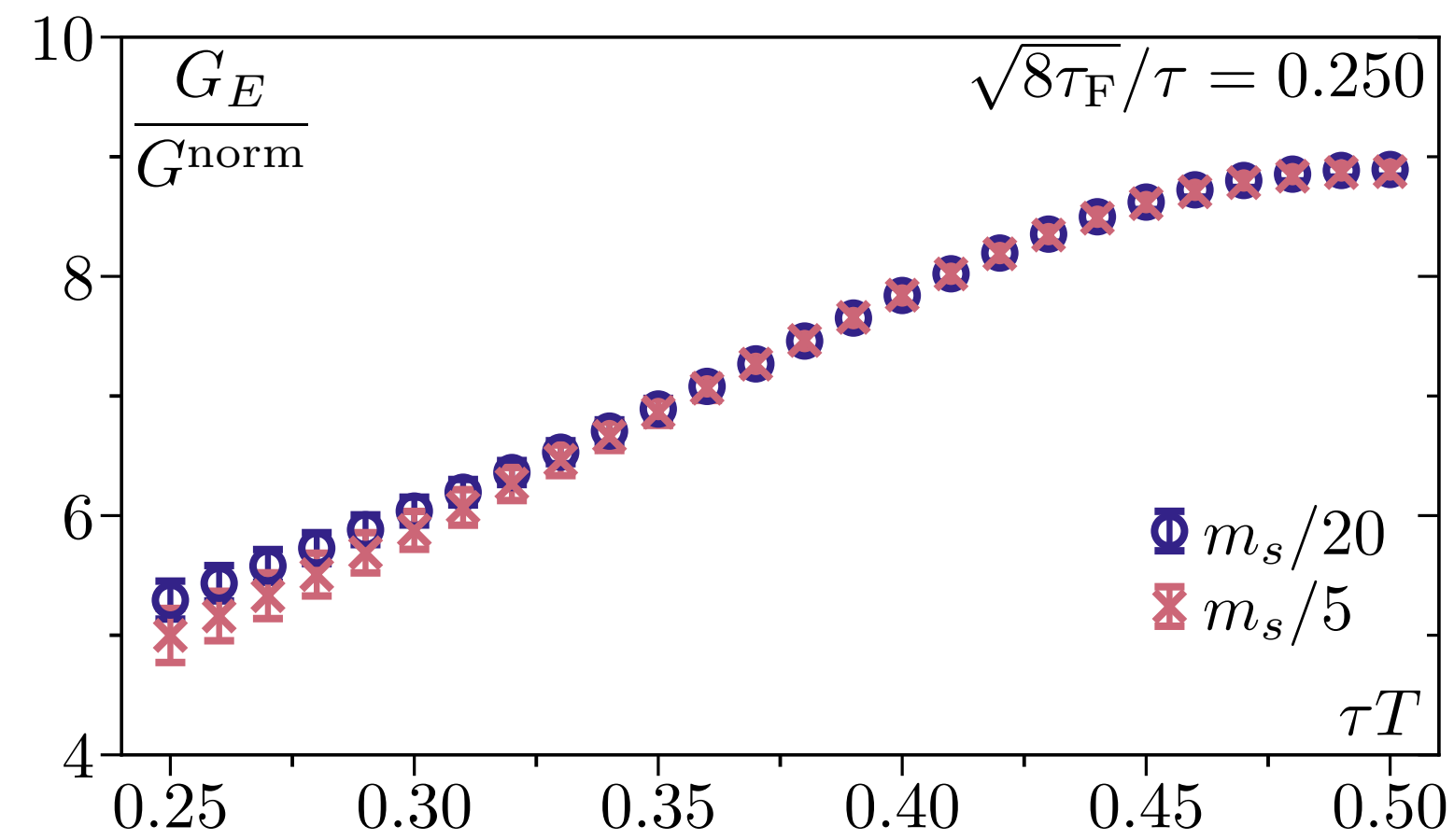
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Interpolation needed

Fixed temperature for several a

NUMERICAL SETUP

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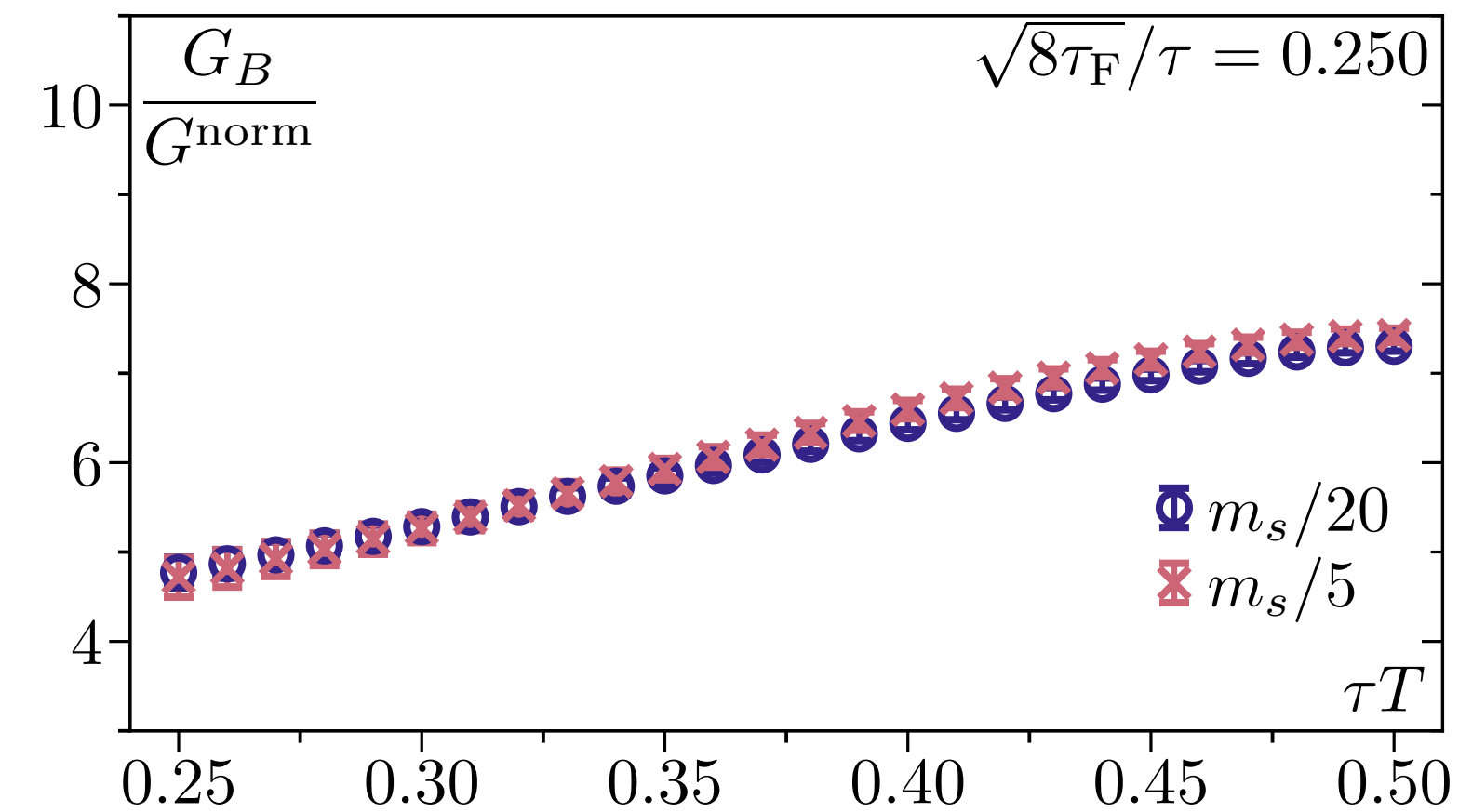
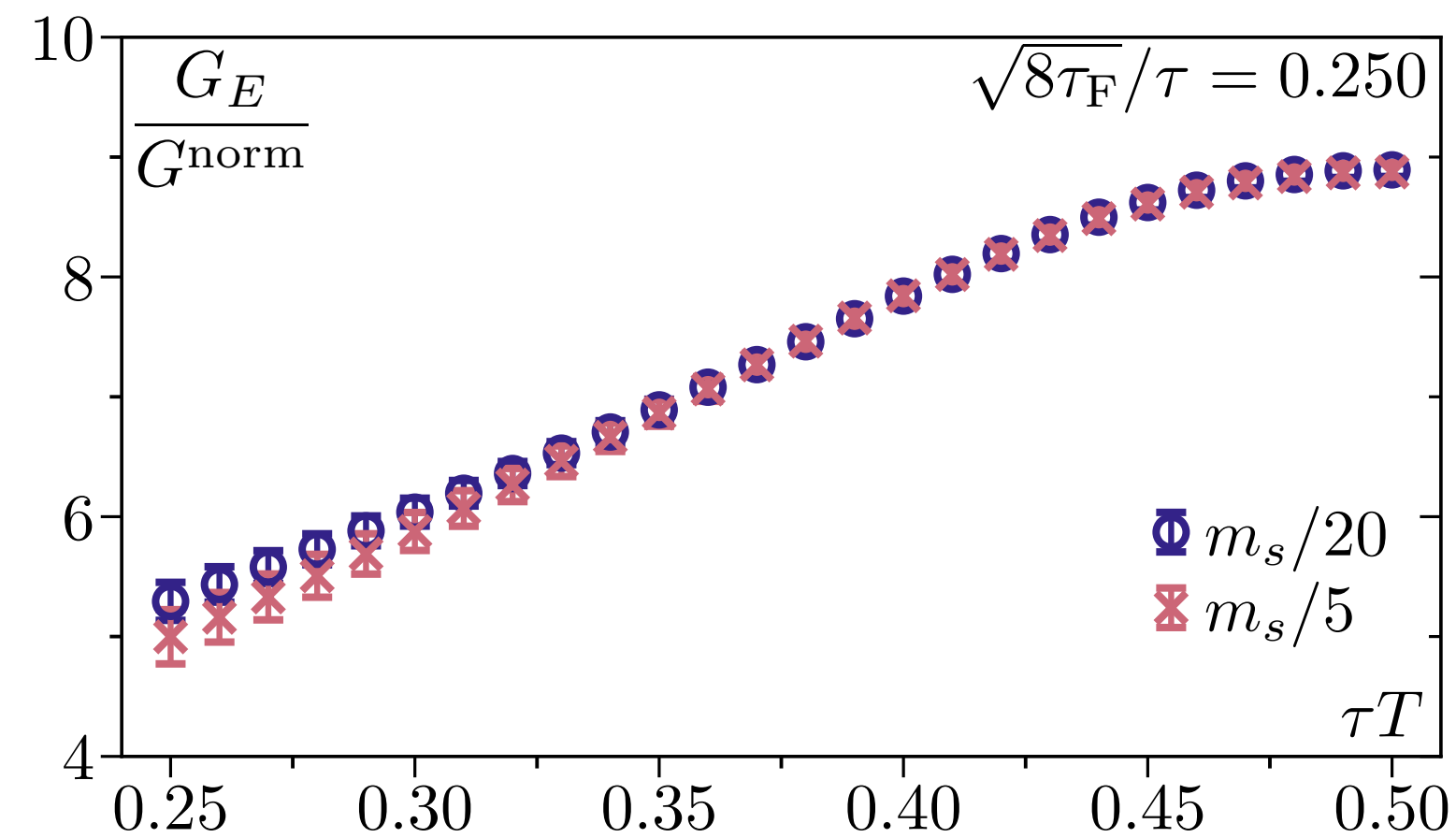


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We target the following temperatures:

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**LIGHT QUARK MASS EFFECTS ARE ALREADY
NEGLECTIBLE AT $T = 195 \text{ MeV}$**

PERTURBATIVE QCD ESTIMATES

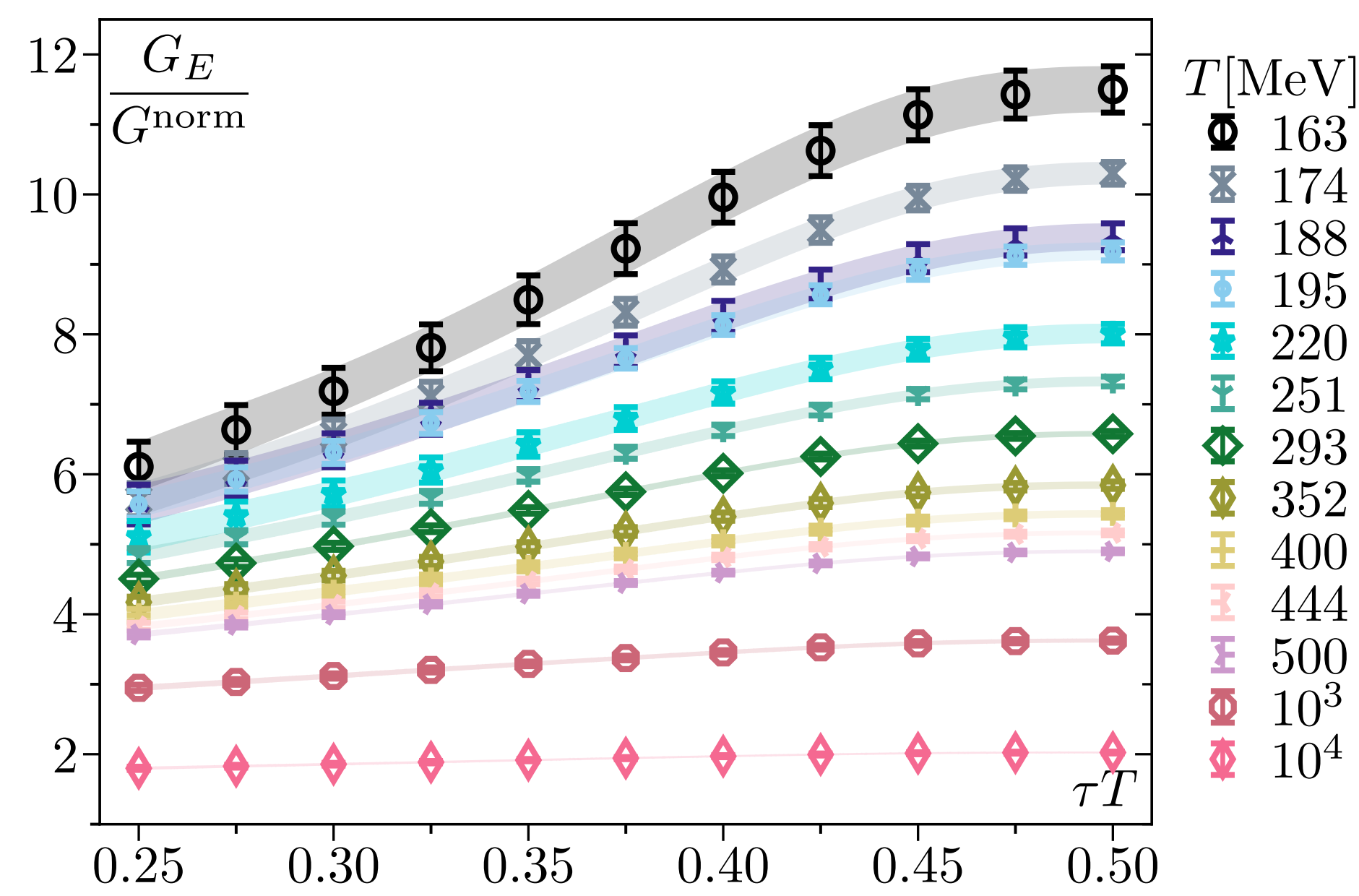
PERTURBATIVE RESULTS FOR THE DIFFUSION COEFFICIENT

Can we use these LO and NLO estimates and scales to normalize the correlators again?

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Electric spectral function :



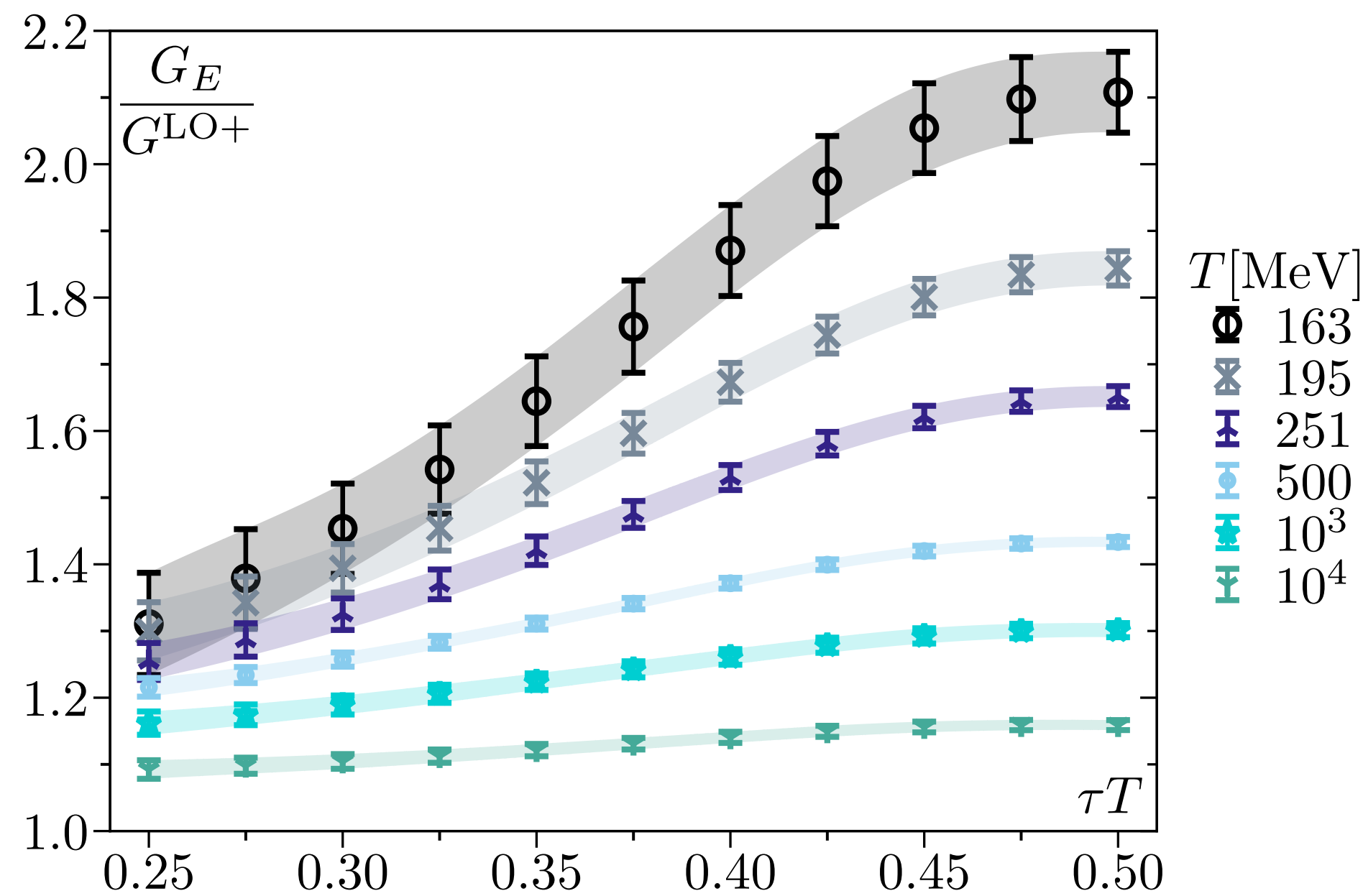
CORRELATORS NORMALIZE BY G^{norm}

Computed with $\rho_E^{\text{LO}}(\omega, \tau)$ but factorizing out the running coupling

PERTURBATIVE RESULTS FOR THE DIFFUSION COEFFICIENT

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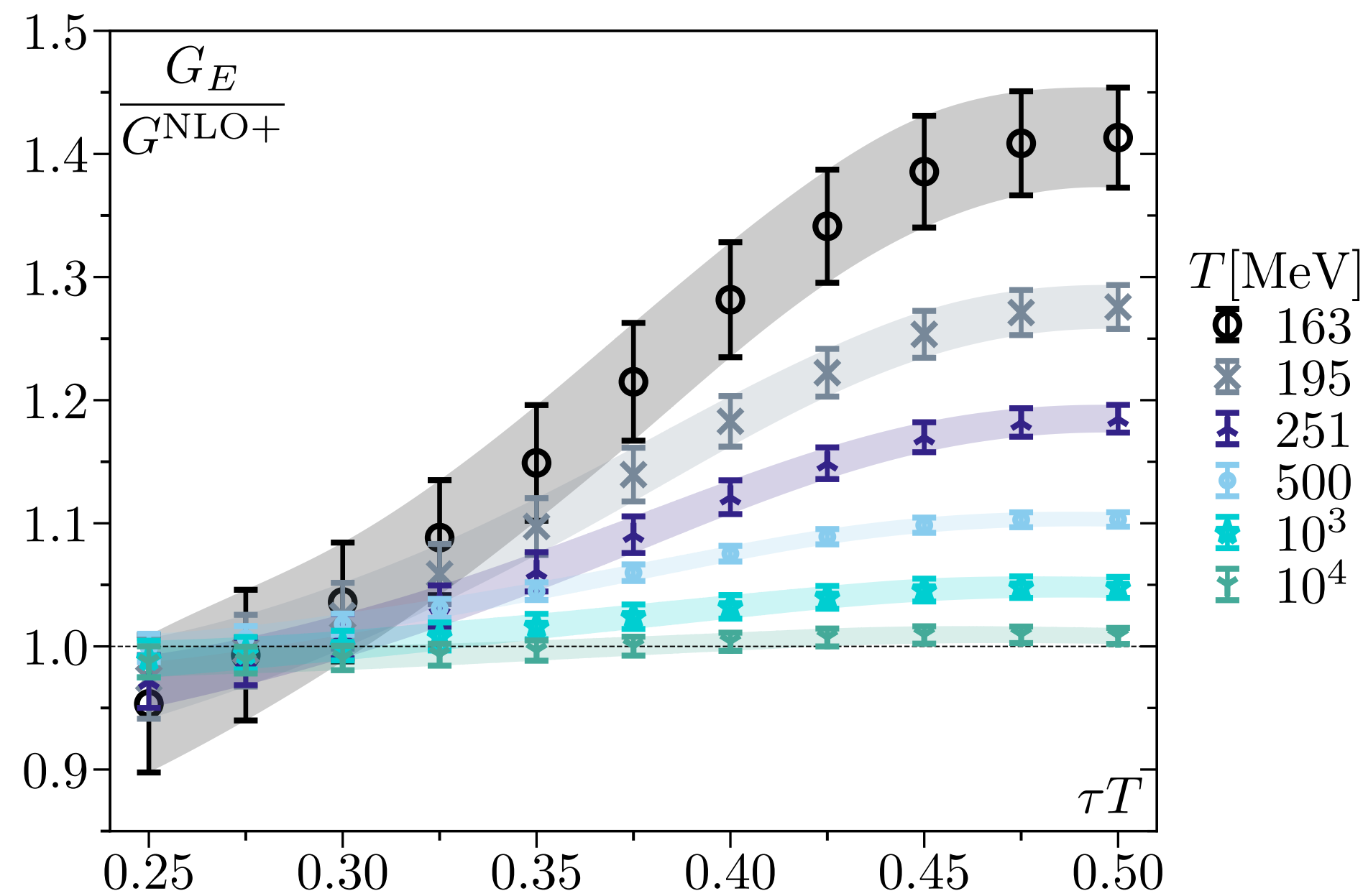
CORRELATORS NORMALIZE BY G^{LO+}

Computed with $\rho_E^{LO}(\omega, \tau)$ and also including the running coupling

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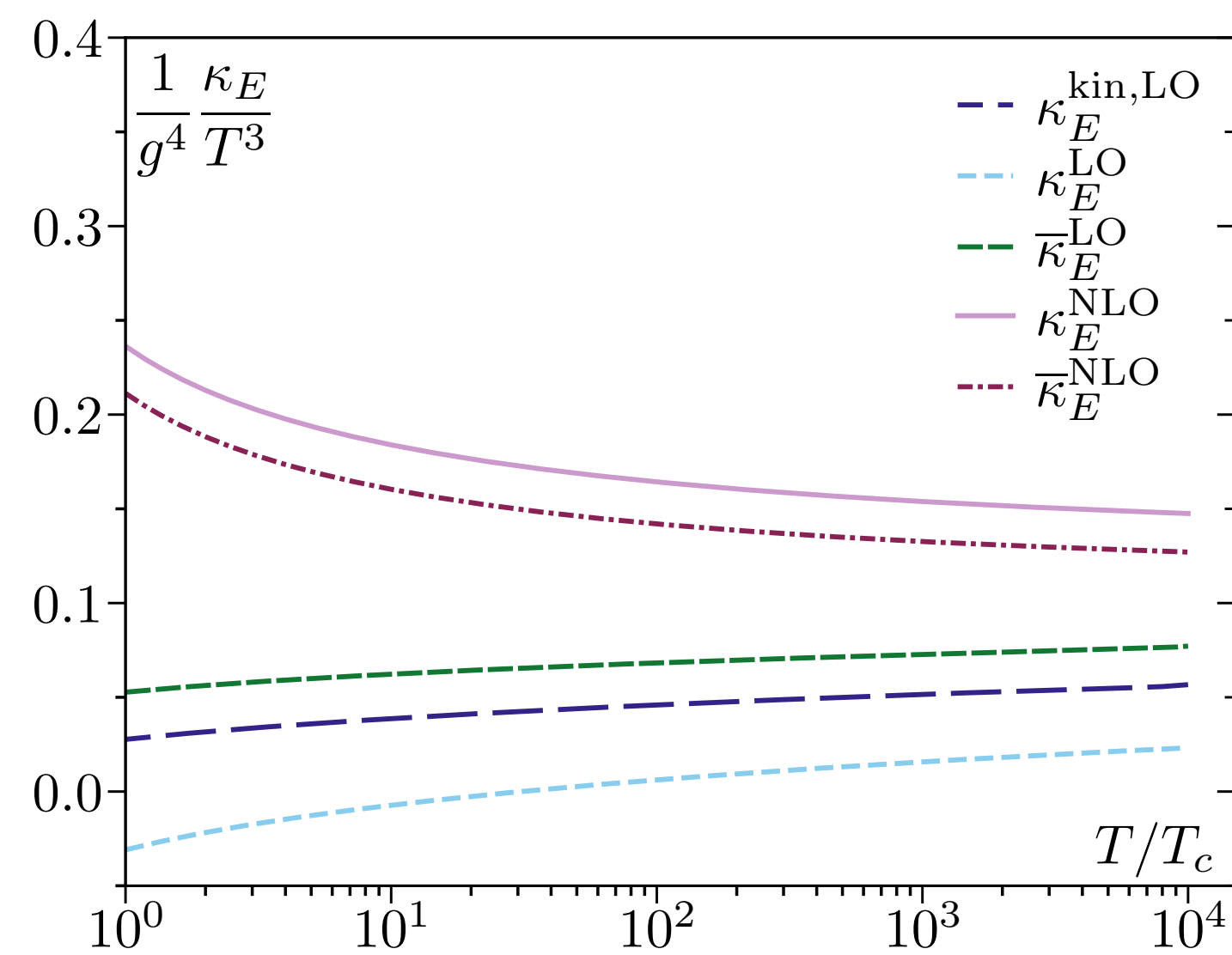
CORRELATORS NORMALIZE BY $G^{\text{NLO}+}$

Computed with $\rho_E^{\text{NLO}}(\omega, \tau)$ and also including the running coupling

THE RATIO IS ALMOST FLAT FOR LARGE T , SO $\kappa \rightarrow 0$

PERTURBATIVE RESULTS FOR THE DIFFUSION COEFFICIENT

We can even compare the results with the direct perturbative calculation of the diffusion coefficient.



QUENCHED QCD

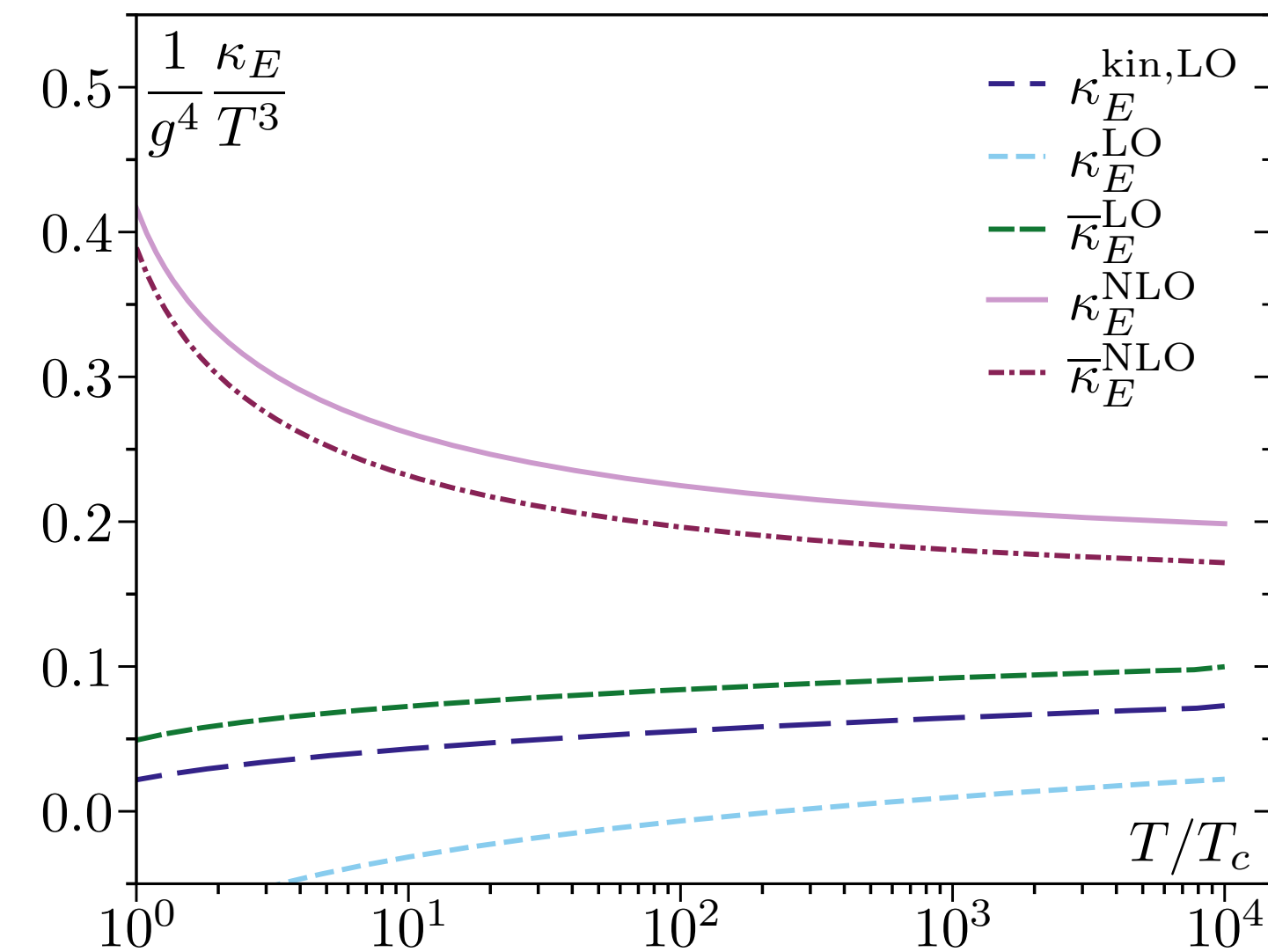
$T_c \simeq 314 \text{ MeV}$

PERTURBATIVE RESULTS FOR THE DIFFUSION COEFFICIENT

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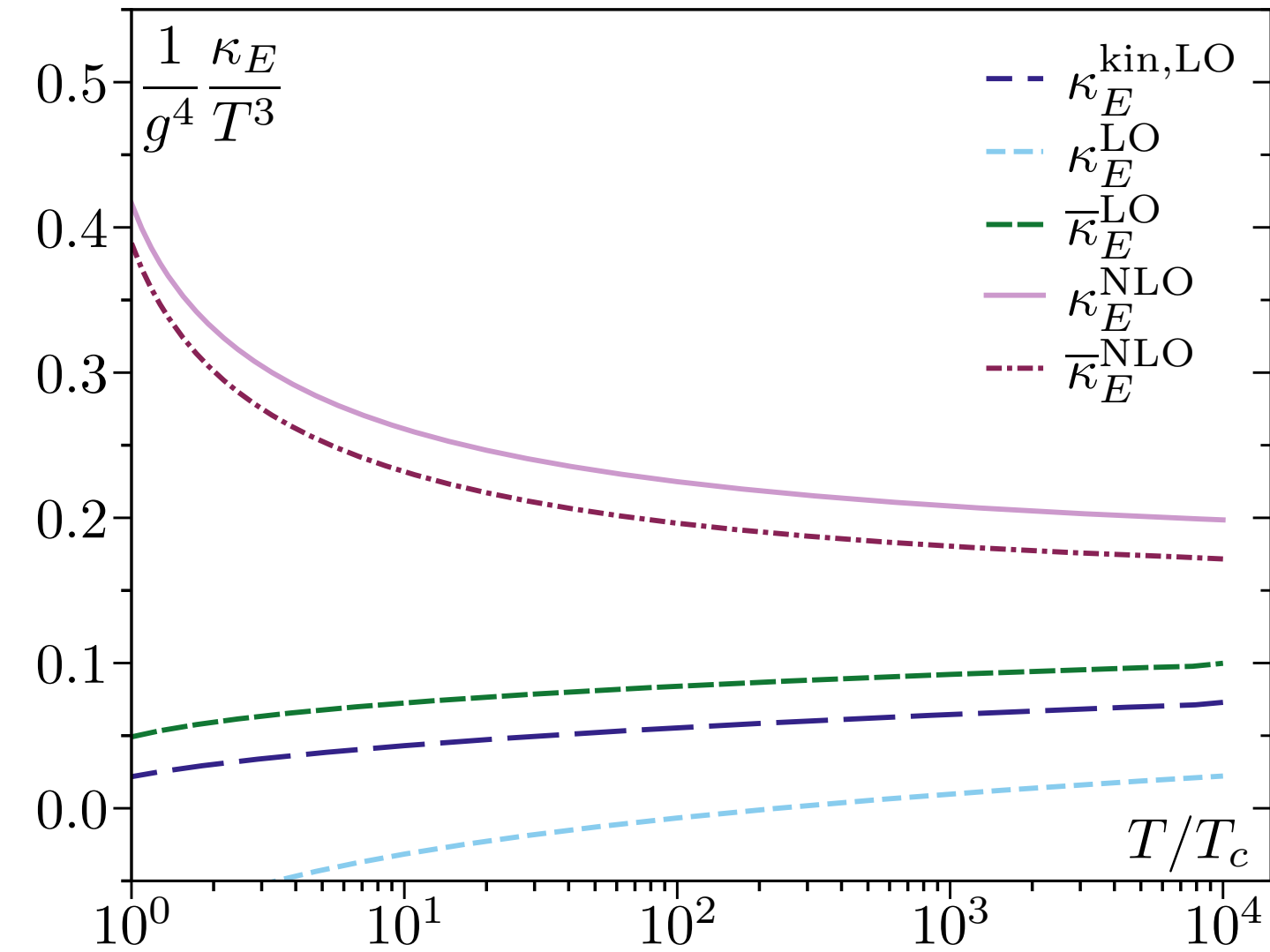
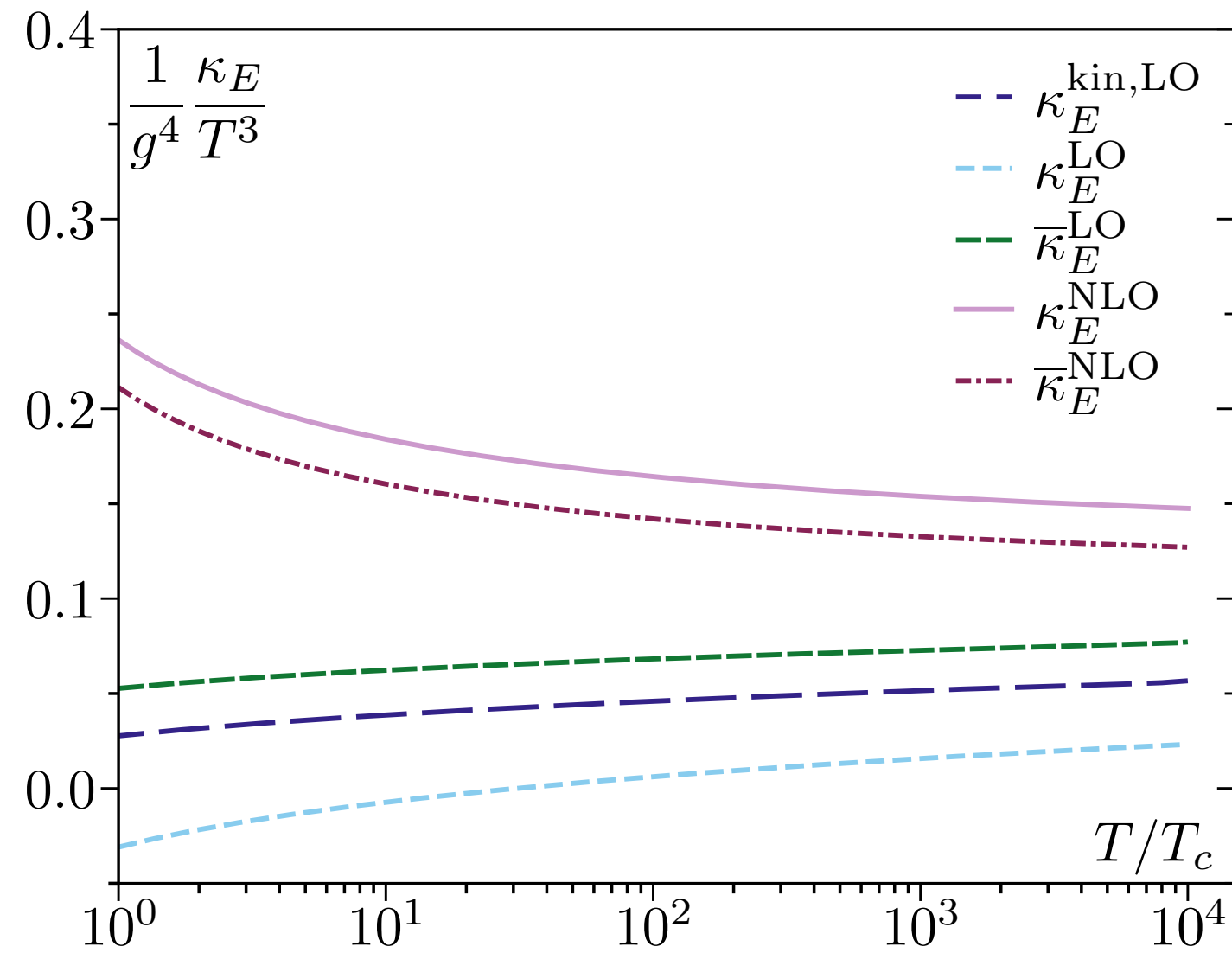
2+1 FLAVOR QCD

$T_c \simeq 154 \text{ MeV}$



PERTURBATIVE RESULTS FOR THE DIFFUSION COEFFICIENT

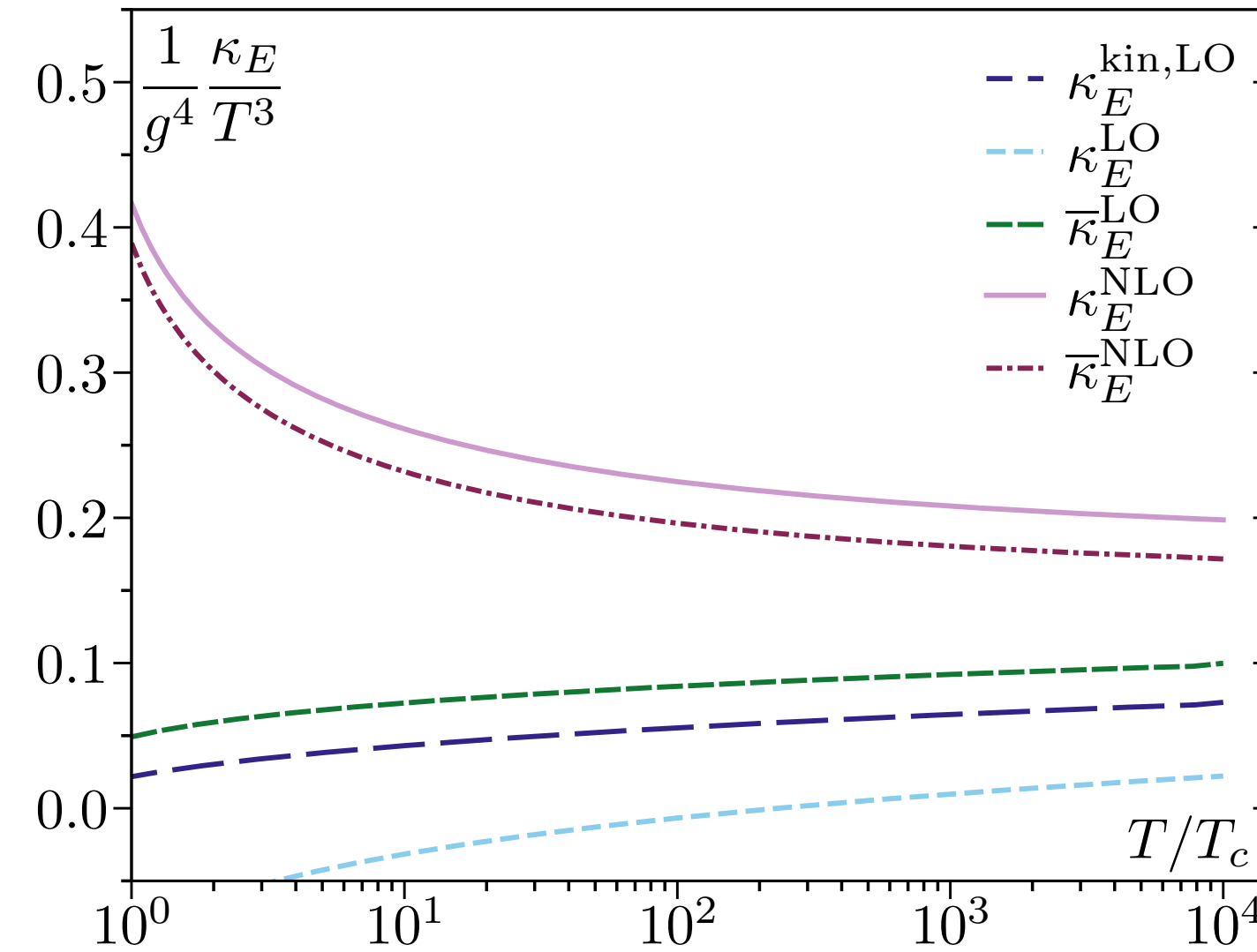
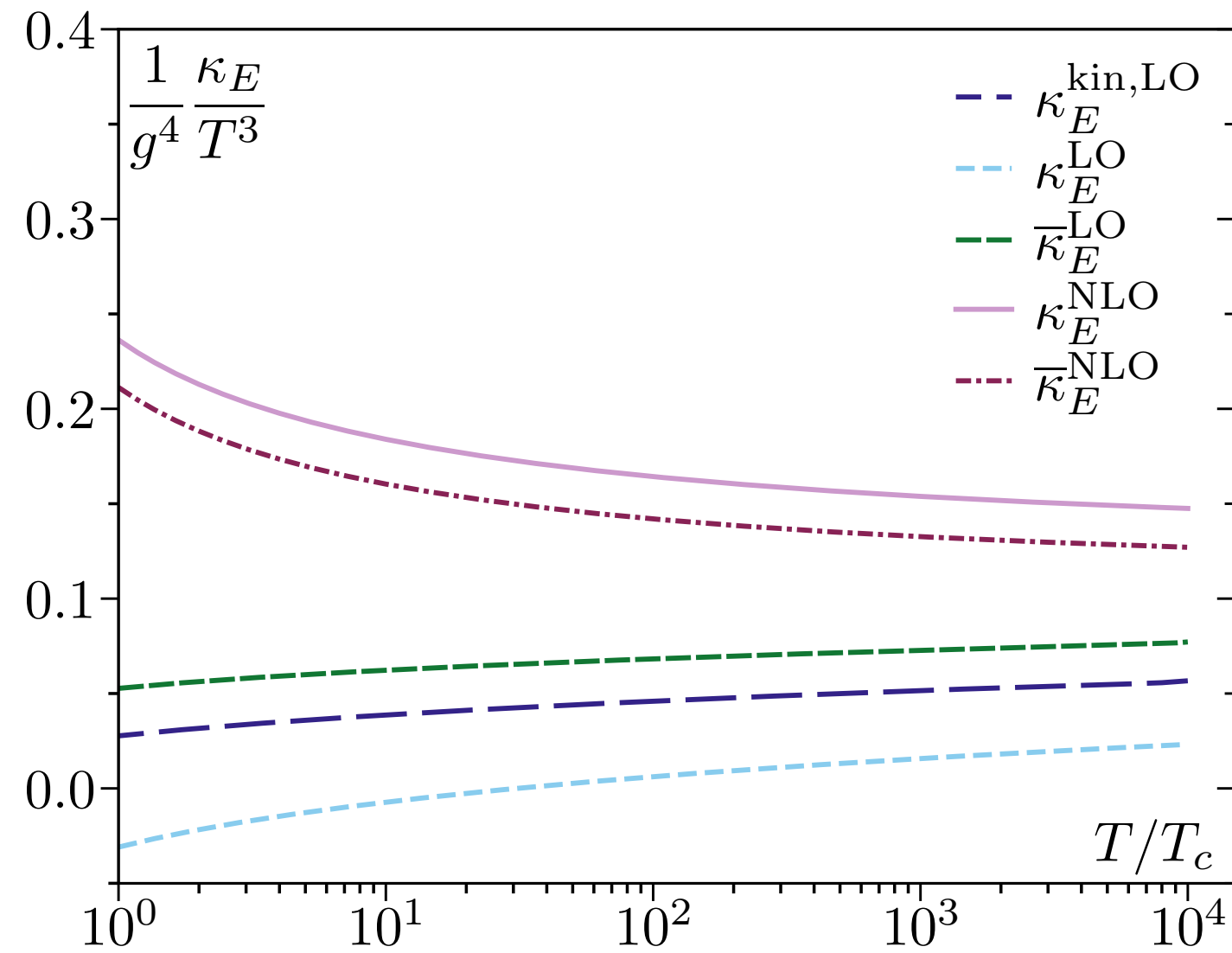
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$$\bar{\kappa}^{\text{LO}} \equiv \frac{g^4 C_F}{12\pi^3} \int_0^\infty dq q^2 \int_0^{2q} dp \frac{p^3}{(p^2 + \Pi_{00}(p, T))^2} \times \left\{ N_f n_F(q)[1 - n_F(q)] \left(2 - \frac{p^2}{2q^2} \right) N_c n_B(q)[n_B(q)] \left(2 - \frac{p^2}{q^2} + \frac{p^4}{4q^4} \right) \right\}$$

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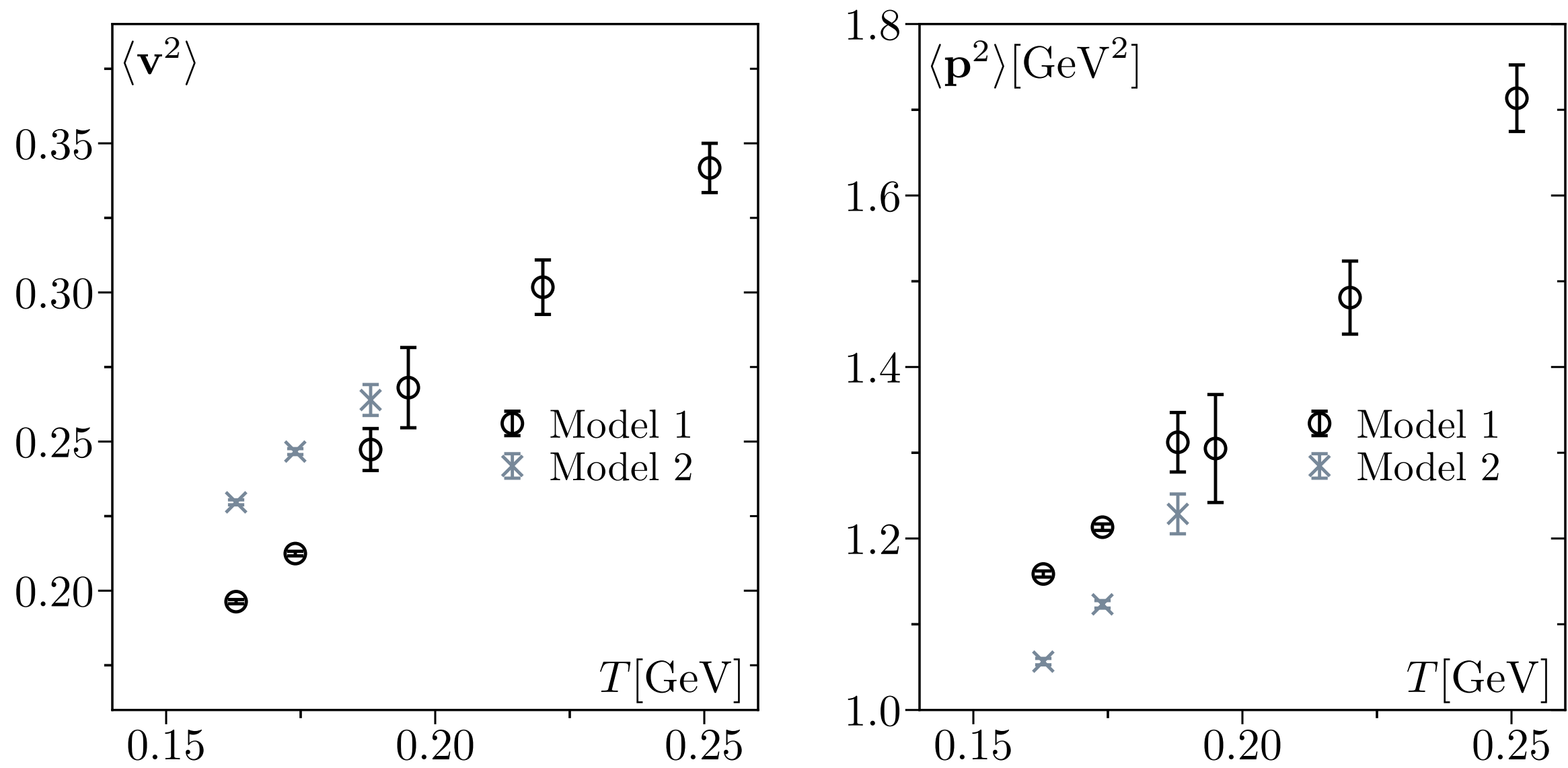
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CAN BE RESUMMED TO IMPROVE THE CONVERGENCE

HEAVY QUARK DIFFUSIONS RESULTS

QUASI-PARTICLE MODEL

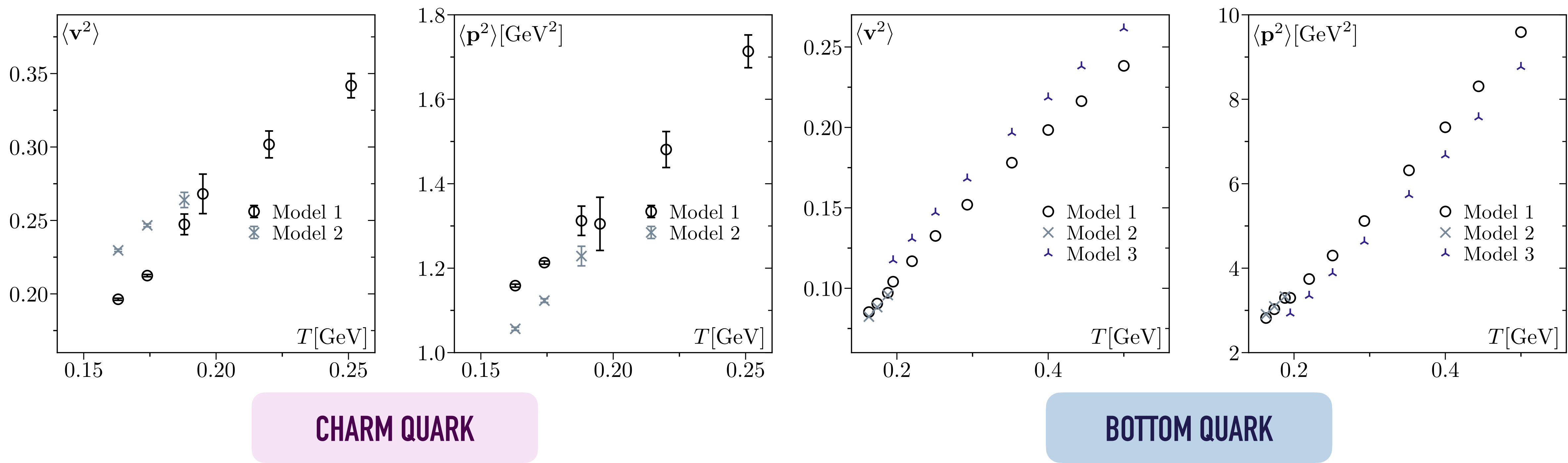
The thermal mean squared-velocity and momentum are then:



CHARM QUARK

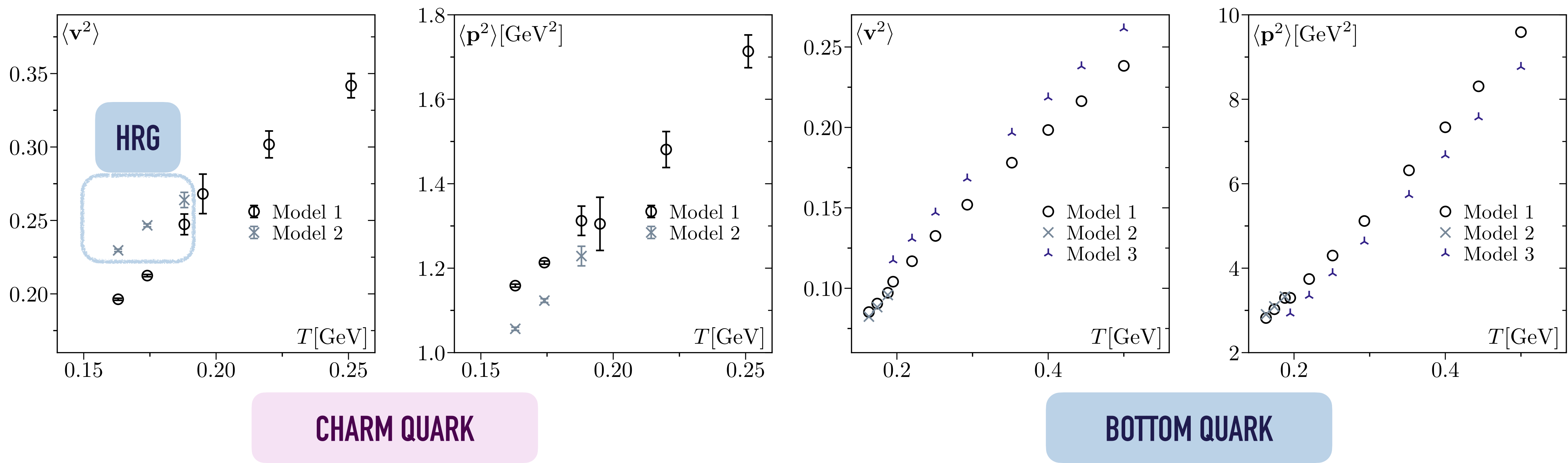
QUASI-PARTICLE MODEL

The thermal mean squared-velocity and momentum are then:



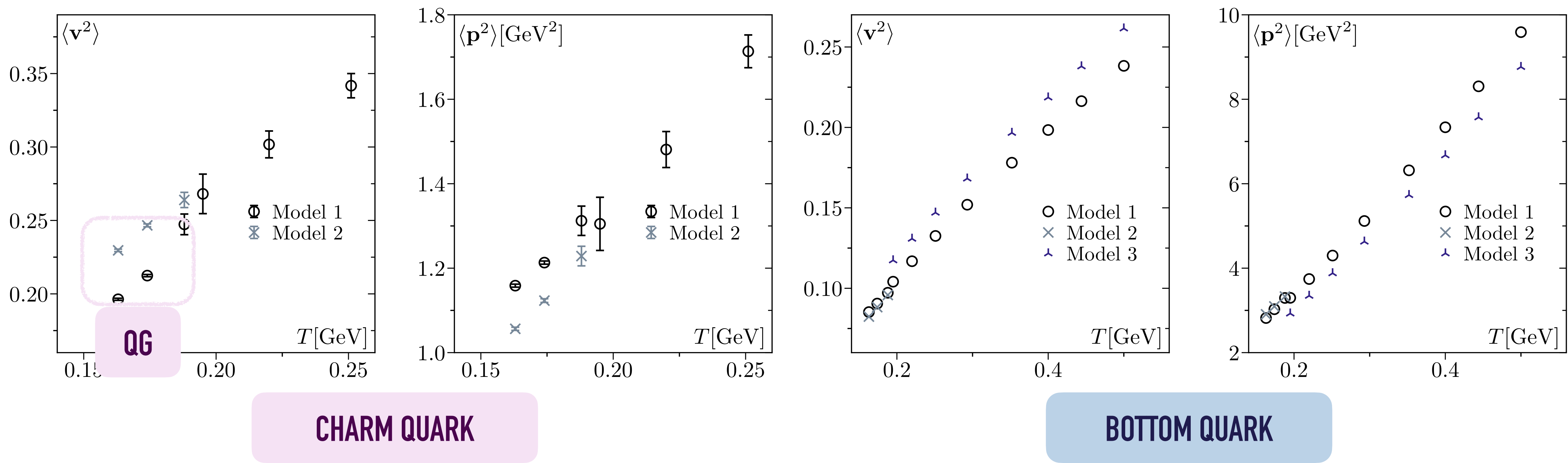
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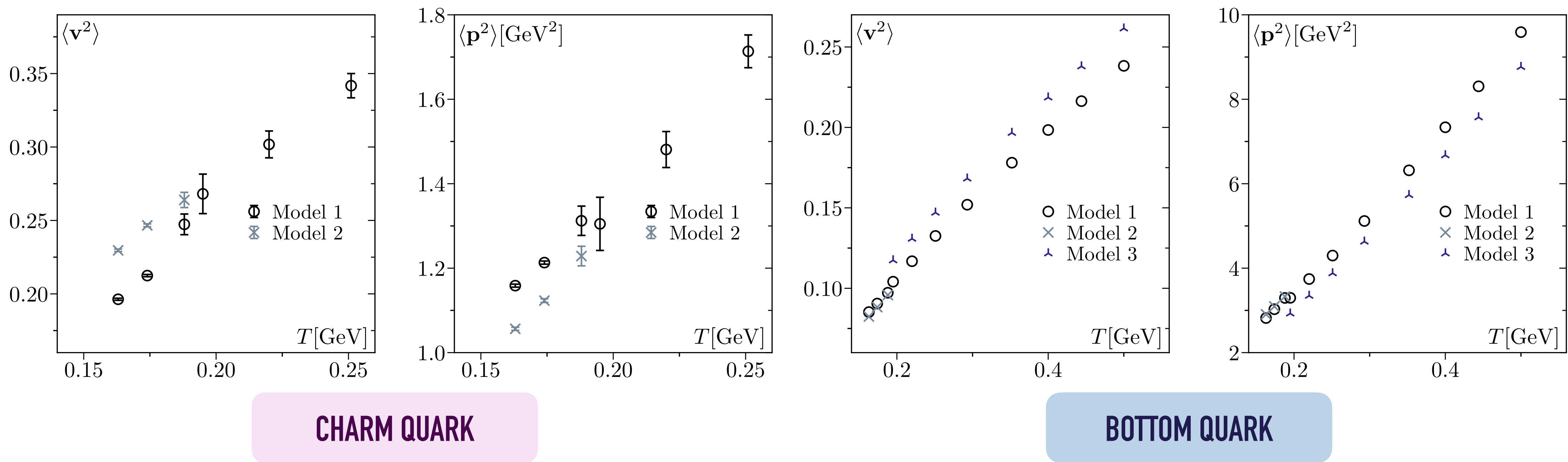
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To assess the sensitivity of our results to the modeling assumptions for $T < 190$ MeV