

Quantum vs. semiclassical description of $\bar{c}c$ -QGP quarkonia in the quantum Brownian regime

HEFTY seminar
BNL
January 7, 2026

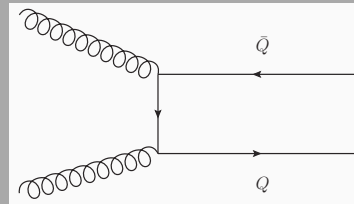
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Introductory remarks

Heavy quarks and quarkonia as 'hard probes' or 'test particles'

Heavy quarks are produced in pairs in the early stages of URHIC. Their number remains constant.



Formation time of a $Q\bar{Q}$ pair is small

$$\Delta t \sim \frac{1}{2M_Q}$$

$$J/\Psi \quad M_c \simeq 1.5 \text{ GeV}$$

$$\Delta t \simeq 0.07 \text{ fm/c}$$

$$\Upsilon \quad M_b \simeq 4.5 \text{ GeV}$$

$$\Delta t \simeq 0.02 \text{ fm/c}$$

Dynamics of heavy quarks is non-relativistic

$$\Upsilon \quad v_0^2 \sim 0.1$$

$$J/\Psi \quad v_0^2 \sim 0.3$$

$$H = \frac{p^2}{M_Q} + V(r) \quad \left(V(r) = \frac{\alpha_s}{r} + \sigma r \right)$$

The potential can be obtained using effective theory (pNRQCD)

[see N. Brambilla, A. Pineda, J. Soto, A. Vairo, NPB566 (2000) 275]

Heavy quark interaction at finite T

Mass is large compared to the typical temperature

$$M_Q \gg T$$

Initial suggestion (Matsui-Satz 86): screening of the potential

$$H = \frac{p^2}{M_Q} + V(r)$$
$$V(r) = -\frac{\alpha}{r} e^{-r m_D(T)} + \sigma(T)r$$

This picture predicts a "suppression" of bound states at high temperature, the most "fragile" ones (bigger, less bound) disappearing first as the temperature increases ("sequential suppression").

Hence the idea of using quarkonia to diagnose the formation of quark-gluon plasma in URHI

A nice idea....

A considerable experimental effort...

But a very difficult many-body problem !

- multifaceted, multi scale dynamics
- a plethora of theoretical approaches

What we need :

A robust and simple picture that encompasses in a coherent framework all the main features of the dynamics

An 'open quantum system'

$$H = H_Q + H_{pl} + H_{int}$$

$$H_{int} = J_Q \cdot A_{pl}$$

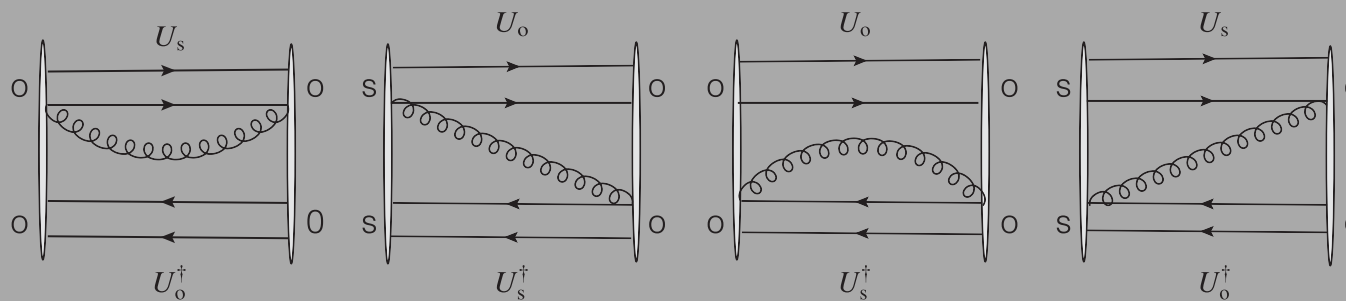
Heavy quark dynamics

- Reduced density matrix for heavy quarks

$$\mathcal{D}_Q(t) = \text{Tr}_{\text{pl}} \mathcal{D}(t)$$

- $\mathcal{D}_Q(t)$ obeys an equation of motion of the form

$$\frac{d}{dt} \mathcal{D}_Q(t) = -i[H_Q, \mathcal{D}_Q(t)] + \int_0^{t-t_0} d\tau \mathcal{L}(\tau) \mathcal{D}_Q(t - \tau)$$



- Non unitary dynamics (dissipation, transport, etc)

[For details, see JPB, M. Escobedo-Espinosa, [1711.10812](#), [1803.07996](#)]

$$\frac{d\mathcal{D}(t)}{dt} = \mathcal{L} [\mathcal{D}(t)] = \sum_{i=0}^4 \mathcal{L}_i \mathcal{D}$$

U
n
i
t
a
r
y

$$\mathcal{L}_0 \mathcal{D} \equiv -i [H, \mathcal{D}]$$

$$\mathcal{L}_1 \mathcal{D} \equiv -\frac{i}{2} \int_{xx'} V(x-x') [n_x n_{x'}, \mathcal{D}]$$

N
o
n

$$\mathcal{L}_2 \mathcal{D} \equiv \frac{1}{2} \int_{xx'} W(x-x') \left(\{n_x n_{x'}, \mathcal{D}\} - 2n_x \mathcal{D} n_{x'} \right)$$

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$$\mathcal{L}_3 \mathcal{D} \equiv -\frac{i}{8T} \int_{xx'} W(x-x') \left(\left\{ \mathcal{D}, [\dot{n}_{x'}, n_x] \right\} + 2\dot{n}_{x'} \mathcal{D} n_x - 2n_x \mathcal{D} \dot{n}_{x'} \right)$$

$$\mathcal{L}_4 \mathcal{D} \equiv \frac{1}{32T^2} \int_{xx'} W(x-x') \left(\{ \dot{n}_x \dot{n}_{x'}, \mathcal{D} \} - 2\dot{n}_x \mathcal{D} \dot{n}_{x'} \right)$$

Typical approximations

(i) Weak coupling between HQ and the plasma

$$H_1 = -g \int_{\mathbf{r}} A_0^a(\mathbf{r}) n^a(\mathbf{r}),$$

gauge potential of plasma HQ density

$$n^a(\mathbf{x}) = \delta(\mathbf{x} - \hat{\mathbf{r}}) t^a \otimes \mathbb{I} - \mathbb{I} \otimes \delta(\mathbf{x} - \hat{\mathbf{r}}) \tilde{t}^a,$$

- The presence of the heavy quarks does not modify significantly the equilibrium state of the plasma.
- The influence of the plasma on the heavy quark dynamics is characterized by simple response functions (correlators)

$$\Delta(t_1, t_2) \equiv \langle A_{\text{pl}}(t_1) A_{\text{pl}}(t_2) \rangle_T = \text{Tr} \left[A_{\text{pl}}(t_1) A_{\text{pl}}(t_2) \mathcal{D}_{\text{pl}} \right]$$

- No weak or strong coupling assumption needs to be made concerning the plasma. The correlators can, in some cases, be obtained from lattice calculations.

(ii) The response of the plasma is "fast"

(Key to obtain a Markovian approximation)

- plasma response characterized by a single energy scale, the Debye mass

$$m_D = CT \quad (C \simeq 2) \quad \text{in strict weak coupling} \quad C = g$$

$$m_D \ll M$$

- collisions with plasma constituents involve small energy transfer

soft gluon exchanges

$$q \lesssim m_D \ll M$$

small energy transfer

$$\frac{q^2}{M} \sim \frac{m_D^2}{M} \ll m_D$$

- the relevant correlator is then generically of the form

$$\Delta(\omega = 0, \mathbf{r}) = \Delta^R(\omega = 0, \mathbf{r}) + i\Delta^<(\omega = 0, \mathbf{r})$$

$$V(\mathbf{r}) = -\Delta^R(\omega = 0, \mathbf{r}), \quad W(\mathbf{r}) = -\Delta^<(\omega = 0, \mathbf{r})$$

Imaginary potential

Static response and "Optical potential"

(*first obtained by M. Laine et al hep-ph/ 0611300)

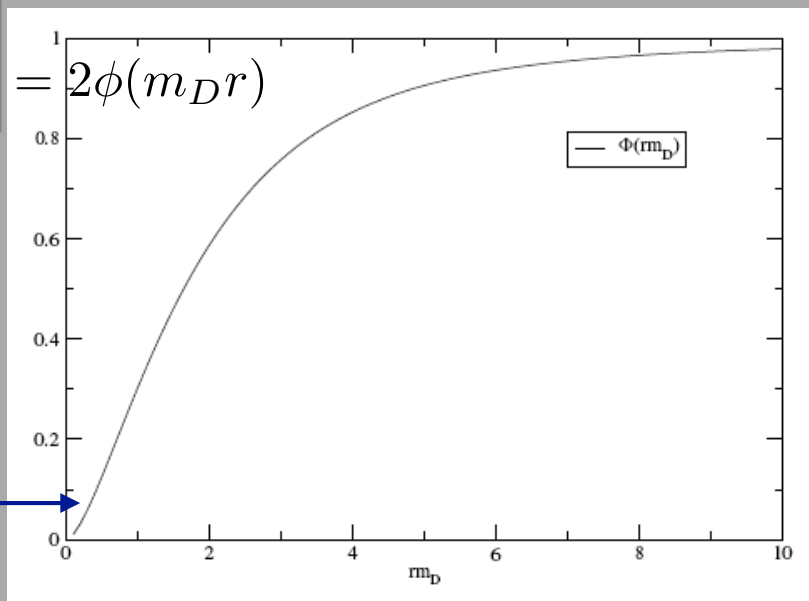
$$\mathcal{V}(r) = V(r) + iW(r)$$

$$\Delta^R(\omega = 0, r) = -V(r)$$

$$\Delta^<(\omega = 0, r) = -W(r)$$

$$\Gamma(r) = W(r) - W(0) = 2\phi(m_D r)$$

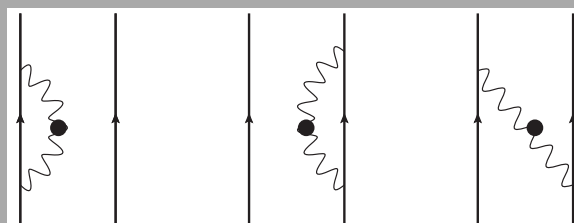
$$\Phi(x) = \frac{x^2}{3} \left(-\ln x + \frac{4}{3} - \gamma_E \right)$$



At large distance the imaginary part is twice the damping rate of the heavy quark

makes density matrix nearly diagonal in coordinate space

At short distance, interference produces cancellation: a small dipole does not "see" the electric field fluctuations.



For one heavy quark

$$\partial_t \langle r | \mathcal{D}_Q | r' \rangle = \dots - \Gamma(r - r') \langle r | \mathcal{D}_Q | r' \rangle$$

(iii) semi-classical approximation

$$M \gg T$$

HQ thermal wavelength $\lambda_{\text{th}} \sim \frac{1}{\sqrt{MT}} \ll \frac{1}{T}$

Density matrix becomes nearly diagonal

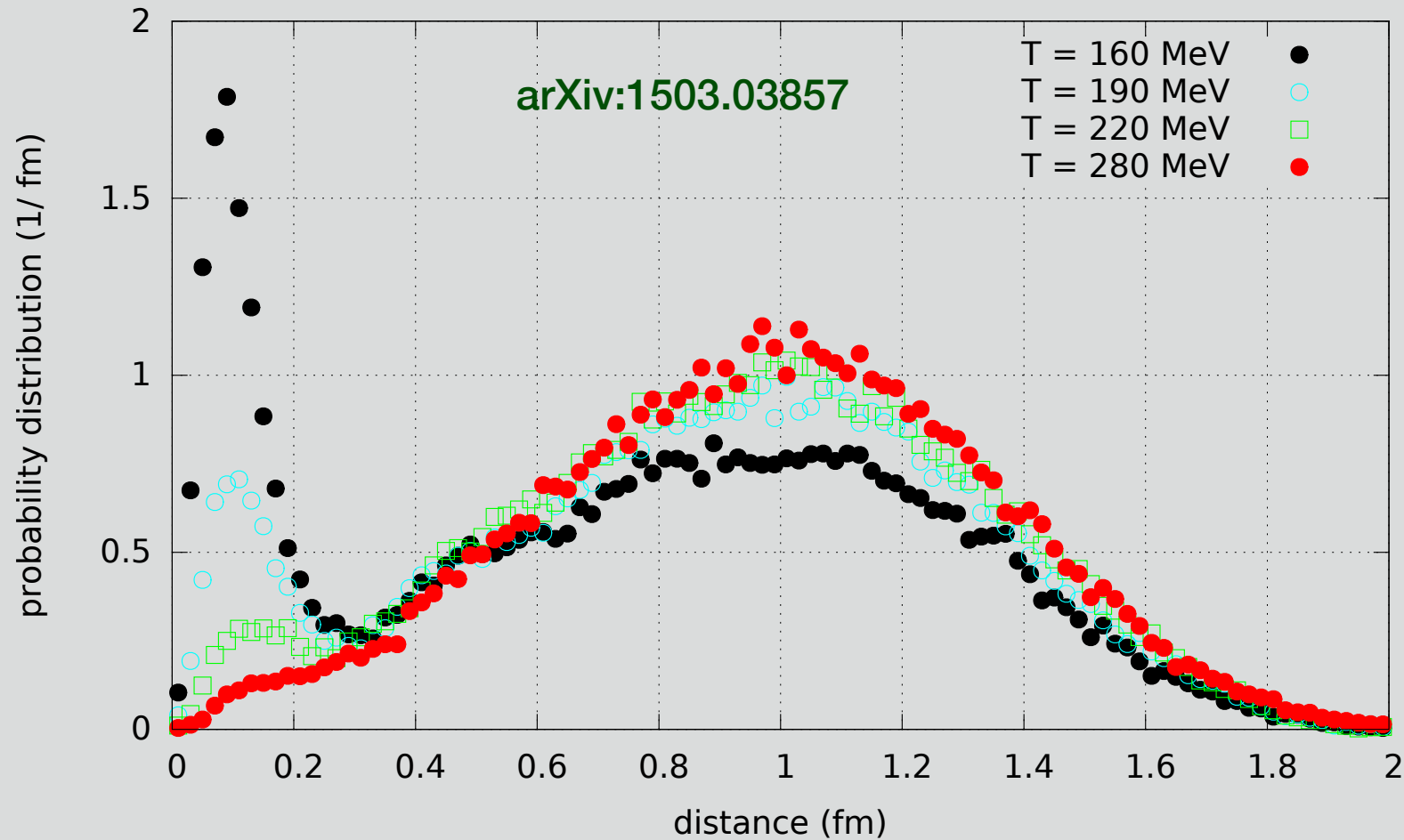
$$\langle \mathbf{r} | \mathcal{D}_Q | \mathbf{r}' \rangle \simeq 0 \quad \text{when} \quad |\mathbf{r} - \mathbf{r}'| \gtrsim \lambda_{\text{th}}$$

Expansion in $|\mathbf{r} - \mathbf{r}'|$  Fokker-Planck and Langevin equations

All three approximations have been implemented successfully in the abelian context. See e.g. JPB, D. De Boni, P Faccioli, G. Garberoglio, NPA 946 (216) 49 [arXiv:1503.03857]

Probability distribution of distance to nearest neighbour

[10 cc pairs in a 4 fm cubic box]



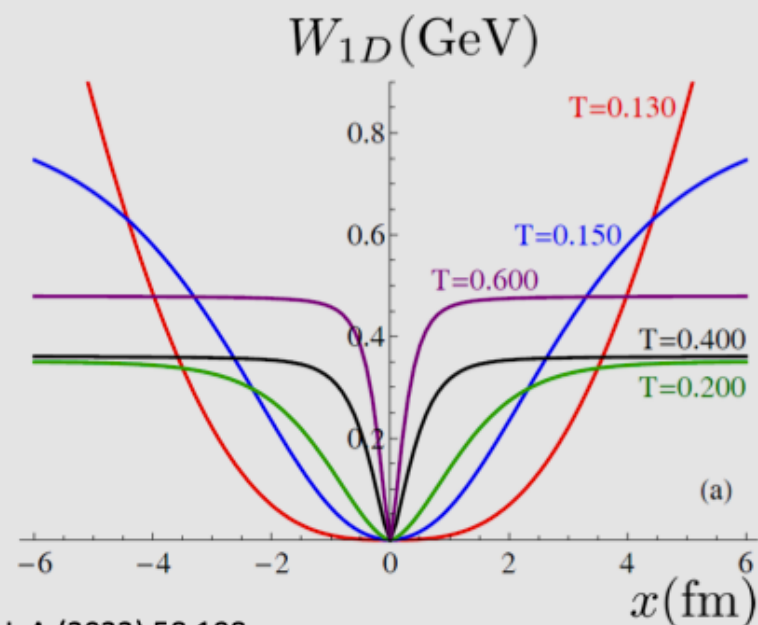
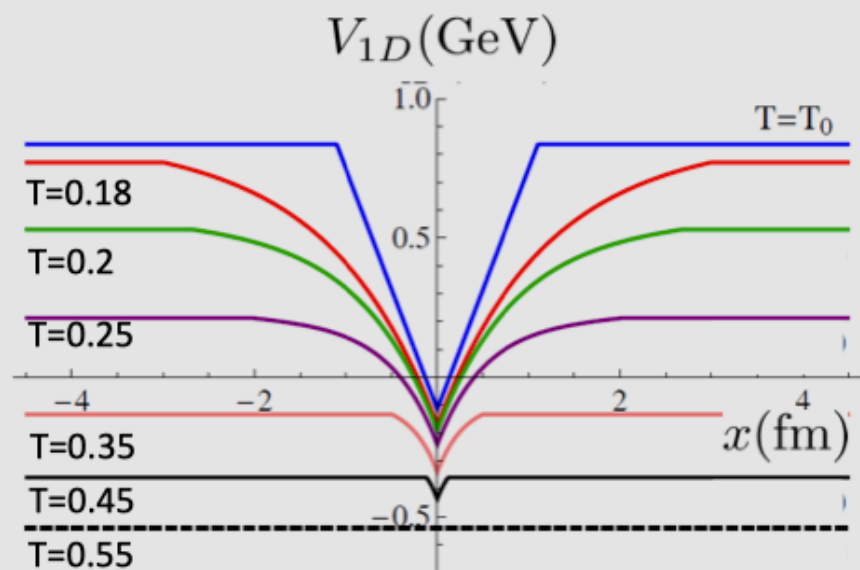
$$P_{q\bar{q}}^{\text{ideal}}(r) = \frac{3}{a} \left(\frac{r}{a} \right)^2 \left(1 - \left(\frac{r}{a} \right)^3 \frac{1}{N} \right)^{N-1} \stackrel{N \gg 1}{\simeq} \frac{3}{a} \left(\frac{r}{a} \right)^2 e^{-(r/a)^{\frac{1}{3}}} \quad a = \left(\frac{3}{4\pi\rho} \right)^{1/3}$$

*Semi-classical approaches
for abelian plasmas
(one-dimensional setting)*

Based on 2506.19194, 2402.04488

A model potential in 1D

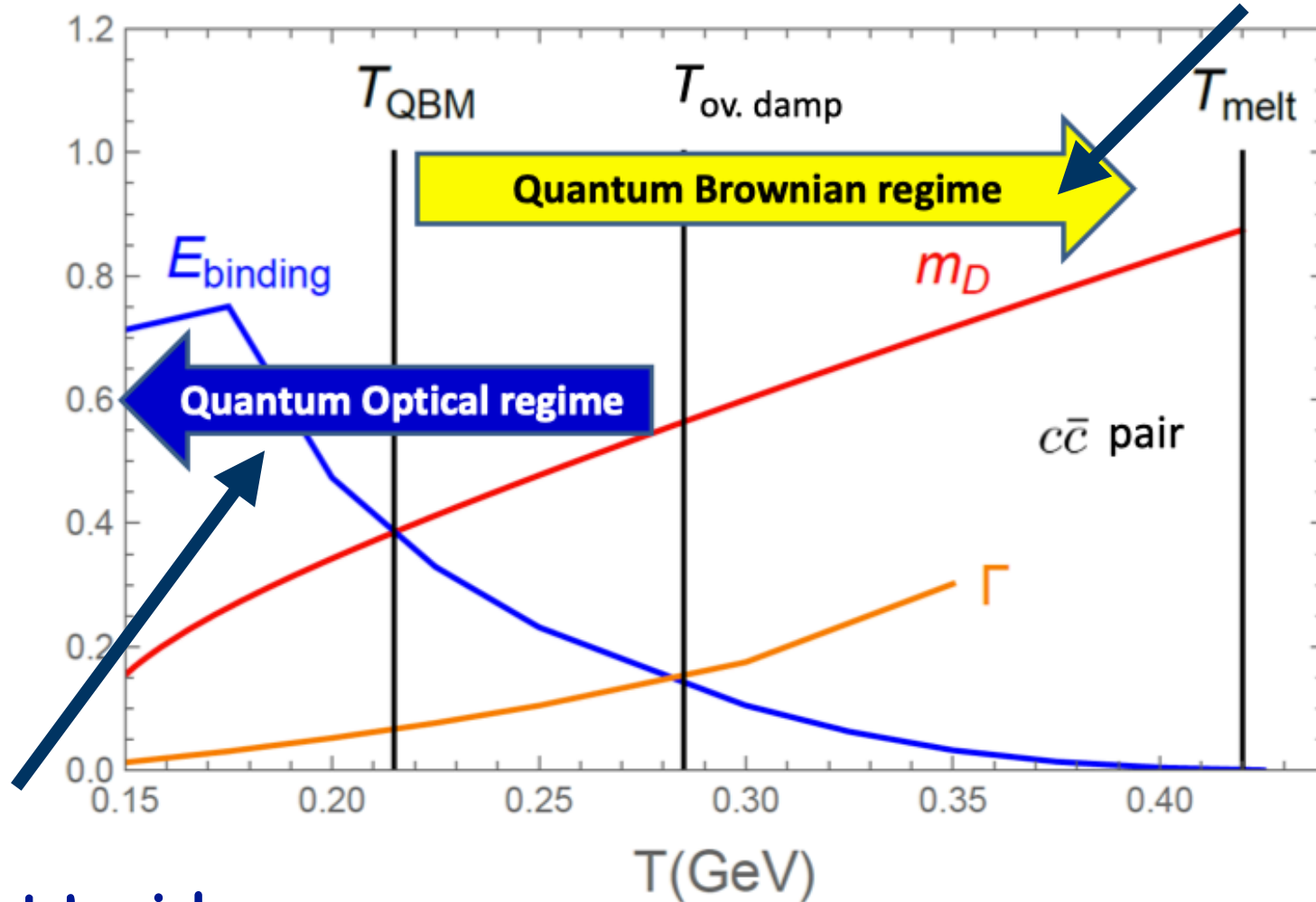
Need to design a realistic 1D bona fide potential $V + iW$ (based on 3D IQCD results, tuned to reproduce 3D mass spectra and decay widths)



1D potential from R. Katz, S. Delorme & PBG, Eur. Phys. J. A (2022) 58:198

Typical regimes in QQS

'Natural basis':
spatial coordinates



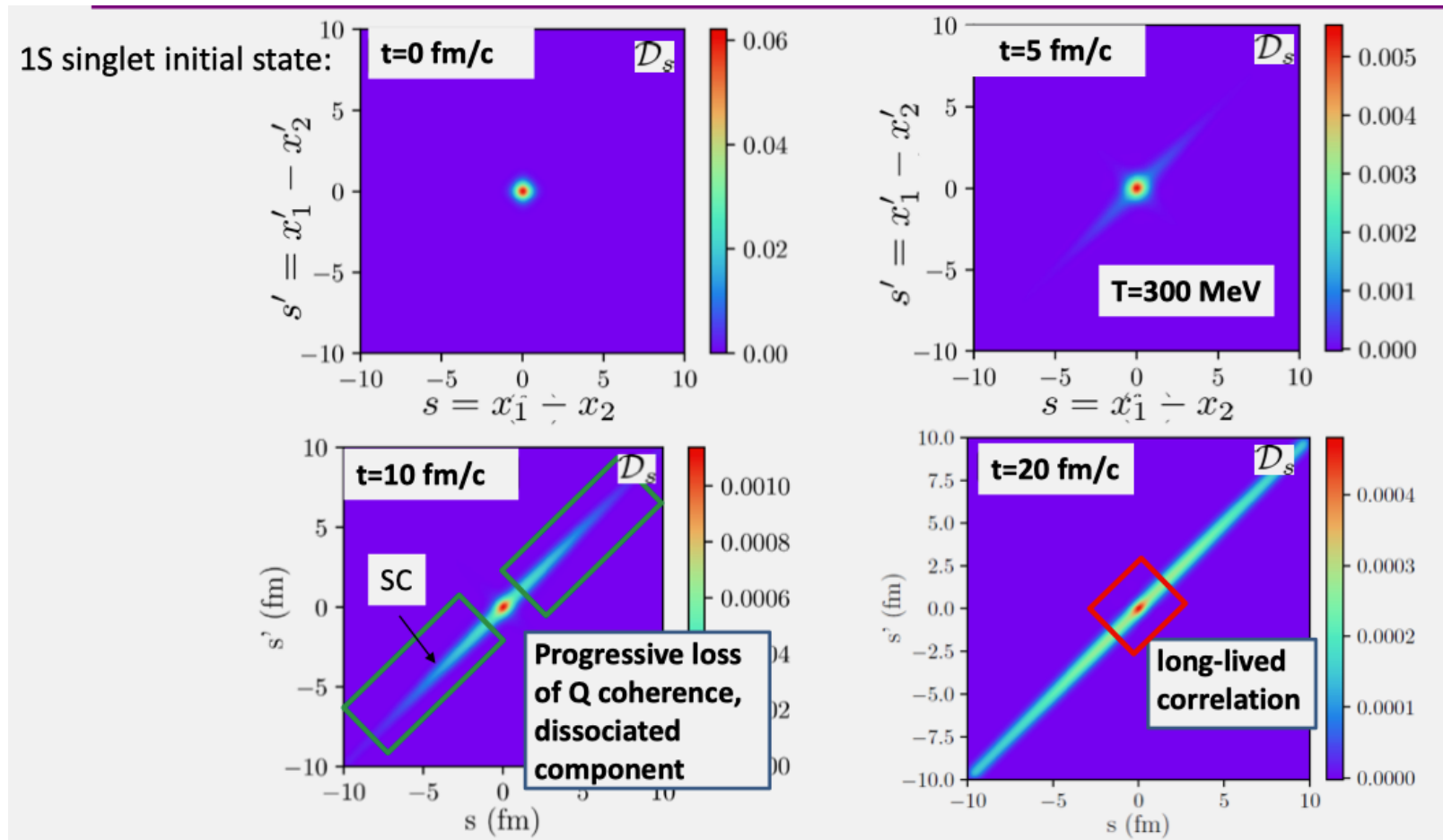
'Natural basis':
stationary states

Typical evolution of the density matrix

Reduced density matrix of a $Q\bar{Q}$ pair: $\langle x_1, x_2 | \mathcal{D} | x'_1, x'_2 \rangle = \mathcal{D}(x_1, x_2; x'_1, x'_2)$

$$\mathcal{D}(x_1, x_2; x'_1, x'_2) \mapsto \mathcal{D}(s, s')$$

$$s = x_1 - x_2 \quad s' = x'_1 - x'_2$$



More on the semi-classical approximation

In the QBM regime $\mathcal{D}(s, s')$ is nearly diagonal

$$r = \frac{s + s'}{2} \quad y = s - s'$$

'Almost' a phase space distribution

$$\mathcal{D}(s, s') \mapsto \mathcal{D}(r, y) \mapsto \mathcal{D}(r, p)$$

Wigner transform

$$(y, \nabla_y) \mapsto (p, \nabla_p)$$

$$\frac{\partial}{\partial t} \mathcal{D}(r, p) = \left[-2 \frac{\vec{p}}{M} \cdot \nabla_r - \nabla_r V \cdot \nabla_p + \frac{\eta(r)}{2} \nabla_p^2 + 2 \frac{\gamma(r)}{M} \nabla_p \cdot \vec{p} \right] \mathcal{D}(r, p)$$

unitary dynamics

fluctuations
decoherence

dissipation

$$\mathcal{L}_0 + \mathcal{L}_1$$

$$\mathcal{L}_2$$

$$\mathcal{L}_3$$

The quantities $\eta(r)$ and $\gamma(r)$ are related to the plasma correlators

Connection to the underlying master equation

$$\frac{d\mathcal{D}(t)}{dt} = \mathcal{L}[\mathcal{D}(t)] = \sum_{i=0}^4 \mathcal{L}_i \mathcal{D}$$

$$\mathcal{L}_0 \mathcal{D} \equiv -i[H, \mathcal{D}]$$

unitary dynamics

$$\mathcal{L}_1 \mathcal{D} \equiv -\frac{i}{2} \int_{xx'} V(x-x') [n_x n_{x'}, \mathcal{D}]$$

fluctuations
decoherence

$$\mathcal{L}_2 \mathcal{D} \equiv \frac{1}{2} \int_{xx'} W(x-x') \left(\{n_x n_{x'}, \mathcal{D}\} - 2n_x \mathcal{D} n_{x'} \right)$$

dissipation

$$\mathcal{L}_3 \mathcal{D} \equiv -\frac{i}{8T} \int_{xx'} W(x-x') \left(\{ \mathcal{D}, [\dot{n}_{x'}, n_x] \} + 2\dot{n}_{x'} \mathcal{D} n_x - 2n_x \mathcal{D} \dot{n}_{x'} \right)$$

positivity issues

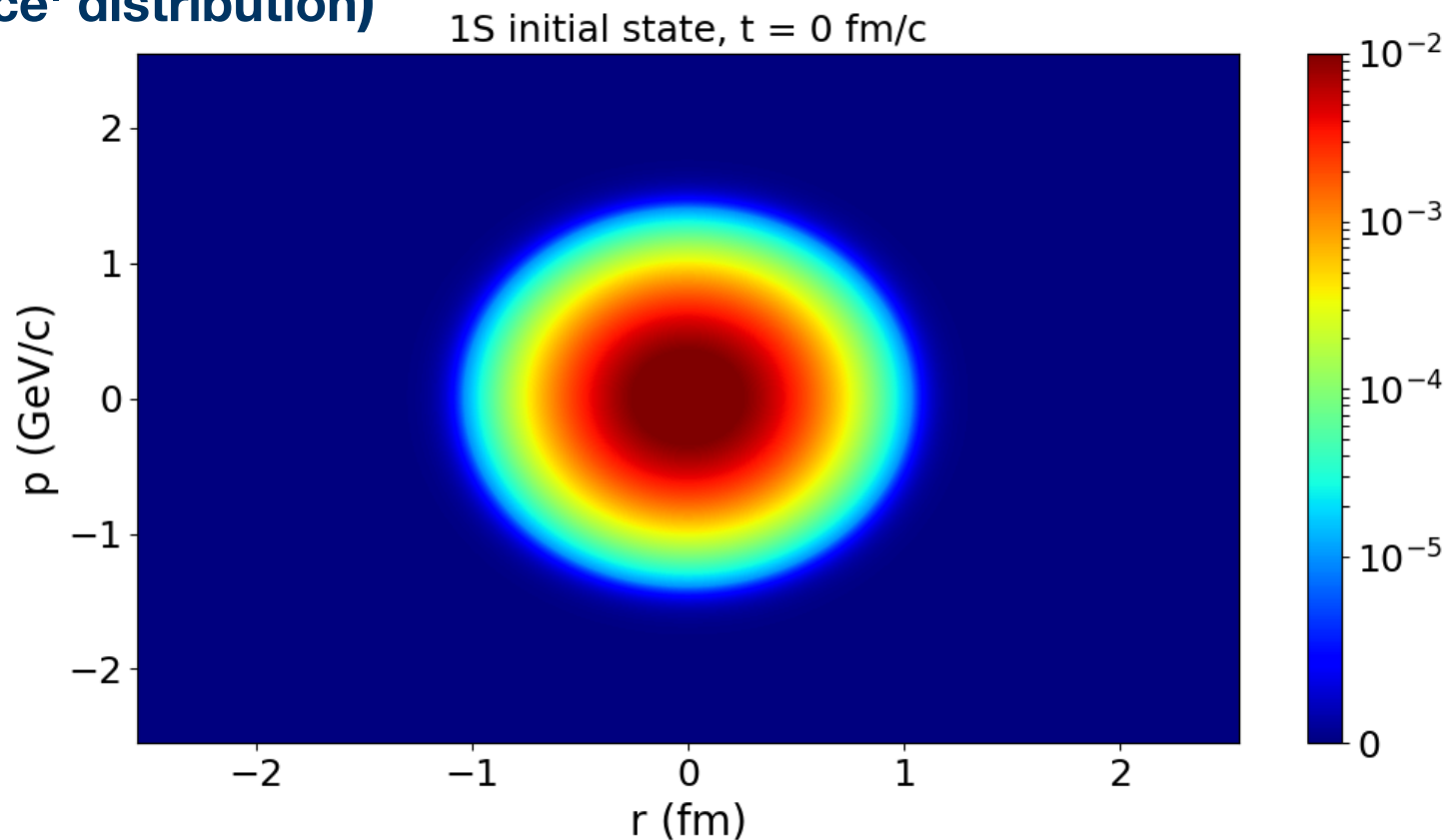
$$\mathcal{L}_4 \mathcal{D} \equiv \frac{1}{32T^2} \int_{xx'} W(x-x') \left(\{ \dot{n}_x \dot{n}_{x'}, \mathcal{D} \} - 2\dot{n}_x \mathcal{D} \dot{n}_{x'} \right)$$

The next figures, taken from [arXiv:2506.19194](#), will illustrate the difference between the evolution of an initial wave packet via the (Lindblad) master equation and its semi-classical approximation (Fokker-Planck)

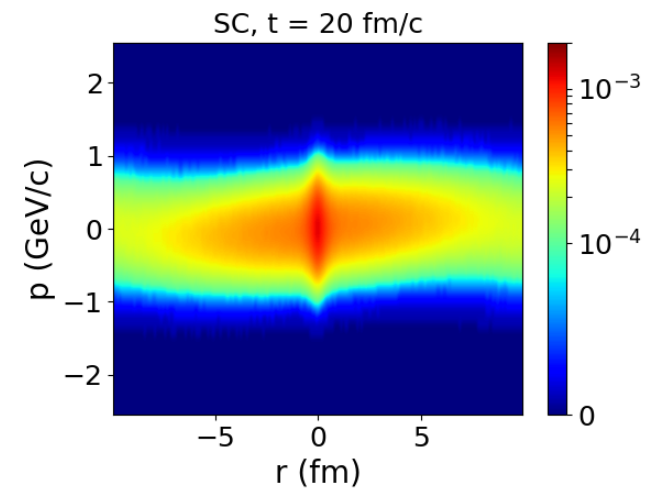
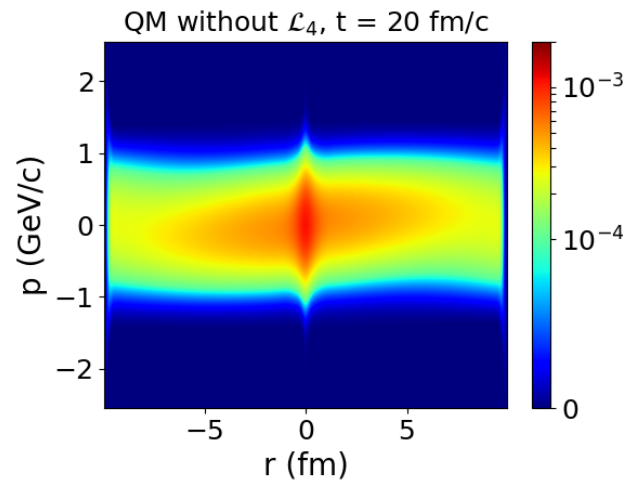
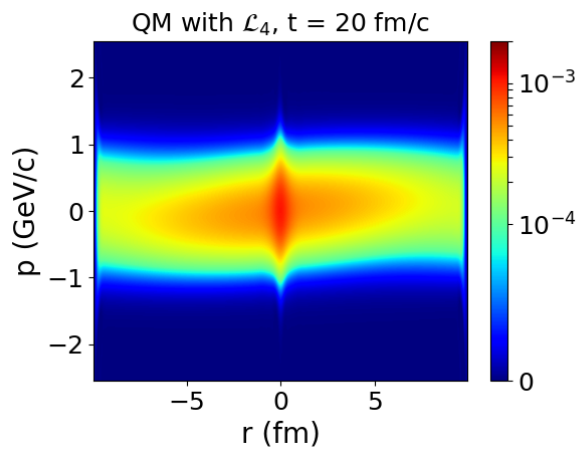
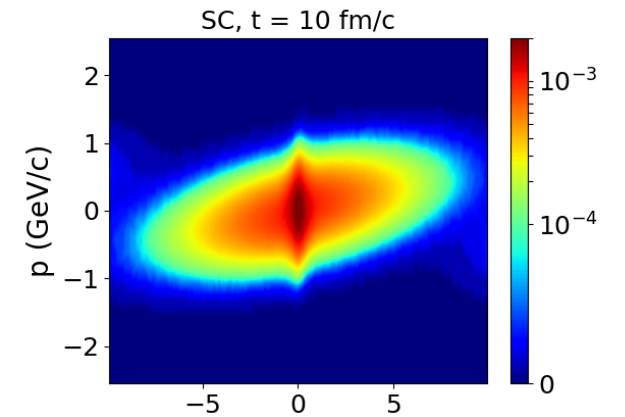
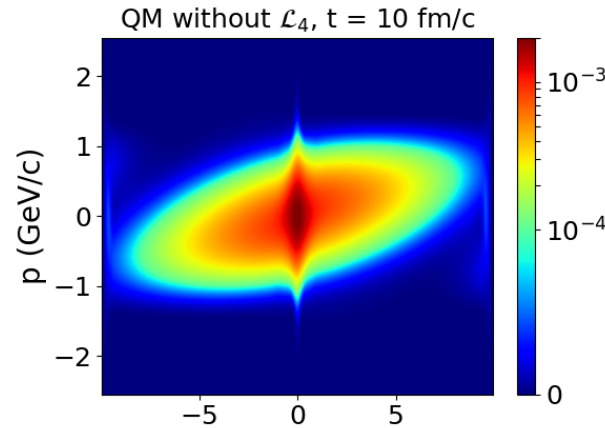
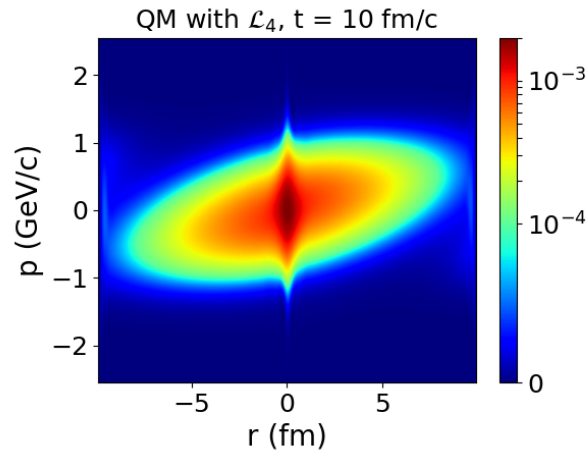
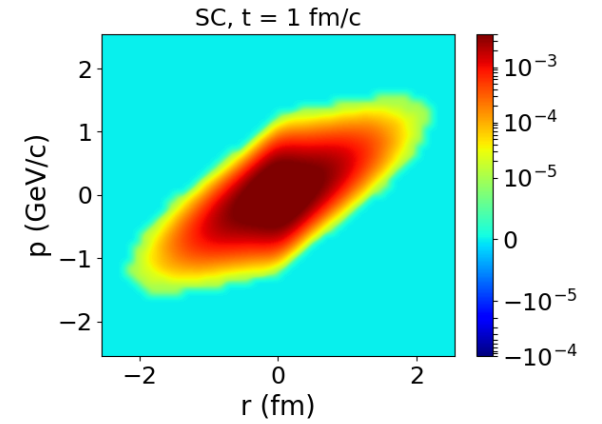
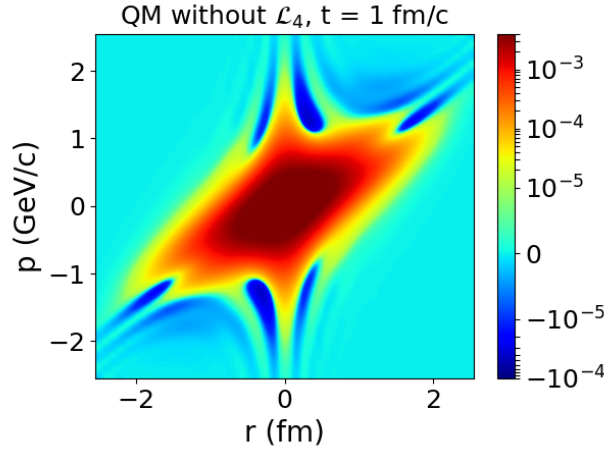
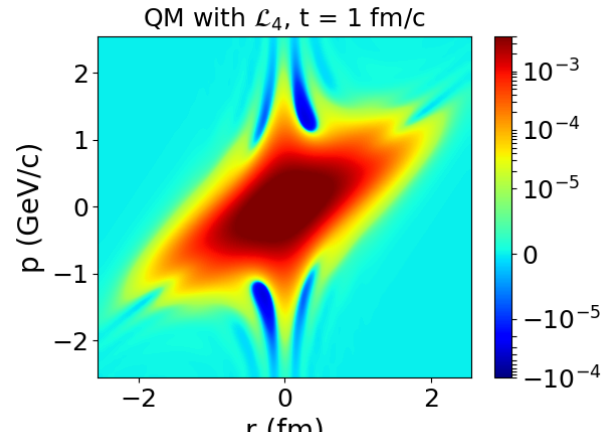
Initial wave packet

$$\psi(x) = \left(\frac{1}{\pi\sigma^2} \right)^{1/4} e^{-\frac{x^2}{2\sigma^2}} \quad (\sigma = 0.38 \text{ fm})$$

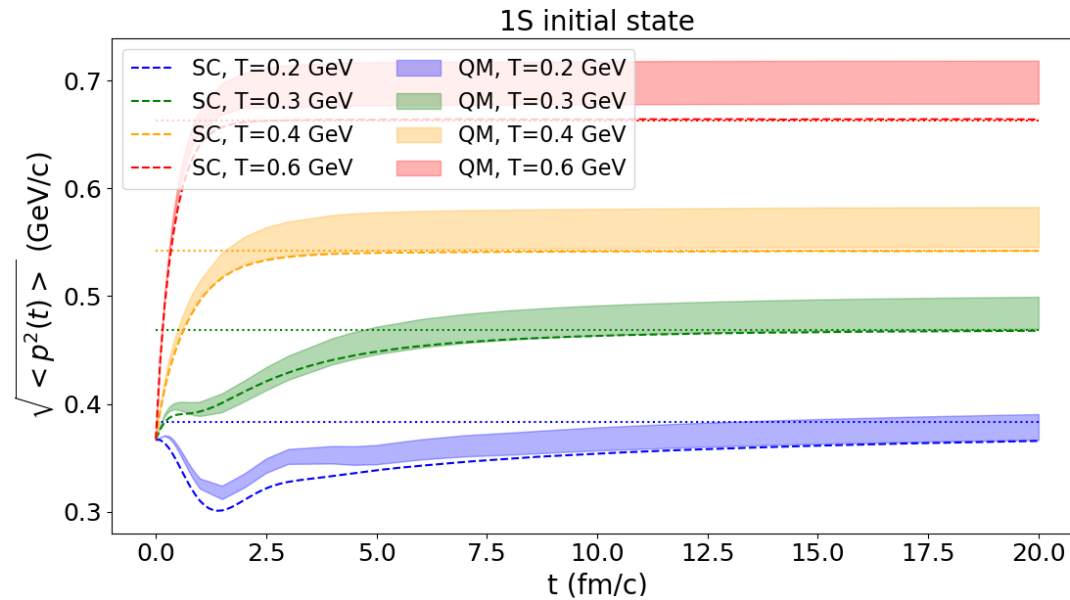
Wigner distribution
(**'phase space'** distribution)



Evolution in QGP with $T=300$ MeV

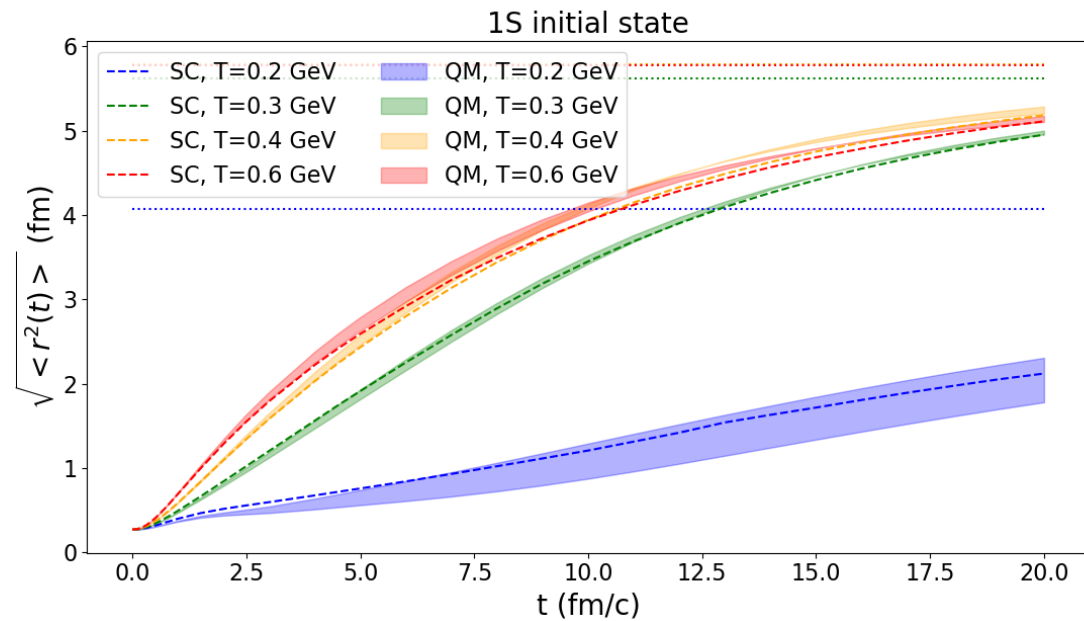


Aspects of thermalisation



**Semi-classical limit
is Boltzmann**

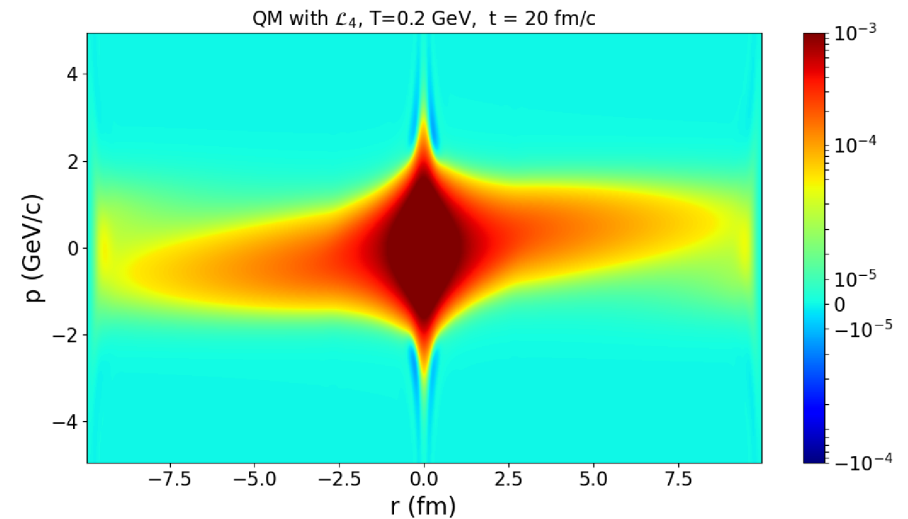
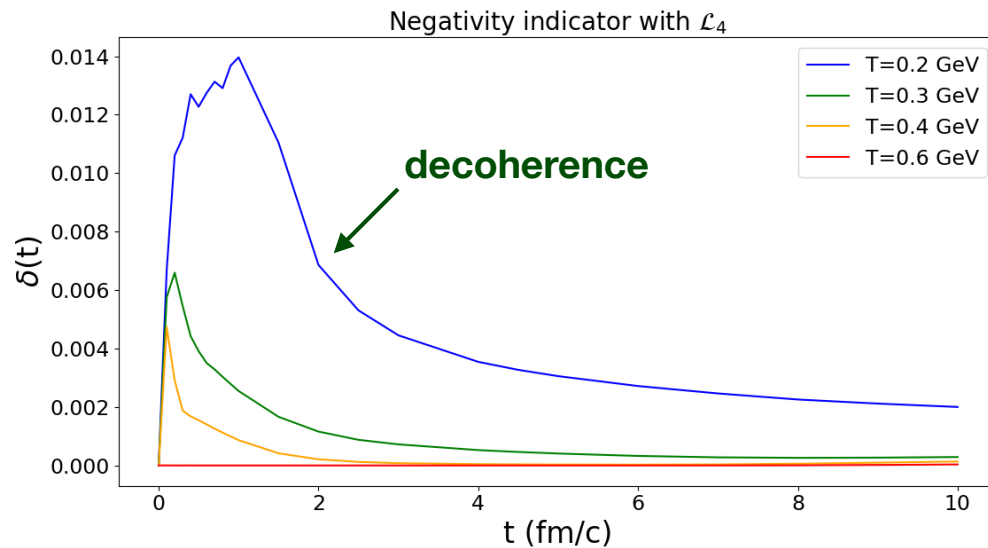
$$\sqrt{\frac{MT}{2}}$$



Looking at finer details

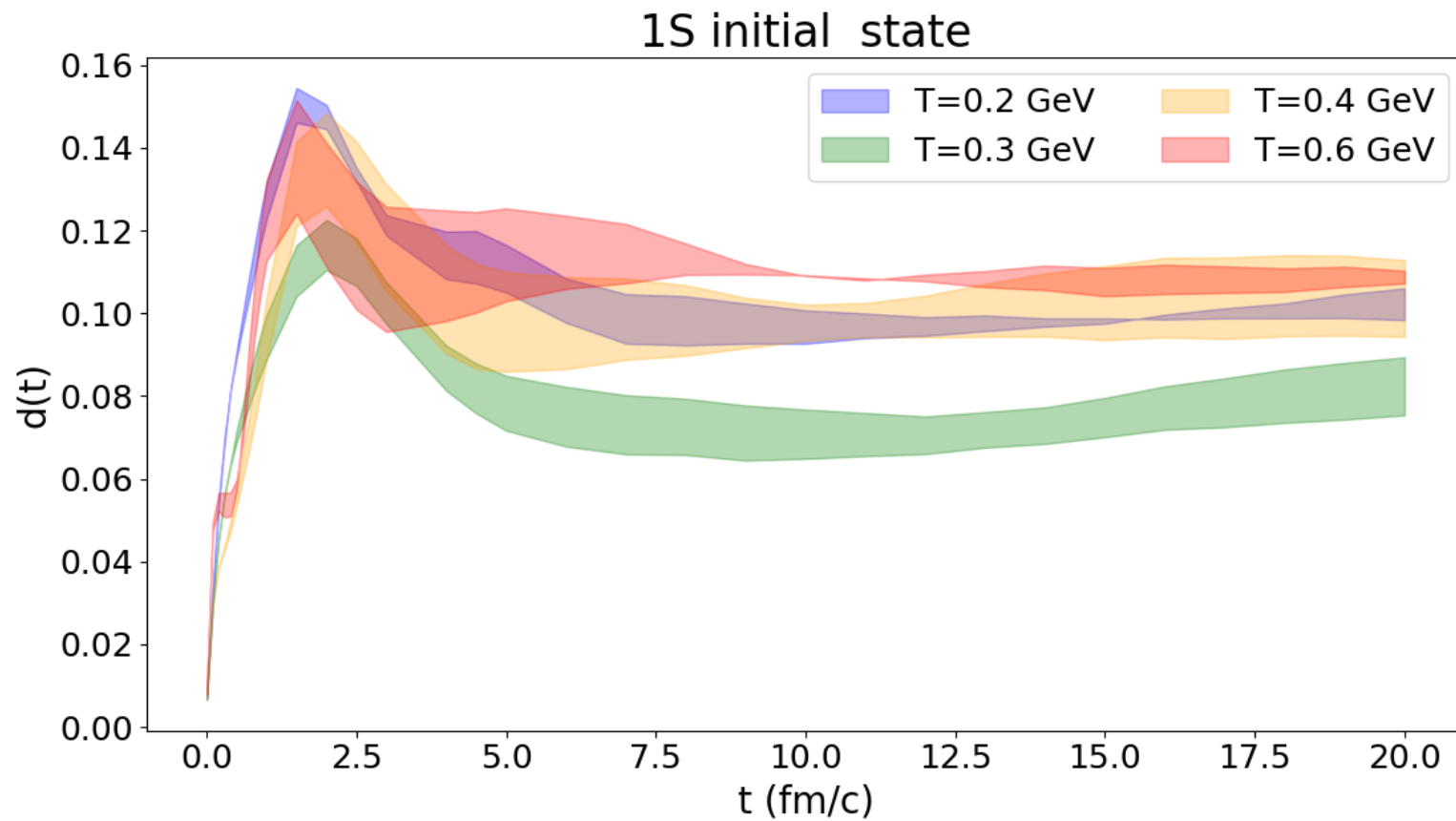
$$\delta(t) = \frac{1}{2\pi\hbar} \int_{r,p} \left(\left| \mathcal{D}(r,p,t) \right| - \mathcal{D}(r,p,t) \right) = \frac{1}{2\pi\hbar} \int_{r,p} \left| \mathcal{D}(r,p,t) \right| - 1.$$

$$\mathcal{D} > 0, \forall t \Rightarrow \delta(t) = 0$$

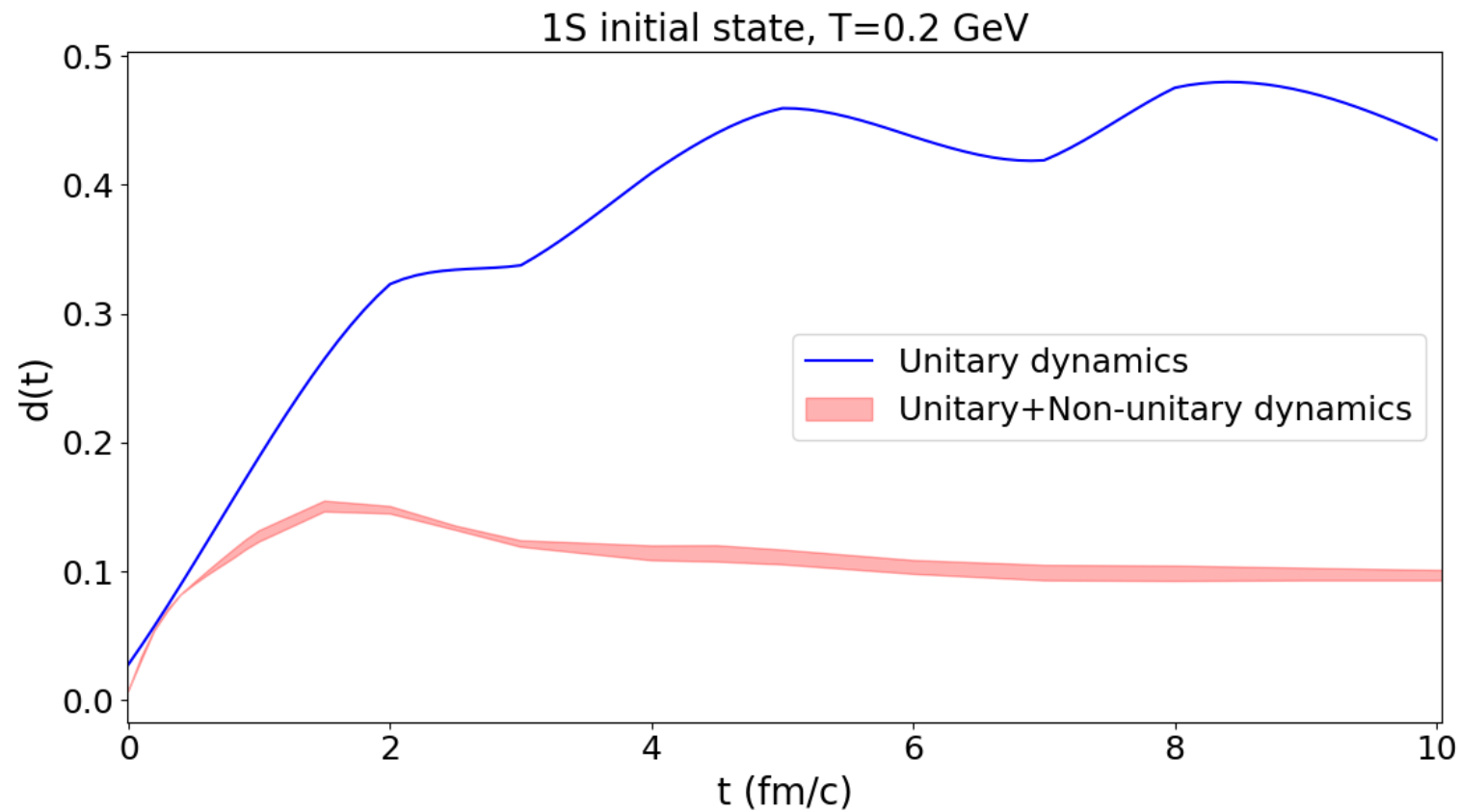


Looking at finer details

$$d(t) = \int \frac{\mathbf{d}r \mathbf{d}p}{2\pi\hbar} \left| \mathcal{D}_{\text{QM}}(t, r, p) - \mathcal{D}_{\text{SC}}(t, r, p) \right|$$

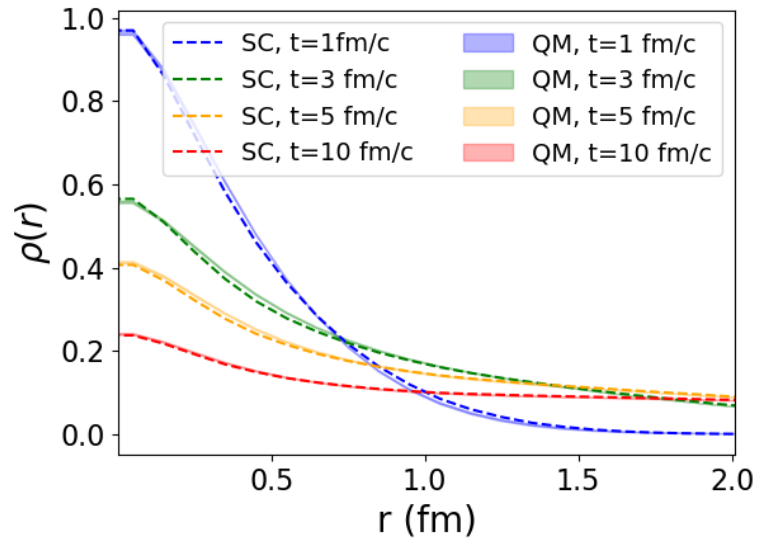


Impact of the non-unitary dynamics

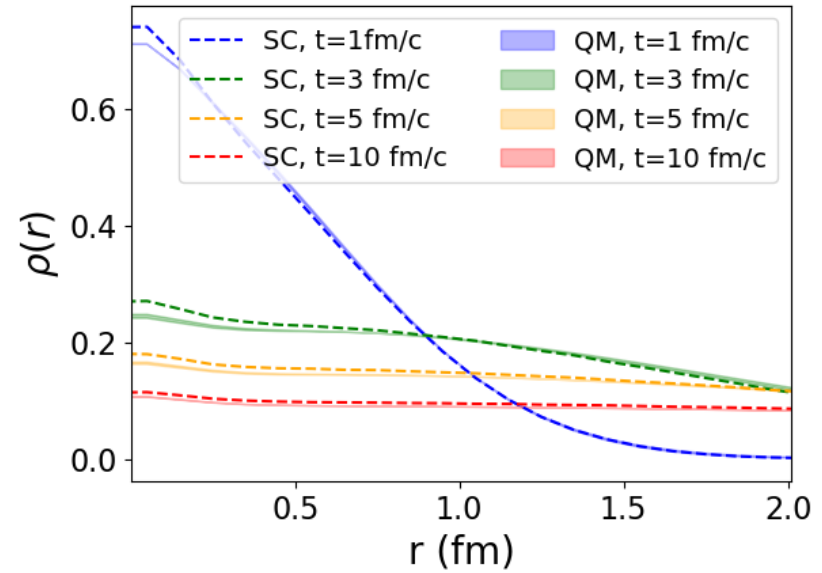


Impact of the temperature

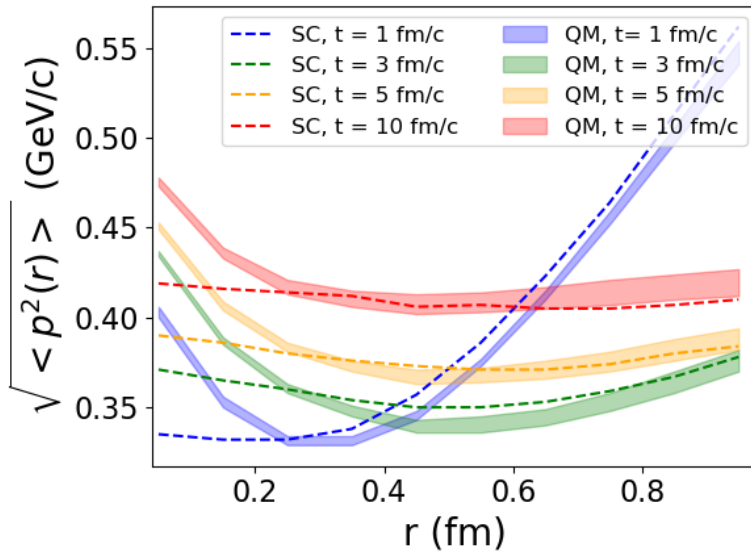
T=0.3 GeV



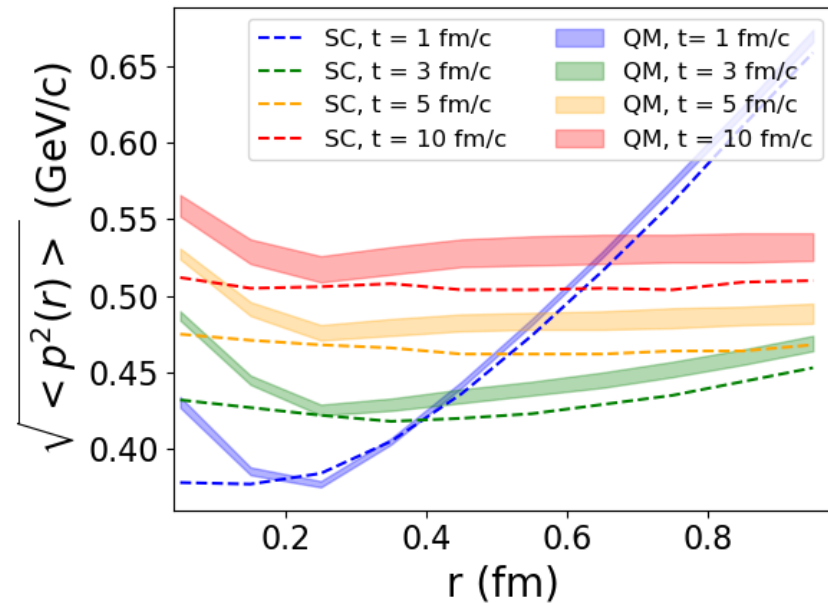
T=0.4 GeV



T=0.3 GeV



T=0.4 GeV



Final remarks

- We have performed a comparative study of the evolution of a $c\bar{c}$ pair immersed in a QGP at temperature T , using a Lindblad equation and the associated Fokker-Planck equation.
- We focussed on the 'Quantum Brownian motion' regime of open quantum systems, with $T \gtrsim \Delta E$. Spatial coordinates represents the 'natural basis' to analyse the dynamics.
- Overall the semi-classical approximation works well, except at early times where quantum interference need to be taken into account. Non unitary processes play an important role.
- In contrast to the Fokker-Planck, the late time evolution of the Lindblad equation is not the usual Boltzmannn distribution.
- Generalisation to QCD remains a challenge.