Probing charm degrees of freedom using generalized charm susceptibilities

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work done in collaboration with Frithjof Karsch and Peter Petreczky

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T30f
Theoretische Teilchen- und Kernphysik

Motivation I

- Strong interaction matter undergoes a chiral crossover at $\mathrm{T_{pc}} = 156.5 \pm 1.5$ MeV.
  [HotQCD Collaboration, 2019; Borsanyi et al., 2020; Kotov et al., 2021]
- In heavy-ion collisions, relevant degrees of freedom change from partonic to hadronic in going from high temperature phase to temperatures below $\mathrm{T_{pc}}$.

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- When do charmed hadrons stop contributing to the total charm pressure?

Motivation II

- Charm fluctuations (cumulants) calculated in the framework of lattice QCD can receive enhanced contributions due the existence of not-yet-discovered open-charm states; it is possible to compare this enhancement to the HRG calculations.

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- $\hat{\mu}_\mathrm{X} = \mu/\mathrm{T}$, $\mathrm{X} \in \{\mathrm{B}, \mathrm{Q}, \mathrm{S}, \mathrm{C}\}$.

Generalized susceptibilities of the conserved charges

$$\mathrm{M_C(T,\overrightarrow{\mu})} = \frac{1}{2\pi^2}\sum_{\mathrm{i}} \mathrm{g_i}\left(\frac{\mathrm{m_i}}{\mathrm{T}}\right)^2 \mathrm{K_2(m_i/T)cosh(Q_i\hat{\mu}_Q + S_i\hat{\mu}_S + C_i\hat{\mu}_C)}$$

- $\mathrm{K_2(x)} \sim \sqrt{\pi/2x}\, e^{-x}\, [1+\mathbb{O}(x^{-1})]$. If $\mathrm{m_i} \gg \mathrm{T}$, then contribution to $\mathrm{P_C}$ will be exponentially suppressed.
- Λ_c^+ mass ~ 2286 MeV, Ξ_{cc}^{++} mass ~ 3621 MeV. At $\mathrm{T_{pc}}$, contribution to $\mathrm{B_C}$ from Ξ_{cc}^{++} will be suppressed by a factor of 10^{-4} in relation to Λ_c^+.

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- Dimensionless generalized susceptibilities of the conserved charges:

$$\chi_{\mathrm{klmn}}^{\mathrm{BQSC}} = \left.\frac{\partial^{(\mathrm{k+l+m+n})} \left[\mathrm{P}\left(\hat{\mu}_{\mathrm{B}}, \hat{\mu}_{\mathrm{Q}}, \hat{\mu}_{\mathrm{S}}, \hat{\mu}_{\mathrm{C}}\right)/\mathrm{T}^4\right]}{\partial\hat{\mu}_{\mathrm{B}}^{\mathrm{k}}\ \partial\hat{\mu}_{\mathrm{Q}}^{\mathrm{l}}\ \partial\hat{\mu}_{\mathrm{S}}^{\mathrm{m}}\ \partial\hat{\mu}_{\mathrm{C}}^{\mathrm{n}}}\right|_{\vec{\mu}=0}$$

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- Dimensionless generalized susceptibilities of the conserved charges using $\mathrm{P_C}$:

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- $\underbrace{\chi^{BC}_{mn}}_{\chi^{\mathrm{BQSC}}_{\mathrm{m00n}}} = \mathrm{B_{C,1}} + 2^{\mathrm{n}}\mathrm{B_{C,2}} + 3^{\mathrm{n}}\mathrm{B_{C,3}} \simeq \mathrm{B_{C,1}} = \mathrm{B_C}, \forall(\mathrm{m}+\mathrm{n}) \in$ even

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- $\chi^{\mathrm{C}}_{\mathrm{m}} = \mathrm{P_C}, \forall \mathrm{m} \in$ even
- At present, we have gone upto fourth order in calculating various cumulants.

Simulation Details

- Partition function of QCD with 2 light, 1 strange and 1 charm quark flavors is :

$$\mathcal{Z} = \int \mathcal{D}[\mathrm{U}]\{\text{det } \mathrm{D}(\mathrm{m_l})\}^{2/4}\{\text{det } \mathrm{D}(\mathrm{m_s})\}^{1/4}\{\text{det } \mathrm{D}(\mathrm{m_c})\}^{1/4}\mathrm{e}^{-\mathrm{S_g}}.$$

  This can be used to calculate susceptibilities in the BQSC basis.
- We used (2+1)-flavor HISQ configurations generated by HotQCD collaboration for $\mathrm{m_s}/\mathrm{m_l} = 27$ and $\mathrm{N}_\tau = 8, 12$ and 16.
- $T = (aN_\tau)^{-1} \implies$ three lattice spacings at a fixed temperature.
- We treated charm-quark sector in the quenched approximation.
- $O((am_c)^4)$ tree level lattice artifacts are removed by adding so-called epsilon-term, which leads to sub-percent errors in observables linked to charm at $am_c \approx 0.5$ or $a \approx 0.1$ fm.

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This can be used to calculate susceptibilities in the BQSC basis.
- Derivatives of the pressure consist of expectation values of various traces comprised of inversions and derivatives of the fermion matrices.
- First derivative w.r.t μ_i will be $\left\langle \text{ Tr } \left(\mathrm{D}(\mathrm{m_i})^{-1}\dfrac{\partial \mathrm{D}(\mathrm{m_i})}{\partial \mu_\mathrm{i}}\right)\right\rangle$.

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- First derivative w.r.t μ_i will be $\left\langle \text{Tr}\left(\mathrm{D(m_i)^{-1}}\frac{\partial \mathrm{D(m_i)}}{\partial \mu_\mathrm{i}}\right)\right\rangle$.
- We used 500 random vectors to do unbiased stochastic estimation:

$$\langle \eta_\mathrm{i} \rangle = \lim_{\mathrm{N_s}\to\infty} \frac{1}{\mathrm{N_S}} \sum_{\mathrm{k}=1}^{\mathrm{N_S}} \eta_\mathrm{ki} = 0 \ , \tag{1}$$

$$\langle \eta_\mathrm{i}\eta_\mathrm{j} \rangle = \lim_{\mathrm{N_s}\to\infty} \frac{1}{\mathrm{N_S}} \sum_{\mathrm{k}=1}^{\mathrm{N_S}} \eta^*_\mathrm{ki}\eta_\mathrm{kj} = \delta_\mathrm{ij} \tag{2}$$

$$\text{Tr}\ (\mathrm{D(m_i)^{-1}}) = \frac{1}{\mathrm{N}} \sum_{\mathrm{k}=1}^{\mathrm{N}} \eta^\dagger_\mathrm{k} \underbrace{\mathrm{D(m_i)^{-1}}\eta_\mathrm{k}}_{\mathrm{x}} \ . \tag{3}$$

Ratios independent of the hadron spectrum

- Irrespective of the details of the baryon mass spectrum, in the validity range of HRG, $\chi_{\mathrm{mn}}^{\mathrm{BC}}/\chi_{\mathrm{kl}}^{\mathrm{BC}} = 1$, $\forall(\mathrm{m}+\mathrm{n}), (\mathrm{k}+\mathrm{l}) \in$ even.

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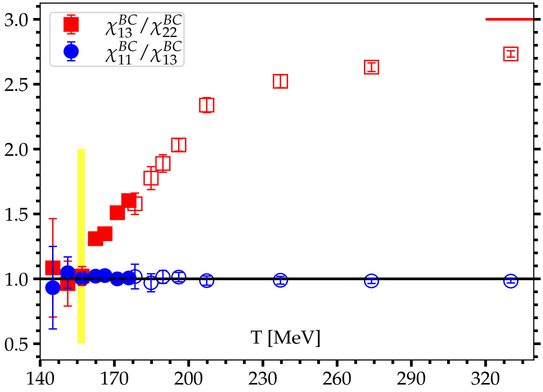
- $\chi_{1\mathrm{n}}^{\mathrm{BC}}/\chi_{1\mathrm{l}}^{\mathrm{BC}} = 1$, $\forall \mathrm{n},\mathrm{l} \in$ odd, for the entire temperature range.

Onset of the charmed hadron melting

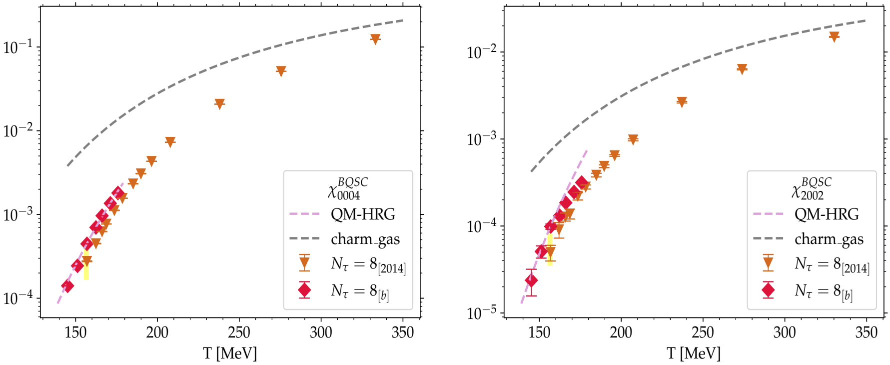


- States with fractional B start appearing near $\mathrm{T_{pc}}$. Is it possible to determine this fractional B?

Approach to free charm-quark gas limit

$$Q_C(T, \vec{\mu}) = \frac{6}{\pi^2}\left(\frac{m_c}{T}\right)^2 K_2(m_c/T)\cosh\left(\frac{2}{3}\hat{\mu}_Q + \frac{1}{3}\hat{\mu}_B + \hat{\mu}_C\right)$$

$m_c = 1.27$ GeV.



Charm degrees of freedom in the intermediate T range

- Based on carrriers of C in low and high-T phase, pose a quasi-particle model consisting of non-interacting meson, baryon and quark-like states:

$$\begin{aligned}\mathrm{P_C}(\mathrm{T},\hat{\mu}_\mathrm{C},\hat{\mu}_\mathrm{B})/\mathrm{T}^4 =& \mathrm{P_M^C}(\mathrm{T})\cosh(\hat{\mu}_\mathrm{C}+...) + \mathrm{P_B^C}(\mathrm{T})\cosh(\hat{\mu}_\mathrm{C}+\hat{\mu}_\mathrm{B}+...)\\ &+\mathrm{P_q^C}(\mathrm{T})\cosh(\frac{2}{3}\hat{\mu}_\mathrm{Q}+\frac{1}{3}\hat{\mu}_\mathrm{B}+\hat{\mu}_\mathrm{C})\end{aligned}$$

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- Use quantum numbers B and C to construct partial pressures:

$$\begin{aligned}\mathrm{P^C_q} &= 9(\chi^{\mathrm{BC}}_{13}-\chi^{\mathrm{BC}}_{22})/2\\ \mathrm{P^C_B} &= (3\chi^{\mathrm{BC}}_{22}-\chi^{\mathrm{BC}}_{13})/2\\ \mathrm{P^C_M} &= \chi^{\mathrm{C}}_4+3\chi^{\mathrm{BC}}_{22}-4\chi^{\mathrm{BC}}_{13}\end{aligned}$$

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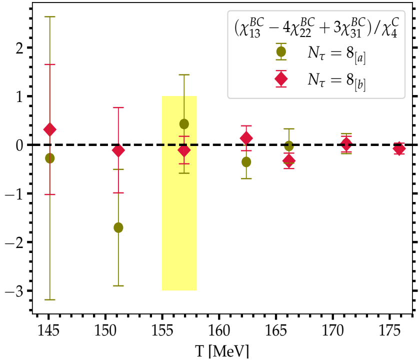
$$\mathrm{P_M^C} = \chi_4^{\mathrm{C}} + 3\chi_{22}^{\mathrm{BC}} - 4\chi_{13}^{\mathrm{BC}}$$

  - Constraint on cumulants in a simple quasi-particle model:

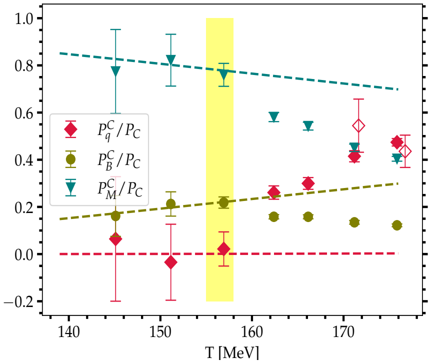
$$\mathrm{c} = \chi_{13}^{\mathrm{BC}} + 3\chi_{31}^{\mathrm{BC}} - 4\chi_{22}^{\mathrm{BC}} = 0$$

Quasi-particle model



The constraint holds true $\implies$ quasi-particle states with $|\mathrm{B}| = 0, 1$ or $1/3$ exist in the intermediate temperature range.

Charm-quark-like excitations in QGP



Right after $\mathrm{T_{pc}}$, $\mathrm{P_q}$ starts contributing to $\mathrm{P_C}$, which is compensated by a reduction (and deviation from HRG) in the fractional contribution of the hadron-like states to $\mathrm{P_C}$.

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- We can use four fourth-order QC correlations to determine partial pressures of the four possible electrically-charged-charm subsectors.

$$\boxed{\mathrm{P}_{\mathrm{C}}^{|\mathrm{Q}|=2/3} = \frac{1}{8}\left[54\chi_{13}^{\mathrm{QC}} - 81\chi_{22}^{\mathrm{QC}} + 27\chi_{31}^{\mathrm{QC}}\right]}$$

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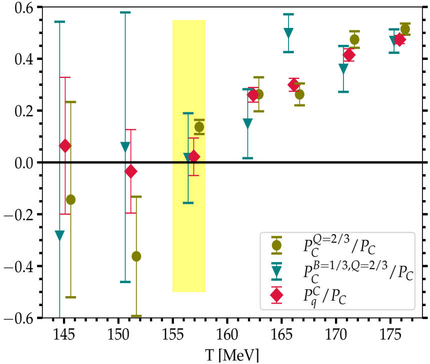
- For the BQC sector there are three possibilities: i)$\{|\mathrm{B}| = 1, |\mathrm{Q}| = 1\}$; ii)$\{|\mathrm{B}| = 1, |\mathrm{Q}| = 2\}$; iii)$\{|\mathrm{B}| = 1/3, |\mathrm{Q}| = 2/3\}$.

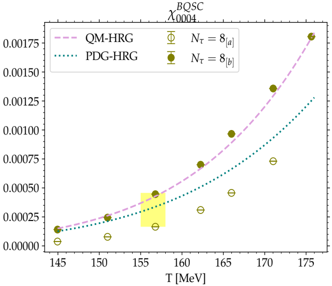
$$\boxed{\mathrm{P}_{\mathrm{C}}^{\mathrm{B}=1/3,\mathrm{Q}=2/3} = \frac{27}{4}\left[\chi_{112}^{\mathrm{BQC}} - \chi_{211}^{\mathrm{BQC}}\right]}$$

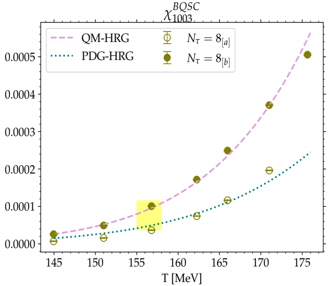
Charm-quark-like excitations in QGP

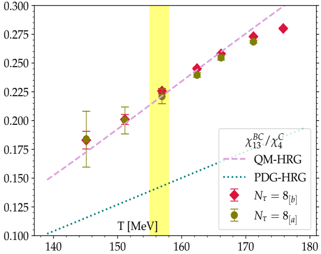


Clear agreement between three independent observables which correspond to the partial pressures of
i) $\mathrm{B} = 1/3$, ii) $\mathrm{Q} = 2/3$, and iii) $\mathrm{B} = 1/3$ and $\mathrm{Q} = 2/3$ charm subsectors.

Ratios calculated using different LCPs

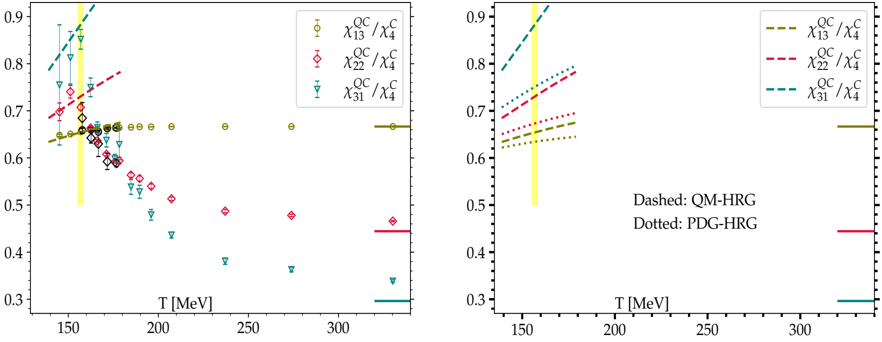






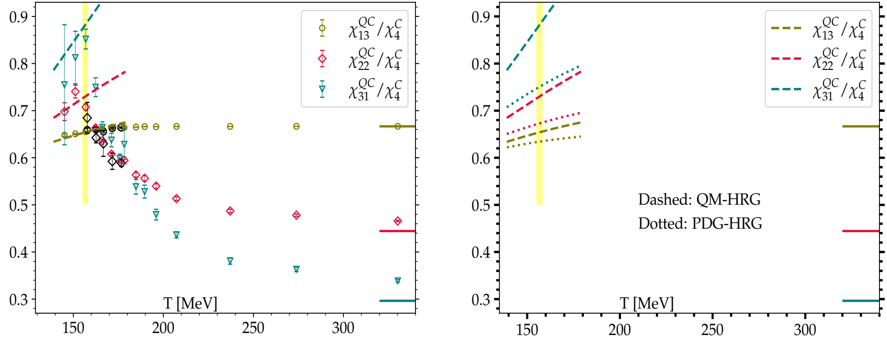
- Sensitivity to the choice of LCP cancels to a large extent in the ratios.
- All previously shown results were based on ratios, and hence valid in the continuum limit.

Electrically-charged-charm subsector



- Ratio of QM-HRG/PDG-HRG increases with increasing Q-moments $\implies |\mathrm{Q}| = 2$ sector more sensitive to 'missing resonances'.
- χ^{QC}_{22} and χ^{QC}_{31} give evidence for 'missing resonances'.

QC correlations for different lattice spacings



- Ratio of QM-HRG/PDG-HRG increases with increasing Q-moments.
- Close to freeze-out, an enhancement over the PDG expectation in the fractional contribution of the $|\mathrm{Q}| = 2$ (Σ_c^{++}) charm subsector to the total charm partial pressure.

History of the lattice setup

- We used (2+1)-flavor HotQCD configurations generated using HISQ action and a Symanzik-improved gauge action for $\mathrm{m_s/m_l} = 27$ and $\mathrm{N}_\tau = 8, 12$ and 16. We treated charm sector in the quenched approximation.

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- The temperature is set using f_K.

$$a f_K(\beta) = \left[\frac{c_0^K f(\beta) + \frac{10}{\beta} c_2^K f^3(\beta)}{1 + \frac{10}{\beta} d_2^K f^2(\beta)}\right], \tag{4}$$

with $c_0^K = 7486(25)$, $c_2^K = 41935(2247)$, $d_2^K = 3273(224)$.
Here, $f(\beta)$ is the 2-loop QCD beta function,
$f(\beta) = \left(\frac{10 b_0}{\beta}\right)^{-b_1/2b_0^2}$ exp $(-\beta/(20 b_0))$, with b_0 and b_1 being its 1-loop perturbative expansion coefficients in 3-flavor QCD.

D. Bollweg et al, arXiv:2107.10011

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- $T = (aN_\tau)^{-1} \implies$ three lattice spacings at a fixed temperature.

History of the lattice setup

- Strange quark mass was (mis)tuned by keeping the mass of fictitious pseudoscalar meson, $\eta_{s\bar{s}}$, fixed to 695 MeV (not 686 MeV). Conversion to MeV was done $r_1 \implies$ strange quark mass is 2.6% larger than its physical value. A. Bazavov et al, arXiv:1111.1710

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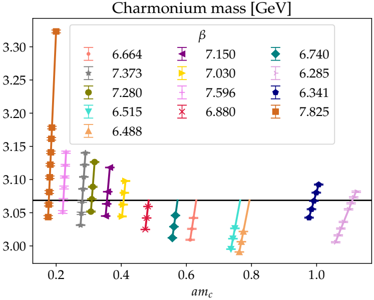
- However for finer lattices corresponding to $\beta > 7.03$, the resulting lattice mass of $\eta_{s\bar{s}}$ is larger than 695 MeV by about 3.5%. This drift from 695 MeV was further corrected by using lowest order χPT such that $M^2_{\eta_{s\bar{s}}} \propto m_s$ but the configuration generation did not take into account this corrected version of LCP. A. Bazavov et al, arXiv:1407.6387

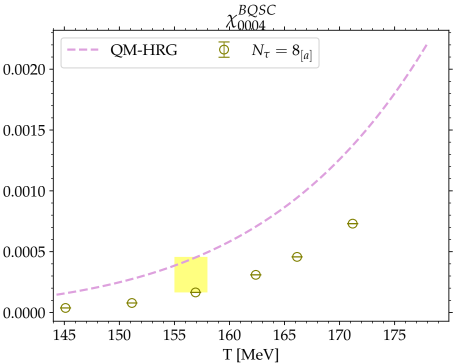
Tuning charm quark mass on LCP[a]



- We used spin-averaged charmonium to tune the bare charm quark at zero temperature.
- We used f_K to convert charmonium mass to physical units.
- The figure on the right fits an RG-inspired ansatz to the intersections of the colored lines and the black line (PDG value) in the left figure.

S.Sharma, arxiv:2212.11148

Quartic charm susceptibility



At T_{pc}, the physical expectation for χ_4^C, based on HRG, is roughly 2.7 times the result obtained for $N_\tau = 8$ $(\mathrm{a} \approx 0.16$ fm) on LCP$_{[a]}$ at the chiral crossover.

Different LCP prescriptions

- In the HRG phase, each charmed hadron contributes as $\left(\frac{m_i}{T}\right)^2 e^{-m_i/T}\,[1+\mathbb{O}((m_i/T)^{-1})]\cosh(B_i\hat{\mu}_B+Q_i\hat{\mu}_Q+S_i\hat{\mu}_S+C_i\hat{\mu}_C)$.

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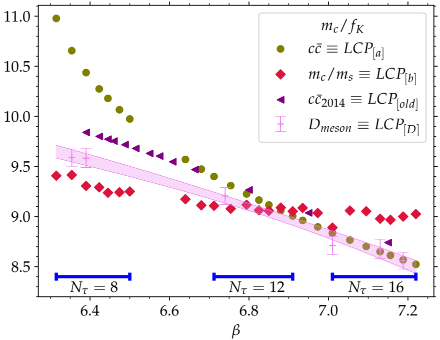
- This also holds true in the QGP phase where charm quarks and charmed hadrons coexist. arxiv:2312.12857
- We removed order $(am_c)^4$ tree level lattice artifacts by adding the so-called epsilon-term, which leads to sub-percent errors in observables linked to charm at $am_c \approx 0.5$ or $a \approx 0.08$ fm. [[HPQCD, UKQCD],2006]

Major source of cutoff effects



- In order to understand the cutoff effects, we investigated the sensitivity of χ_4^C to four different LCP prescriptions used to tune am_c values: LCP$_{[b]}$ fixes $m_c/m_s = 11.76$, LCP$_{[D]}$ is defined by the physical D-meson mass, LCP$_{[old]}$ is also defined by the charmonium mass A. Bazavov et al, arXiv:1404.4043

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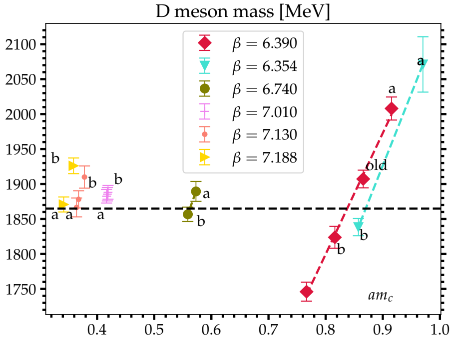
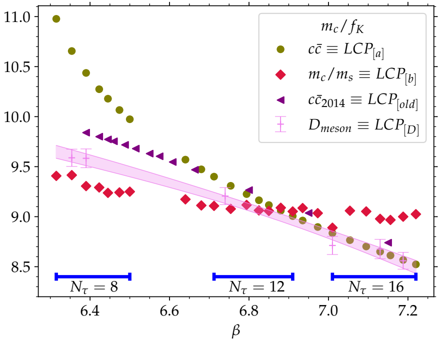


- The difference between mass of the most thermodynamicaly dominant hadron, i.e., D-meson, calculated on $\mathrm{LCP}_{[a]}$ and its physical value is less than 2% only for $\beta > 6.74$ (finer lattices).

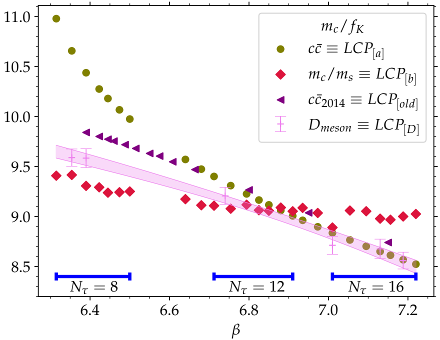
Bare charm quark mass on $\mathrm{LCP}_{[D]}$



$$am_c(\beta) = af_K(\beta)\frac{M_c^{\mathrm{RGI}}}{f_K}\left(\frac{20b_0}{\beta}\right)^{\frac{4}{9}}\left[\frac{1+\frac{10}{\beta}m_{1c}f^2(\beta)}{1+\frac{10}{\beta}dm_{1c}f^2(\beta)}\right] \quad ,$$

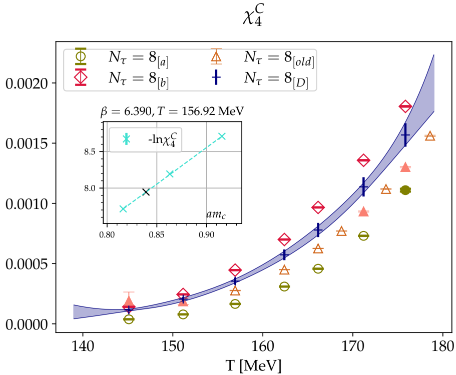
$M_c^{\mathrm{RGI}} = 1.528$ GeV, $f_K = (155.7/\sqrt{2})$ MeV

Cutoff effects for different LCPs



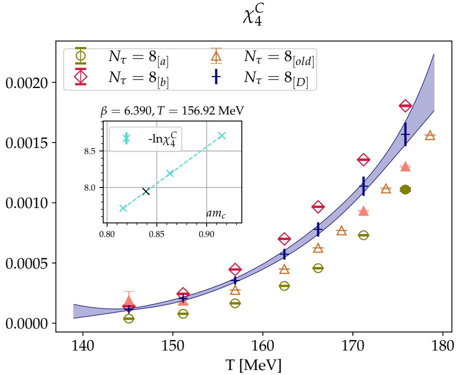
- Cutoff effects dependent on am_c are smaller on LCP$_{[D]}$ in comparison to LCP$_{[a]}$ for smaller β.
- Both LCP$_{[D]}$ and LCP$_{[a]}$ converge in the continuum limit.
- Mistuning of am_s reflects in am_c on LCP$_{[b]}$ for larger beta.

Interpolating quartic susceptibility in am_c at $N_\tau = 8$



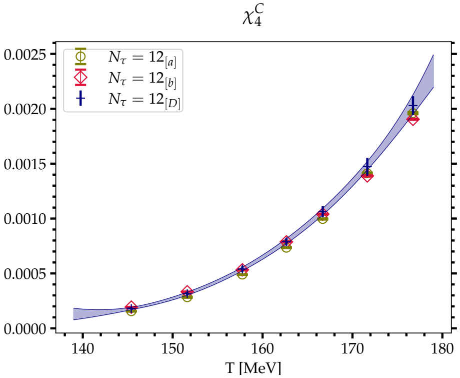
- χ_n^C is a proxy of partial charm pressure, and at a given T, receives dominant contribution from the lightest charmed state, whose Boltzmann weight is $\propto e^{-am_c/aN_\tau}$.

Interpolating quartic susceptibility in am_c at $N_\tau = 8$



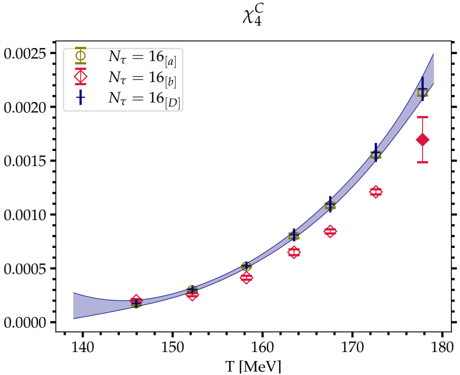
- At each T, linear interpolation of $-\ln \chi_4^C$ in am_c gives χ_4^C on LCP$_{[D]}$ – blue plus markers which incorporate bootstrap error and error from uncertainity of am_c on LCP$_{[D]}$.
- Blue band is a [2,2] Pade interpolation.

Interpolating quartic susceptibility in am_c at $N_\tau = 12$



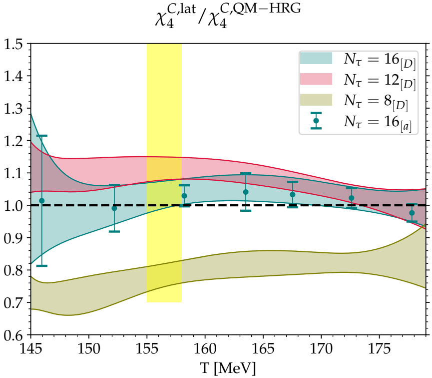
- Ordering of χ_4^C on different LCPs can a understood from the ordering of am_c values $\implies$ heavier the charmed state mass, smaller the thermodynamic contribution to the partial charm pressure.

Interpolating quartic susceptibility in am_c at $N_\tau = 16$



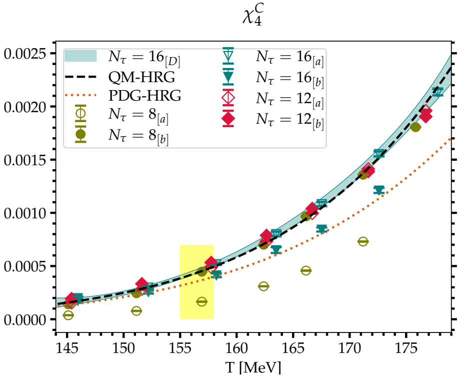
- LCP$_{[a]}$ is effectively LCP$_{[D]}$.
- mistuning of am_s also reflects in χ_4^C.

Continuum limit of quartic susceptibility



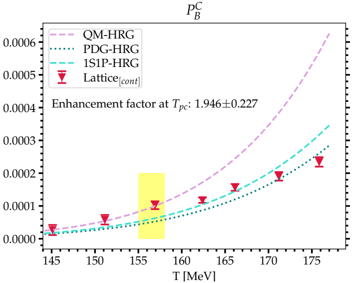
- LCP$_{[D]}$ bands for $N_\tau = 12$ and 16 overlap $\implies$ use $N_\tau = 16_{[D]}$ as the continuum estimate.

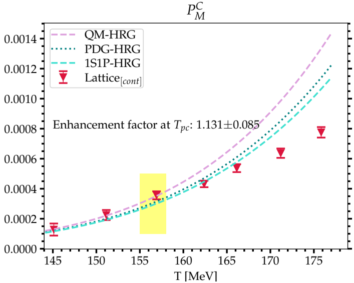
Continuum limit of quartic susceptibility

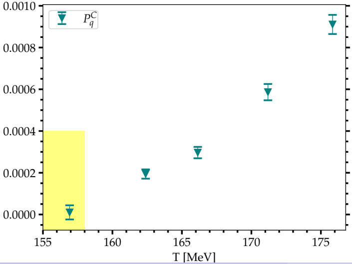


- Note: χ_4^C is not sensitive to the change in charm degrees of freedom and agreement with QM-HRG above chiral crossover is accidental.

Absolute partial charm pressures

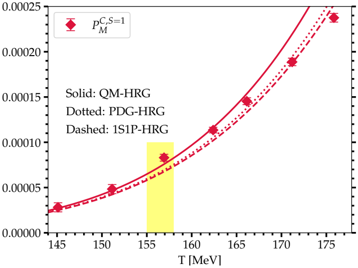


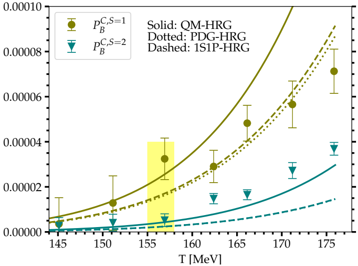




- P. Braun-Munzinger, et. al., JHEP, 04:058, 2025 used enhancement factor for P_B^C to describe observed experimental yield of open-charmed baryons.

Strange charmed hadrons

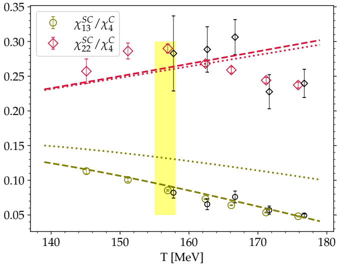


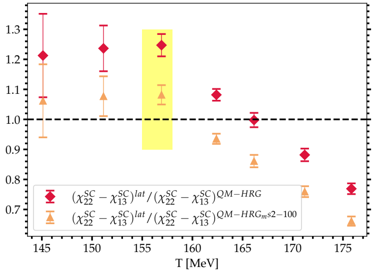


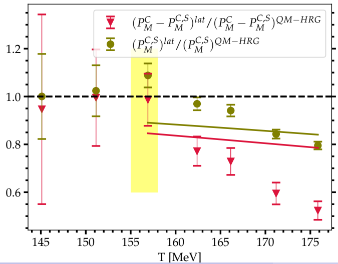
$$
\begin{aligned}
P_M^{C,S=1} &= \chi_{13}^{SC} - \chi_{112}^{BSC} \\
P_B^{C,S=1} &= \chi_{13}^{SC} - \chi_{22}^{SC} - 3\chi_{112}^{BSC} \\
P_B^{C,S=2} &= (2\chi_{112}^{BSC} + \chi_{22}^{SC} - \chi_{13}^{SC})/2
\end{aligned}
$$

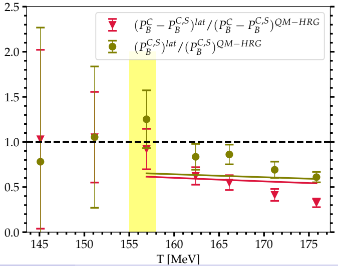
- QM-HRG underpredicts $|S| = 2$ lattice data above T_{pc}. Incomplete knowledge of $|S| = 2$ sector?
- Deviation from HRG occurs at a slower rate than non-strange charmed sector.

Strange charmed hadrons







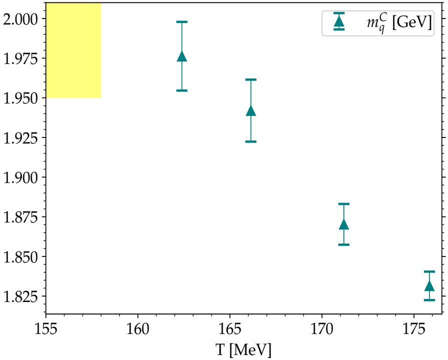


Conclusions

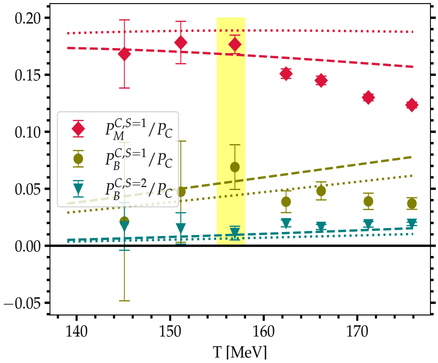
- Charmed hadrons start dissociating at the chiral crossover temperature, leading to the appearance of charm quark degrees of freedom.
- Hadron-like excitations survive in QGP.
- At T_{pc}, only 50% contribution to charmed baryon pressure comes from experimentally known states.
- Incomplete knowledge of the doubly-strange charmed baryon sector.

thermal mass of charm quark-like excitation

$$\mathrm{P_q^C}(\mathrm{T}, \vec{\mu}) = \frac{6}{\pi^2}\left(\frac{\mathrm{m_q^C}}{\mathrm{T}}\right)^2 \mathrm{K_2}(\mathrm{m_q^C}/\mathrm{T})\cosh\left(\frac{2}{3}\hat{\mu}_{\mathrm{Q}} + \frac{1}{3}\hat{\mu}_{\mathrm{B}} + \hat{\mu}_{\mathrm{C}}\right)$$



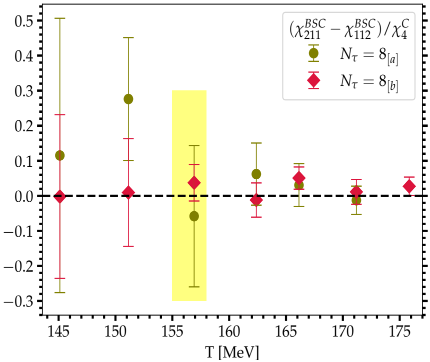
Backup slide I



Why not consider only charm-quark-like excitations?

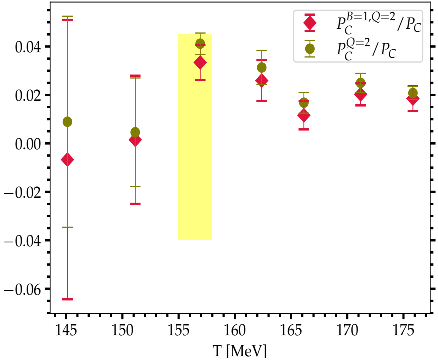
$\implies$ Contribution from SC sector above $\mathrm{T_{pc}}$ – these states can not be quark-like.

Backup slide II: Fate of diquarks



If strange-quark diquarks exist in QGP, their contribution to P_C has to be less than 20%.

Backup slide III



Only $|\mathrm{B}| = 1$ sector contributes to partial pressure from $|\mathrm{Q}| = 2$ charm subsector.