Recent Developments in Quarkonium Quantum Transport

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18th March 2026, Online seminar series "Heavy Flavor Probes of QCD Matter"

Outline

Heavy quarkonium in heavy-ion collisions

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- Heavy quarks can only be created at the beginning of the collision. It is a hard process.
- However, the existence of a medium changes the probability that a bound state is formed and its lifetime.
- Measuring R_{AA}, the ratio of quarkonium states measured in heavy-ion collisions divided by the naive extrapolation of pp data, we can extract information about the medium.

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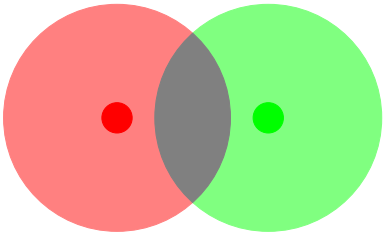
At finite temperature



Debye radius

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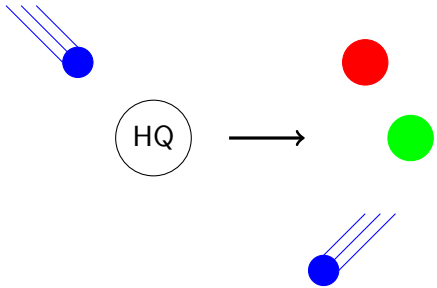
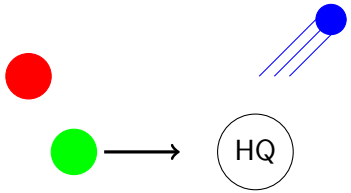


Recombination



Two heavy quarks coming from different origin may recombine to form a new quarkonium state.

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- In some cases, decays and recombination can be described with rate or Boltzmann equation in the semi-classical approximation. However, this is not always the case.
- When thermal effects are important, we need to describe all three effects taking into account quantum effects.

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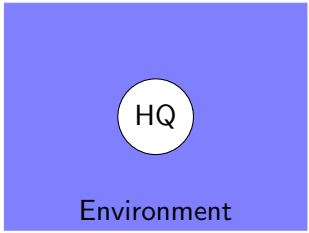

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- We can recover the Schrödinger equation and the Boltzmann equation as limits of the master equation in specific regimes.
- We need to derive the master equation from QCD. This has been done in:
  - Perturbation theory. Akamatsu (2015,2020), Blaizot and Escobedo (2017,2018).
  - Potential non-relativistic QCD (pNRQCD) in the $\frac{1}{r} \gg T$ regime. Brambilla et al. (2016,2017).
  - Schrödinger-Langevin equation (Gossiaux and Katz, 2016)

The Lindblad equation

Any master equation that is:

- Markovian
- Preserves the properties that a density matrix must fulfil (Hermitian, positive semi-definite, trace is conserve).

Can be written as a Lindblad (or GKSL) equation (Lindblad (1976), Gorini, Kossakowski and Sudarshan (1976)).

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In the case of quarkonium, the Markovian limit corresponds to the case in which the energy of the particles in the environment is larger than the binding energy.

Problems of the Lindblad equation

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- *Computational cost*. Since the state of the system is represented by a density matrix, if we use a lattice with the double of points, the cost is multiplied by four.
- *Black box*. From the Lindblad equation it is difficult to get a physical picture of what is the leading effect.
- *Solution*. Use the Quantum Trajectories method (Dalibard, Castin and Molmer, 1992). Monte Carlo technique to solve the Lindblad equation using wave functions instead of the density matrix.

The Monte-Carlo Wave Function method

Take the Lindblad equation

$$\partial_t \rho = -i[H(\gamma), \rho] + \sum_k (C_k(\kappa)\rho C_k^\dagger(\kappa) - \frac{1}{2}\{C_k^\dagger(\kappa)C_k(\kappa), \rho\})$$

Let us define

$$\Gamma_n = C_n^\dagger C_n \qquad \Gamma = \sum_n \Gamma_n$$

and

$$H_{eff} = H - i\frac{\Gamma}{2}$$

$\rho(t) = \sum_n p_n |\Psi_n(t)\rangle\langle\Psi_n(t)|$. If we know how to evolve the case $\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$, it is straightforward to generalize.

The Monte-Carlo Wave Function method

The algorithm to evolve from t to $t + dt$

- With probability $1 - \langle\Psi(t)|\Gamma|\Psi(t)\rangle dt$.
  - Evolve the wave-function with $(1 - iH_{eff}\,dt)|\Psi(t)\rangle$. In our case, this implies solving a 1D Schrödinger equation because H_{eff} does not mix states with different color or angular momentum.
- With probability $\langle\Psi(t)|\Gamma_n|\Psi(t)\rangle dt$.
  - Take a quantum jump, $|\Psi(t)\rangle \to C_n|\Psi(t)\rangle$.
  - Only here transitions between different color and angular momentum are allowed.
- Normalize the resulting wave-function.

The average of this stochastic evolution of the wave-function is equivalent to the Lindblad equation for the density matrix.

How does the quantum trajectory method encode each effect?

- Screening. Through the Hermitian part of the Hamiltonian. If there are no bound states the heavy quark and antiquark will separate.
- Decay width. Through the Non-hermitian part of the Hamiltonian. Possibility to take a quantum jump to an unbound state.
- Recombination. Through jump operator. Finite probability to jump back to a bound state.

The $\frac{1}{r} \gg T, m_D \gg E$ regime

pNRQCD approach. Brambilla, M.A.E., Soto and Vairo (2017-2018)

Because all the thermal scales are smaller than $\frac{1}{r}$ but bigger than E the evolution equation is of the Lindblad form. [1]

$$\partial_t \rho = -i[H(\gamma), \rho] + \sum_k (C_k(\kappa)\rho C_k^\dagger(\kappa) - \frac{1}{2}\{C_k^\dagger(\kappa) C_k(\kappa), \rho\})$$

$$\kappa = \frac{g^2}{6\, N_c} \mathrm{Re} \int_{-\infty}^{+\infty} ds \, \langle \mathrm{T}\, E^{a,i}(s, \mathbf{0}) E^{a,i}(0, \mathbf{0}) \rangle$$

$$\gamma = \frac{g^2}{6\, N_c} \mathrm{Im} \int_{-\infty}^{+\infty} ds \, \langle \mathrm{T}\, E^{a,i}(s, \mathbf{0}) E^{a,i}(0, \mathbf{0}) \rangle$$

[1] We use a redefined field $E(t, \mathbf{x})$ such that we do not need to write explicitly the Wilson lines.

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- The pNRQCD Lindblad equation can be efficiently solved using a publicly available code called Qtraj.

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- In the dipole limit, quantum jumps can only change the angular momentum by one unit.
- The computational cost of quantum transport is much higher than that of classical transport. For phenomenology, we need to average over many centralities and medium profiles. We need very efficient codes.

Outline

- E/T corrections and the approach to thermal equilibrium.
- Beyond the dipole approximation.
- The $X(3872)$ state.

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- In QCD, E/T corrections have been computed in perturbation theory (Blaizot and Escobedo (2017)) and in pNRQCD.

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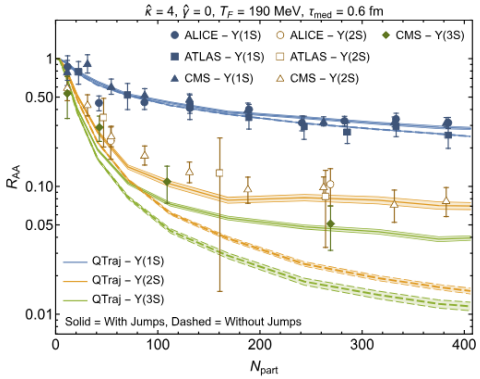
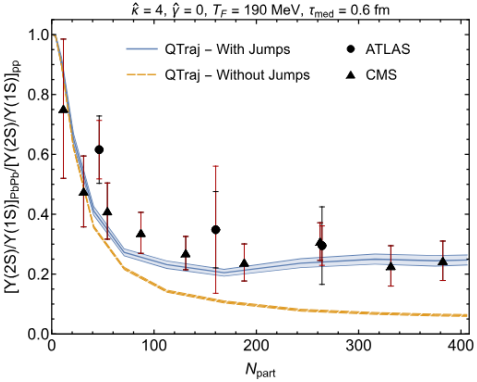
- E/T expansion seems to converge for $T > 190\, MeV$.
- The decay width is reduced. Results are less sensitive to κ and more sensitive to γ.

E/T corrections in pNRQCD



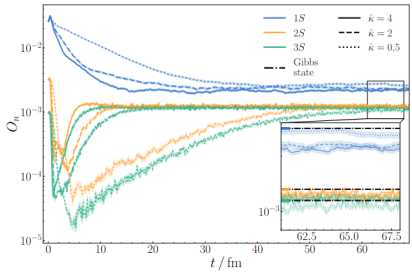


Picture taken from Brambilla, Escobedo, Islam, Strickland, Tiwari, Vairo and Vander Griend (2023).

Thermalization in pNRQCD

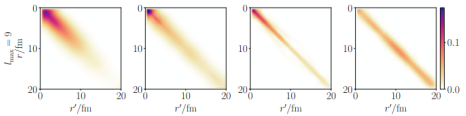
Brambilla, Magorsch and Vairo (2025)

- The steady state is approximately a thermal state.
- Thermalization occurs at timescales much larger than the lifetime of the quark-gluon plasma.
- Corrections to the Gibbs state become smaller as the temperature increases.



Decoherence in pNRQCD

- A typical wave function does not have a well defined position.
- However, the interaction with the medium localizes the wave function in position space.
- The density matrix in position space becomes approximately diagonal.
- In the Abelian limit, this would be the regime in which the Langevin equation is valid.



Picture taken from Brambilla, Magorsch and Vairo (2025). The times are (from left to right) $t = 0.1\,fm$, $t = 1\,fm$, $5\,fm$ and $t = 50\,fm$.

Decoherence, classicality and thermalization in HTL

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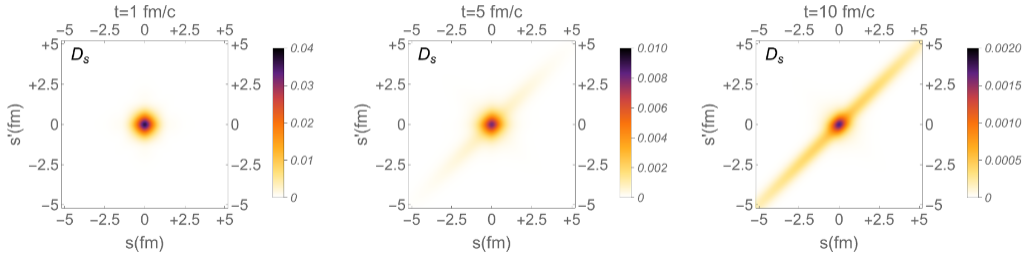
- The system approximately thermalizes.
- HTL and pNRQCD give a coherent picture.

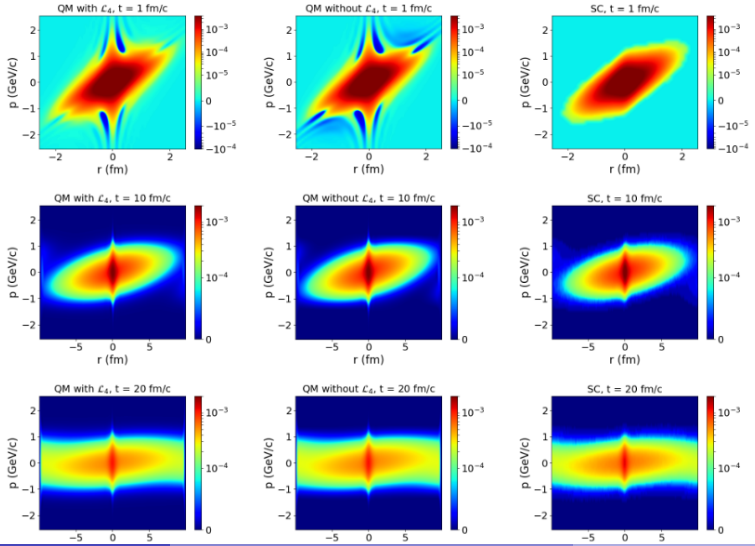
Decoherence in HTL

Delorme, Katz, Gousset, Gossiaux and Blaizot (2024)



Classicality in HTL

Daddi Hammou, Blaizot, Gossiaux and Gousset (2026)



Thermalization in HTL

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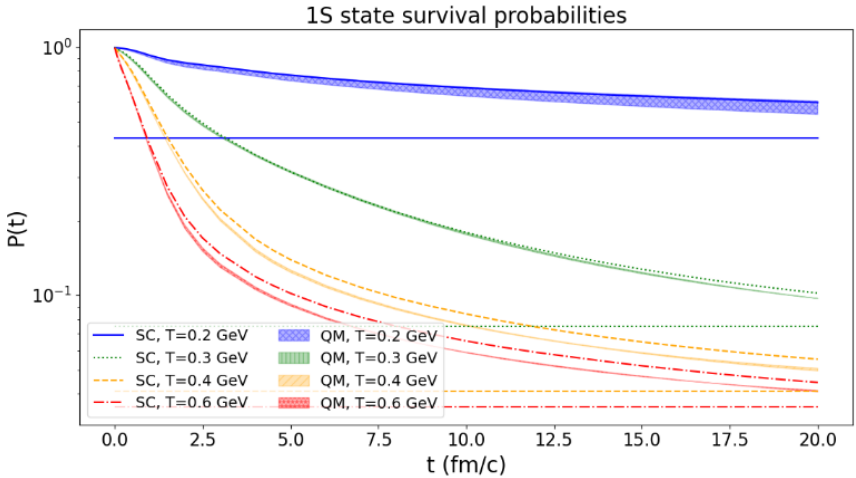

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- How much do the results depend on the potential used?
- Can we get even closer to thermal equilibrium? How important is that taking into account that the lifetime of the QGP is smaller than the thermalization time?

Work done in collaboration with Andrea Beraudo, Paolo Parotto and Jorge Manuel Martínez Vera.

Limitations of the dipole approximation

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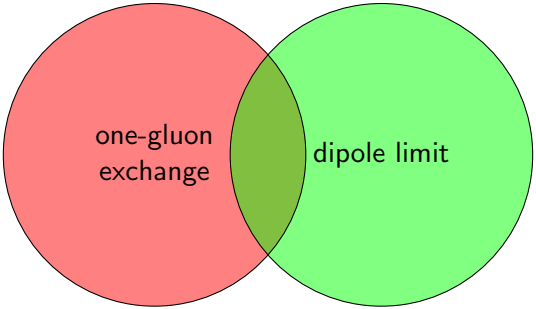
- This is a good approximation for bottomonium at the temperatures reached in current heavy-ion collisions, but not for charmonium.
- Goal: go beyond the dipole approximation in an efficient way.

Motivation

We wish to be able to efficiently simulate a larger set of possible master equations. Better description of charmonium and excited states.



We take the HTL one-gluon exchange case (Blaizot and Escobedo, 2018). However, the framework would be easily adapted to any one-gluon exchange model.

The master equation

$$\frac{d\rho_Q}{dt} = -i\left[H'_Q, \rho_Q\right] + \sum_{i=0}^{2} \int \frac{d^3q}{(2\pi)^3} \left(C_q^i \rho_Q C_q^{i\dagger} - \frac{1}{2}\{C_q^{i\dagger} C_q^i, \rho_Q\} \right),$$

where

$$H'_Q \equiv \begin{pmatrix} H'_s & 0 \\ 0 & H'_o \end{pmatrix} \equiv \begin{pmatrix} -C_F \dfrac{\alpha_s}{r} e^{-m_D r} & 0 \\ 0 & \dfrac{1}{2N_c} \dfrac{\alpha_s}{r} e^{-m_D r} \end{pmatrix}$$

Note also that the are infinitely many jump operators.

The master equation

Collapse operators

$$\frac{d\rho_Q}{dt} = -i\left[H'_Q, \rho_Q\right] + \sum_{i=0}^{2} \int \frac{d^3q}{(2\pi)^3} \left(C_q^i \rho_Q C_q^{i\dagger} - \frac{1}{2}\{C_q^{i\dagger} C_q^i, \rho_Q\} \right),$$

where

$$C_q^0 \equiv \begin{pmatrix} 0 & \frac{1}{\sqrt{2N_c}} L_q \\ \sqrt{C_F} L_q & 0 \end{pmatrix} \quad C_q^1 \equiv \sqrt{\frac{N_c^2 - 4}{4N_c}} \begin{pmatrix} 0 & 0 \\ 0 & L_q \end{pmatrix}$$

$$C_q^2 \equiv \sqrt{\frac{N_c}{4}} \begin{pmatrix} 0 & 0 \\ 0 & \overline{L}_q \end{pmatrix}$$

Collapse operators

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where

$$L_q \equiv g\sqrt{\Delta^>(\omega = 0, q)} S_{\mathbf{q}\cdot\mathbf{r}} \quad \text{and} \quad \overline{L}_q \equiv g\sqrt{\Delta^>(\omega = 0, q)} C_{\mathbf{q}\cdot\mathbf{r}}$$

and $\Delta^> = \langle A_0 A_0 \rangle$ in the HTL approximation, $S_{\mathbf{qr}} = 2\sin\left(\frac{\mathbf{qr}}{2}\right)$ and $C_{\mathbf{qr}} = 2\cos\left(\frac{\mathbf{qr}}{2}\right)$.

The spherical harmonics expansion

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Approach

$$S_{\mathbf{qr}} = 2\sin\left(\frac{\mathbf{qr}}{2}\right) = -i\left(\mathrm{e}^{i\frac{\mathbf{qr}}{2}} - \mathrm{e}^{-i\frac{\mathbf{qr}}{2}}\right)$$

and then we can make use of the identity

$$\mathrm{e}^{i\mathbf{kr}} = 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^{l} i^l j_l(kr) Y_l^m(\Omega_k) Y_l^{m*}(\Omega_r)$$

Rewriting the Lindblad equation

$$\int \frac{d^3q}{(2\pi)^3} \to \int_0^\infty dq q^2 \sum_{t=0}^{\infty} \sum_m$$

$$L_{\mathbf{q}} \to g\sqrt{\frac{2\Delta^{>}(0,q)}{\pi}} j_{2t+1}\left(\frac{qr}{2}\right) Y^m_{2t+1}(\Omega_r)$$

and

$$\overline{L}_{\mathbf{q}} \to g\sqrt{\frac{2\Delta^{>}(0,q)}{\pi}} j_{2t}\left(\frac{qr}{2}\right) Y^m_{2t}(\Omega_r)$$

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Equivalent

Same physics. We just trade a 3-D parameter $\mathbf{q}$ by a continous parameter q and two discrite ones (t and m).

Example. Plane wave expansion of Laine's potential

$$\mathrm{Im}\, V_s(r) = -\frac{g^2 C_F}{8\pi^2} \int_0^\infty dq q^2 \Delta^>(0,q) \left(1 - \frac{\sin(qr)}{qr}\right) =$$

$$= -\frac{g^2 C_F}{4\pi^2} \sum_{t=0}^{\infty} (4t+3) \int_0^\infty dq q^2 \Delta^>(0,q) \left(j_{2t+1}\left(\frac{qr}{2}\right)\right)^2$$

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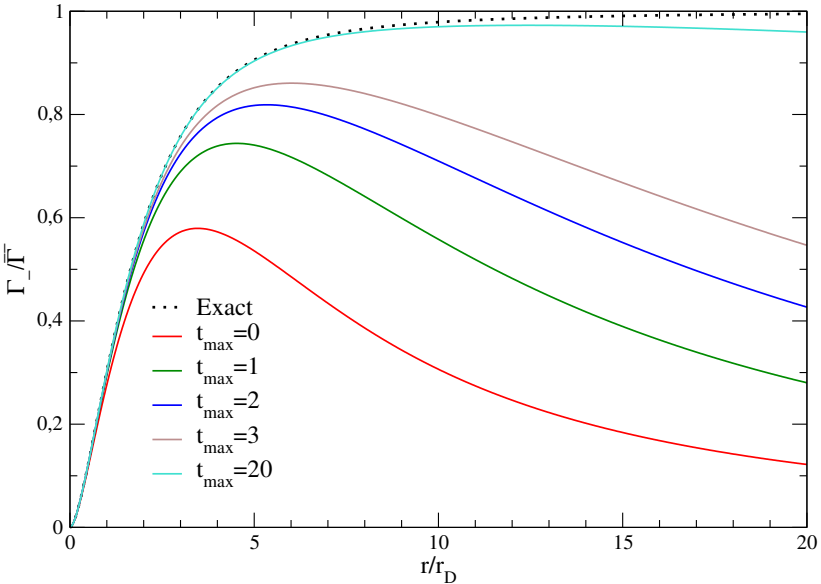
Cross-check

Can be also shown to be true using the following identities from Abramowitz and Stegun

$$\sum_{n=0}^{\infty}(2n+1)(j_n(z))^2 = 1 \quad \sum_{n=0}^{\infty}(-1)^n(2n+1)(j_n(z))^2 = \frac{\sin(2z)}{2z}$$

t expansion of Laine's potential



Structure of a jump

A jump is parametrized by the parameters of the corresponding jump operator producing it plus the out-going angular momentum

$$P_{jump}(i, q, t, l)$$

To simplify the implementation of the code, we can think of it in the following way

$$P(i)P(q|i)P(t|q, i)P(l|t, q, i)$$

$P(i)$

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- $\Gamma_o = \sum_{i=0}^{2} \Gamma_o^i$. We generate a random number x between 0 and 1.
  - If $x < \frac{\langle \Psi | \Gamma_o^0 | \Psi \rangle}{\langle \Psi | \Gamma_o | \Psi \rangle}$, then $i = 0$.
  - Elseif $x < \frac{\langle \Psi | \Gamma_o^0 + \Gamma_0^1 | \Psi \rangle}{\langle \Psi | \Gamma_o | \Psi \rangle}$, then $i = 1$.
  - Else, $i = 2$.

Example: $P_s(q|0)$

$$P_s(q|0) = \frac{\frac{g^2 C_F}{4\pi^2} q^2 \Delta^{>}(0,q) \langle \Psi | 1 - \frac{\sin(qr)}{qr} | \Psi \rangle}{\langle \Psi | \Gamma_s | \Psi \rangle}$$

Probability distribution. We can get random values of q following this distribution using standard techniques. For example, we use the accept-reject method.
However, this is slow. We can do better by understanding this distribution as a 2D distribution for q and r. Using accept-reject we can get q and r values. Then, we take only the q value.

$P(t|q_c, 0)$

$$P(t|q_c,0) = (4t+3)\frac{\langle\Psi|\left(j_{2t+1}\left(\frac{q_c r}{2}\right)\right)^2|\Psi\rangle}{2\langle\Psi|1-\frac{\sin(q_c r)}{q_c r}|\Psi\rangle}$$

Similarly to $P(i)$, we can decide which t is taken generating a random number.

Orbital angular momentum

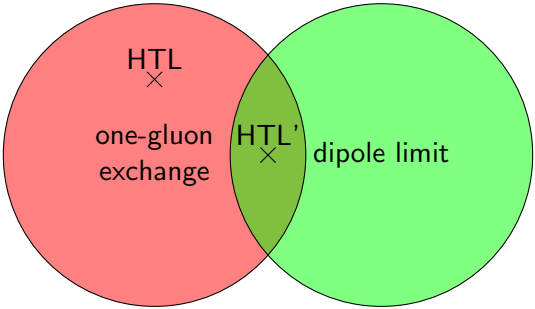
Once we know t, we know that the jump operator is going to act with Y^m_{2t+1}. Using the Wigner-Eckart theorem, we can prove that

$$\sum_{m,m'} Y^m_{2t+1} |l'm'\rangle\langle l'm'| Y^{m*}_{2t+1} = \sum_{m''} p_{l'\to l} |lm''\rangle\langle lm''|$$

$p_{l'\to l}$ is a probability that can be computed using Crebsch-Gordon coefficients.

$$p_{l'\to l} = |\langle l, 0; 2t+1, 0 | l', 0\rangle|^2$$

Cross-check, recovering the dipole limit



We need to propose a new master equation (HTL') that can be modeled with both versions of *Qtraj*. It doesn't need to represent any physical situation.

HTL in the $rm_D \ll 1$ limit

The naive expansion

$$\Gamma_s(\hat{r}) = \frac{g^2 C_F}{2\pi^2} \int dq q^2 \Delta^<(q) \left(1 - \frac{\sin(q\hat{r})}{q\hat{r}}\right)$$

If we expand naively for small r

$$\Gamma_s(\hat{r}) \approx \frac{g^2 C_F \hat{r}^2}{12\pi^2} \int dq q^4 \Delta^<(q)$$

However, the integral diverges in the UV. This is because the HTL approximation is not valid for large q. However, taking the full one-loop $\Delta^<(q)$ in thermal field theory would be too complicated for our purposes.

HTL in the $rm_D \ll 1$ limit

Integration by regions

We can split the integral in two regions. Regulate each piece in dimensional regularization and then add them. One regions is $q \sim m_D$ and the other $q \sim \frac{1}{r}$.

$$\Gamma_s(\hat{r}) = \kappa(\hat{r})\hat{r}^2$$

with

$$\kappa(\hat{r}) = \alpha_s C_F \frac{m_D^2 T}{6}\left[\frac{1}{3}(4 - 3\gamma_E - 3\log 2) - \log\left(\frac{m_D \hat{r}}{2}\right)\right]$$

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- The answer is no.

The modified HTL

There are several ways to modify HTL such that in the $rm_D \ll 1$ limit we get $\Gamma_s = \kappa \hat{r}^2$ with κ independent of r.

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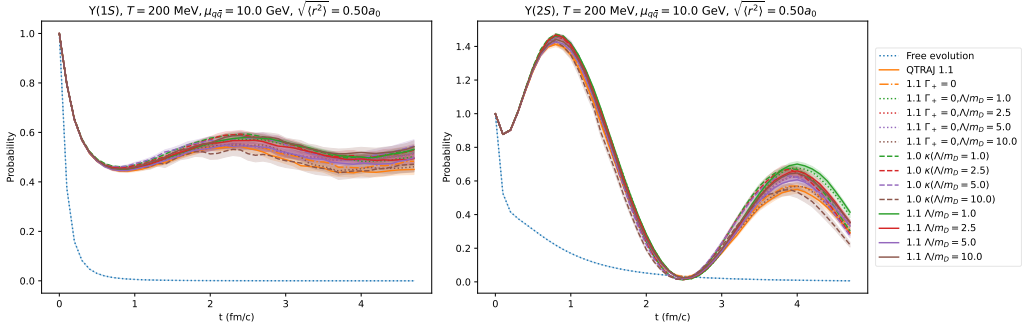
1. Take $\Gamma_s = \kappa(a_0)\hat{r}^2$ with a_0 the Bohr radius of the ground state. We did not follow this approach.
2. Introduce an UV cut-off in all q integrals. $\Delta^<(q) \to \Delta^<(q)\theta(\Lambda_{UV} - q)$.

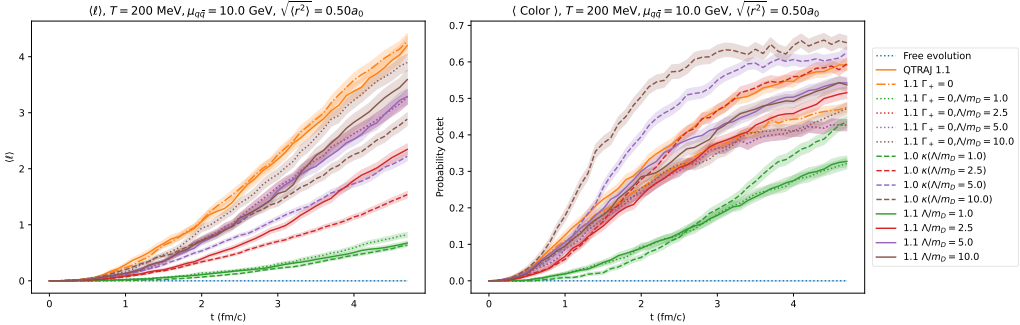
Comparison of both versions of Qtraj

Survival probability at constant temperature.



Comparison of both versions of Qtraj

Average orbital momentum and color.



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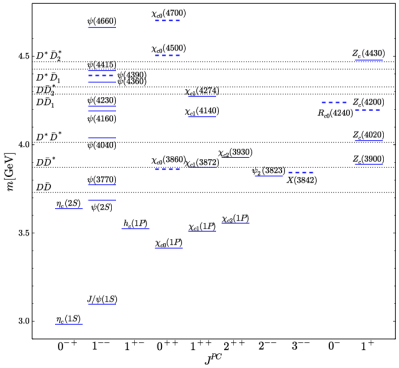
  - However, relaxing the dipole approximation opens an additional channel. An octet to octet jump operator proportional to $\overline{L}_q$. Note that this goes to a constant in the dipole limit.
- The code is working as expected.

Work done in collaboration with Nestor Armesto, Elena G. Ferreiro and Victor López-Pardo. Phys.Lett.B 854 (2024) 138760 and arXiv:2512.11539.

Quarkonia exotics

- There are Quarkonium-like particles that can not be explained by the simple quark-antiquark model.



Picture taken from Physics Reports 873 (2020)

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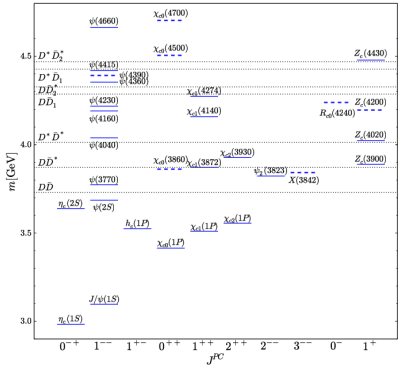

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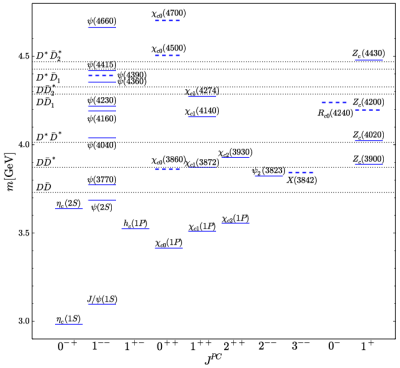

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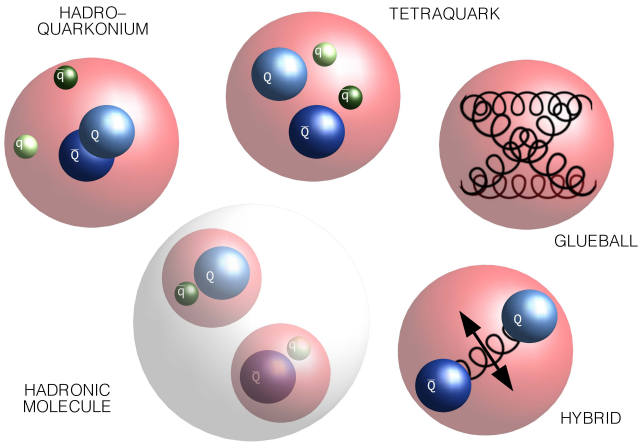

Picture taken from `https://www.fz-juelich.de/en/ias/ias-4/research/exotic-hadrons/exotics_pad.jpg`.

The internal structure of the $X(3872)$

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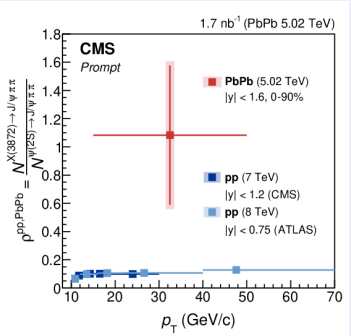
The $X(3872)$ in heavy-ion collisions



Picture taken from Phys.Rev.Lett. 128 (2022) 3, 032001

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- The effect of light particles and gluons can be encoded in a potential computed assuming that the heavy quarks are frozen and separated a given distance r.
- Two step-approximation:
  - Compute the potential taking the heavy quarks as static color sources.
  - Solve the Schrödinger equation.

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- We use data taken from Phys. Rev. D 99(3), 034502 (2019).

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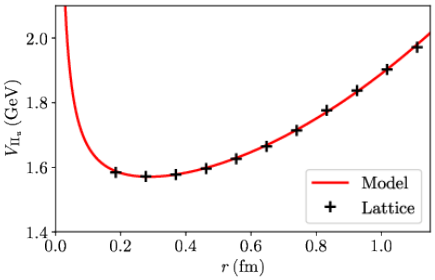

This potential is well fitted by the formula

$$V(r,0) = \frac{A_{-1}}{r} + A_0 + A_2 r^2 \,.$$

Lattice data is not sensitive to large r behavior. But, using Effective String Theory results, we know that at very large r it only grows linearly.

The finite T potential

The real part

We use the following assumption

$$V(\mathbf{p}) = \frac{V_{vac}(\mathbf{p})}{\epsilon(\mathbf{p}, m_D)},$$

where ϵ is the medium permittivity in the HTL approximation and m_D is the Debye mass.

$$\Re[V(r, m_D)] = A_{-1}\left(m_D + \frac{e^{-m_D r}}{r}\right) + A_0$$
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Rationale

This model was able to describe lattice quarkonium potential at finite T using m_D as a fitting parameter (Phys. Rev. D 101(5), 056010 (2020)).

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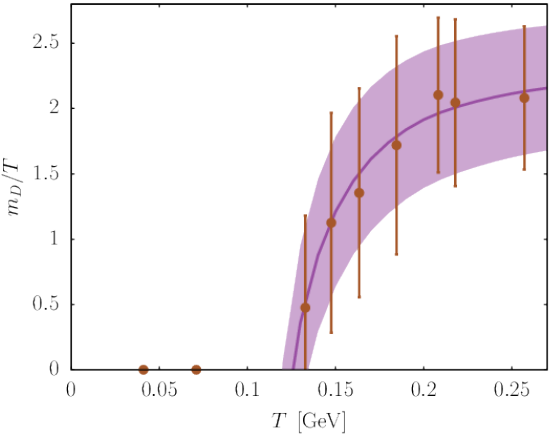

Plot taken from Phys. Rev. D 101(5), 056010 (2020)

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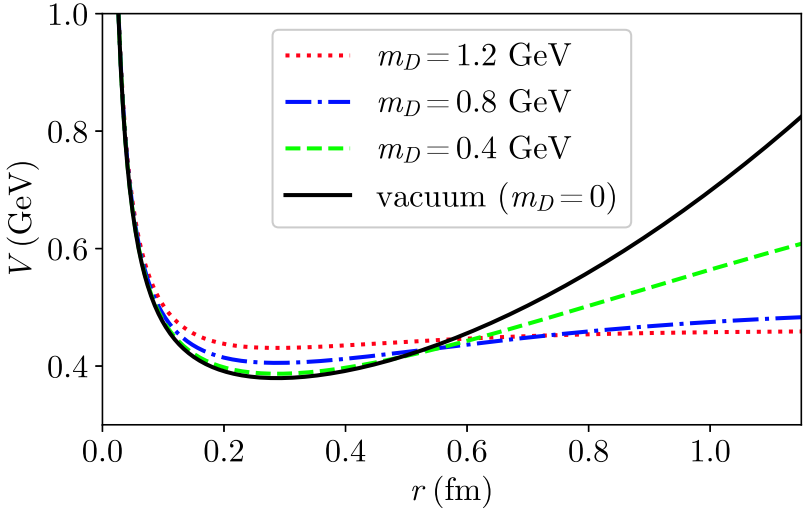

The finite T potential

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The imaginary part of the potential of *conventional* quarkonium has the following properties:

The finite T potential

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The imaginary part of the potential of *conventional* quarkonium has the following properties:

- At short distances it goes like r^2 because the medium sees quarkonium as a small dipole.

The finite T potential

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The imaginary part of the potential of *conventional* quarkonium has the following properties:

- At short distances it goes like r^2 because the medium sees quarkonium as a small dipole.
- At long distances, the heavy quarks are not correlated. Therefore, the imaginary part of the potential is equal to $-i$ times the decay width of a single heavy quark.

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- Between these two limits it is a smoothly increasing function.

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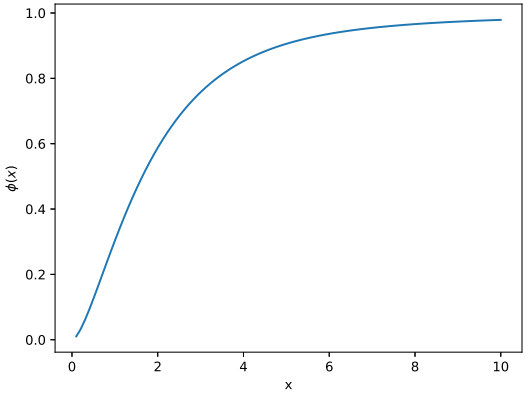
For example, in the HTL approximation

$$\Im V(r) = -\alpha_s C_F T \phi(m_D r)\,.$$



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In a tetraquark state treated in the BO approximation, the heavy quarks are in an octet state.

- When $r \to 0$ the medium sees the heavy quark pair as a non-relativistic heavy gluon. The imaginary part of the potential will be $-i/2$ times the decay width of a heavy gluon.

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- At intermediate distances we expect that the imaginary part of the potential is a smooth function that interpolates between the two regimes.
- In the large N_c limit the decay width of a heavy gluon is equal to that of two heavy quarks.
- Therefore, we can take the imaginary part of the potential to be a constant.

The Decay Width

$$\Gamma = A_{-1}\, T + A_2 \frac{T}{m_D^3}\,.$$

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- Dimensional analysis.
- It is a educated guess. However, note that our aim is to get a qualitative understanding.

Coalescence from the Lindblad equation

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- Molecular chaos hypothesis. Unbound heavy quarks are uncorrelated.
- How many unbound states jump into bound states per unit of time? From these, how many survive until freeze-out?

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1. The effective Hamiltonian can be diagonalized in *bound* and *free* states.
2. Using projectors P_b and P_s, we can regard the Lindblad equation as two coupled equations for the bound and free density matrices.

$$\begin{aligned}\partial_t \rho_b &= -iH_{\text{eff}}\rho_b + i\rho_b H_{\text{eff}}^\dagger \\ &\quad + \sum_n (P_b C_n \rho_f C_n^\dagger P_b + P_b C_n \rho_b C_n^\dagger P_b), \\ \partial_t \rho_f &= -iH_{\text{eff}}\rho_f + i\rho_f H_{\text{eff}}^\dagger \\ &\quad + \sum_n (P_f C_n \rho_f C_n^\dagger P_f + P_f C_n \rho_b C_n^\dagger P_f).\end{aligned}$$

Production of bound states

The transition rate throught channel n is given by

$$\mathrm{Tr}\Big(C_n^\dagger P_b C_n \rho_f\Big)$$

we can define the *target* density matrix as

$$\rho_{t,n} = \frac{C_n^\dagger P_b C_n}{Z_{t,n}}, \qquad Z_{t,n} = \mathrm{Tr}\Big(C_n C_n^\dagger P_b\Big) = \sum_i \langle i|\, C_n C_n^\dagger |i\rangle.$$

We can rewrite this term in terms of Wigner functions as a convolution of the thermalized free quark Wigner distribution with the Wigner distribution of the bound states *dressed* by the jump operator C_n.

From recombination time to freeze-out

After a recombination event driven by channel n, the density matrix is

$$\rho_n = \frac{P_b C_n \rho_f C_n^\dagger P_b}{R_n}$$

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- We have a model for $\Gamma = \sum_n C_n^\dagger C_n$, but not for the individual C_n's.
- Remember than in our model Γ is a constant.

The adiabatic theorem

Given a slowly varying Hamiltonian $H(t)$, if the system is initially in an eigenstate of $H(0)$, it will remain in the instantaneous eigenstate of $H(t)$ at later times, up to a phase factor. Therefore, the probability to have a given state at freeze-out is equal to the probability to recombine to this state at recombination time multiplied by its survival probability.

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Rate of recombination to a 1S state

$$R_{1S}(t) = \sum_n \langle 1S|\rho_n(t)|1S\rangle \, R_n(t) = \langle 1S| \sum_n C_n \rho_f \, C_n^\dagger |1S\rangle$$

We assume $R_{1S} = \Gamma_{1S}$.

- They are related by the fluctuation-dissipation theorem.
- This can be improved with a more detailed microscopic model.

Other challenges

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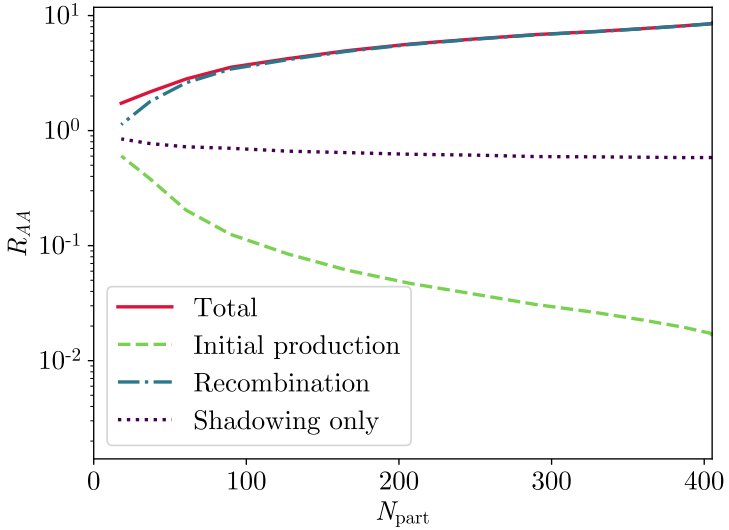
- There is no $X(3872)$ pp cross-section data at $\sqrt{s} = 5.02$ TeV. We need to extrapolate from other energies.
- Results are very sensitive to the volume of the medium. The heavy quarks need to find each other to recombine. We use a simple Bjorken model.
- Results depend quadratically on the $c\bar{c}$ cross-section. This results in an increased sensitivity to shadowing effects compared to primordial production.

Results


10^1
10^0
10^{-1}
10^{-2}
R_{AA}
0
100
200
300
400
N_{part}
Total
Initial production
Recombination
Shadowing only

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- E/T corrections allow to get close to thermalization.
- The dipole approximation can be relaxed without much increase in the computational cost.
- The $X(3872)$ state can be studied using the BO approximation and the Lindblad equation.

Thank you for your attention!

Questions?