Progress of the BELFEM code part I: New features and code validation

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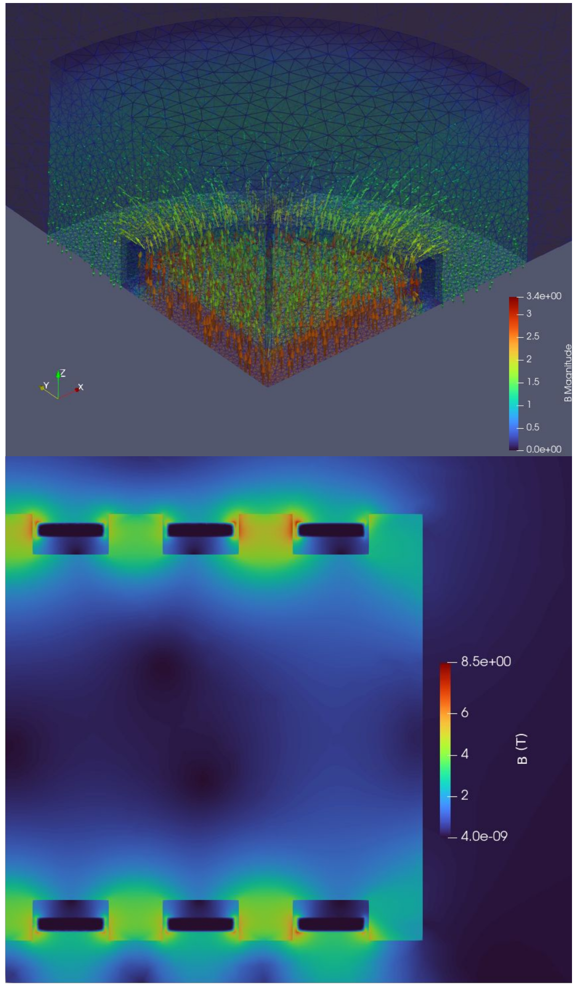
OUTLINE

- **Past milestones**
- New BELFEM features
  - 3-D thin shell model
  - Cohomology and circuit coupling
- Validation with analytical solutions
- Next steps

PAST MILESTONES

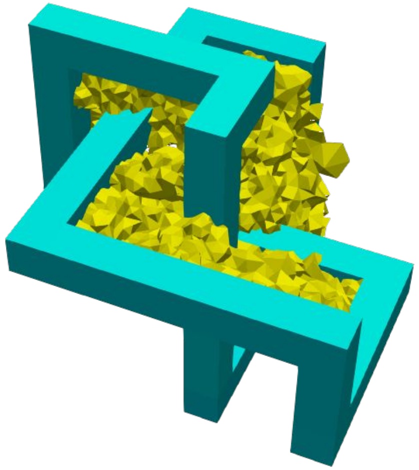
2-D and 3-D simulations with bulk conductors



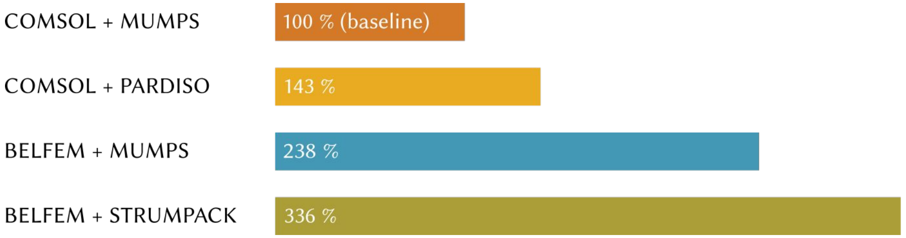
Automatic cohomology for current conditions in h-φ

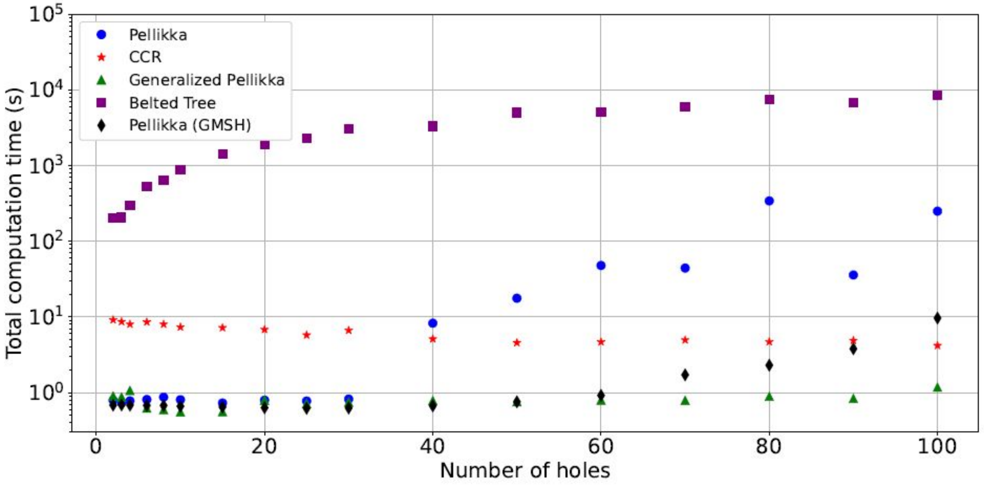
```
boundary condition
{
    current
    {
        input terminals : 4;
        type : sine ;
        amplitude : 4.5 kA ;
        frequency : 50 Hz;
        phase : 0 deg ;
    }

    bearing
    {
        nodes : 16;
    }
}
```



Comparable/advantageous performance compared to commercial software





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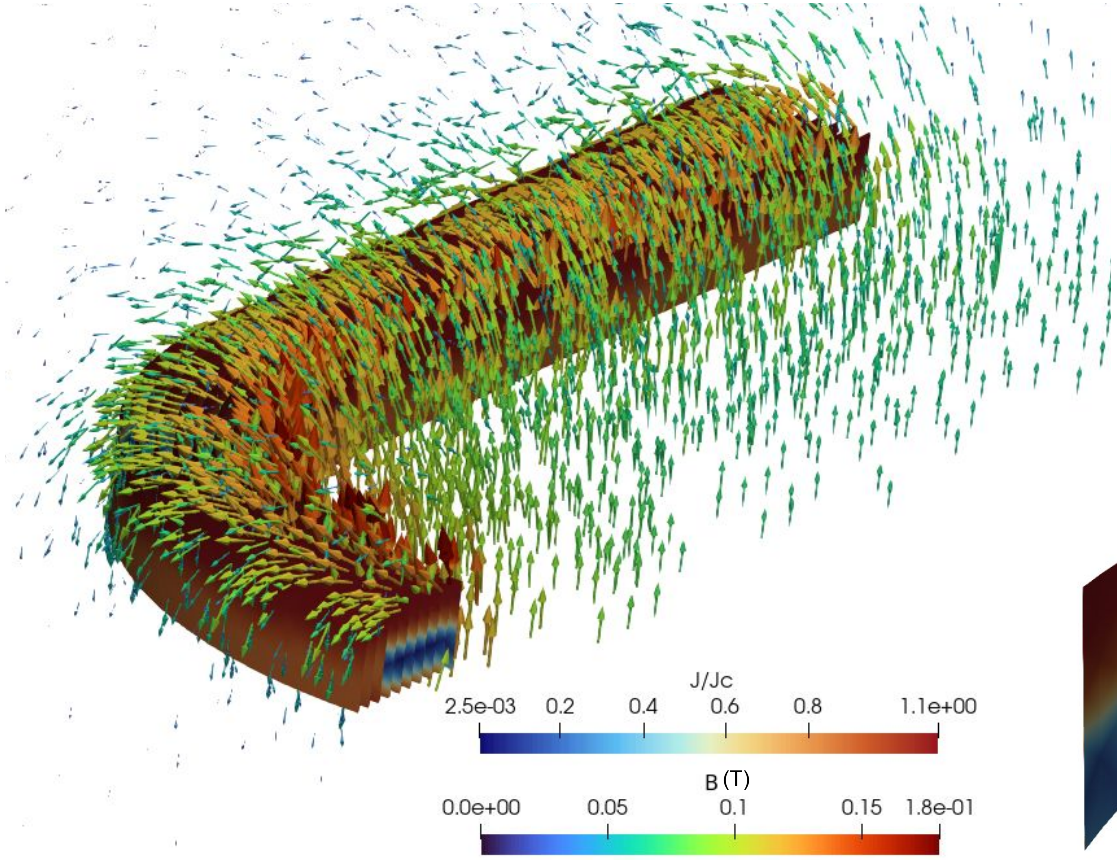
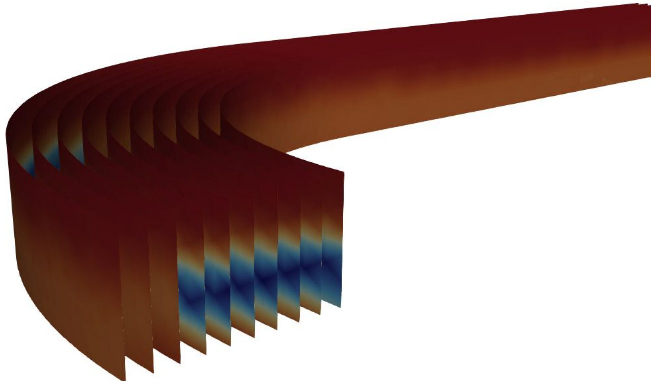
3-D THIN SHELL MODEL

Model extremely useful to optimize simulations with HTS tapes. First application of thin shells to real cable and real magnet cases (Racetrack dipole)

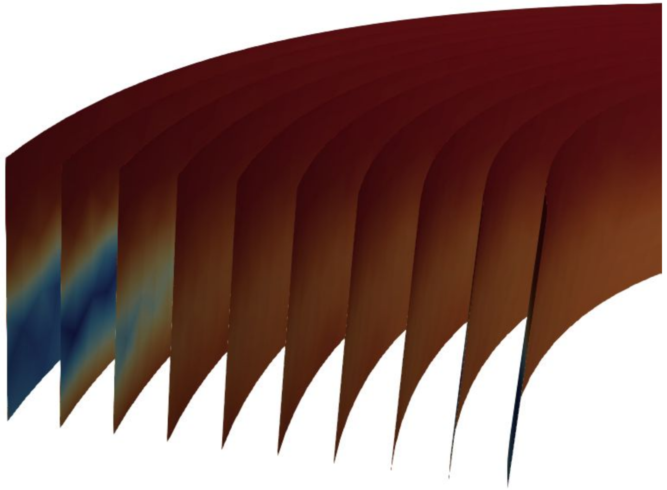


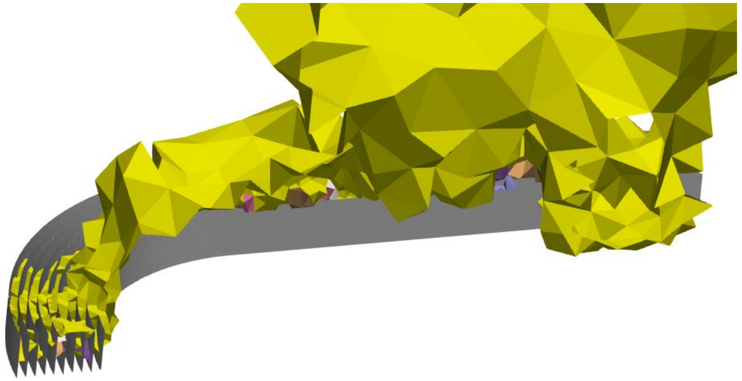


10 tapes racetrack dipole

5 thin shell layers/tape

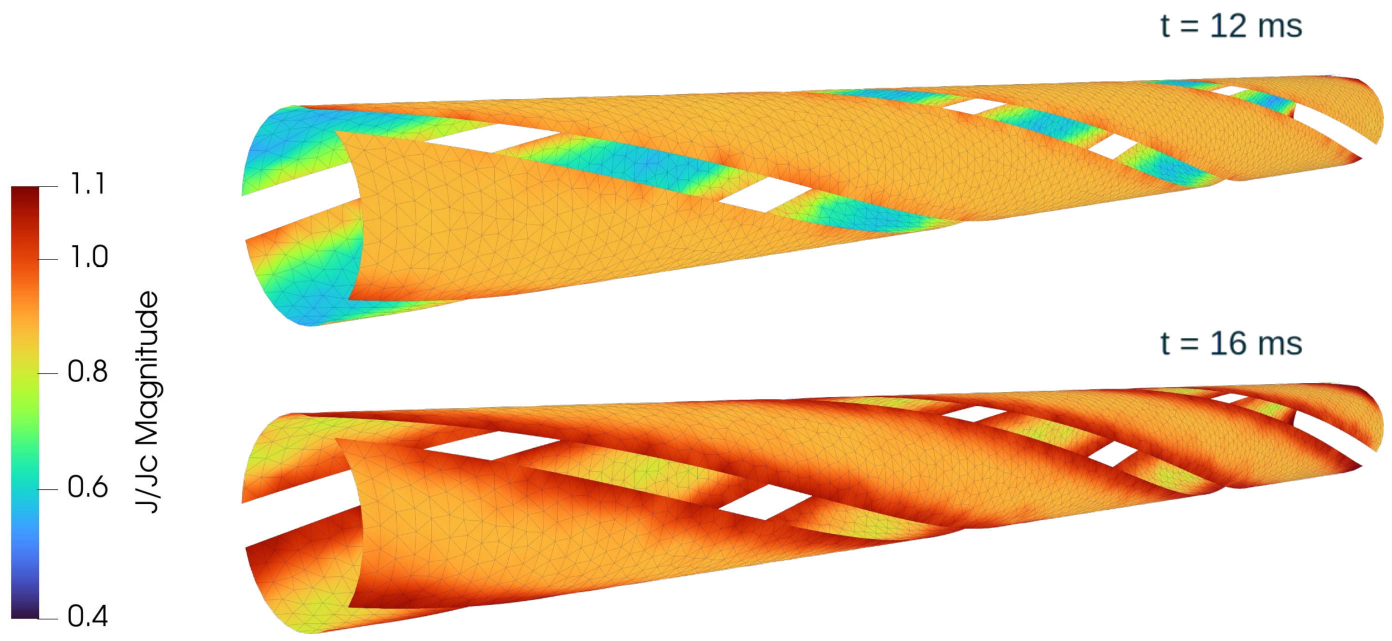
1/8th of the geometry modelled

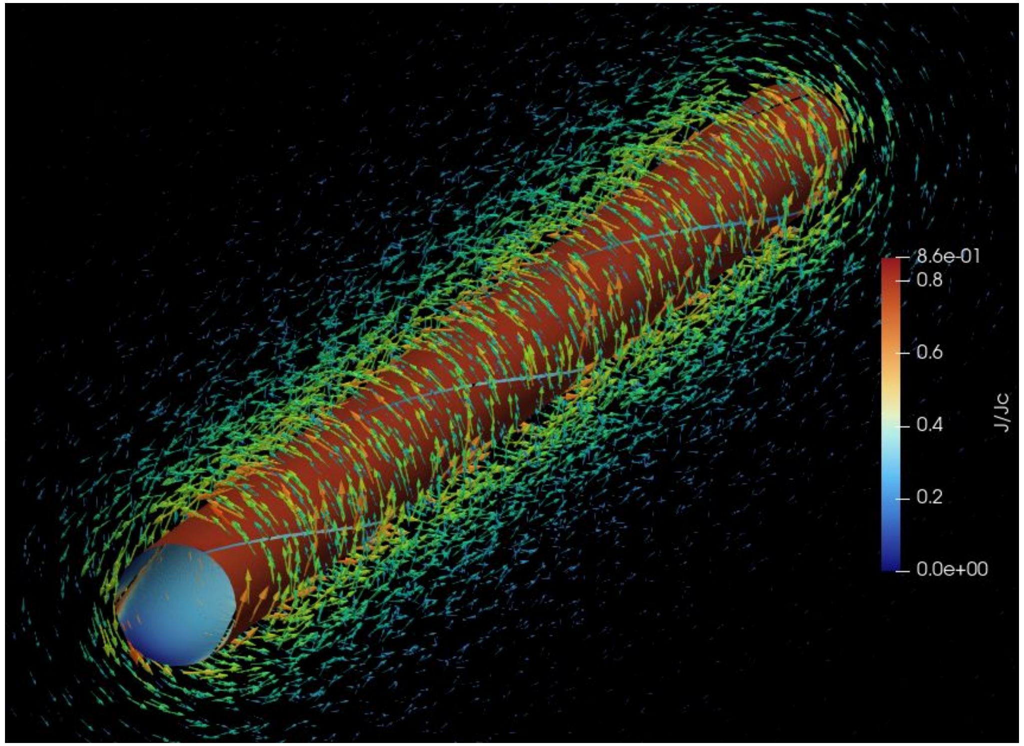




3-D THIN SHELL MODEL

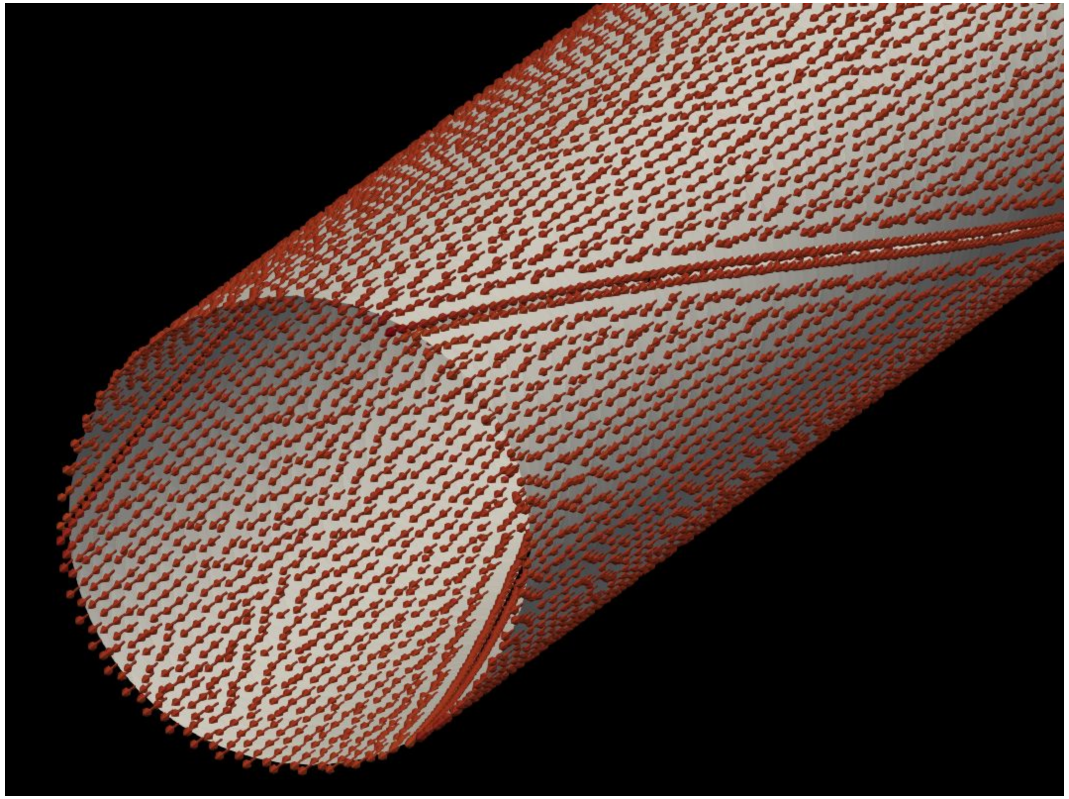
Model extremely useful to optimize simulations with HTS tapes. First application of thin shells to real cable and real magnet cases (CORC cable)



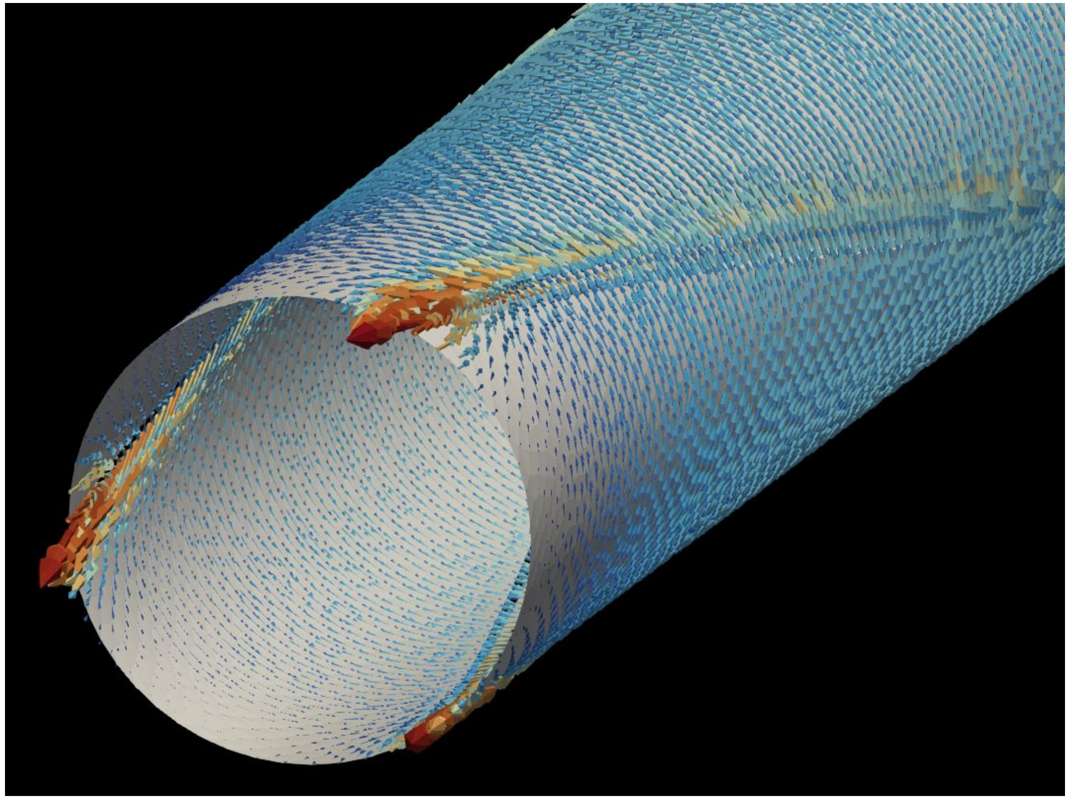


3-D THIN SHELL MODEL

Thin shells can still capture complex phenomena such as the Garber effect in CORC (current density along z on the outer layers and along φ in the inner layers)



Outer layer



Inner layer

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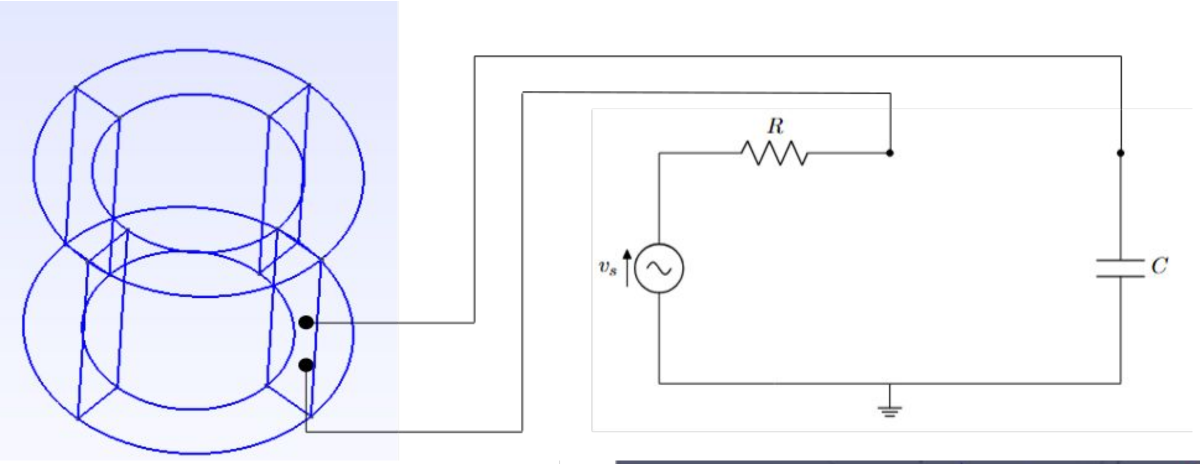
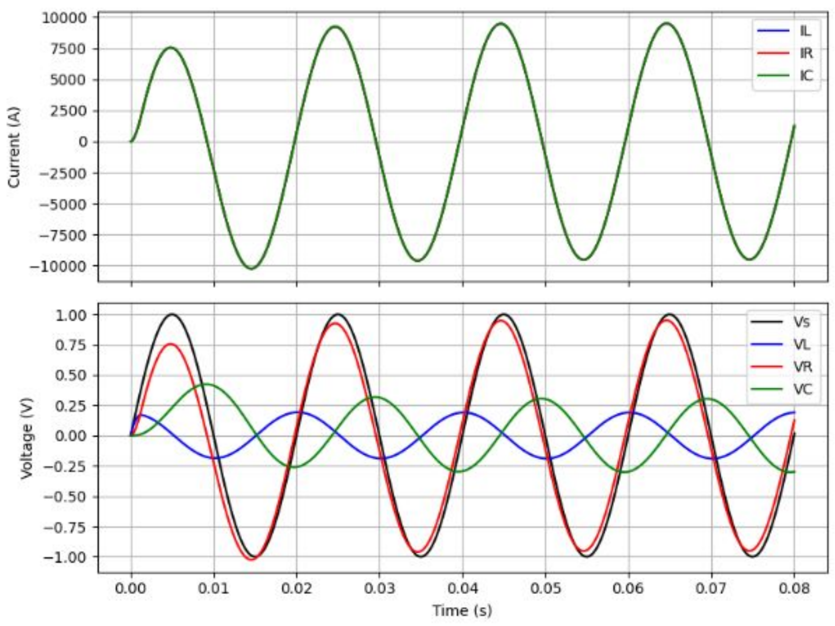
COHOMOLOGY AND CIRCUIT COUPLING

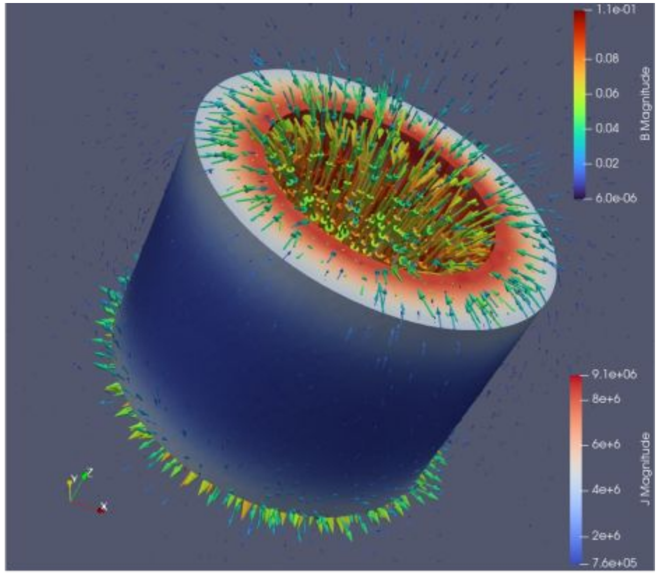
Cohomology cuts can also be used to impose a voltage.

Alternatively, the voltage can be extracted from the cut.

Using that information, we implemented a simple circuit coupling module in BELFEM within the time stepping scheme







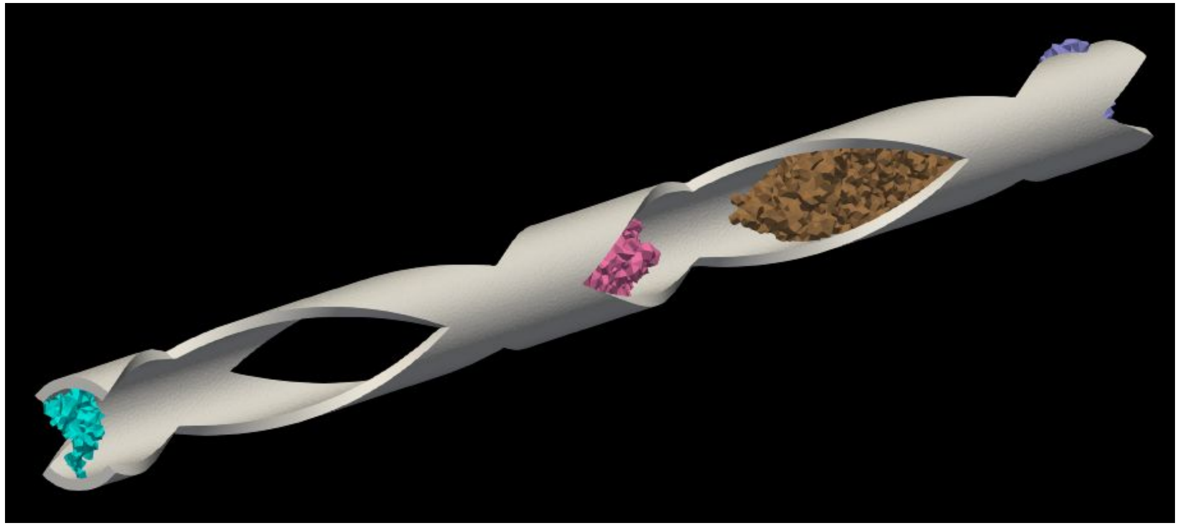
COHOMOLOGY AND CIRCUIT COUPLING

However, complex geometries with multiple terminals can also lead to “internal” or “hanging” cuts.

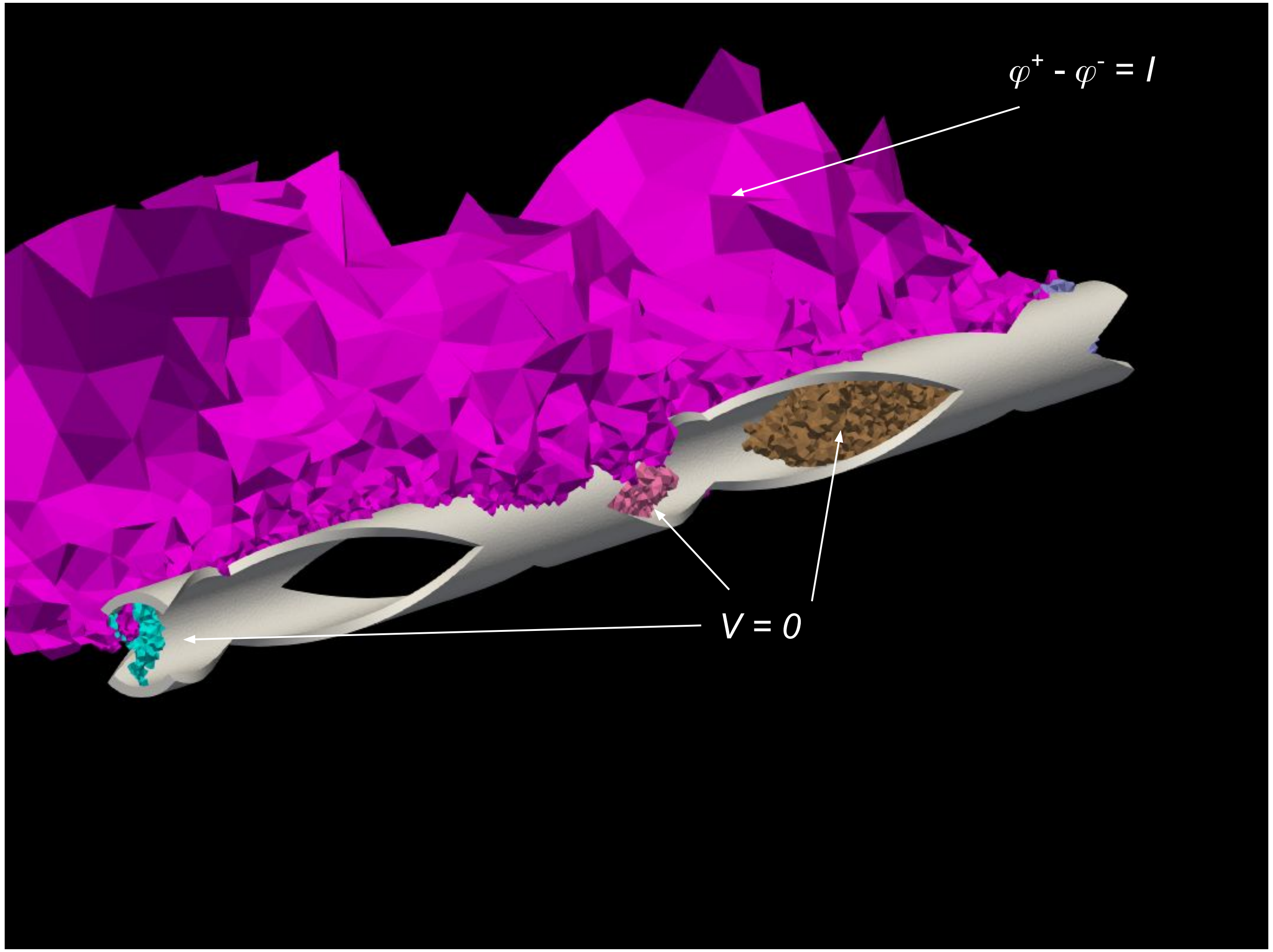
These cuts still require either a current or voltage condition, otherwise the FEM system is underdetermined

Although, we still want the user to define only pairs of terminals (in/out) for the required physics

Using only this information, we implemented a strategy to constraint every cuts for any given domain, provided a valid set of terminals from the user



COHOMOLOGY AND CIRCUIT COUPLING



The user-defined pairs of terminals allow us to find a linear combination of cuts for the driving current.

All the other cuts (inner/hanging) with a condition *V* = *0* for current conservation

We call those “free” cuts

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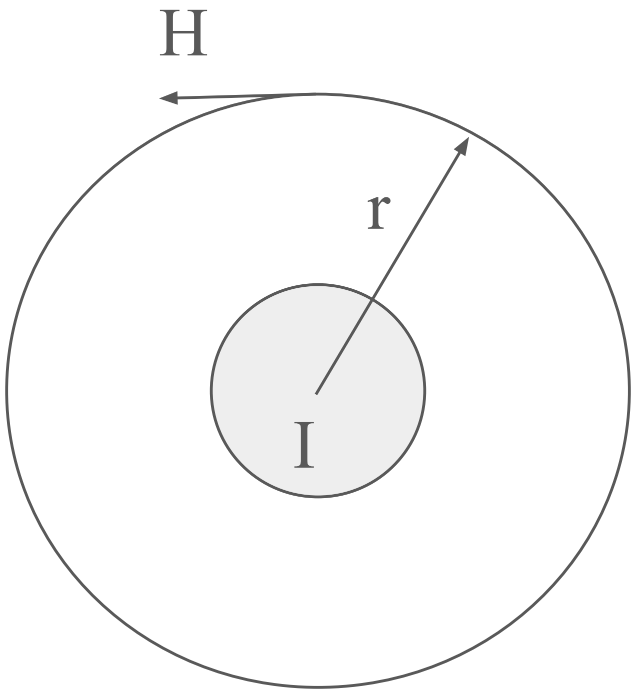
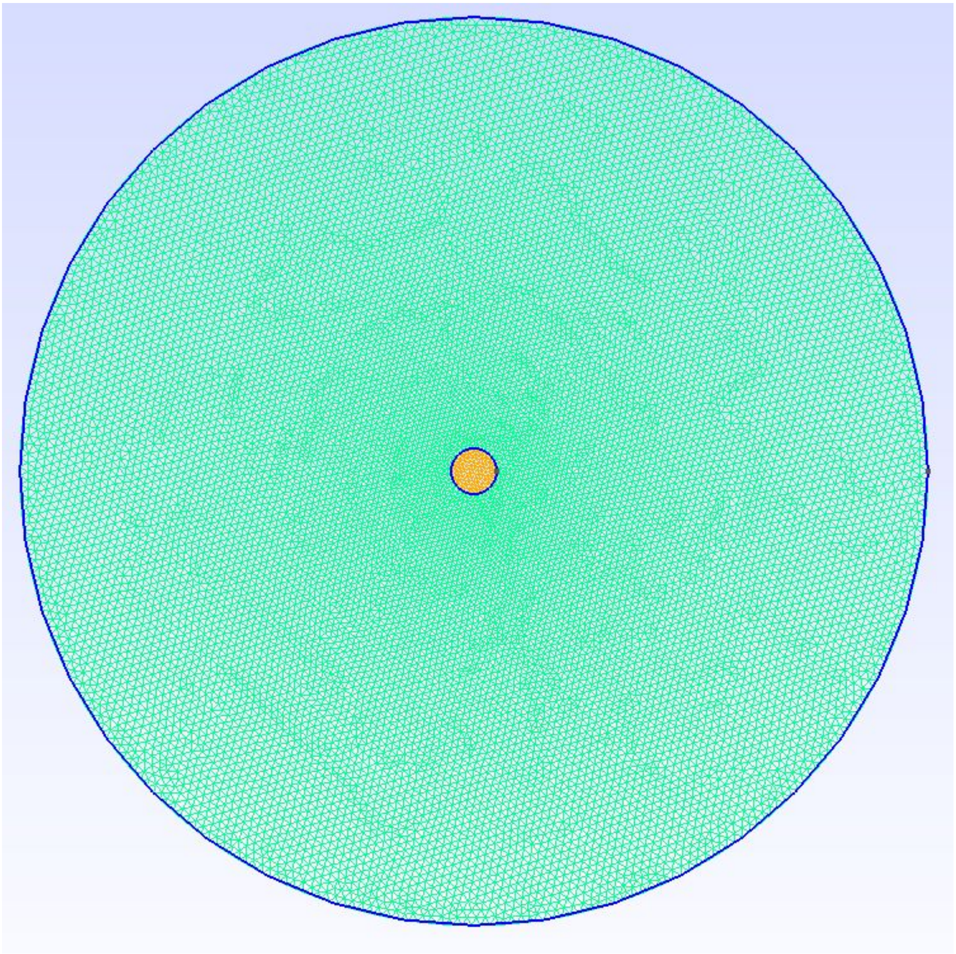
VALIDATION WITH ANALYTICAL SOLUTIONS

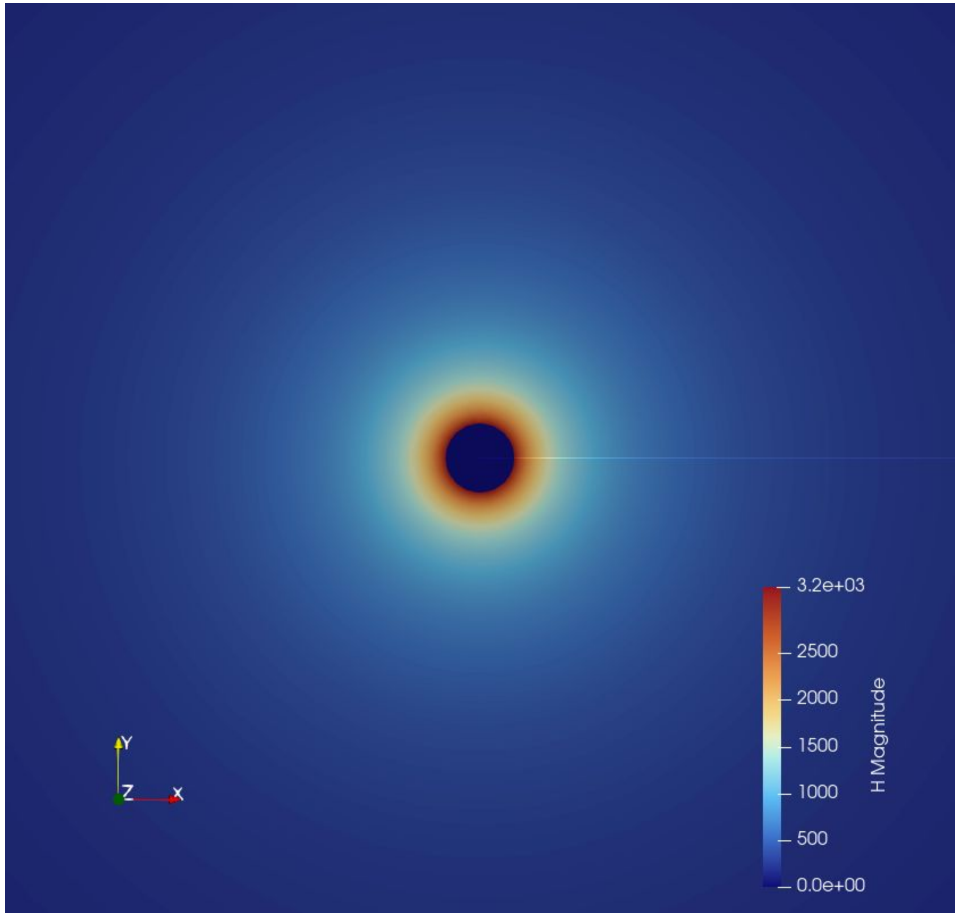
Problem 1: Infinite wire carrying a current

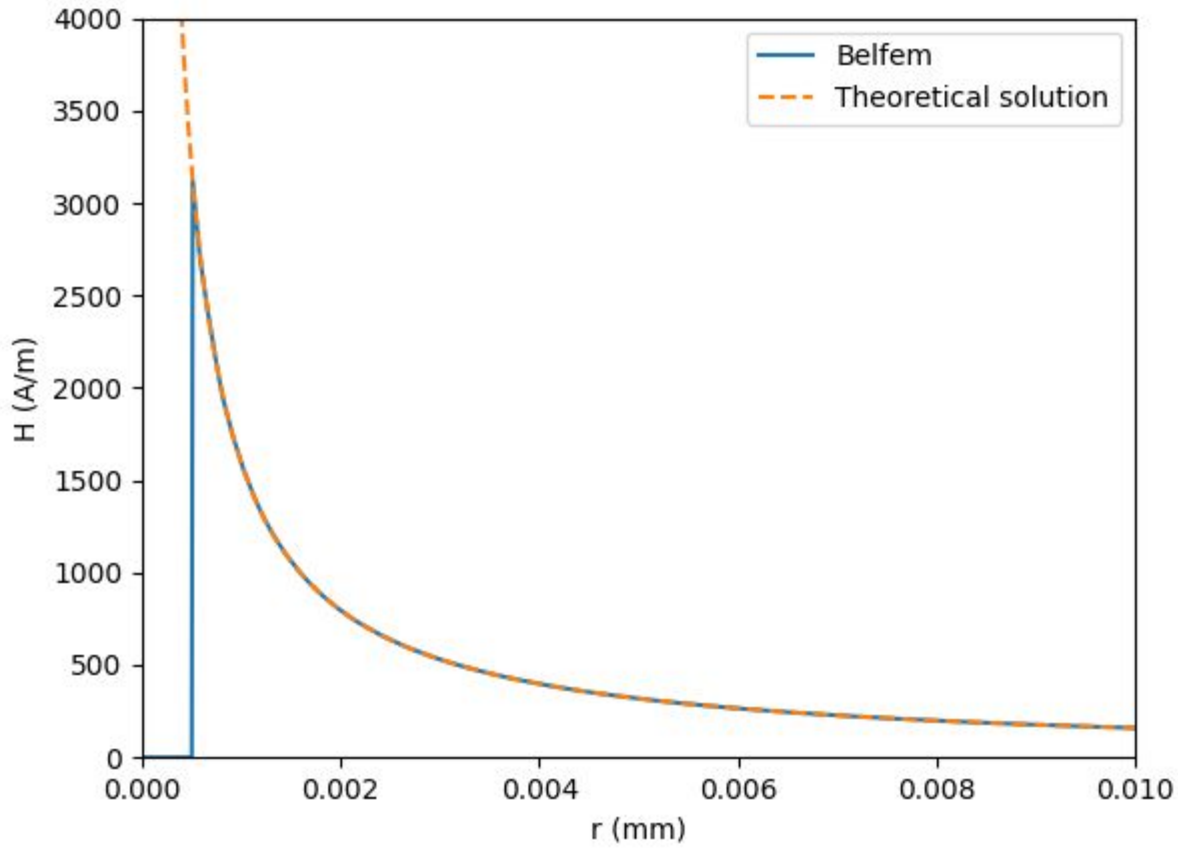
$$\oint \mathbf{H} \cdot d\mathbf{l} \Rightarrow H = \frac{I}{2\pi r}$$





VALIDATION WITH ANALYTICAL SOLUTIONS



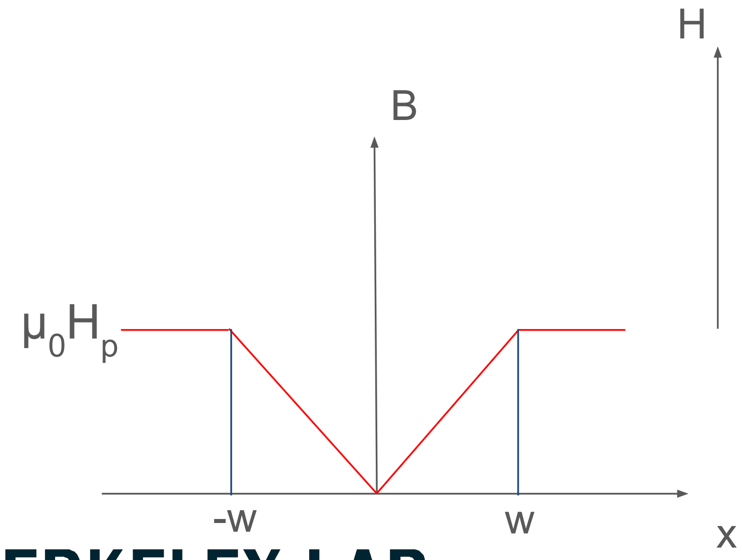


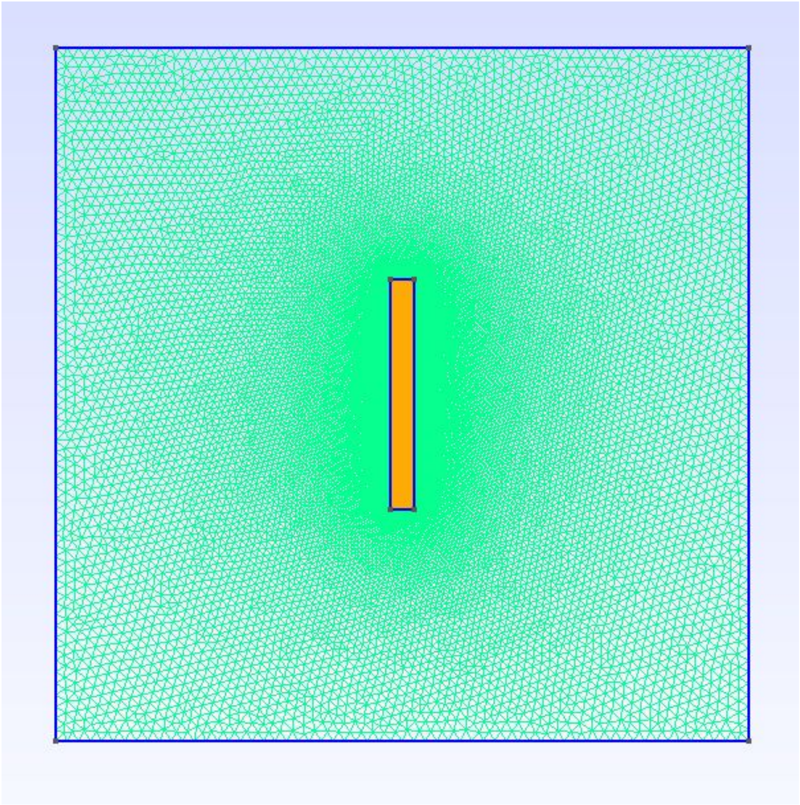
VALIDATION WITH ANALYTICAL SOLUTIONS

Problem 2: Magnetization of a type II superconducting slab

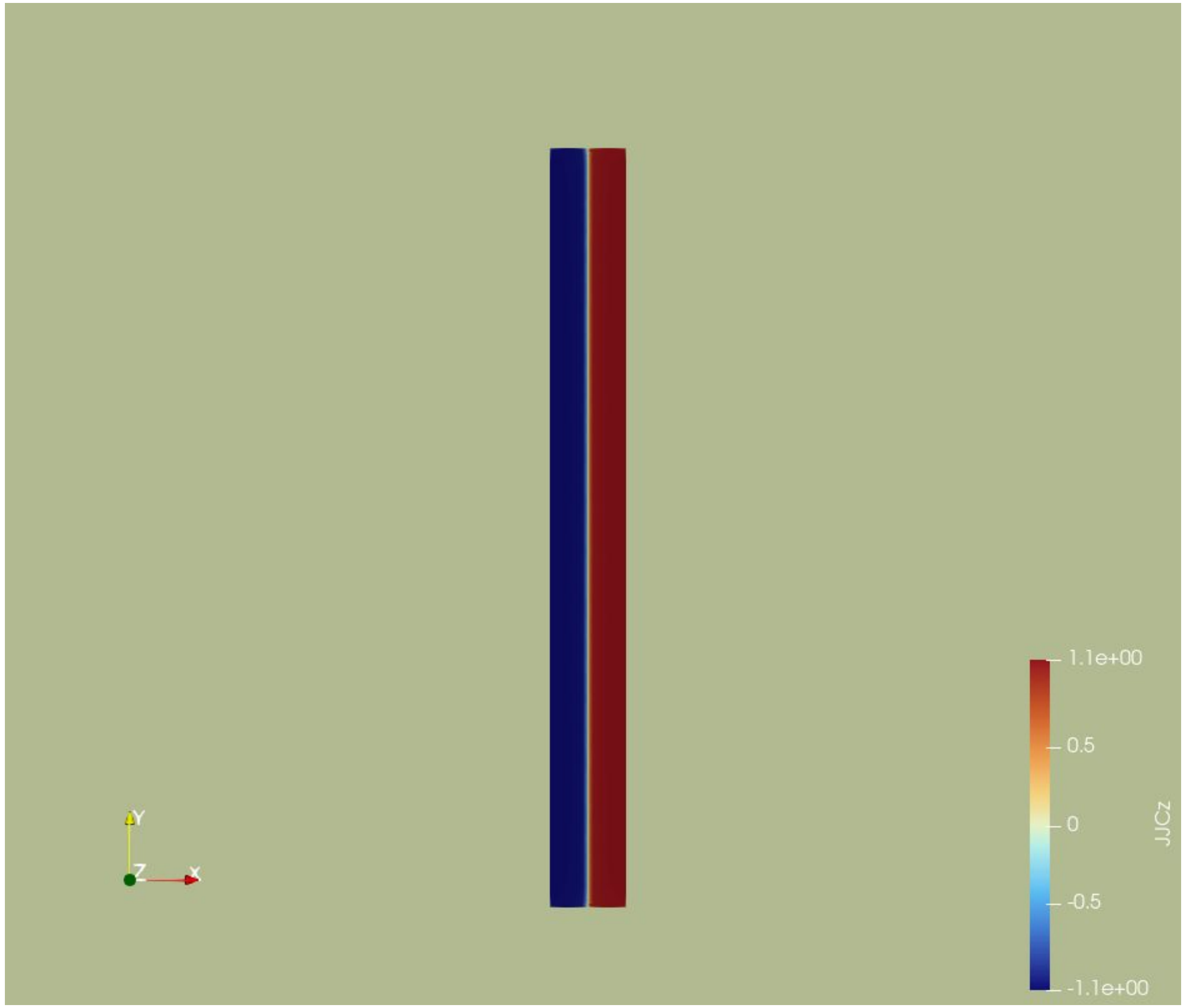
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J_c}$$

$$\frac{\partial B_z}{\partial x} = -\mu_0 J_c$$

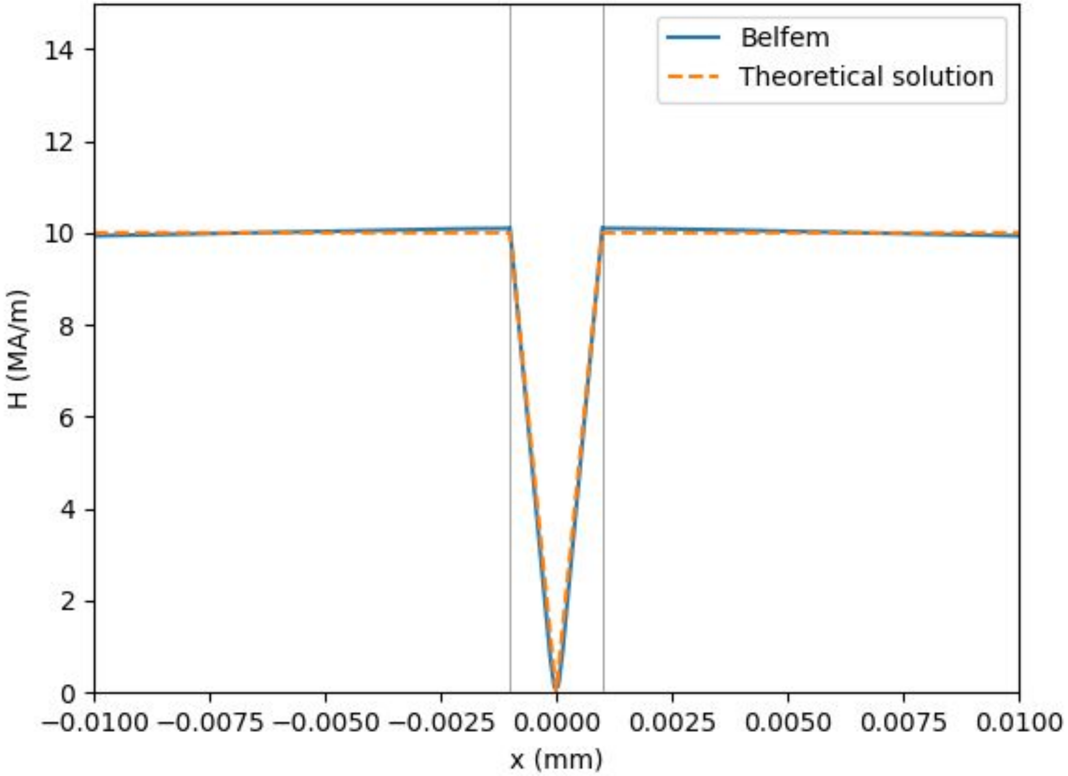




VALIDATION WITH ANALYTICAL SOLUTIONS



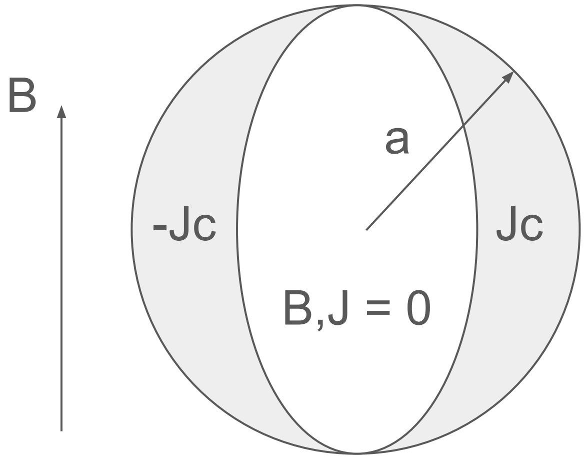
$$H_p = wJ_c \Rightarrow H_p = 10^{-3} \cdot 10^{10} = 10^7 \mathrm{A/m}$$

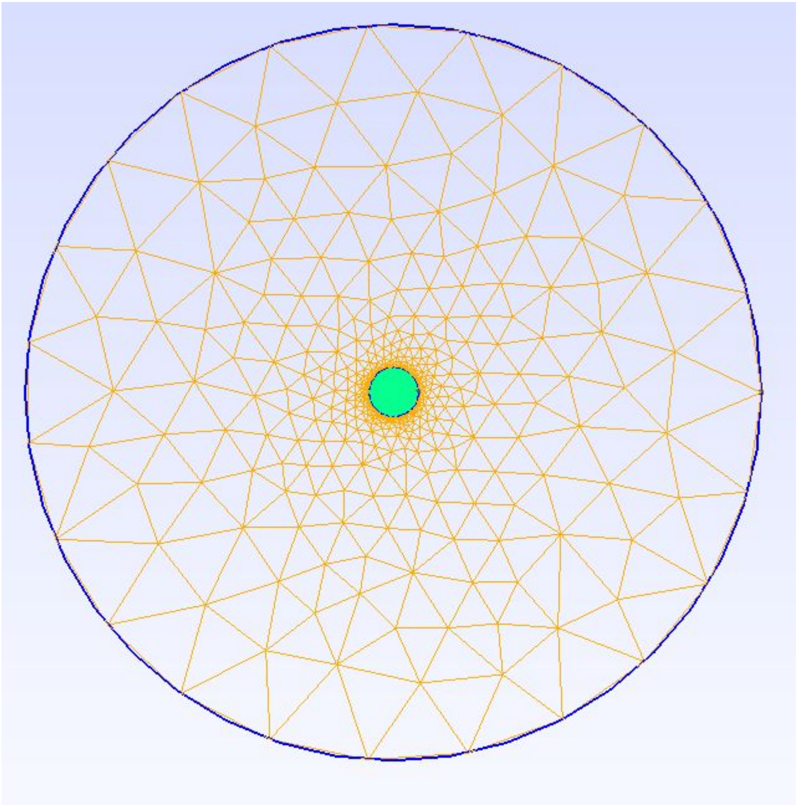


VALIDATION WITH ANALYTICAL SOLUTIONS

Problem 2: Magnetization of a type II superconducting cylindrical wire (solution from Wilson, M. N. (1983). Superconducting magnets. p 167.)

$$H_p = \frac{2aJ_c}{\pi}$$





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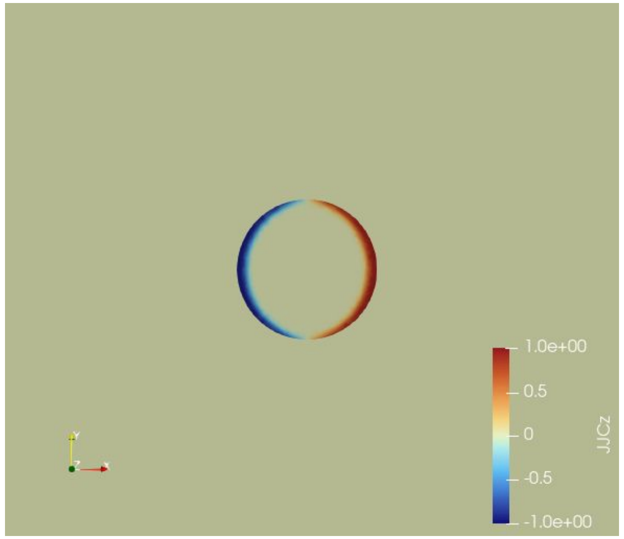
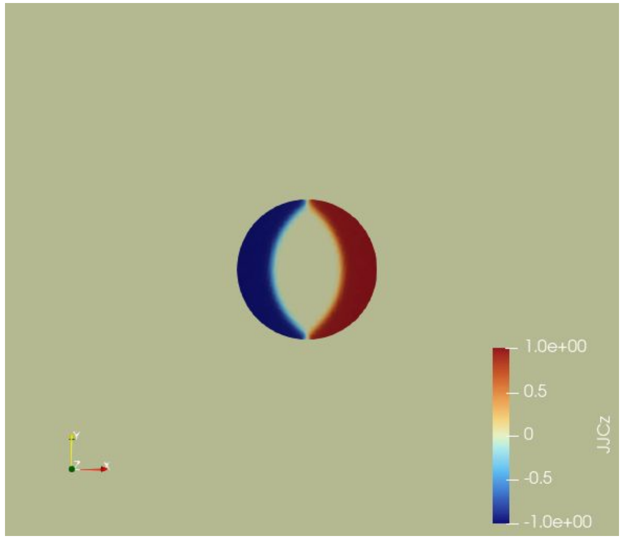
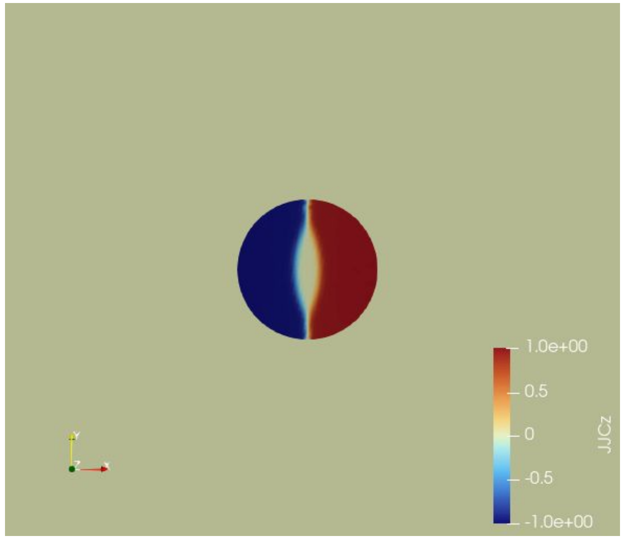
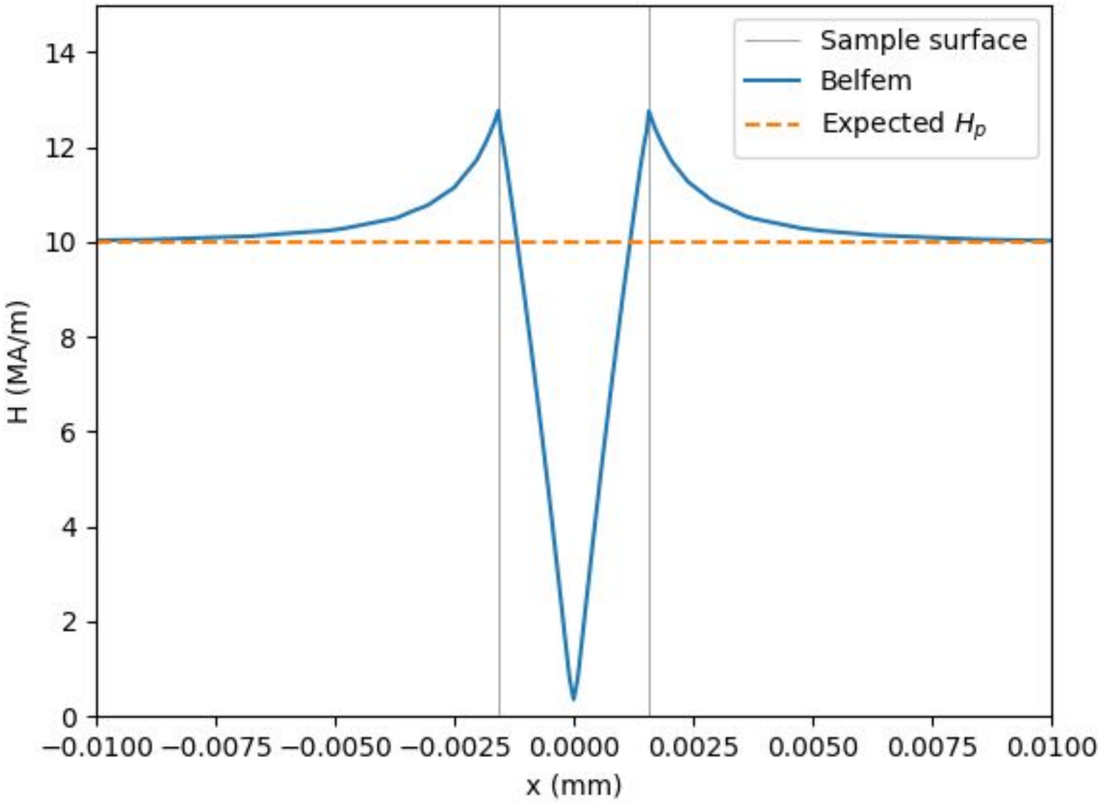






$$H_p = \frac{2J_c r}{\pi} \Rightarrow H_p = \frac{2(10^{10})(10^{-3}\pi/2)}{\pi} = 10^7 \mathrm{A/m}$$



VALIDATION WITH ANALYTICAL SOLUTIONS

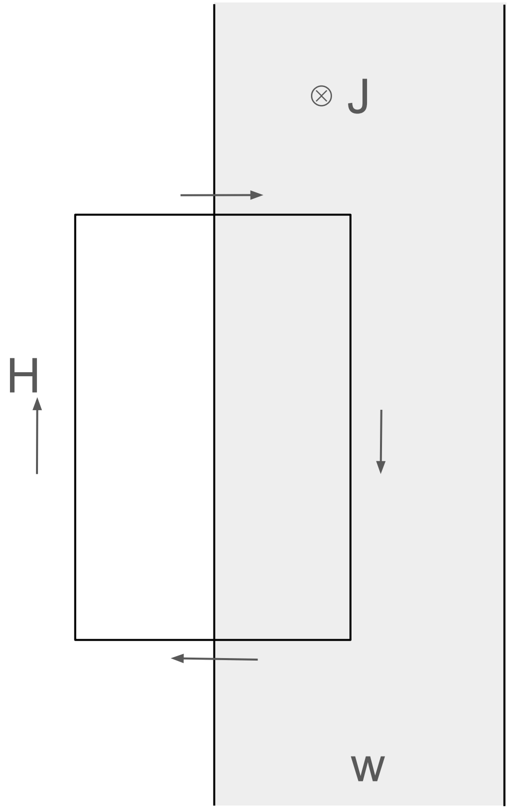
Problem 3: Field produced by planes carrying parallel and anti-parallel currents

$$\oint \mathbf{H} \cdot d\mathbf{l} = \int\int \mathbf{J} \cdot d\mathbf{s}$$

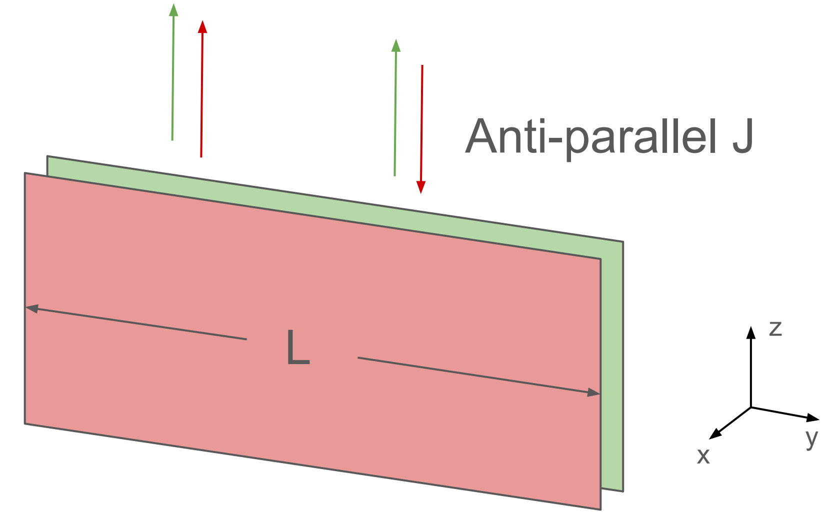
Single plane: $$H = \frac{w}{2}J$$

Double planes: $$H = \frac{w}{2}J \pm \frac{w}{2}J$$

Single plane

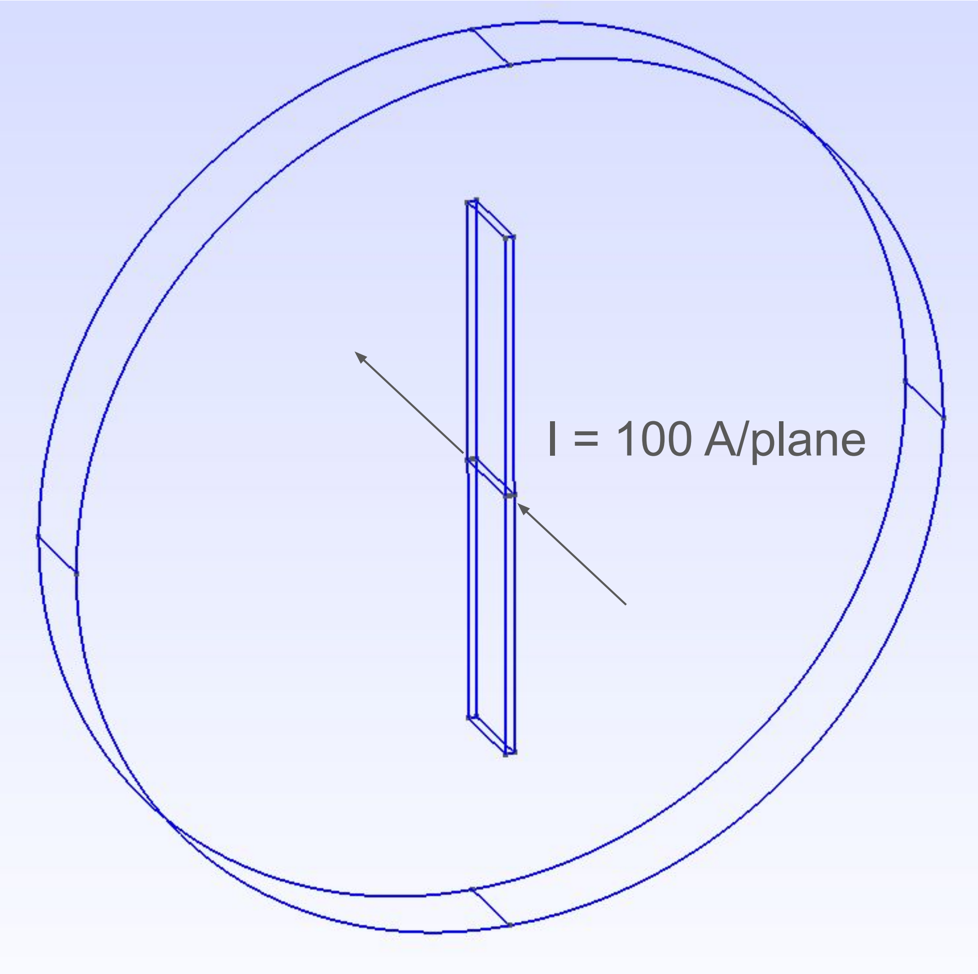


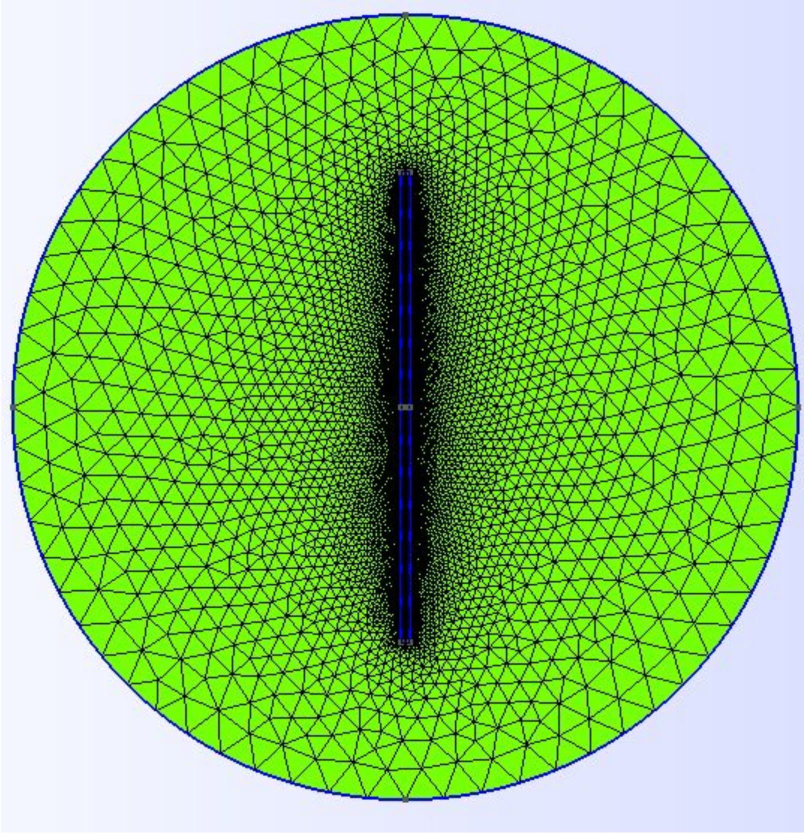
Double planes



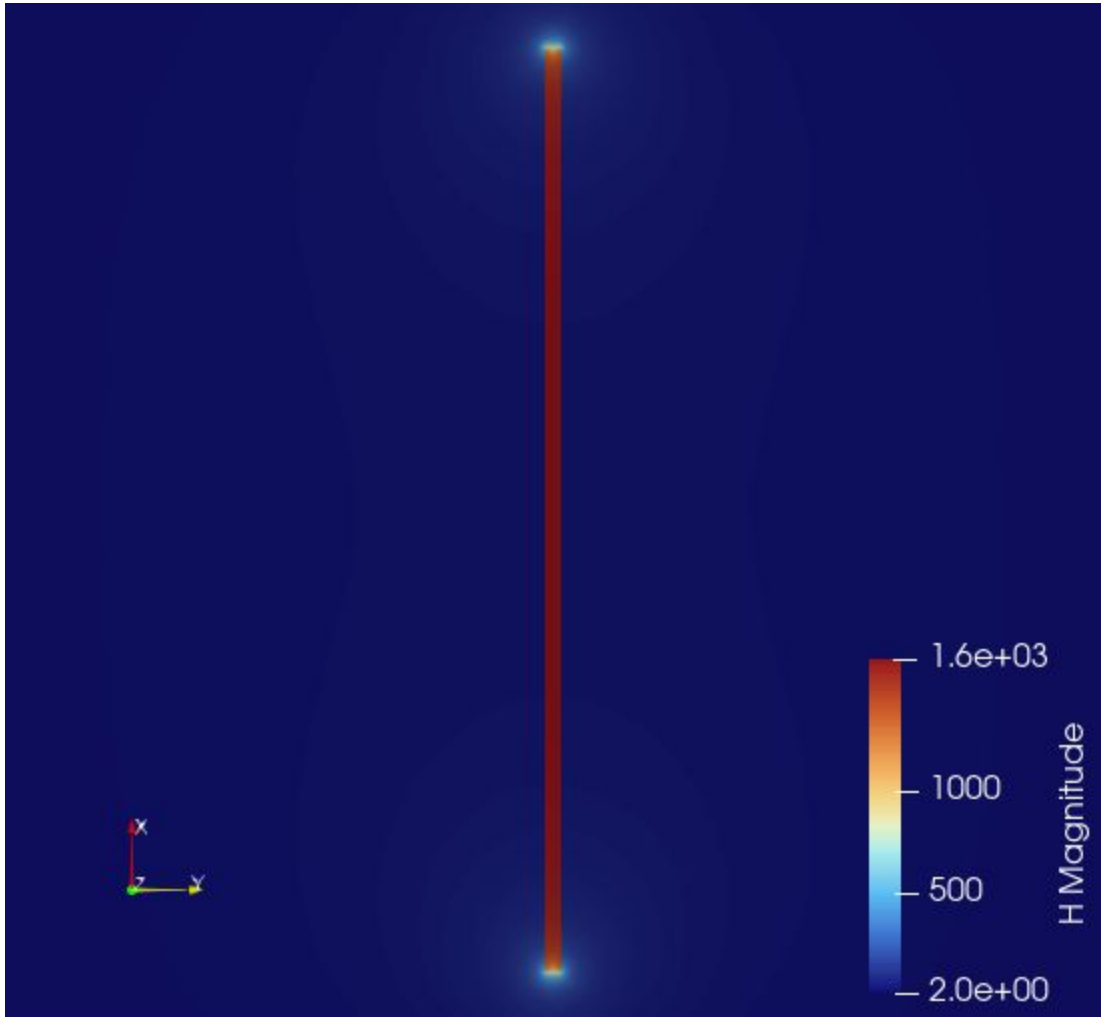
Planes infinite along **z**

VALIDATION WITH ANALYTICAL SOLUTIONS

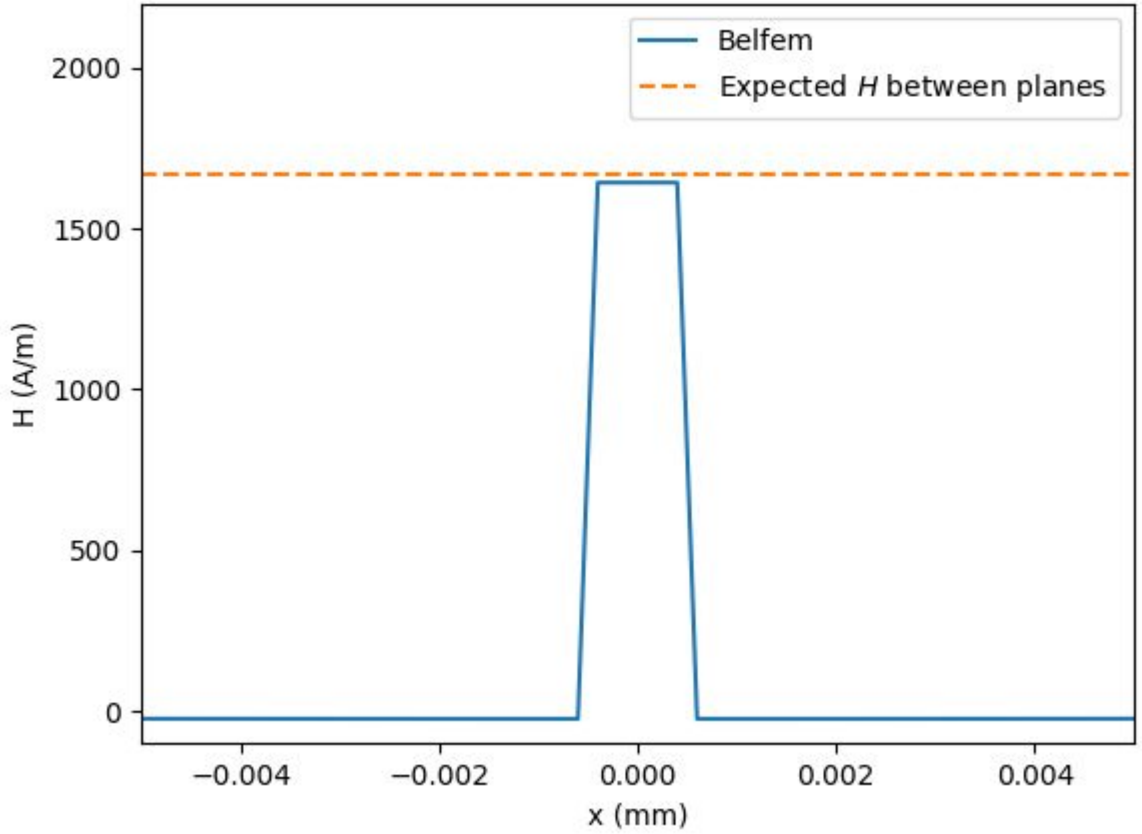




VALIDATION WITH ANALYTICAL SOLUTIONS



$$H_{center} = wJ = \frac{wI}{wL} = \frac{100}{0.06} = 1666\mathrm{A/m}$$



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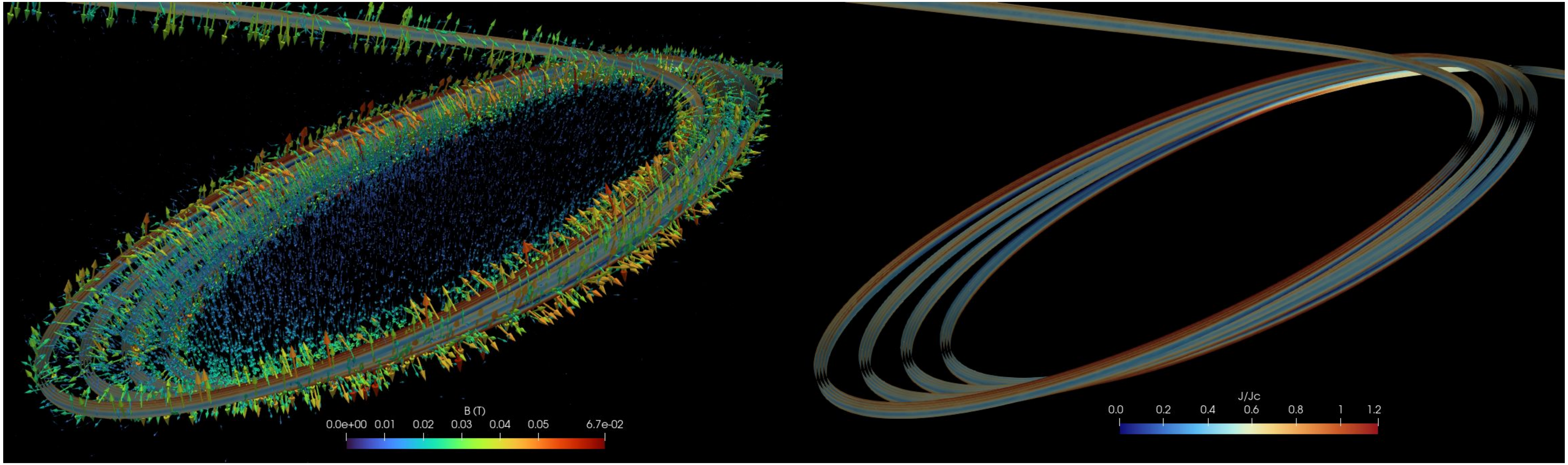
NEXT STEPS

Teaser: CCT magnet

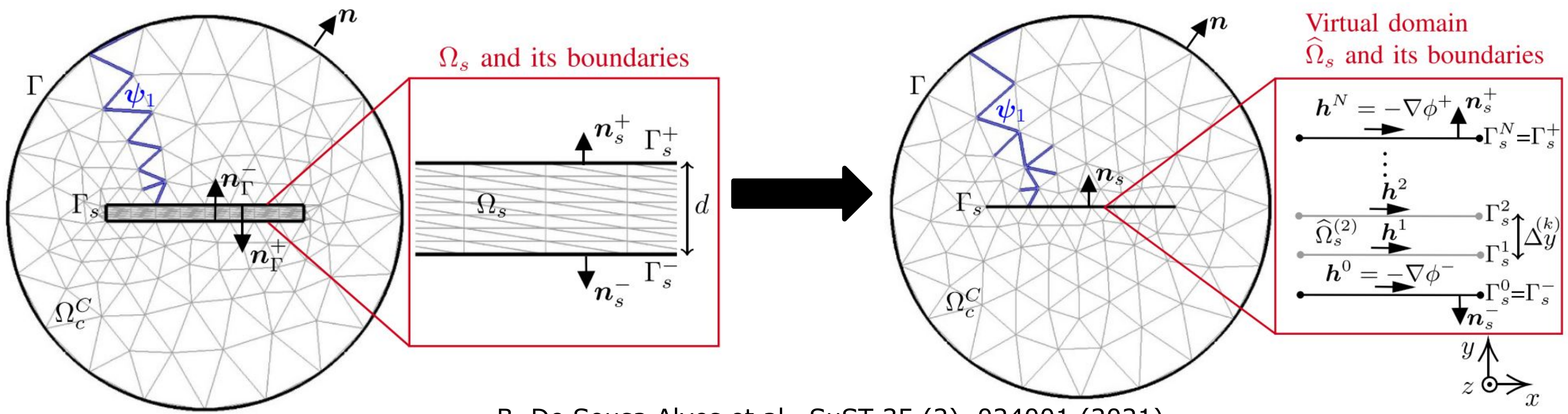


BONUS SLIDES

3-D THIN SHELL MODEL

For HTS tapes, we implemented a thin-shell (T-S) model to avoid the necessity of a very fine mesh, while keeping the accuracy.

2-D:



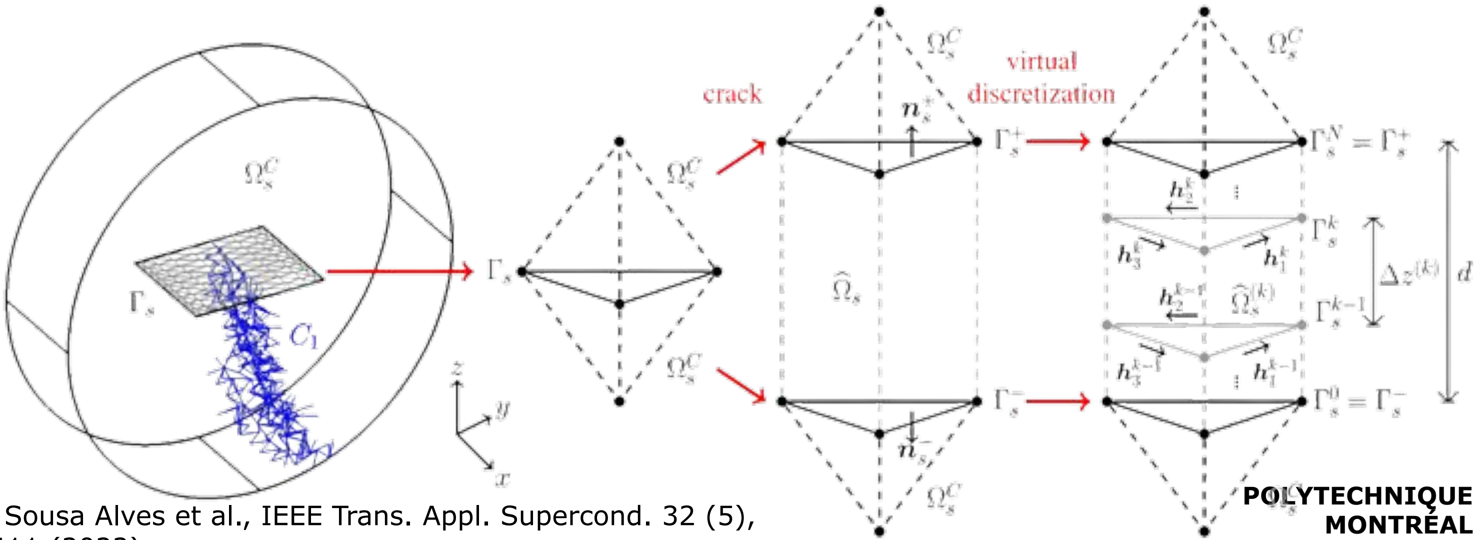
B. De Sousa Alves et al., SuST 35 (2), 024001 (2021)

3-D:

$$\boldsymbol{j} = \nabla \times \boldsymbol{h} = \begin{bmatrix} -\partial_z h_y \\ \partial_z h_x \\ \partial_x h_y - \partial_y h_x \end{bmatrix}$$

In-plane current

Out-of-plane current



B. De Sousa Alves et al., IEEE Trans. Appl. Supercond. 32 (5), 7500411 (2022)