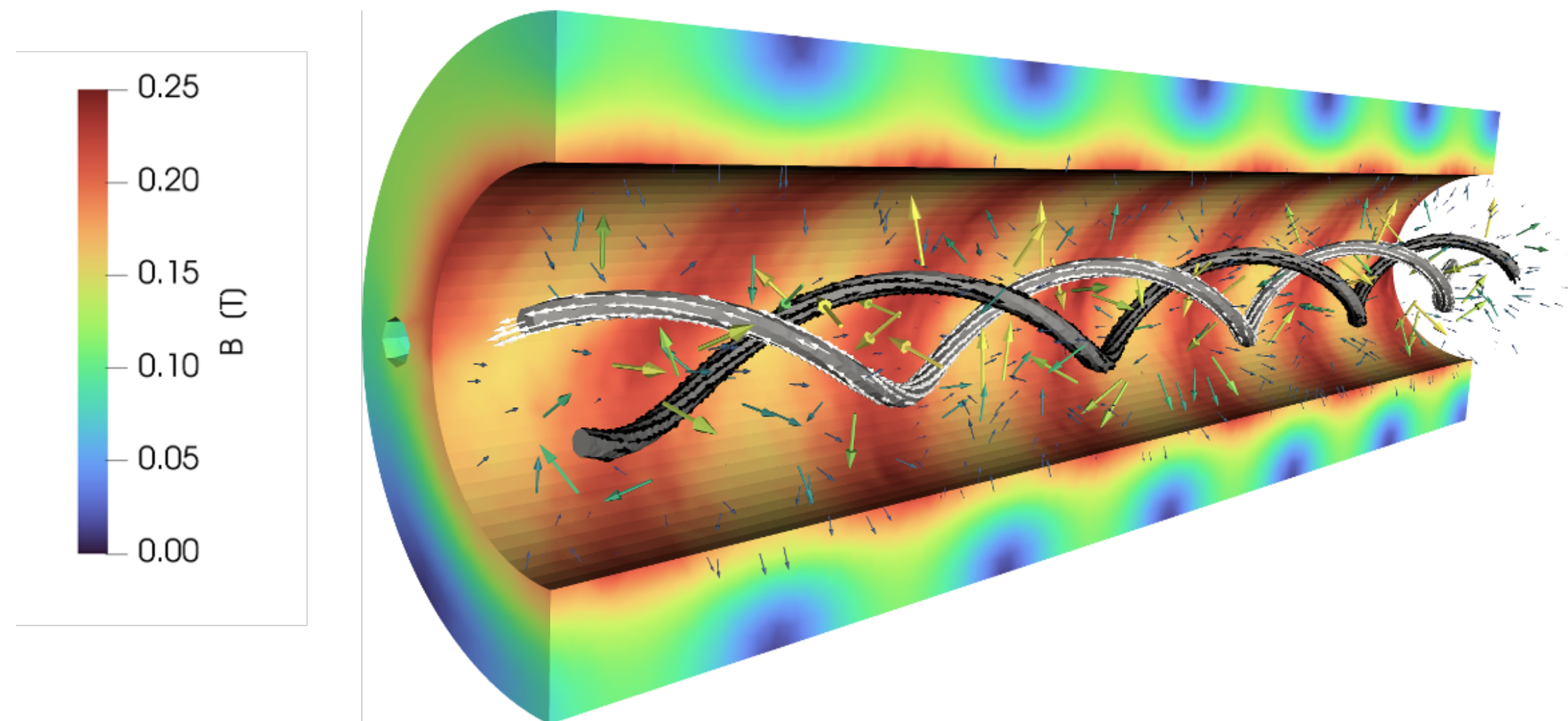


# Progress of the BELFEM code part II: Development and Status

Christian Messe, Gregory Giard, Frédéric Sirios, Giorgio Vallone, Lucas Brouwer



## General

- workflow / user interface
- performance benchmark
- code structure
- current work packages

## Thermal Modeling

- coupling with EM-Model
- proof of concept
- thin shell approach

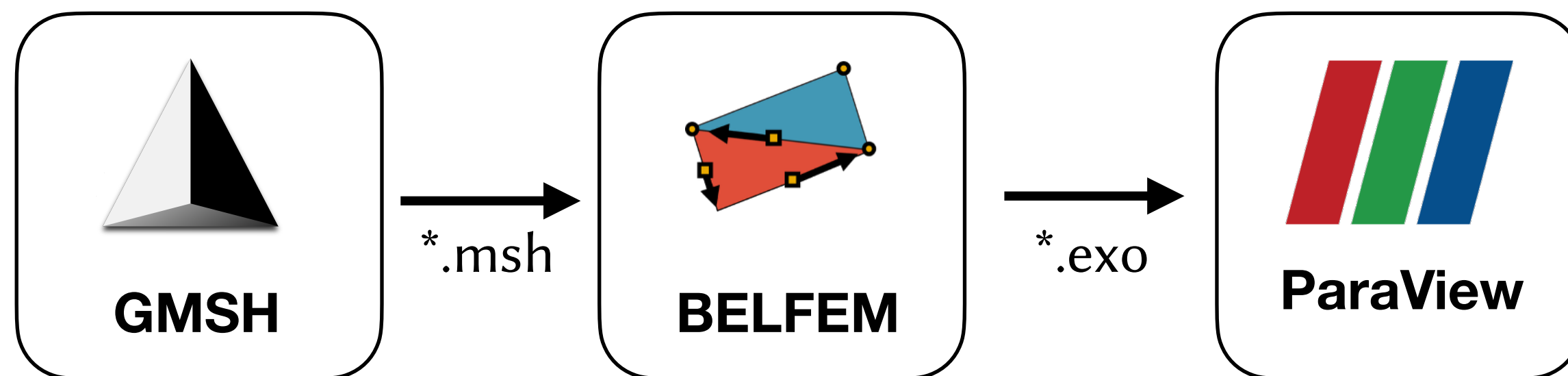
## Material Database

- Bulk data using splines
- $J_c$ -scaling for REBCO

## Goals of the BELFEM Project:

- provide state-of-the art framework for quench analysis and circuit model integration
- solve types of problems not yet supported in other codes
- be fast!
- user friendly code tailored to HTS cable and magnet design
- targeted to magnet engineers

## Open source workflow:

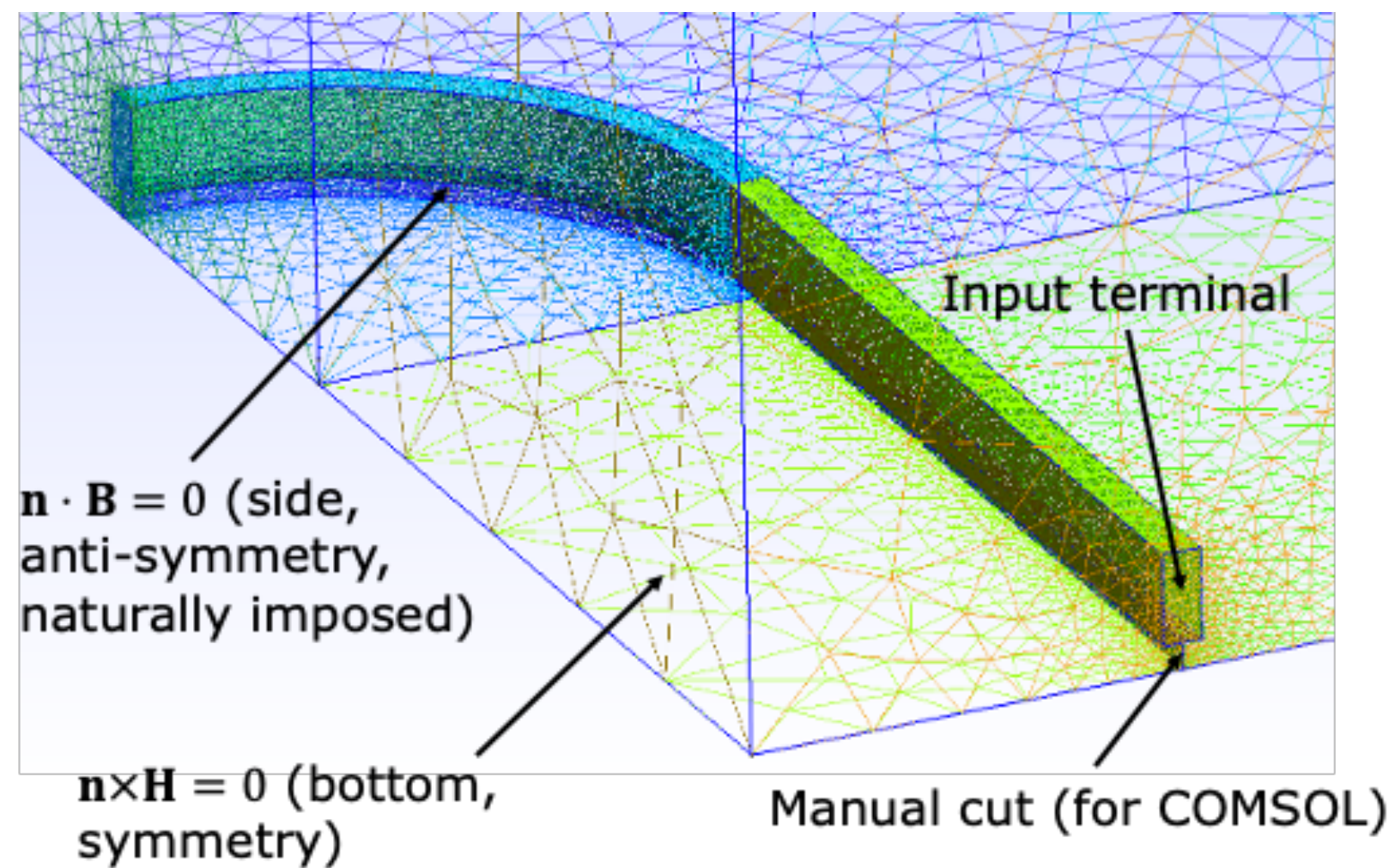


## text-based input file:

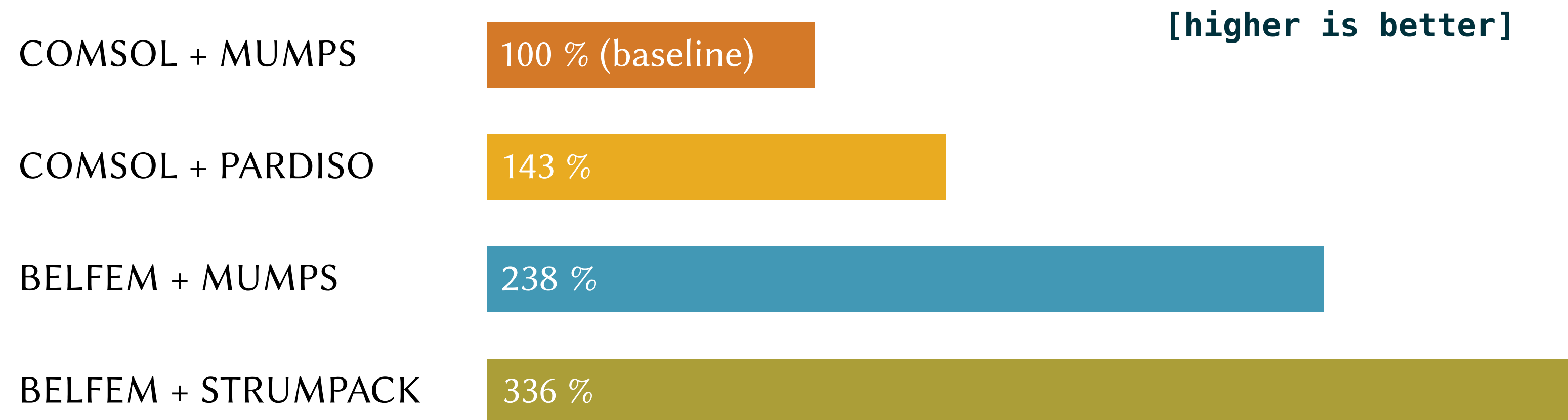
```

topology
{
    thinshell : tape
    {
        sidesets : 1:3 ;
    }
    air
    {
        blocks : 1:2 ;
    }
}

boundary conditions
{
    current
    {
        input terminals : 7@tape ;
        output terminals : 8@tape ;
        type : sine ;
        amplitude : 200 A ;
        period : 0.1 s ;
        phase : 0 deg ;
    }
}
  
```



- bulk conductor
- same hardware and procs
- same mesh

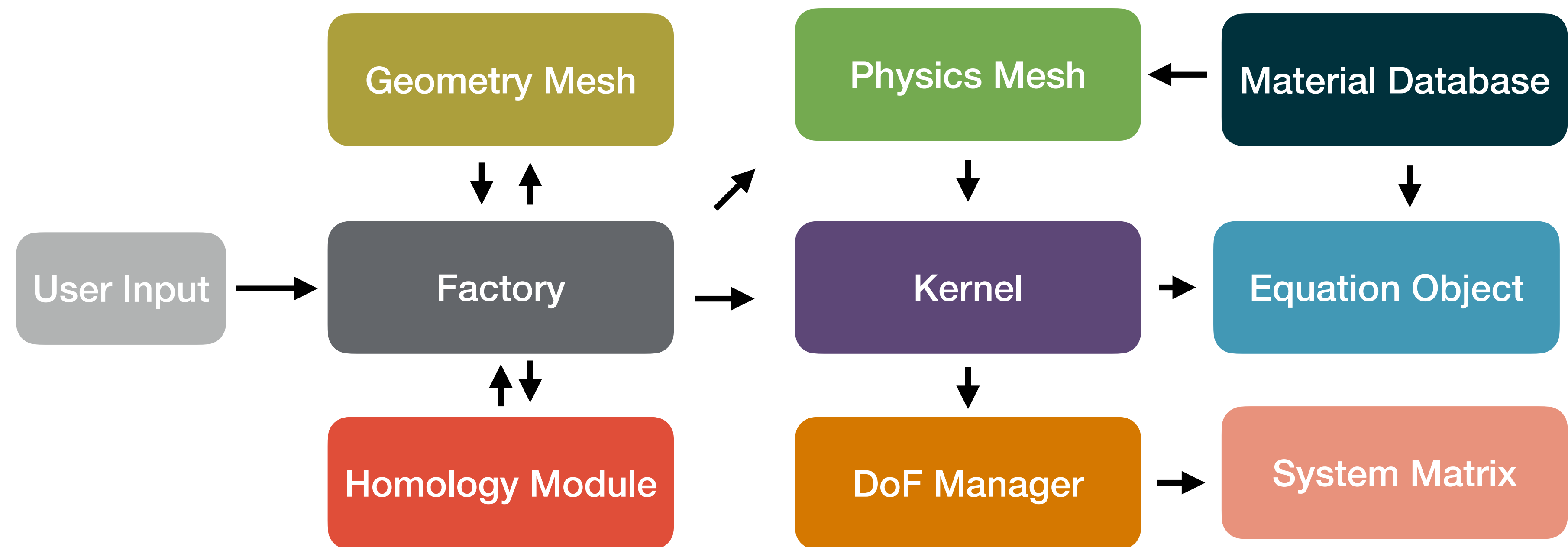


- reduced overhead due to traditional element-wise implementation rather than equation based
- using of static condensation instead of Lagrange multipliers
- hierarchical matrix compression and low-rank approximations using STRUMPACK

# Code Organization

## Core Features

- high modularization  
→ easier to maintain and extend
- relying on standard open source libraries in scientific computing  
→ e.g. BLAS/LAPACK, PETSc, ...
- parallelization using MPI  
→ ability to run on HPC



# Work Packages for Q4 2025

finished:

New Parallel Engine

Thermal Bulks

Mat DB API

Currently in progress (needed for current sharing)

Thermal Shells

Mat DB Data

Next on the list (needed for quenching)

Nedelec-Nedelec  
TS-Solid coupling

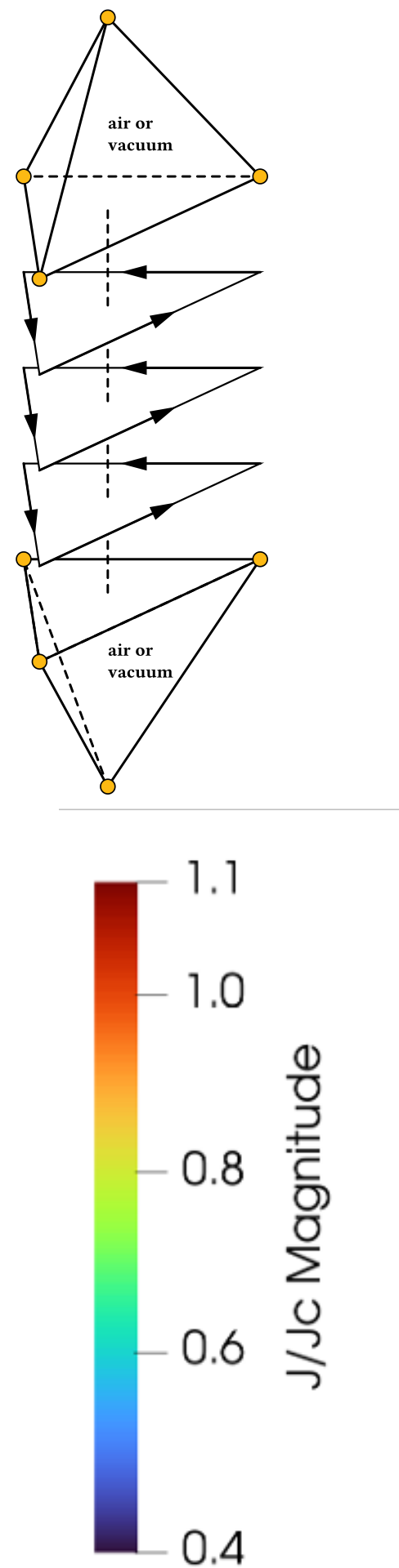
Periodic BCs

Pending (needed for clean open-sourcing)

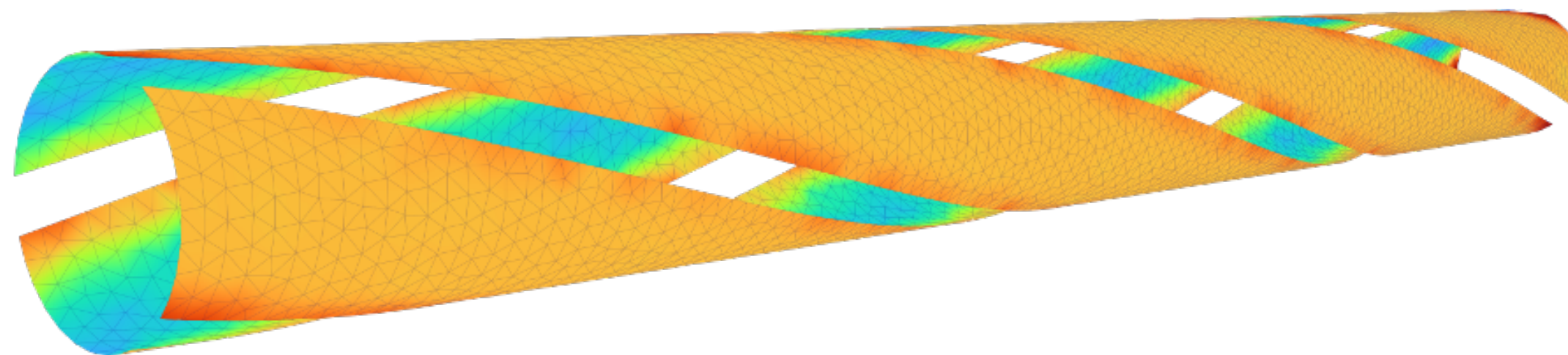
Software  
Repository

CMake Setup

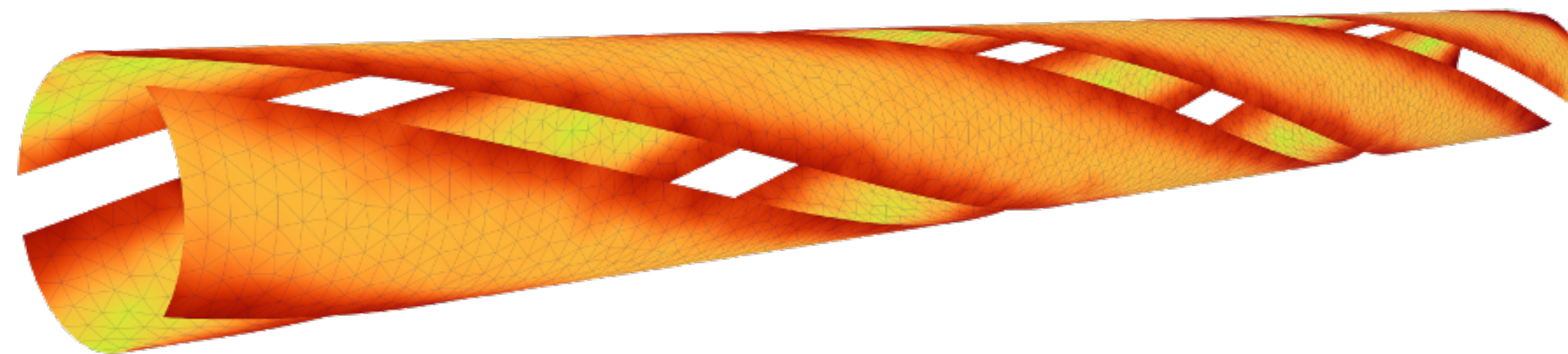
# h- $\phi$ Thin Shell Model



$t = 12 \text{ ms}$



$t = 16 \text{ ms}$

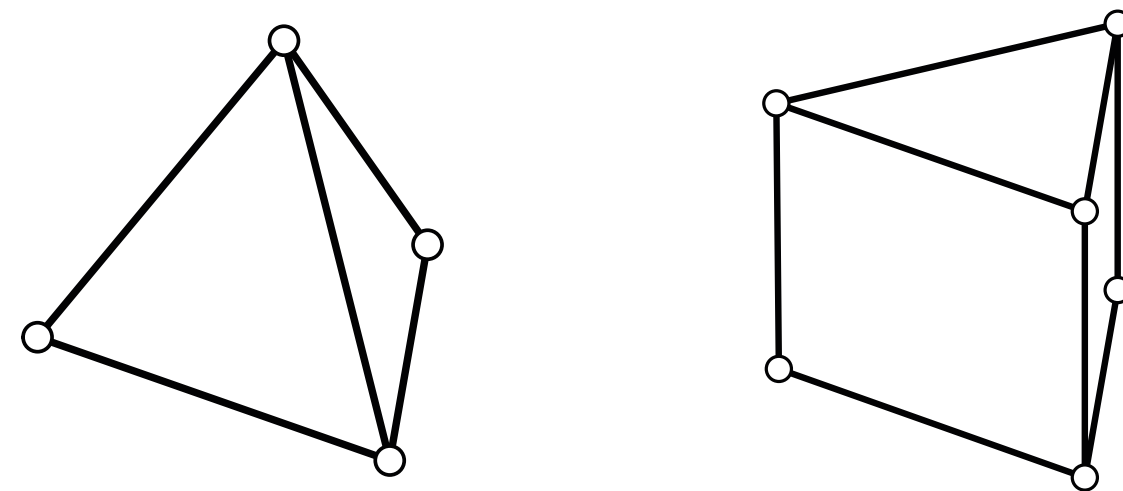


## Model overview:

- based on model from Alves et al, 2021\*
  - perpendicular h-components are assumed constant along thickness, in-plane component varies
  - but: can capture perpendicular current!
- domain interface using static condensation
  - significantly better matrix conditioning than Lagrange multipliers
- cable layers resolved in model
  - no homogenization methods needed
- supported boundary conditions per tape
  - current (Dirichlet)
  - voltage (Neumann)
  - integral constraint, eg.  $\sum I_i = I$
  - coupling to electronic circuit network model.

- **sequential iteration within timestep iteration:**
  - EM and thermal model have access to each other's fields
  - models select specialized database APIs for HTS and Metal-type materials

- **Modeling approach**
  - Solids : classic Lagrange-Type finite elements
  - Shells: considering finite volume model for better performance and stability



## EM-Model (h- $\phi$ formulation)

$$\rho(\vec{j}, \vec{b}, \vec{n}, T)$$

$$\begin{aligned} \dot{q}_v &= \rho j^2 \\ \vec{b} &= \mu_0 \vec{h} \end{aligned} \quad \downarrow \quad \uparrow T$$

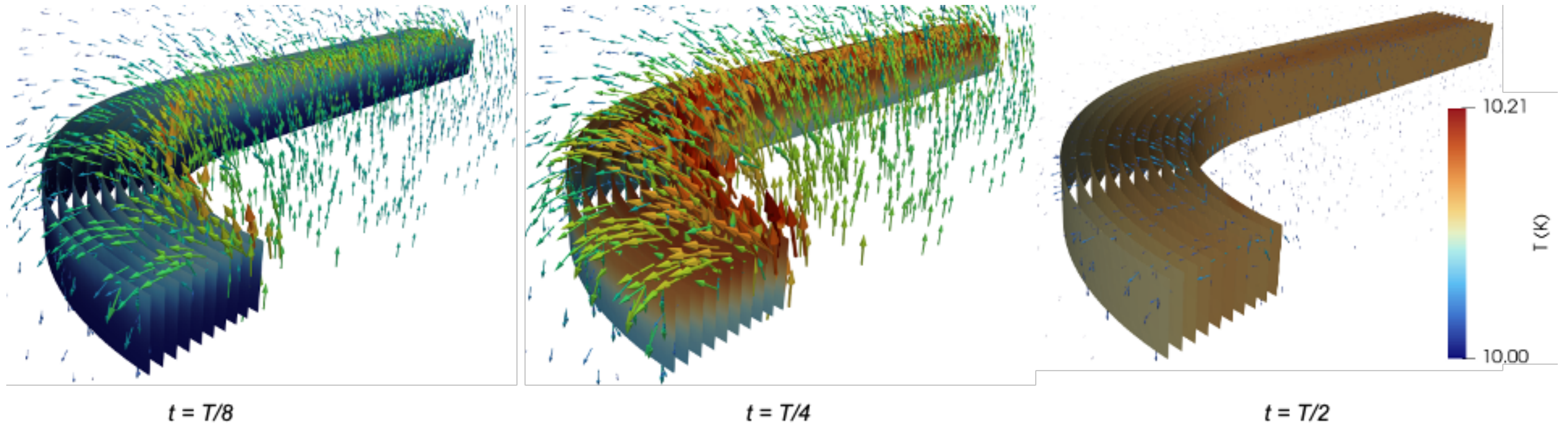
## Thermal Model

$$c_p(T)$$

$$\lambda(\vec{j}, \vec{b}, \vec{n}, T)$$

**Proof of concept for thermal coupling**

- using solid elements for thermal modeling
- using constant material properties for testing
- demonstrated code's ability to handle multiphysical problem

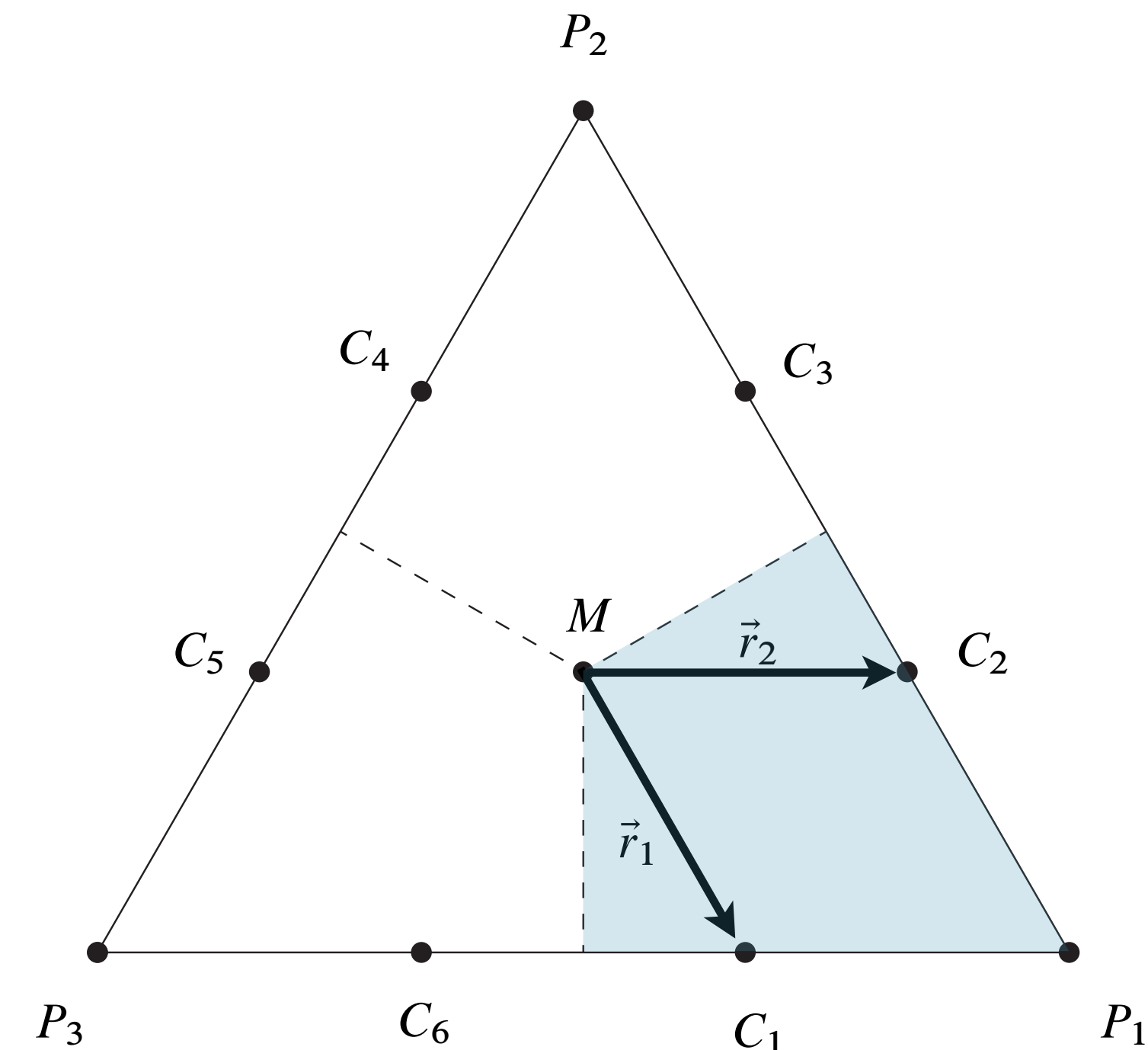


## Why not FEM for thermal modeling?

- magnetic degrees of freedom sit on edges, but thermal degrees of freedom sit on nodes
  - noise due to mismatch
  - risk of numerical instabilities
- node-bound dofs could damp extreme temperature regions in the quench region
  - model might not catch temperature spike

## Advantages of Finite Volumes (MPFA-O)

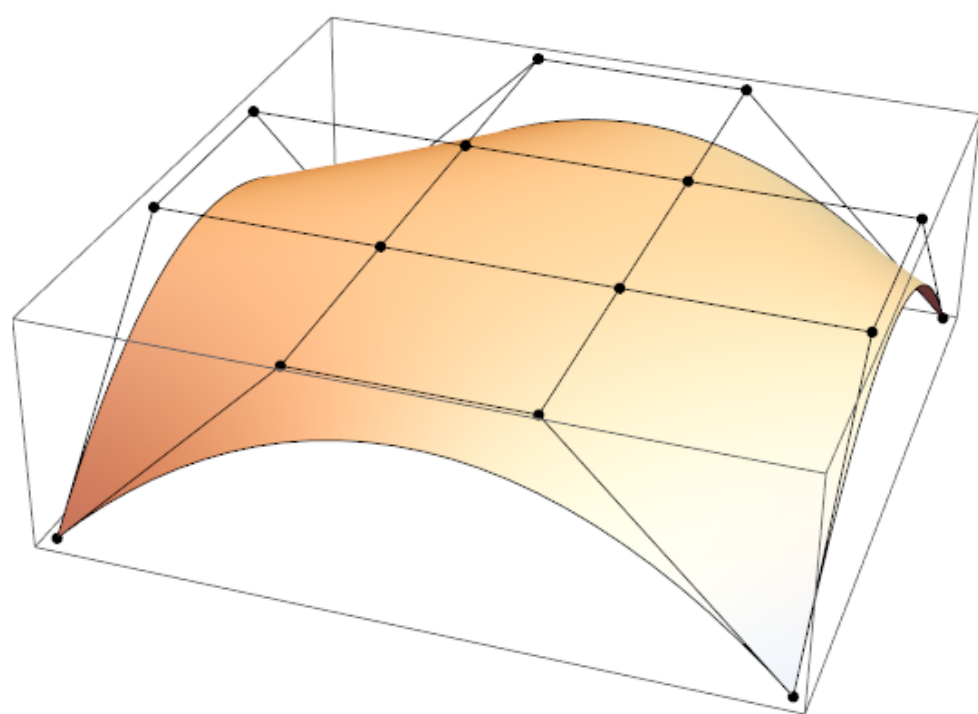
- extremely robust
  - great with highly nonlinear material properties
- can capture high temperature gradients
  - better handling of hot-spots
- enforces flux consistency between elements
  - more conservative than FEM



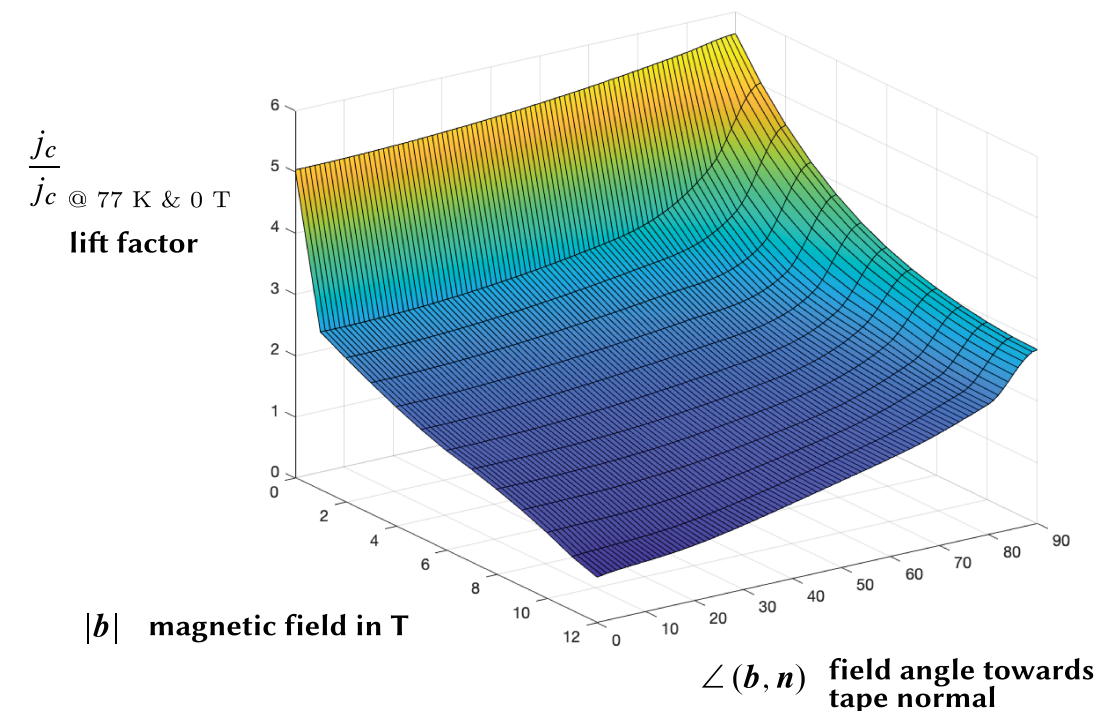
implementation is work in progress

## Core Features

- Spline-based lookup tables
  - very fast
  - C2-continuity results in better convergence of timestep iteration.
- special interfaces for
  - Copper, Silver : j-B-dependency in  $\rho$  and  $\lambda$  (Kohler Model)
  - HTS model : specialized  $j_c$ -functions or precomputed lookup tables



B-Spline surface



smoothed BSCCO dataset

## Property Functions

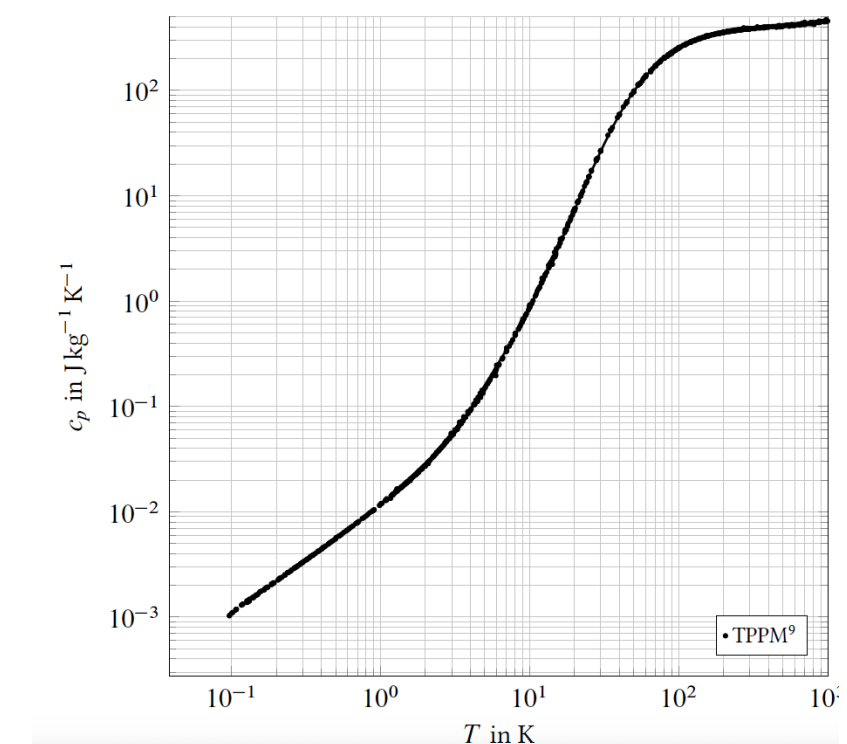
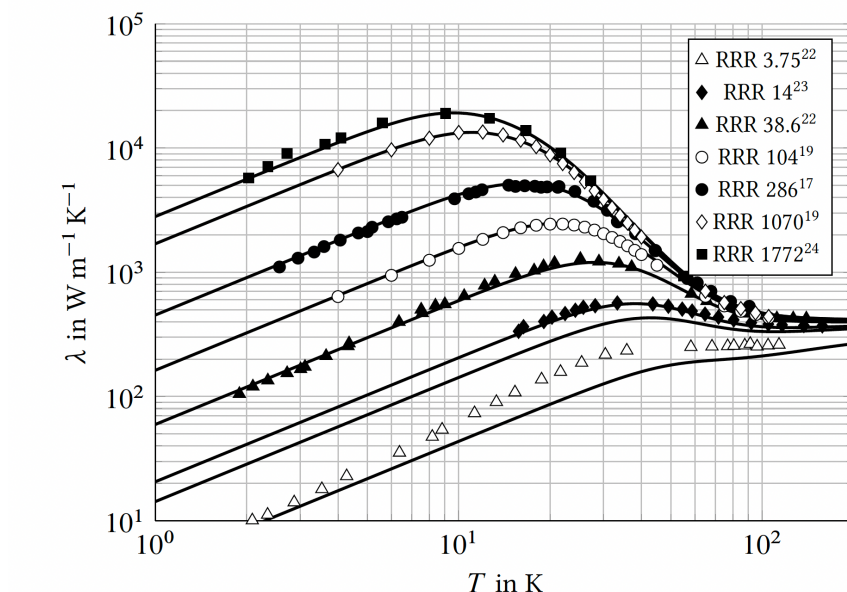
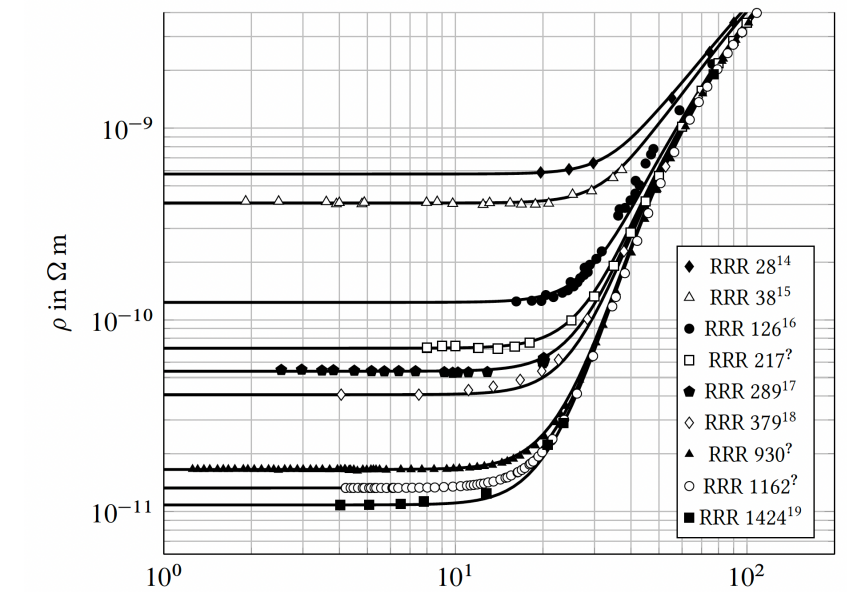
- $j_c(T, B, j, n)$
- $\rho(T, B, j, n)$
- $\lambda(T, B, j, n)$
- $c_p(T)$
- $E(T), \nu(T)$
- $\alpha(T)$

## Bulk Material Properties

- YBCO
- Copper, arbitrary RRR
- Silver, arbitrary RRR
- Hastelloy C276

## B-H Curves

- ROXIE default iron
- AISI 1010





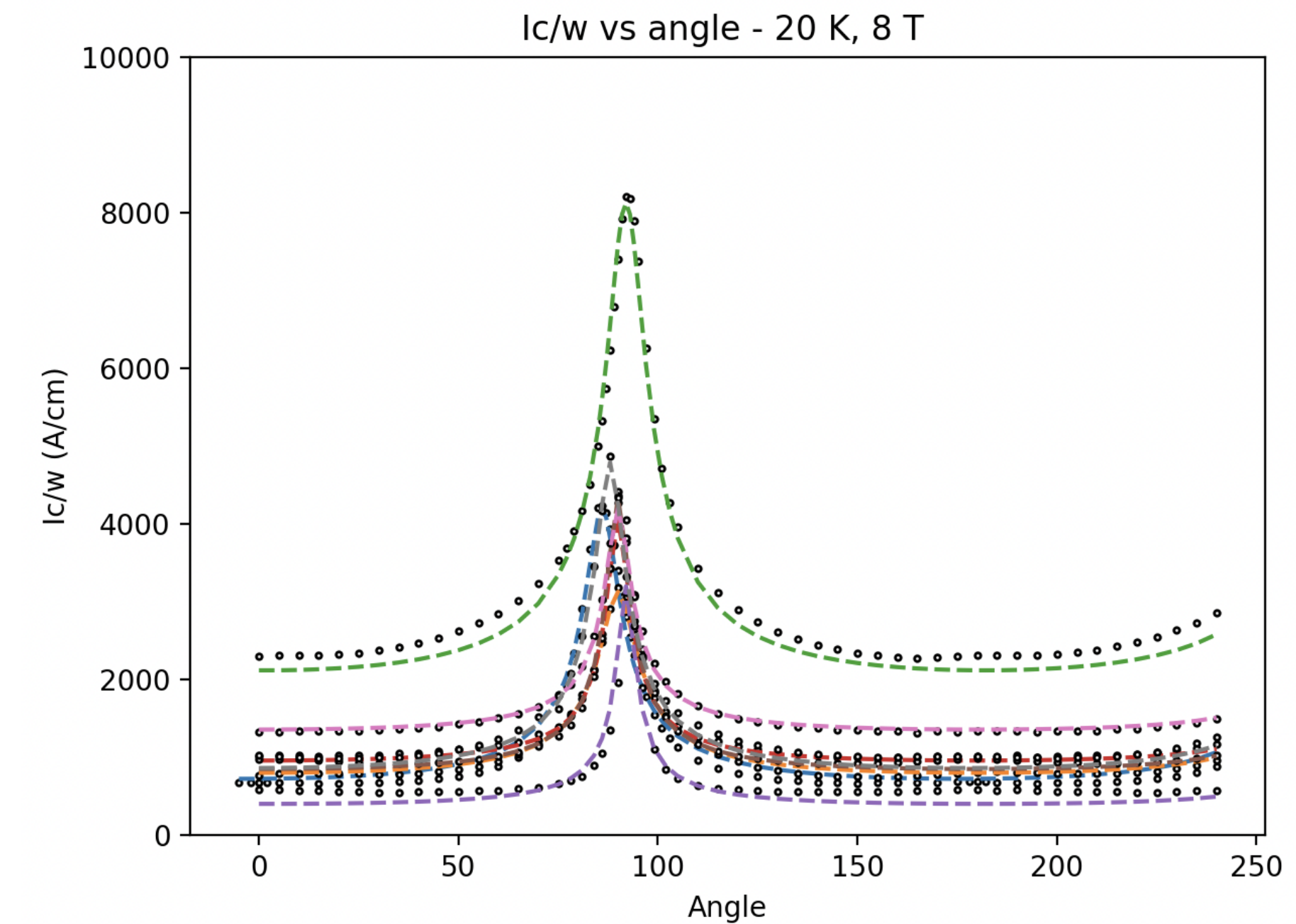
## Core Features

- Extension of Hilton model for Single-Peak behavior\*
- tailored for high field magnet applications
  - temperature and field dependent data
  - considers field angle
- fitted to experimental data from 8 different tape suppliers
- conservative extrapolations to unmeasured fields strengths up to 20 T and down to 4 K

$$I_c(B, T, \theta) = \underbrace{I_{c,min}(B, T) + [I_{c,max}(B, T) - I_{c,min}(B, T)]}_{\text{Temperature and magnetic field magnitude}} \underbrace{\frac{\omega f(\omega, \theta, \theta_0) - 1}{\omega - 1}}_{\text{Magnetic field orientation}}$$

Temperature and magnetic field magnitude

Magnetic field orientation



[ curtesy of Jean-François Crouteau ]

## EUCAS 2025:

- poster presentation and invited flash talk + podium discussion

→ very encouraging feedback

## Work in Progress

- currently working on thermal model and material database

→ proof of concept results very promising

## Next Big Tasks

- periodic boundary conditions

- inter-tape current sharing

→ quench simulation


**BERKELEY LAB**


**U.S. DEPARTMENT OF ENERGY**  
Office of Science

**Recent advancements in the Berkeley Lab Finite Element Framework**  
 Ch. Messe, G. Giard, L. Brower, G. Vallone, F. Sirois and M. Laforest


**POLYTECHNIQUE MONTRÉAL**

### Modeling

**Berkeley Lab Finite Element Framework**

- a special purpose finite element code for HTS cables and magnets, written in C++
- long term goal: model quench behavior in for complex problems
- leverages the mixed h- $\phi$  formulation with thin shell extension
- ongoing development at LBL, aimed to be published as open source once sufficiently mature

**Fundamentals**

- conducting domains: weak form of Faraday's law  

$$\int_D \mathbf{E}^T \cdot \frac{\partial(\mathbf{E} \cdot \mathbf{h})}{\partial t} dV + \int_D \mathbf{E}^T \cdot \nabla^T \mathbf{p} \cdot \nabla \times \mathbf{h} dV - \int_D \mathbf{h}^T (\nabla \times \mathbf{e}) dS = 0$$
- substituting h with  $-\nabla\phi$  yields the formulation for non-conducting domains  

$$\int_D \mathbf{E}^T \nabla^T \cdot \frac{\partial(\mathbf{p} \cdot \nabla \phi)}{\partial t} dV + \int_D \mathbf{E}^T \nabla^T (\nabla \times \mathbf{e}) dS = 0$$

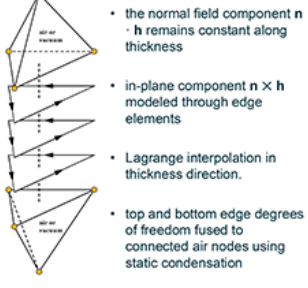
**Thin Shell Approach**

- the normal field component n · h remains constant along thickness
- in-plane component n × h modeled through edge elements
- Lagrange interpolation in thickness direction.
- top and bottom edge degrees of freedom fused to connected air nodes using static condensation

**Cohomologies**

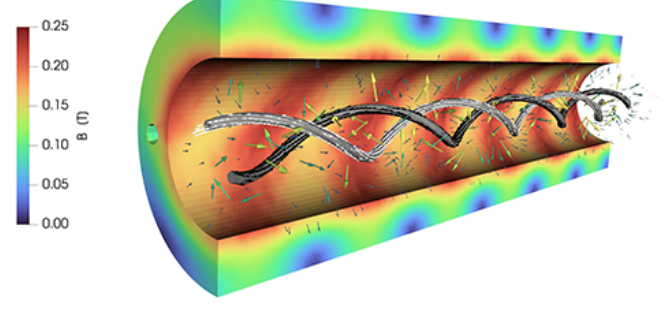
- current boundary conditions are imposed using Ampere's circuital law  

$$\oint_C \mathbf{h} \cdot d\mathbf{l} = I_c$$
- a generalized Peikka algorithm identifies edges on the mesh which a loop must cross an odd number of times (thick cut)
- Relying on the Poincaré-Lefschetz duality theorem, we extract the surfaces (thin cuts)
- Along the surfaces, degrees of freedom are duplicated and statically condensed with the current boundary condition



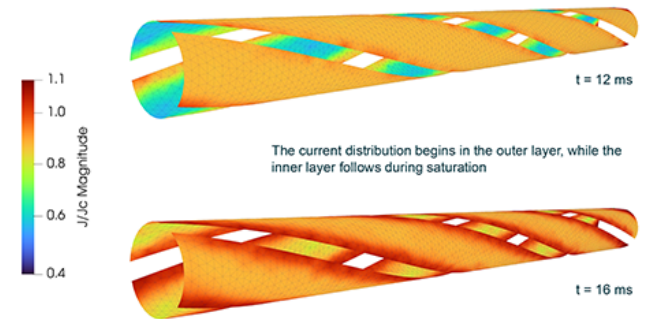
### Double Helix Example (solid elements)

This example shows a twisted HTS wire and an iron yoke, modeled using solid elements. As with the air, a  $\phi$ -formulation is used in the ferromagnetic domain, while a h-formulation is used in the wire.



### CORC® Geometry (thin shell elements)

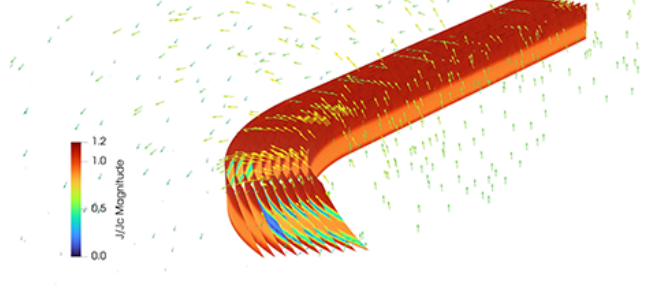
The multi-layered thin shell approach allows a three-dimensional resolution of the tape structure:



The current distribution begins in the outer layer, while the inner layer follows during saturation

### Racetrack Geometry (thin shell elements)

This example shows 1/8 of a racetrack geometry. As with the CORC cable, the tapes were resolved with multiple layers. The picture shows a timestep prior saturation where the current distribution differs significantly between the top and the bottom side of each individual tape.



### User Friendly Input File

```

materials
{
  yoke
  {
    power law
    {
      n : 35 ;
      k : 4.76e05 A/m^m^2 ;
      m : 1e-4 V/m ;
    }
    model : constant ;
  }
  iron
  {
    b-h curve: Radiation ;
  }
}

tapes : tape
{
  yoke : 0.55 mm ; // we resolve the tape
  yoke : 0.55 mm ; // in three layers
  yoke : 0.55 mm ; // along thickness
}

topology
{
  shellset : tape
  {
    elements : 13 ;
  }
  air
  {
    blocks : 1,2 ;
  }
}

boundary conditions
{
  current
  {
    input terminals : 7@tape ;
    output terminals : 8@tape ;
    type : sine ;
    amplitude: 200 A ;
    period : 0.1 s ;
    phase : 0 deg ;
  }
}
    
```

### Performance Benchmark

| Geometry          | COMSOL | STRUMPACK | Ratio |
|-------------------|--------|-----------|-------|
| CORC - MUMPS      | 20.5   | 0.5       | 41    |
| CORC - FEMGEN     | 20.5   | 0.5       | 41    |
| BELEM - MUMPS     | 20.5   | 0.5       | 41    |
| BELEM - STRUMPACK | 20.5   | 0.5       | 41    |

We simulated a 3D superconducting racetrack geometry, testing various mesh resolutions. The same meshes were solved in COMSOL and BELFEM code. Using the MUMPS solver, BELFEM outperformed COMSOL by a factor of 2.4. Using Berkeley Lab's STRUMPACK solver, BELFEM outperformed by a factor of 3.4.

This performance gain stems from three factors:

- reduced overhead due to traditional element-wise implementation rather than equation based
- usage of static condensation instead of Lagrange multipliers
- hierarchical matrix compression and low-rank approximations using STRUMPACK

### Next Steps Towards Quench:

- extend material database
- inter-tape contact resistivity
- thermal coupling