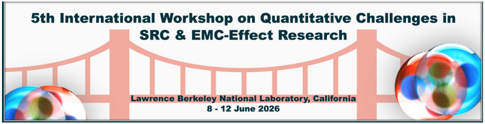


האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

Factorization and Universality in Nuclear Physics

Nir Barnea

The Hebrew University, Jerusalem, Israel

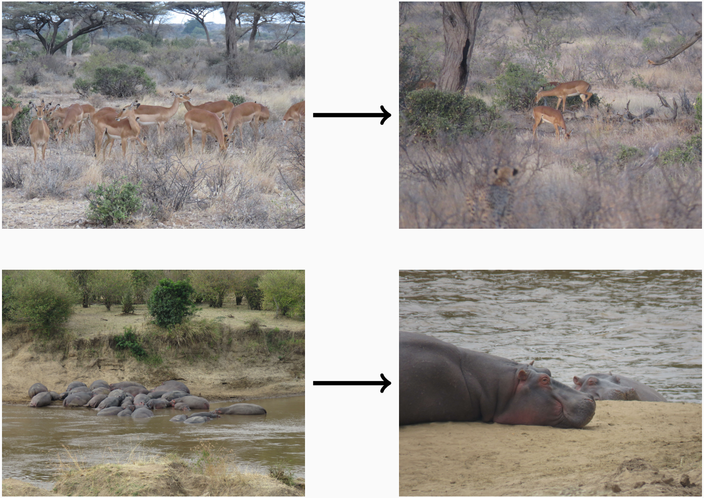
LBNL, Berkeley

8-12 June 2026

Short Range Correlations in a many-body systems

Kenya (2016).

Short Range Correlations in many-body systems



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Short Range Correlations in many-body systems

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Length scales:

The interparticle distance:

$$d \propto n^{1/3}$$

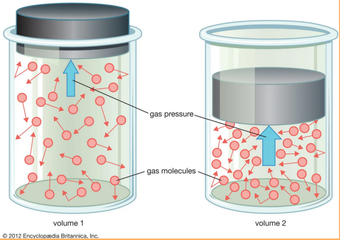
Interaction range:

$$\sigma \approx \pi R^2$$

$V(r) \to 0$ at typical distance $R \ll d$

I. Particles move freely most of the time

II. Equilibrium resluts from **short range** collisions



Ideal Gas

Getting closer to home

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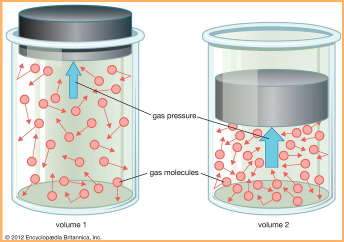


J. S. Levinger
"The high energy nuclear photoeffect", Phys. Rev. **84**, 43 (1951).

Quasi-deuteron photoabsorption

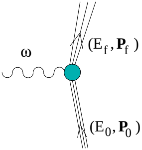
- **Factorization** -

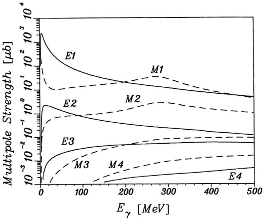
$$\Psi(12\ldots A) \to \varphi(12)A(34\ldots A)$$

- **Universality** -
  φ independent of system and energy
- Dipole $(E1)$ dominance $\Rightarrow$ np pair
- The resulting cross-section

$$\sigma_A(\omega) = L\frac{NZ}{A}\sigma_d(\omega)$$

- L is the Levinger Constant





H. Arenhovel, and M. Sanzone, Few-Body Syst. (1991).

The short range wave function - Universality

The 2-body Schrodinger ...

$$\boxed{\left[-\frac{\hbar^2}{m}\nabla^2 + V(\boldsymbol{r})\right]\psi = E\psi}$$

Vanishing distance, $r \longrightarrow 0$

- The energy becomes negligible $E \ll \hbar^2/mr^2$
- The w.f. ψ assumes an asymptotic **energy independent** form φ

$$\boxed{\left[-\frac{\hbar^2}{m}\nabla^2 + V(r)\right]\varphi(\boldsymbol{r}) = 0} \qquad \boxed{r\varphi(r) = 0|_{r=0}}$$

- φ **is a universal function**
  (in the **weak** sense - model dependent)

Theoretical develpoments in nuclear physics

(with personal bias)

- **Levinger - Photoabsorption**

  J. S. Levinger, Phys. Rev. 84, 43 (1951).

- **Amado, Woloshyn - Momentum Distribution**

  R. D. Amado, Phys. Rev. C 14, 1264 (1976).

  R. D. Amado and R. M. Woloshyn, Phys. Lett. B 62, 253 (1976).

- **Zabolitsky - Coupled Cluster**

  J. G. Zabolitsky and W. Ey, Phys. Lett. B 76, 527 (1978).

- **Frankfurt, Strikman - Factorization**

  L. Frankfurt, and M. Strikman, Phys. Rep. **160**, 235 (1988).

- **Ciofi degli Atti - Electron scattering**

  C. Ciofi degli Atti, Phys. Rep. 590, 1 (2015).

- **Bogner, Roscher - Factorization**

  S. K. Bogner and D. Roscher, Phys. Rev. C **86**, 064304 (2012).

Universality and Factorization - QFT

Operator product expansion (OPE)

Established for conformal theories, provides a natural framework for studying SRCs

K. G. Wilson and W. Zimmermann, Commun. Math. Phys. 24, 87 (1972).

Zero range, infinite scattering length

Universality and factorization were derived from OPE

E. Braaten and L. Platter, Phys. Rev. Lett. 100, 205301 (2008).

Coulomb systems

Application, without a-priori justification

J. Hofmann, M. Barth, W. Zwerger, PRB 87, 235125 (2015)

The Contact


A message from deep space.
Who will be the first to go?
A journey to the heart of the universe.
CONTACT

The 2-body system

$$\psi(\boldsymbol{r}) \xrightarrow[r \to 0]{} \varphi_0(\boldsymbol{r})$$

The N-body system

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N) \xrightarrow[r_{12} \to 0]{} \varphi_0(\boldsymbol{r}_{12}) A(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_N)$$

Short Range Observables

The Contact - Tan, Braaten & Platter,...

The 2-body system

$$\psi(\boldsymbol{r}) \xrightarrow[r \to 0]{} \varphi_0(\boldsymbol{r})$$

$$O_{12} \approx \delta(\boldsymbol{r}_{12}) \Longrightarrow \langle \psi | O_{12} | \psi \rangle \approx C_2 \langle \varphi_0 | O_{12} | \varphi_0 \rangle$$

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$$\langle \Psi | \sum O_{ij} | \Psi \rangle \approx \underbrace{\tfrac{N(N-1)}{2} \langle A | A \rangle}_{C_N} \langle \varphi_0 | O_{12} | \varphi_0 \rangle$$

The 2-body system

$$\tilde{\psi}(\boldsymbol{k}) \xrightarrow[k \to \infty]{} \tilde{\varphi}_0(\boldsymbol{k})$$

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$$\tilde{\Psi}(\boldsymbol{k}_1, \boldsymbol{k}_2, \ldots, \boldsymbol{k}_N) \xrightarrow[k_{12} \to \infty]{} \tilde{\varphi}_0(\boldsymbol{k}_{12})\tilde{A}(\boldsymbol{K}_{12}, \boldsymbol{k}_3, \ldots, \boldsymbol{k}_N)$$

Short Range Observables - Momentum space

The Contact - Tan, Braaten & Platter,...

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Momentum distributions

Tan, Braaten & Platter, Hofmann, ...

- **A system of spin half neutral fermions** $|a|^{-1} \ll k \ll R^{-1}$

$$n_\sigma(\boldsymbol{k}) \longrightarrow \frac{C}{k^4}$$

- Non polarized electron gas

$$n(\boldsymbol{k}) \longrightarrow C_{\uparrow\downarrow}\left(\frac{k_F}{k}\right)^8$$

- Polarized, ferromagnetic, electron gas

$$n(\boldsymbol{k}) \longrightarrow C_{\uparrow\uparrow}\left(\frac{k_F}{k}\right)^{10}$$

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Bosonic Helium Clusters

Bazak, Valiente, Barnea

Normalized n-body densities

$$\rho_n^{(N)}/C_n^{(N)}$$

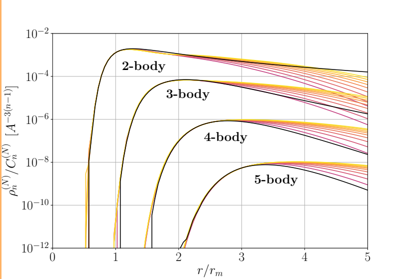
N - system size,

$N = 10, 15, \ldots 50$

$n = 2$ - 2-body correlations

$n = 3$ - 3-body correlations

$\ldots$



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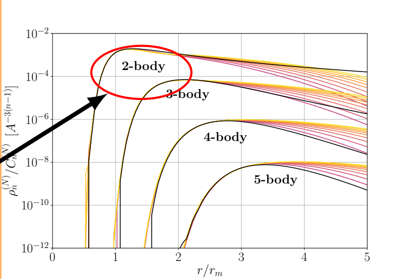

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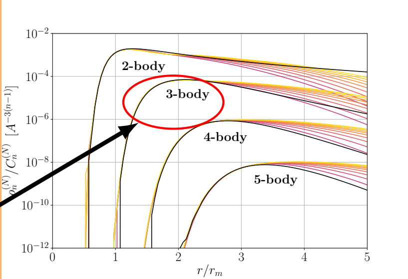

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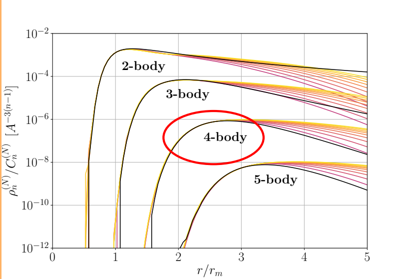

The Nuclear Contact(s)

Weiss, Bazak, ...

- In nuclear physics we have **3** possible particle pairs

$$ij = \{pp,\ nn,\ pn\}$$

- For each pair there are different channels,

$$\alpha = (s,\ell)jm$$

- We take the asymptotic form

$$\Psi \xrightarrow[r_{ij}\to 0]{} \sum_\alpha \varphi_\alpha(\boldsymbol{r}_{ij}) A_{ij}^\alpha(\boldsymbol{R}_{ij},\{\boldsymbol{r}_k\}_{k\neq i,j})$$

with φ_α the **zero-energy 2-body nuclear WFs**
$\rightarrow$ the **off-shell zero-energy T-matrix**

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- For each pair we define the contact matrix

$$C_{ij}^{\alpha\beta} \equiv N_{ij} \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle$$

using normalization $\int_{k_F}^{\infty} \frac{d\boldsymbol{k}}{(2\pi)^3} |\tilde{\varphi}_\alpha(\boldsymbol{k})|^2 = 1$

- **The contact $C_{ij}^{\alpha\alpha}$ counts the number of SRC pairs in channel α**
- For $\ell = 0$ we need consider **4** contacts

$$\{C_{pp}^{S=0}, C_{nn}^{S=0}, C_{np}^{S=0}, C_{np}^{S=1}\}$$

- The deuteron contact $C_{np}^{S=1} \approx a_2$
- If **isospin symmetry holds** the number of contacts is **2**,

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It doesn't ... (for heavy nuclei)

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The Generalized Contact Formalism

We look for a systematic approach to:

- Obtain the factorization ansatz
- Understand the rule of Higher-body SRCs
- Derive the universal function φ_0
- Calculate the contacts

To achieve these goals we use the coupled-cluster (CC) method

The CC method describes correlations naturally

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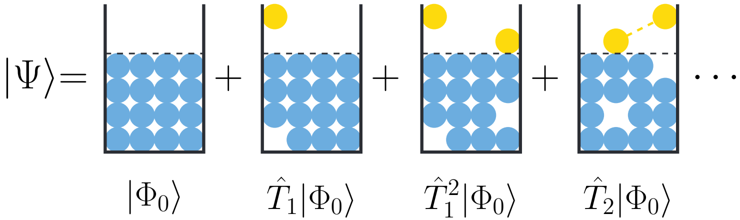
Coupled cluster

The wave-function is expanded in **clusters**

$$|\Psi\rangle = e^{\hat{T}}|\Phi_0\rangle \qquad \hat{T} = \hat{T}_1 + \hat{T}_2 + \hat{T}_3 + \cdots$$

$\hat{T}_n$ operator excites n particles from the Slater determinant Φ_0



R. J. Bartlett and M. Musial, Rev. Mod. Phys. 79, 291 (2007)

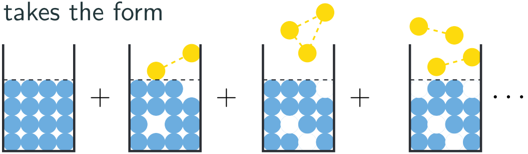
The complete 2- and 3-body CC equations

$\hat{T}_n$ is the *npnh* operator

$$T_n = \frac{1}{n!^2} \sum t^{a_1 \ldots a_n}_{i_1 \ldots i_n} \boldsymbol{a}_1^\dagger \boldsymbol{a}_2^\dagger \cdots \boldsymbol{i}_2 \boldsymbol{i}_1$$

For nuclear matter $T_1 = 0$,

and the wave-function takes the form

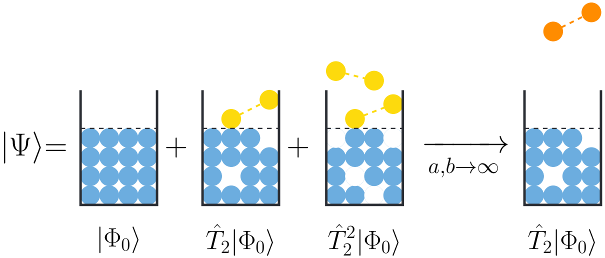
$|\Psi\rangle =$ 

$|\Phi_0\rangle \quad + \quad t^{ab}_{ij}|\Phi^{ab}_{ij}\rangle \quad + \quad t^{abc}_{ijk}|\Phi^{abc}_{ijk}\rangle \quad + \quad t^{ab}_{ij} t^{cd}_{kl}|\Phi^{abcd}_{ijkl}\rangle \cdots$

The Coupled-Cluster equations **2-body:**

$$0 = \langle \Phi^{ab}_{ij} | \hat{V} + [\hat{H}_0, \hat{T}_2] + [\hat{V}, \hat{T}_2] + \frac{1}{2}[[\hat{V}, \hat{T}_2], \hat{T}_2] + [\hat{V}, \hat{T}_3] + [\hat{V}, \hat{T}_4] | \Phi \rangle$$

The CC equations are coupled and non-linear



Zabolitsky derived the T_2 equation in the **low density** $k_F \to 0$, **high momentum** $\boldsymbol{k} \to \infty$ limit.

$$T_2(\boldsymbol{k}) = \frac{-1}{k^2}\left[V(\boldsymbol{k}) + \int \frac{d\boldsymbol{k}'}{(2\pi)^3} V(\boldsymbol{k}-\boldsymbol{k}')T_2(\boldsymbol{k}')\right]$$

where

$$T_2(\boldsymbol{k}) \equiv t_{\mathbf{0},\mathbf{0}}^{\boldsymbol{k},-\boldsymbol{k}} = \langle \boldsymbol{k}, -\boldsymbol{k} | T_2 | \mathbf{0}, \mathbf{0} \rangle$$

J.G. Zabolitsky and W. Ey, PLB 76, 527 (1978)

The Nuclear Contact Ansatz

Beck, Weiss, Barnea

Substituting

$$\varphi_2(\boldsymbol{k}) = T_2(\boldsymbol{k}) + (2\pi)^3\delta(\boldsymbol{k})$$

We get

$$\boxed{\varphi_2(\boldsymbol{k}) = (2\pi)^3\delta(\boldsymbol{k}) - \frac{1}{k^2}\int\frac{d\boldsymbol{k}'}{(2\pi)^3}V(\boldsymbol{k}-\boldsymbol{k}')\varphi_2(\boldsymbol{k}')}$$

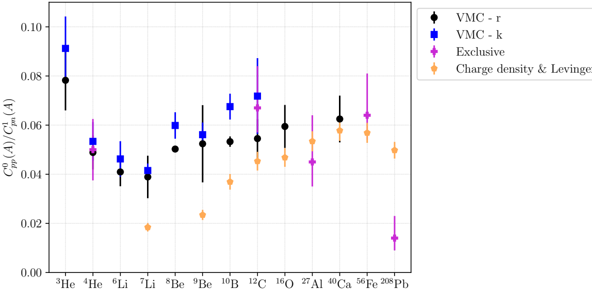
Which is just the **zero-energy Lipmann-Schwinger** equation!

The resulting momentum distribution

$$n(\boldsymbol{k}) = \langle\Phi_0|e^{T^\dagger}a^\dagger_{\boldsymbol{k}}a_{\boldsymbol{k}}e^{T}|\Phi_0\rangle \longrightarrow \langle\Phi_0|T_2^\dagger a^\dagger_{\boldsymbol{k}}a_{\boldsymbol{k}}T_2|\Phi_0\rangle \propto |T_2(\boldsymbol{k})|^2$$

The nuclear contacts

The Contacts - system dependence

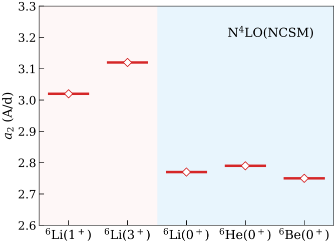


The Contact

$$C_{ij}^{\alpha\beta} = N_{ij}\langle A_{ij}^{\alpha}|A_{ij}^{\beta}\rangle$$

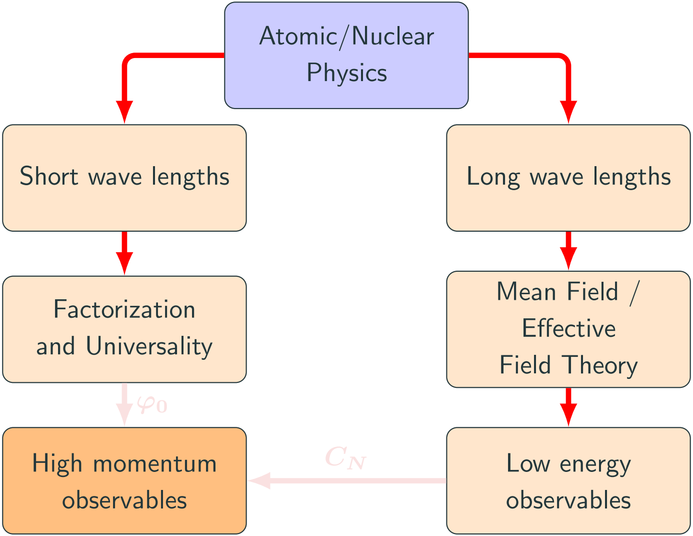
The number of SRC pairs depends on:

- density, size, ...
- N, Z
- nucleus state



H. Shang et al., PLB 871, 139976 (2025)

Short and Long



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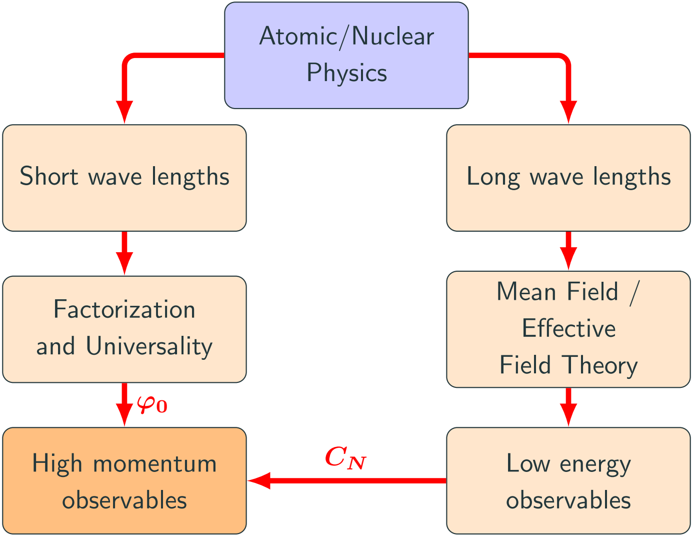


The Nuclear Contact

The nuclear contact relations/applications

- Photoabsorption cross-section
- Momentum distributions
- The nuclear spectral function
- Electron scattering
- Symmetry energy
- Two-body density
- Double beta-decay

and more ...

Numerical Verifications

Momentum distributions

1-body neutron and proton momentum distributions

$$\rho_n(\boldsymbol{k}), \;\; \rho_p(\boldsymbol{k})$$

2-body nn, np, pp momentum distributions

$$\rho_{nn}(\boldsymbol{k}), \;\; \rho_{pn}(\boldsymbol{k}), \;\; \rho_{pp}(\boldsymbol{k})$$

Momentum distributions - asymptotic relations

Using the factorization ansatz

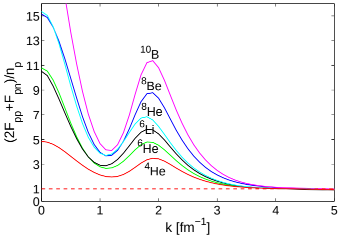
$$\Psi \xrightarrow[r_{ij}\to 0]{} \sum_{\alpha} \varphi_{\alpha}(\boldsymbol{r}_{ij}) A_{ij}^{\alpha}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$

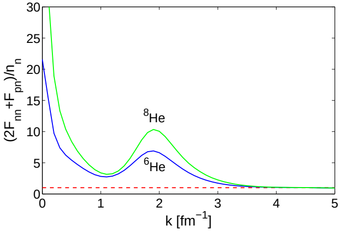
We can derive **asymptotic** relations between the 1-body and 2-body momentum distributions

$$\boxed{\rho_p(\boldsymbol{k}) \xrightarrow[k\to\infty]{} 2\rho_{pp}(\boldsymbol{k}) + \rho_{pn}(\boldsymbol{k})}$$

$$\boxed{\rho_n(\boldsymbol{k}) \xrightarrow[k\to\infty]{} 2\rho_{nn}(\boldsymbol{k}) + \rho_{pn}(\boldsymbol{k})}$$

These are **model independent** relations, that hold regardless of the specific form of φ_{α} and without any assumptions on $\{\alpha\}$





VMC calculations of light nuclei

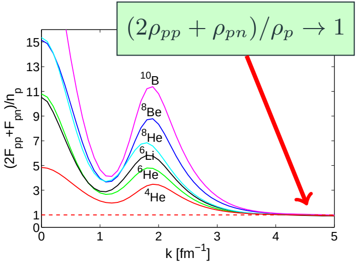
R. B. Wiringa, *et al.*, PRC **89**, 024305 (2014)

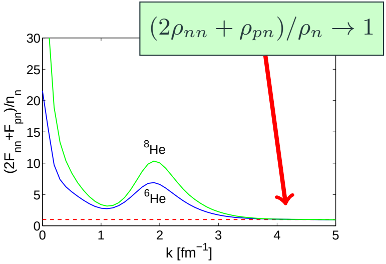
- 1-body, 2-body momentum distributions .
- Data available for $2 \leq A \leq 10$ and $A = 12, 16, 40$
  https://www.phy.anl.gov/theory/research/momenta/
- Potential - AV18+UX.

The momentum relations holds for $4\ \mathrm{fm}^{-1} \leq k \leq 5\ \mathrm{fm}^{-1}$

Numerical verification of the momentum relations

Weiss, Bazak, Barnea





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Further numerical verifications

Weiss et al

The resulting **asymptotic** 1-body momentum distribution

$$\rho_n(\boldsymbol{k}) \longrightarrow C_{np}^{s=0}|\tilde{\varphi}_{np}^{s=0}(\boldsymbol{k})|^2 + C_{np}^{s=1}|\tilde{\varphi}_{np}^{s=1}(\boldsymbol{k})|^2 + 2C_{nn}^{s=0}|\tilde{\varphi}_{nn}^{s=0}(\boldsymbol{k})|^2$$

Comparing with the VMC data:

Surprisingly, the agreement holds for $k_F \leq k \leq 6 \text{ fm}^{-1}$

Further numerical verifications

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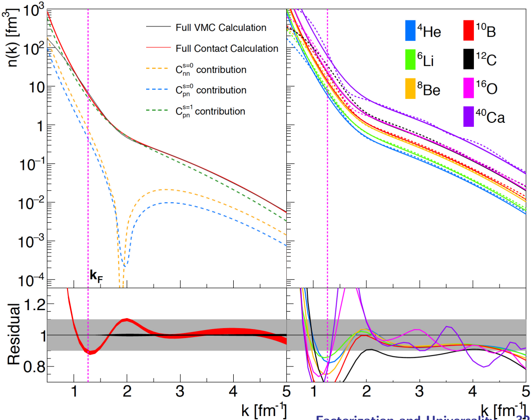


Surprisingly, the agreement holds for $k_F \leq k \leq 6 \text{ fm}^{-1}$

2-body densities

The **asymptotic** 2-body density of a NN pair in quantum state α

$$\rho^{\alpha}_{pp}(\boldsymbol{r}) \xrightarrow[r\to 0]{} C^{\alpha}_{pp}|\varphi^{\alpha}_{pp}(\boldsymbol{r})|^2$$

$$\rho^{\alpha}_{np}(\boldsymbol{r}) \xrightarrow[r\to 0]{} C^{\alpha}_{np}|\varphi^{\alpha}_{np}(\boldsymbol{r})|^2$$

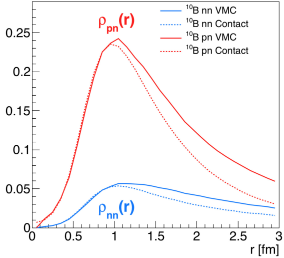
$$\rho^{\alpha}_{nn}(\boldsymbol{r}) \xrightarrow[r\to 0]{} C^{\alpha}_{nn}|\varphi^{\alpha}_{nn}(\boldsymbol{r})|^2$$

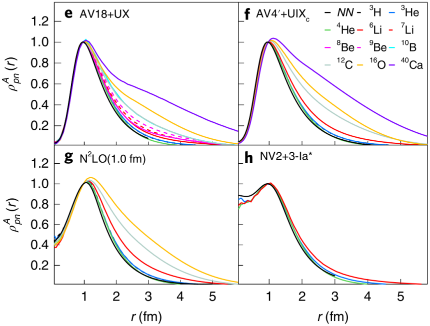
2-body densities

Compare 4 realistic $NN + 3N$ interactions:

- Phenomen. - **AV18 + UX**
- Phenomen. - **AV4' + UIX$_c$**
- χEFT - **N^{2}LO cutoff 1.0 fm**
- χEFT - **N^{2}LO cutoff 1.2 fm**

Cruz-Torres, Lonardoni, et al., Nature Phys. 17, 306 (2021)



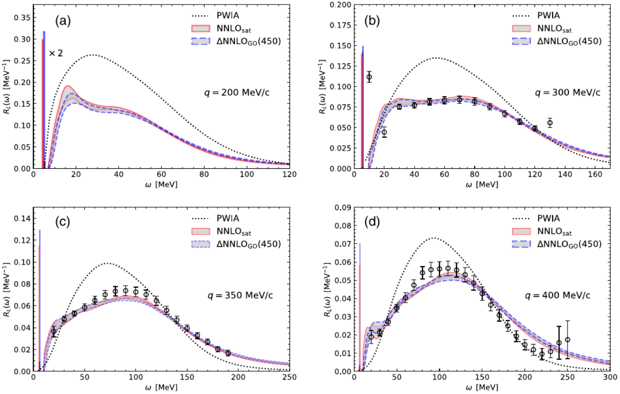


Experimental Verifications

The cross-section around the quasi elastic peak

CC-LIT method - Longitudinal response of ^{40}Ca

PHYSICAL REVIEW LETTERS **127**, 072501 (2021)



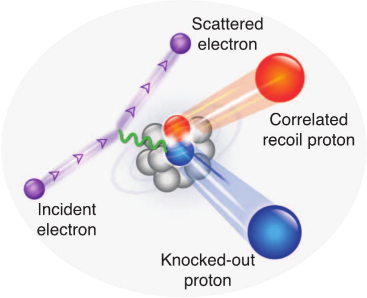
J. E. Sobczyk, B. Acharya, S. Bacca, and G. Hagen

Electron scattering more-body dynamics

The Bjorken scaling parameter - x_B

$$x_B = \frac{Q^2}{2m\omega}$$

Where the virtual photon carries $(\boldsymbol{q}, \omega)$ and $Q^2 = -q^2$



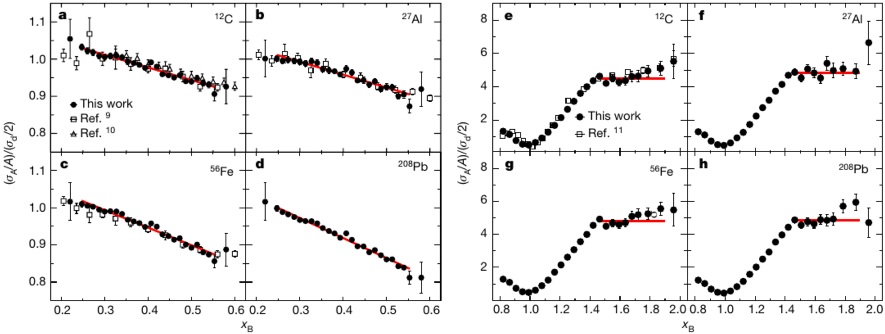
- Kinematical considerations:

  For $n - 1 < x_B \leq n$

  the **"active subsystem"** must include n-nucleons.
- The cross-section ratio $a_2 \equiv (\sigma_A/A)/(\sigma_D/2)$ is (almost) flat for $1.4 < x_B \leq 2$.

Electron scattering - scaling

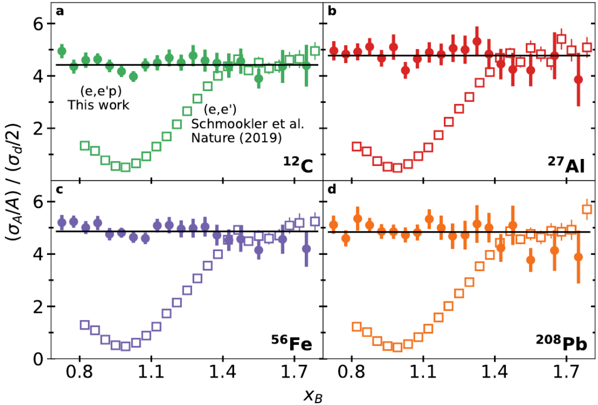
The cross-section ratio $a_2 \equiv (\sigma_A/A)/(\sigma_D/2)$



B. Schmookler, M. Duer, et al. (CLAS collaboration), Nature 566, 354 (2019)

Electron scattering - scaling

Large missing momentum: $p_{miss} \geq 350\text{MeV/c}$



Korover et al, PRC 107, L061301 (2023)

np dominance

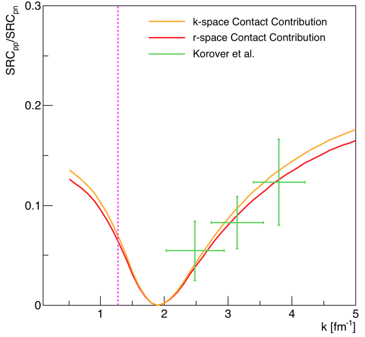
Electron scattering

The ratio of short range pp and np pairs is given by

$$\frac{SRC_{pp}(k)}{SRC_{pn}(k)} \approx \frac{\rho_{pp}(k)}{\rho_{pn}(k)} = \frac{C_{pp}^{s=0}|\tilde{\varphi}_{pp(nn)}^{s=0}(k)|^2}{C_{pn}^{s=0}|\tilde{\varphi}_{pn}^{s=0}(k)|^2 + C_{pn}^{s=1}|\tilde{\varphi}_{pn}^{s=1}(k)|^2}$$

np dominance

- np pairs much more common than pp or nn pairs
- The pp/np ratio is attributed to the node in the pp w.f.
- The tensor force fills this node in the np w.f.



Korover et al., PRL 113, 022501 (2014)

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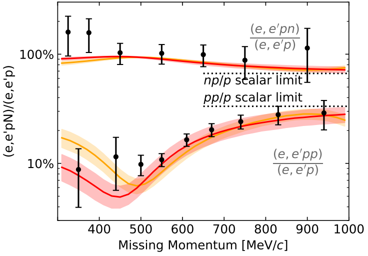
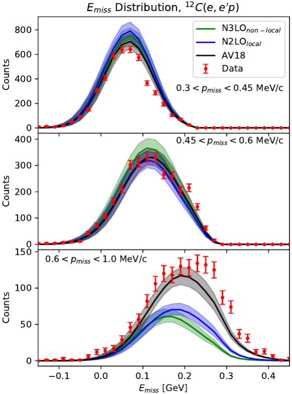


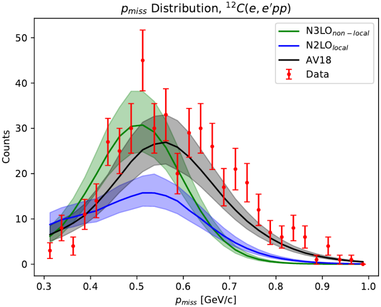
Korover et al., PLB 820, 136523 (2021)



Contacts taken from **ab-initio** calculations

σ_{CM} taken from previous **experiments**.

E^*_{A-2} is modified in the range $(0, 30)$MeV.



A. Schmidt *et al.* (CLAS Collaboration), Nature 578 (2020) 7796, 540-544

Conclusions

Summary and Conclusions

Factorization and universality in nuclear physics

- The contact formalism was generalized and applied to NP $\rightarrow$ **GCF**.
- **Surprisingly, it seems to be working...**
- Many relations were derived. Many more still await.
- Coupled Cluster theory provides the **"missing link"** between the contact formalism and the underlying many-body physics.

What's next

- 3-body SRCs
- Neutron-neutron correlations
- Different observables

The Team

Israel

R. Yankovich, S. Beck, N. Goldberg,
B. Bazak, E. Pazy, E. Piasetzky, I Korover

US

R. Weiss, G. Miller,
D. Lonardoni, R. Cruz-Torres,
J. Pybus, A. W. Denniston,
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O. Hen, B. Wiringa,
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M. Strikman

China

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Thank you for your attention to this important matter !