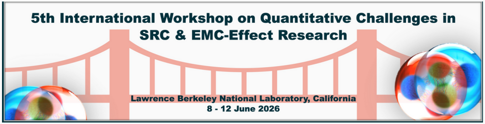

האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

Relative Abundance of Correlated Nucleon Pairs

Raz Yankovich, Ehoud Pazy, Nir Barnea

The Hebrew University, Jerusalem, Israel

LBNL, Berkeley

8-12 June 2026

Short Range Observables

The Contact - Tan, Braaten & Platter,...

The 2-body system

$$\psi(\boldsymbol{r}) \xrightarrow[r \to 0]{} \varphi_0(\boldsymbol{r})$$

The N-body system

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N) \xrightarrow[r_{12} \to 0]{} \varphi_0(\boldsymbol{r}_{12}) A(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_N)$$

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$$O_{12} \approx \delta(\boldsymbol{r}_{12}) \Longrightarrow \langle\psi|O_{12}|\psi\rangle \approx C_2\langle\varphi_0|O_{12}|\varphi_0\rangle$$

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$$\langle\Psi|\sum O_{ij}|\Psi\rangle \approx \underbrace{\frac{N(N-1)}{2}\langle A|A\rangle}_{C_N} \langle\varphi_0|O_{12}|\varphi_0\rangle$$

The Nuclear Contact(s)

- For each nucleon pair we define the contact matrix

$$C_{ij}^{\alpha\beta} \equiv N_{ij} \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle$$

using normalization $\int_{k_F}^{\infty} \frac{d\boldsymbol{k}}{(2\pi)^3} |\tilde{\varphi}_\alpha(\boldsymbol{k})|^2 = 1$

- **The contact $C_{ij}^{\alpha\alpha}$ counts the number of SRC pairs in channel α**
- For $\ell = 0$ we need consider **4** contacts

$$\{C_{pp}^{S=0}, C_{nn}^{S=0}, C_{np}^{S=0}, C_{np}^{S=1}\}$$

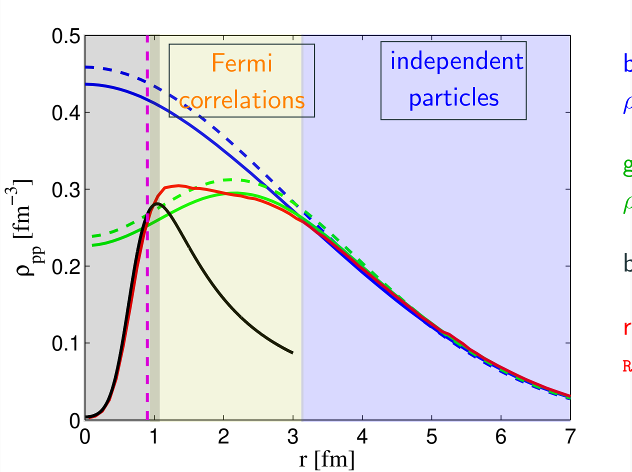
- The deuteron contact $a_2 \approx C_{np}^{S=1}$
- If **isospin symmetry holds** the number of contacts is **2**,

$$C_{np}^{S=1} \qquad\qquad C_{np}^{S=0} = C_{pp}^{S=0} = C_{nn}^{S=0}$$

It doesn't ... (for heavy nuclei)

The contact - many-body calcs.

The pp density of ^{40}Ca



blue - $\rho_{pp} = \rho_p \rho_p$

green - $\rho_{pp} = \rho_p \rho_p \times$ Fermi

black - SRCs

red - VMC

R. B. Wiringa, *et al.*

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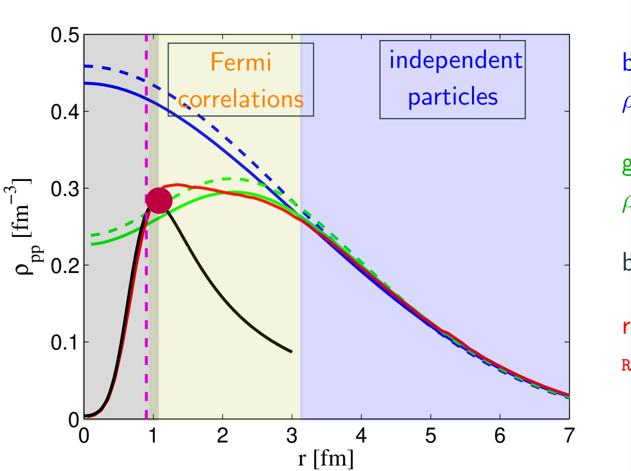

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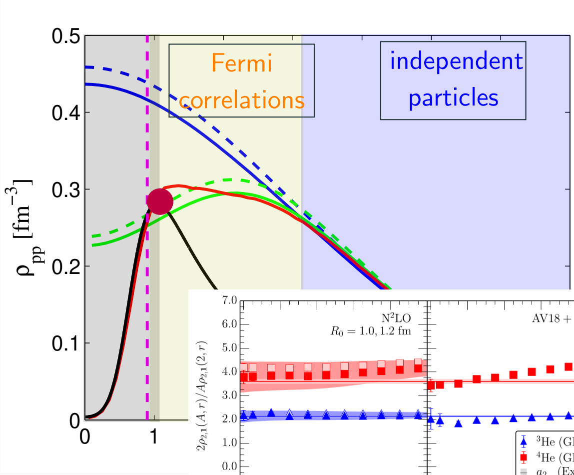

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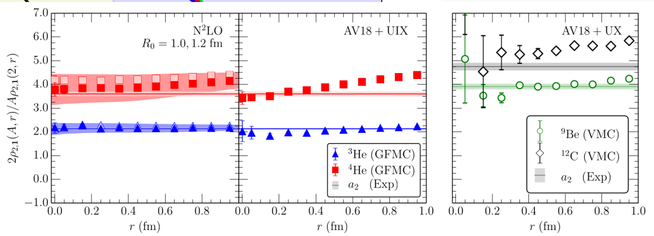
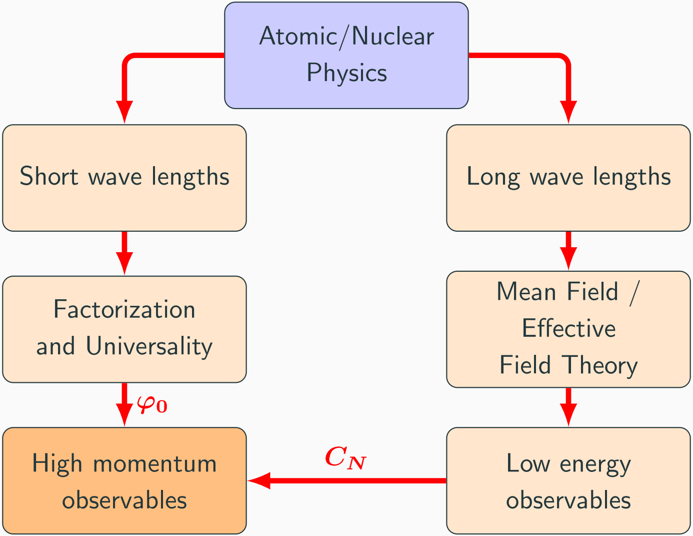


J.W. Chen, W. Detmold, J.E. Lynn, and A. Schwenk, PRL 2016

Short and Long



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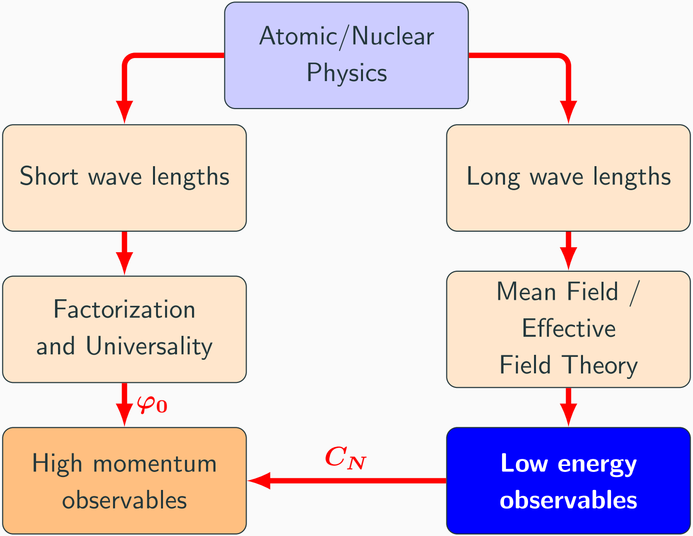


Mean Field

Contacts - Scale and Scheme dependence

Model dependence of contact ratios

$$\frac{C^{\alpha}_{NN}(A)}{C^{\alpha}_{NN}(B)} = \frac{\langle \Psi_A | \hat{O}^{\alpha} | \Psi_A \rangle}{\langle \Psi_B | \hat{O}^{\alpha} | \Psi_B \rangle}$$

ratios **weakly** depend on interaction

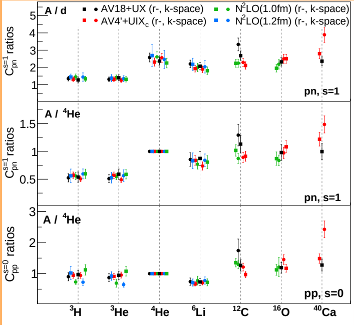
Conjecture (I. Talmi):

$$\frac{C^{\alpha}_{NN}(A)}{C^{\alpha}_{NN}(B)} \cong \frac{\langle \Phi_A | \hat{O}^{\alpha} | \Phi_A \rangle}{\langle \Phi_B | \hat{O}^{\alpha} | \Phi_B \rangle} \; .$$

with $|\Phi_A\rangle, |\Phi_B\rangle$ single Slater determinant **shell-model** w.f.

Cruz-Torres, Lonardoni, et al.

Nature Phys. 17, 306 (2021)



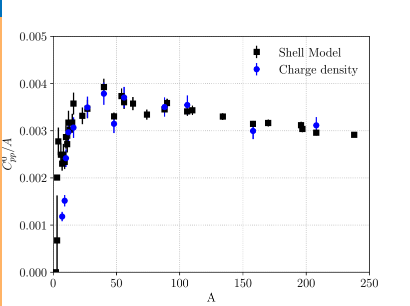
Verification

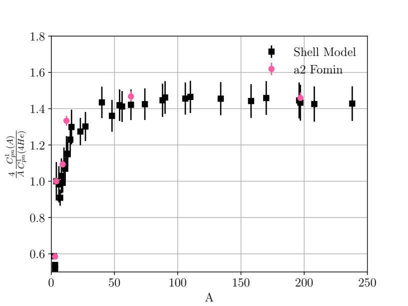
Conjecture:

$$\frac{C^{\alpha}_{NN}(A)}{C^{\alpha}_{NN}(B)} \cong \frac{\langle \Phi_A | \hat{O}^{\alpha} | \Phi_A \rangle}{\langle \Phi_B | \hat{O}^{\alpha} | \Phi_B \rangle} \; .$$

$|\Phi_A\rangle, |\Phi_B\rangle$ single Slater determinant
shell-model w.f.

Yankovich, Pazy, Barnea, PRC (2025)



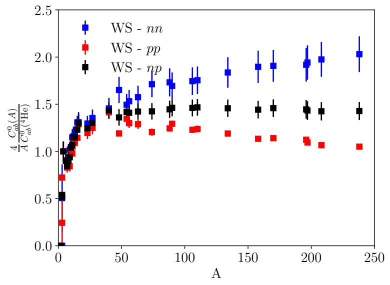


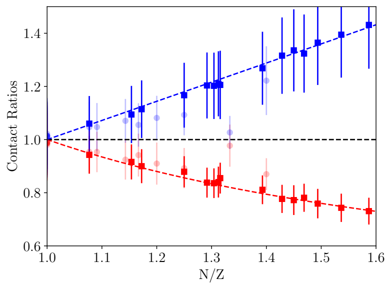
Spin zero contacts

For nuclei with $A > 50$ the contact ratio $C^0_{nn}(A)/C^0_{np}(A)$ presents a rather linear dependence on N/Z, Same for $C^0_{pp}(A)/C^0_{np}(A)$ with Z/N.

$$\frac{C^0_{nn}}{C^0_{np}} \approx \left[1+\beta\left(\frac{N}{Z}-1\right)\right]$$
$$\frac{C^0_{pp}}{C^0_{np}} \approx \left[1+\beta\left(\frac{Z}{N}-1\right)\right] \ ,$$

with $\beta \approx 0.72$.





Contacts - A simple model

Conjecture

- Assume N_a, N_b particles of species a, b in volume $\Delta\Omega$
- Further assume that the correlation volume is v_{ab}

Then averege number of correlated pairs is

$$\Delta C_{ab} = N_a N_b \frac{v_{ab}}{\Delta\Omega} = \rho_a \rho_b v_{ab} \Delta\Omega$$

ρ_a, ρ_b - densities of types a, b

Replacing v_{ab} by L^s_{ab} (honoring Levinger)

$$\boxed{C^S_{ab}(A) = L^S_{ab} \int d\Omega \; \rho_a \rho_b \approx L^S_{ab} \rho_a \rho_b \Omega_{ab}}$$

Ω_{ab} - the relevant nuclear volume

Contacts - A simple model

Yankovich, et al (2025)

$$\boxed{C^s_{ab}(A) = L^s_{ab}\ \rho_a \rho_b \Omega_{ab}}$$

ρ_a, ρ_b - densities of types a, b

Ω_{ab} - the relevant nuclear volume

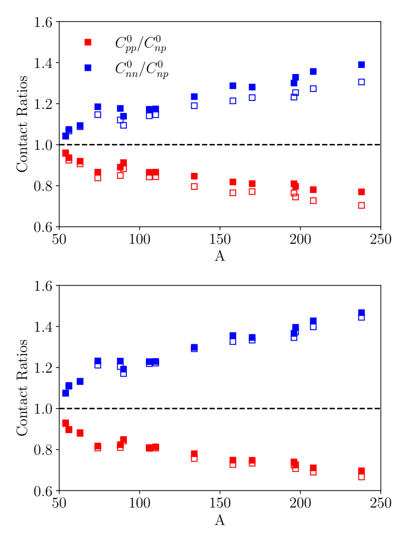
$$C^0_{nn}(A) = L^0_{nn} \frac{N^2}{R_N^3}$$

$$C^0_{np}(A) = L^0_{np} \frac{NZ}{R_N^3 R_P^3} R^3$$

$$C^0_{pp}(A) = L^0_{pp} \frac{Z^2}{R_P^3}$$

top - Universal WS parameters

bottom - SWV parameters.



HFB Calculations

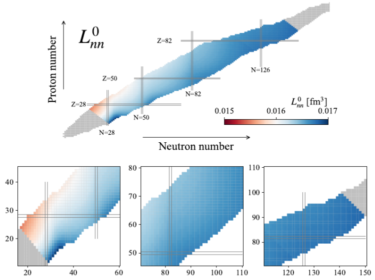
T. Liang et al, PLB 863, 139351 (2025)

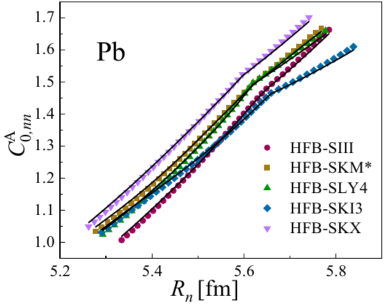
HFB many-body approch to calculate the nuclear w.f.
Used the matching method to calculate the contacts.

$$C^0_{nn}(A) = L^0_{nn}\frac{N^2}{R_N^3}$$

$$C^0_{np}(A) = L^0_{np}\frac{NZ}{R_N^3 R_P^3}R^3$$

$$C^0_{pp}(A) = L^0_{pp}\frac{Z^2}{R_P^3}$$





Conclusions

Summary and Conclusions

The nuclear contacts

- The contacts are system and state dependent quantities
- They are low energy quantities tough they describe SRCs
- We have used mean-field theory to evaluate the contacts
- A simplified model seems to capture the main features

What's next

- 3-body contacts
- ...

Thank you for your attention to this important matter !