Entanglement Entropy of Short-Range Correlations

5-th International Workshop on Quantitative Challenges in Short-Range Correlations and the EMC Effect Research

LBNL
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Ehoud Pazy
NRCN

How quantum is the nucleus?

How quantum is the nucleus?

Not a well posed question

Maybe rather:

How quantum are

Our models of the nucleus?

How Quantum are the models for the ? Nucleus

- This is actually not a well defined question.
- Defining the what is mean by “QuAnatum”:

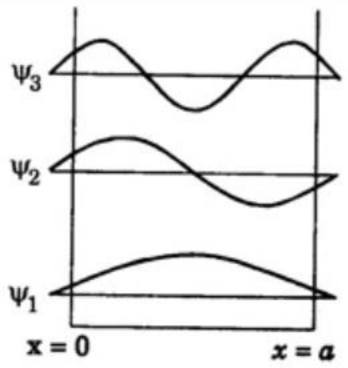
Entanglement versus discrete levels.

- A measure for “Quntum”: Entanglement Entropy.
- Calculation of entanglement entropy: GCF to calculate entanglement entropy.
- Surprise result.

Entanglement versus Discrete Energy Levels

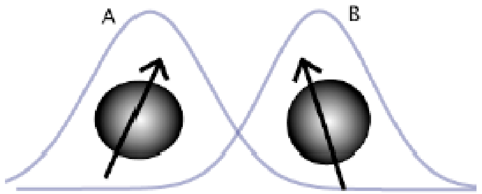
Discrete energy levels as one can observe in the Shell model are:

- A result of wave confinement\boundary conditions.
- Can be achieved by a single particle.
- Has a partial classical analog- standing waves.



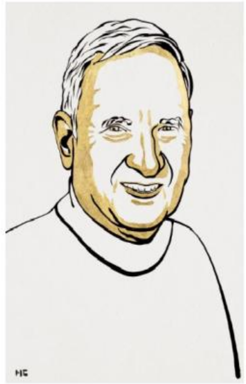
Entanglement:

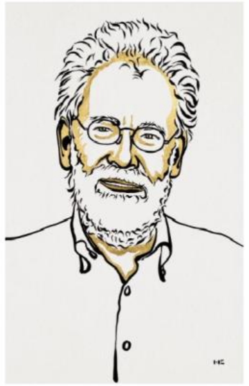
- Non-separability of multi-particle states.
- Requires ≥ 2 particles.
- Has no classical analog.
- It's a property of multi-particle systems: when two or more particles interact, their quantum states can become correlated in a way that cannot be decomposed into independent individual states.



Entanglement

The Physics Nobel Prize 2022





Caveat: this talk is not centered around the entanglement between the Constitutes of the Short Range Correlated pair. I will not consider them as an EPR pair.

I will introduce of a measurement on the entanglement between the SRC pairs and the Fermi sea nucleons

Entanglement Entropy a Measure of Entanglement

Divide a given quantum system into two parts A and B.
Then the total Hilbert space becomes factorized

$$H_{tot} = H_A \otimes H_B \quad .$$

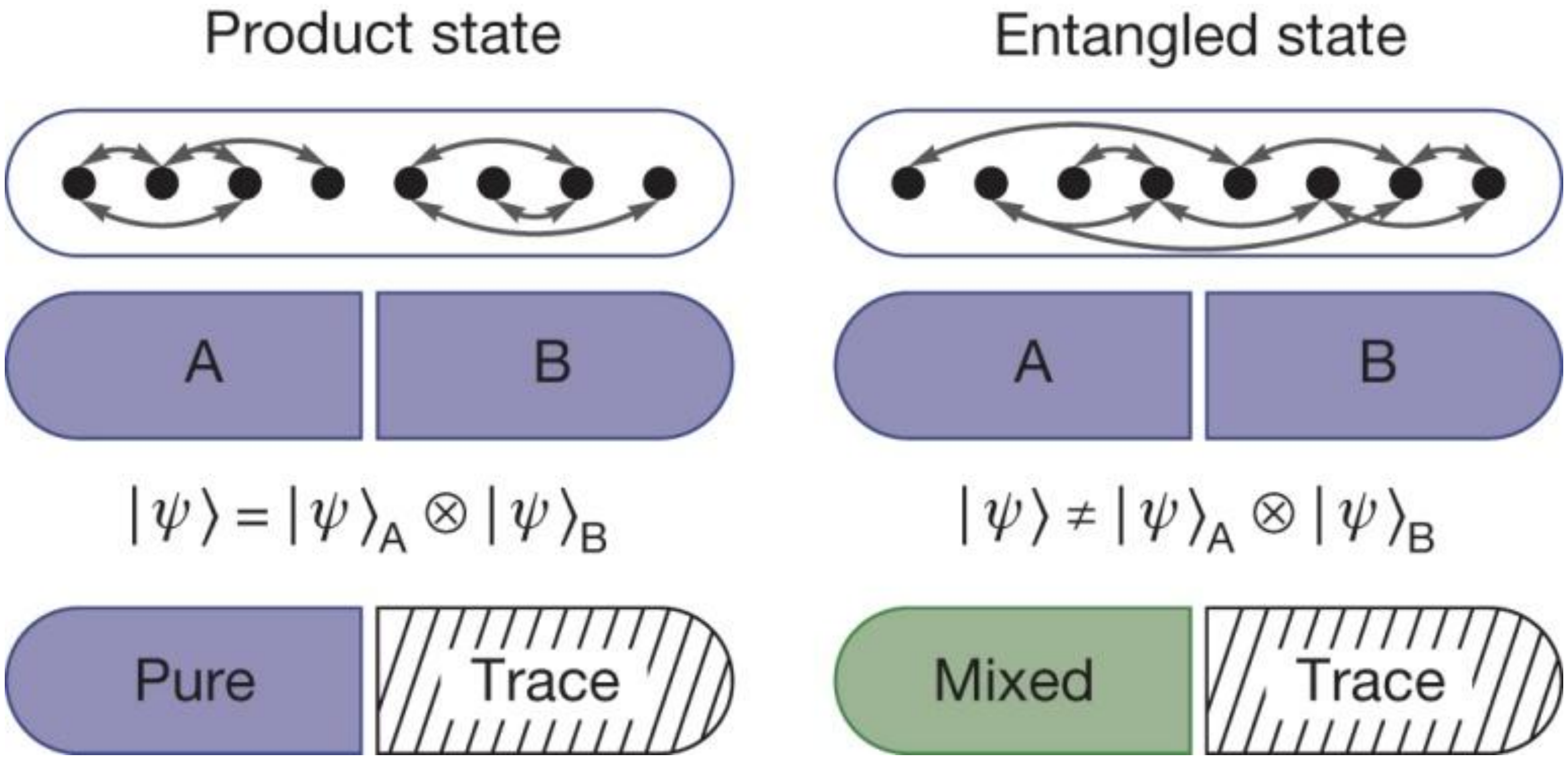
We define the reduced density matrix ρ_A for A by

$$\rho_A = \mathrm{Tr}_B \rho_{tot} \quad ,$$

Tracing over the Hilbert space of B .
Now the entanglement entropy S_A is defined by the von Neumann entropy

$$\boxed{S_A = -\mathrm{Tr}_A \, \rho_A \log \rho_A \quad .}$$

Product verses Entangled



S=0

S>0

R Islam *et al. Nature* **528,** 78 (2015) doi:10.1038/nature15750

No Entanglement Entropy for the Shell Model

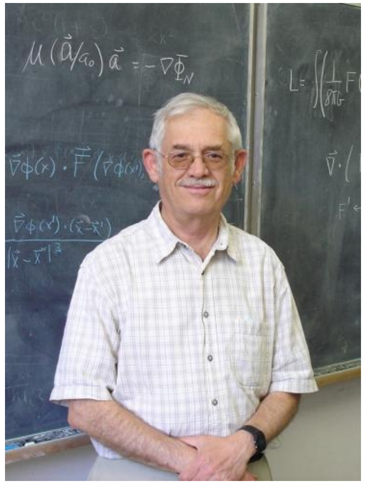
The shell model is an independent particle model
Hence it does not have any entanglement entropy

Therefore the entanglement entropy between the Fermi sea and the short range correlations shell be calculated

Finally a well posed question:

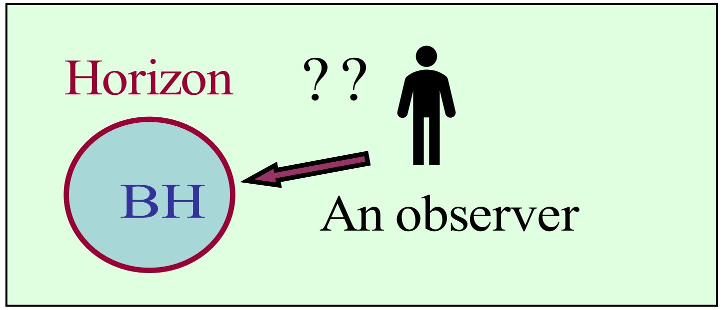
How do we increase the entanglement entropy by introducing Short pair correlations to the nuclear model?

The Entanglement Entropy Holographic Principle and Black Hole Entropy



An analogy with Black hole entropy

The boundary region ∂A~ the event horizon



Entangled Entropy **Area Law**

In many systems the entanglement entropies are proportional to the area that separates the system from the environment

Why is Entanglement Entropy Important for Nuclear Structure?

ntanglement entropy, single-particle occupation probabilities, and short-range correlations

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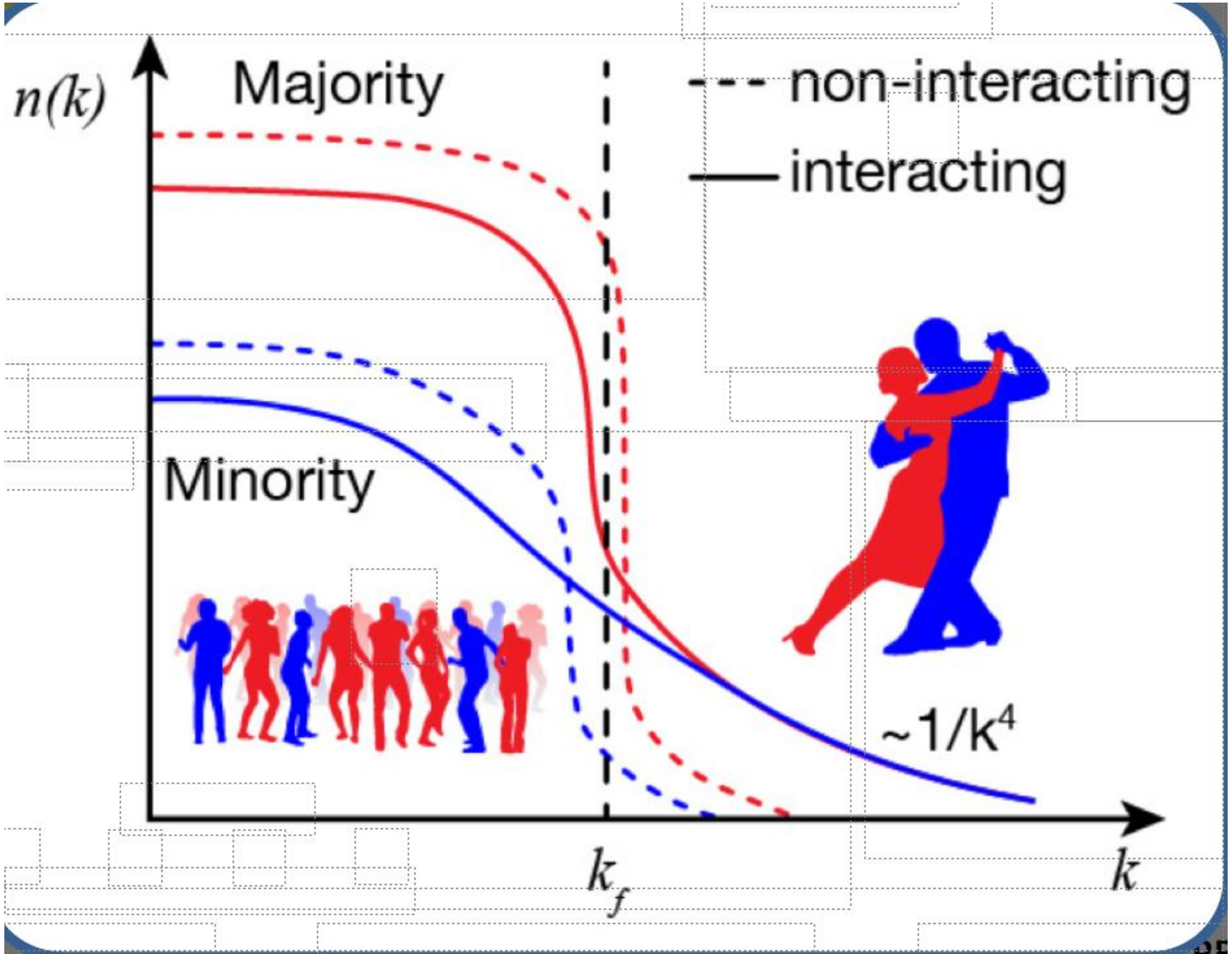
(Dated: June 5, 2023)

For quantum many-body systems with short-range correlations (SRCs), the intimate relationship between their magnitude, the behavior of the single-particle occupation probabilities at momenta larger than the Fermi momentum, and the entanglement entropy is a new qualitative aspect not studied and exploited yet. A large body of recent condensed matter studies indicate that the time evolution of the entanglement entropy describes the non-equilibrium dynamics of isolated and strongly

It is shown here that short-range correlations, which induce high-momentum tails of the single-particle occupation probabilities, increase the entropy of fermionic systems, which in its turn will affect the dynamics of many reactions, such as heavy-ion collisions and nuclear fission.

Relates to the Uncertainty in Single Particle Number



A surge of Interest in Estimating the Entanglement in the Nucleus

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Quantum magic and multipartite entanglement in the structure of nuclei

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Letter

Exploring Short-Range Correlations in symmetric nuclei: contacts and entanglement entropy

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Entanglement entropy of nuclear systems

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We study entanglement entropies between the single-particle states of the hole space and its complement in nuclear systems. Analytical results based on the coupled-cluster method show that entanglement entropies are proportional to the particle number fluctuation and the depletion num-

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Quantum complexity fluctuations from nuclear and hypernuclear forces

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Quantum Complexity and New Directions in Nuclear and High-Energy Physics Phenomenology

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Entanglement entropy, single-particle occupation probabilities, and short-range correlations

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(Dated: June 5, 2023)

General Contact Formalism (GCF)


$$\Psi \xrightarrow[r_{ij}\to 0]{\text{ansatz}} \varphi(\boldsymbol{r}_{ij}) A(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$

A function of the A-2 remaining nucleons
also in terms of:
Center of mass coordinate

$\boldsymbol{R}_{ij} = (\boldsymbol{r}_i + \boldsymbol{r}_j)/2$

Two Body Universal wave function
in terms of: Relative coordinate $\boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j$

The Contact →

$$C_{pn} = N(A,Z)\langle A_{pn}^{\alpha_1} | A_{pn}^{\alpha_1}\rangle$$

The contact traces over all of the degrees of freedom aside from the pair

$$\langle A|A\rangle = \int d^3R_{ij} \prod_{k\neq i,j} d^3r_k \; A^\dagger(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}) \, A(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$

The Generalized Contact Formalism is the ideal method for calculating the Entanglement entropy

Calculating the SRC Entanglement Entropy

$$\rho^{(i)} = \begin{pmatrix} 1-\gamma_{ii} & 0 \\ 0 & \gamma_{ii} \end{pmatrix} \longrightarrow \quad S_i^{(1)} = -Tr[\rho^{(i)} \ln \rho^{(i)}] = -\sum_{k=1}^{2} \omega_k^{(i)} \ln \omega_k^{(i)}$$

One Orbital Density Matrix

One SRC Density Matrix

The Entanglement entropy for SRC

$$\rho^C = \begin{pmatrix} 1-C_{pn} & 0 \\ 0 & C_{pn} \end{pmatrix} \longrightarrow \quad S_{pn}^{SRC} = -\left[c_{pn} \ln\left(\frac{c_{pn}}{1-c_{pn}}\right) + \ln(1-c_{pn})\right]$$

SRC Entanglement Entropy is a sum of the Entanglement Entropy of Single SRC

Assumption: the total SRC entanglement entropy is the sum of the number of SRC pairs-
Meaning assuming SRC pairs are not entangled

$$C_{pn} = N(A,Z)\langle A_{pn}^{\alpha_1} | A_{pn}^{\alpha_1} \rangle \quad N(A,Z=A/2) = \frac{A}{2} \quad \frac{\int_{k_F}^{\infty} n(k)dk}{\int_0^{\infty} n(k)dk} = C_{pn}/(A/2)$$

$$c_{pn} \equiv \frac{C_{pn}}{(A/2)}$$

Calculating the SRC Entanglement Entropy

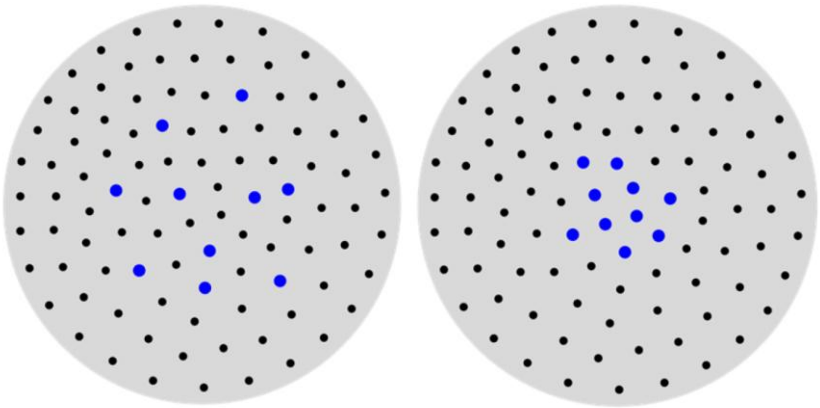
$$S_A^{SRC} = -\frac{A}{2}\left[c_{pn}\ln\left(\frac{c_{pn}}{1-c_{pn}}\right) + \ln\left(1-c_{pn}\right)\right]$$

The SRC Entanglement Entropy is extensive ~A

E. Pazy, Orbital Entanglement Entropy of Short Range Correlated Pairs in Nuclear Structure, Phys. Rev. C **107**, 054308 (2023)

Similar Results at the Same Time

Entanglement Entropies of Nuclear Systems Grow as the Volume of those Systems



Gu, C., Sun, Z.H., Hagen, G., and Papenbrock, T., Entanglement entropy of nuclear systems. *Physical Review C* **108**, 054309 (2023). [DOI: 10.1103/PhysRevC.108.054309]

Results are Equivlant

Entanglement entropy, single-particle occupation probabilities, and short-range correlations

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$$S = -g\int \frac{d^3k}{(2\pi)^3} n(k)\ln n(k) - g\int \frac{d^3k}{(2\pi)^3}[1-n(k)]\ln[1-n(k)], \qquad (5)$$

The Entropy in Terms of the Canonical momentum occupation function

Summary

- A general simple expression was obtained for the SRC Entanglement Entropy

$$S_A^{SRC} = -\frac{A}{2}\left[c_{pn}\ln\left(\frac{c_{pn}}{1-c_{pn}}\right) + \ln\left(1-c_{pn}\right)\right]$$

- The SRC Entanglement Entropy was found to be extensive.

Thank You for your Attention