Spectral Functions for Light Nuclei

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5th International Workshop on Quantitative Challenges in SRC-EMC Research

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THE GEORGE WASHINGTON UNIVERSITY
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Spectral functions are ground state distributions of both momentum and separation energy.

- Spectral Function: $\mathcal{S}(\vec{p}_m, E_m)$

- Missing momentum: $\vec{p}_m = \pm(\vec{q} - \vec{p}_N)$
- Missing energy: $E_m = \omega - T_N - T_{A-1}$

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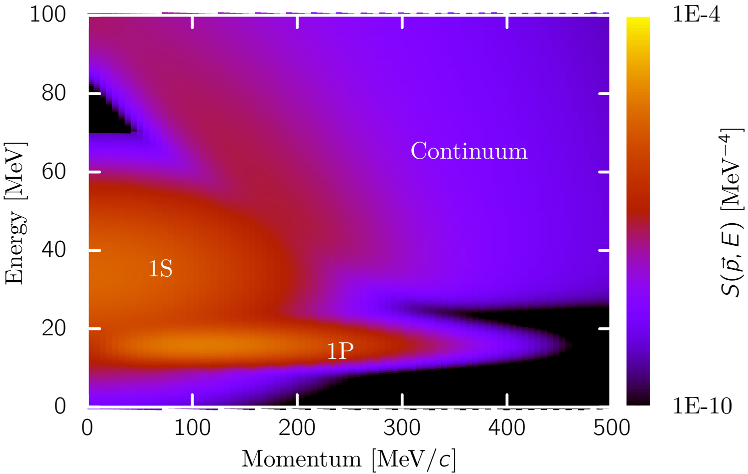
$$\begin{aligned} E_m &= m_N - m_A + m_{A-1} \\ &= m_N - m_A + \sqrt{E_{A-1}^2 - p_{A-1}^2} \\ &= m_N - m_A + \sqrt{(\omega + m_A - m_N)^2 - p_m^2} \end{aligned}$$

In this convention, missing energy is fixed when $A - 1$ system is left intact in a state of fixed energy.

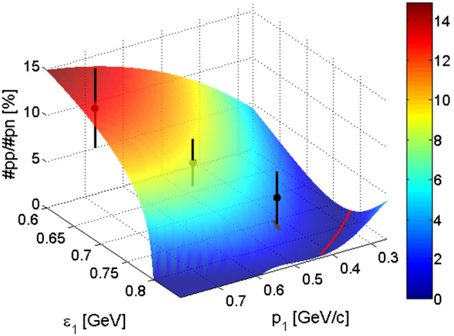
Example spectral function for ^{12}C



Calculation by N. Rocco, A. Lovato

Spectral functions from Contact Formalism was a big breakthrough for SRC research.

$$\mathcal{S}(\vec{p}_m, E_m) = \sum_\alpha C_\alpha \cdot \frac{1}{4\pi} \int \frac{d^3\vec{p}_2}{(2\pi)^3} \left|\tilde{\varphi}_\alpha(\vec{p}_{\text{rel.}})\right|^2 P_\alpha\left(\vec{p}_m + \vec{p}_2\right) \delta(E_m - E'_m(\vec{p}_2))$$



R. Weiss et al., PLB 791, p. 242 (2019)

Plane Wave Impulse Approximation

Quasi-elastic $(e, e'p)$ scattering:

$$\frac{d^6\sigma}{d\Omega_e dE_e d\Omega_N dE_n} = p_N E_N \cdot \sigma_{eN} \cdot \mathcal{S}_N(\vec{p}_m, E_m)$$

- σ_{eN}: single-nucleon cross section (e.g., deForest σ_{cc1})
- $\mathcal{S}_N(\vec{p}_m, E_m)$: spectral function for nucleon of type N

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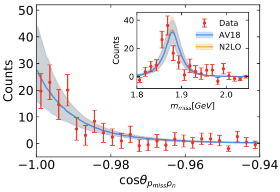
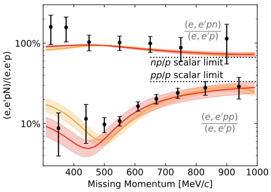
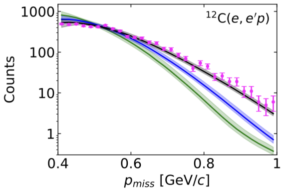
PLB 805, 135429 (2020)



PLB 820, 136523 (2021)



Nature 578, 540 (2020)



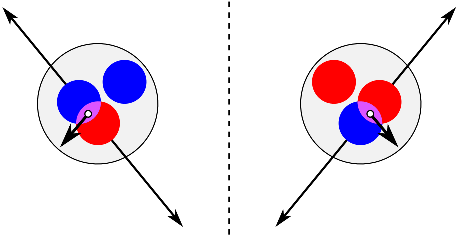
In my talk today:

- (e, e') from ^{3}He and ^{3}H
  - Sizeable low-E_m contribution at high x_B
  - Schmidt, Denniston, Seroka et al., PRC 109, 054001 (2024)

- Two-body spectral function for recoil tagging in ^{4}He
  - Different off-shell modification functions lead to
  - Ratliff, Schmidt, EPJA 60:5 (2024)

- Spectral functions from Quantum Monte Carlo
  - A word of caution about SRC contributions
  - . . . and what I think should be done.

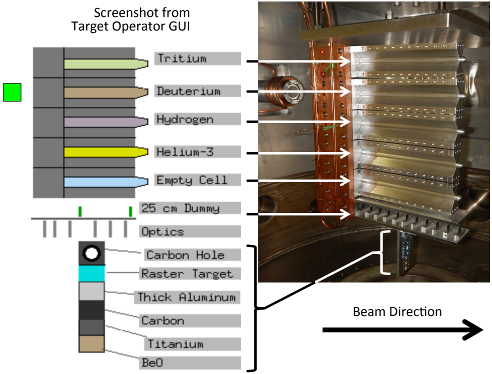
The $A = 3$ system is a very interesting test case.



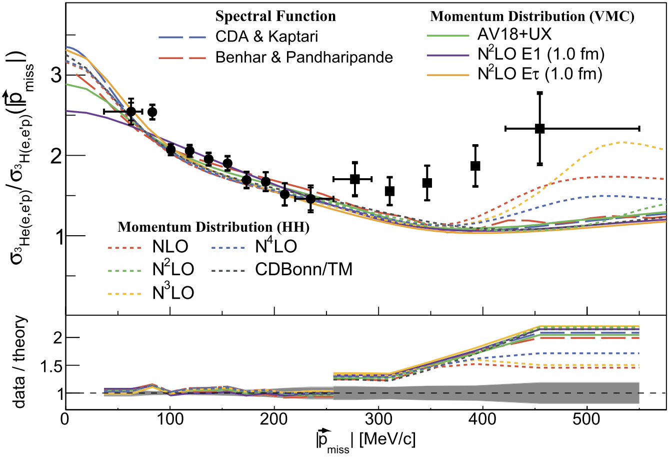
- Isospin-1/2 doublet: ^{3}He and ^{3}H (tritium)
- Huge proton-neutron asymmetry
- Relatively easy to make calculations
- **Problem: tritium is radioactive**

Jefferson Lab Hall A Tritium Run (2017–2018)

- Experiments:
  - MARATHON (F_2^n/F_2^p)
  - Inclusive $x > 1$
  - $(e, e'p)$
- Targets:
  - ^{3}H, ^{3}He, d, p
  - Sealed cell design
- High-Res. Spectrometers
  - 6.7 msr ang. acc.
  - $\pm$4.5% mom. acc.
  - $\Delta p/p \sim 10^{-4}$

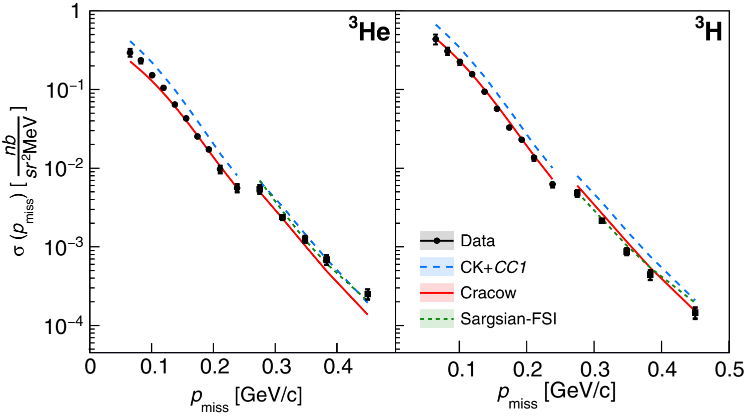


We saw surprisingly high $(e, e'p)$ ^{3}He/tritium ratios.



R. Cruz-Torres et al., PLB (2019)

Things started to make sense when we studied the absolute cross sections.



R. Cruz-Torres, D. Nguyen et al., PRL (2020)

$A = 3$ Theory Calculations

- CK+CC1
  - C. Ciofi degli Atti, L. P. Kaptari, PRC 71, 024005 (2005)
  - Spectral-function including continuum FSI between spectator nucleons
  - Based on ^{3}He wave function developed by Pisa group, assuming AV18 (no 3-body force!)
- Cracow
  - J. Golak et al., Phys. Rep. 415, pp. 89–205 (2005)
  - Fadeev calculation assuming CDBonn+UIX, plane-wave scattering
  - Includes continuum FSI between spectator nucleons
- Sargsian FSI
  - M. M. Sargsian et al., PRC 71, 044614 (2005)
  - M. M. Sargsian et al., PRC 71, 044615 (2005)
  - Includes FSI between struck and spectator nucleons in Generalized Eikonal Approximation framework
  - Does not include single-charge exchange FSI

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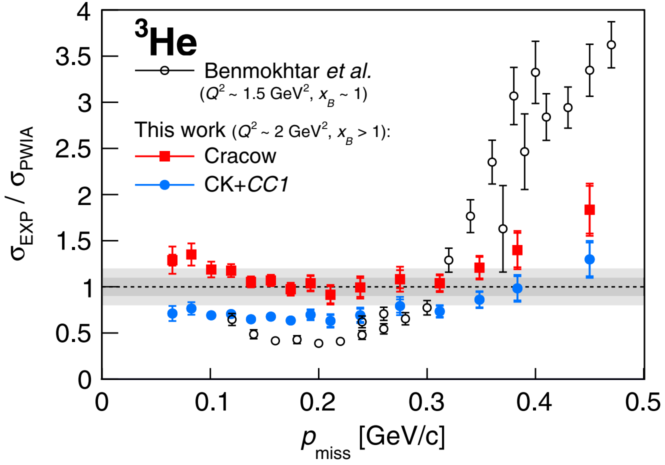

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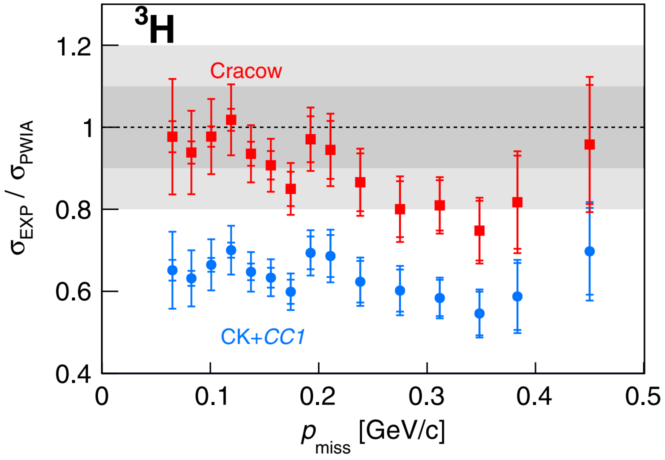
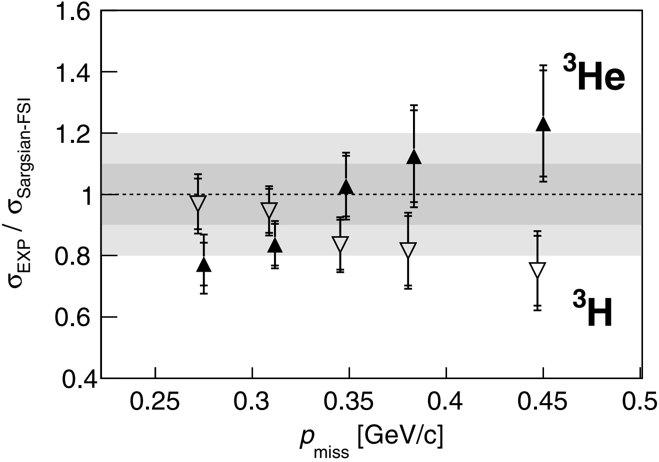


R. Cruz-Torres, D. Nguyen et al., PRL (2020)

Final state interactions are slightly different between ^3H and ^3He.



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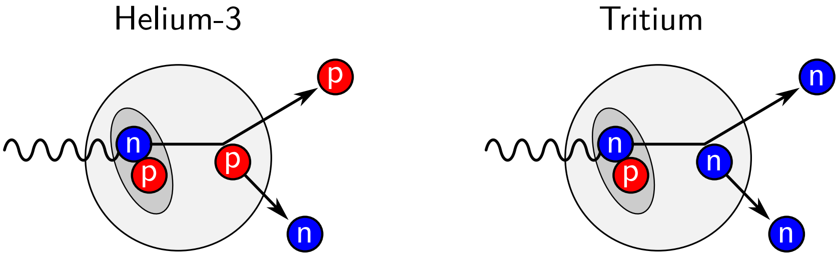

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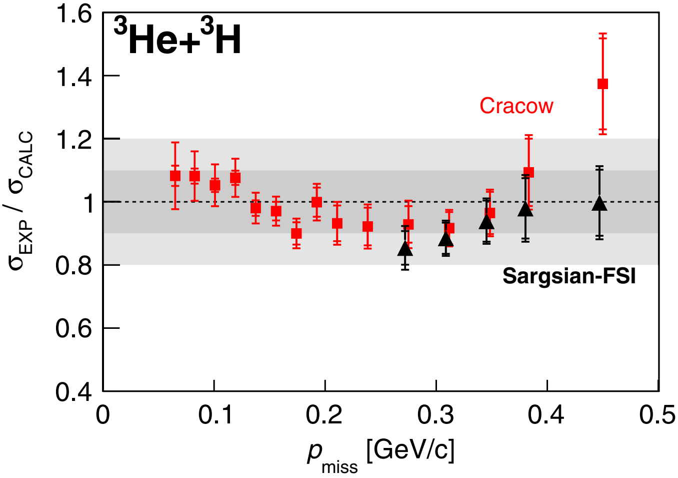
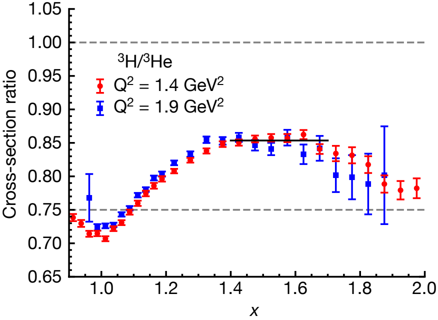


Inclusive scattering offers another way to probe *pp* vs *pn* vs *nn* pairing.



S. Li et al., Nature 609, p. 41 (2022)

$$\frac{\sigma_{\mathrm{T}}}{\sigma_{\mathrm{He}}} = \frac{R_{np/pp}(1+\sigma_{p/n})+2}{R_{np/pp}(1+\sigma_{p/n})+2\sigma_{p/n}}$$

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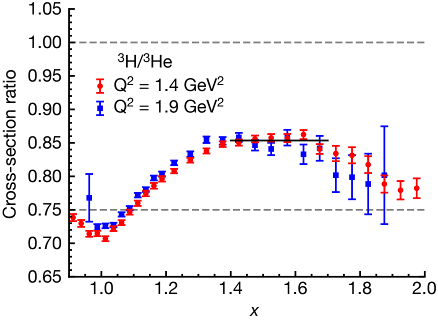

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$$\frac{\sigma_{\mathrm{H}}}{\sigma_{\mathrm{He}}} = 0.854 \pm 0.010$$

$$\frac{N_{pp}}{N_{np}} = 0.228 \pm 0.023$$

$$\frac{N_{np}}{N_{pp}} = 4.39^{+0.49}_{-0.41}$$

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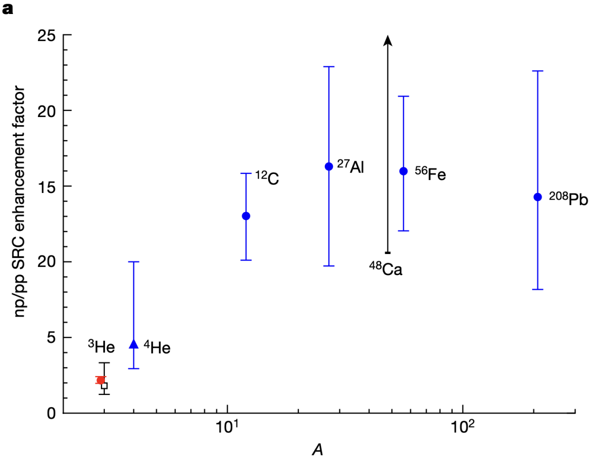
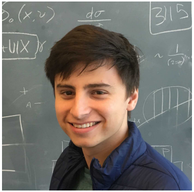


S. Li et al., Nature 609, p. 41 (2022)

$A = 3$ (e, e') $x_B \geq 1$ cross section ratios and the isospin structure of short range correlations



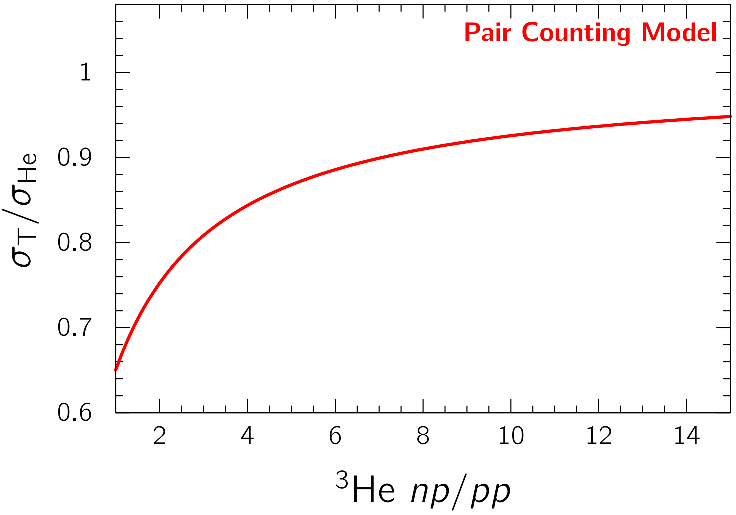
Andrew Denniston, MIT

A. Schmidt, A. W. Denniston, E. M. Seroka,
N. Barnea, D.W. Higinbotham, I. Korover, G.A. Miller,
E. Piasetzky, M. Strikman, L.B. Weinstein, R. Weiss, O. Hen
Phys. Rev. C 109, 054001 (2024)

Erin Seroka, GW

Data from:
S. Li et al., Nature 609, p. 41 (2022)

What if there is uncertainty in pair counting itself?



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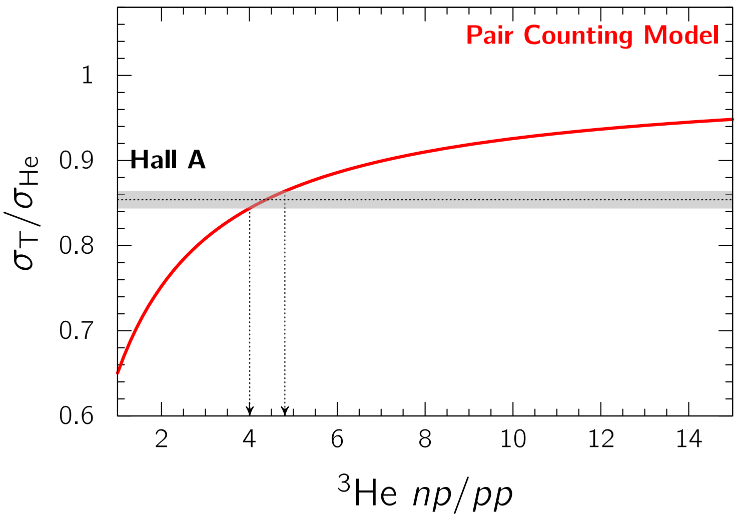

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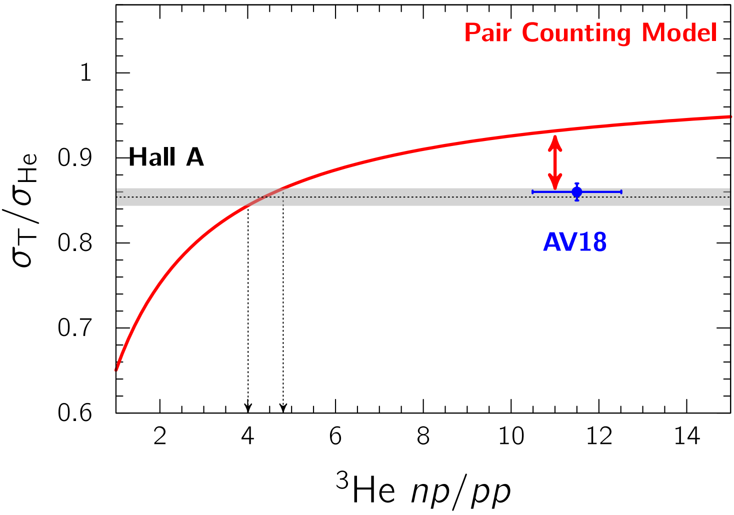

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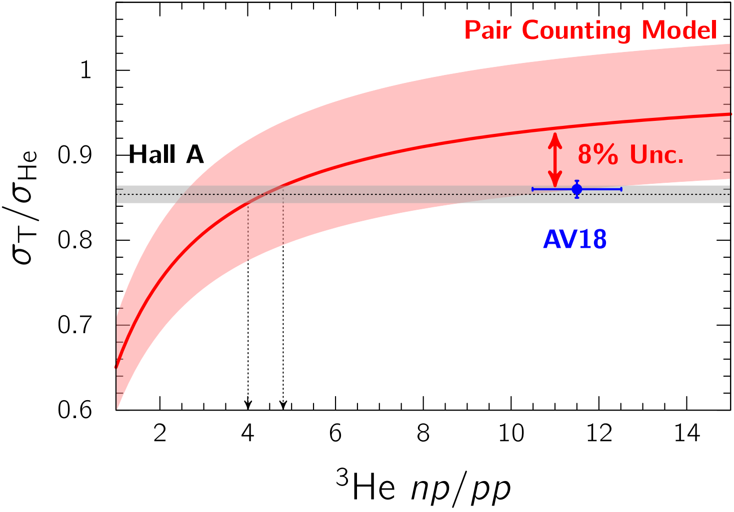

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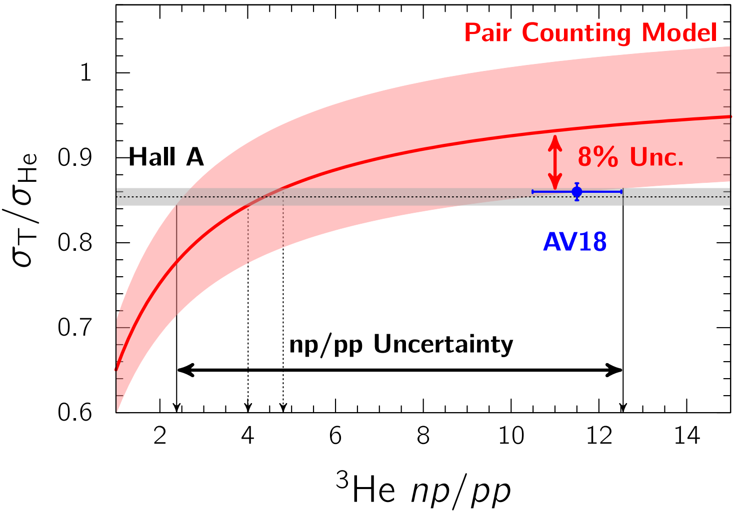
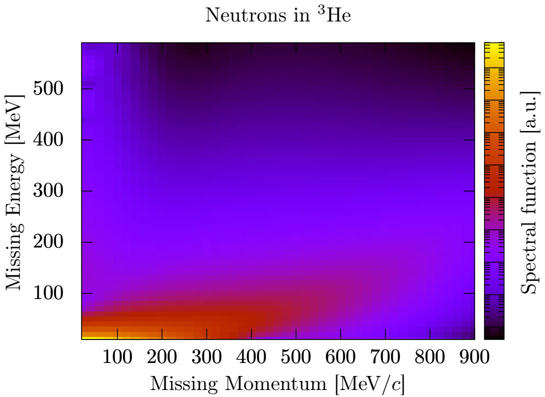


Integrating $(e, e'N)$ should reproduce (e, e').

$A = 3$ Spectral Function



C. Ciofi degli Atti, L. P. Kaptari

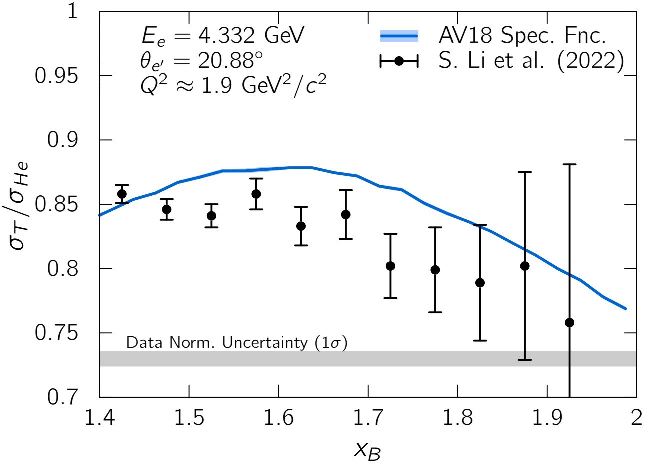
Phys.Rev.C 71, 024005 (2005)

PWIA:

$$\frac{d^6\sigma}{d\Omega_e dE_e d\Omega_N dE_N} = p_N E_N \sigma_{eN} \mathcal{S}(\vec{p}_m, E_m)$$

3-body wave function assuming **AV18**

Theoretical spectral function + PWIA does a decent job of reproducing the data.



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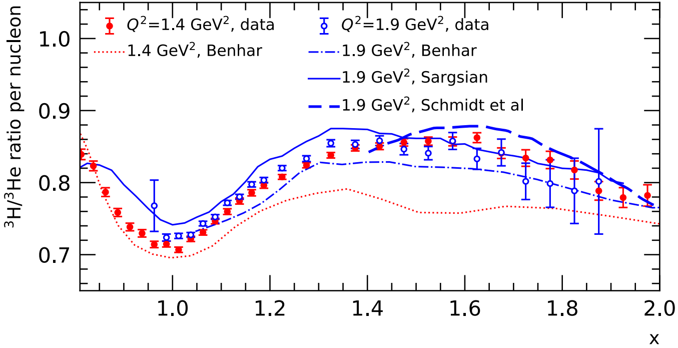
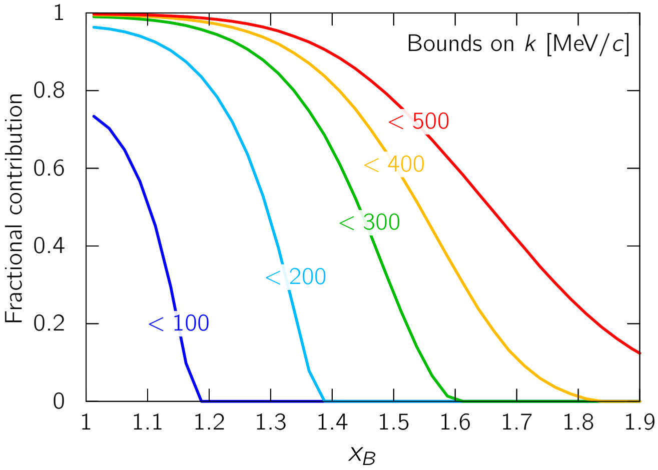


Long-Range Outlook for Short-Range Correlations
arxiv:2601.09568

The spectral function allows us to look "under the hood."



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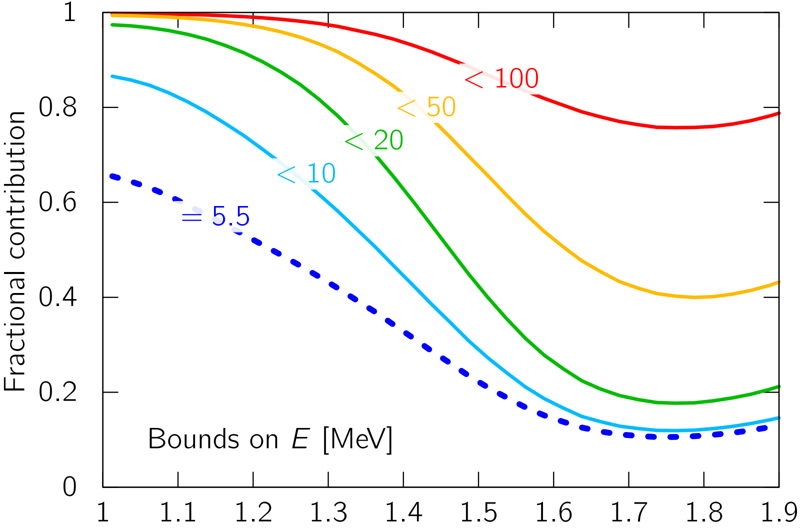
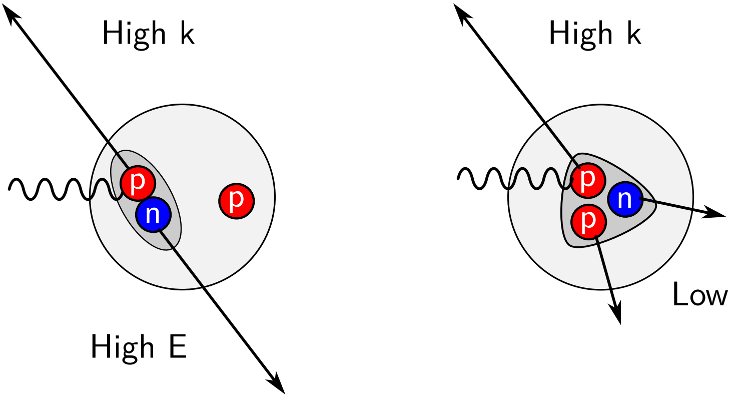


High p_m, low E_m is not a ‘pair’ in the traditional sense.

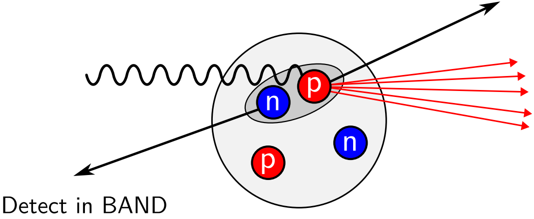


Studying the impact of virtuality-dependent nucleon structure modification on spectator-tagged deep inelastic scattering

Spectator-tagged DIS

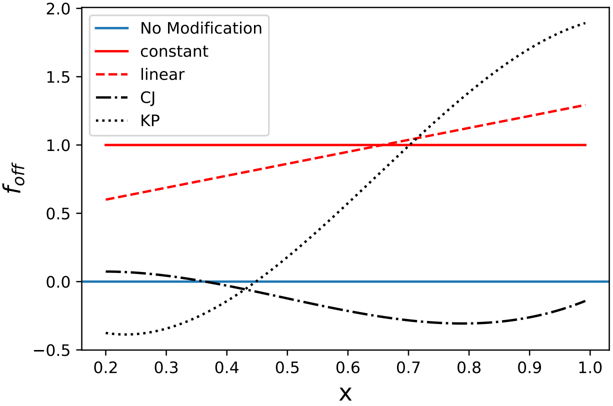
Ratliff, Schmidt, EPJA 60:5 (2024)

Sara Ratliff, GW



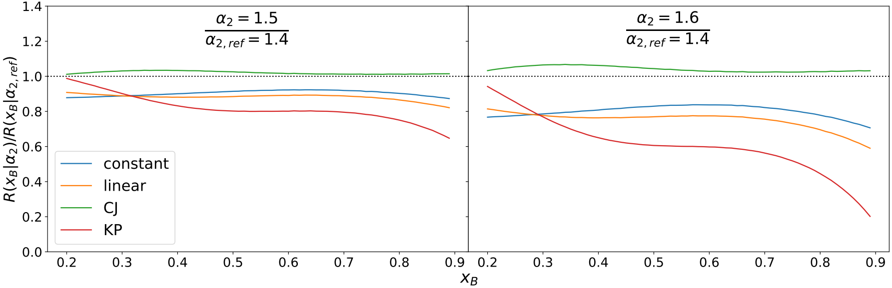
- 2-nucleon "decay function" from GCF
- Light-front convolution formalism → F_2
- Modification proportional to struck nucleon "virtuality"

We tried several models of x-dependence of the modification.



Double ratio results $(F_2^{\text{bound}}/F_2^{\text{free}})/(F_2^{\text{bound}}/F_2^{\text{free}})_{\text{ref.}}$

Data could help distinguish models.



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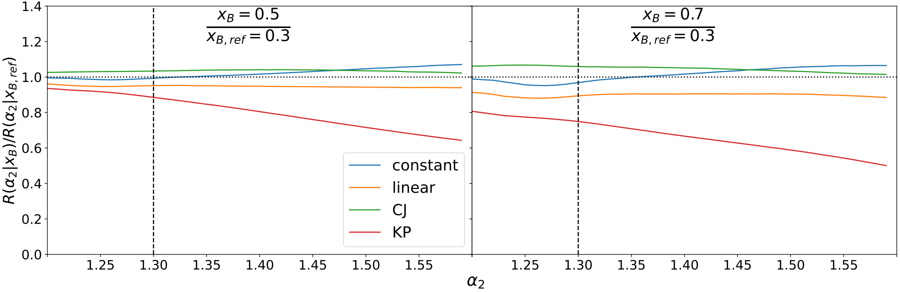

Rocco/Lovato calculation of the "correlated" part of the 1-nucleon spectral function.

$$\mathcal{S}(p_1, E_m) = \int \frac{d^3\vec{p}_2}{(2\pi)^3} \sum_n \left| \left\langle \psi_0^A \middle| \vec{p}_1 \otimes \vec{p}_2 \otimes \psi_n^{A-2} \right\rangle \right|^2 \delta(E_m - \ldots)$$

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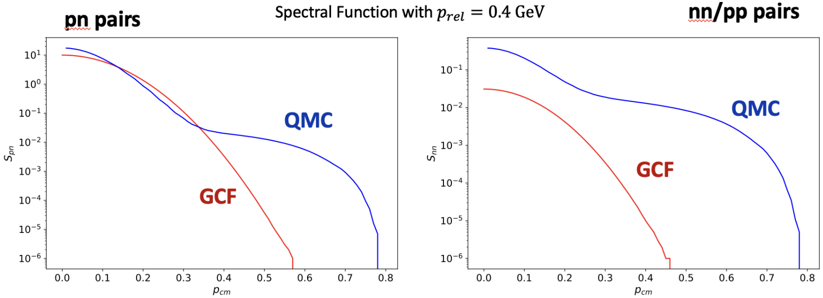
$$= \int \frac{d^3\vec{p}_2}{(2\pi)^3} n_{2N}^A(\vec{p}_1, \vec{p}_2) \delta(E_m - \ldots)$$

The QMC SF is counting *all* pairs not just correlated ones.



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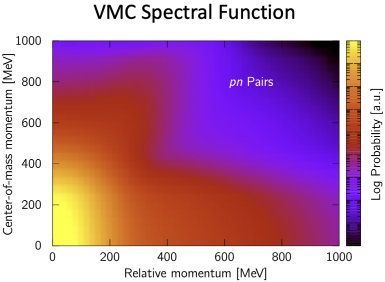

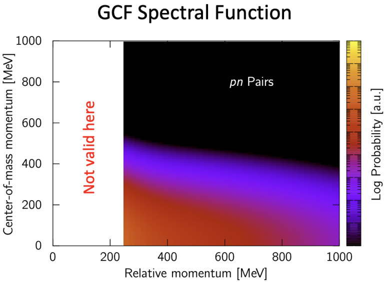

The solution is to orthogonalize $|p_2 \otimes \psi_n^{A-2}\rangle$ with respect to $|\psi_0^{A-1}\rangle$.

The problem is that:

$$\left\langle p_2 \otimes \psi_n^{A-2} \middle| \psi_0^{A-1} \right\rangle \neq 0$$

We can build an orthogonal wave function:

$$\left| p_2 \otimes \psi_n^{A-2} \right\rangle_{\text{ortho.}} = \left| p_2 \otimes \psi_n^{A-2} \right\rangle - \left\langle \psi_0^{A-1} \middle| p_2 \otimes \psi_n^{A-2} \right\rangle \left| \psi_0^{A-1} \right\rangle$$

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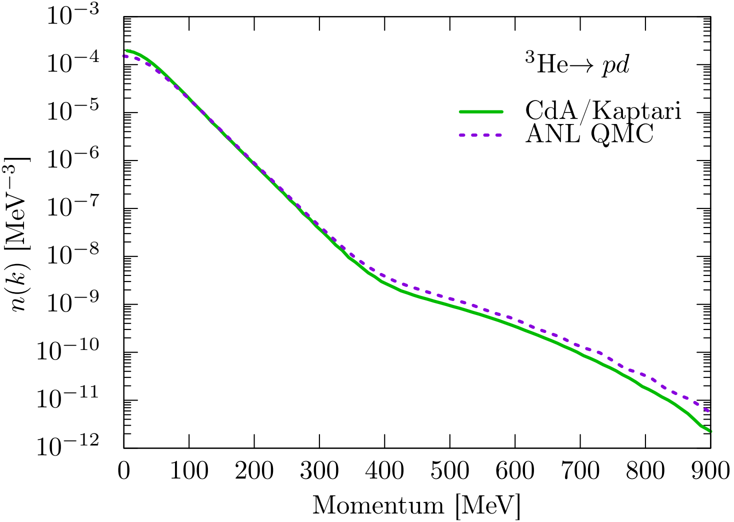
$$\mathcal{S}(p_1, p_2, E_m) = \sum_n \Big| \left\langle \psi_0^A \middle| p_1 \otimes p_2 \otimes \psi_n^{A-2} \right\rangle$$

$$- \left\langle \psi_0^{A-1} \middle| p_2 \otimes \psi_n^{A-2} \right\rangle \left\langle \psi_0^A \middle| p_1 \otimes \psi_0^{A-1} \right\rangle \Big|^2 \cdot \delta(E_m - \dots)$$

This simplifies somewhat for the $A = 3$ case but work is still on-going.



Take aways

- Ab initio $A = 3$ spectral function shows a modest "non-pair-like" contribution for $x \to 2$.

- GCF spectral function for ^{4}He shows that spectator tagged DIS can help resolve different models of medium modification.

- Naive QMC approach to building a "correlated" function have questionable E_m dependence due to considering *all* pairs, not just correlated pairs.

- There is a need for properly orthogonalized correlated spectral functions.

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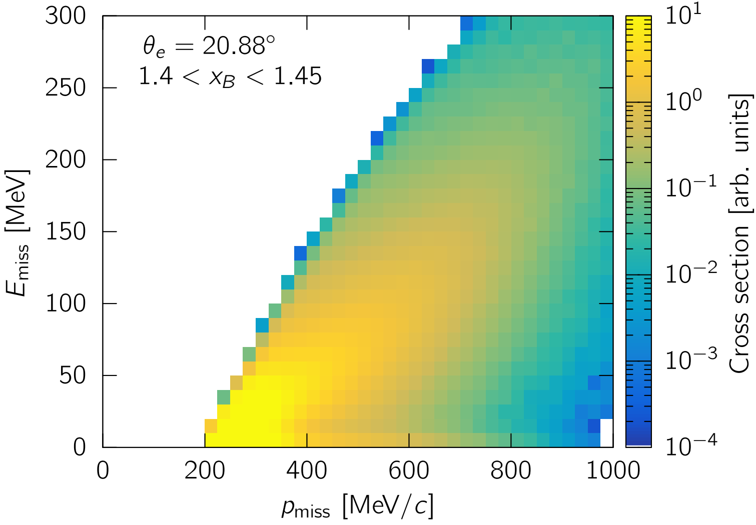
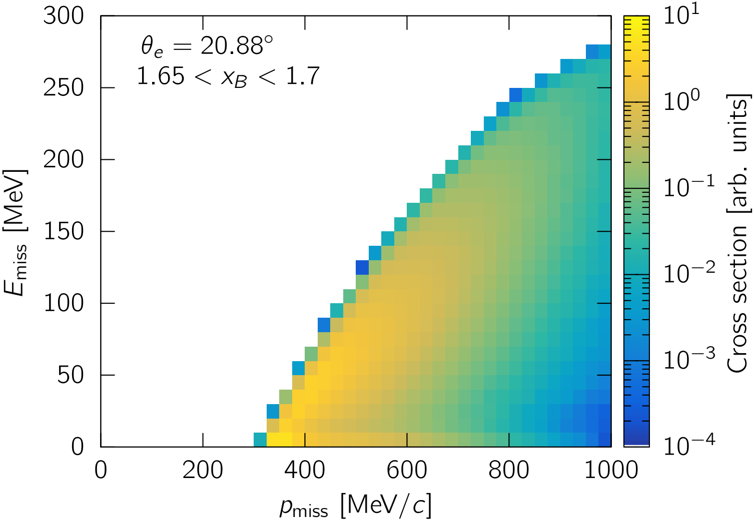
- Ab initio $A = 3$ spectral function shows a modest "non-pair-like" contribution for $x \to 2$.
- GCF spectral function for ^{4}He shows that spectator tagged DIS can help resolve different models of medium modification.
- Naive QMC approach to building a "correlated" function have questionable E_m dependence due to considering *all* pairs, not just correlated pairs.
- There is a need for properly orthogonalized correlated spectral functions.
- Although, after yesterday, may I should be spending my time worrying more about FSIs.

BACK-UP

E_m and p_m content per x_B bin



E_m and p_m content per x_B bin



Spectral function calculation versus data