Study of 3N SRCs using ab-initio calculations

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Washington University in St. Louis

SRC & EMC workshop, LBNL, June 2026

SRC - Theory

- Understanding **the impact of short-range physics** on observables
- **Validate theories** against experimental data: **Structure + Reaction** (nuclear interactions, PWIA, FSI, factorization...)
- Studies of **relativistic effects**, **non-nucleonic degrees of freedom**, medium modifications, ...
- ...

Methods:

Ground-state ab-initio calculations	Approximated factorized approaches	Studies of few-body systems	QMC for reactions	Light front methods

and more...

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and more...

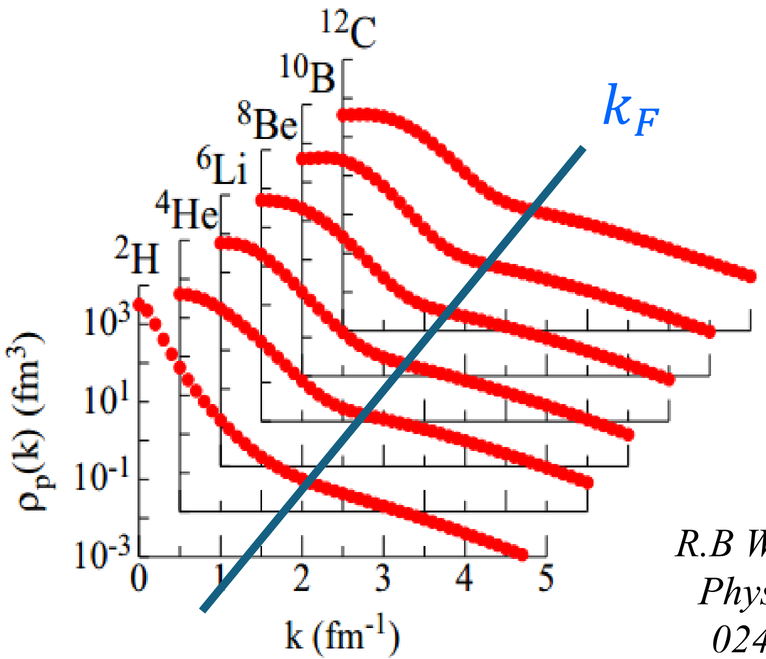
Ab-initio calculations: 2N SRCs

- Calculations based on **exact solution** of the Schrodinger equation
- Learn about **main SRC features**
- **Light-medium** nuclei ($A \leq 40$) [with QMC]
- **Ground-state quantities** (e.g., mom. distributions)
- **Input/validation for approximated methods to compare with experiments**

Ab-initio calculations: 2N SRCs

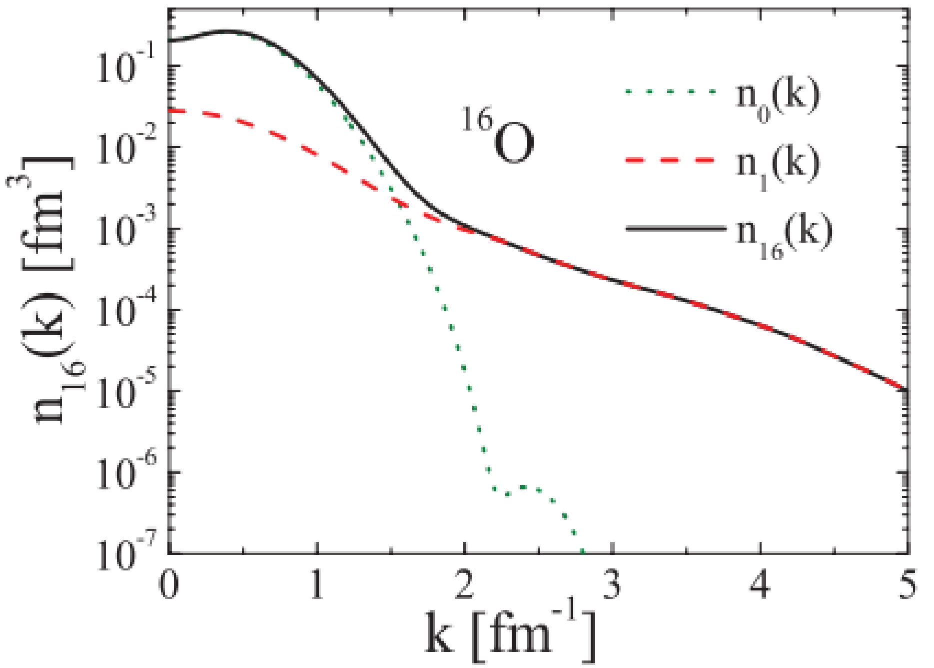
High-momentum tails

Similar shapes
$15 - 20\%$ for $k > k_F$



R.B Wiringa et. al., Phys. Rev. C 89, 024305 (2014)

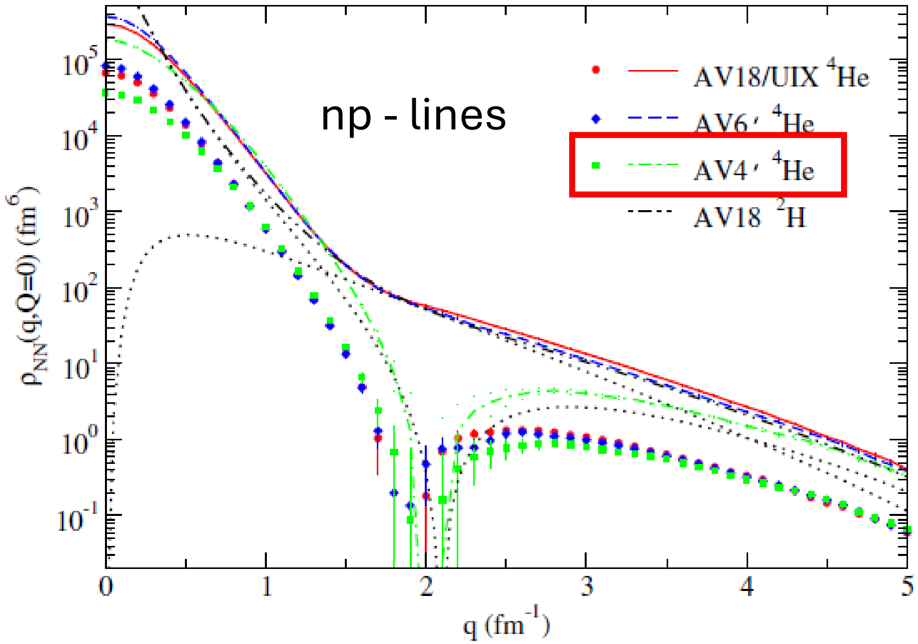
contribution from excited $A - 1$ system



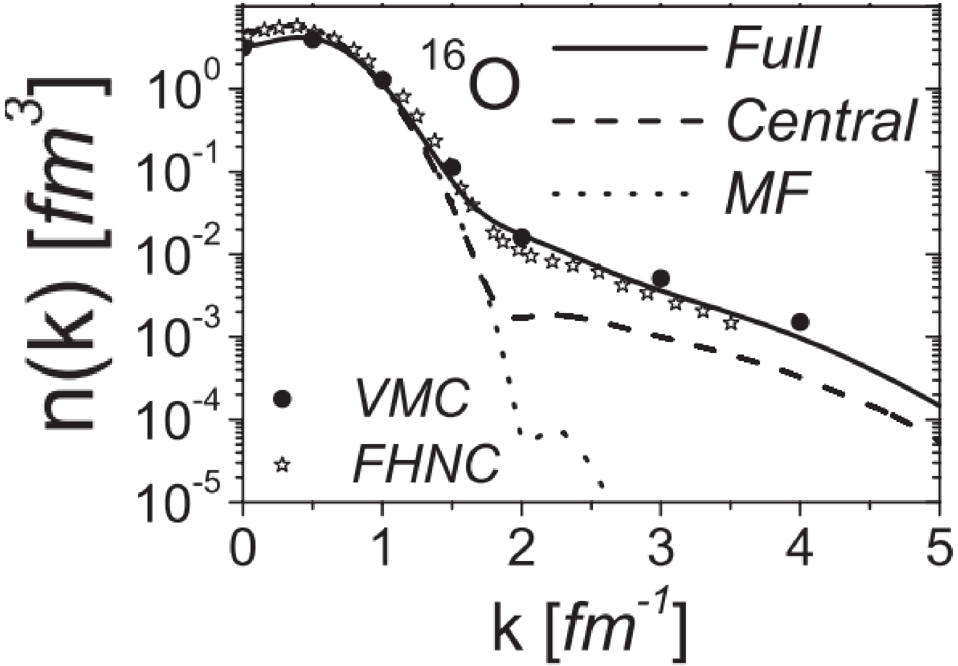
M. Alvioli et al. PRC 87, 034603 (2013)

Ab-initio calculations: 2N SRCs

np dominance & tensor force



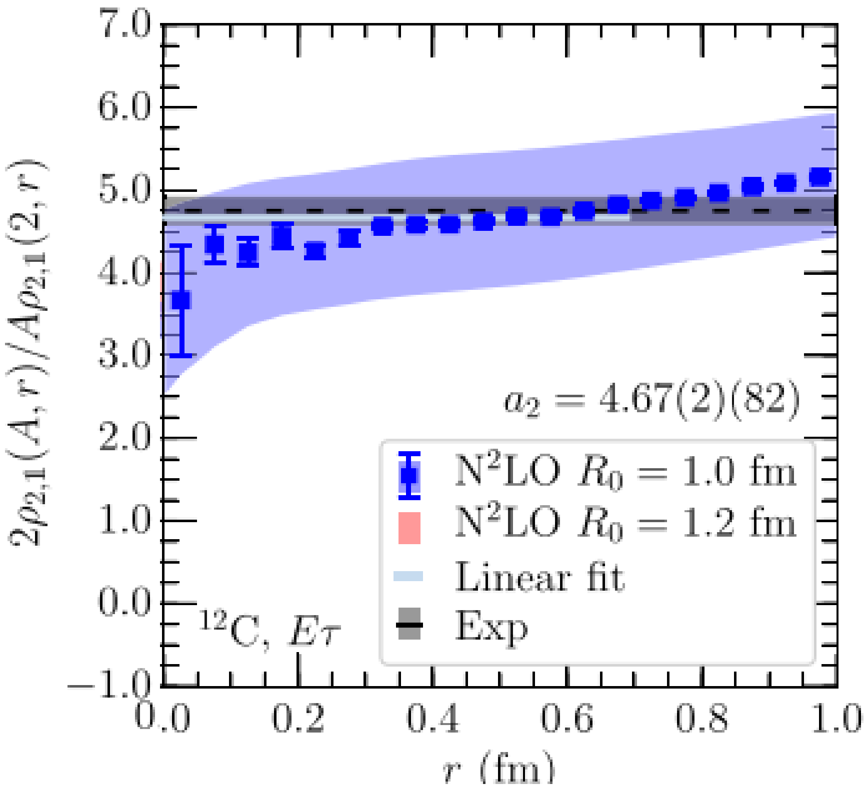
R. Schiavilla et al. PRL 100, 162503 (2008)



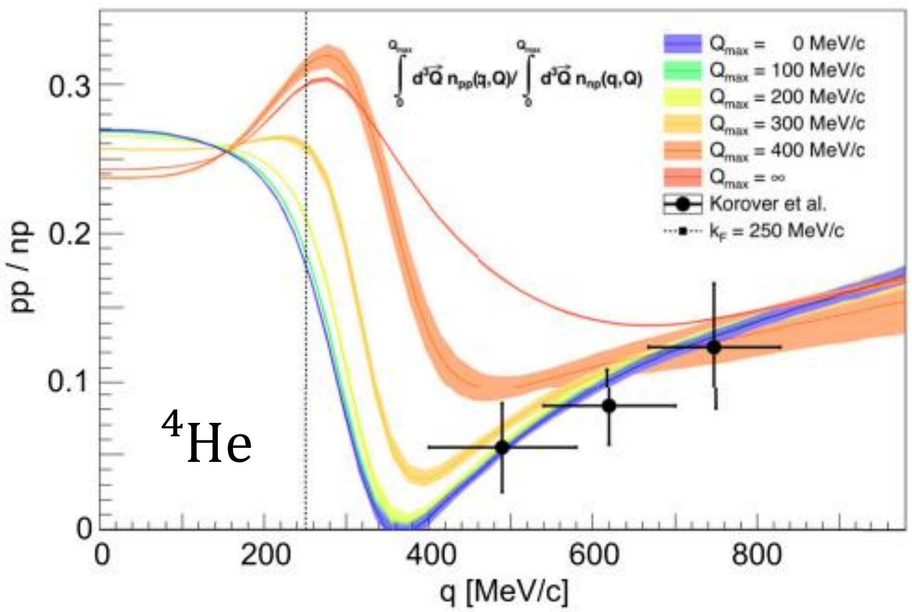
M. Alvioli et al. PRL 100, 162503 (2008)

Ab-initio calculations: 2N SRCs

Direct comparison to data



Lynn et al, J. Phys. G: Nucl. Part. Phys. 47 045109 (2020)



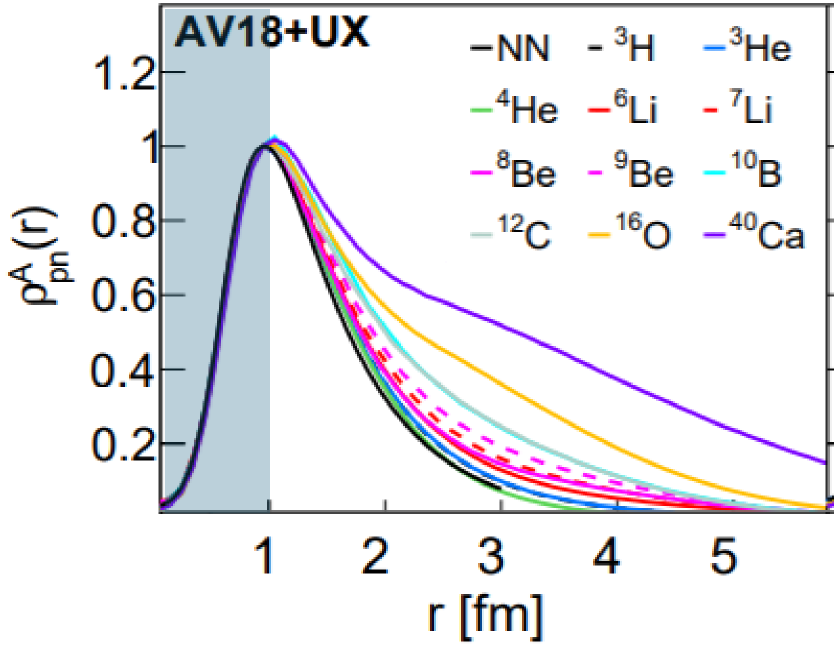
Data: Korover et al., 2014, Calculations: Wiringa et al., 2014, Figure: Hen et al., 2017

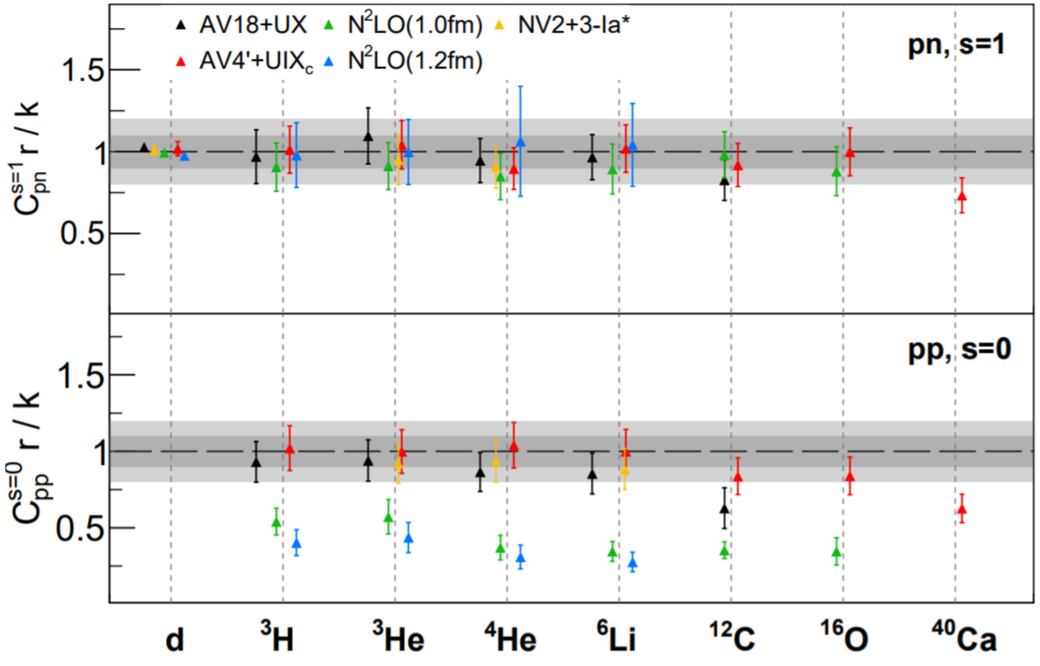
Ab-initio calculations: 2N SRCs

Factorization

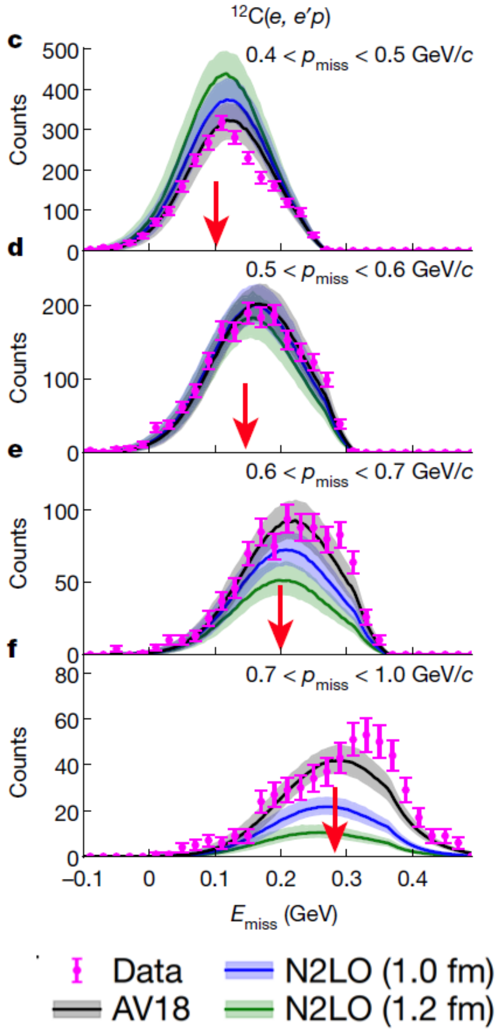
$$\Psi \rightarrow \varphi_2 \Phi_{A-2}$$

r/k equivalence





R. Cruz-Torres, D. Lonardoni, RW, et al., Nature Physics (2020)



Schmidt, et al. Nature 578, 540 (2020)

3N SRCs

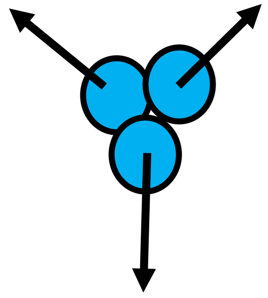
Coordinate space RW, S. Gandolfi, Phys. Rev. C 108, L021301 (2023)

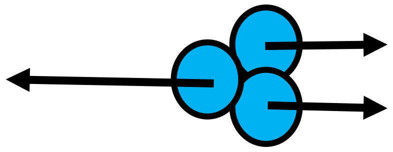
Momentum space RW, L. Andreoli, S. Pastore (in preparation)

Three-body correlations

Various open questions:

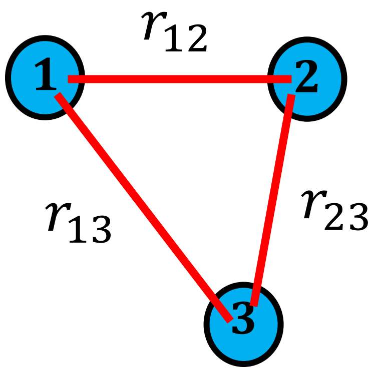
- **Are there 3N SRCs** in nuclei? $(k_1, k_2, k_3 > k_F; K_{cm} < k_F)$
- Are they **universal**? Wave-function **factorization**?
- What is their **abundance**?
- What are the **dominant configurations**?
- Are 3N SRCs sensitive to the **three-body force**?
- What is their **contribution to different observables**?





3N SRCs: coordinate space

We performed first ab-initio calculations of 3N SRC



$$\rho_3(r_{12}, r_{13}, r_{23}) \equiv \binom{A}{3} \langle\Psi|\delta(|\boldsymbol{r}_1 - \boldsymbol{r}_2| - r_{12})\delta(|\boldsymbol{r}_1 - \boldsymbol{r}_3| - r_{13})\delta(|\boldsymbol{r}_2 - \boldsymbol{r}_3| - r_{23})|\Psi\rangle$$

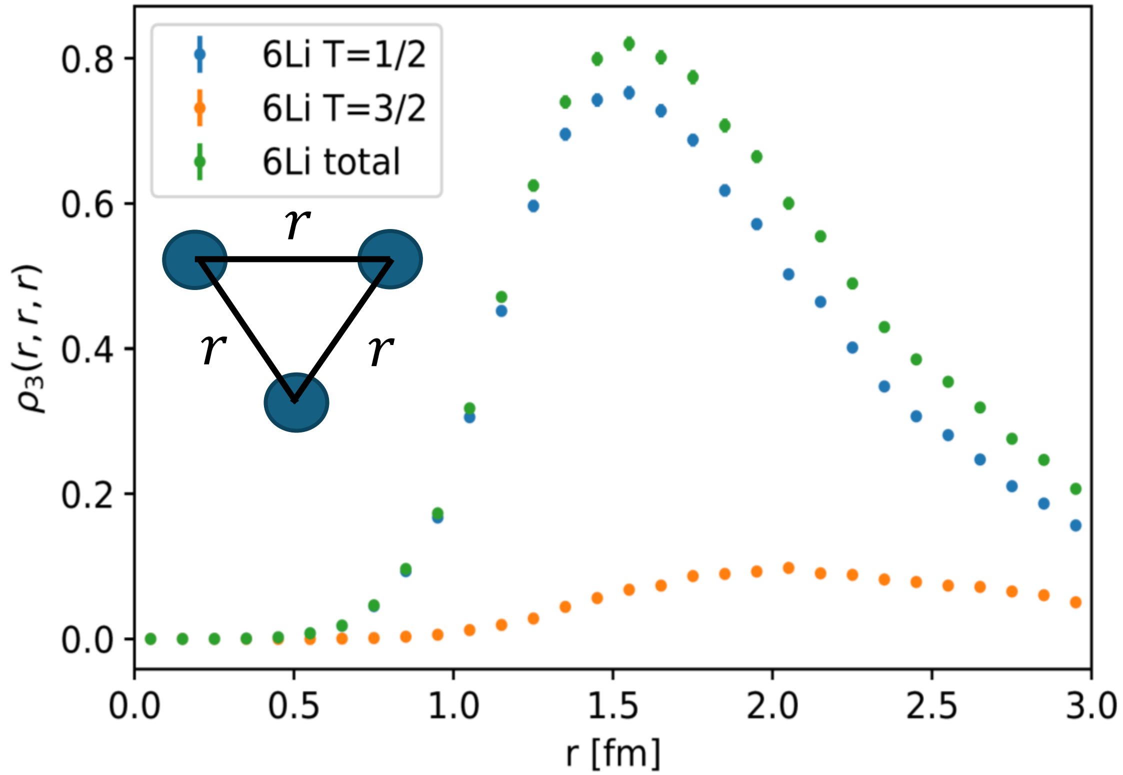
- **QMC method** (AFDMC)
- N2LO($R = 1.0$ fm)E1 local chiral interaction
- Nuclei: ^{3}He, ^{4}He, ^{6}Li, , ^{16}O

RW, S. Gandolfi, Phys. Rev. C 108, L021301 (2023)

3N SRCs: coordinate space

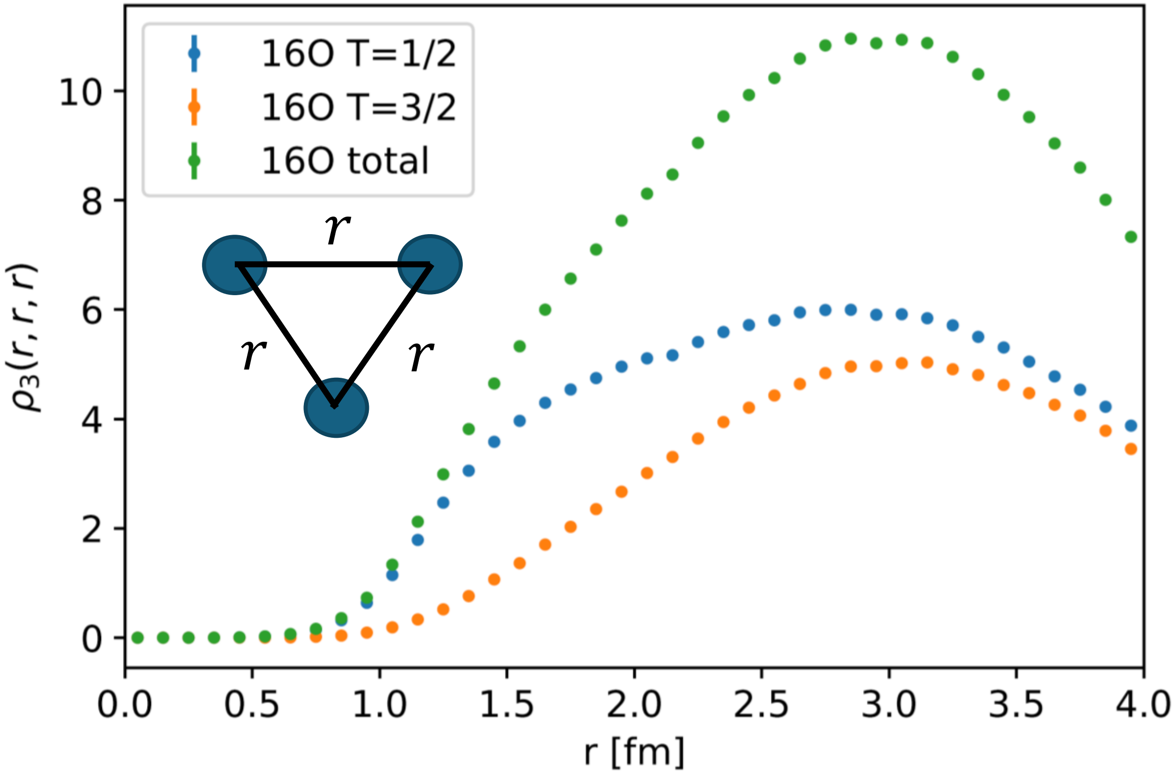
$T = 1/2$ vs $T = 3/2$

^{6}Li



Total number of triplets: $T = \frac{1}{2}: 16$; $T = \frac{3}{2}: 4$

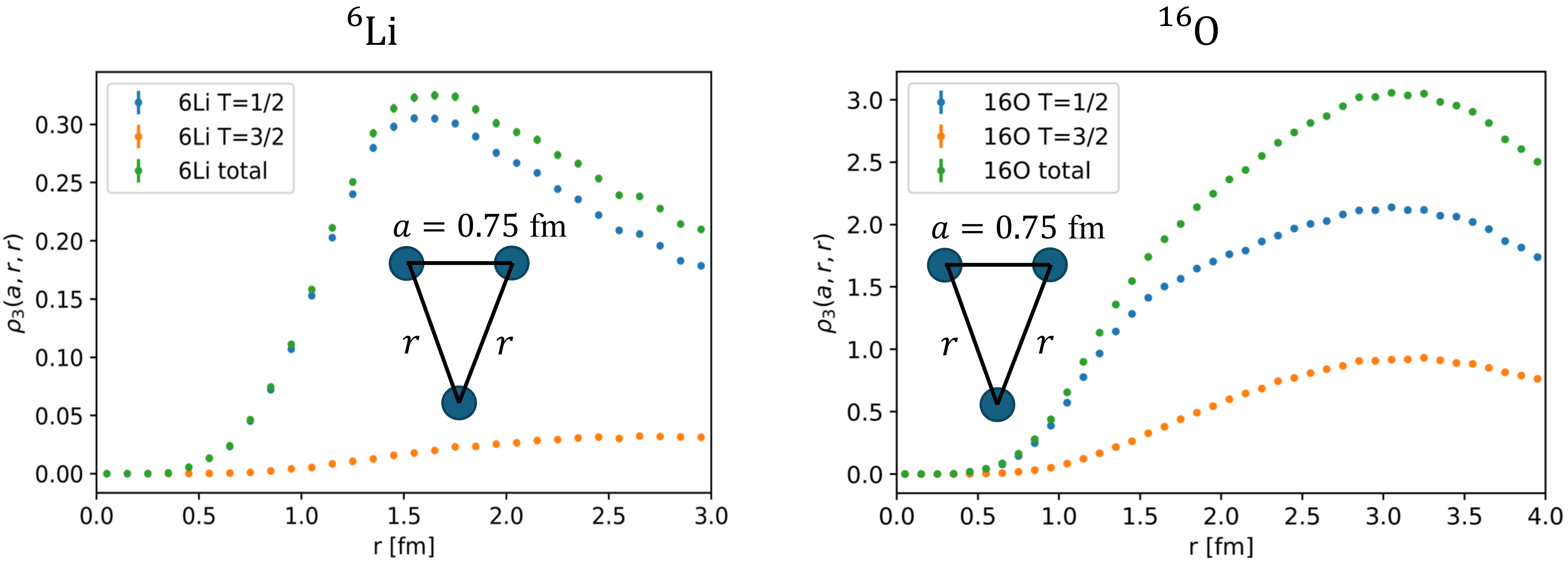
^{16}O



Total number of triplets: $T = \frac{1}{2}: 336$; $T = \frac{3}{2}: 224$

3N SRCs: coordinate space

$T = 1/2$ vs $T = 3/2$



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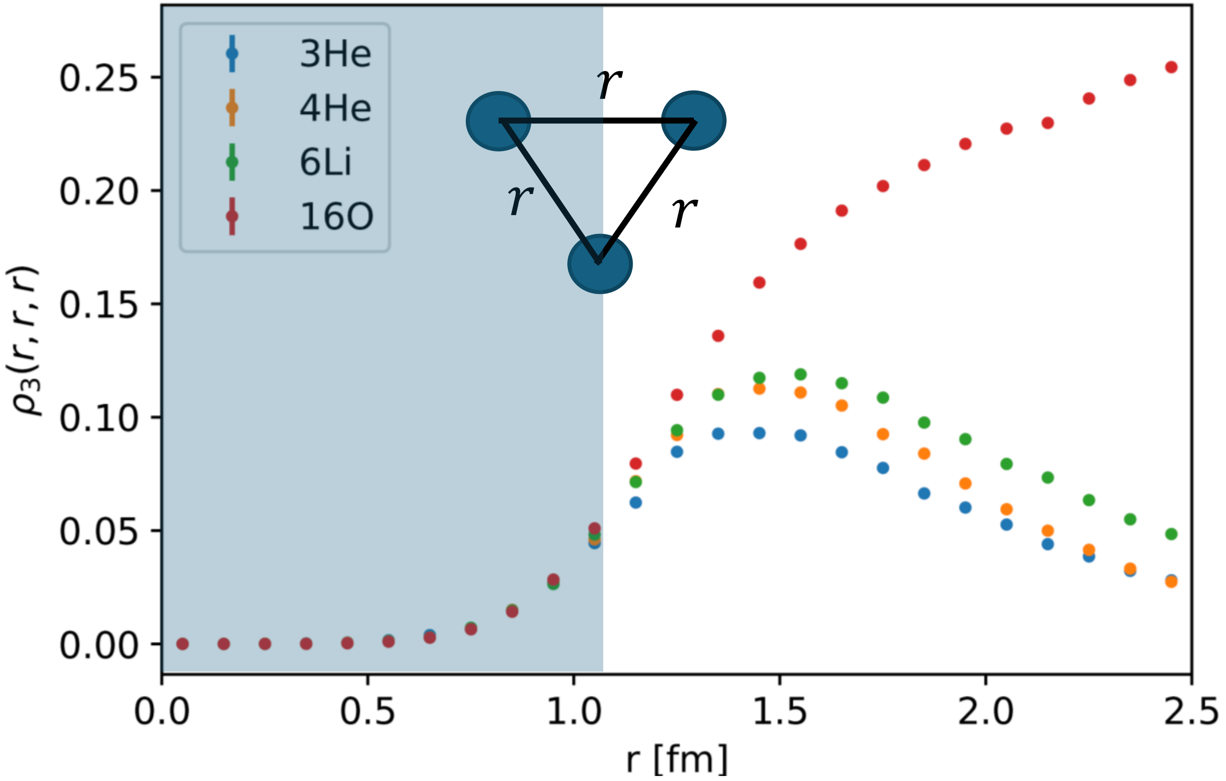
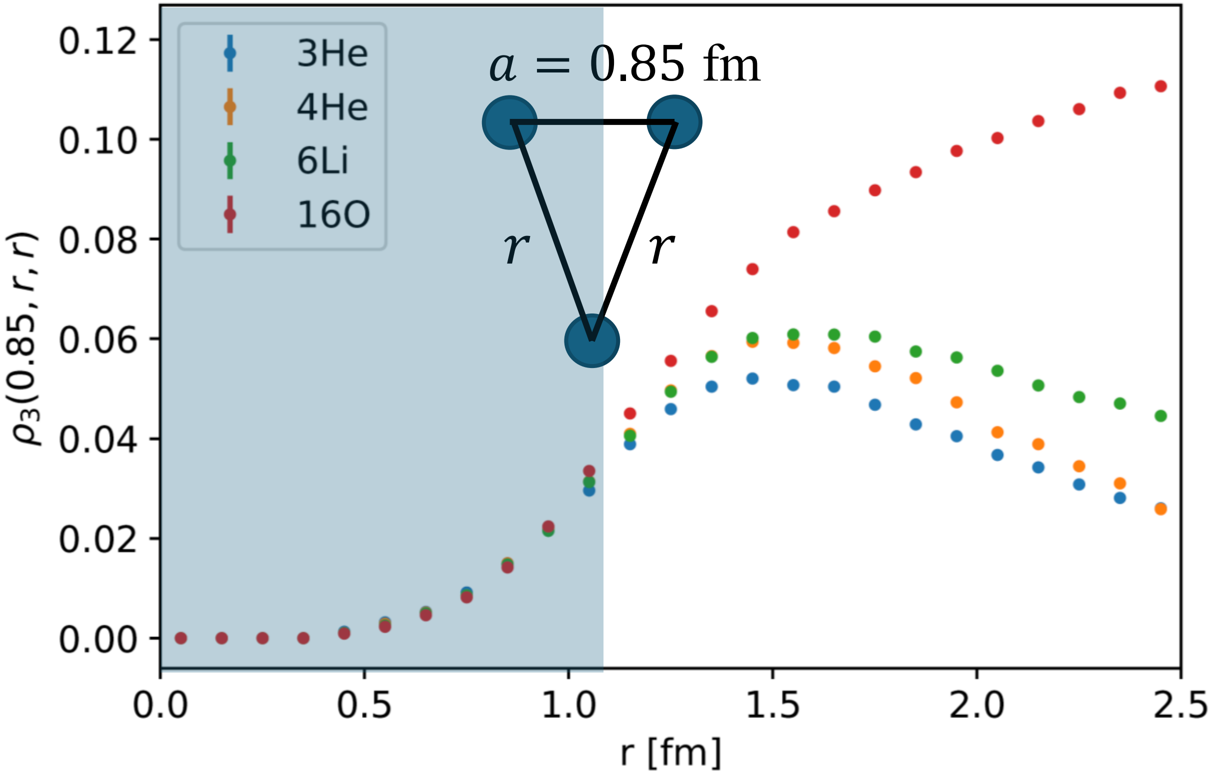
Three-body density

Universality

Same scaling factor for all geometries!





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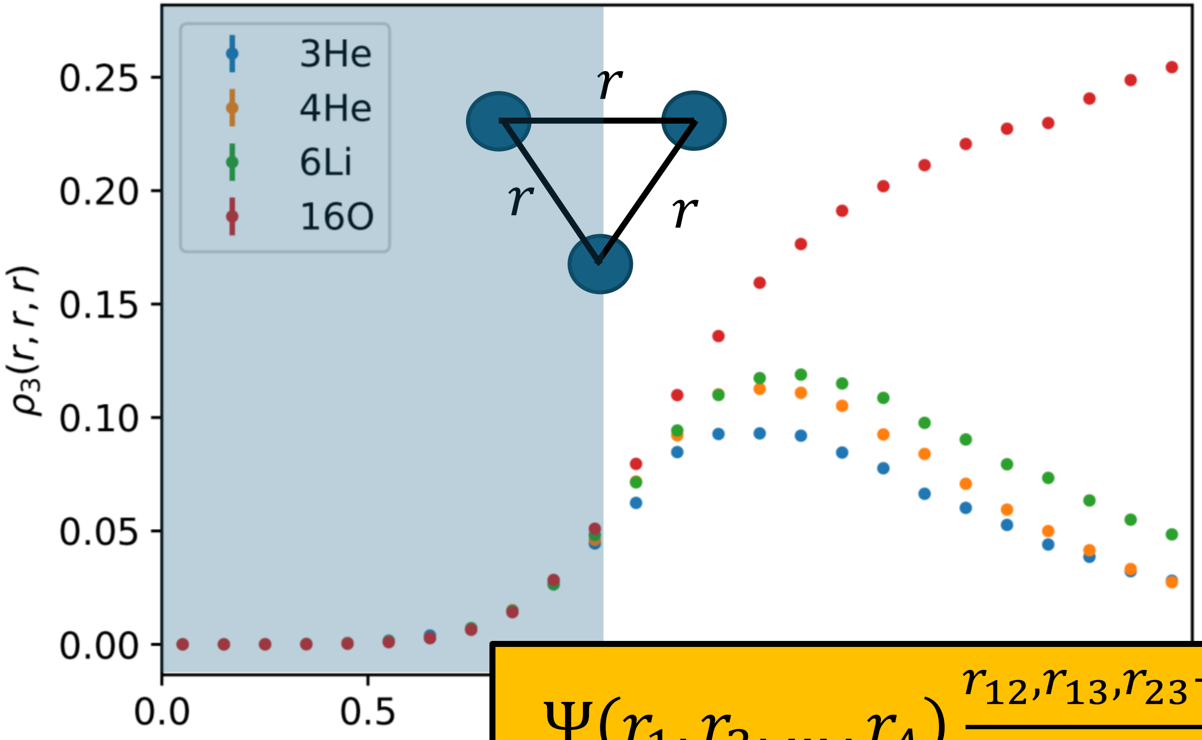

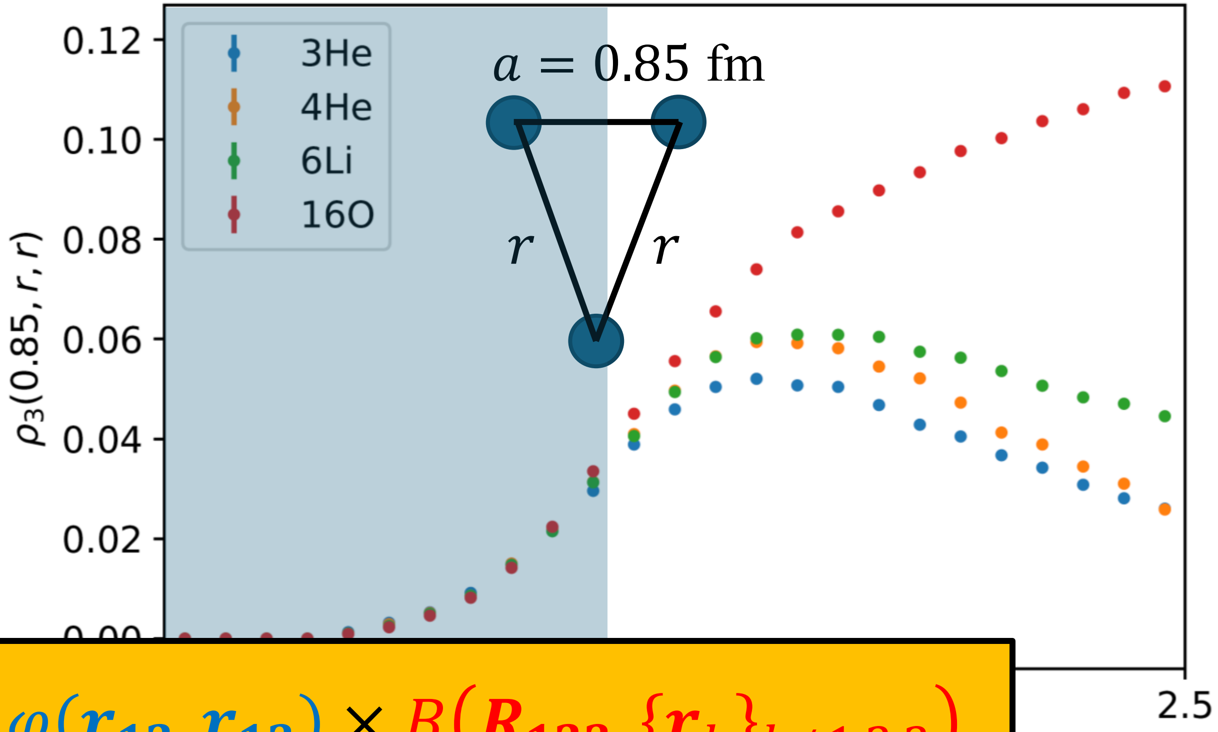

$$\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{12}, r_{13}, r_{23} \to 0} \varphi(\boldsymbol{r}_{12}, \boldsymbol{r}_{13}) \times B\left(\boldsymbol{R}_{123}, \{\boldsymbol{r}_k\}_{k \neq 1,2,3}\right)$$

universal function

Three-body contact values

Abundance of 3N correlations

$$\frac{C(\,^4\mathrm{He})}{C(\,^3\mathrm{He})} \times \frac{3}{4} = 3.8 \pm 0.3$$

$$\frac{C(\,^6\mathrm{Li})}{C(\,^3\mathrm{He})} \times \frac{3}{6} = 3.1 \pm 0.3$$

$$\frac{C(^{16}\mathrm{O})}{C(\,^3\mathrm{He})} \times \frac{3}{16} = 4.2 \pm 0.5$$

$$\frac{C(^{12}\mathrm{C})}{C(\,^3\mathrm{He})} \times \frac{3}{12} = 4.1 \pm 0.3$$

New Result for ^{12}C

Can be compared **to inclusive cross section ratios** for a **symmetric nucleus *A***

$$a_3(A) \equiv \frac{3}{A} \frac{\sigma_{eA}}{(\sigma_{e\,^3\mathrm{He}} + \sigma_{e\,^3\mathrm{H}})/2}$$

$$a_3(A) \approx \frac{3}{A} \frac{C(A)}{C(\,^3\mathrm{He})}$$

Under some assumptions/caveats...

Three-body contact values

or with respect to ^{4}He (and symmetric nucleus A): $\frac{4}{A}\frac{\sigma_{eA}}{\sigma_{e\,^4\mathrm{He}}}$

$$\frac{C(\,^6\mathrm{Li})}{C(\,^4\mathrm{He})} \times \frac{4}{6} = 0.82 \pm 0.10 \qquad \frac{C(^{12}\mathrm{C})}{C(\,^4\mathrm{He})} \times \frac{4}{12} = 1.08 \pm 0.12 \qquad \frac{C(^{16}\mathrm{O})}{C(\,^4\mathrm{He})} \times \frac{4}{16} = 1.11 \pm 0.16$$

See talks by **Jordan O'Kronley** and **Burcu Duran:** 3N SRC experimental study (inclusive)

3N SRCs

Momentum space

RW, L. Andreoli, S. Pastore (in preparation)

3N SRCs: momentum space

- **Momentum distribution – which?**
  - $n(k_1, k_2, k_3, angles)$ – expensive and noisy
  - $n(k_1, k_2, \theta_{12})$ – incomplete but should be sensitive to 3N SRCs

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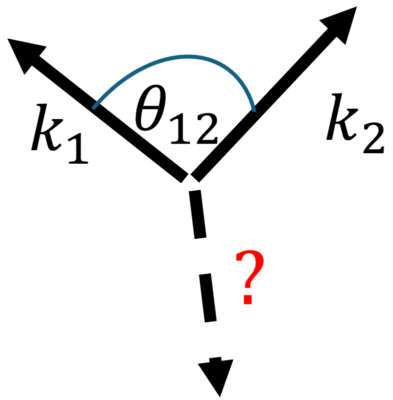


RW, L. Andreoli, S. Pastore (in preparation)

3N SRCs: momentum space

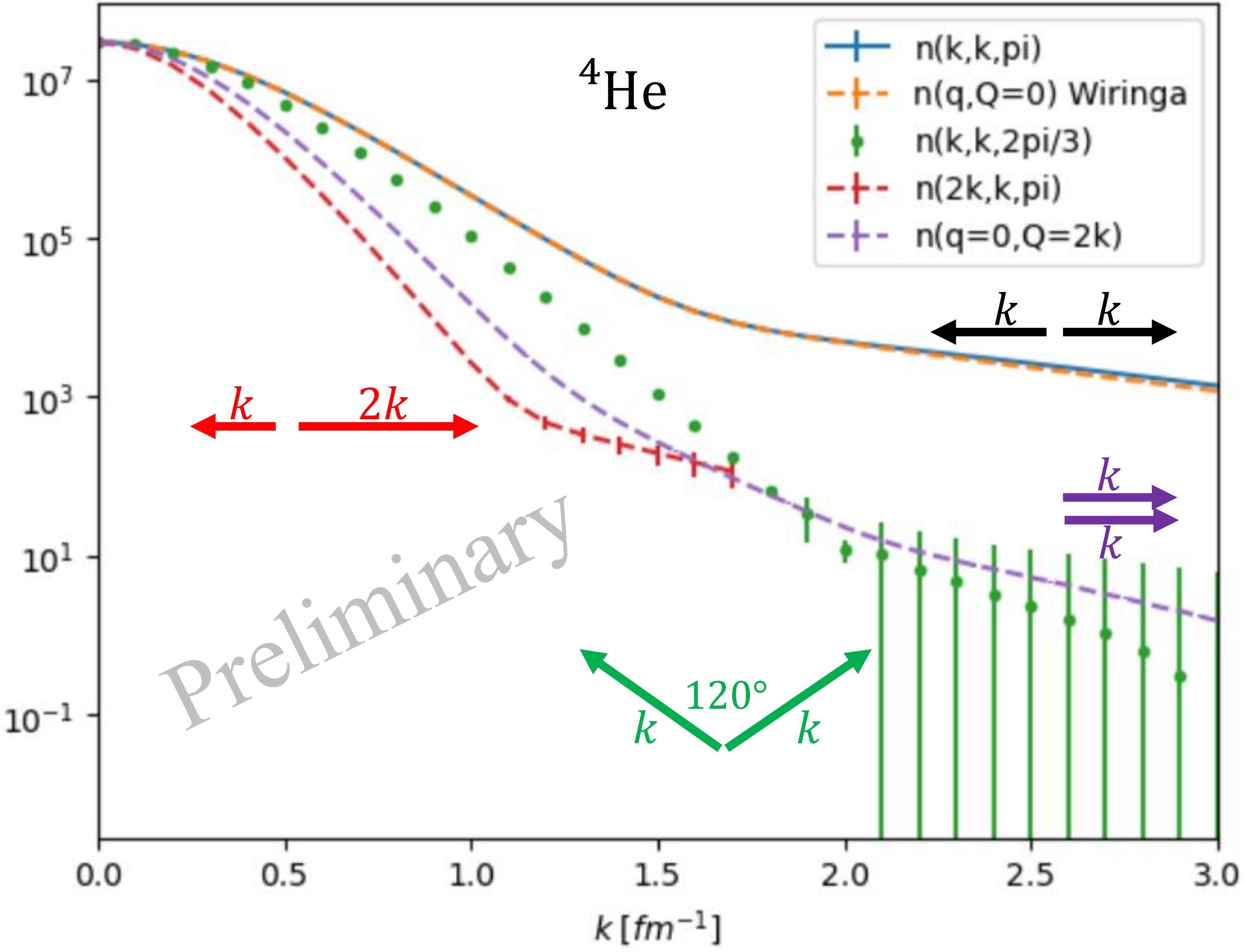
- $n(k_1, k_2, \theta_{12})$:
  - Method: **VMC**
  - Required **theoretical formulation,** and **coding**.
  - Still much more **expensive** than $n(k_1, k_2)$ or $n(q_{12}, Q_{12})$.
  - AV18 (2N interaction) +UIX (3N interaction)
  - Nuclei: ^{3}He, ^{4}He.

RW, L. Andreoli, S. Pastore (in preparation)

Results

(Preliminary)

3N SRCs?

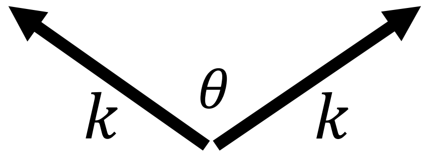


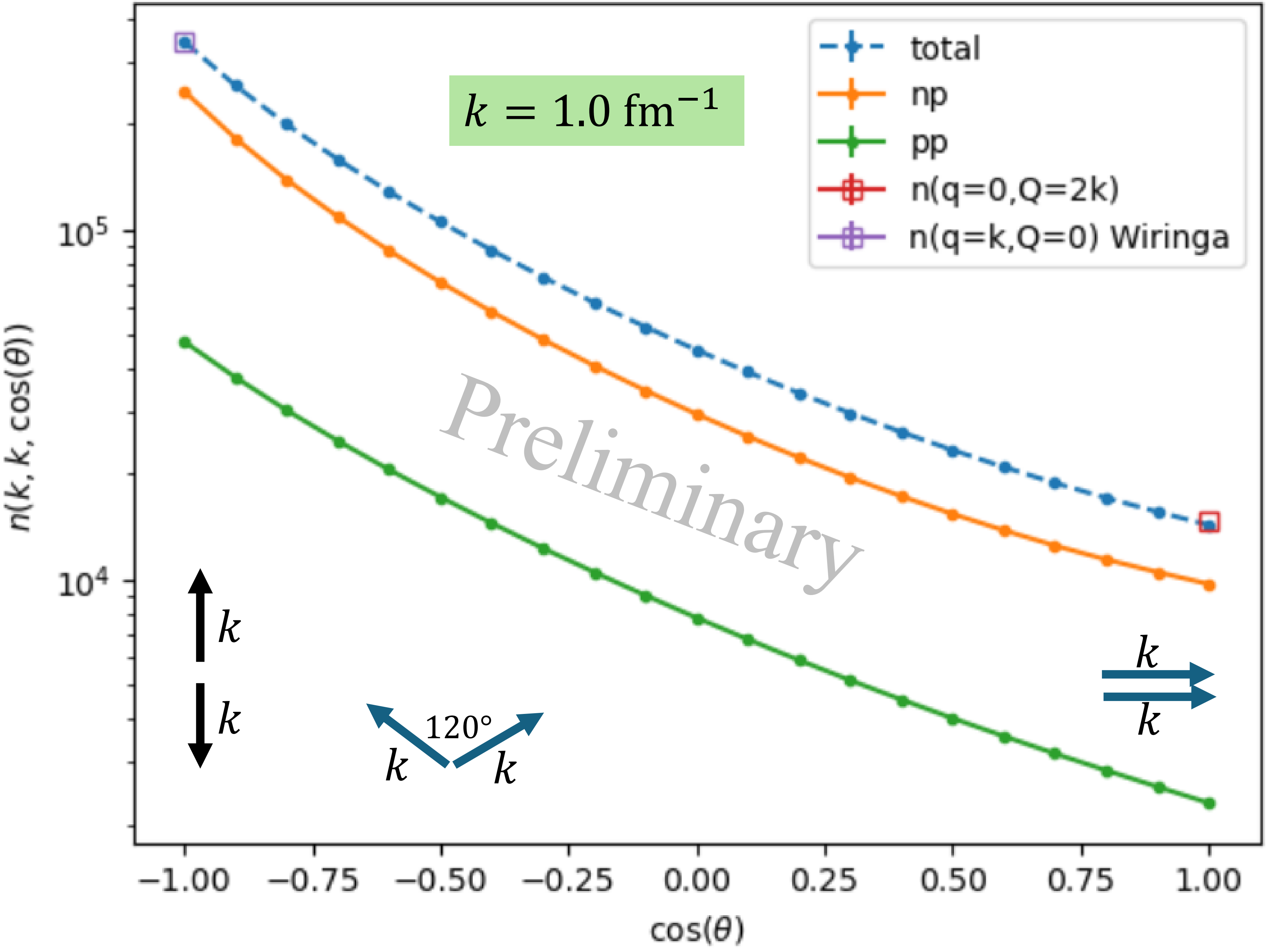
Results ^{4}He

(Preliminary)

Angle dependence

$n(k, k, \cos(\theta))$





Results ^{4}He

(Preliminary)

Angle dependence

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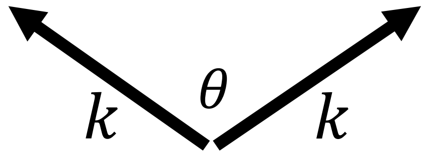
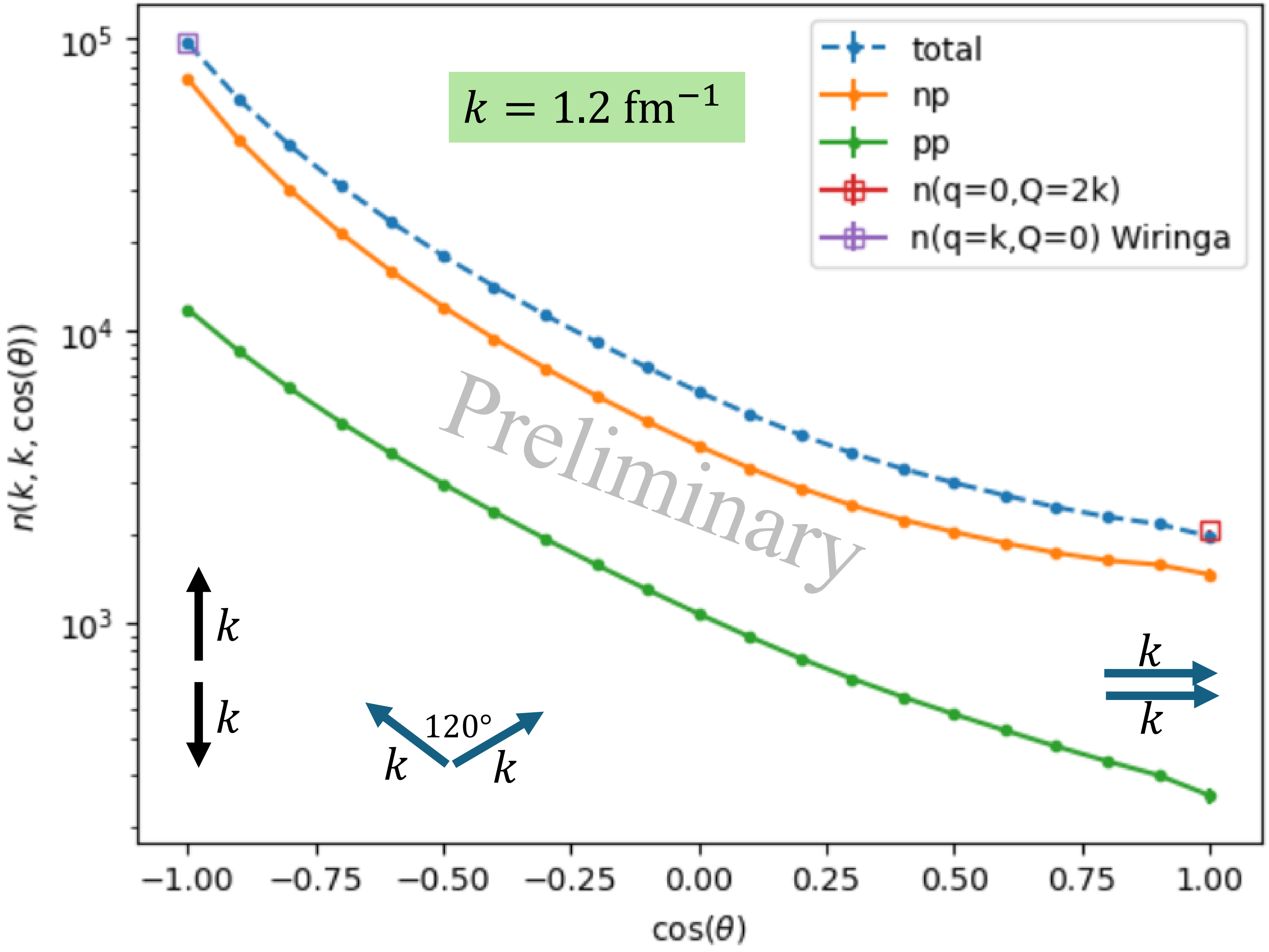


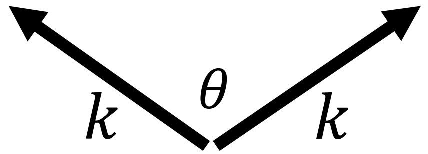


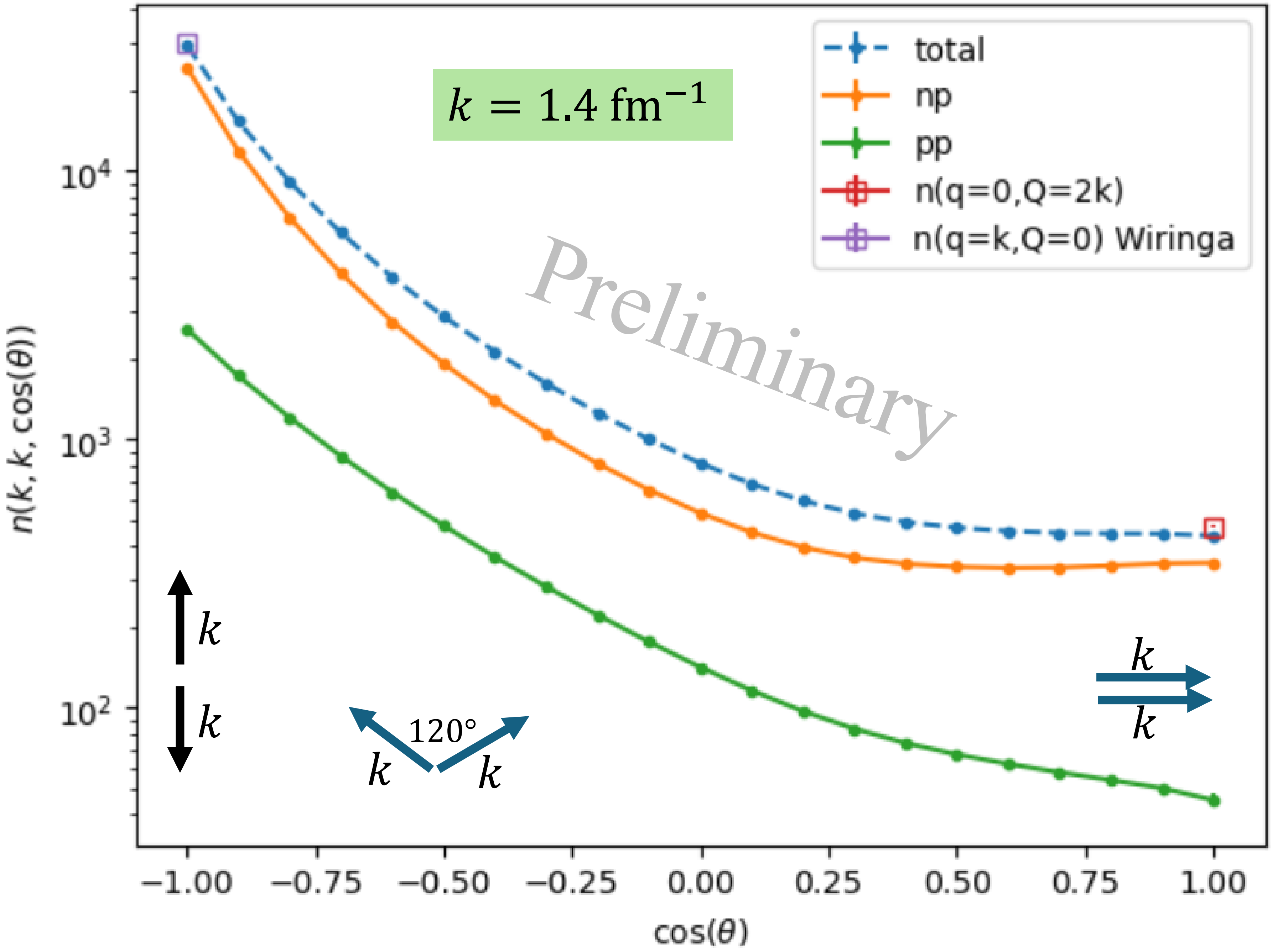
Results ^{4}He

(Preliminary)

Angle dependence

$$n(k, k, \cos(\theta))$$



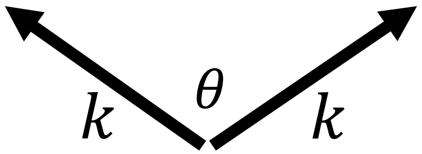


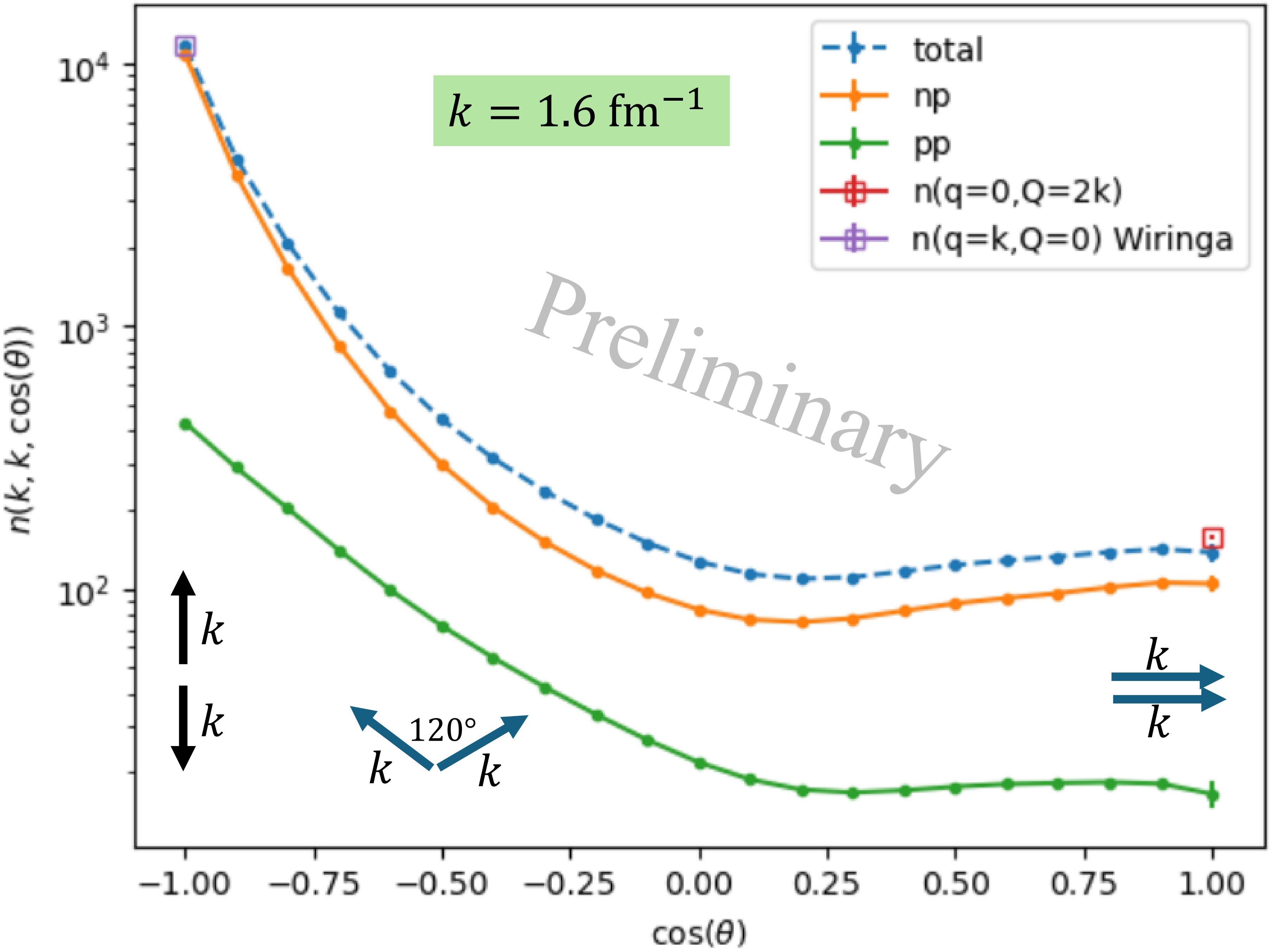
Results ^{4}He

(Preliminary)

Angle dependence

$n(k, k, \cos(\theta))$





Results ^{4}He

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Angle dependence

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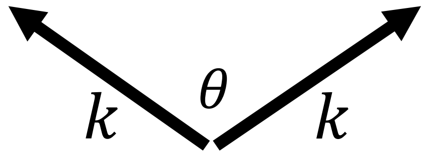

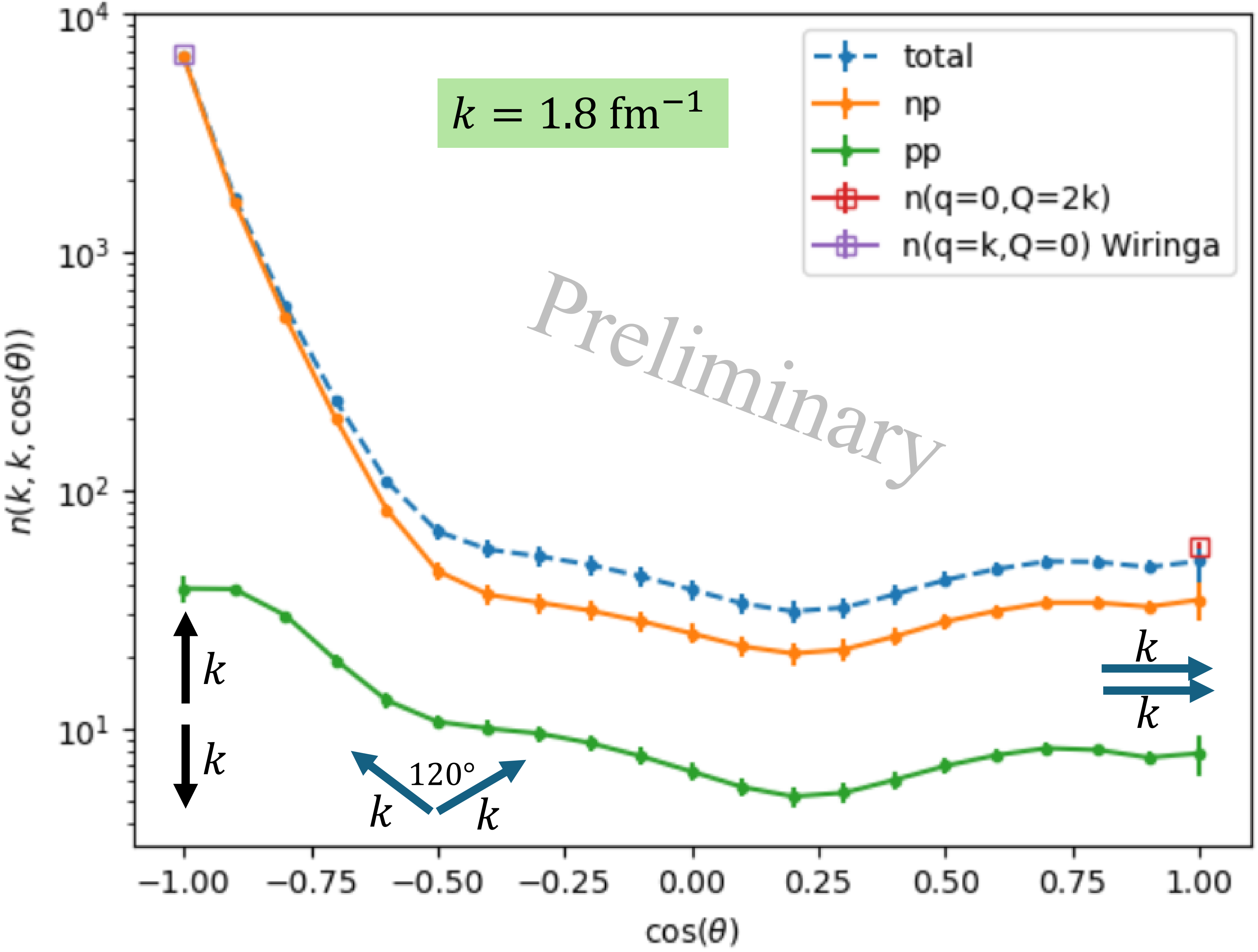

Results ^{4}He

(Preliminary)

Angle dependence

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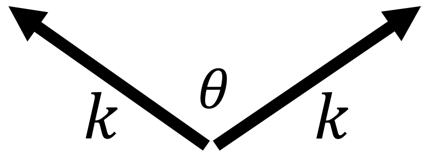
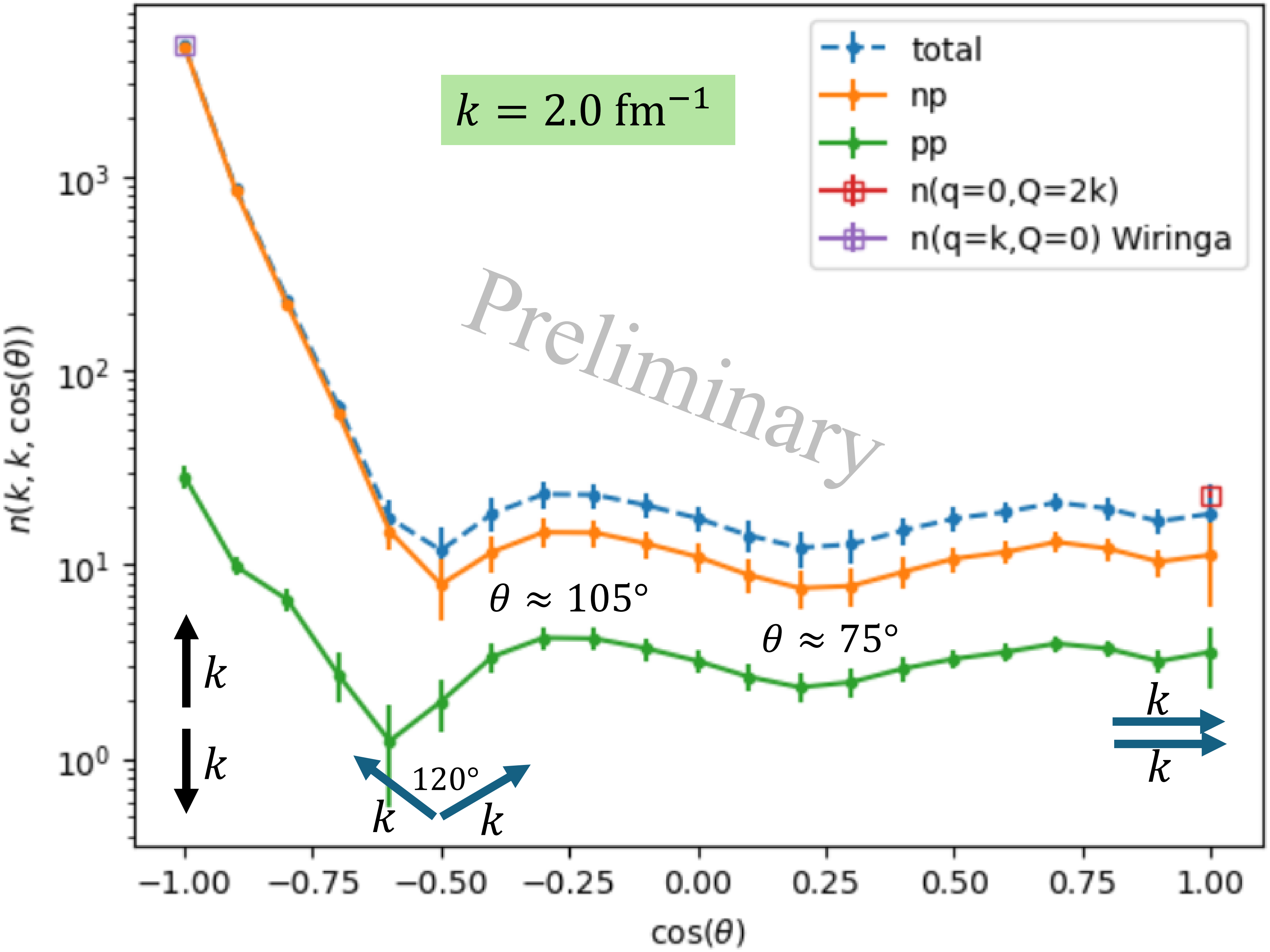


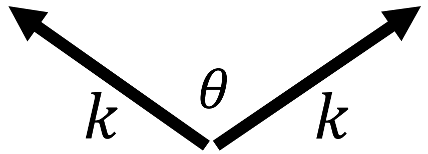


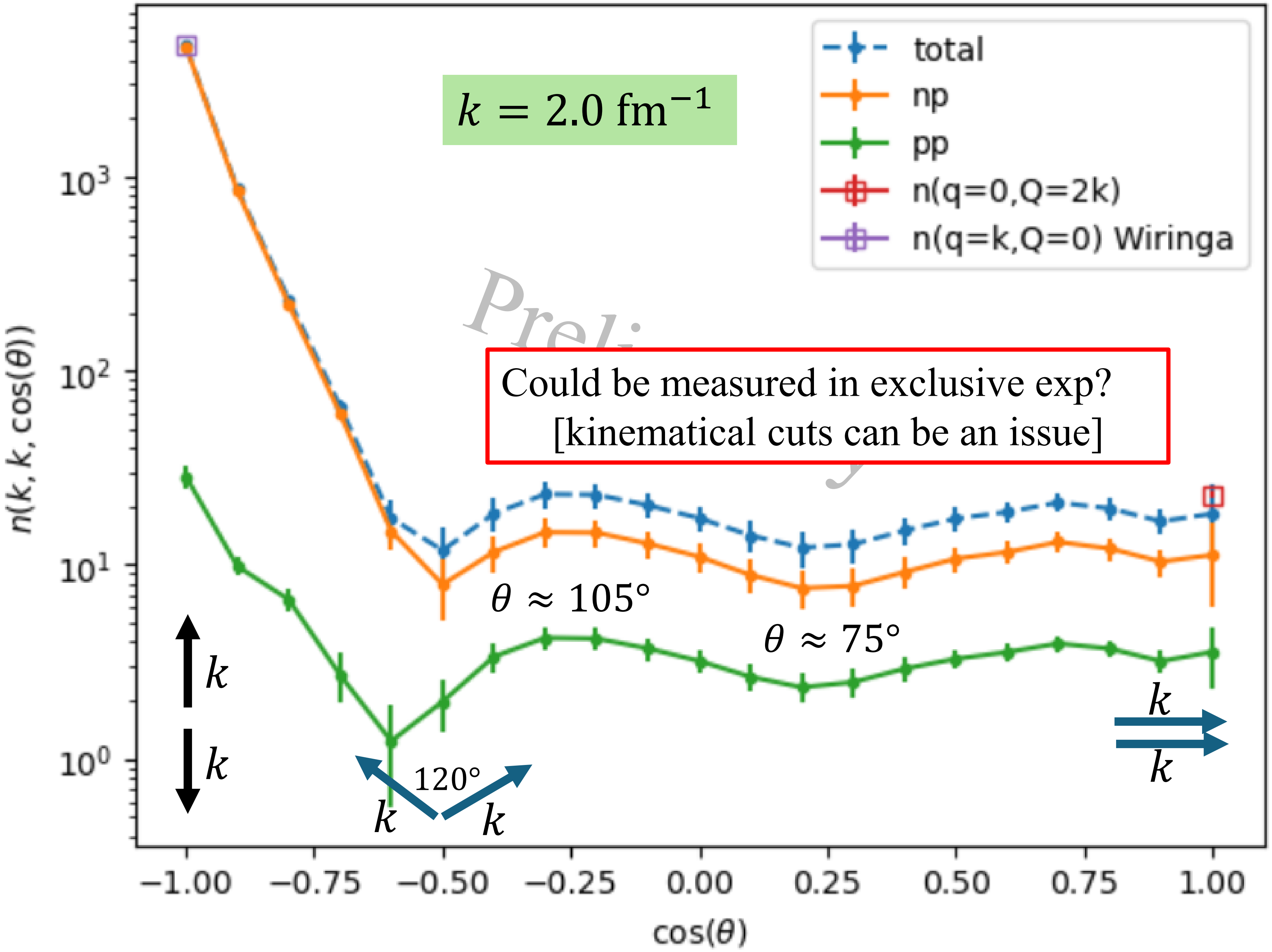
Results ^{4}He

(Preliminary)

Angle dependence

$n(k, k, \cos(\theta))$

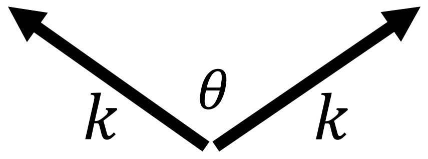


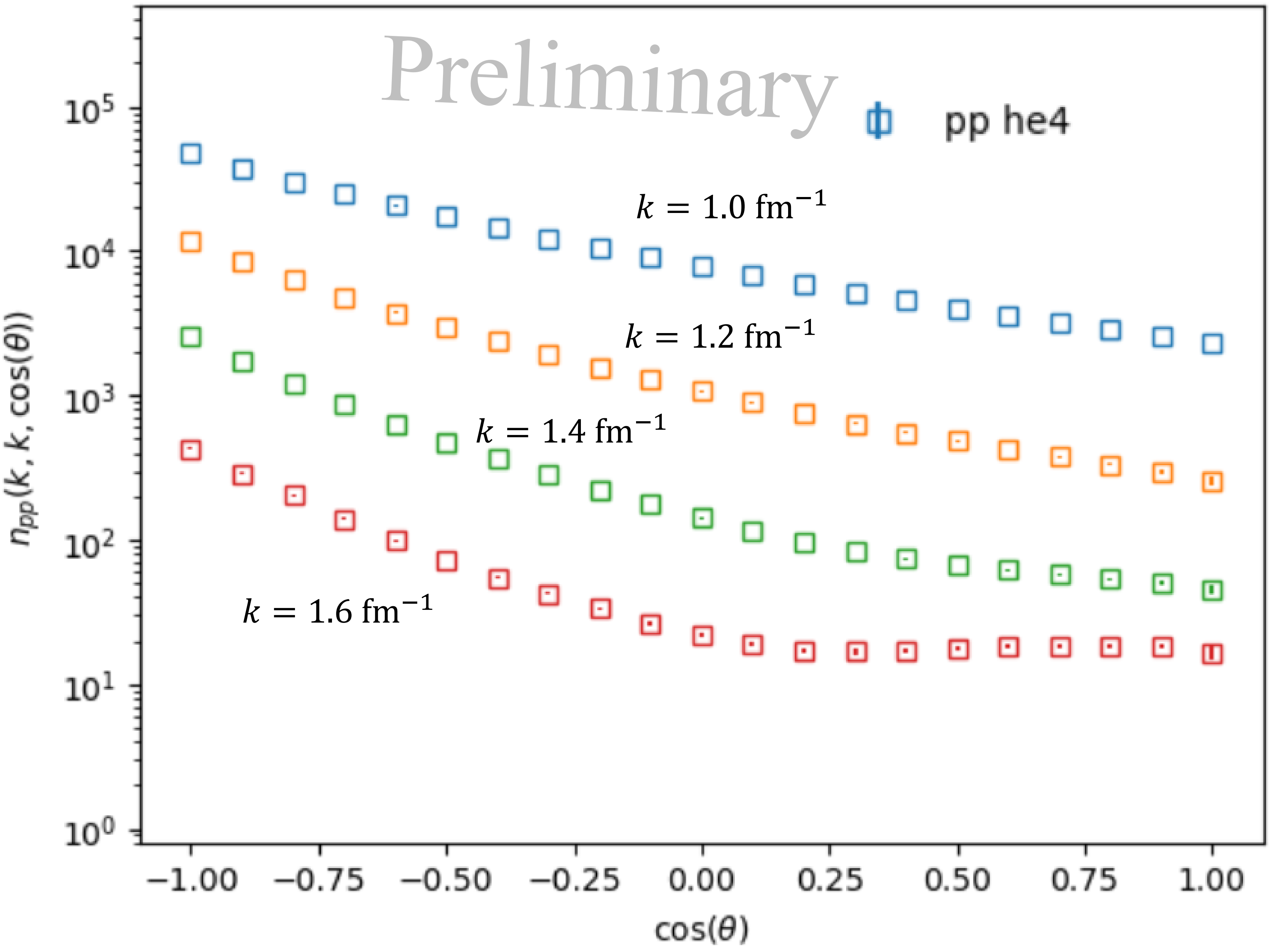


Results

(Preliminary)

^{4}He vs ^{3}He

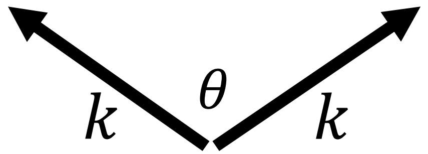




Results

(Preliminary)

^{4}He vs ^{3}He

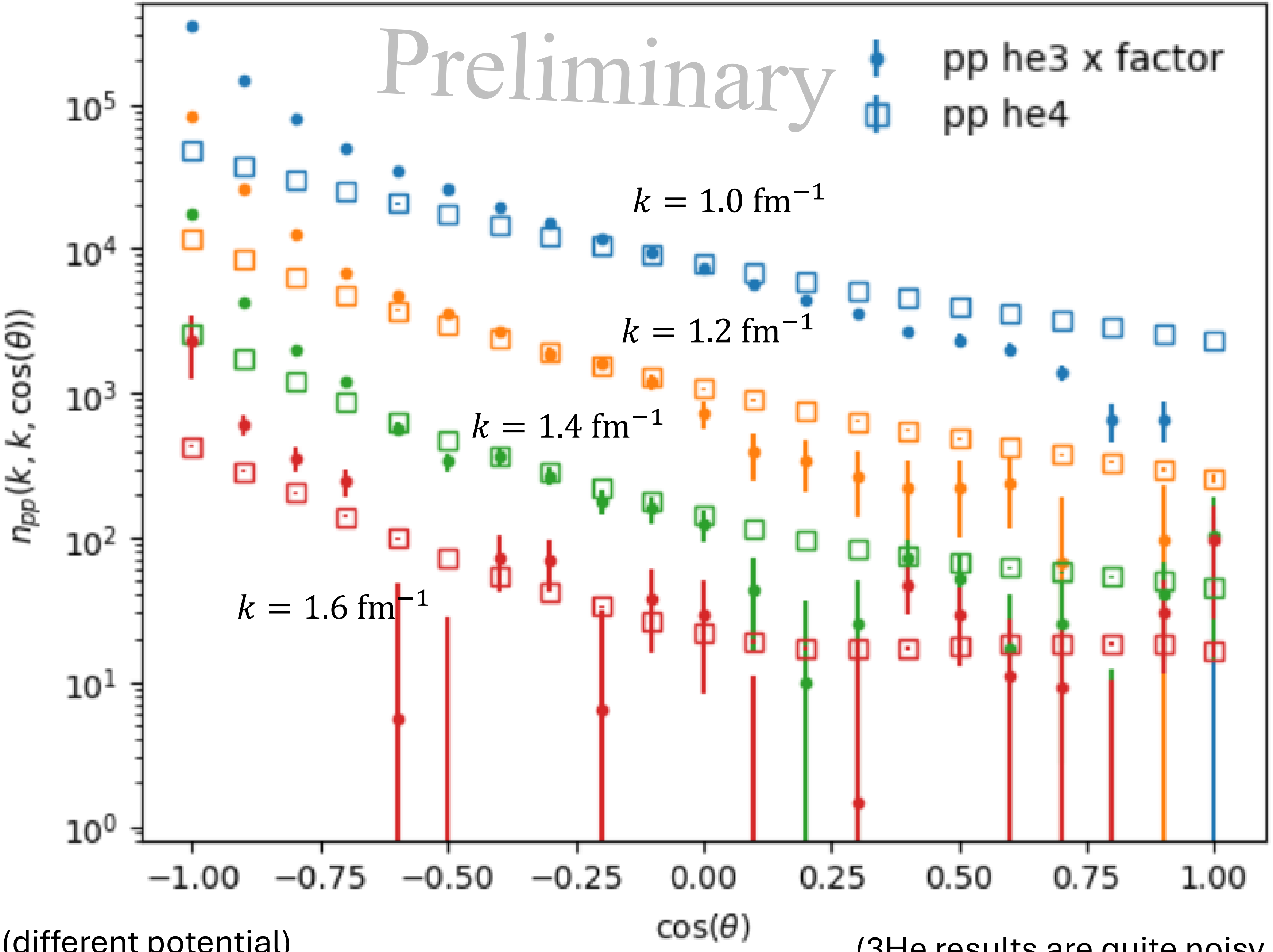


Factor = $\frac{4}{3} \times 3.5$

From coordinate space:

$$\frac{C(^4\mathrm{He})}{C(^3\mathrm{He})} = \frac{4}{3} \times (3.8 \pm 0.3)$$

(different potential)

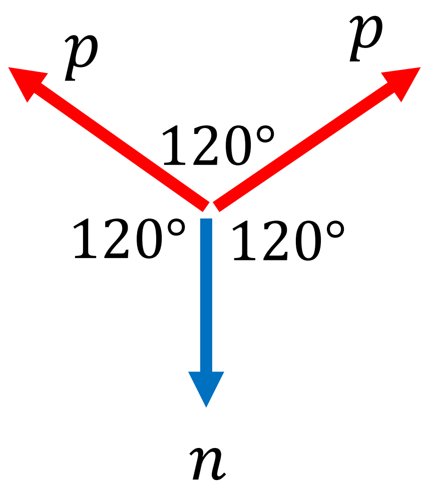


(3He results are quite noisy...)

Results

(Preliminary)

"Mercedes" relation



If each $(\boldsymbol{k}, \boldsymbol{k}, \mathbf{120°})$ pair is part of a "Mercedes" configuration:

$$\#np \leftrightarrow 2 \times \#pp$$

Results

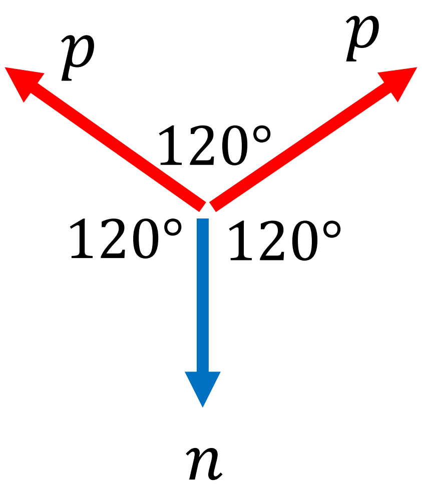
(Preliminary)

"Mercedes"

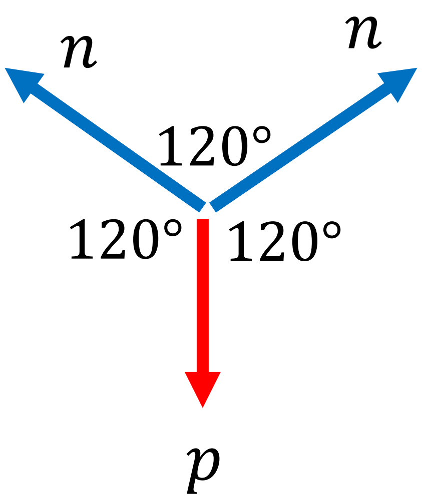
relation

$\#np = 2 \times \#nn + 2 \times \#pp$

If each $(\boldsymbol{k}, \boldsymbol{k}, 120°)$ pair is part of a "Mercedes" configuration:



$\#np \leftrightarrow 2 \times \#pp$



$\#np \leftrightarrow 2 \times \#nn$

(neglecting ppp and nnn triplets)

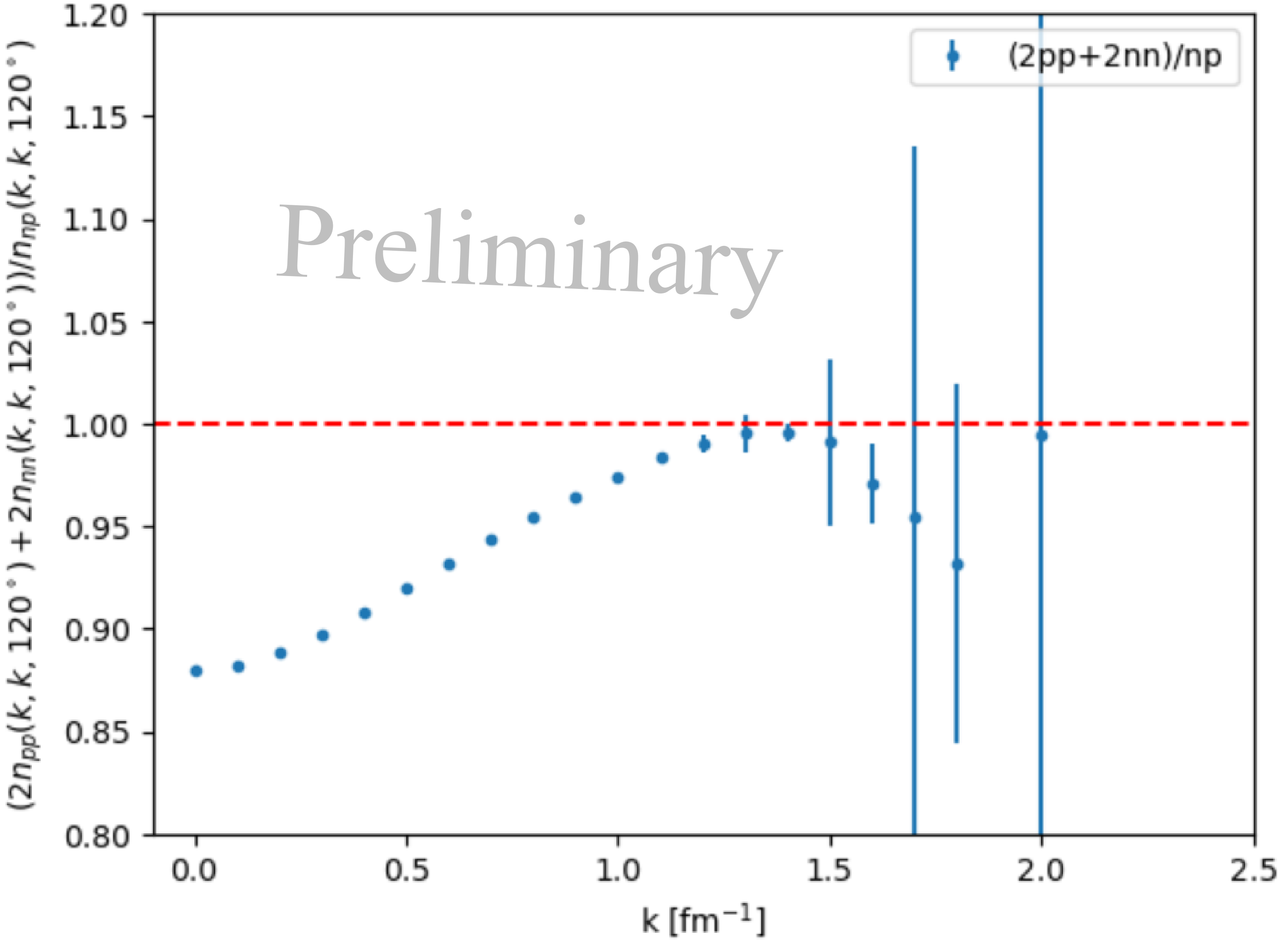
Results ^{4}He

(Preliminary)

"Mercedes"

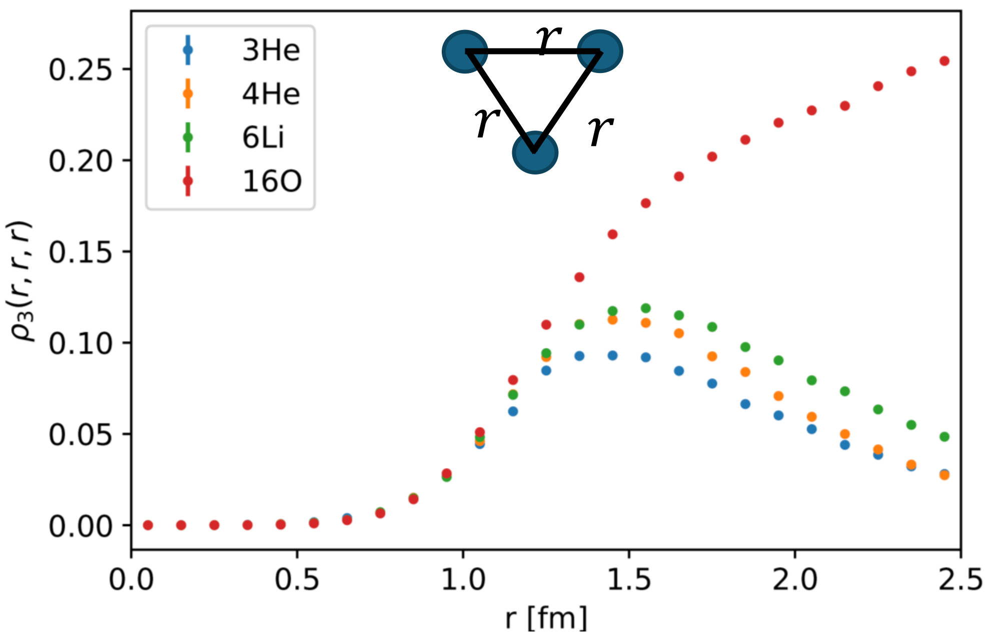
relation

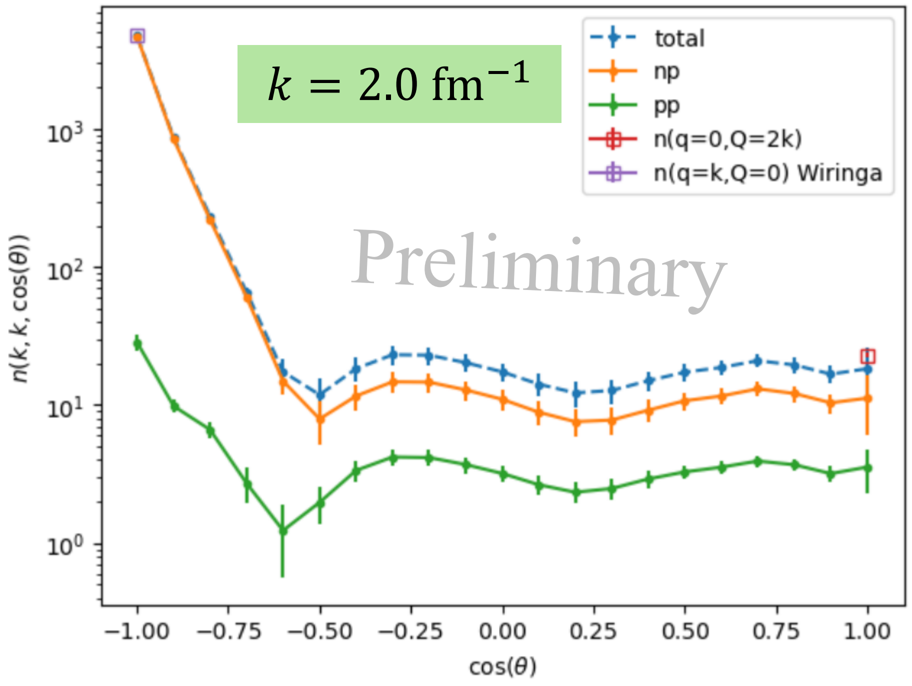
$$\#np = 2 \times \#nn + 2 \times \#pp$$



Conclusions: 3N SRCs

- **Coordinate space:**
  - **Short-range factorization**
  - **Universality**
  - **Scaling factors extracted**
- **Momentum space:**
  - **QMC calculations of $n(k_1, k_2, \cos(\theta))$**
  - **Angle dependence: 3N SRC features**
  - **^{4}He $\propto$ ^{3}He, r/k equivalence**
  - **“Mercedes” relation holds**

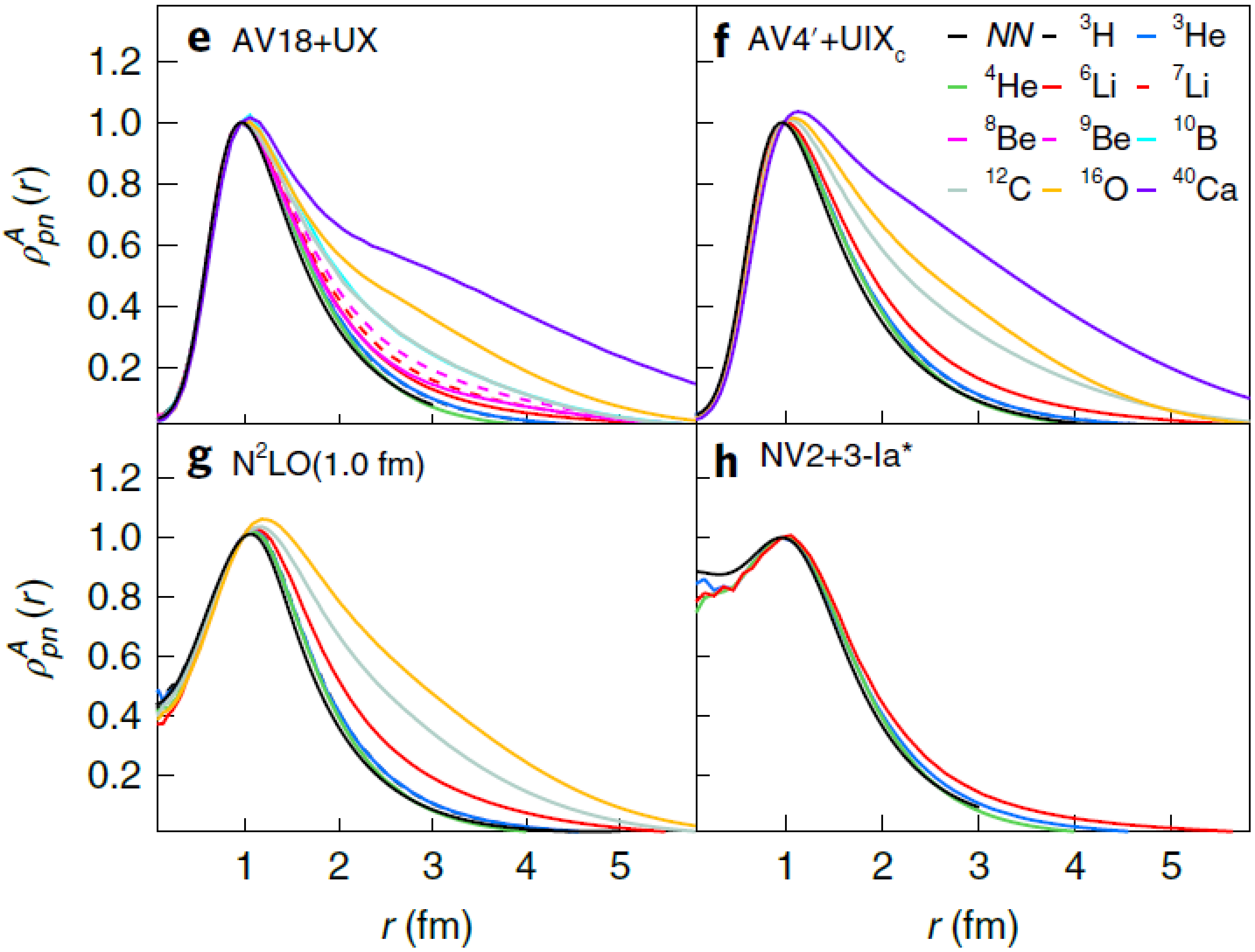




Next:

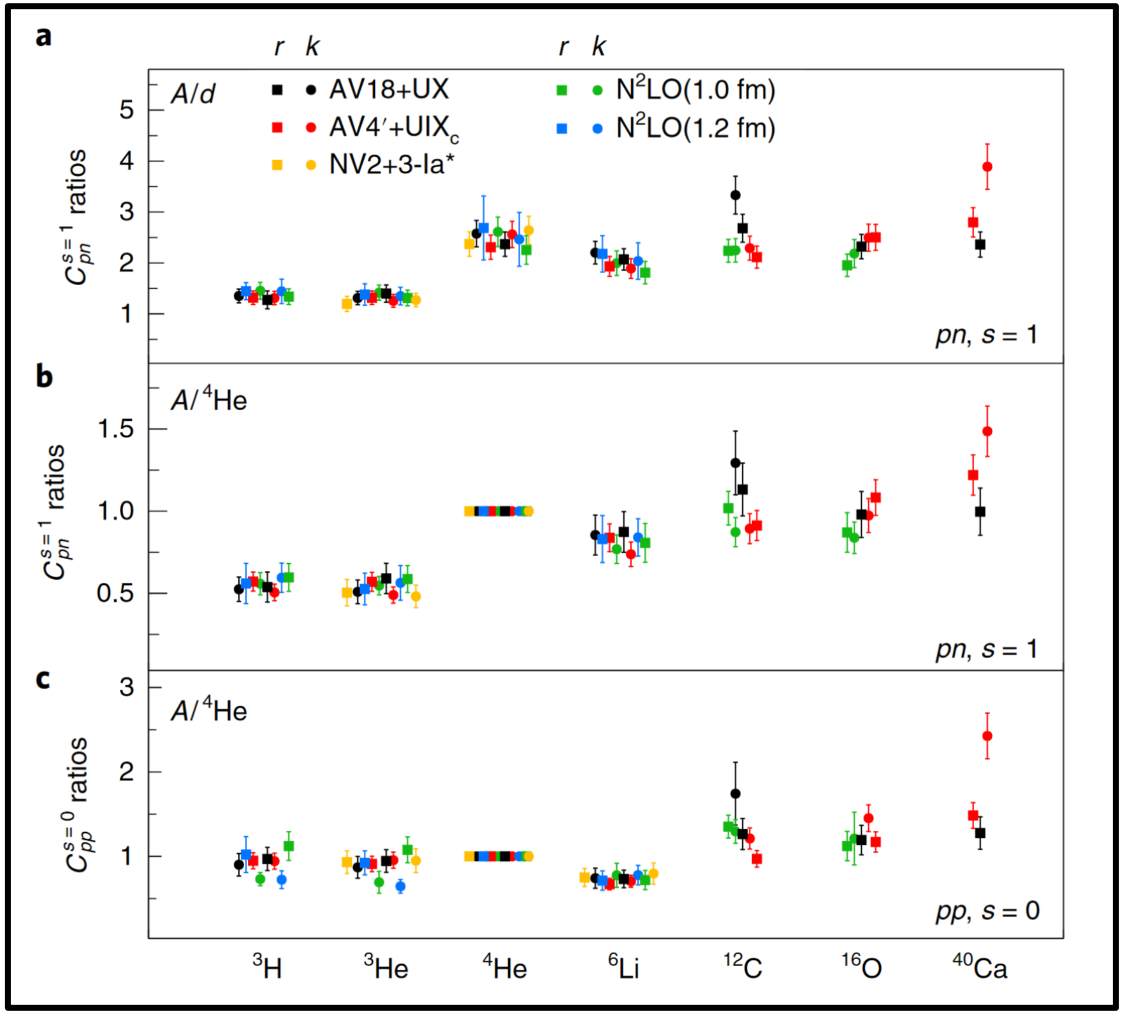
- Improve statistics
- Impact of 3N force, repulsive core, tensor force…
- More of the full phase space $n(k_1, k_2, \theta)$
- Model (in)dependence
- Comparison with experiments? (inclusive, exclusive)
- One-body momentum distribution (2N+3N SRCs)
- Spectral function, Reactions… (GCF)
- …

BACK UP



e AV18+UX
f AV4′+UIX$_c$
g N^{2}LO(1.0 fm)
h NV2+3-Ia*
NN
^{3}H
^{3}He
^{4}He
^{6}Li
^{7}Li
^{8}Be
^{9}Be
^{10}B
^{12}C
^{16}O
^{40}Ca
$\rho^A_{pn}(r)$
r (fm)
0.2
0.4
0.6
0.8
1.0
1.2
1
2
3
4
5

Model independence of contact ratios



$$\frac{C^{V_1}(X)}{C^{V_1}(Y)} = \frac{C^{V_2}(X)}{C^{V_2}(Y)}$$