Nuclear Light-Front Wave Functions with Short Ranged Correlations and High-x Cross Sections

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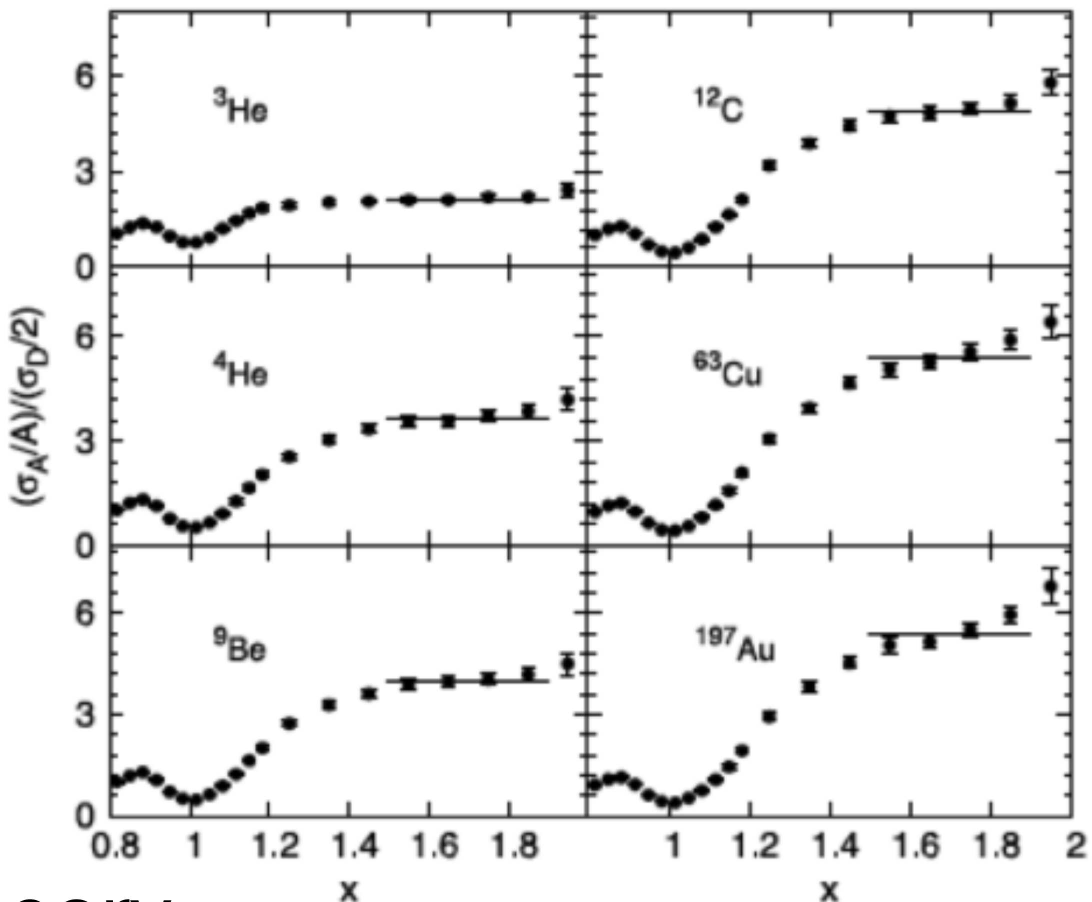
Want to handle EMC (DIS) and SRC physics - relativistic treatment needed

Inclusive Electron Nucleus Scattering Ratios



These ratios are known as a_2

Fomin et al

1 Theory

***Ab initio* short-range-correlation scaling factors from light to medium-mass nuclei**

J. Phys. G: Nucl. Part. Phys. **47** (2020) 045109 (20pp)

J E Lynn[1,2], D Lonardoni[3,4], J Carlson[4], J-W Chen[5,6], W Detmold[6], S Gandolfi[4] and A Schwenk[1,2,7]

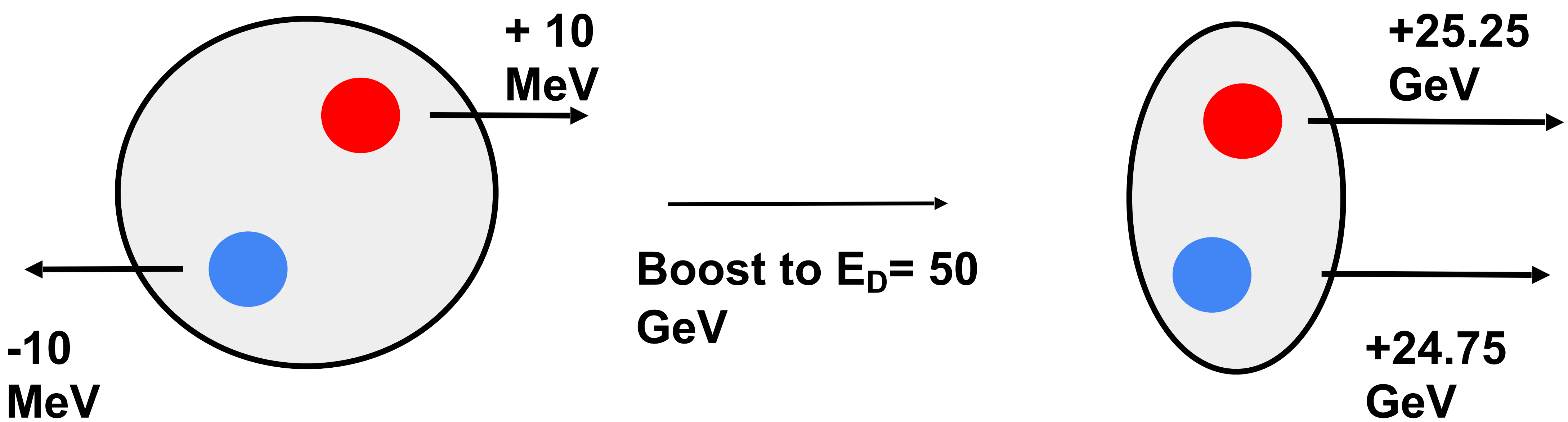
$$a_2(A/d) = \lim_{r \to 0} \frac{2}{A} \frac{\rho_{2,\mathbb{1}}(A, r)}{\rho_{2,\mathbb{1}}(d, r)}, \quad \neq \frac{2}{A}\sigma(A)/\sigma(D)$$

$$\rho_{2,\mathbb{1}}(A, r) = \frac{1}{4\pi r^2} \langle \Psi | \sum_{i<j}^{A} \delta(r - r_{ij}) | \Psi \rangle.$$

Will show calculations of cross sections here

Why use light front?

Example: Boosting the Deuteron



$$|k_{rel}| = 10 \text{ MeV} \longrightarrow |k_{rel}| = 250 \text{ MeV}$$

The relative momentum has changed

Need formulation where starting point is boost invariant: Light Front

Light-Front (LF) Coordinates

$$p^{\pm} = p^0 \pm p^3, \mathbf{p}_{\perp} = (p_x, p_y)$$

Longitudinal LF Boosts

$$p^+ \to e^{\omega} p^+$$

$$p^- \to e^{-\omega} p^-$$

$$\boldsymbol{p}^{\perp} \to \boldsymbol{p}^{\perp}$$

$$e^{\omega} = \sqrt{(1-v)/(1+v)}$$

$$P^+ = \sum_i p_i^+$$

Transverse LF Boosts

$$x_i = \frac{p_i^+}{P^+} \qquad \boldsymbol{k}_i^{\perp} = \boldsymbol{p}_i^{\perp} - x_i \boldsymbol{P}^{\perp}$$

$$\boldsymbol{P}^{\perp} = \sum_i \boldsymbol{p}_i^{\perp}$$

$$\sum_i x_i = 1 \qquad \sum_i \boldsymbol{k}_i^{\perp} = 0$$

How to use light front for finite-sized nuclei

- Need light front treatment of wave function including SRC
- None exist
- Start with Mean Field

Light-front Dirac equation

Solution

$$\left[i\boldsymbol{\alpha}^{\perp} \cdot \boldsymbol{\partial}^{\perp} + i\alpha^3 \partial^+ + \gamma^0 m_N + \Sigma_H(\boldsymbol{x}) \right] \psi_i(\boldsymbol{x}) = p_i^- \Lambda^+ \psi_i(\boldsymbol{x})$$

Technique: Blunden, Burkardt and Miller PRC 59,R2998, 60,055211

Lagrangian Lalazissis, Niksic, Vretenar, Ring Phys. Rev. C (2005) 71, 024312

Walecka model-density dependent coupling constants

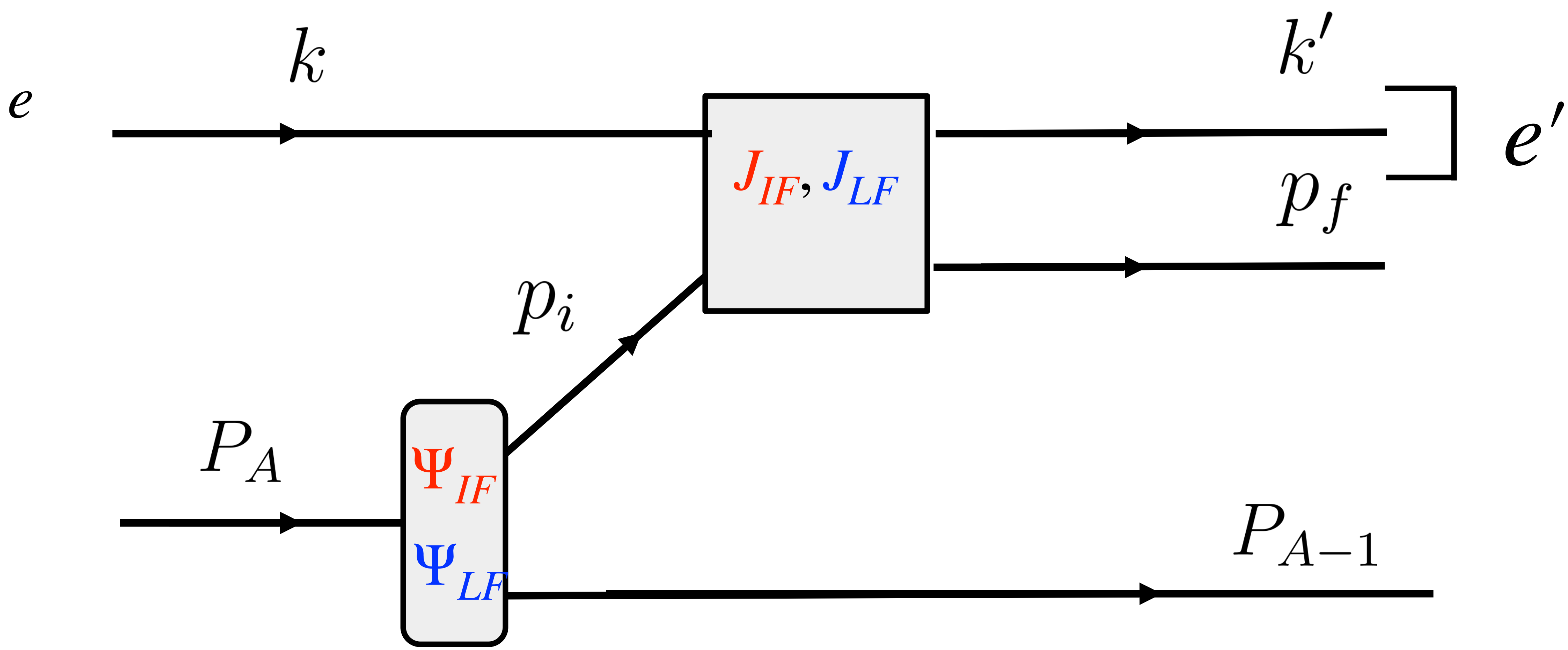
Nucleus	BE (MeV)	r_c (fm)	$r_n - r_p$ (fm)
^{16}O	127.801 (127.619)	2.727 (2.730)	−0.03
^{40}Ca	342.741 (342.052)	3.464 (3.485)	−0.05
^{48}Ca	414.770 (415.991)	3.481 (3.484)	0.18
^{208}Pb	1638.426 (1636.446)	5.518 (5.505)	0.19 (0.20)

Vacuum is trivial in LF, but complicated in Instant Form (t, x, y, z)

Next step- LF vs IF in mean field impact on (e,e’)

Inclusive Electron-Nucleus Scattering (RMF)

Impulse approximation

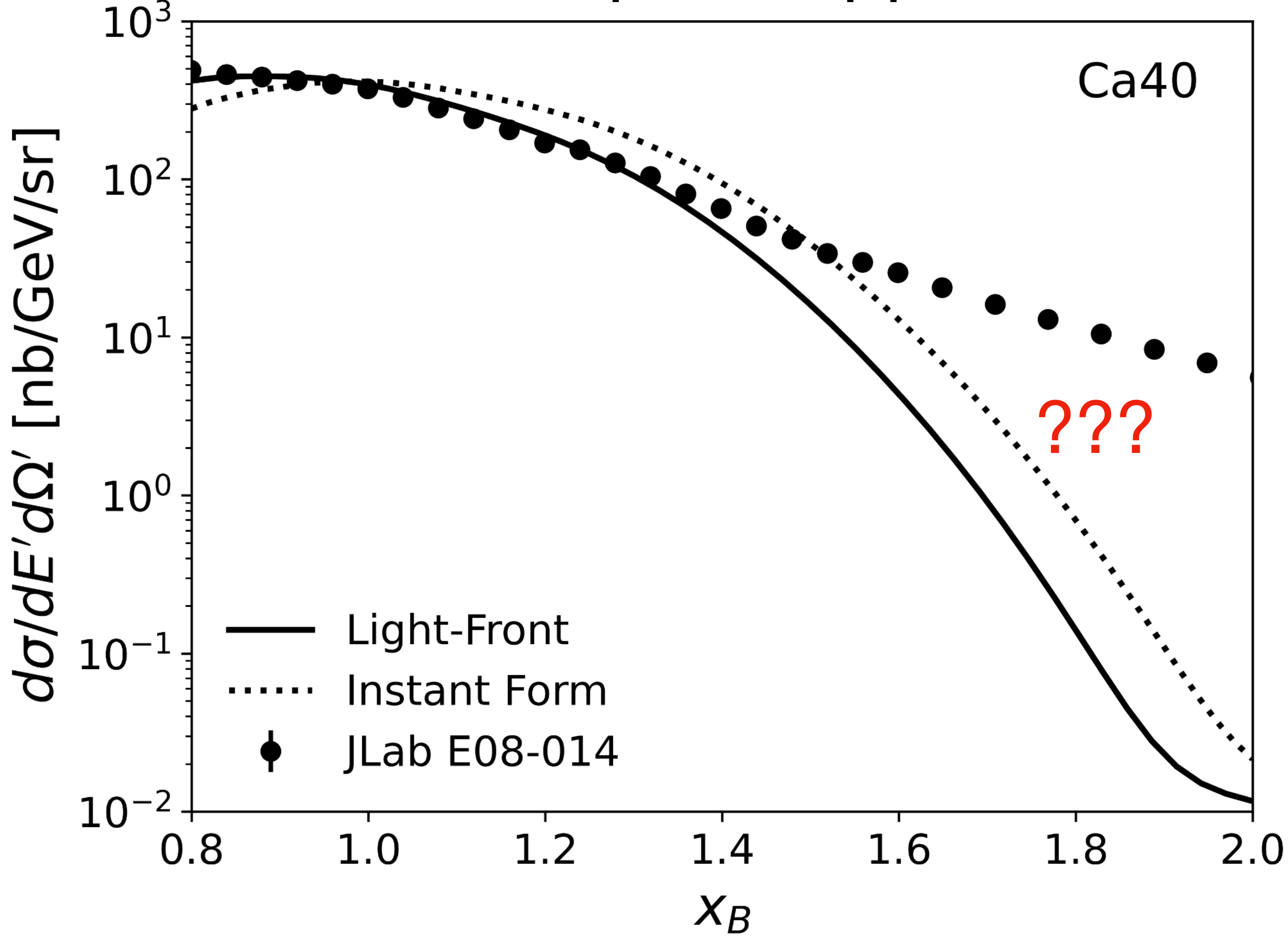


Off-shell effects smaller with J_{LF} Vera & Sargsian

Boost of P_{A-1} nucleus treated better with Ψ_{LF}

Conventional (IF) vs Light-Front Relativistic Mean Field Theory

Impulse approximation



Current and boosts treated better in LF

$x < 1$ due to inelastic excitations, $x \approx 1$ impulse approx

$x > 1$, something missing - Short ranged correlations

Including SRC -Low resolution vs high resolution

- Mean field is low resolution picture
- Impulse approximation assumes high resolution
- Similarity Renormalization Group (SRG) bridges the gap

  A. J. Tropiano, S. K. Bogner, R. J. Furnstahl, M. A. Hisham, A. Lovato, and R. B. Wiringa. High-resolution momentum distributions from low-resolution wave functions. *Phys. Lett. B*, 852:138591, 2024.

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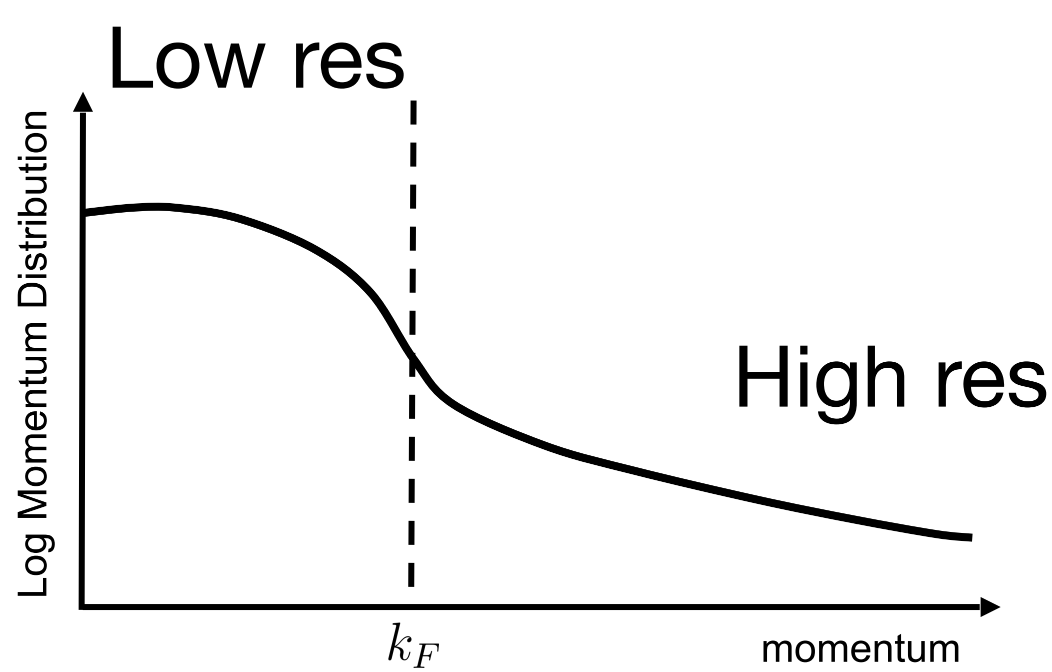
- Evolve from high res to low res - exact low resolution wave function approximated by the mean field

$H(\lambda), \lambda$ maximum momenta in low-res wave functions



How to Study SRC Physics using Mean Field Theory

High res

$$\langle \Psi_A | \hat{\mathcal{O}} | \Psi_A \rangle = \underbrace{\langle \Psi_A | U_\lambda^\dagger}\underbrace{U_\lambda \hat{\mathcal{O}} U_\lambda^\dagger}\underbrace{U_\lambda | \Psi_A \rangle}$$

$$= \langle \Psi_A(\lambda) | \hat{\mathcal{O}}(\lambda) | \Psi_A(\lambda) \rangle$$

$$\approx \langle \Psi_A^{MF} | \hat{\mathcal{O}}(\lambda_{MF}) | \Psi_A^{MF} \rangle \qquad \lambda_{MF} = 2\,\mathrm{fm}^{-1}$$

Observable

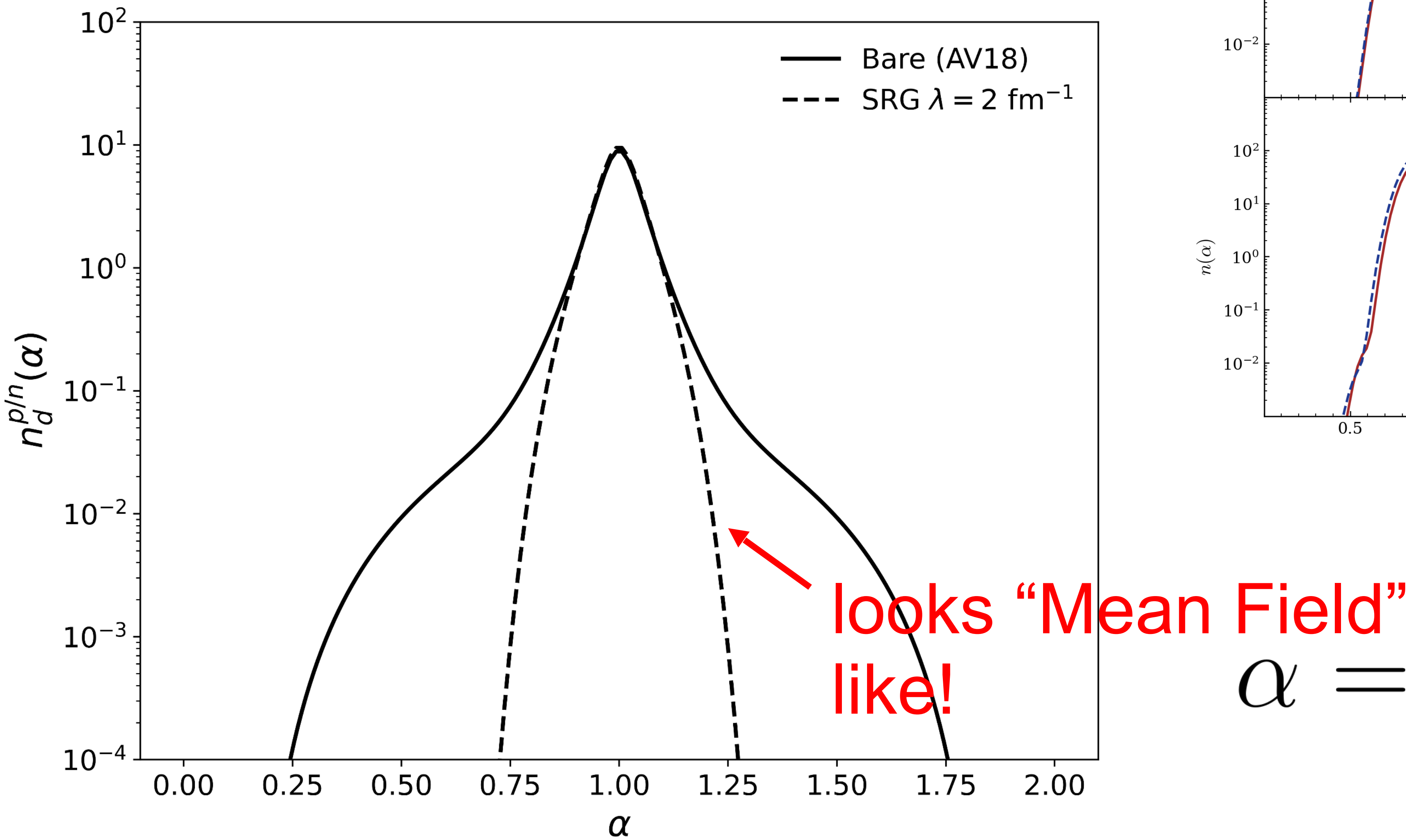
$\hat{\mathcal{O}}$ includes effects of SRC

$$\hat{\mathcal{O}} = \hat{n}(p^+, \mathbf{p}_\perp)$$

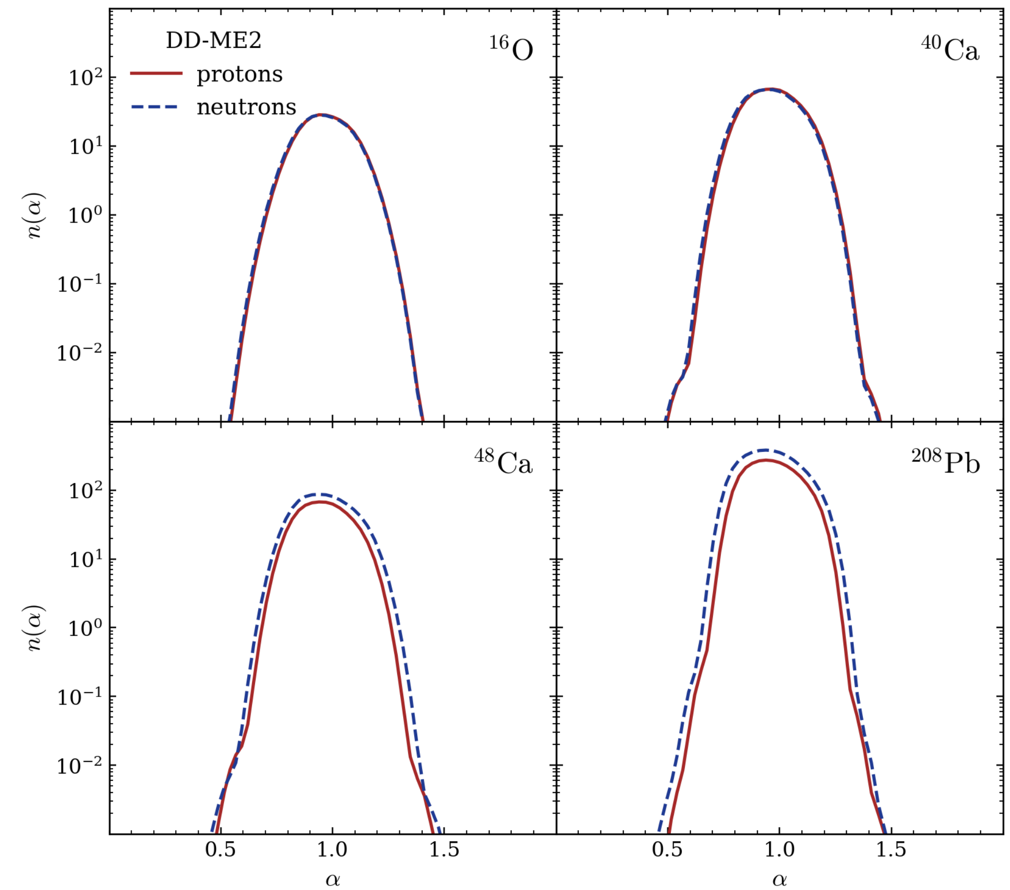
$$\hat{\mathcal{O}} = \int d^4\xi e^{iq\cdot\xi} J^\mu(\xi) J^\nu(0)$$

Nuclear light-front momentum distributions

Deuteron Example



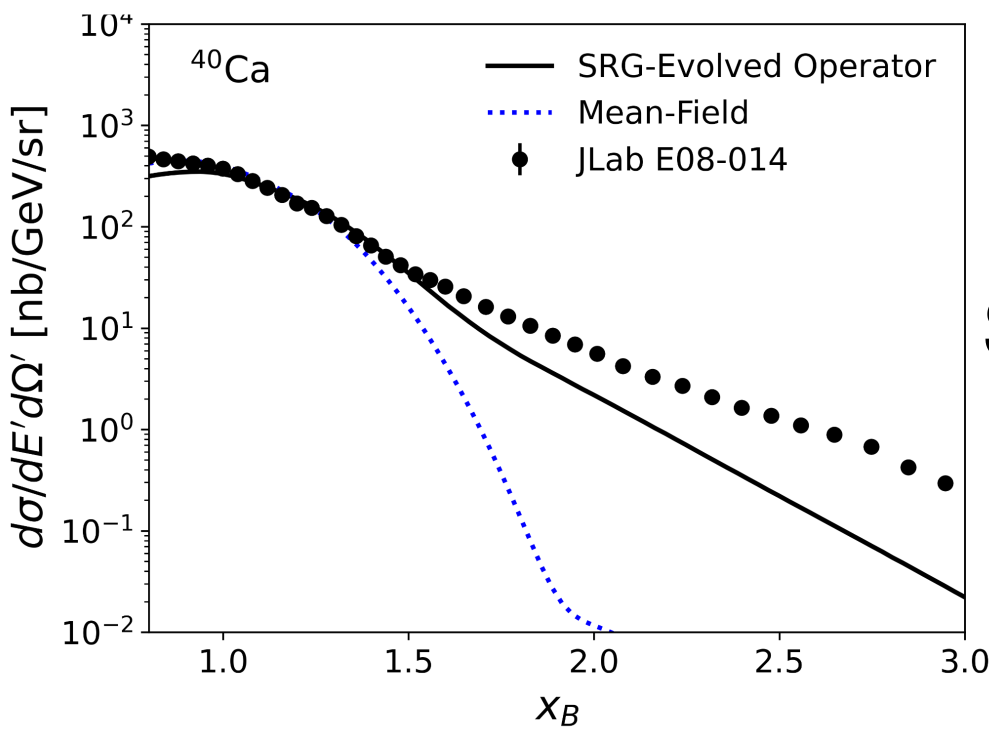
LF mean field



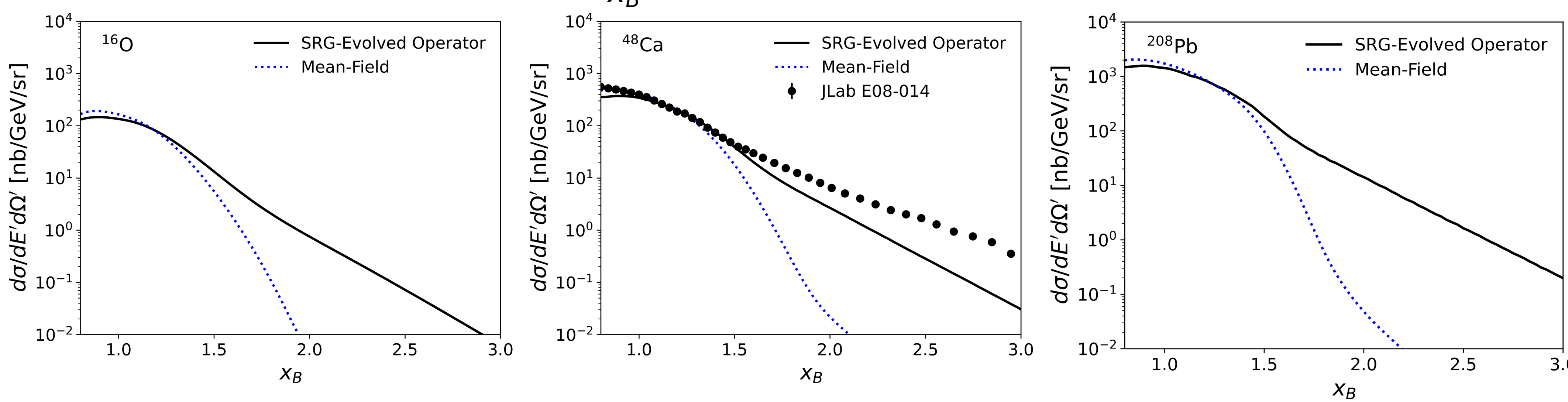
$\alpha =$ Light-Front 'longitudinal' momentum fraction carried by the nucleon in the nucleus, weighted by mass number A

(e, e') Results including SRC via SRG $\lambda = 2\,\mathrm{fm}^{-1}$

SRC are important



SRC effects very important



Something is still missing!

What's missing

- Final state interactions C Degli Atti &Simula 1994, FSI in correlated pair is needed to fit data- indicates density ratio not sufficient. We do not reproduce this finding- effect is small
- More likely -inelastic excitations at high x!

Conservation of 4-momentum for pion creation
Threshold value of s for a nucleus A

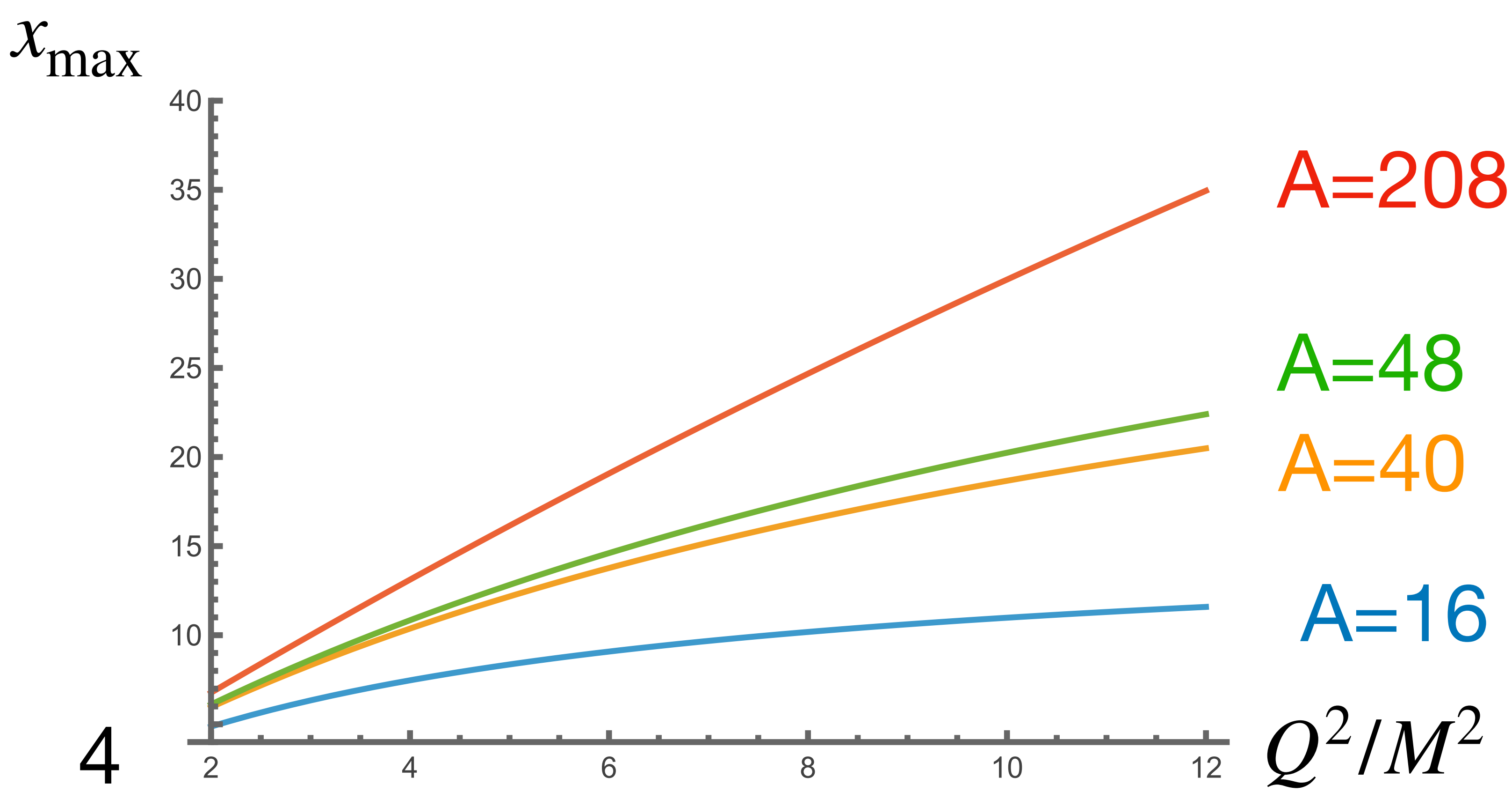
$$s_A = W_A^2 = (\nu + MA)^2 - \vec{q}^2 = (MA + m_\pi)^2$$

$$\frac{Q^2}{M^2}\left(\frac{A}{x_{\max}} - 1\right) = 2A\frac{m_\pi}{M}$$

M is nucleon mass

Large A allows larger value of x to produce pion (excite nucleon)

x_{max} - largest Bj x to produce a pion



More than two nucleons participate at large values of x even

at x=2

Summary

- Calculation of nuclear light front wave functions including SRC is possible
- Ratios of cross sections are not ratios of two-body densities
- Nucleon excitation at high Bj x is possible and probably important
- Several nucleons participate at high x