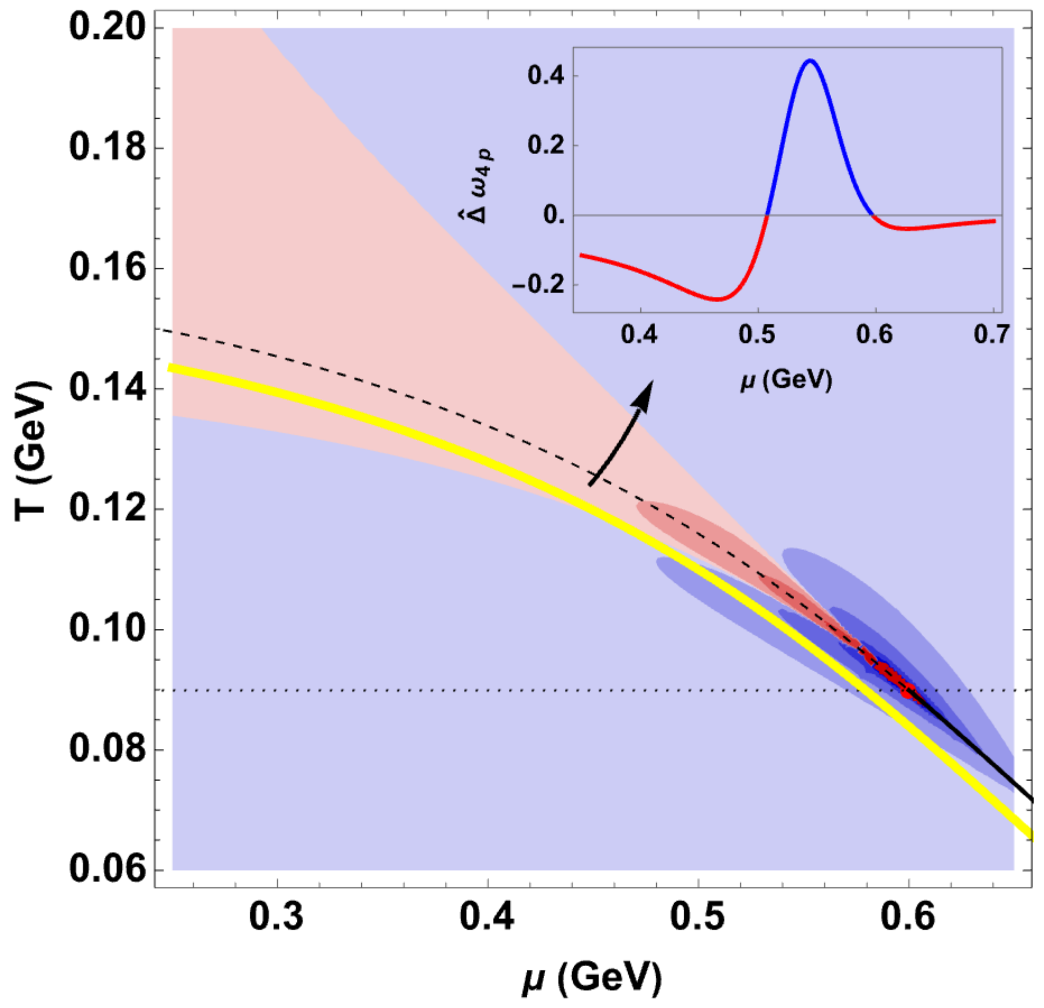
Hydrodynamic Correlations from Theory To Particle Multiplicity Cumulants in experiment

Pre-SQM Workshop 2026 held at LBNL



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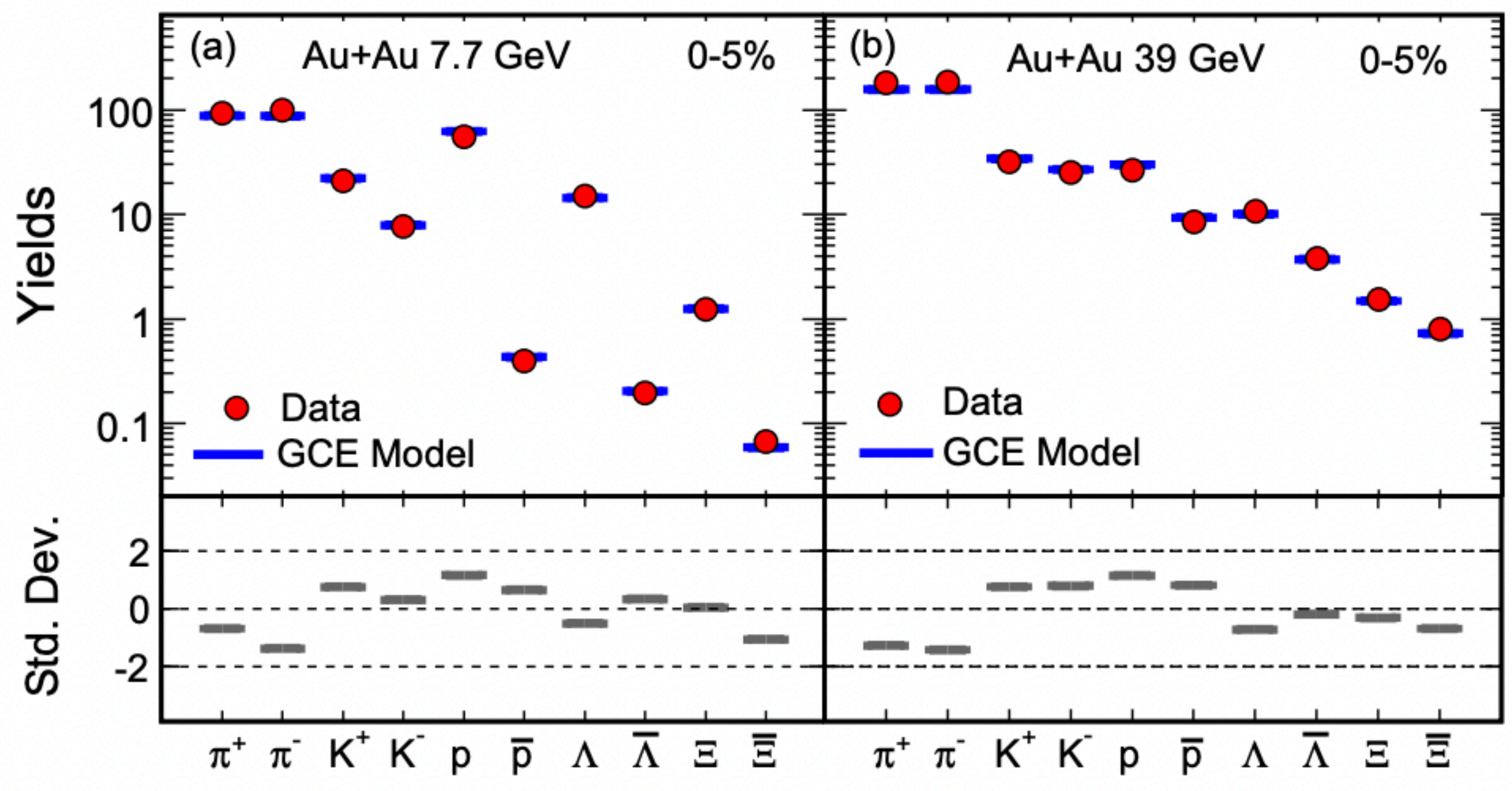
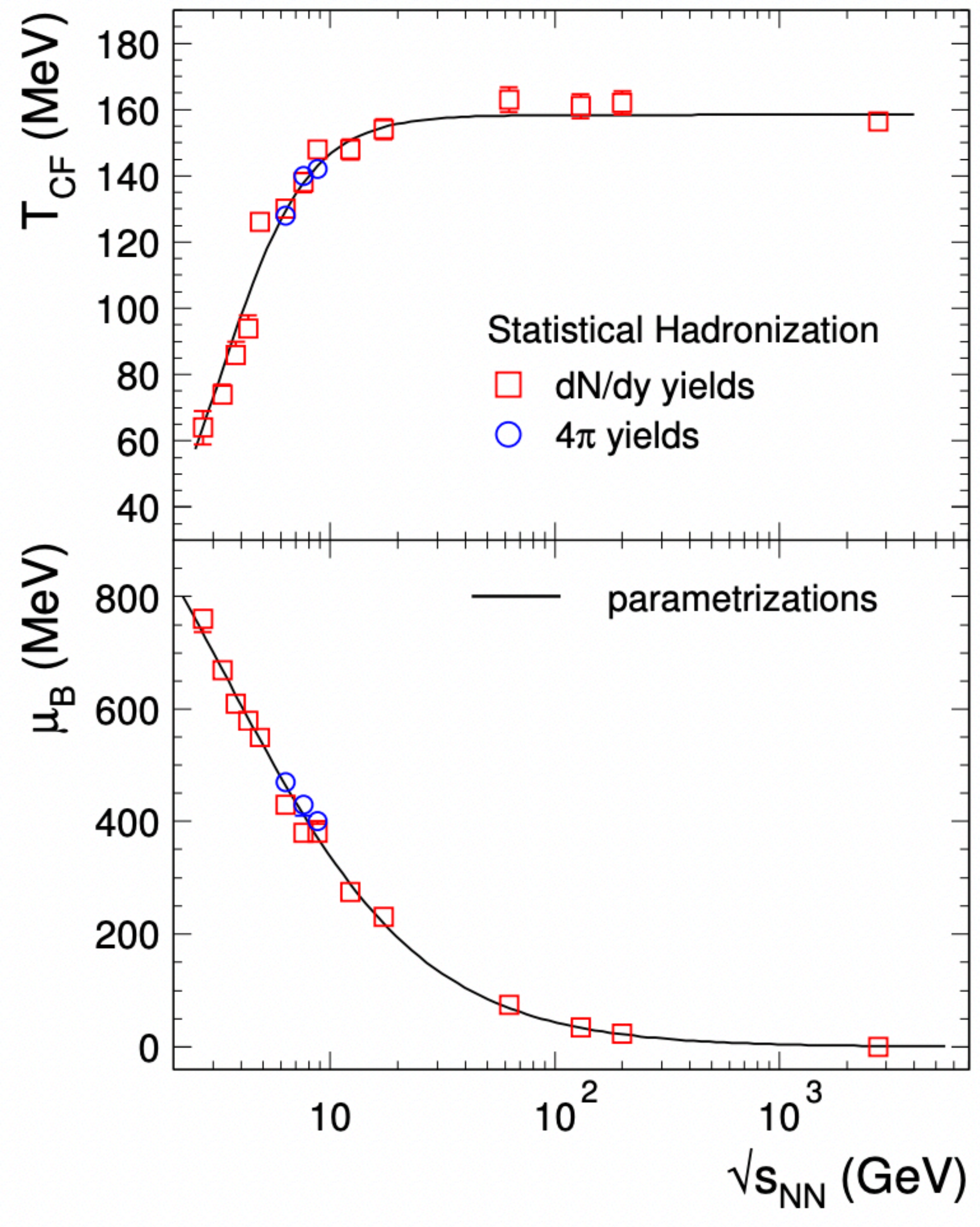
Hadron Resonance Gas at freeze-out



STAR Collaboration,Phys. Rev. C 96, 044904 (2017)

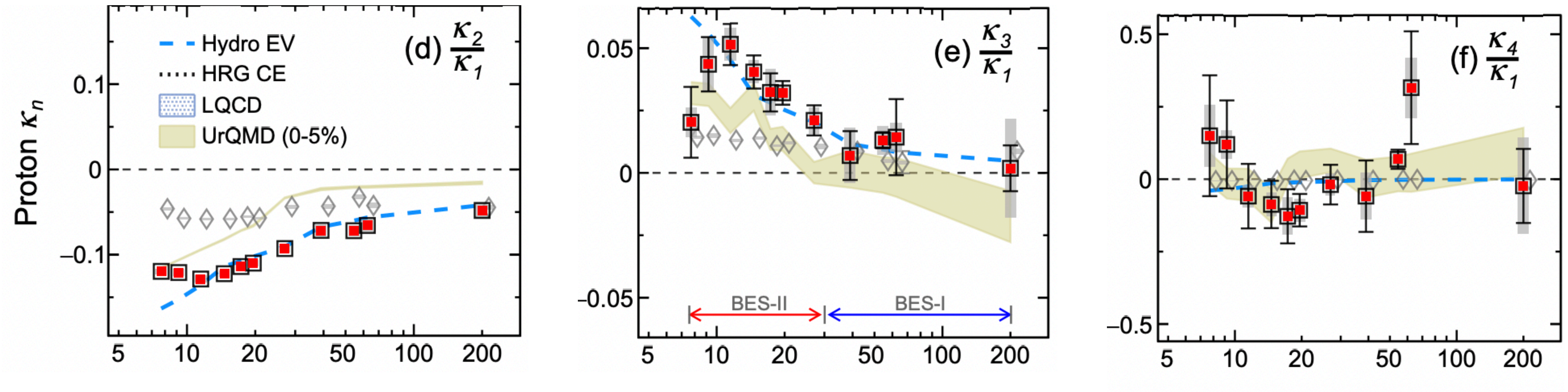


Andronic et al., Nature, 2018

- Mid-rapidity yields consistent with a Statistical Hadron Resonance Gas model
- What about event-by-event fluctuations?
- Deviations from a quantum gas of hadrons can be due to non-trivial EoS, CP, non-equilibrium dynamics etc..

Non-trivial correlations in hadron gas : Factorial cumulants of particle multiplicities

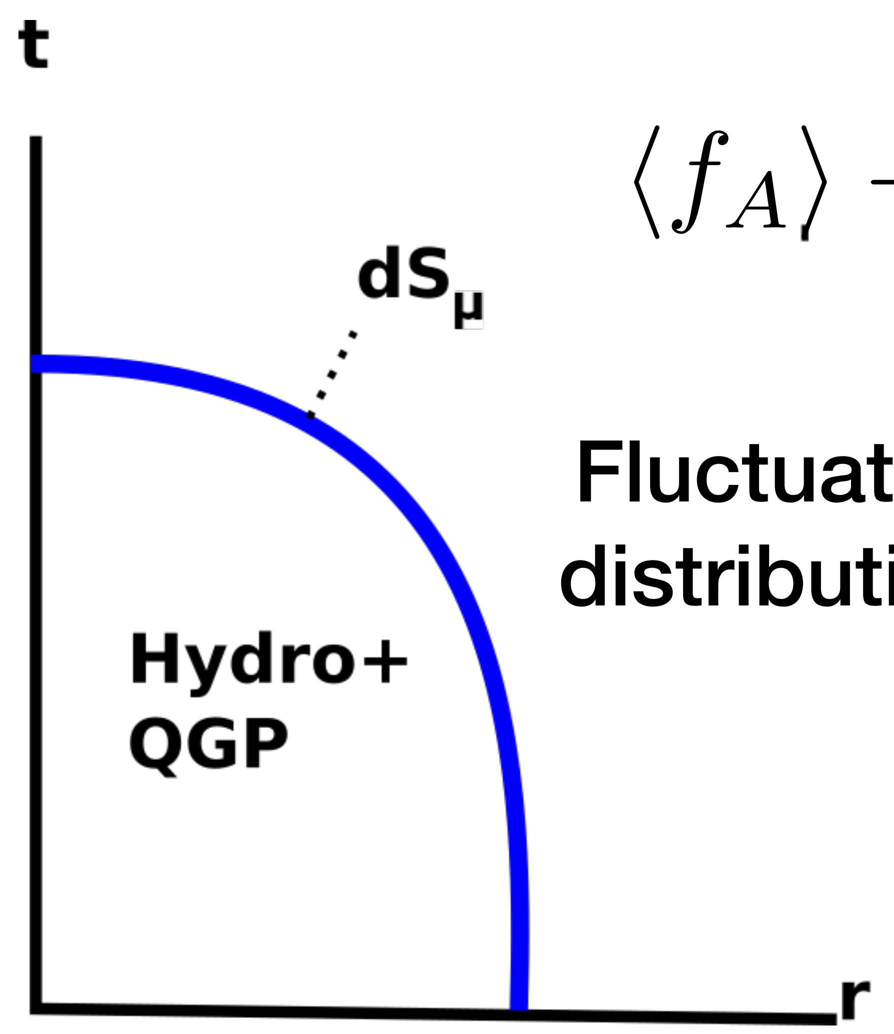
arXiv: 2504.00817, STAR collaboration, PRL



Collision Energy (GeV)

- Non-trivial deviations from a non-interacting gas of particles
- Sources of this correlations?
- Do they show any indications of a critical point in the vicinity of the heavy-ion trajectory in the phase diagram?
- Hope : Bound the location of the CP, if it exists and constrain the EoS near it

Connecting hydrodynamic correlations to particle multiplicity fluctuations at freeze-out



$$\langle f_A \rangle + \delta f_A$$

Fluctuating particle distribution function

MP, Stephanov, PRL 23

Most likely phase space distribution of particles after freeze-out that

- **Maximizes the entropy of the hadron resonance gas ensemble**
- **Constraint - Average values of all conserved densities and their correlators in hadron gas match with hydrodynamics**

The maximum entropy formula for factorial cumulants of proton multiplicities

$$\widehat{\Delta}\omega_{kp} = \frac{\widehat{\Delta} H_{a_1 \ldots a_k} \left(\sum_{B_1} \Gamma_{B_1 \to p} P^{a_1}_{B_1} \right) \ldots \left(\sum_{B_k} \Gamma_{B_2 \to p} P^{a_k}_{B_k} \right)}{\sum_B \Gamma_{B \to p} \langle N_B \rangle}$$

Karthein, MP, Rajagopal, Stephanov,Yin 25, PRD

Input to the freeze-out prescription for hydro fluctuations

- Hydrodynamic correlations from the hydro (hydro+) simulation
- Freeze-out hyper surface
- Masses, charges and branching ratios of different resonances

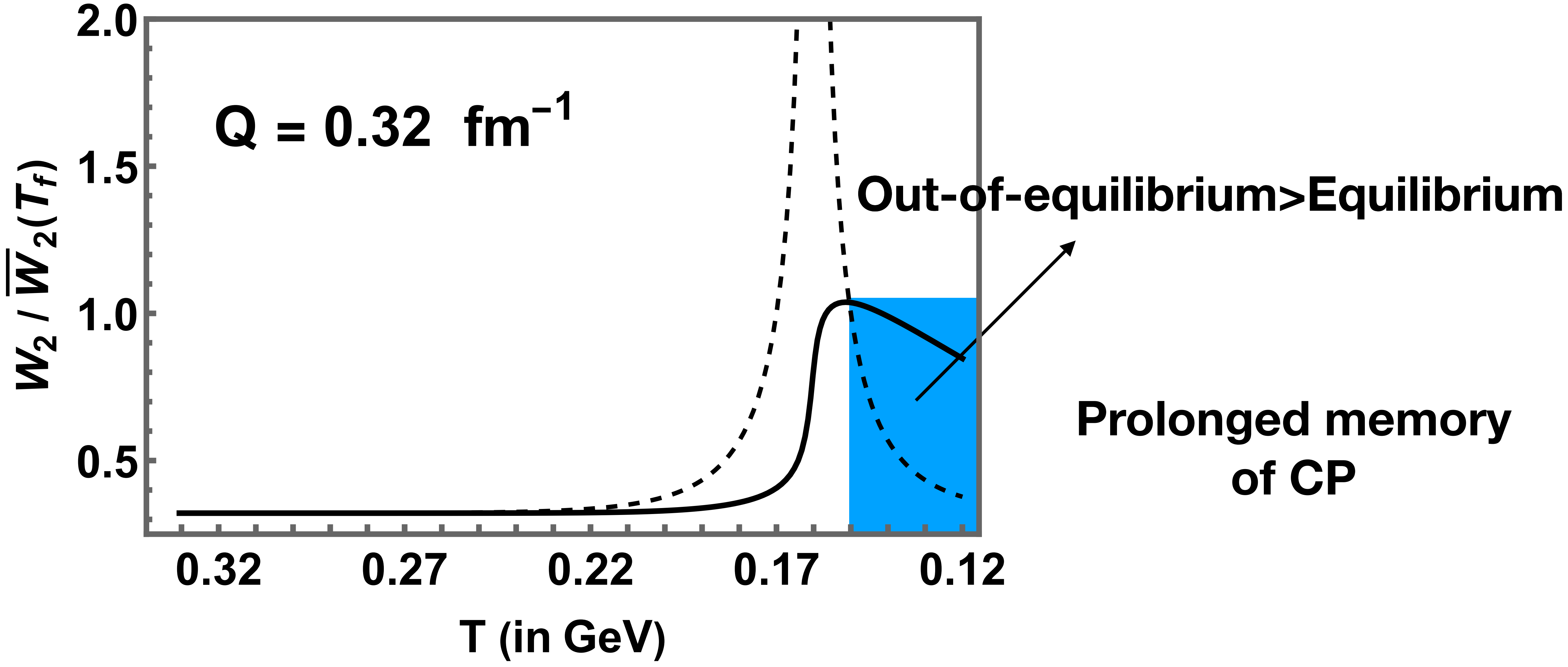
Hydrodynamic correlations at freeze-out

$$\langle \delta\hat{s}(x_+)\delta\hat{s}(x_-)\rangle = \int e^{i\mathbf{Q}\cdot\mathbf{\Delta x}}\, W_2(\mathbf{Q})\,,\ \mathbf{\Delta x} = x_+ - x_-$$

Persistence of critical imprints in the fluctuation observables until freeze-out

D_0= 1 fm

Suppression of peak

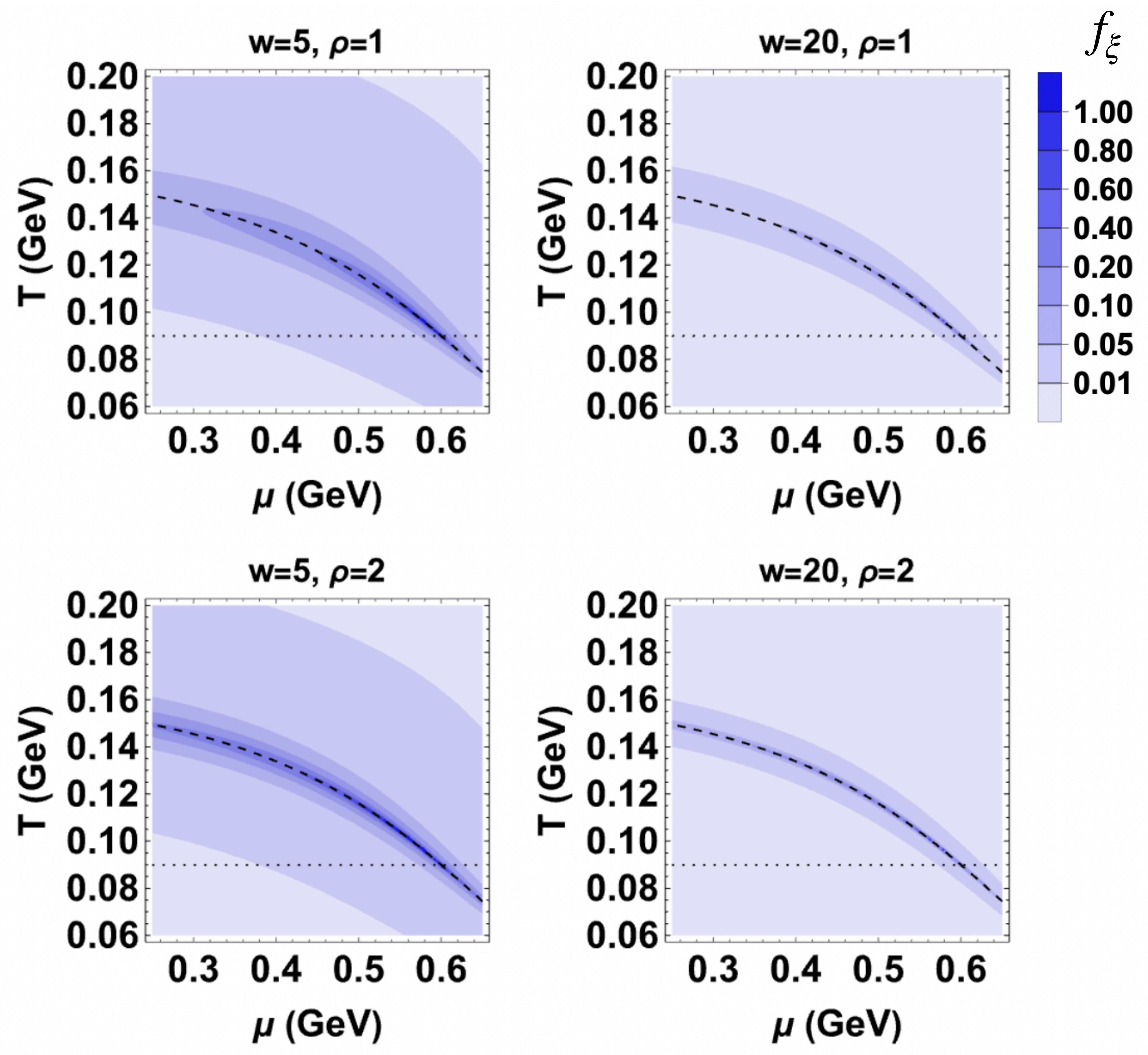


Prolonged memory of CP

Rajagopal,Ridgway,Weller,Yin, PRD,19
Du,Heinz,Rajagopal, Yin, PRC, 20
MP, Rajagopal, Stephanov, Yin, PRD 22
Mukherjee, Venugopalan, Yin PRC,15

$$\Gamma(\boldsymbol{Q}) = \frac{2D_0\xi_0}{\xi^3_{\rm QCD}} K(Q\xi_{\rm QCD}),$$

Relaxation rate of the slowest mode near the critical point



Karthein, MP, Rajagopal, Stephanov,Yin 26, PRD

$$\Gamma(\boldsymbol{Q}) = \frac{2D_0\xi_0}{\xi^3_{\mathrm{QCD}}} K(Q\xi_{\mathrm{QCD}}),$$

$$\xi_{\mathrm{QCD}} = f_\xi\, \xi.$$

- Rate depends on distance to CP
- Rate at a certain point on the phase diagram depends on the scale factors D_0, ρ, w and f_ξ

Consider a situation where relaxation is sufficiently fast for the local hydrodynamic correlations to be given by their equilibrium values.

Fast relaxation of correlations

$$\widehat{\Delta}\omega_{kp} = \frac{\widehat{\Delta} H_{a_1 \ldots a_k} \left(\sum_{B_1} \Gamma_{B_1 \to p} P_{B_1}^{a_1}\right) \ldots \left(\sum_{B_k} \Gamma_{B_2 \to p} P_{B_k}^{a_k}\right)}{\sum_B \Gamma_{B \to p} \langle N_B \rangle}$$

Karthein, MP, Rajagopal, Stephanov,Yin 26, PRD

$$H_{(j)\epsilon(k-j)n} \equiv \langle (\delta\epsilon)^j (\delta n)^{k-j} \rangle^{\text{equilibrium}}_{\text{connected}} = \frac{(-1)^j}{V^{k-1}} \frac{\partial^k (\beta P)}{\partial \beta^j \partial \hat{\mu}^{k-j}} \, .$$

- Hydrodynamic correlations are given by their local equilibrium conditions, directly related to susceptibilities

Hydrodynamic correlations in QCD near the CP

$$P(\mu, T) = \bar{P}(\mu, T) + \Delta P(\mu, T)\,,$$

$$H_{(j)\epsilon(k-j)n} = \bar{H}_{(j)\epsilon(k-j)n} + \Delta H_{(j)\epsilon(k-j)n}$$

$$\bar{P} = P_{\mathrm{HRG}}.$$

$$\Delta H_{kn} \equiv \frac{V^{k-1}}{T_c^3} \Delta H_{(k)n},$$

- $$\Delta P = P^{\mathrm{singular}}(\mu, T) = -T_c^4\, G_{\mathrm{Ising}}(r(\mu, T)\, h(\mu, T))$$

Simplification

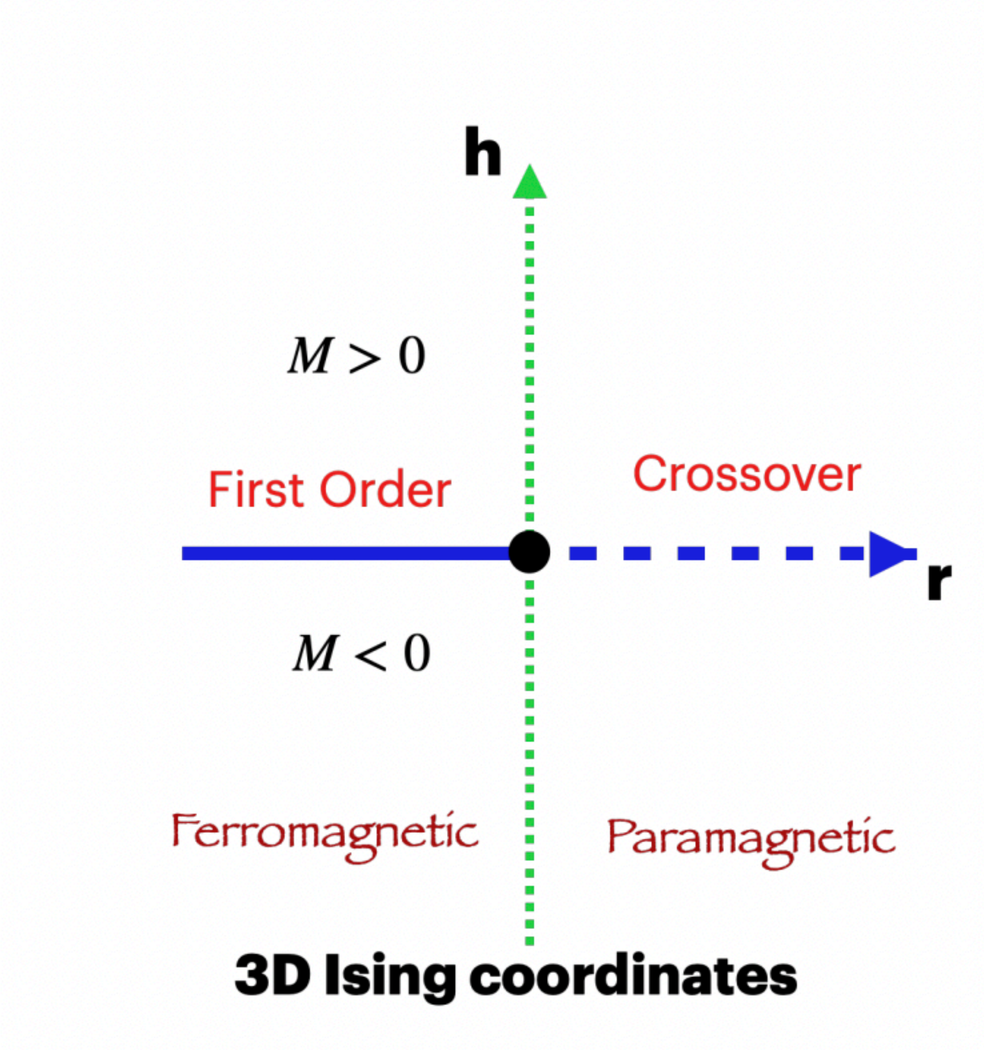
$$\Delta H_{kn} = \frac{T_c}{T} \frac{\partial^k G_{\mathrm{Ising}}}{\partial \hat{\mu}^k}$$

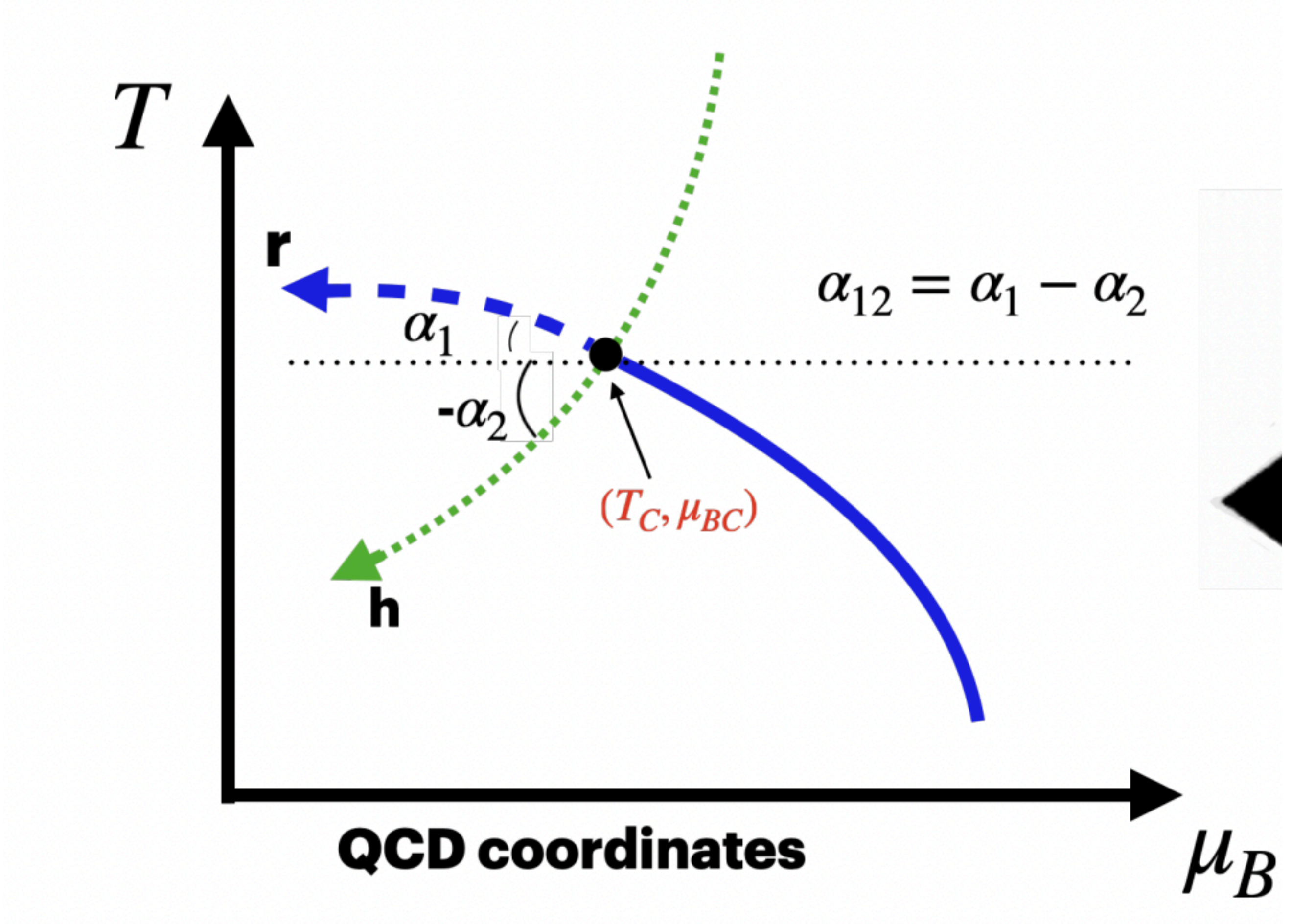
Karthein, MP, Rajagopal, Stephanov, Yin 26, PRD

Basar, MP, Stephanov 26 (To be on arXiv next week)

A general class of candidate EoSs

$$P_{\mathrm{QCD}}(\mu, T) = P_{\mathrm{BG}}(\mu, T) + A\,G(r(\mu, T), h(\mu, T))$$





Independent & non-universal parameters

$$\mu_c\,,\ \alpha_{12}\,,\ \rho\,,\ w$$

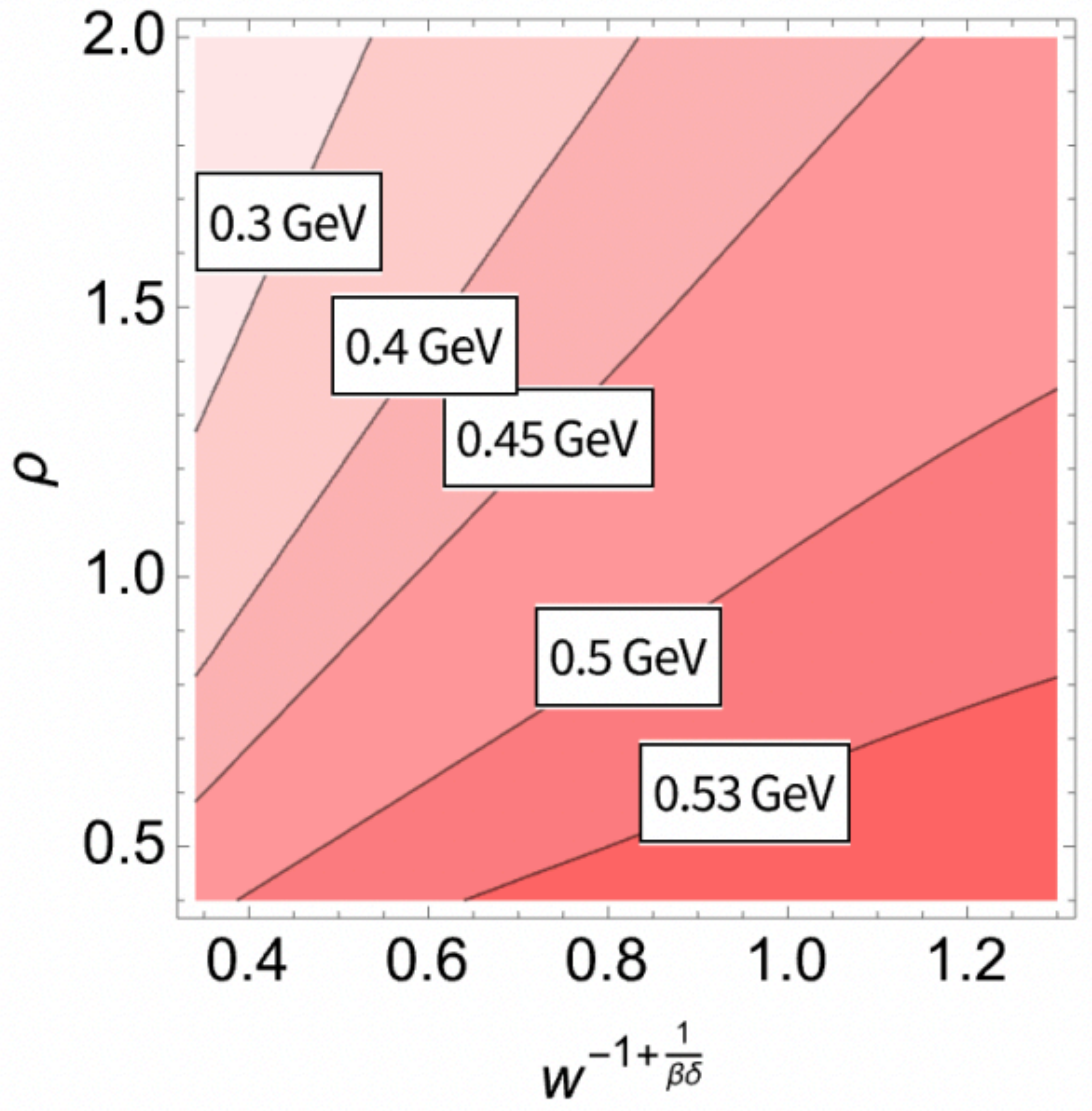
Weakly constrained in the chiral limit

MP, Stephanov, PRD, 19

Kahangirwe et al., PRD 24

Affect of mapping to Ising model on the observable particle cumulants

Karthein, MP, Rajagopal, Stephanov, Yin 26, PRD



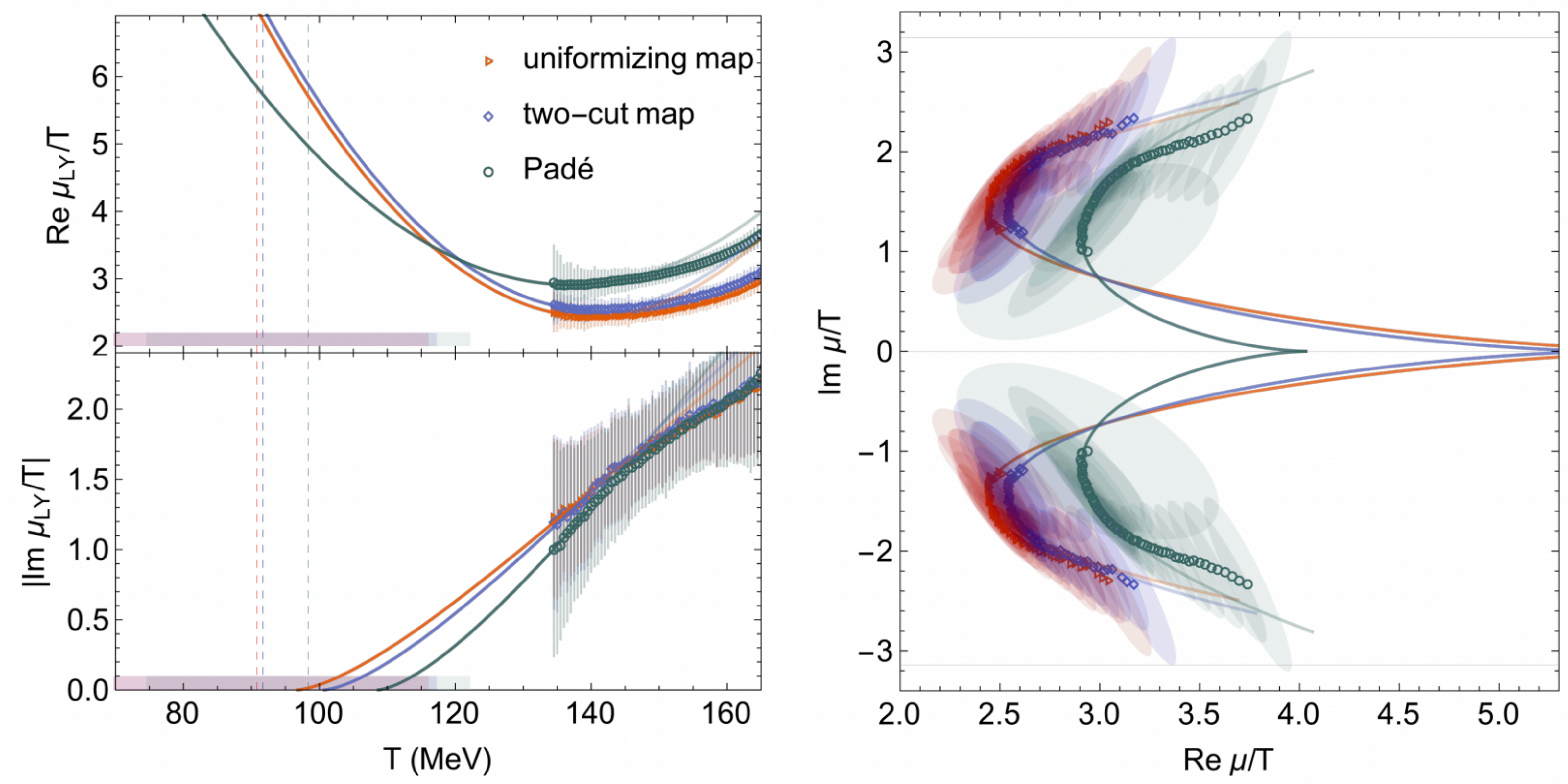
$$\widehat{\Delta}\omega_{kp}(\mu, T; \bar{\rho}, \lambda^{-1} w) = \lambda^{1+\frac{1}{\delta}}\,\widehat{\Delta}\omega_{kp}(\mu, T; \bar{\rho}, w)$$

Basar, MP, Stephanov 26 (To be on arXiv next week)

- $\bar{\rho} = \rho\, w^{0.36}$ controls the position of peaks/dips along a freeze-out curve
- $\hat{\Delta}\omega_p^k \sim w^{-1.2} \Delta T_f^{1.2-k} \sim \xi^{k(5-\eta)/2-3}$

Constraining the non-universal mapping parameters using Lee-Yang singularities

Basar, PRC, 24



$$\mu_{LY}(T) \approx \mu_c - c_1(T - T_c) \pm i x_{LY} c_2 (T - T_c)^{\beta\delta},$$

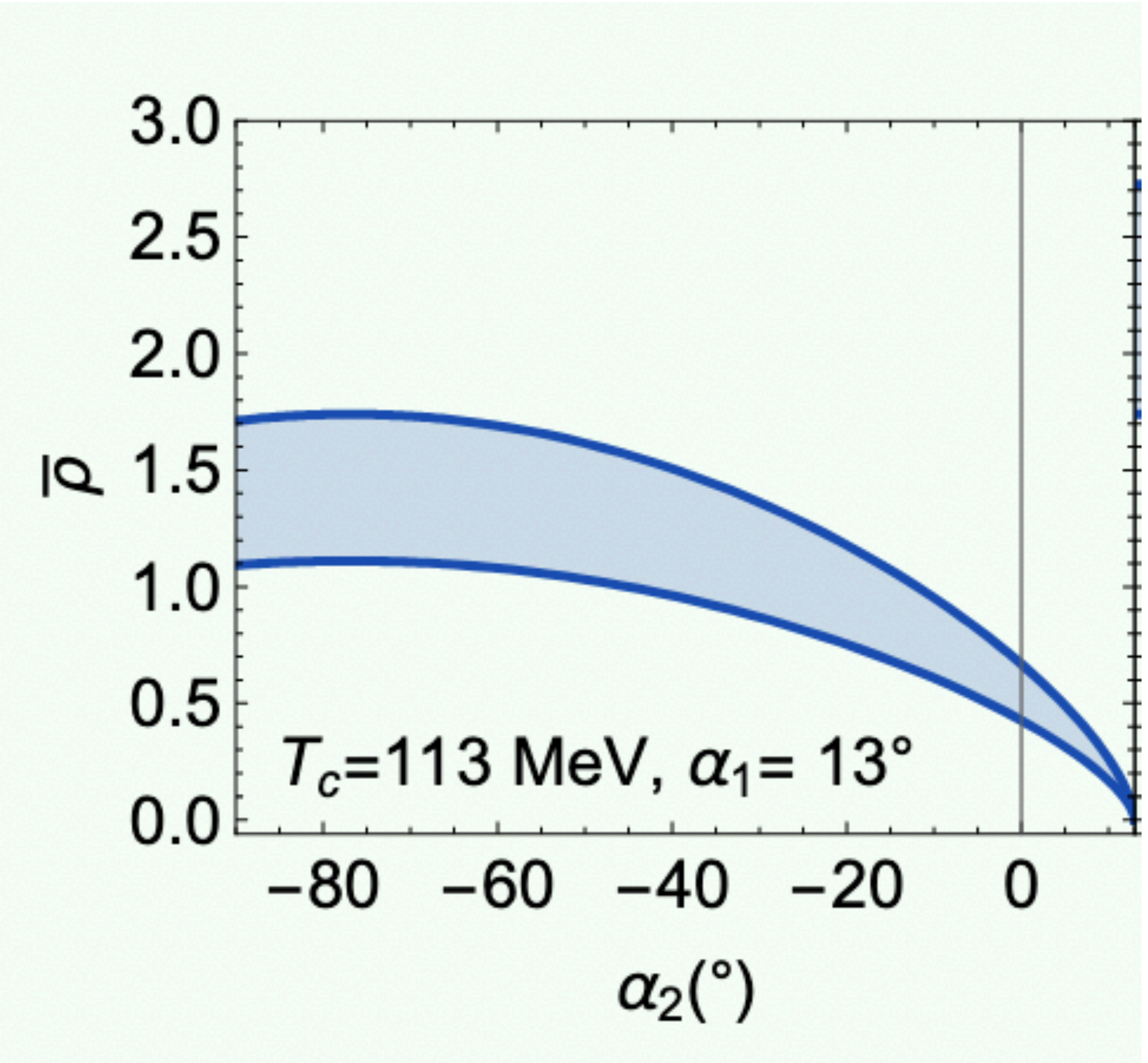
where $c_1 := \frac{h_T}{h_\mu} := \tan\alpha_1 \quad c_2 := \frac{{r_\mu}^{\beta\delta}}{h_\mu}\left(\frac{r_T}{r_\mu} - \frac{h_T}{h_\mu}\right)^{\beta\delta}.$

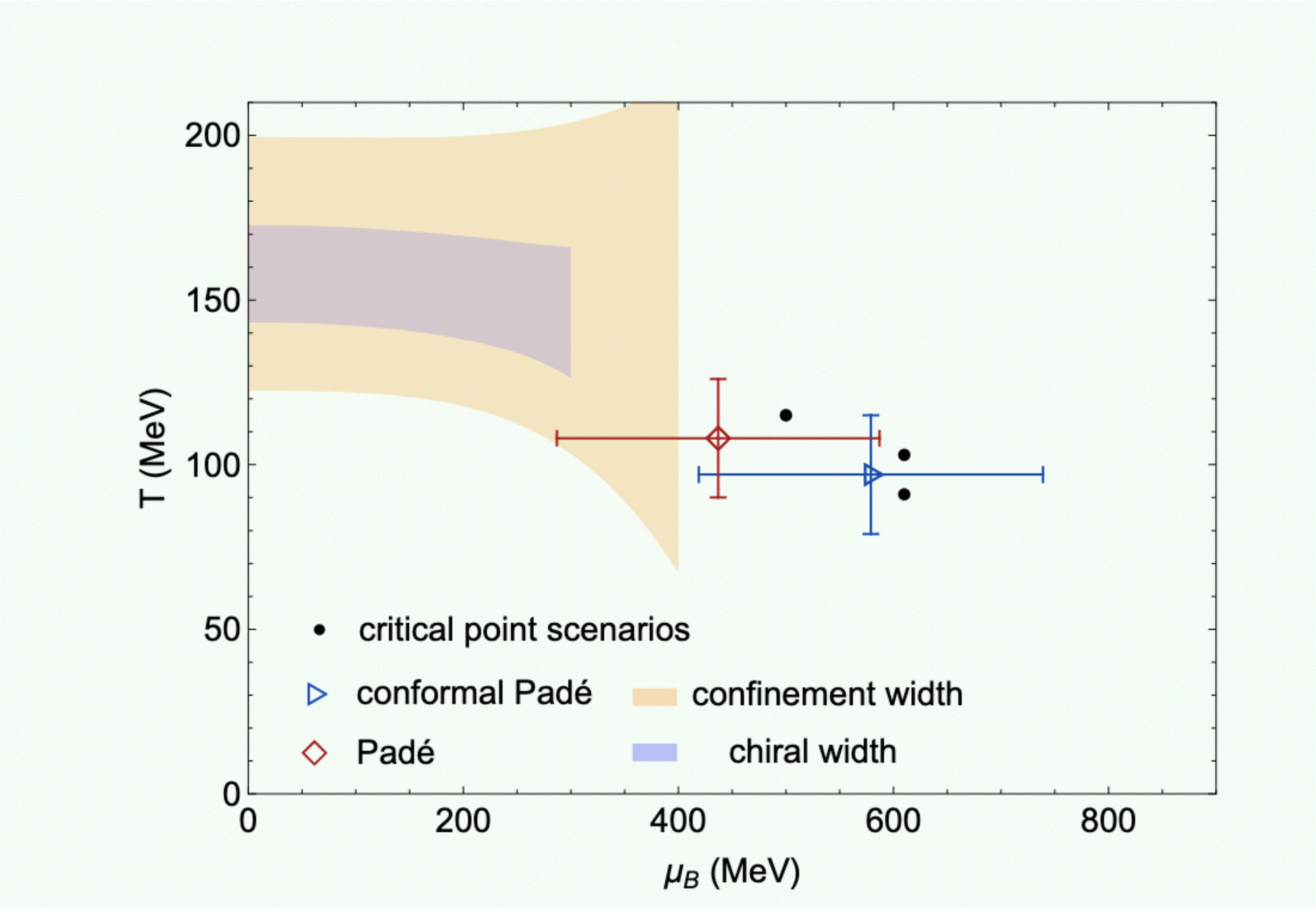
	T_c (MeV)	μ_c (MeV)	crossover slope (α_1)	c_2 (MeV$^{1-\beta\delta}$)
uniformizing	97^{+18}_{-18}	579^{+172}_{-160}	$9.40^\circ\ ^{+3.89}_{-3.81}$	$2.22^{+0.52}_{-0.86}$
two-cut	100^{+18}_{-18}	557^{+175}_{-150}	$8.69^\circ\ ^{+3.91}_{-3.83}$	$2.56^{+0.58}_{-1.21}$
Padé	108^{+21}_{-21}	437^{+114}_{-50}	$4.55^\circ\ ^{+3.41}_{-3.37}$	$3.35^{+0.82}_{-1.37}$

Basar, MP, Stephanov 26 (To be on arXiv next week)

$$\bar{\rho} = \left(\frac{1}{c_2 T_c^{\beta\delta-1}} \frac{|\sin\alpha_{12}|}{|\sin\alpha_1|^{\beta\delta+1}}\right)^{1/\beta\delta}$$

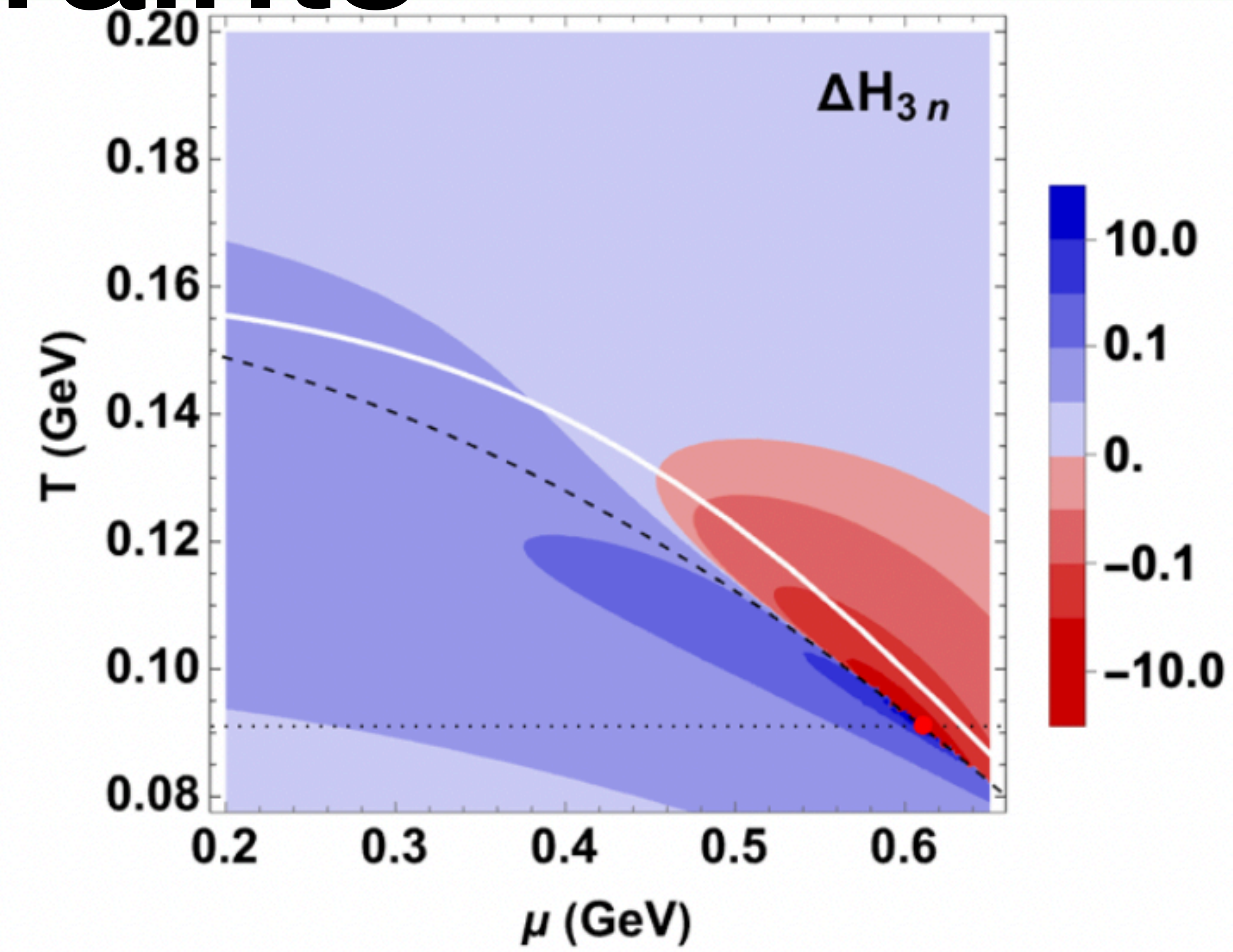
Lattice informed choices for map from QCD to 3D Ising model



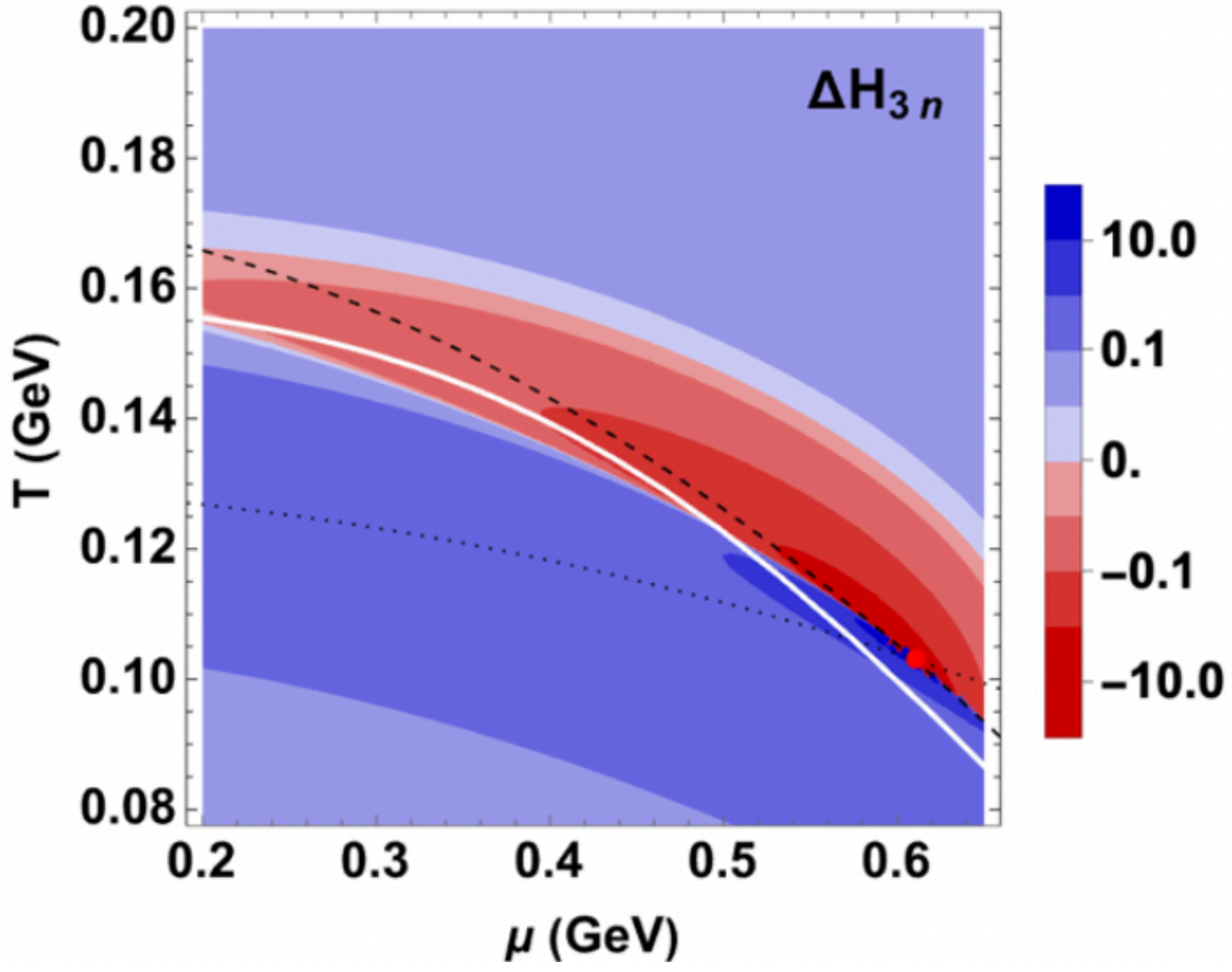


Basar, MP, Stephanov 26 (To be on arXiv next week)

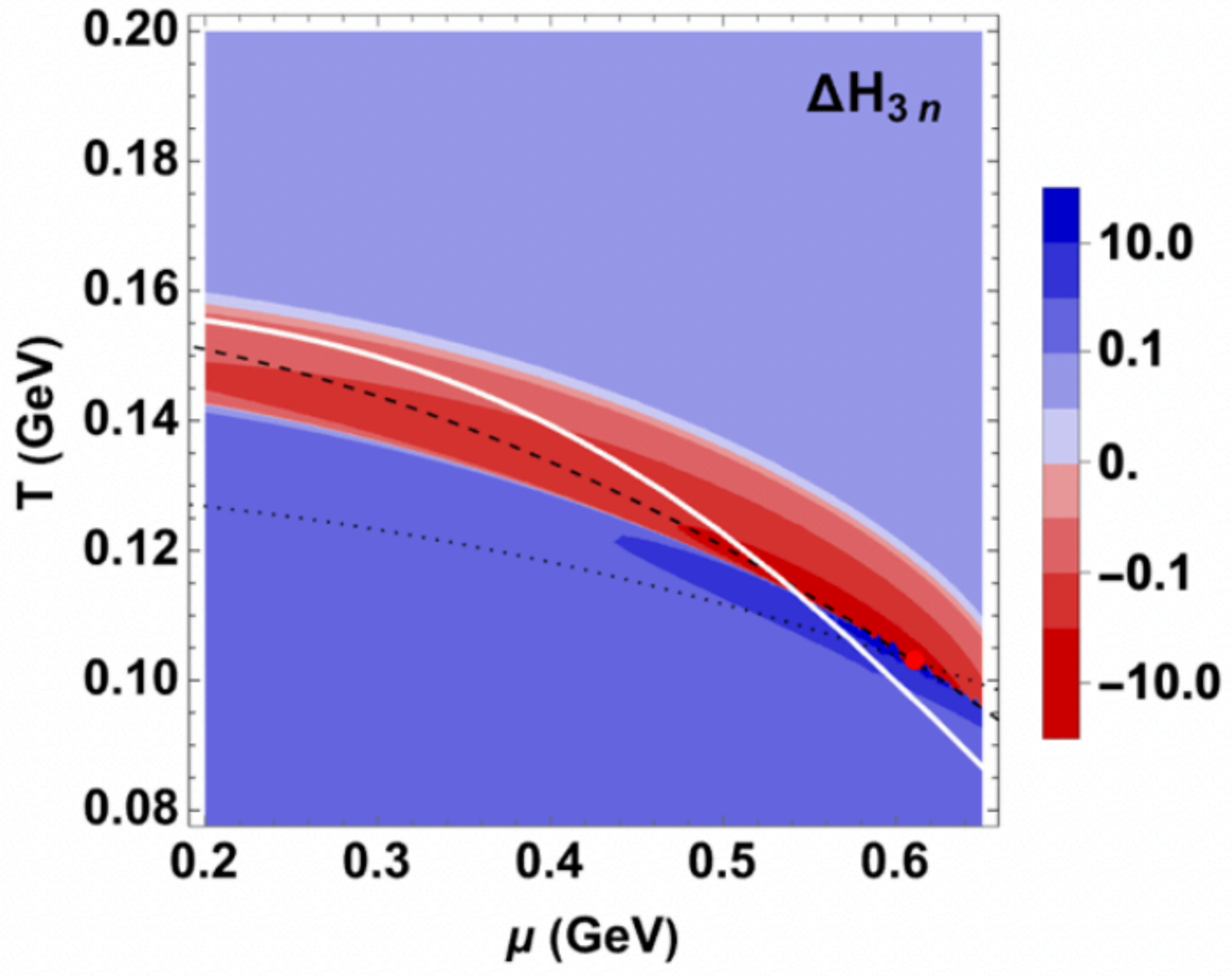
Four topologically different scenarios for freeze-out consistent with these constraints



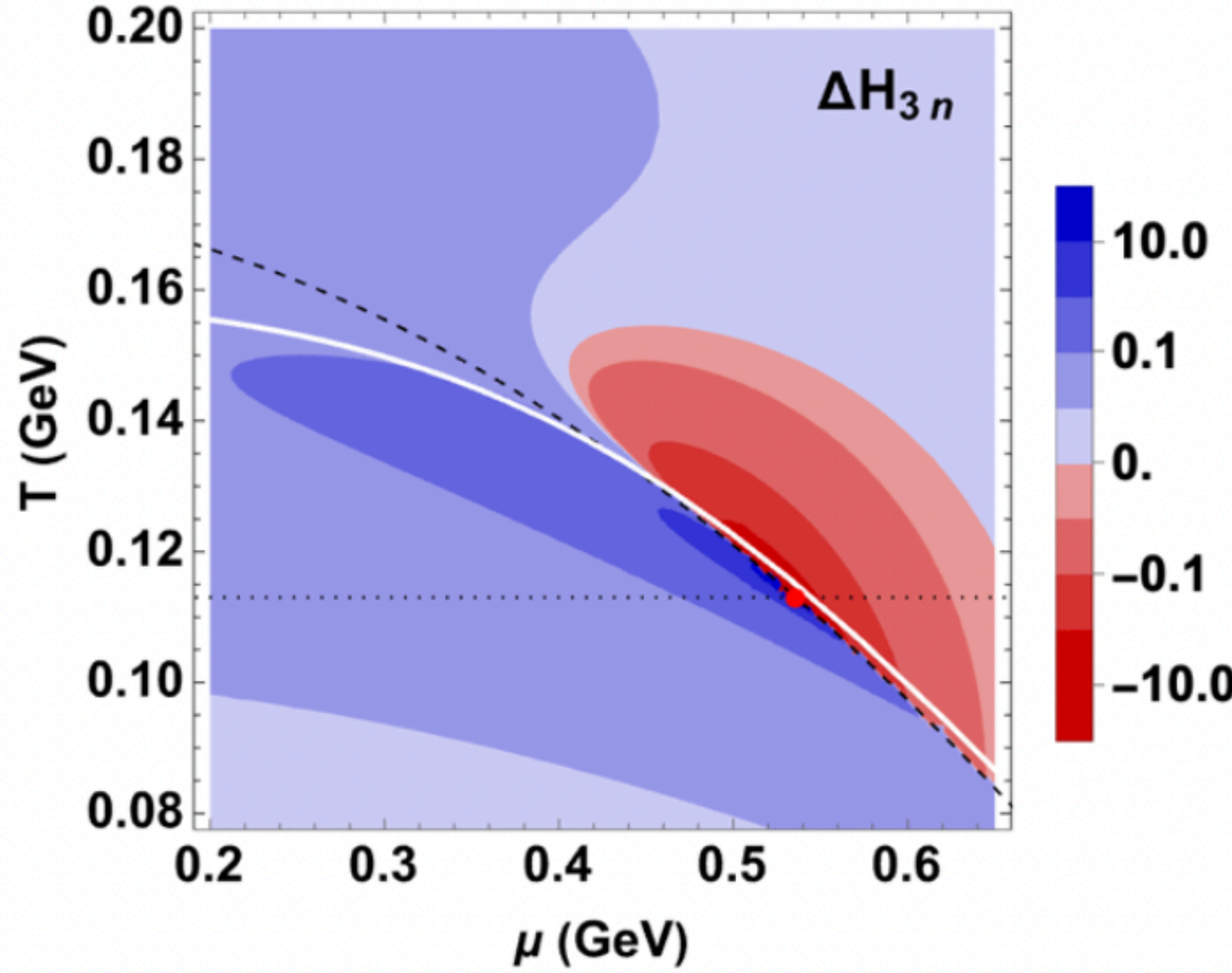
Hot critical point without crossing (H)



Hot critical point with crossing (HX)



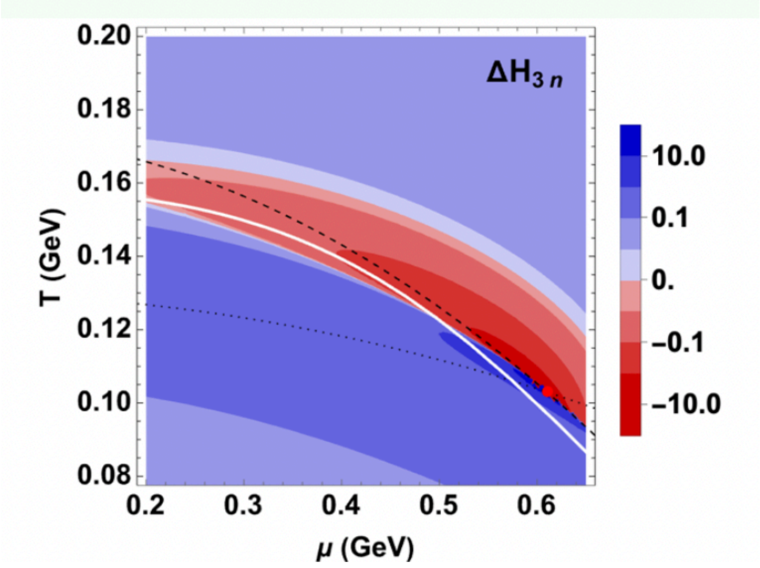
Cool critical point without crossing (C)



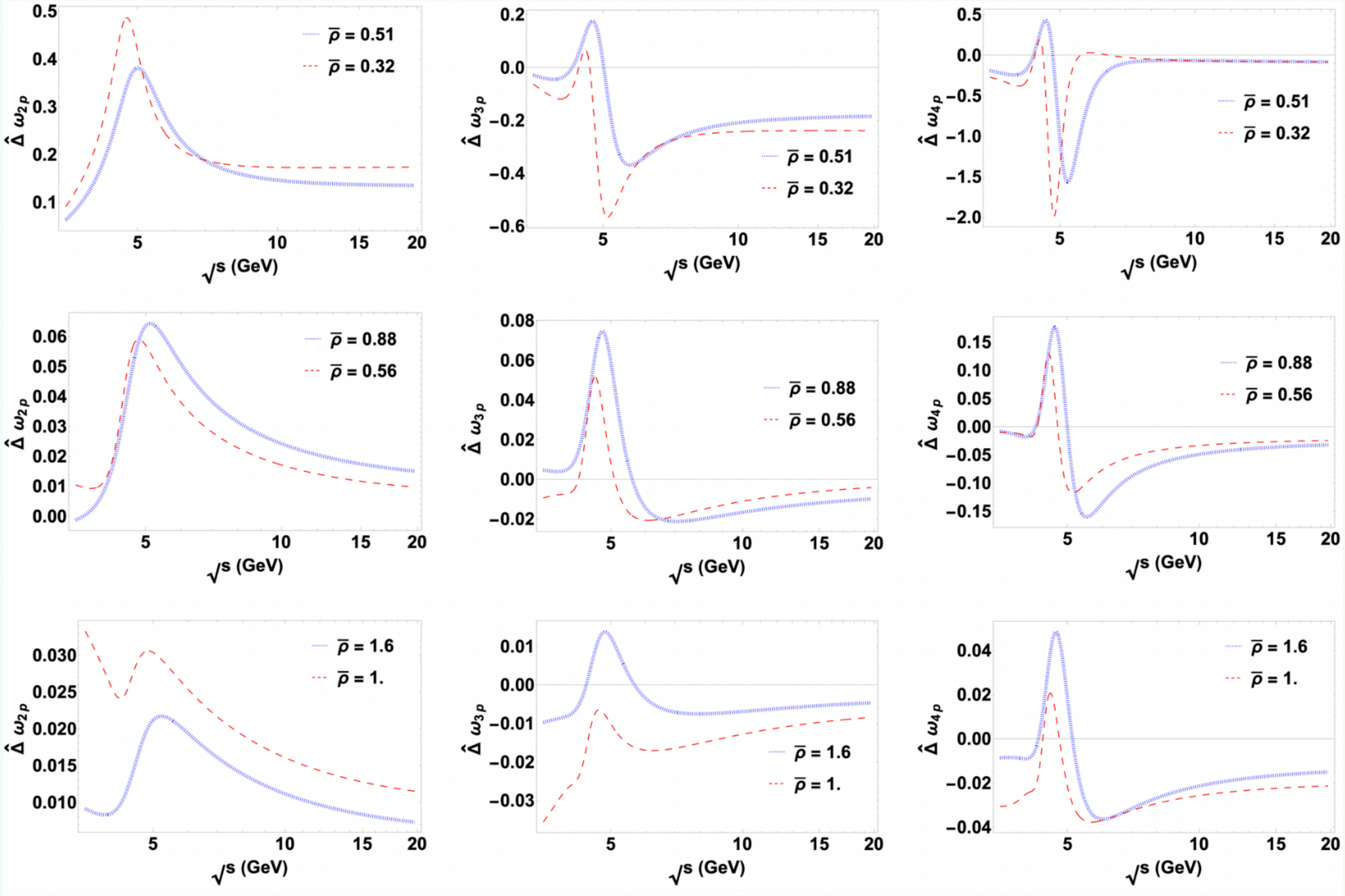
Basar, MP, Stephanov 26 (To be on arXiv next week)

Cool critical point with crossing (CX)

Hot critical point without crossing (H)

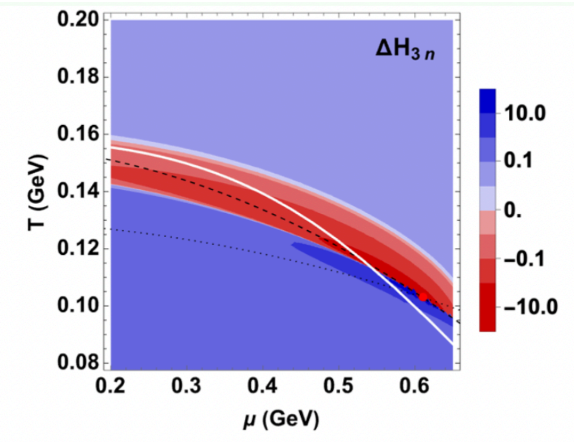


Basar, MP, Stephanov 26 (To be on arXiv next week)

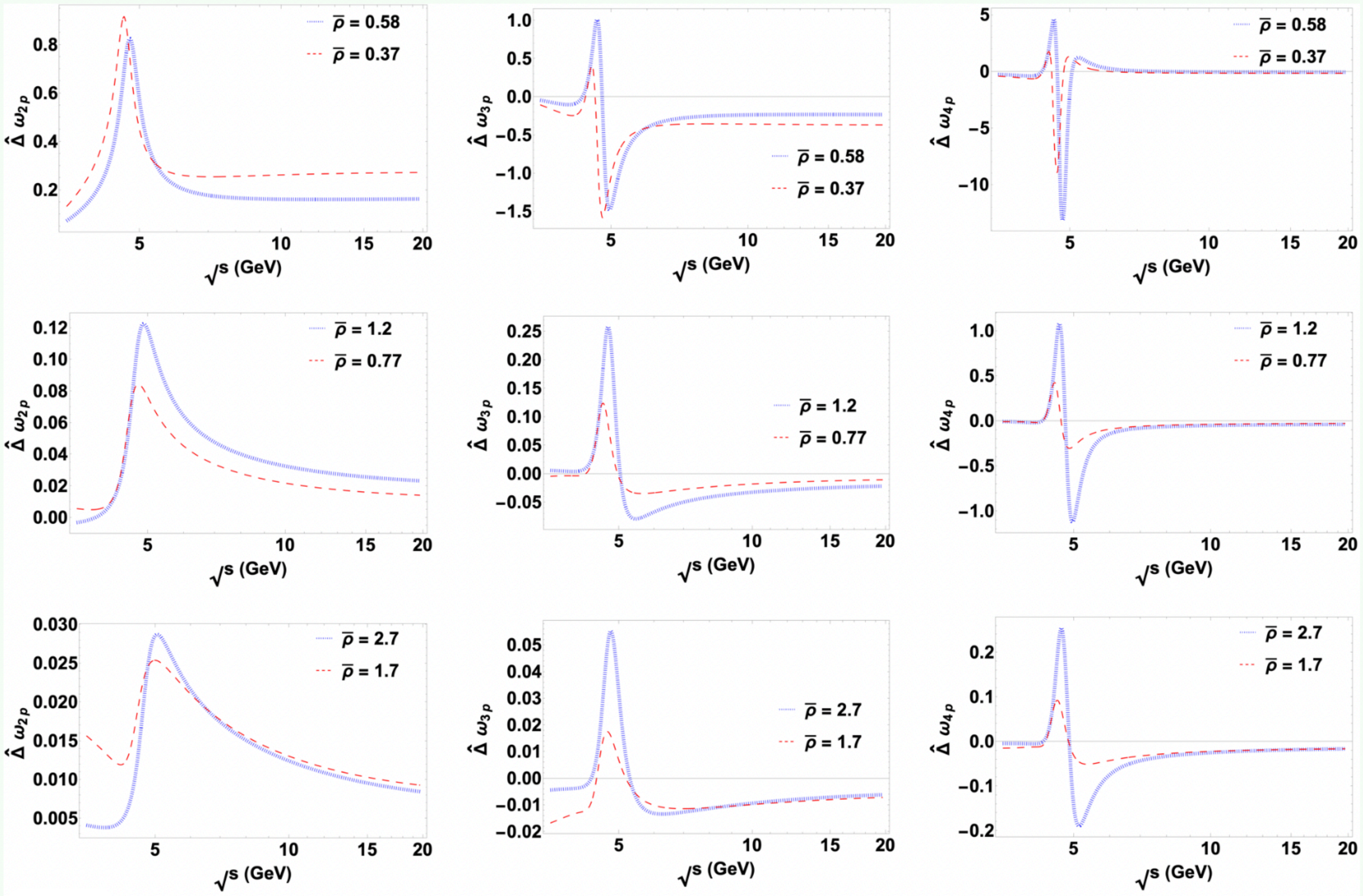


Scenario H: $\widehat{\Delta}\omega_{2p}$, $\widehat{\Delta}\omega_{3p}$, and $\widehat{\Delta}\omega_{4p}$ for $\mu_c = 610\,\text{MeV}$, $T_c = 103\,\text{MeV}$, $\alpha_1 = 13°$, and $w = 5$. Top row: $\alpha_2 = 5°$, middle row: $\alpha_2 = -6°$, bottom row: $\alpha_2 = -89°$

Hot critical point with crossing (HX)



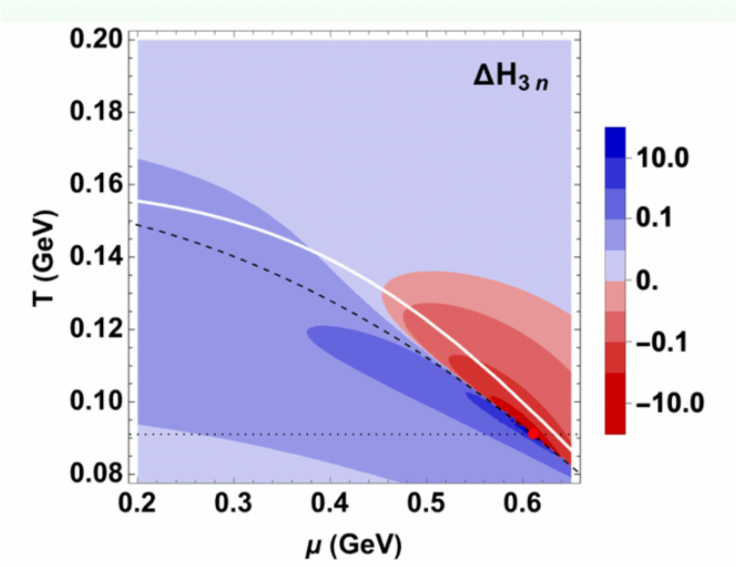
Basar, MP, Stephanov 26 (To be on arXiv next week)



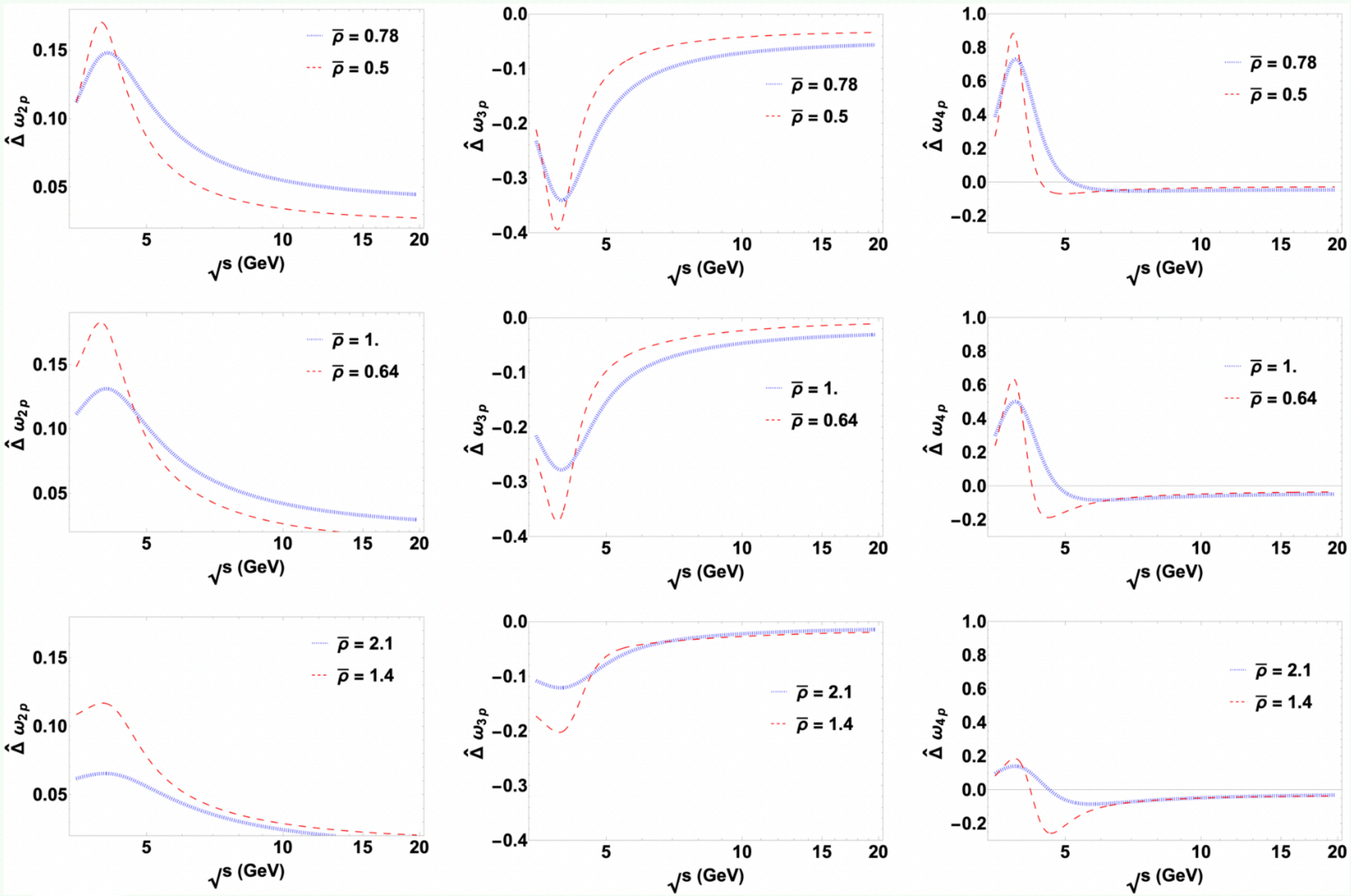
Scenario HX: $\widehat{\Delta}\omega_{2p}$, $\widehat{\Delta}\omega_{3p}$, and $\widehat{\Delta}\omega_{4p}$ for $\mu_c = 610\,\mathrm{MeV}$, $T_c = 103\,\mathrm{MeV}$, $\alpha_1 = 10°$, and $w = 5$. Top row: $\alpha_2 = 5°$, middle row: $\alpha_2 = -6°$, bottom row: $\alpha_2 = -89°$.

Rapid changes from +ve to -ve with crossing

Cool critical point without crossing (C)



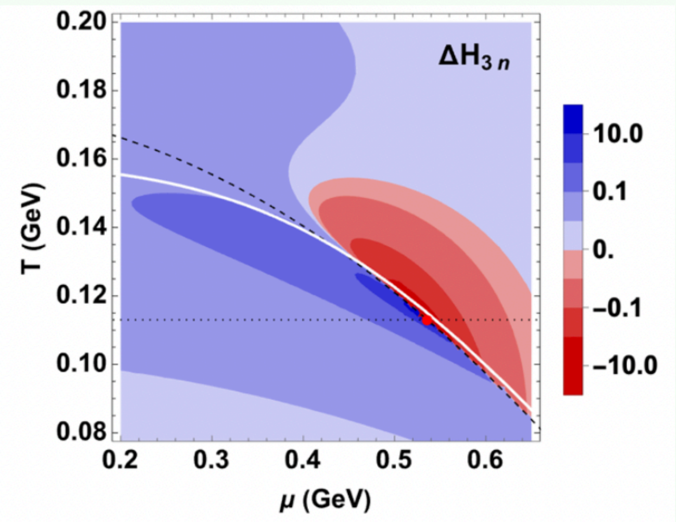
Basar, MP, Stephanov 26 (To be on arXiv next week)



Scenario C: $\widehat{\Delta}\omega_{2p}$, $\widehat{\Delta}\omega_{3p}$, and $\widehat{\Delta}\omega_{4p}$ for $\mu_c = 610\,\text{MeV}$, $T_c = 91\,\text{MeV}$, $\alpha_1 = 12°$, and $w = 5$. Top row: $\alpha_2 = 0°$, middle row: $\alpha_2 = -6°$, bottom row: $\alpha_2 = -89°$.

Dip in skewness with a cool critical point

Cool critical point with crossing (CX)



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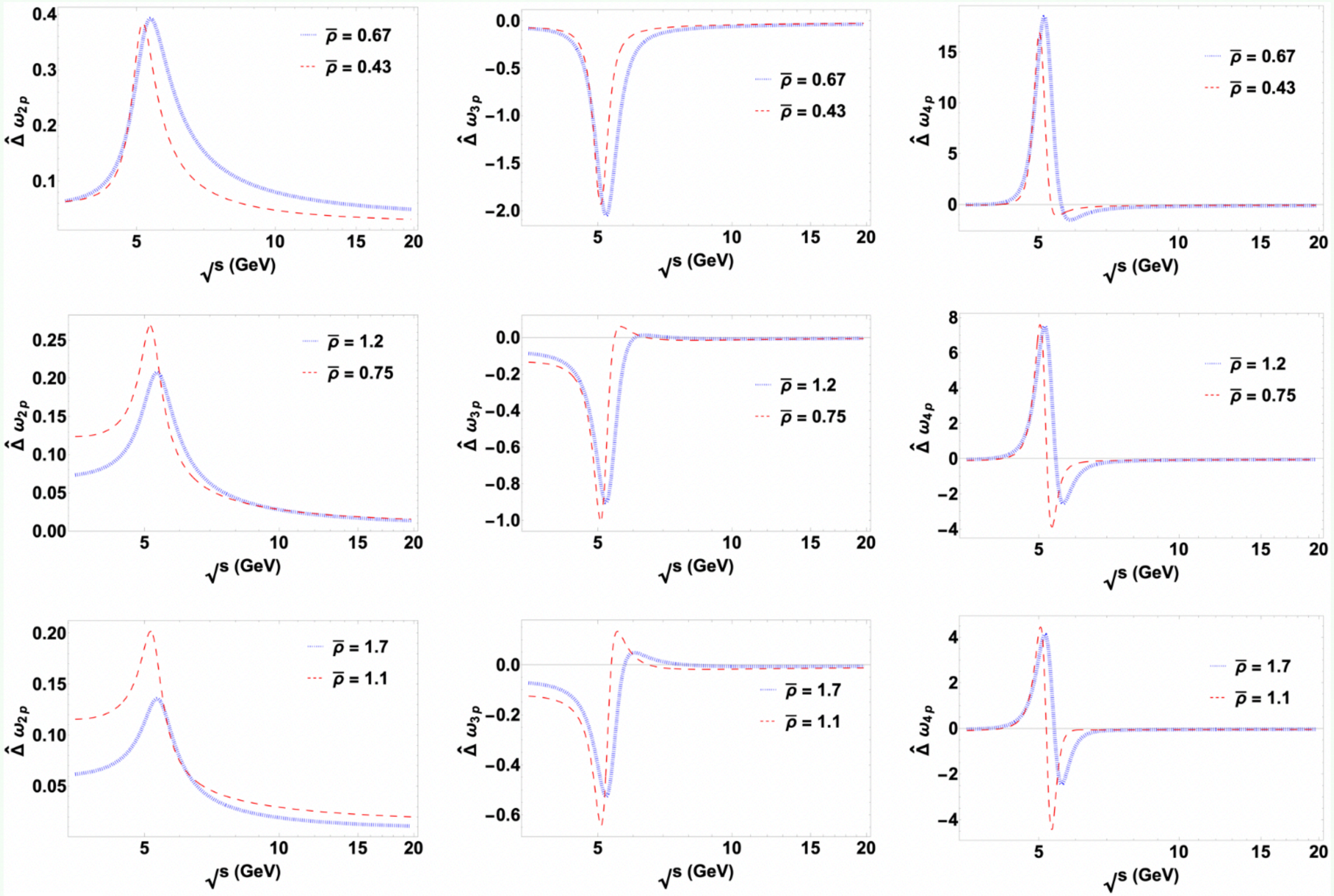
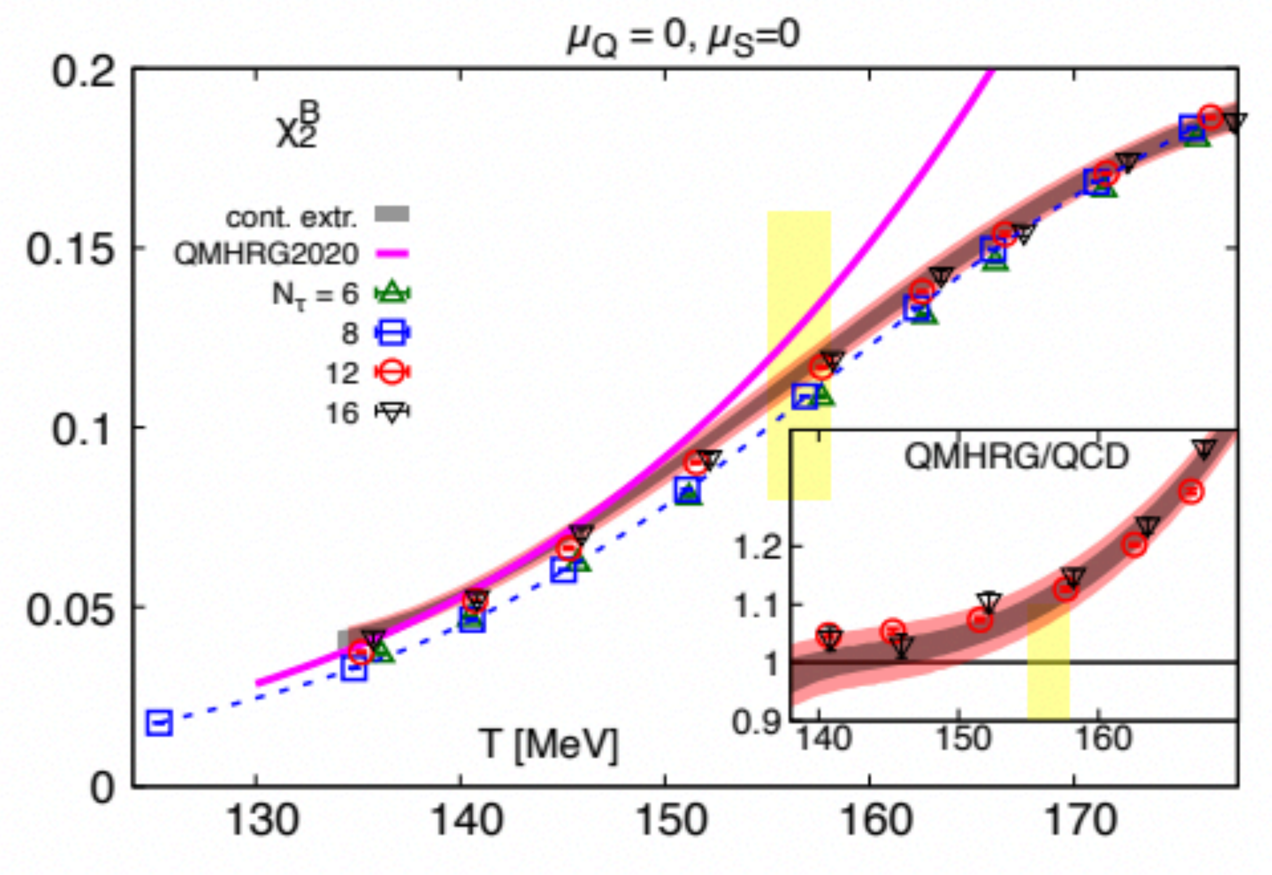


: Scenario CX: $\widehat{\Delta}\omega_{2p}$, $\widehat{\Delta}\omega_{3p}$, and $\widehat{\Delta}\omega_{4p}$ for $\mu_c = 536\,\text{MeV}$, $T_c = 113\,\text{MeV}$, $\alpha_1 = 13°$, and $w = 5$. Top row: $\alpha_2 = 0°$, middle row: $\alpha_2 = -20°$, bottom row: $\alpha_2 = -89°$.

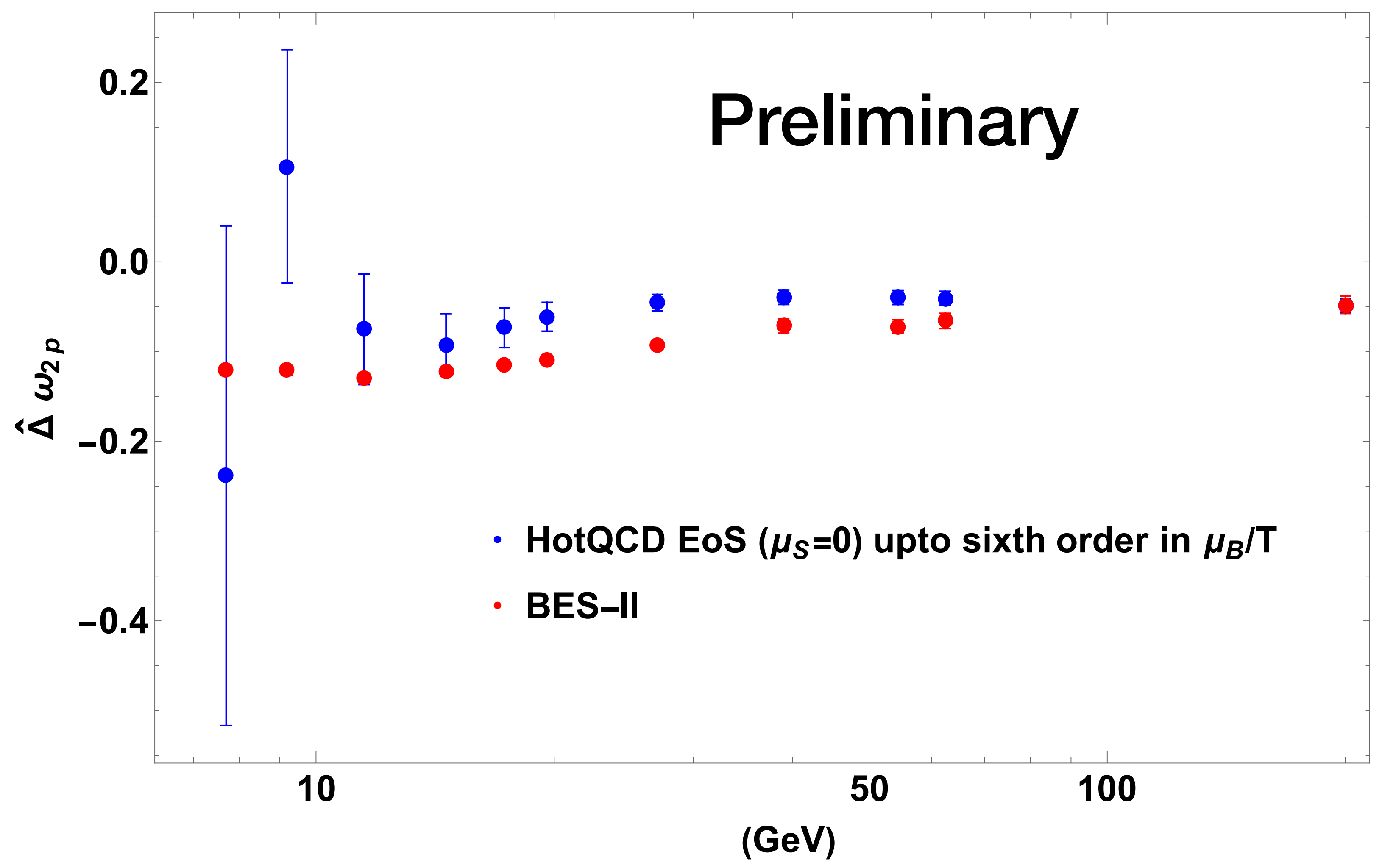
Location of peaks and dips significantly constrained by Lattice QCD input.

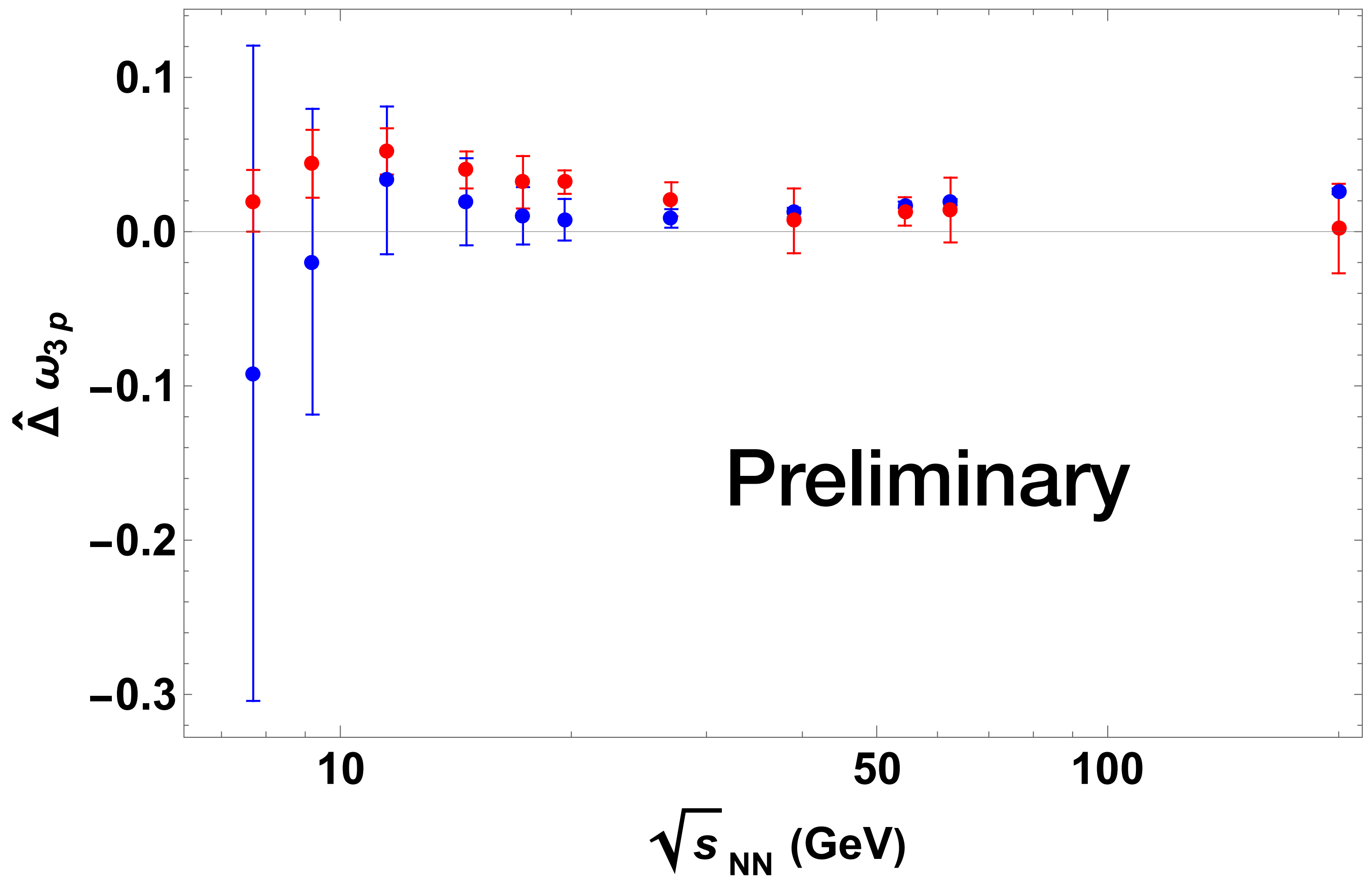
Proton multiplicity cumulants from lattice EoS (Work in progress)

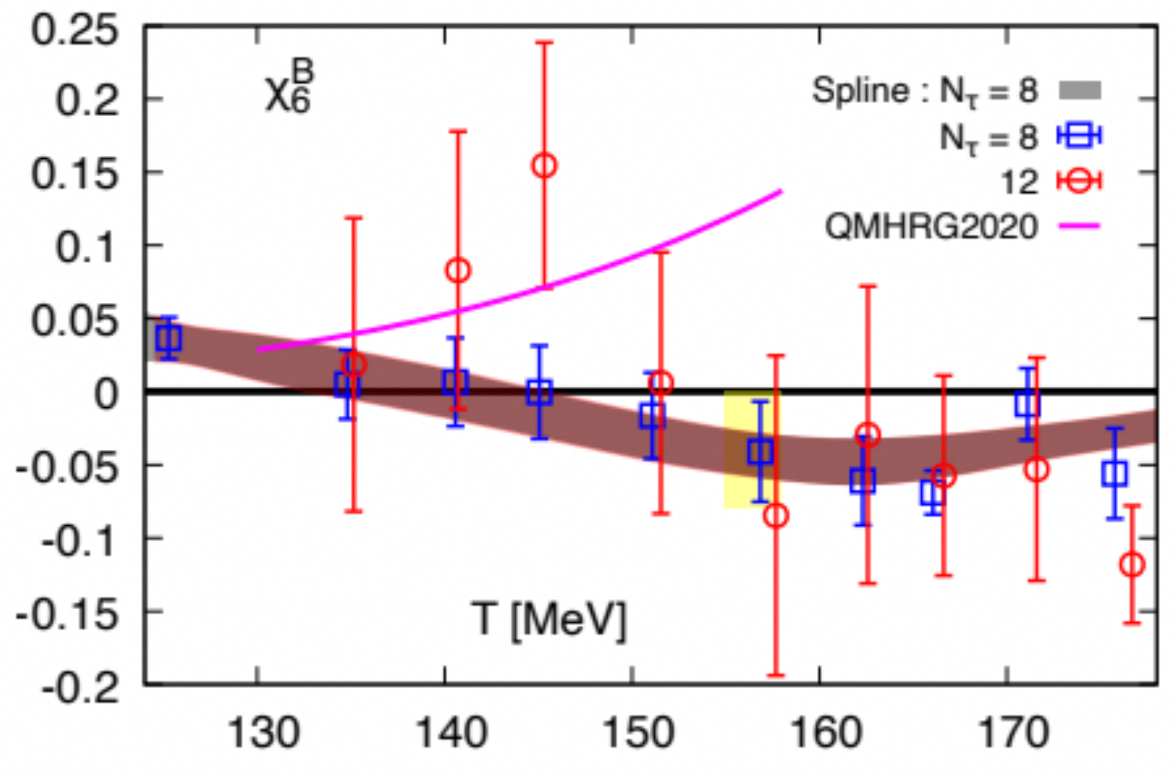
MP, Stephanov, Goswami, Basar (in preparation)

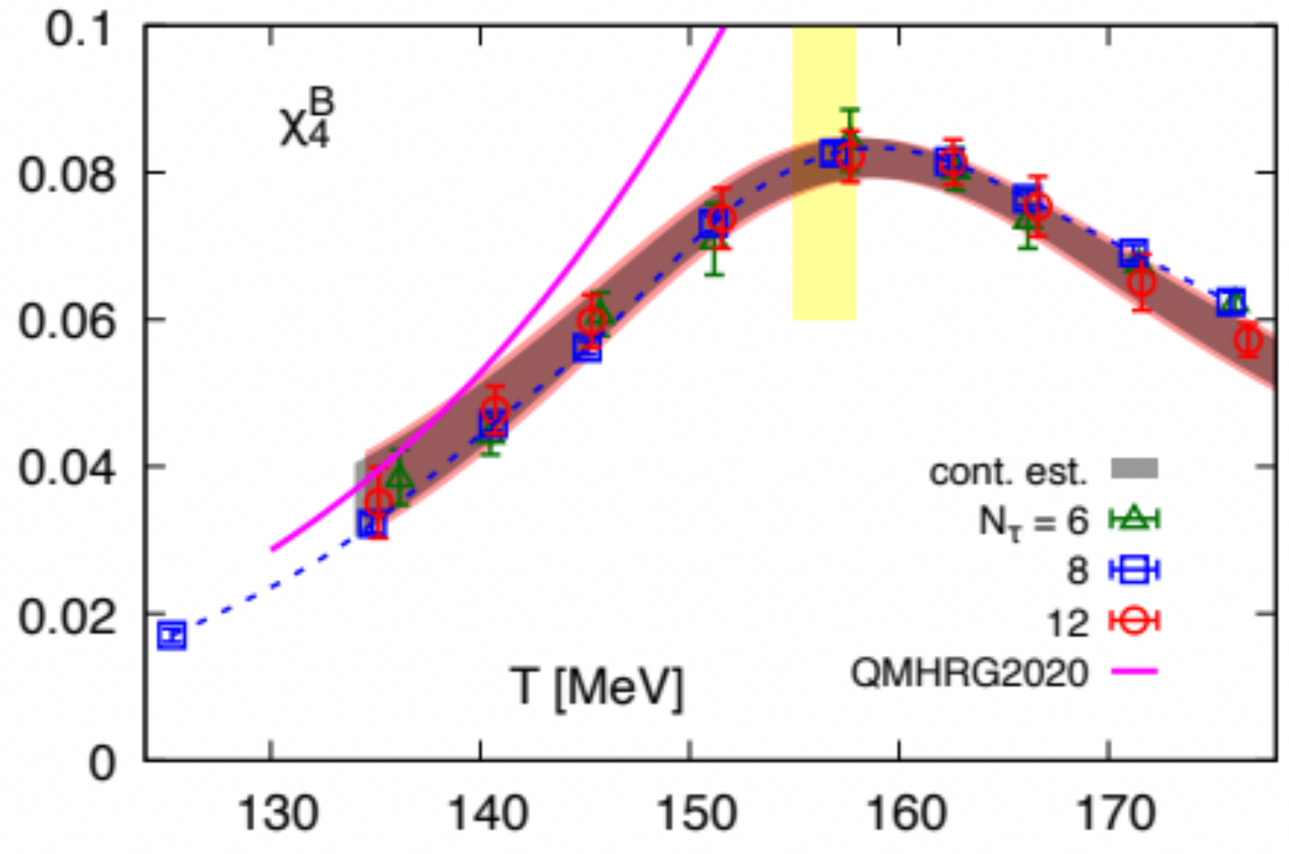


HotQCD, *Phys.Rev.D* 105 (2022) 7, 074511









Summary and Looking forward..

- **Theory landscape**
  - Lattice, FRG/DSE, and QCD-like models → constrained, almost converged on the region where a possible CP may exist

- **Key Goal**
  - Quantitative agreement between theories/models and experiment to put bounds on CP and constrain the EoS near it

- **Maximum entropy Freeze-out prescription for fluctuations -** Connects hydrodynamic fluctuations from theory to particle multiplicity distributions observed in experiments

- **New constraints from lattice**
  - Lee–Yang extrapolation ⇒ tight limits on **factorial cumulant structure** along freeze-out

- **Ongoing efforts**
  - Observables directly from lattice EoS
  - Bayesian analysis of BES data (with a theoretical model that includes an Ising-like critical point and fluctuations in equilibrium)

- **Dynamics of fluctuations out of equilibrium**
  - Out-of-equilibrium fluctuation evolution (Schäefer, Teaney, Chattopadhyay, Basar…)
  - Hydro+ simulations already gives qualitative and semi-quantitative guidance in simplistic models

- Future→ Unify dynamics + freeze-out (ME) → comparison to data

Thank you!