Model simulations for BES-II

Iurii Karpenko

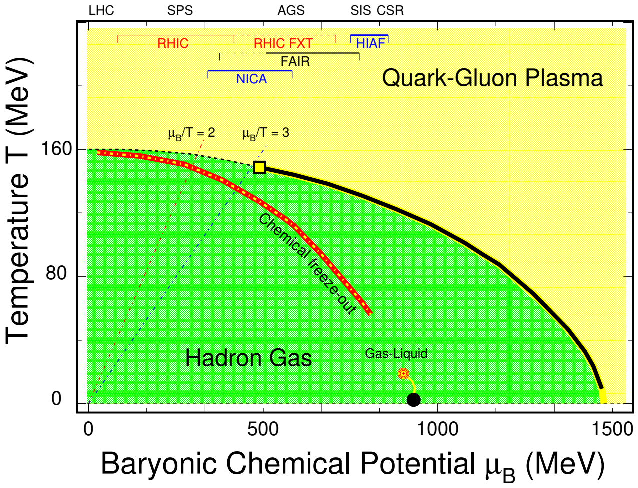
Czech Technical University in Prague

Simulations = dynamical simulations

= hydro or transport (hadronic cascades)

The goal

Heavy-ion collisions <==> hydro simulations



<==>

Possible signals of the **1st order phase transition** in:

- Shape of net proton distribution
- Flow
- HBT
- …

Model construction

Hydro-based models for RHIC BES region include: UrQMD+vHLLE (2015), SMASH+vHLLE (2022), EPOS4 2024), 3D Glauber+MUSIC (2018?), SMASH+MUSIC (a.k.a. JETSCAPE, 2025), CCAKE+SMASH (a.k.a. MUSES, 2025)

The recipe is mostly imported from higher energies:

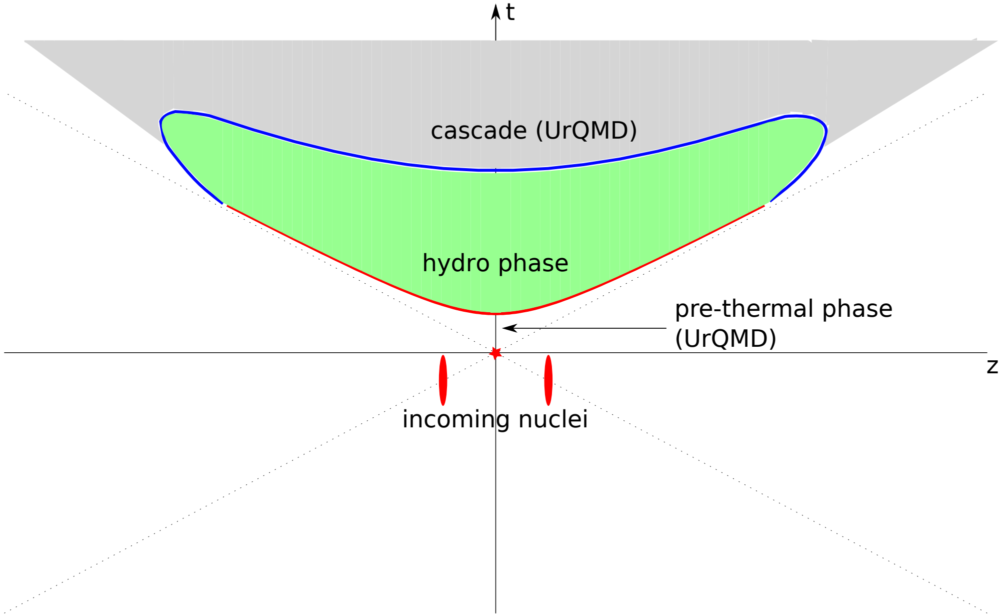
- 3D initial state: from transport or parametrized
  Start hydro later (nuclei are not thin pancakes)
- 3D hydro with finite viscosity (not a challenge)
  and conserved charges (not a challenge either)
- EoS at finite muB – an input to the modelling
- Same or slightly adapted Cooper-Frye
- final-state hadronic cascade

Most of basic observables are reproduced (see next slides)

Modelling is complex

- multi-step simulation, e.g. (SMASH) + (vHLLE) + (smash-hadron-sampler) + (SMASH)
- 3 to 4 different codes stacked one after another
- Need to take care of every interface

The first interface: initial state to hydro



The easy choice: start fluid-dynamic picture at $\tau=\tau_0$

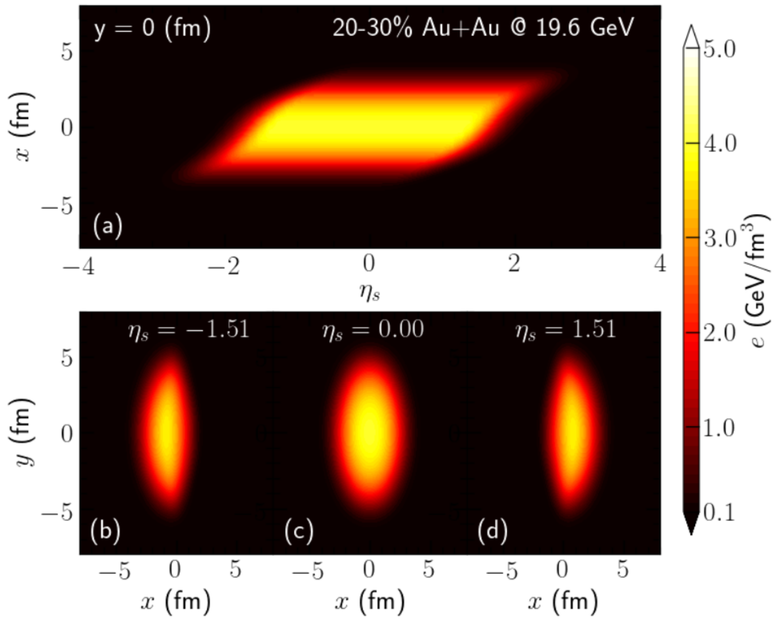
Motivated at high collision energies:

- Longitudinal boost invariance
- Physics changes as a function of τ

Parametrized 3D initial state at τ=τ$_0$

superMC code (?)

$$e(x, y, \eta_s; y_{\rm CM}) = \mathcal{N}_e(x, y) \times \exp\left[-\frac{(|\eta_s - y_{\rm CM}| - \eta_0)^2}{2\sigma_\eta^2}\theta(|\eta_s - y_{\rm CM}| - \eta_0)\right]$$



$\mathcal{N}_e(x, y)$ determined from the full energy count

Parameter tuning:

$\sqrt{s_{\rm NN}}$ (GeV)	τ_0 (fm/c)	η_0	σ_η	$\eta_{B,0}$	$\sigma_{B,\rm in}$	$\sigma_{B,\rm ou}$
AuAu & dAu @ 200	1.0	2.5	0.6	3.5	2.0	0.1
AuAu & dAu @ 62.4	1.0	2.25	0.3	2.7	1.9	0.2
AuAu & dAu @ 39	1.3	1.9	0.3	2.2	1.6	0.2
AuAu@27	1.4	1.6	0.3	1.8	1.5	0.2
AuAu & dAu @ 19.6	1.8	1.3	0.3	1.5	1.2	0.2
AuAu@14.5	2.2	1.15	0.3	1.4	1.15	0.2
AuAu@7.7	3.6	0.9	0.2	1.05	1.0	0.1
PbPb@17.3	1.8	1.25	0.3	1.6	1.2	0.2
PbPb@8.77	3.5	0.95	0.2	1.2	1.0	0.1

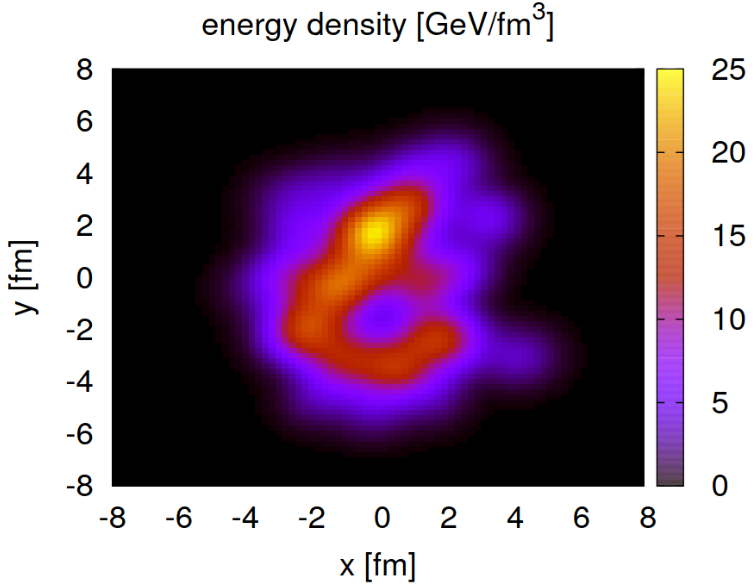
Shen, Alzhrani, Phys.Rev.C 102 (2020) 1, 014909

+ follow-up by Lipei du et al

3D initial state at $\tau=\tau_0$ from transport

...which hopefully has fewer parameters as initial state dynamics dictates the shape

(UrQMD, SMASH, EPOS, JAM)



Karpenko, Huovinen, Petersen, Bleicher, Phys.Rev.C 91 (2015) 6, 064901

- Patch a transport code to stop evolution or disable interactions after after $\tau=\tau_0$
- Collect hadrons at $\tau=\tau_0$
- Assume that hadrons melt into fluid:

$$\Delta P^{\alpha}_{ijk} = P^{\alpha} \cdot C \cdot \exp\left(-(\Delta x_i^2 + \Delta y_j^2)/R_\perp^2 - \Delta\eta_k^2\gamma_\eta^2\tau_0^2/R_\eta^2\right)$$
$$\Delta N^{0}_{ijk} = N^{0} \cdot C \cdot \exp\left(-(\Delta x_i^2 + \Delta y_j^2)/R_\perp^2 - \Delta\eta_k^2\gamma_\eta^2\tau_0^2/R_\eta^2\right)$$

$$E/\Delta V = (\epsilon + p)(u^0)^2 - p, \quad P^i/\Delta V = (\epsilon + p)u^0 u^i$$

- 3D initial state with finite n_B, n_Q, n_S

Does the medium look like a fluid when it is fluidized?

In case of event averaged initial state, in some cases the answer is yes.

Gabriele Inghirami, Hannah Elfner, Eur. Phys. J. C 82, 796 (2022)

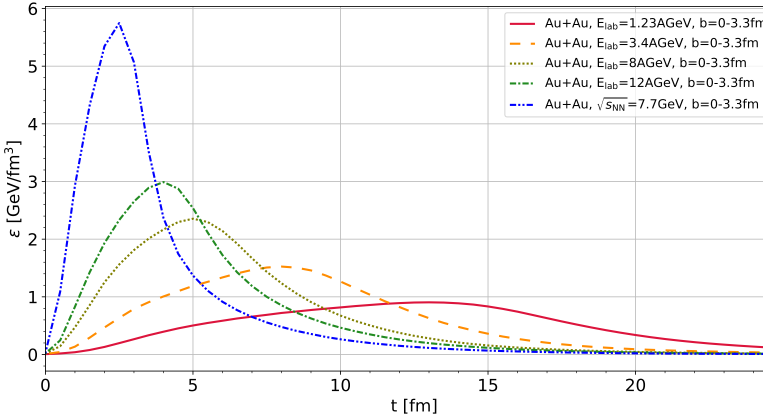
From $E_{\text{lab}} = 1.23$ GeV up to $\sqrt{s} = 7.7$ GeV

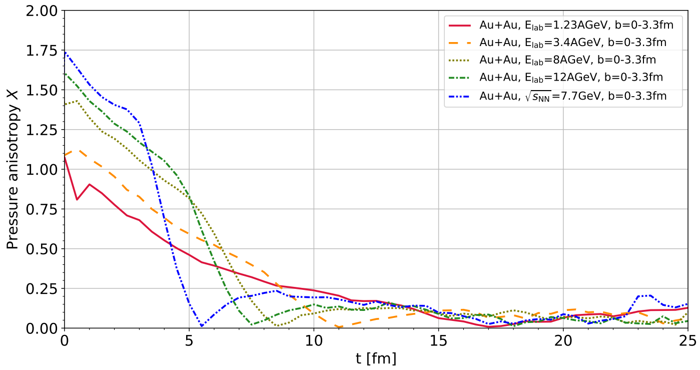
Examining the pressure anisotropy:

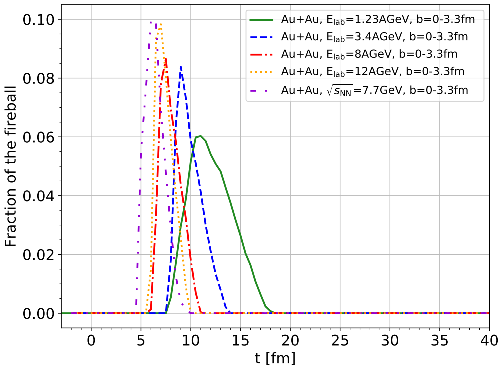
$$T^{\mu\nu} = \sum_i \frac{p_i^\mu p_i^\nu}{p_i^0} K(\mathbf{r} - \mathbf{r}_i, \mathbf{p_i})$$

$$X \equiv \frac{|\langle T_L^{11}\rangle - \langle T_L^{22}\rangle| + |\langle T_L^{22}\rangle - \langle T_L^{33}\rangle| + |\langle T_L^{33}\rangle - \langle T_L^{11}\rangle|}{\langle T_L^{11}\rangle + \langle T_L^{22}\rangle + \langle T_L^{33}\rangle},$$

$$Y \equiv \frac{3(|\langle T_L^{12}\rangle| + |\langle T_L^{23}\rangle| + |\langle T_L^{13}\rangle|)}{\langle T_L^{11}\rangle + \langle T_L^{22}\rangle + \langle T_L^{33}\rangle}$$

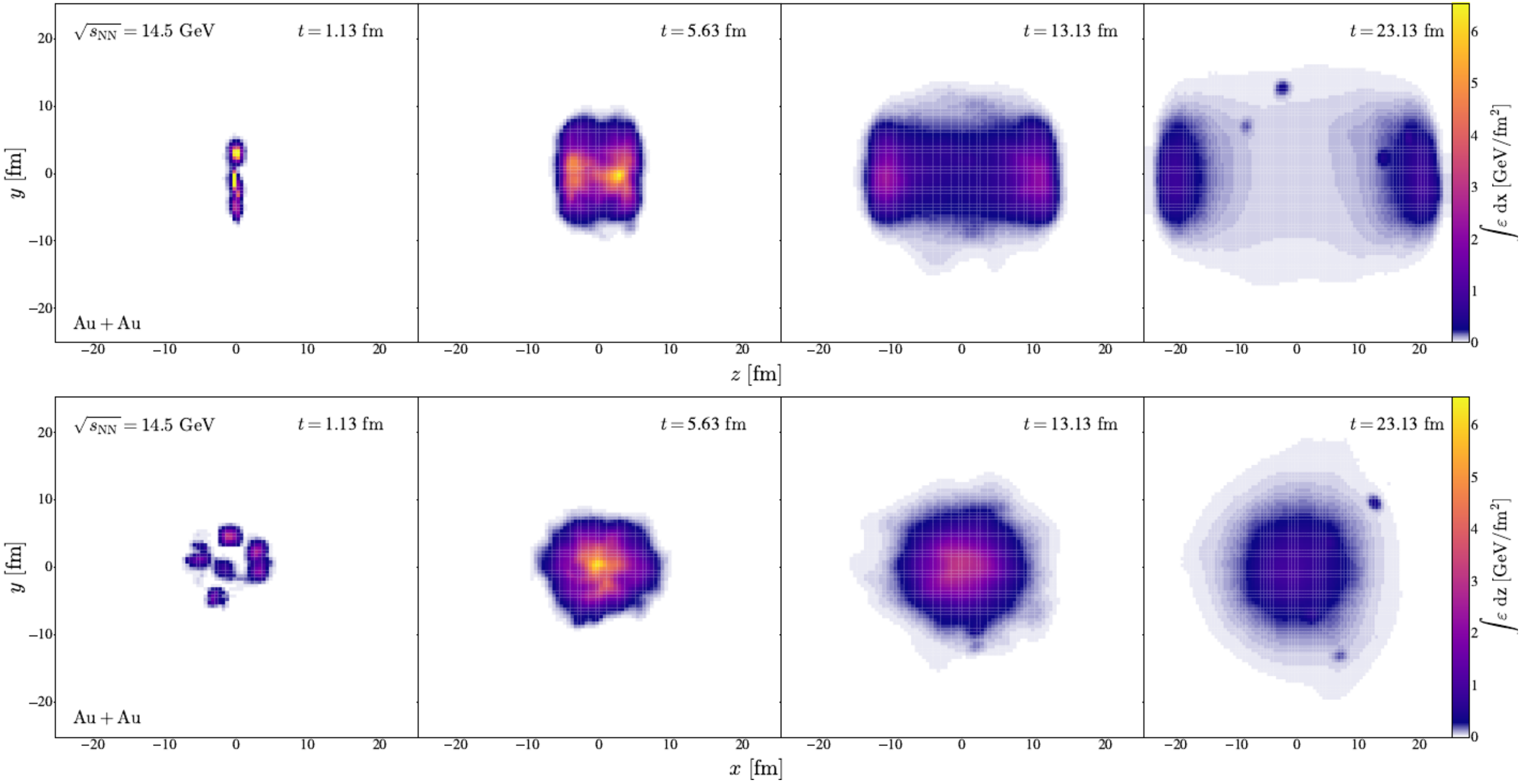






A more sophisticated switching idea: dynamical fluidization

The idea: once the energy density in LRF is high enough *locally*, fluidize this part.



Góes-Hirayama, Egger, Paulínyová, Karpenko, Elfner, 2507.19389

`JAM` + dynamical fluidisation + hydro + `JAM`

Y. Akamatsu, M. Asakawa, T. Hirano, M. Kitazawa, K. Morita, K. Murase, Y. Nara, C. Nonaka, A. Ohnishi, Phys. Rev. C 98, 024909 (2018)

JAM IS: HIJING string excitation + PYTHIA6 fragmentation + rescatterings of produced hadrons.

Hadrons are converted to fluid if the local energy density $e > e_f = 0.5$ GeV/fm^3.

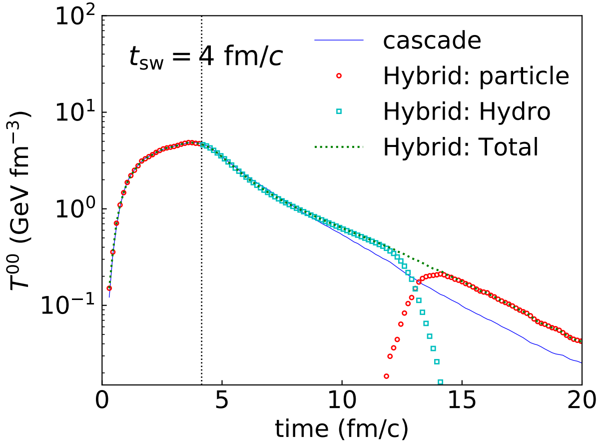
$$\partial_\mu T_f^{\mu\nu} = J^\nu, \quad \partial_\mu N_f^\mu = \rho$$

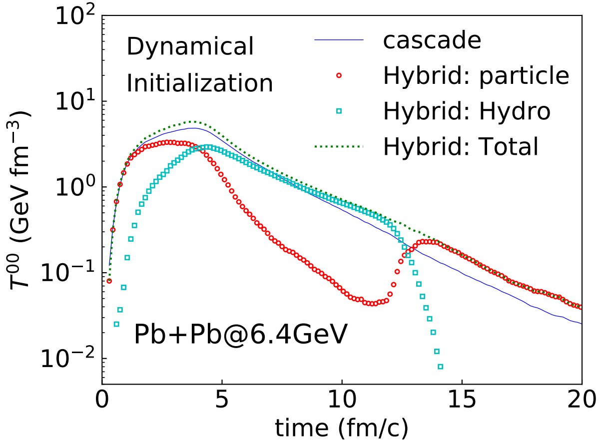
$$J^\mu(r) = \frac{1}{\Delta t}\sum_i p_i^\mu G(r - r_i(t))$$

$$\rho(r) = \frac{1}{\Delta t}\sum_i B_i G(r - r_i(t))$$

$G(r)$ is a Gaussian smearing profile.

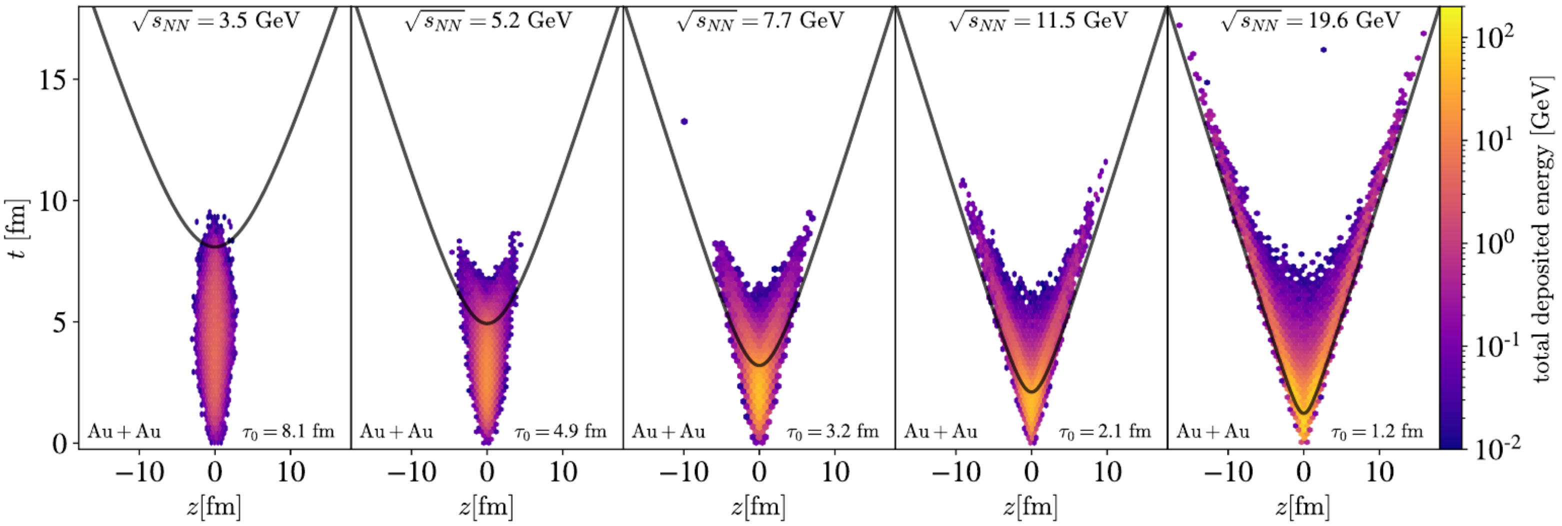
EoS: EoS Q, a.k.a. BM+HRG EoS





As a result, the simulation does not wait for the two nuclei to completely pass through each other, which is a long time at low √s

Fluid approximation still applies for the most dense part of the evolution



Góes-Hirayama, Egger, Paulínyová, Karpenko, Elfner, 2507.19389

Skipping the first switching/interface altogether: *multi-fluid approach*

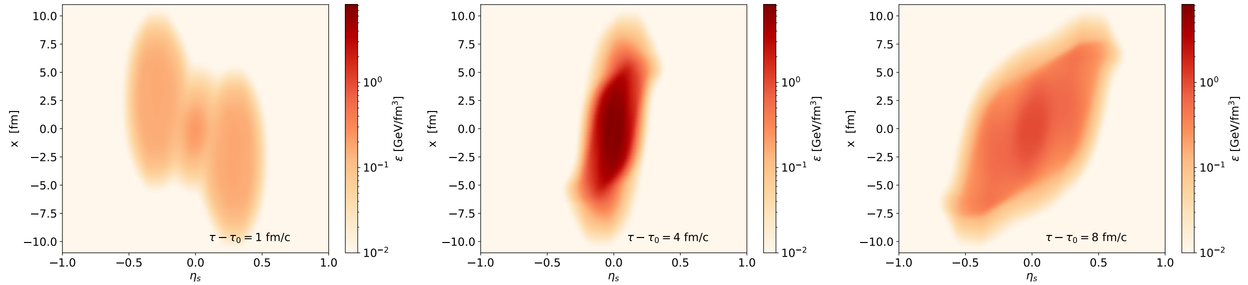
- Incoming nuclei = two fluids labelled as projectile and target
- Interaction of the fluids (slowing down) via "friction" terms
- Friction transports energy and momentum into the third fluid labelled as fireball
- It is a minimal setup to reproduce baryon stopping at low $\sqrt{s}$ and baryon transparency at high $\sqrt{s}$.

$$\partial_\mu T_{\rm p}^{\mu\nu}(x) = -F_{\rm p}^{\nu}(x) + F_{\rm fp}^{\nu}(x),$$
$$\partial_\mu T_{\rm t}^{\mu\nu}(x) = -F_{\rm t}^{\nu}(x) + F_{\rm ft}^{\nu}(x),$$
$$\partial_\mu T_{\rm f}^{\mu\nu}(x) = F_{\rm p}^{\nu}(x) + F_{\rm t}^{\nu}(x) - F_{\rm fp}^{\nu}(x) - F_{\rm ft}^{\nu}(x),$$

Cimerman, IK, Tomasik, Huovinen, [Phys.Rev.C 107 (2023) 4, 4]

Werthmann, IK, Huovinen, Phys.Rev.C 113 (2026), 034908

Reichert, Spieles, Bleicher



Snapshots of multi-fluid evolution in x-η_s plane, Au-Au collision at $\sqrt{s_{\rm NN}} = 7.7$ GeV

(Hadronic) Transport

✅ natively 3D

✅ EoS can be emulated with potentials

✅ Exact energy-momentum, charge conservation

✅ no interfaces (to/from hydro)

Caveats:

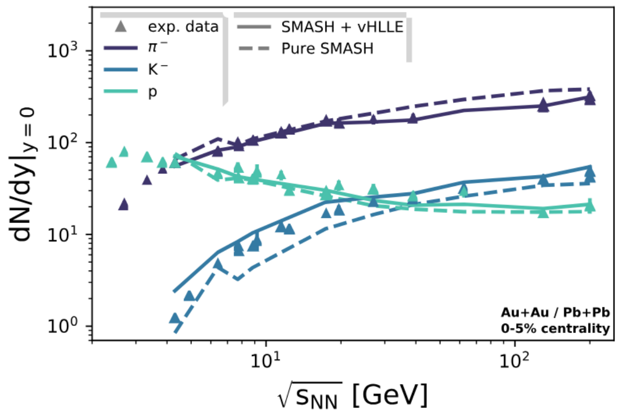
⚠️ No change in the degrees of freedom (UrQMD, SMASH, HSD, JAM)

Unless: partonic phase + coalescence to hadrons (AMPT, PHSD, PHQMD(?))

⚠️ No good agreement with the data above sqrt(s)>10 GeV (except for JAM!)

What does the modelling tell us

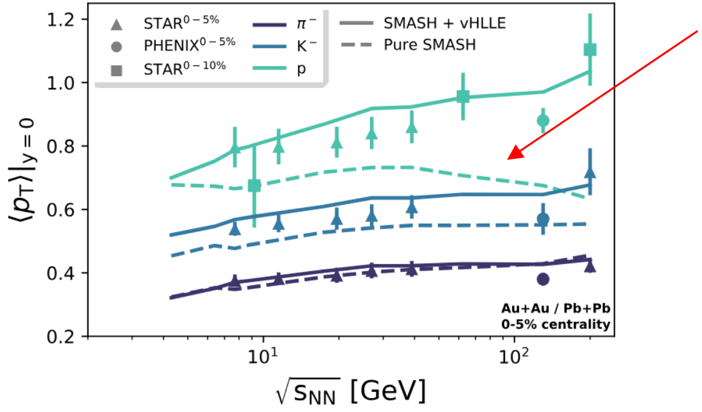
Aggregate conclusions from the hydro modelling

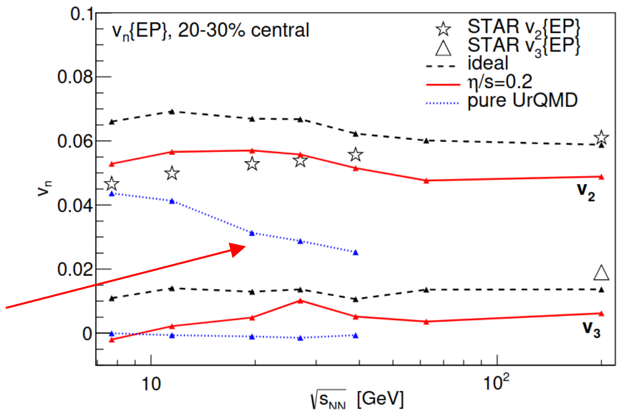


❗ Fluid phase improves agreement with the data; pure cascade fails to describe flow and mean pT above √s~7 GeV

Schäfer, Karpenko, Wu, Hammelmann, Elfner, [Eur.Phys.J.A 58 (2022) 11, 230]

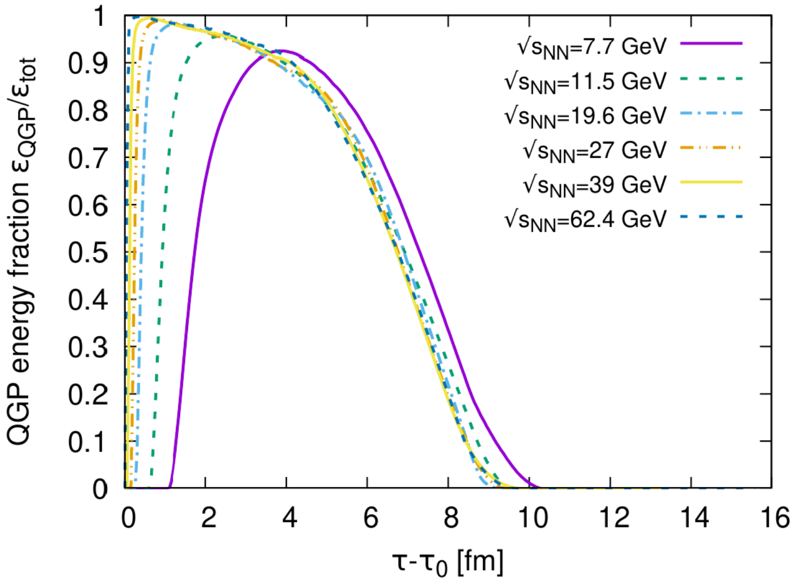
Karpenko, Huovinen, Petersen, Bleicher, [Phys.Rev.C 91 (2015) 6, 064901]



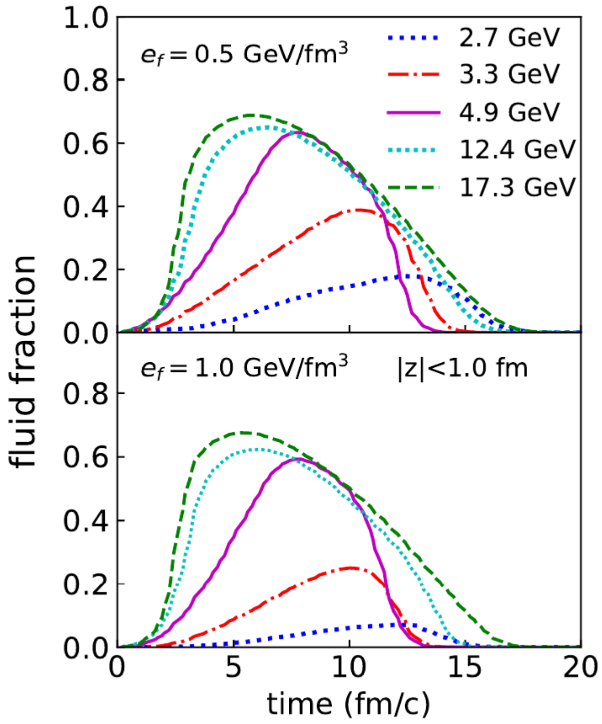


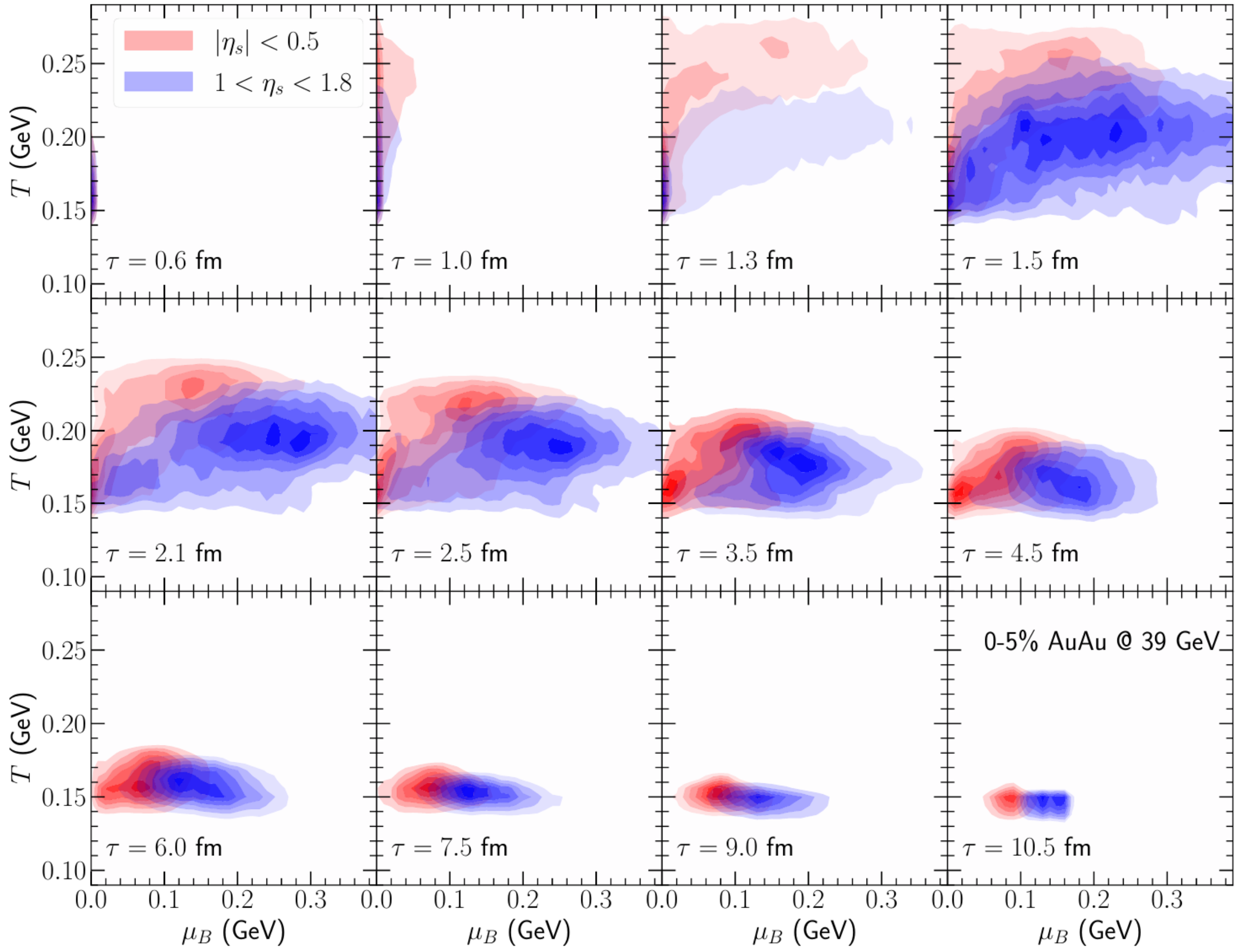
❗ High densities are reached at low collision energies

MUFFIN 1.0: Cimerman, Karpenko, Tomasik, Huovinen, [Phys.Rev.C 107 (2023) 4]()



JAM+hydro: Yasushi Nara et al





❗ The fireball is quite inhomogeneous

A √s point does not map to a (T, μ_B) point.

This plot is taken from Chun Shen, [2108.04987]

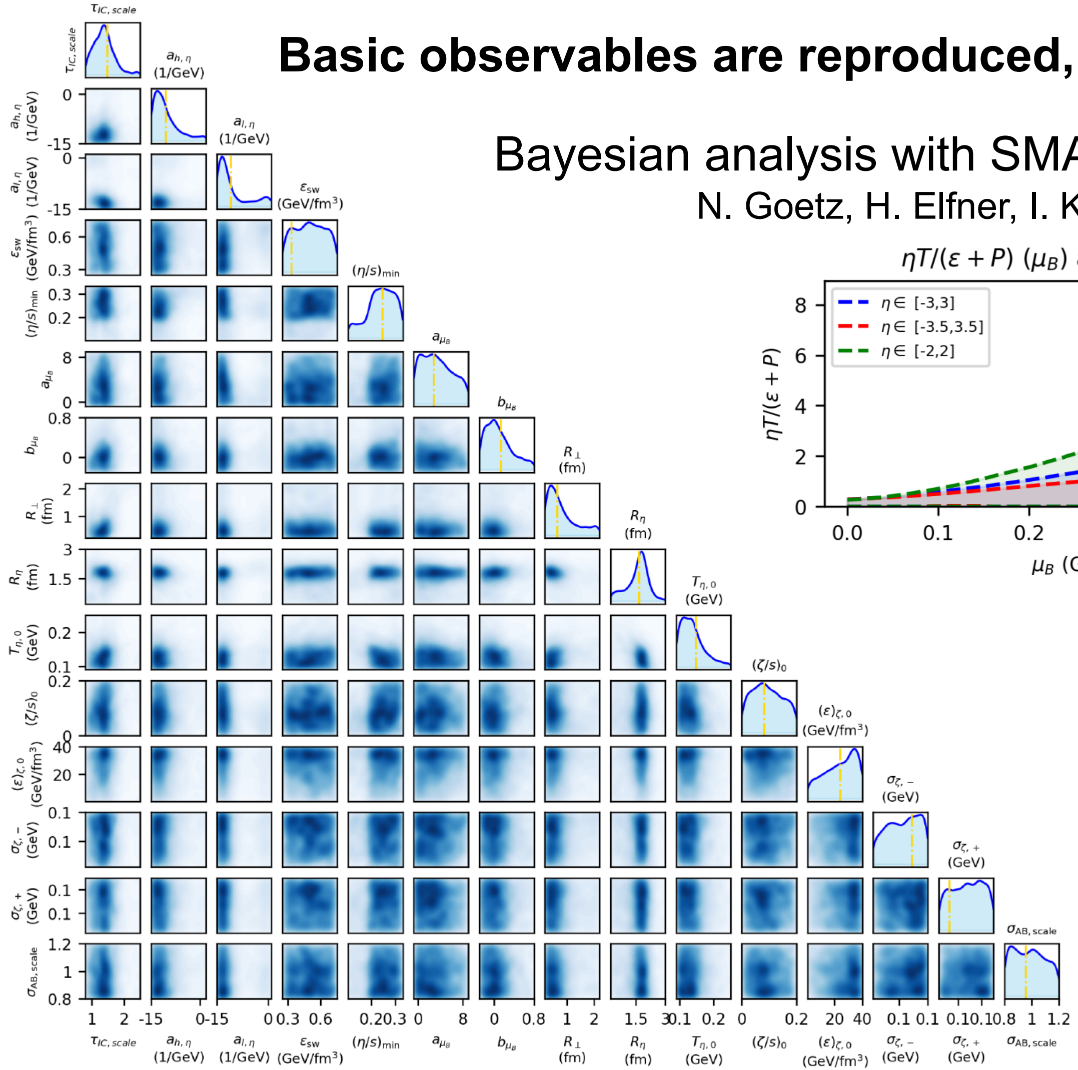
Other hybrids show a similar picture

For particle number ratios, the whole system is integrated out, which results in one point at the phase diagram

Evolution movies: https://smash-transport.github.io/movies-hybrid.html

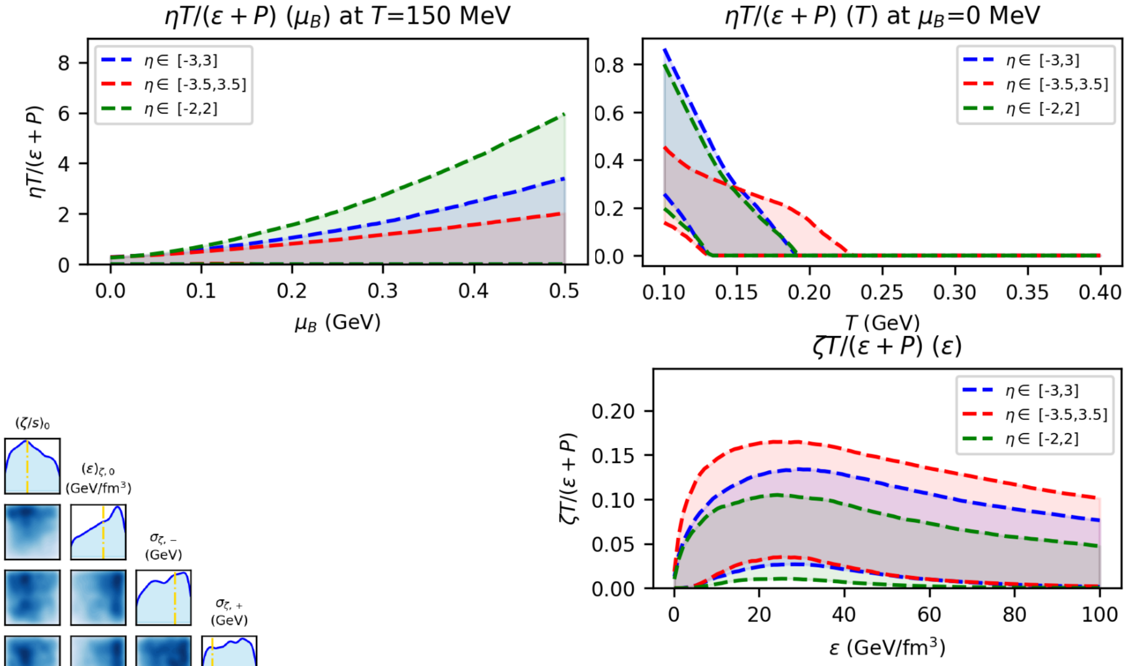
Bayesian analyses

Basic observables are reproduced, models constrained via BA



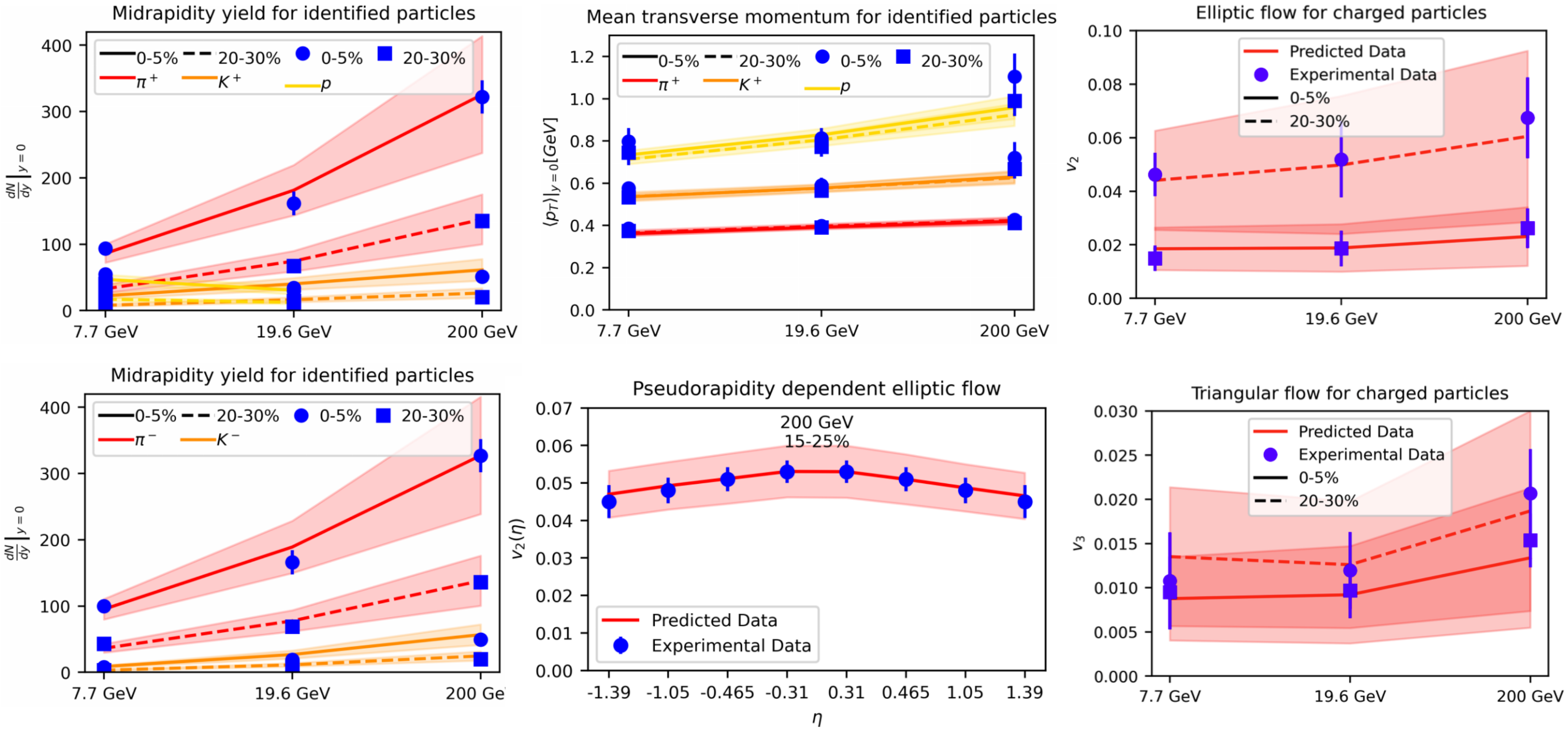
Bayesian analysis with SMASH+vHLLE, sqrt(s)=7.7…200 GeV

N. Goetz, H. Elfner, I. Karpenko, Phys.Rev.C 112 (2025) 1, 014910



Parameter space is large,
Bayesian analysis is challenging,
exp. data is not always high quality

What experimental data is reproduced in (being used for) the Bayesian analysis:

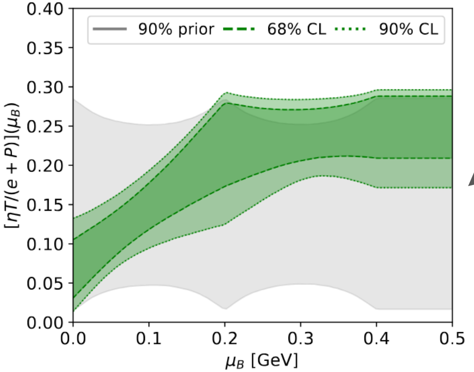


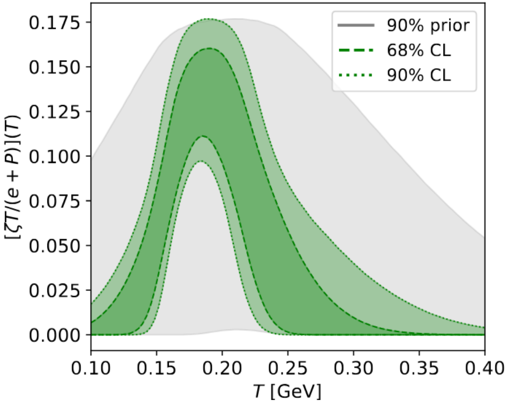
N. Goetz, H. Elfner, I. Karpenko, [Phys.Rev.C 112 (2025) 1, 014910]

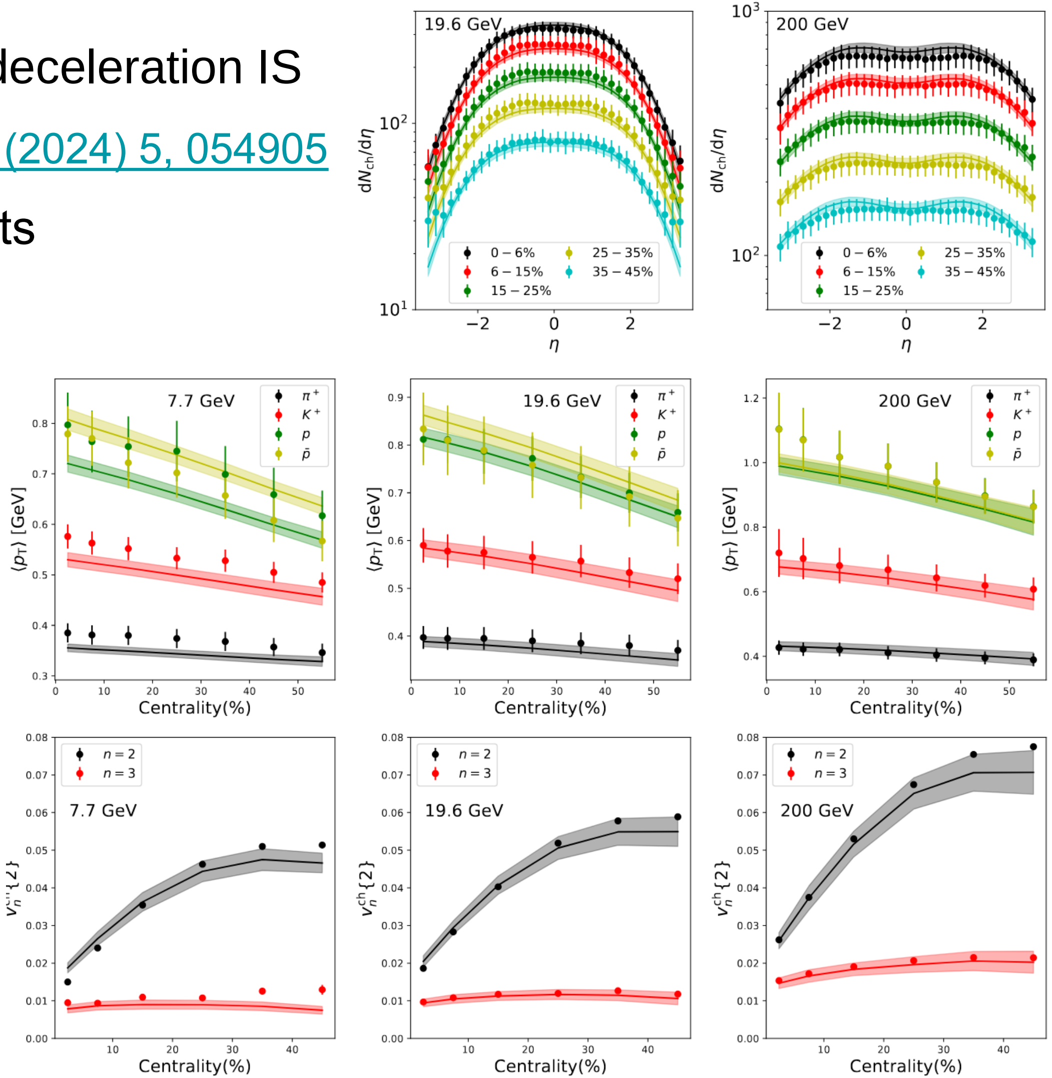
Another Bayesian analysis: string deceleration IS

Jahan, Roch, Shen, Phys.Rev.C 110 (2024) 5, 054905

Constraints







Caveats for the BAs above:

- no agreement so far in the extracted viscosities

- tied to particular initial state(s)

EoS sensitivity / constraints?

There is no simple answer for that.

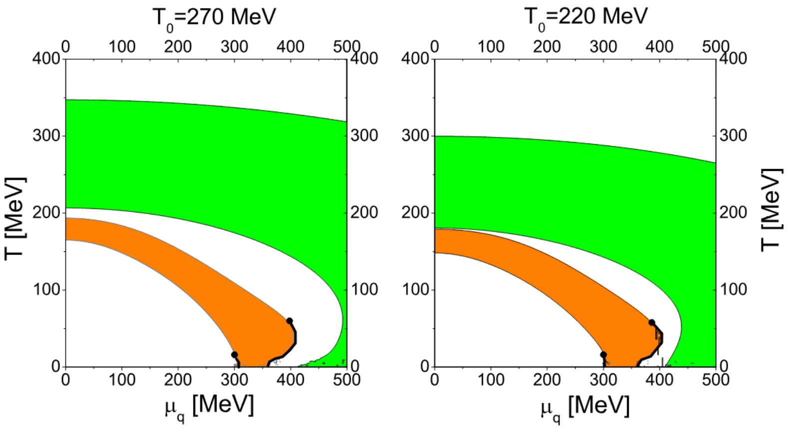
- EoS sensitivity in the models can be obscured by other sensitivities (just like at high energies).
- E.g. EoS constraining via directed flow does not seem to work.
- State-of-the-art EoS with CP location as a free parameter are applied in *some* fluid-dynamic models (MUSIC code, NEOS).
- But there is no BA constraining the EoS at non-zero baryon density yet (?)

New EoS at finite n_B (optionally n_Q, n_S) are coming

EoS can include first-order PT but only Maxwell construction

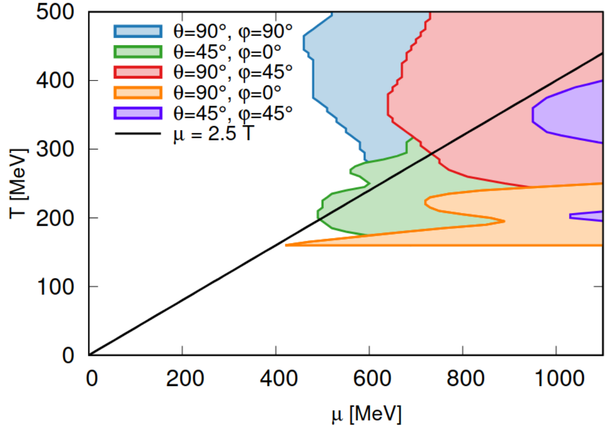
EoS has to be known/tabulated down to very low T and high mub → limits the candidate EoS to use.

Older choices: Chiral model EoS



Phase diagram of the EoS

State-of-the-art: **MUSES**



White region is where the used expansion works

Steinheimer, Schramm, J.Phys.G 38 (2011) 035001

- Energy and baryon density (2D)
- Compatible with latticeQCD at μ_B=0
- Reproduces ground state of nucleus (?!)
- Crossover transition at all μ_B

Ahmed Abuali et al (MUSES), Phys.Rev.D 112 (2025) 5, 054502

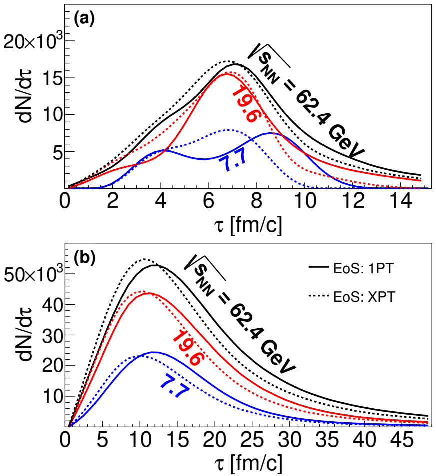
- 4D (T, mub, muq, mus)
- latticeQCD at μ_B=0
- CP location parameterized

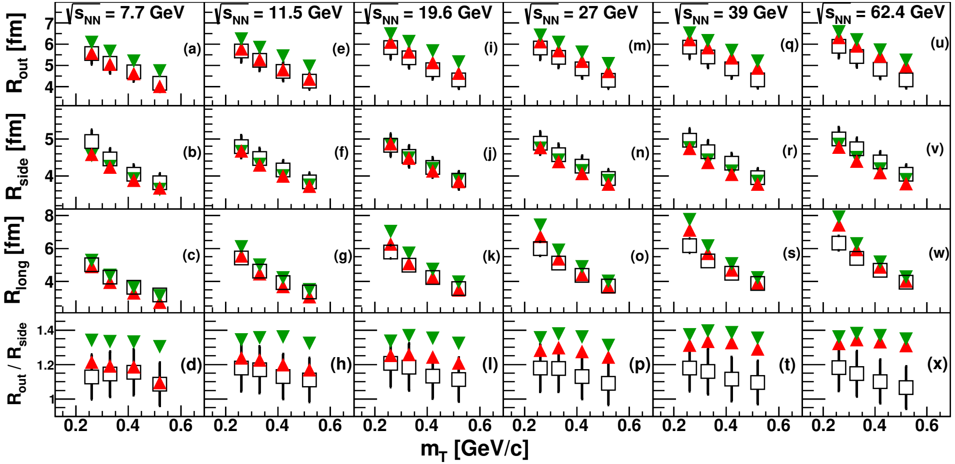
EoS/PT sensitivity via HBT/femtoscopy

Initial idea: enhancement of $R^2_{\text{out}} - R^2_{\text{side}}$ would signal a passage through the mixed phase.

"Reality check":
`UrQMD` (initial state) + fluidisation at fixed τ_0 + `vHLLE` (hydro) + `UrQMD`

Batyuk, IK, Lednicky, Malinina, Mikhaylov, Rogachevsky, Wielanek, Phys. Rev. C 96, 024911 (2017)





1PT = 1st order PT, XPT = crossover; crossover EoS is red, 1PT EoS is green

There is weak EoS sensitivity, crossover EoS is preferred.

Directed flow indicates a cross-over deconfinement transition in relativistic nuclear collisions

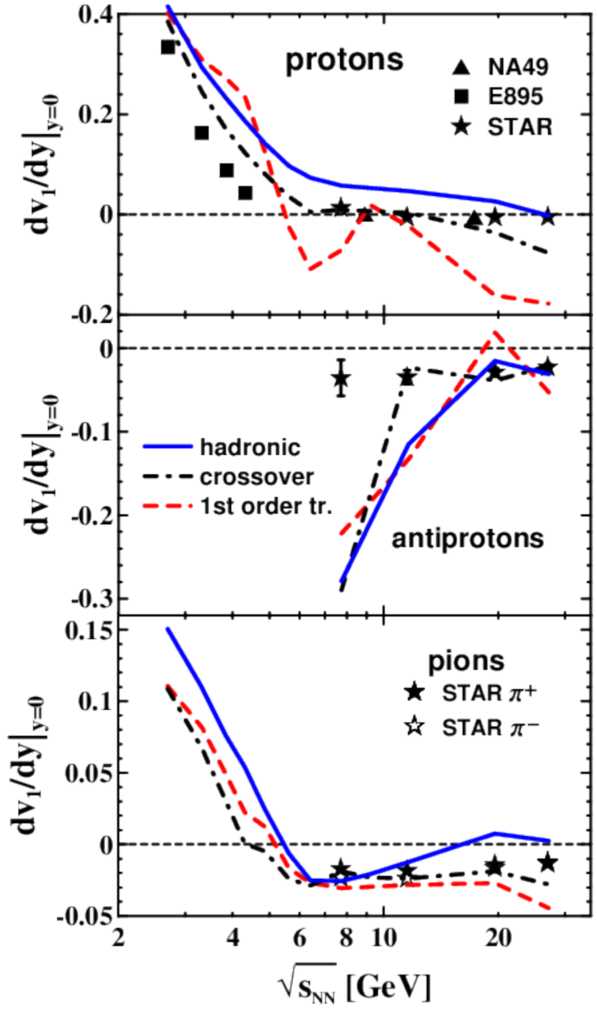
Yu. B. Ivanov[1,2,*] and A. A. Soldatov[2,†]

Show more

Phys. Rev. C **91**, 024915 – **Published 26 February, 2015**

DOI: https://doi.org/10.1103/PhysRevC.91.024915

Crossover-type EoS preferred



Summary

- There is a zoo of models for RHIC BES energies, covering a region of $\sqrt{s}$=5-200 GeV.
- There is WIP to extend the picture further down to a few GeV, with either dynamical fluidisation or multi-fluid dynamics.
- Most of models cover most of basic observables: rapidity distributions, net protons, radial flow, elliptic flow, (triangular flow). HBT seems to be not much off from the data.
- There has been a few Bayesian analyses that constrain
- The studies above are done mostly with crossover-type EoS. There has been no constraints on the EoS yet.
- Hadronic cascades allow to emulate EoS softening (~1PT EoS) but fundamentally their applicability stops (gradually) at around sqrt(s)=10 GeV.