

# From the Sun to the Cosmos: Solving the Neutrino Flavor Equations

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*Looking for nu physics on Earth and in the Cosmos  
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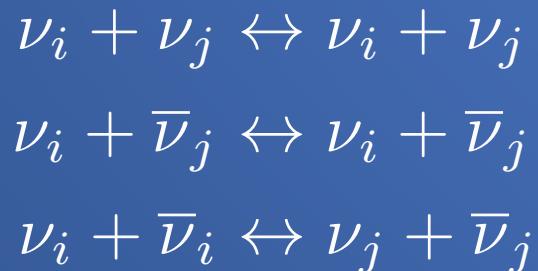


# Weak Decoupling: Overview

1. Initially: neutrinos at the same temperature as electrons and positrons
2. Electrons and positrons annihilate to produce photon pairs, slightly raising temperature of plasma
3. Two processes create heat flow between neutrinos and plasma



4. Three processes redistribute energy within neutrino seas



5. End result: neutrinos cooler than photons

# QKEs in the Early Universe

$$\epsilon = E_\nu/T_{\text{cm}} \simeq E_\nu a$$

Change array dimensions (Majorana or Dirac):

$$\{f_i(\epsilon)\}, \{\bar{f}_i(\epsilon)\} \rightarrow f_{ij}(\epsilon), \bar{f}_{ij}(\epsilon)$$

2 Generalized  $3 \times 3$   
density matrices ( $\phi=0$ )

Equations of motion for neutrinos:

$H$ : Hamiltonian-like  
potential (coherent)

$$\frac{df}{dt} = -i[H, f] + \hat{C}(f, \bar{f})$$

$\hat{C}$ : Collision term from  
Blaschke & Cirigliano (2016)

Nonlinear coupled ODEs

# Coherent Term in the Early Universe (Matrices)

$$H = H_V + H_A + H_S$$

$$H_V = \frac{1}{2\epsilon T_{\text{cm}}} U M^2 U^\dagger$$

Vacuum Oscillations

$$H_A = \sqrt{2} G_F (L + \tilde{L})$$

Asymmetric Term  
(proportional to number difference)

$$H_S = -\frac{8\sqrt{2}G_F\epsilon T_{\text{cm}}}{3m_W^2} (E + \cos^2 \theta_W \tilde{E})$$

Symmetric term  
(proportional to energy density)

# Collisions

*Positron-Electron Annihilation*

$$\nu(k)\bar{\nu}(q_3) \rightarrow e^+(q_2)e^-(q_1)$$

*Loss Potential (Blaschke & Cirigliano 2016)*

$$\Pi_R^+(k) = \frac{-32G_F^2}{|\vec{k}|} \int d\vec{q}_1 d\vec{q}_2 d\vec{q}_3 (2\pi)^4 \sum_{I=L,R} \left[ (1 - f_{e,1})(1 - \bar{f}_{e,2}) \right. \\ \left. \times Y_I \bar{f}_3 \left( 2Y_I \mathcal{M}_I^R(q_1, -q_2, -q_3, k) - Y_{J \neq I} \mathcal{M}_m(q_1, -q_2, -q_3, k) \right) \right]$$

*Amplitudes*

$$\mathcal{M}_I^R(q_1, q_2, q_3, k) = \left( \delta_I^L(q_3 q_1)(k q_2) + \delta_I^R(q_3 q_2)(k q_1) \right) \delta^{(4)}(k - q_3 - q_1 + q_2)$$
$$\mathcal{M}_m(q_1, q_2, q_3, k) = m_e^2(k q_3) \delta^{(4)}(k - q_3 - q_1 + q_2)$$

## Collisions (Cont.)

*Matrices in Weak eigenbasis*

$$Y_L = \begin{bmatrix} \frac{1}{2} + \sin^2 \theta_W & 0 & 0 \\ 0 & -\frac{1}{2} + \sin^2 \theta_W & 0 \\ 0 & 0 & -\frac{1}{2} + \sin^2 \theta_W \end{bmatrix}$$

$$Y_R = \sin^2 \theta_W \times \mathbb{1}$$

*Collision Term*

$$C[f(\epsilon)] = \frac{1}{2} \{\Pi_R^+, f(\epsilon)\} - \frac{1}{2} \{\Pi_R^-, 1 - f(\epsilon)\}$$

# Let's stop and think here....

1. Have all the terms, integrate the network of coupled ODEs with explicit methods

$$\frac{df}{dt} = -i[H, f] + \hat{C}(f, \bar{f})$$

2. Is there a better way? Think about the physics.

# The Solar Neutrino Problem:

## Haxton, Annu. Rev. Astron. Astrophys. 1995

1. Schrodinger Picture, weak eigenbasis:

$$|\nu(x)\rangle = a_e(x)|\nu_e\rangle + a_\mu(x)|\nu_\mu\rangle,$$

2. Equation of Motion:

$$i \frac{d}{dx} \begin{pmatrix} a_e \\ a_\mu \end{pmatrix} = \frac{1}{4E} \times \quad (20)$$

$$\begin{pmatrix} 2E\sqrt{2}G_F\rho(x) - \delta m^2 \cos 2\theta_v & \delta m^2 \sin 2\theta_v \\ \delta m^2 \sin 2\theta_v & -2E\sqrt{2}G_F\rho(x) + \delta m^2 \cos 2\theta_v \end{pmatrix} \begin{pmatrix} a_e \\ a_\mu \end{pmatrix},$$

# Change the Basis

## 1. Instantaneous Mass Basis:

$$\begin{aligned} |\nu_L(x)\rangle &= \cos\theta(x)|\nu_e\rangle - \sin\theta(x)|\nu_\mu\rangle, \\ |\nu_H(x)\rangle &= \sin\theta(x)|\nu_e\rangle + \cos\theta(x)|\nu_\mu\rangle. \end{aligned}$$

$$\begin{aligned} \sin 2\theta(x) &= \frac{\sin 2\theta_v}{\sqrt{X^2(x) + \sin^2 2\theta_v}}, \\ \cos 2\theta(x) &= \frac{-X(x)}{\sqrt{X^2(x) + \sin^2 2\theta_v}}, \end{aligned}$$

## 2. Rewrite the wavefunction:

$$|\nu(x)\rangle = a_H(x)|\nu_H(x)\rangle + a_L(x)|\nu_L(x)\rangle,$$

# New Equations of Motion

## 3. Transform Equations of Motion:

$$i \frac{d}{dx} \begin{pmatrix} a_H \\ a_L \end{pmatrix} = \begin{pmatrix} \lambda(x) & i\alpha(x) \\ -i\alpha(x) & -\lambda(x) \end{pmatrix} \begin{pmatrix} a_H \\ a_L \end{pmatrix}$$

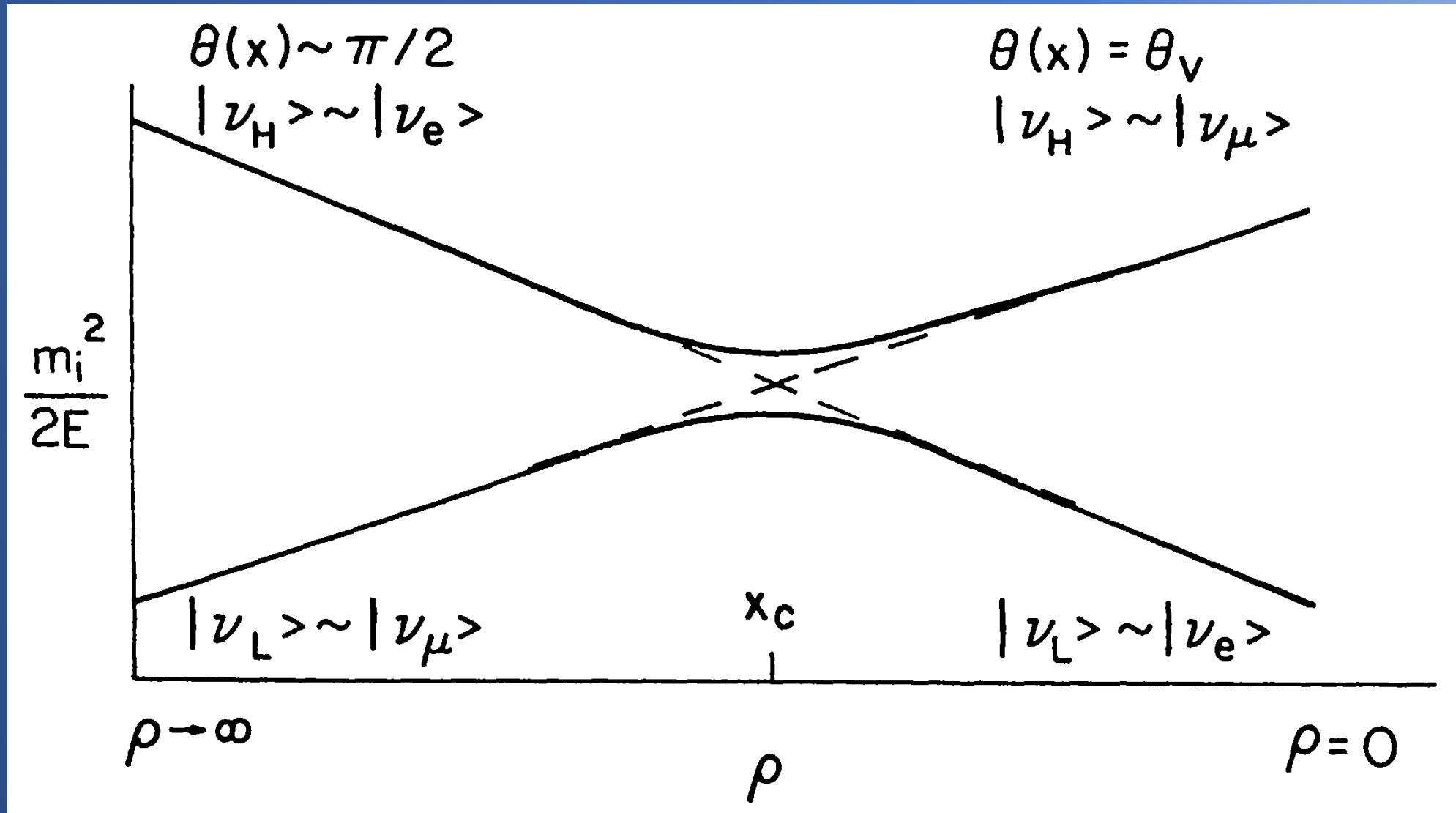
$$2\lambda(x) = \frac{\delta m^2}{2E} \sqrt{X^2(x) + \sin^2 2\theta_v}$$

$$\alpha(x) = \left( \frac{E}{\delta m^2} \right) \frac{\sqrt{2} G_F \frac{d}{dx} \rho(x) \sin 2\theta_v}{X^2(x) + \sin^2 2\theta_v}.$$

## 4. Adiabatic Limit

$$\gamma(x) = \left| \frac{\lambda(x)}{\alpha(x)} \right| \gg 1$$

# Level Crossing Diagram



# Back to the Early Universe

1. Regroup the QKEs:

$$\frac{df(\epsilon)}{dt} = -i[\mathcal{H}_V, f(\epsilon)] + \mathcal{C}_S[\epsilon; f, \bar{f}]$$

2. Epsilon and time dependent transformation:

$$\tilde{f}(\epsilon) = e^{i\Omega_V(\epsilon,t)} f(\epsilon) e^{-i\Omega_V(\epsilon,t)}$$

# Corotating Frame Transformation

1. Relationship between transformation matrix and the Hamiltonian matrix:

$$\frac{d\Omega}{dt} = \mathcal{H}_V$$

$$\mathcal{H}_V = \frac{1}{2\epsilon T_{\text{cm}}} U M^2 U^\dagger$$
$$T_{\text{cm}} \propto t^{-1/2}$$

2. New QKE:

$$\frac{d\tilde{f}}{dt} = e^{i\Omega_V(\epsilon, t)} \mathcal{C}_S[\epsilon; f, \bar{f}] e^{-i\Omega_V(\epsilon, t)}$$

# 3-flavor Preliminary Calculation

## 1. Only Vacuum Oscillations

$$H = H_V$$

## 2. PDG values for mixing

$$\delta m_{21}^2 = 7.53 \times 10^{-5} \text{ eV}^2$$

$$\delta m_{31}^2 = 2.59 \times 10^{-3} \text{ eV}^2$$

$$\theta_{12} = 33.6^\circ$$

$$\theta_{13} = 8.37^\circ$$

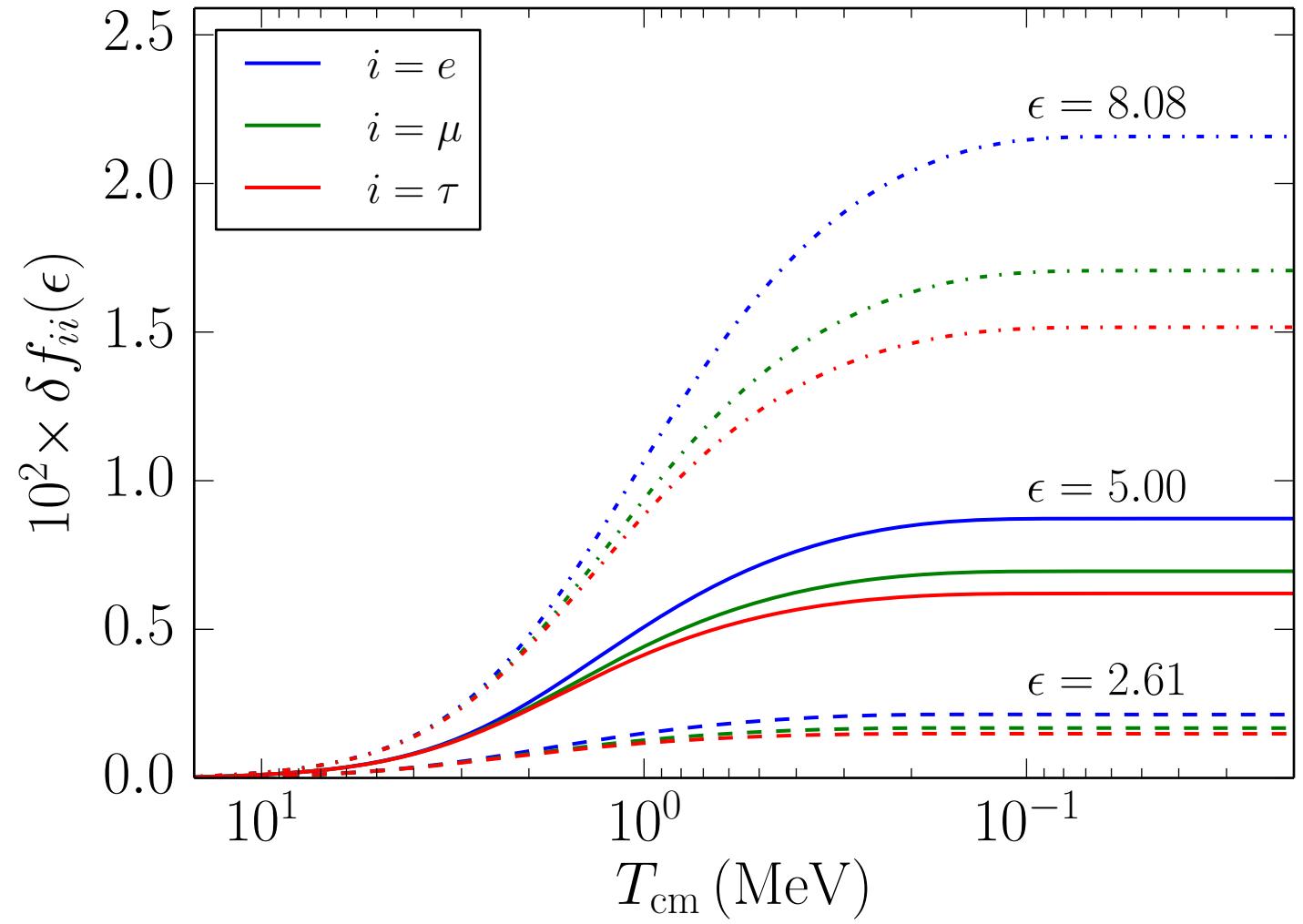
$$\theta_{23} = 40.2^\circ$$

Normal Hierarchy

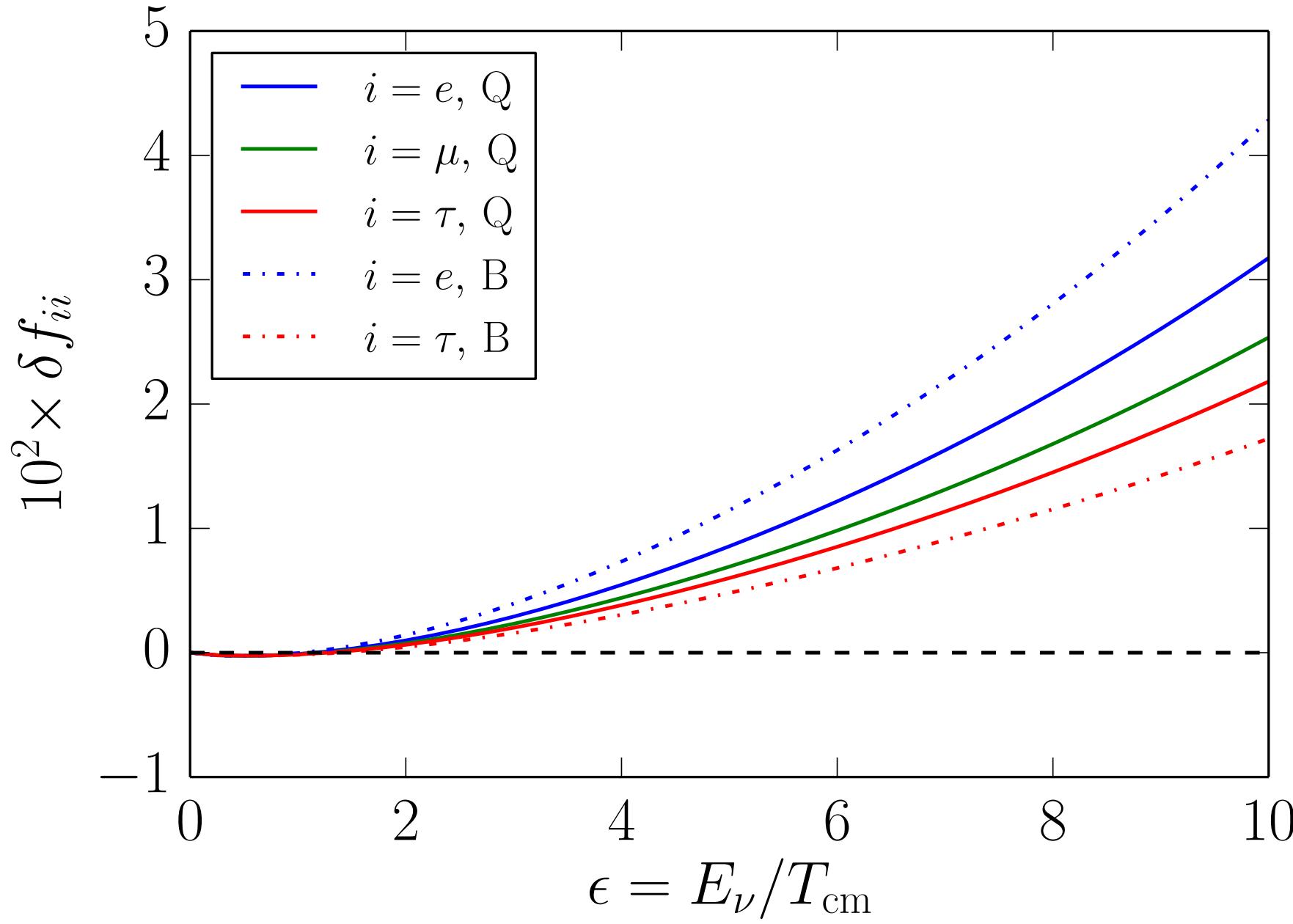
No  $CP$  Violation

## 3. Full collision term





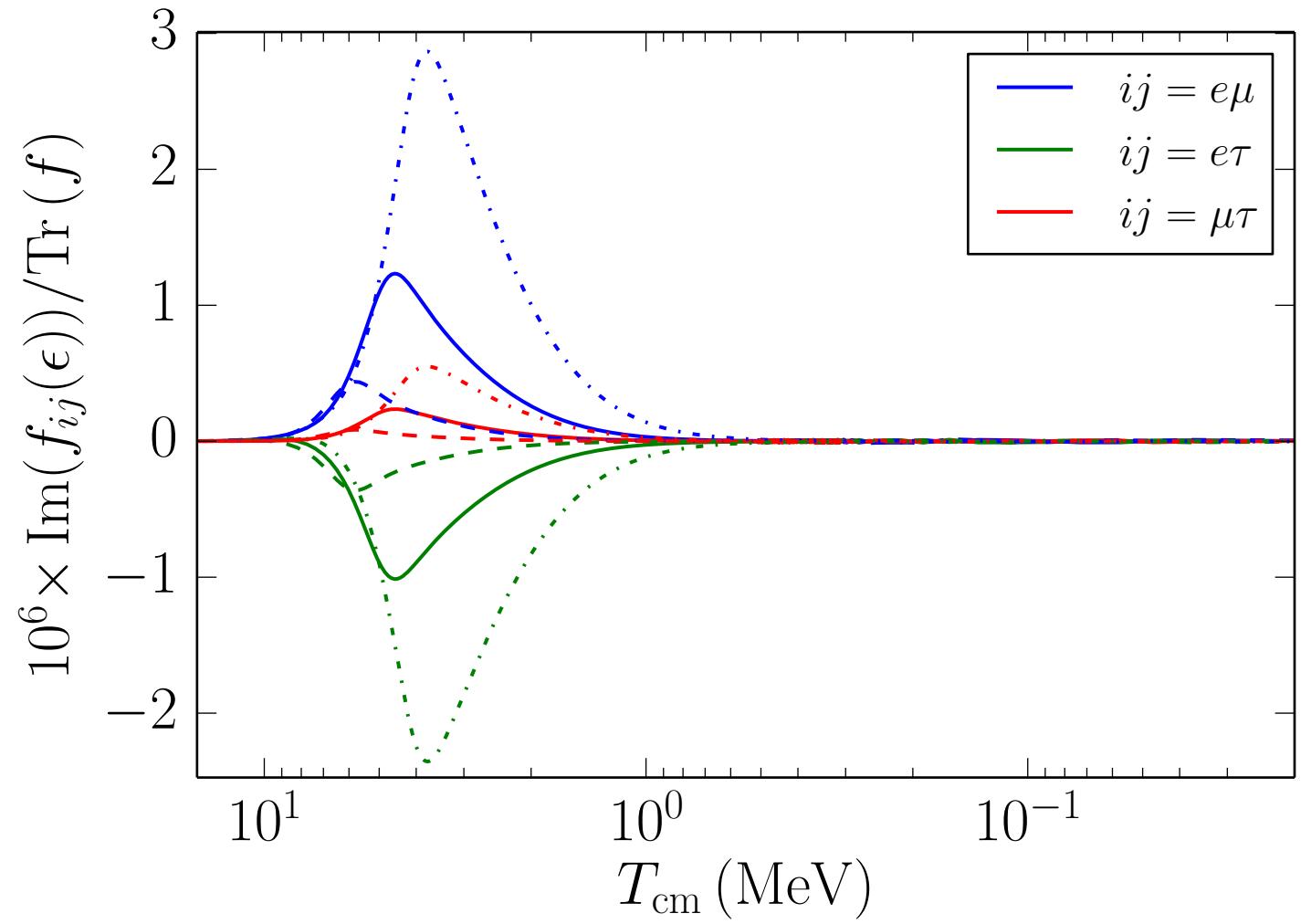
## Freeze-Out Spectra

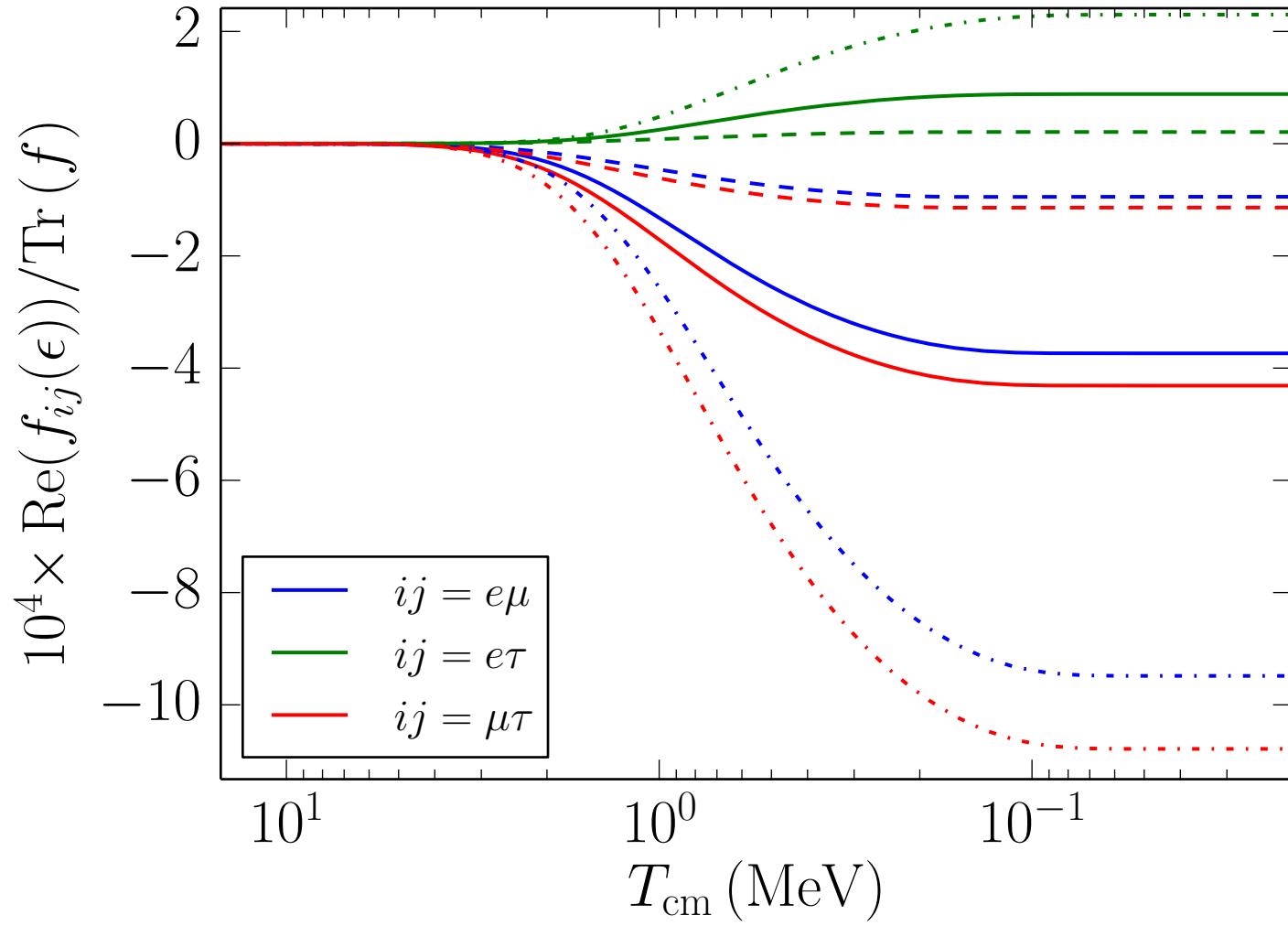


Full collision term,  
vacuum Potential  
in QKE calc.

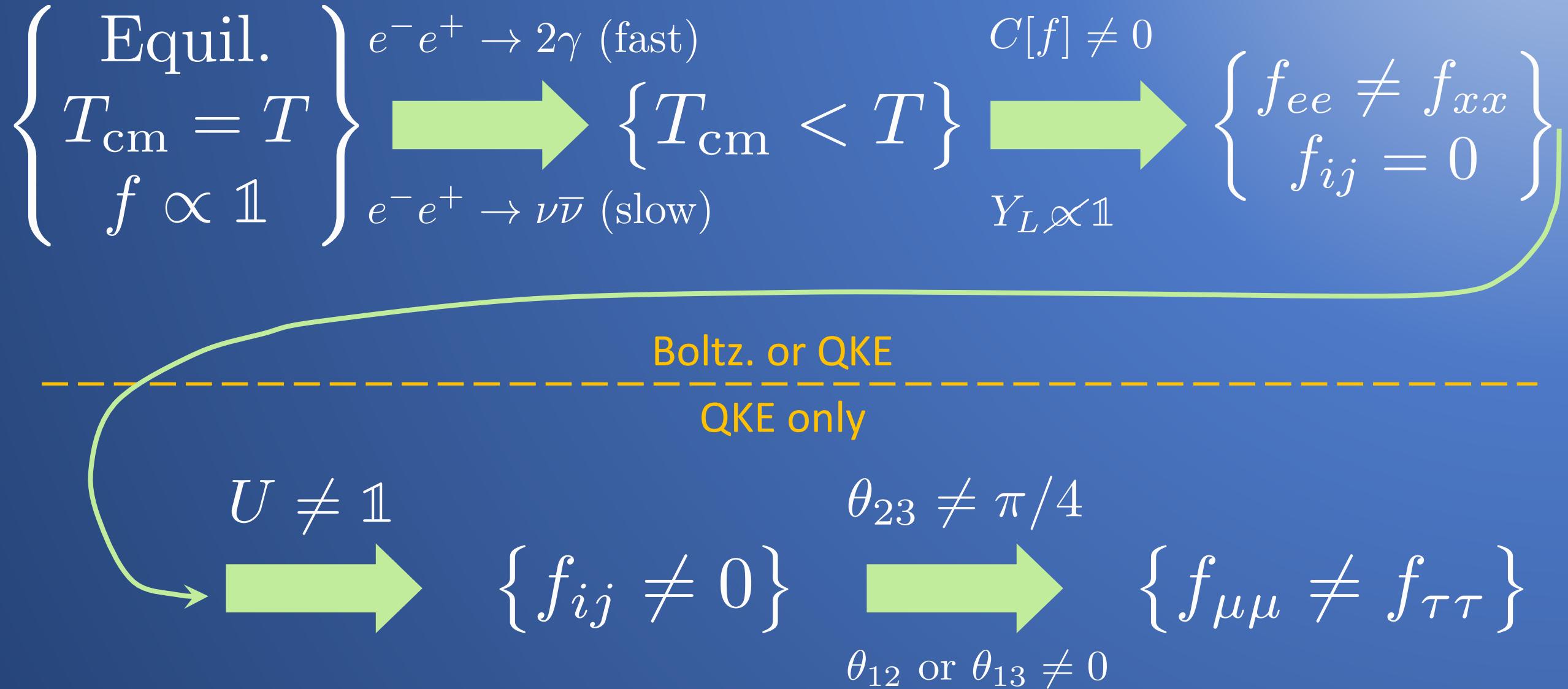
Full collision term  
in Boltzmann  
transport calc.

Preliminary Calc.





# Logical Progression of Neutrino Decoupling



# Summary/Lessons Learned

1. Homogeneity & Isotropy implies symmetry, NOT simplicity
2. Early Universe QKEs is a tractable problem
3. The physics guides the numerics (computation still important – GPUs with Ken M.)
4. Thank you Wick!