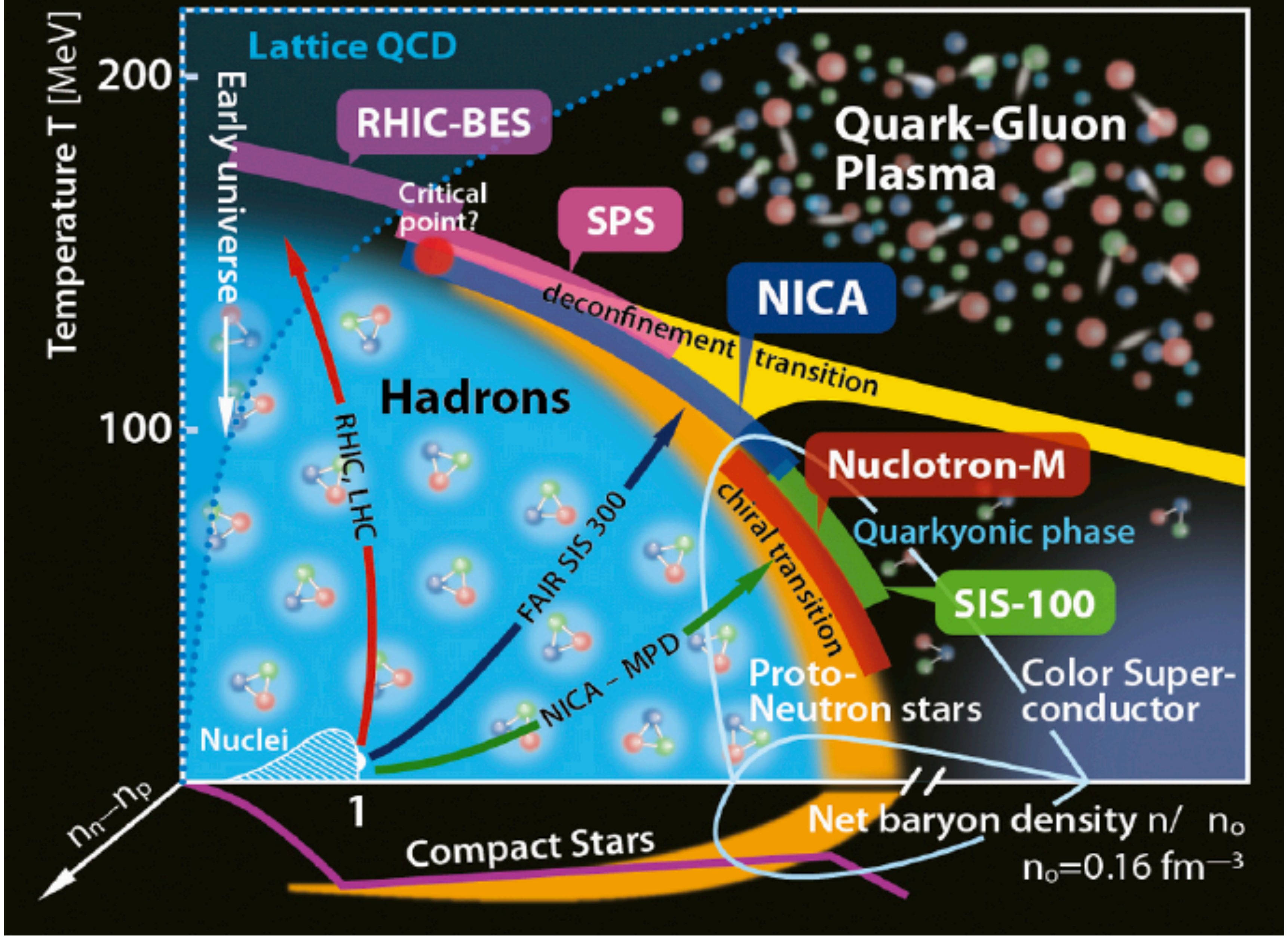


We only have compelling knowledge of the properties of QCD matter at high T and very small baryon density,.

We have strong evidence for the equation of state for very small T and high T from static properties of neutron stars and their collisions.

All the rest is at best educated conjecture and at worst idle speculation. Later in the talk I will engage in some speculation

It is not what you don't know that gets you in trouble, it is what you think you know but you don't. Mark Twain

Work with Sanjay Reddy PRL 122 (2019) 122701;
with Srimoyee Sen and Kiesang Jeon, arXiv:1908.04799

with Pisarski

Quarkyonic Matter:

Confinement at finite temperature disappears because the Debye screening length become shorter than the confinement scale. Gluons give

$$1/r^2_{Debye} \sim g^2 N_c T^2$$

At large number of colors

$$g^2 N_c = g^2_{tHooft}$$

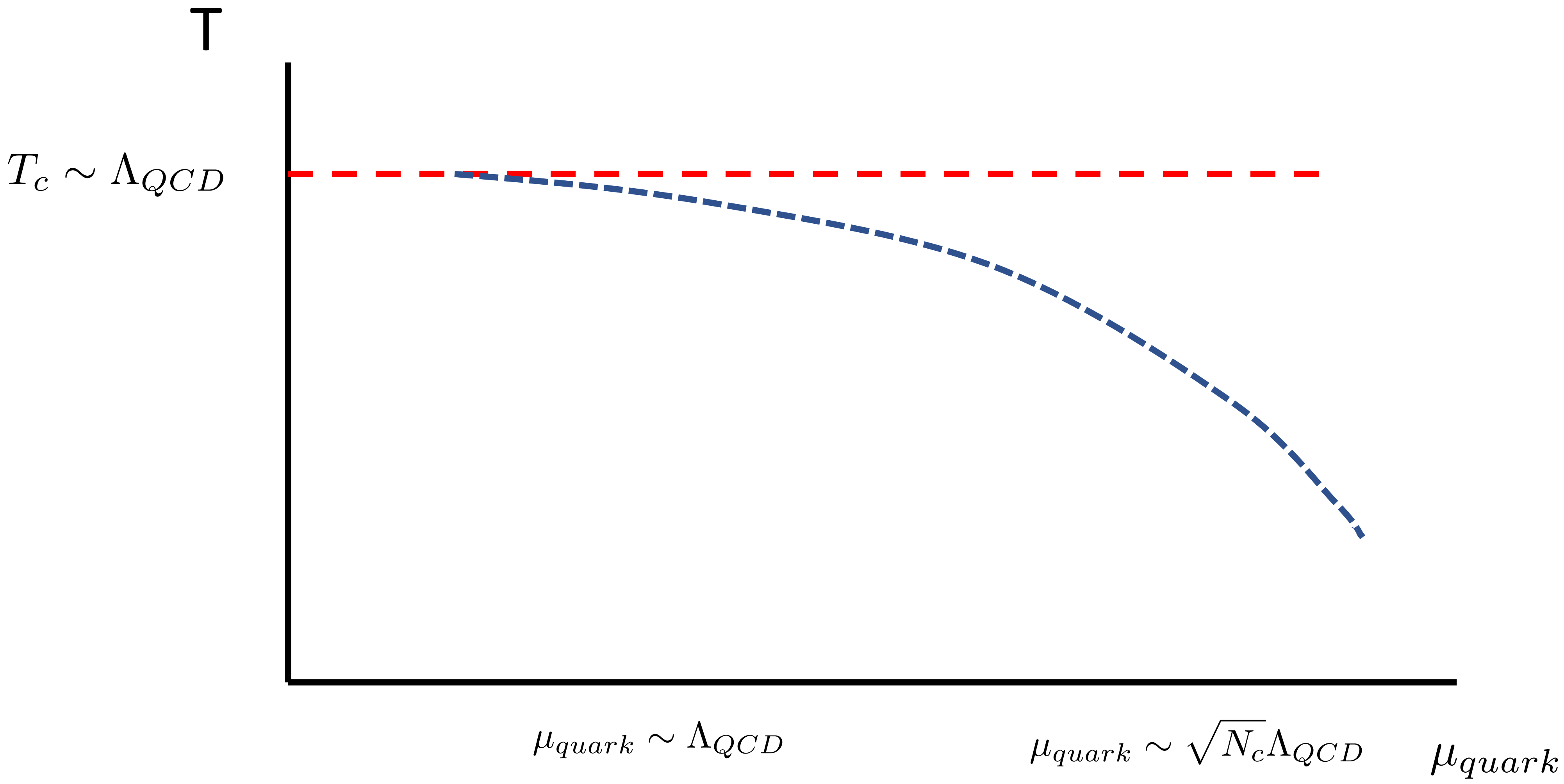
\is held fixed, so that the deconfinement temperature is of order the QCD scale

At finite density on the other hand because there are only N_c quarks

$$1/r^2_{Debye} \sim g^2 \mu^2_{quark} \sim g^2_{tHooft}\ \mu^2_{quark}/N_c$$

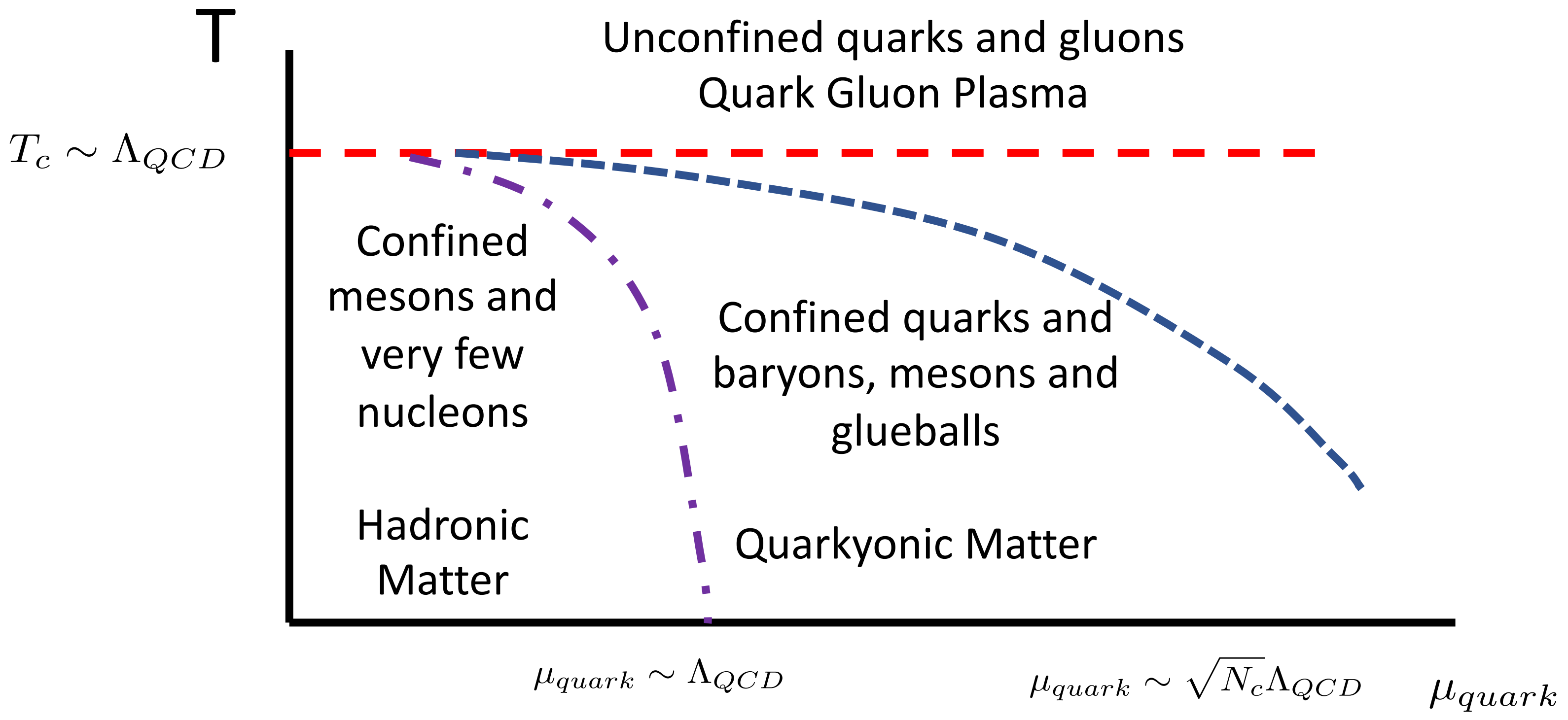
In the large N_c limit, at finite density and zero temperature limit,
the deconfinement chemical potential is

$$\mu_{qaurk} \sim \sqrt{N_c} \Lambda_{QCD} >> \Lambda_{QCD}$$

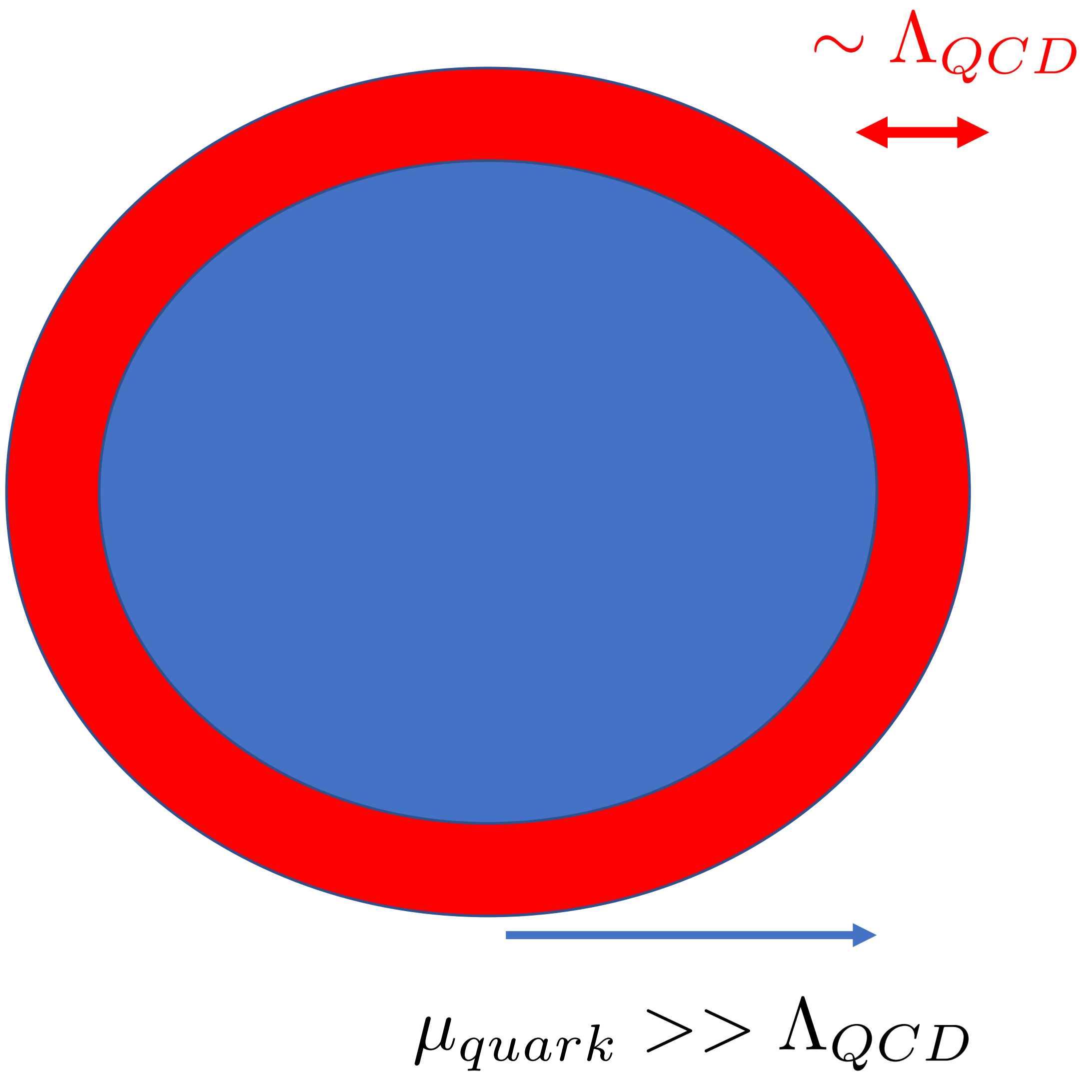

T
$T_c \sim \Lambda_{QCD}$
$\mu_{quark} \sim \Lambda_{QCD}$
$\mu_{quark} \sim \sqrt{N_c}\Lambda_{QCD}$
μ_{quark}

$$n_{baryon} \sim e^{(\mu_B - E)/T} \sim e^{N_c(\mu_Q - E_Q)/T}$$

No baryons for $\mu_{qaurk} < M_{nucleon}/N_c \sim \Lambda_{QCD}$



Fermi Surface is Non-perturbative



Fermi Surface
Interactions sensitive to infrared
Degrees of freedom: baryons,mesons and glueballs

Fermi Sea: Dominated by exchange interactions which are less sensitive to IR.
Degrees of freedom are quarks

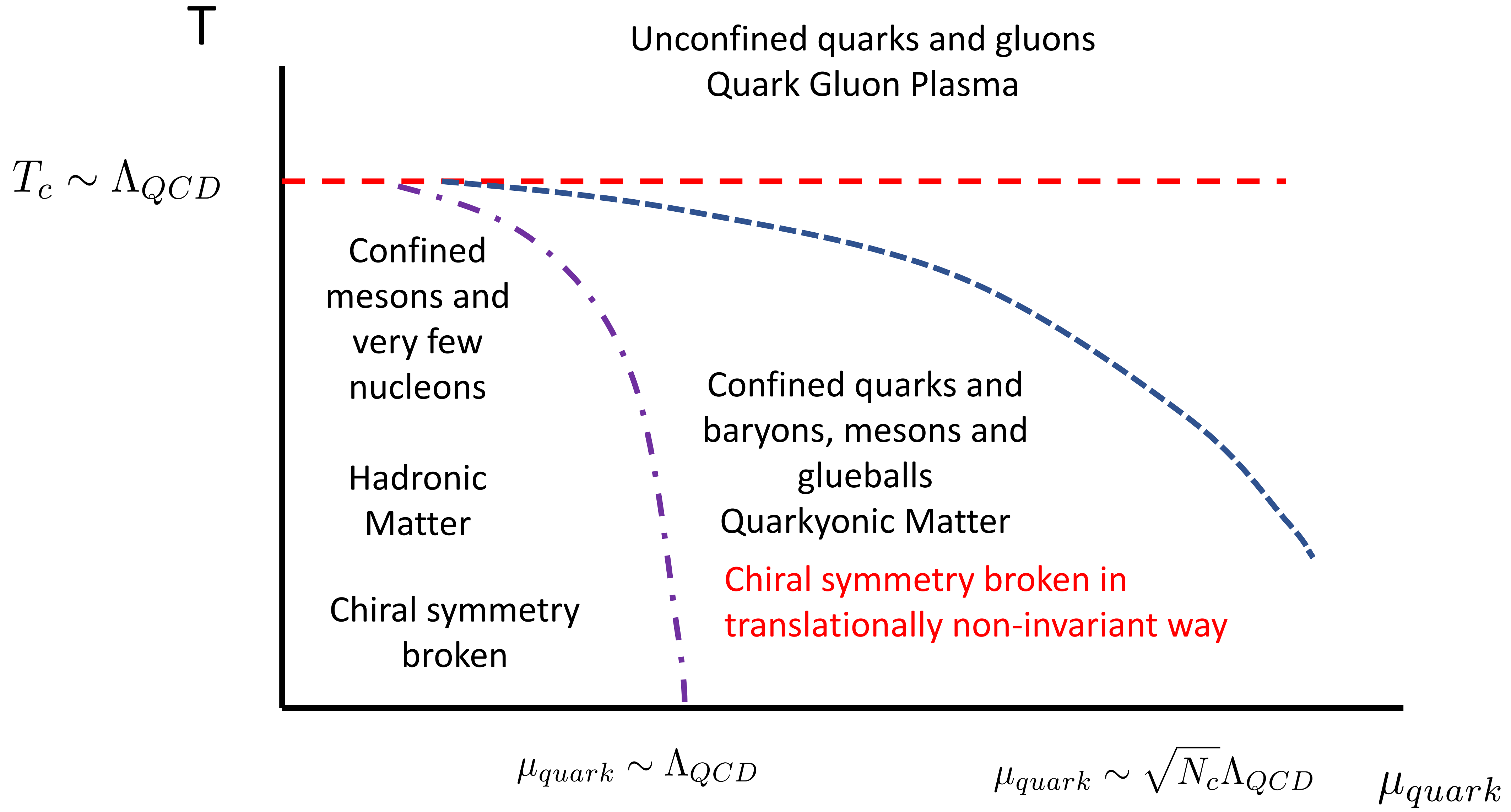
Chiral Symmetry Breaking and the Quarkyonic Phase

Scalar sigma meson condensation near the Fermi surface: particle hole

A small relative momentum of particle antiparticle requires that particle and antiparticle each have momentum of order the fermi momentum

Condensate breaks translational and rotational invariance

In fact there are many possibilities of patching together a discrete number of such condensates, as seen in model computations by Kojo, Pisarski and Tsvelik


T
$T_c \sim \Lambda_{QCD}$
Unconfined quarks and gluons
Quark Gluon Plasma
Confined
mesons and
very few
nucleons
Hadronic
Matter
Chiral symmetry
broken
Confined quarks and
baryons, mesons and
glueballs
Quarkyonic Matter
Chiral symmetry broken in
translationally non-invariant way
$\mu_{quark} \sim \Lambda_{QCD}$
$\mu_{quark} \sim \sqrt{N_c}\Lambda_{QCD}$
μ_{quark}

Simple large N_c considerations

Near nuclear matter density

$$k_F \sim \Lambda_{QCD}$$

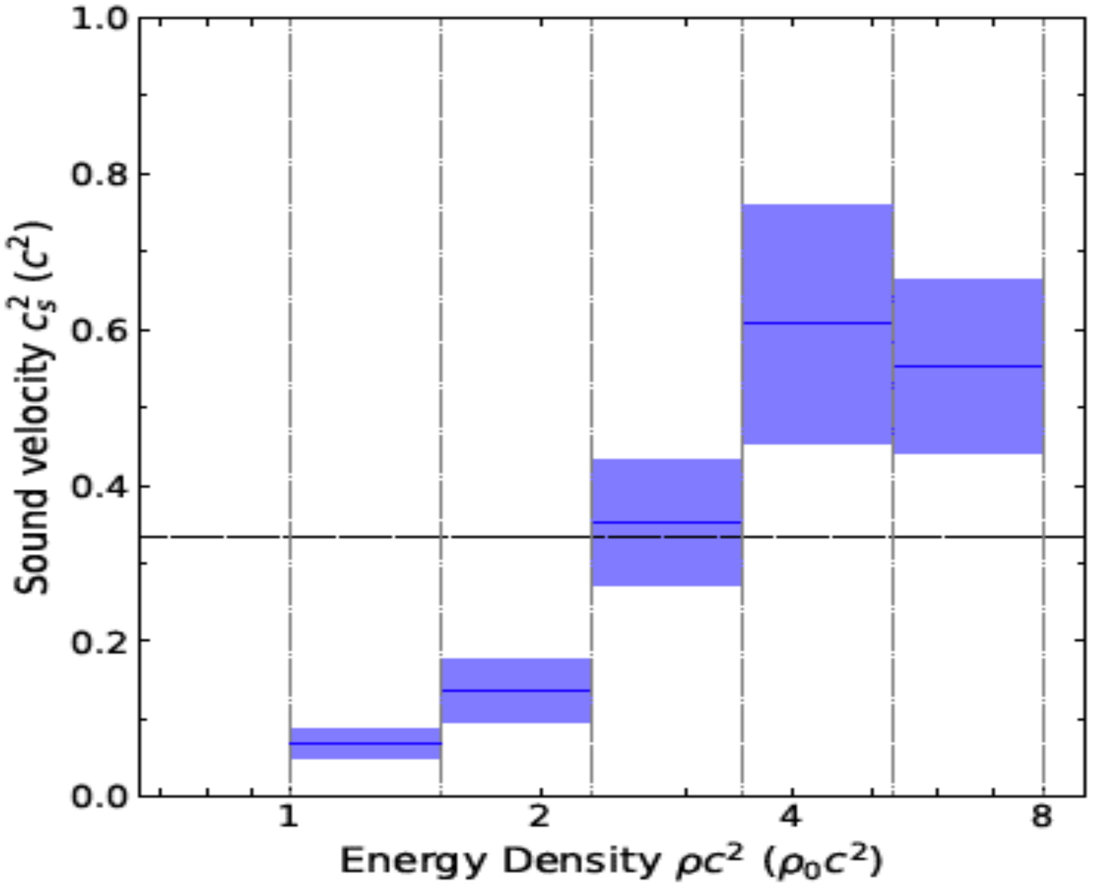
$$\epsilon/n - M_N \sim \Lambda^2_{QCD}/2M_N \sim \Lambda_{QCD}/N_c$$

On the other hand, short distance QCD interactions are of order N_c

$$\epsilon/n - M_N \sim N_c \Lambda_{QCD}$$

But the density of hard cores is also parametrically of order

$$\Lambda^3_{QCD}$$



Y. Fujimoto, K. Fukushima,
K. Murase

As a result of LIGO experiments, and more precise measurement of neutron star masses, the equation of state of nuclear matter at a few time nuclear matter density is tightly constrained

Typically sound velocity approaches and perhaps exceeds

$$v_s^2 = 1/3$$

at a few times nuclear matter density

Tews, Carlso, Gandolfi and Reddy; Kojo; Anala, Gorda, Kurkela and Vorinen

Sound velocity of order one has important consequences
For zero temperature Fermi gas:

$$\frac{n_B}{\mu_B dn_B/d\mu_B} = v_s^2$$

where the baryon chemical potential includes the effects of nucleon mass

$$\frac{\delta\mu_B}{\mu_B} \sim v_s^2 \frac{\delta n_B}{n_B}$$

So if the sound velocity is of order one, an order one change in the baryon density generates a change in the baryon number chemical potential of order the nucleon mass

For nuclear matter densities

$$\mu_B - M \sim \frac{\Lambda_{QCD}^2}{2M} \sim 100\ MeV$$

Large sound velocities will require very large intrinsic energy scales, and a partial occupation of available nucleon phase space because density is not changing much while Fermi energy changes a lot

The obvious problem is that how can the density stay fixed if the Fermi momentum goes up. It is solved by having the nucleon sit on a shell of varying thickness as the density increases. The added baryon number has to come in the form of new degrees of freedom: quarks, that fill up a Fermi sea

This can be understood in large N_c arguments:

$k_f \sim \Lambda_{QCD}$ requires $\mu_B - M_N \sim \Lambda_{QCD}/N_c$

Is it possible to get relativistic degrees of freedom with quarks?
Quarks should become important when

$$\mu_Q = \mu_B/N_c \sim \Lambda_{QCD}$$

The hypothesis of quarkyonic matter implies there is no confining transition, nor need there be any chiral transition that separates baryon matter from quark matter. Might they be smoothly connected? This would require a transition when the baryon Fermi energy is very close to the nucleon mass, so the transition may in principle occur quite close to nuclear matter density. To understand this, we do N_c counting

$$n_B^n = \frac{2}{3\pi^2} k_n^{F\ 3}$$

Density is of order one in power of N_c for baryon density computed both by both quark and nucleon degree of freedom

$$n_B^q = \frac{2}{3\pi^2} k_f^{q\ 3}$$

The problem is that if we take a constituent quark model

$$k_n^F = \sqrt{\mu_B^2 - M_N^2} = \sqrt{N_c^2 \mu_q^2 - N_c^2 M_q^2} = N_c k_q^F$$

If there is a continuous transition then the baryon density will have to remain fixed, so the chemical potential will change by of order N_c. The sound velocity is changing for a very non-relativistic system to a very relativistic one. The equation of state is therefore very stiff which is what is required from observation

$$\epsilon_B = M_N \Lambda_{QCD}^3 \sim N_c \Lambda_{QCD}^4$$

But quark density is of order N_c (each quark carries baryon number 1/N_c), so energy density can also be continuous

$$\epsilon_Q \sim \Lambda_{QCD} n_q \sim N_c \Lambda_{QCD}^4$$

$$P \sim \frac{k_F}{M_B}\epsilon_N$$

The pressure on the other hand must jump by order N_c squared

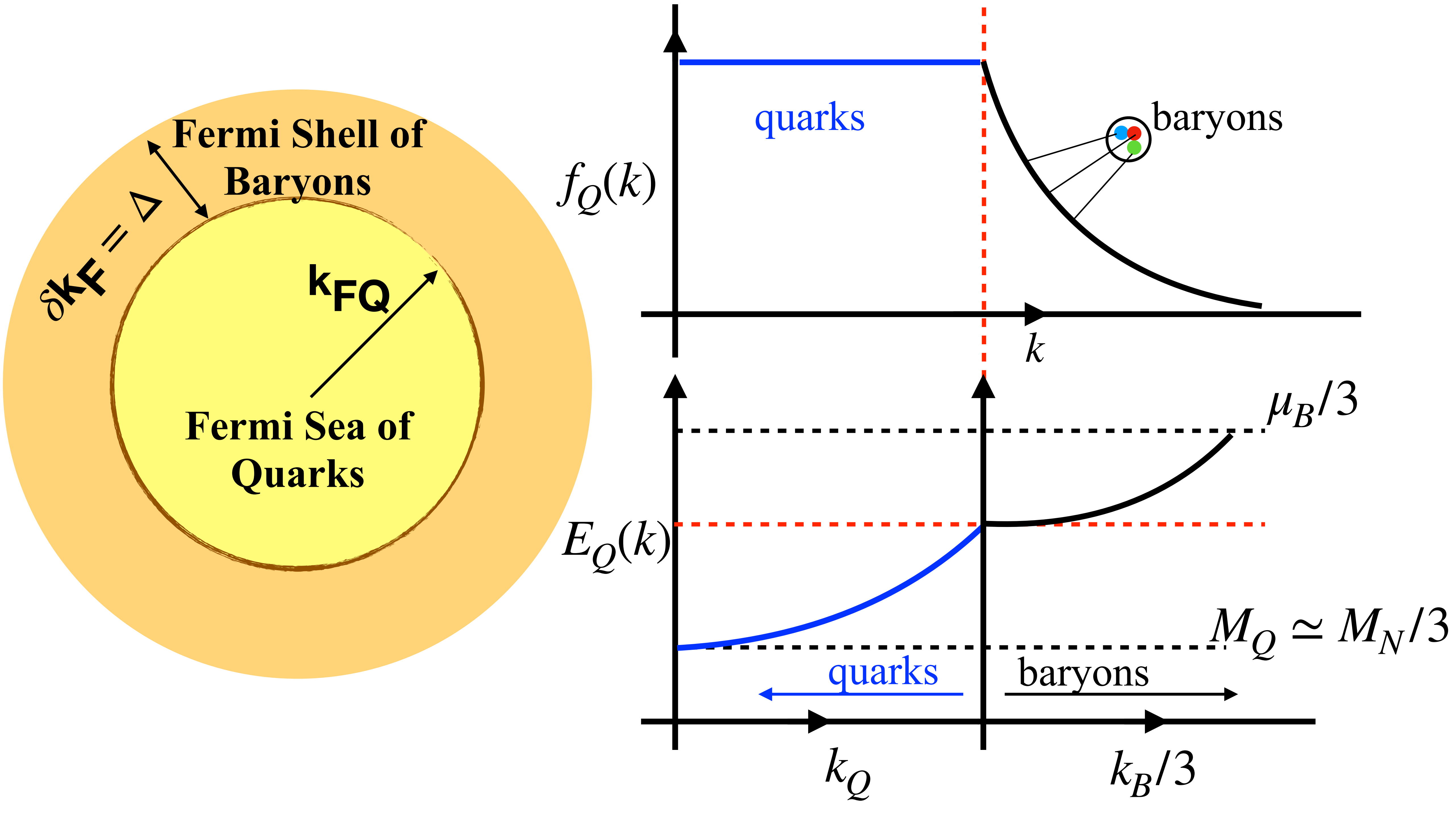
$$P \sim \epsilon_q$$

In ordinary first order phase transitions, the energy density and density jump but the pressure and chemical potential remain fixed

Here the energy density and density do not jump but the pressure and chemical potential jump.

The transition from nuclear matter to quarkyonic matter therefore involves a

First Order Un-Phase Transition



Fermi Shell of Baryons
$\delta k_F = \Delta$
k_{FQ}
Fermi Sea of Quarks
$f_Q(k)$
quarks
baryons
k
$\mu_B/3$
$E_Q(k)$
$M_Q \simeq M_N/3$
quarks
baryons
k_Q
$k_B/3$

Simplest model for a First Order Un-Phase Transition with Quarkyonic Matter

McLerran and Reddy

Above some Fermi momentum, assume a Fermi surface shell develops. For momenta

$$k_F^B > k > k_F^B - \Delta$$

the degrees of freedom are nucleons. Near the Fermi surface there can be non-perturbative low momentum interactions.

For momentum

$$k < k_F^B - \Delta$$

The degrees of freedom are quarks.

When the Fermi shell first appear, Delta is of the order of the QCD scale. At high densities, it must narrow to be of order 1/N_C^2 because we require that we smoothly match to a QCD limit for a a finite shell thickness

$$k_F^{B\ 3} - (k_F^B - \Delta)^3 \sim k_F^{B\ 2}\Delta \sim N_C^2 \Lambda^2 \Delta \sim \Lambda^3$$

A reasonable parameterization is

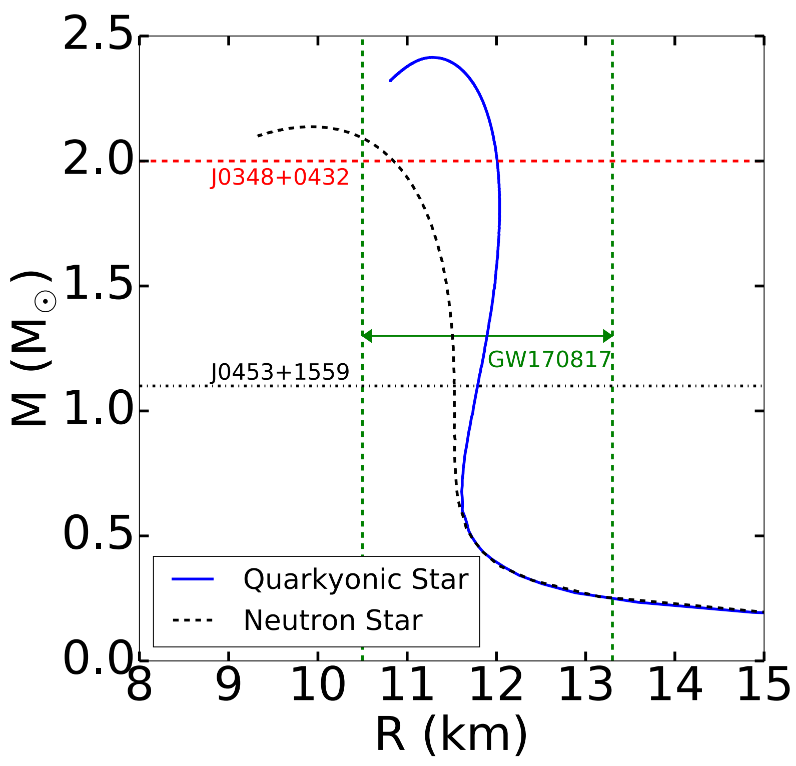
$$\Delta = \frac{\Lambda^3}{k_B^2}$$

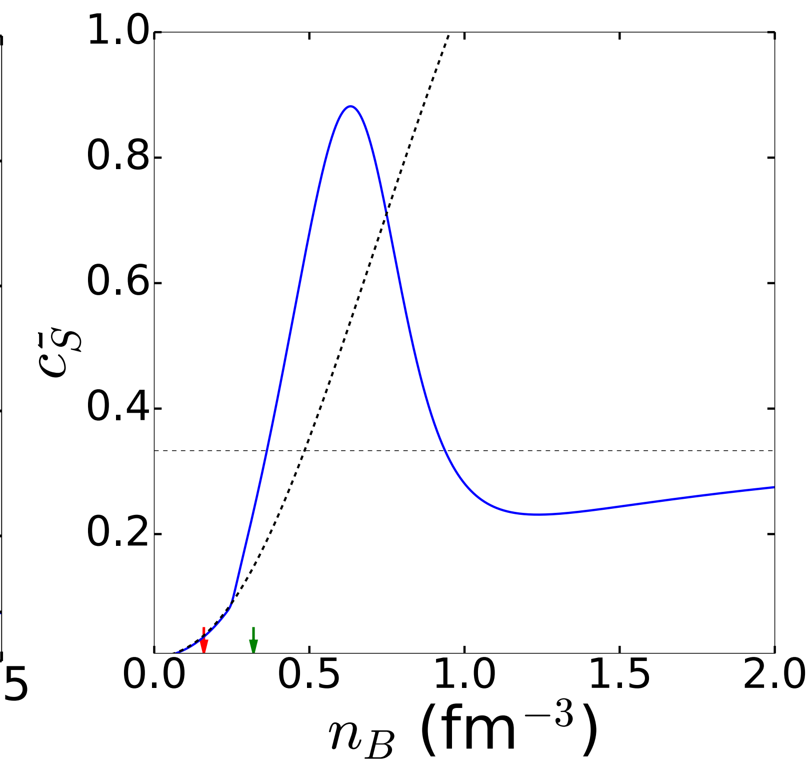
But if the density really becomes a constant then the sound velocity diverges.
to have a finite limit with v^2 < 1 need to include a correction

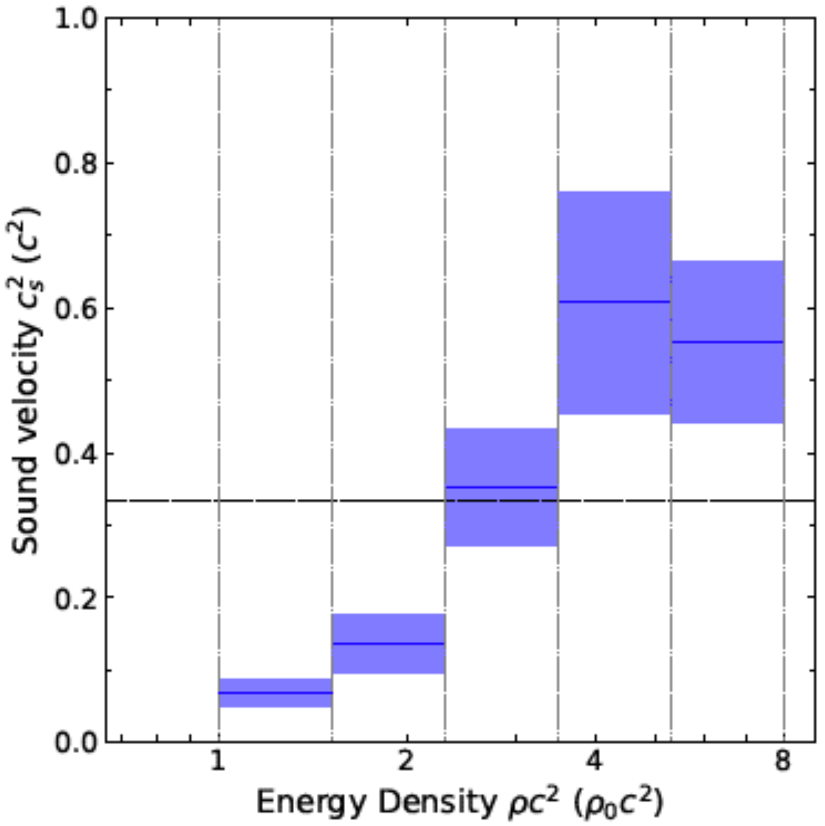
$$\Delta = \frac{\Lambda^3}{k_B^2} + \kappa \frac{\Lambda}{N_c^2}$$

This correction is important only at very high density where the quark contribution is of the order of the baryons

In explicit computation, we used a phenomenological equation of state for nuclear matter that is hard, and matched at a few times nuclear density







Y. Fujimoto, K. Fukushima, K. Murase

With Kiea-song Jeon and Srimoyee Sen

Treat nucleons as free gas in excluded volume determined by volume not included in hard cores. Put in a Fermi shell near the fermi surface

Quark in a sphere below the fermi surface

Minimize the energy to determine shell thickness

Acceptable equations of state with maximum in sound velocity near hard core density

J. Magueron and
With Jeon, Duarte and Hernandez:

beta equilibrium including hard core effect in progress

Main conclusion:

If a maximum in the sound velocity is conclusively determined from neutrons star studies, it very probably is due to a hard core nucleon density at about the density of the maximum, and the physics at and beyond the maximum involves quarks.

\

How to experimentally see this singular behavior of sound velocity and Fermi momentum distribution in lab experiments?

How to measure Fermi momentum distribution at high density?

Theoretically, getting the sound velocity less than 1 is clumsy, and in a really good treatment it should be automatic.

Meh?

Shrug?

I think not!