Retrospective
(with a hefty dash of perspective)

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Introduction

1978: Having read a review of neutrinoless vs 2–υ double beta decay, I suggested a quick and easy project to Wick; using existing tools, calculate the relevant nuclear matrix elements.

You may use this information to calibrate any projections that I may make.

After some years and encounters with many fascinating scientists we were asked to prepare a review of the subject by Sir Denys Wilkinson. Wick wrote it, I argued over every line and lost every argument.

The discussion of the neutrino mass matrix lays out the structure of a (6×6) mass matrix spanned by three active and three sterile neutrinos coupled by the Higgs (i.e. the Dirac mass) with the possibility of additional terms appearing in the sterile sector.

The following comments, under the guise of prospective, are all based on recent work with Terry Goldman.

Charged Leptons

In the Weyl basis, the form of the mass matrix here is

$$\widetilde{M}_\iota = (\iota(\eta_a^*)^T, \iota(\eta_s^*)^T) \begin{pmatrix} 0 & \widetilde{M}_D \\ \widetilde{M}_D{}^\dagger & 0 \end{pmatrix} \begin{pmatrix} \eta_a \\ \eta_s \end{pmatrix}$$

where $\widetilde{M}_D$ is a 3 x 3 Hermitean matrix, η_a (active) is a three component vector of the left chiral representation of Weyl spinors carrying negative electric charge and weak charge -½ and η_s (sterile) is a three component vector of the left chiral representation of Weyl spinors carrying positive electric charge and weak charge 0. In the mass eigen basis,

$$\eta_a = \begin{pmatrix} \eta_{e-} \\ \eta_{\tau-} \\ \eta_{\mu-} \end{pmatrix} \text{ and } \eta_s = \begin{pmatrix} \eta_{e+} \\ \eta_{\tau+} \\ \eta_{\mu+} \end{pmatrix}$$

Charged Leptons

The ordering choice is influenced by the PMNS matrix as described by the Particle Data Group

In the mass basis, the 6×6 mass matrix becomes

$$\widetilde{M}_l = m_l \times \begin{pmatrix} 0 & 0 & 0 & \varepsilon_l\delta_l & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \varepsilon_l \\ \varepsilon_l\delta_l & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon_l & 0 & 0 & 0 \end{pmatrix}$$

Charged Leptons

The same structure applies to the quarks, with color indices suppressed and the 1's and ε's interchanged. For the three cases, the parameters are:

Name	***Charge***	m_0	ε	δ
τ	-1	1.776 GeV/c^2	0.0595	0.0048
b	-⅓	4.18 GeV/c^2	0.0073	0.0222
T	+⅔	172.9 GeV/c^2	0.0073	0.0502

Neutrinos

Following the discussion in the last section, we assume that the full 6 × 6 neutrino mass matrix resembles the mass matrix for the charged leptons in the current representation with one important distinction,

$$\widetilde{M}_{\upsilon} = (\iota(\varphi_a^*)^T, \iota(\varphi_S^*)^T) \begin{pmatrix} 0 & \widetilde{M}_D \\ \widetilde{M}_D{}^{\dagger} & \widetilde{M}_S \end{pmatrix} \begin{pmatrix} \varphi_{\mathrm{a}} \\ \varphi_{\mathrm{s}} \end{pmatrix}$$

namely, the 3 × 3 matrix $\widetilde{M}_{\mathrm{S}}$ is not zero. This structure is the basis for the see-saw model which, assuming no singularities, leads to a prescription for mostly active neutrino masses through the matrix

$$\tilde{\mu}_a = \widetilde{M}_D \widetilde{M}_S^{-1} \widetilde{M}_D^{\dagger}$$

Neutrinos

However, that relation is reflexive so information about the mostly active Majorana masses, coupled with our assumptions about the form of $\widetilde{M}_{\mathrm{D}}$, leads to restrictions on the form of the mostly sterile mass matrix, $\widetilde{M}_{\mathrm{S}}$, through

$$\widetilde{M}_S = \widetilde{M}_D^{\dagger} \tilde{\mu}_a^{-1} \widetilde{M}_D$$

The mostly active Majorana neutrino mass eigenstates, as constrained by solar and atmospheric oscillation data, may be described by two mass hierarchies.

Neutrinos

The eigenstates are labelled 1, 2, and 3, where 1 is predominantly composed of the electron-neutrino current eigenstate, 2 is an approximately even mixture of the current eigenstates and 3 has little electron current eigenstate admixture. In the limit of the PNMS matrix as the TBM matrix these relations would be

$$|1> = \frac{1}{\sqrt{6}}\left(2|\upsilon_e> - |\upsilon_\mu> - |\upsilon_\tau>\right)$$

$$|2> = \frac{1}{\sqrt{3}}\left(|\upsilon_e> + |\upsilon_\mu> + |\upsilon_\tau>\right)$$

$$|3> = \frac{1}{\sqrt{2}}\left(-|\upsilon_\tau> + |\upsilon_\mu>\right)$$

Neutrinos

We choose to parametrize this description in the mass eigenstate representation as

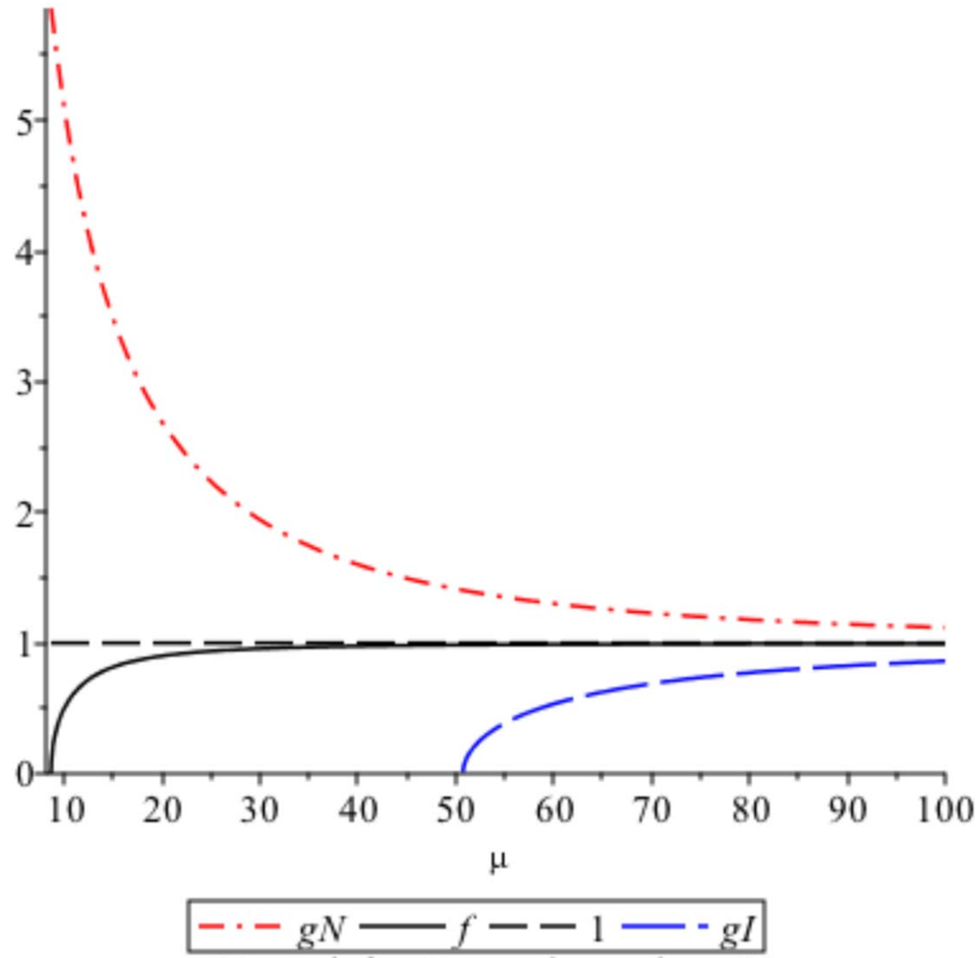
$$\tilde{\mu}_a = \mu \times \begin{pmatrix} f & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & g \end{pmatrix}$$

where the Normal Hierarchy (NH) has ($f < 1 < g$) and the Inverted Hierarchy (IH) has ($g < f < 1$).

Neutrinos

For this exercise, we take Δm^2_{solar} = 75 $(meV/c^2)^2$ and Δm^2_{atmos} = 2500 $(meV/c^2)^2$. This gives us two relations among f; g and μ. Demanding that the smallest mass be greater than or equal to zero requires that, for NH, $\mu > 8.66$ meV/c^2, while for IH, $\mu > 50.74$ meV/c^2. Cosmologogical constraints suggest that $\mu \leq 100$ meV/c^2. This is shown in the next figure.

Neutrinos



g-Normal, f, g-Inverted vs. μ in meV

Neutrinos

What may we expect these restrictions to look like in general? Using the facts that the determinant of any mass matrix is the product of its eigenvalues, that determinants are invariant under unitary transformations and that the determinant of a product of matrices is the product of the individual determinants, we may evaluate each factor in its mass eigenstate basis and arrive at (where we define a factor of m_d as the overall scale of the matrix, $\widetilde{M}_D$, for the neutrino Dirac masses, analogous to the m_l factor for the charged lepton masses)

Neutrinos

$$\det\left(\widetilde{M}_S\right) = (\varepsilon^2 \delta m_d^3)\left(\frac{1}{fg\mu^3}\right)(\varepsilon^2 \delta m_d^3)$$

Or, defining $m_S = \frac{m_d^2}{\mu}$,the product of the three eigenvalues becomes

$$\Pi_i \lambda_i = \frac{(m_S)^3}{fg}\epsilon^4\delta^2$$

Neutrinos

With the expectation that ε and δ are both the order of 10^{-2} or less, we expect that, whatever m_{s}, f, g are, the ratios of sterile neutrino masses should vary proportionally to the squares of ratios similar to those of the charged fermion masses, with details depending on the actual hierarchy of active neutrinos and the scale of mostly active masses. The details of the matrix structure can only affect the coefficients of the eigenmass values proportional to m_{s}, $\varepsilon^2 m_s$ and $\varepsilon^2\delta^2 m_s$.

Neutrinos

Implementing all of this, including the PMNS matrix relating mostly active mass eigenstates to flavor eigenstates we find the eigenvalues for the mostly sterile eigenstates to be

$$M_{large} = Y_{22} m_S \qquad M_{middle} = \frac{W_{23}}{Y_{22}} \varepsilon^2 m_S$$

$$M_{small} = \frac{1}{W_{23} f g} \varepsilon^2 \delta^2 m_S$$

Where Y_{22} and W_{23} are functions of μ and a Cabbibo-like angle θ which describes the misalignment of the lightest mostly sterile eigenfunction in the $(\upsilon_\mu - \upsilon_e)$ plane.

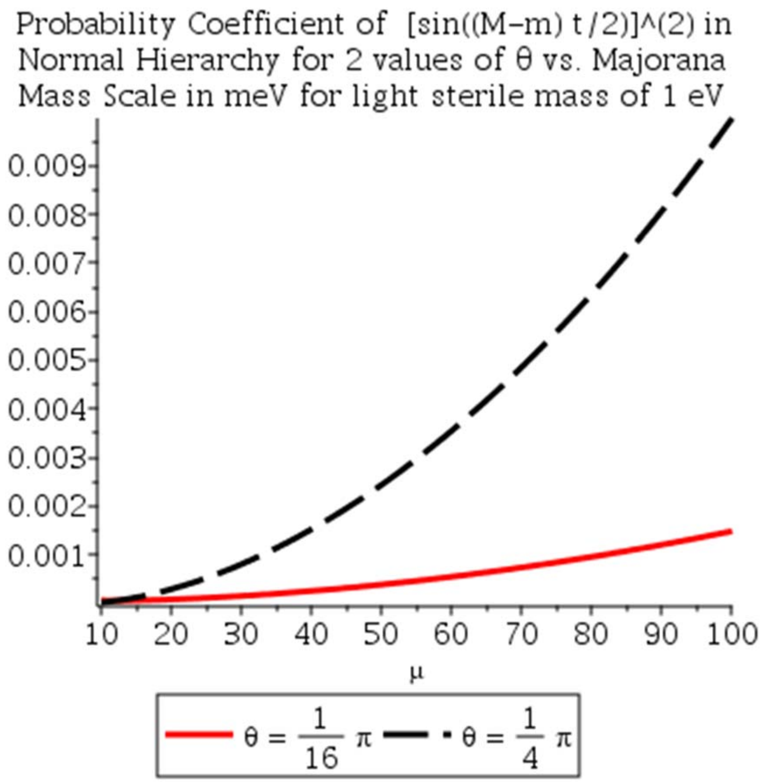
Neutrinos

- The admixture of υ_μ and υ_e are then given, in lowest order, by the action of $\widetilde{M}_D$ divided by M_{small}. Each of these amplitudes is proportional to

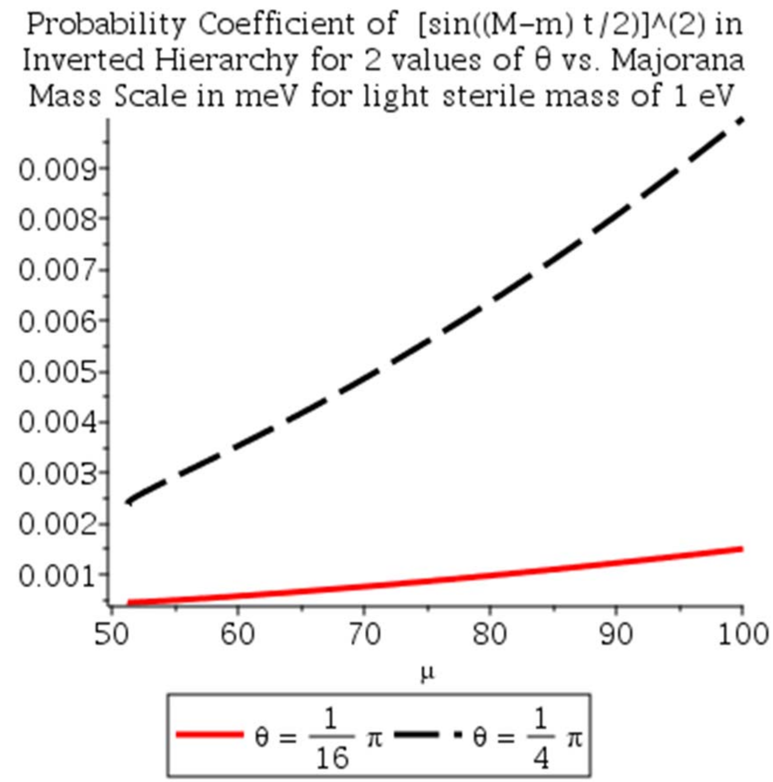
$$\frac{m_D \varepsilon \delta}{m_S \varepsilon^2 \delta^2}$$

The probability to go from $\upsilon_\mu \rightarrow \upsilon_e$ as a function of μ and θ is plotted in the next figure.

Neutrinos



Probability Coefficient of [sin((M−m) t/2)]^(2) in Normal Hierarchy for 2 values of θ vs. Majorana Mass Scale in meV for light sterile mass of 1 eV
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
10 20 30 40 50 60 70 80 90 100
μ
θ = 1/16 π
θ = 1/4 π

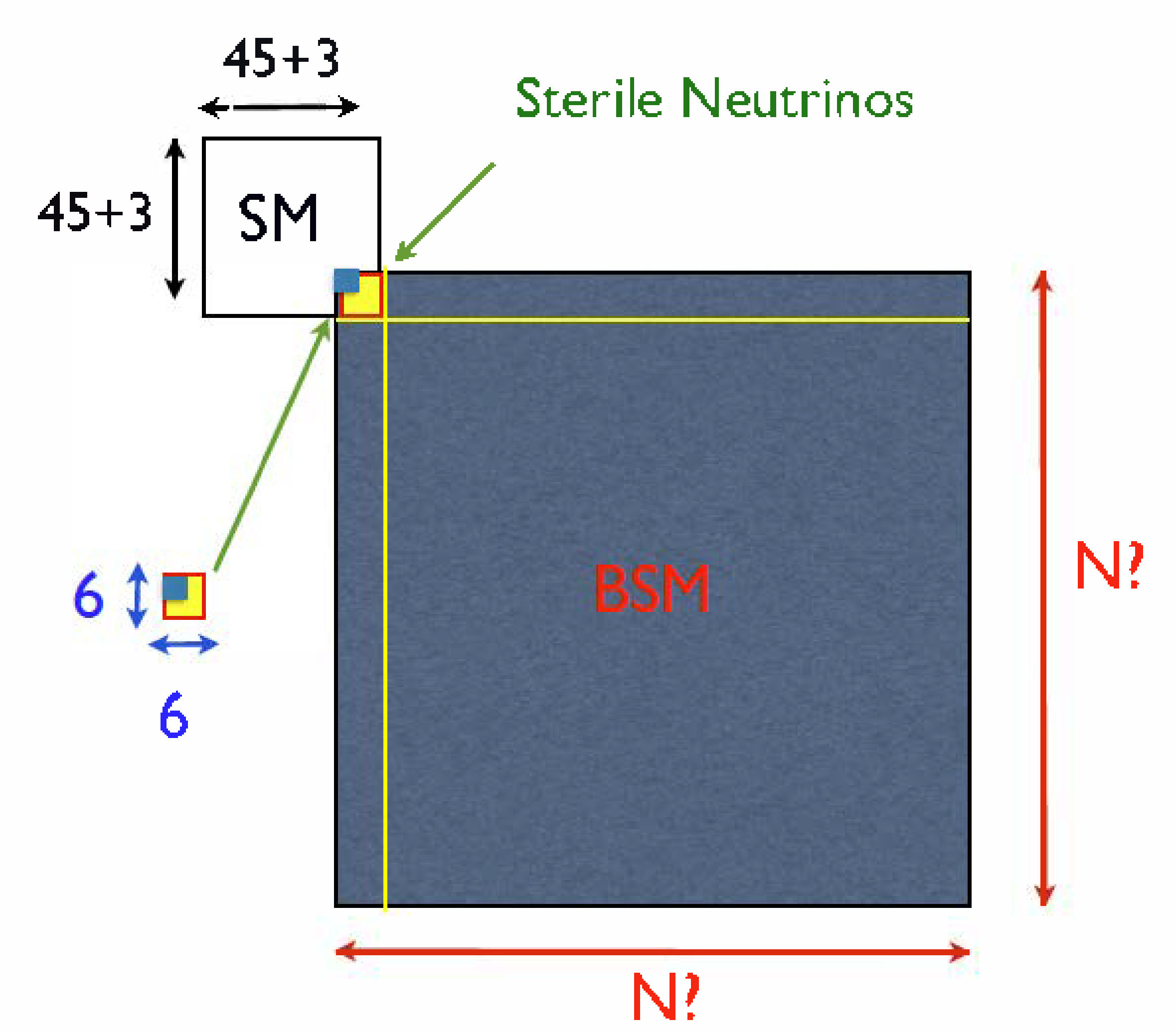


Probability Coefficient of [sin((M−m) t/2)]^(2) in Inverted Hierarchy for 2 values of θ vs. Majorana Mass Scale in meV for light sterile mass of 1 eV
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
50 60 70 80 90 100
μ
θ = 1/16 π
θ = 1/4 π

Neutrinos

How should we think about these mostly sterile neutrinos? They could be components of dark matter. Since they all have (different) components of active neutrinos, they would all exhibit Pal-Wolfenstein decay which is, in principle, observable. The action of the Higgs field on the active neutrino states may lead to mass eigenstates, as is implicit in the analysis just described, or, as with the Giant Dipole resonance in Nuclear Physics, lead to a distribution over mass eigenstates, as shown in the next figure.

Neutrinos



Conclusion

If any of this proves true, neutrino physics not only holds great promise, even after all these years, in its own right, but might provide a direct window into some of the properties of Dark Matter. That may even be realized within some of our lifetimes, but do heed my earlier warning.