Hamiltonian approaches to describing collective phenomena in beams

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WG4 — Connections to other GARD roadmaps (cross-cutting)

HEP GARD Accelerator and Beam Physics Workshop

LBNL, 10 Dec. 2019

The Low Action — a starting point for Hamiltonian collective effects

$$\mathcal{A} = \int dt \int d\vec{\Omega}_0 \Bigg[\underbrace{-mc^2 \sqrt{1 - \frac{1}{c^2}\left(\frac{\partial \mathbf{x}}{\partial t}\right)^2}}_{\mathcal{L}_{\text{ptcl}}} \underbrace{- q\phi + \frac{q}{c}\frac{\partial \mathbf{x}}{\partial t} \cdot \mathbf{A}}_{\mathcal{L}_{\text{p-c}}} \Bigg] f(\vec{\Omega}_0) +$$

By making all approximations to the action, the resulting models are guaranteed to be symplectic. For example:

$$\underbrace{\frac{1}{8\pi} \int d\mathbf{x} \left(-\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \nabla\phi\right)^2 - (\nabla \times \mathbf{A})^2}_{\mathcal{L}_{\text{EM}}}$$

Field Modal Decomposition

$$A_i = \sum_\sigma Q_\sigma \Psi_i(\mathbf{x})$$

Fields on a Grid

$$\varphi = \sum_\sigma \phi_\sigma \Psi(\mathbf{x} - \mathbf{x}_\sigma)$$

Introduce Macroparticles

$$f = \sum_j w_j \Lambda(\mathbf{q} - \mathbf{q}_j)\delta(\dot{q} - \dot{q}_j)$$

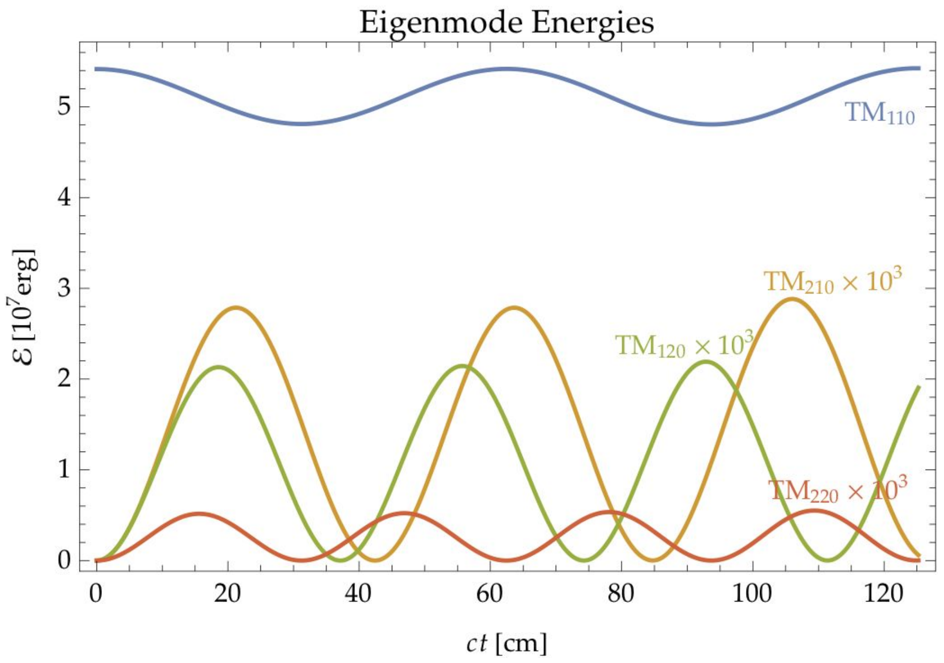
Quasi-static approximation

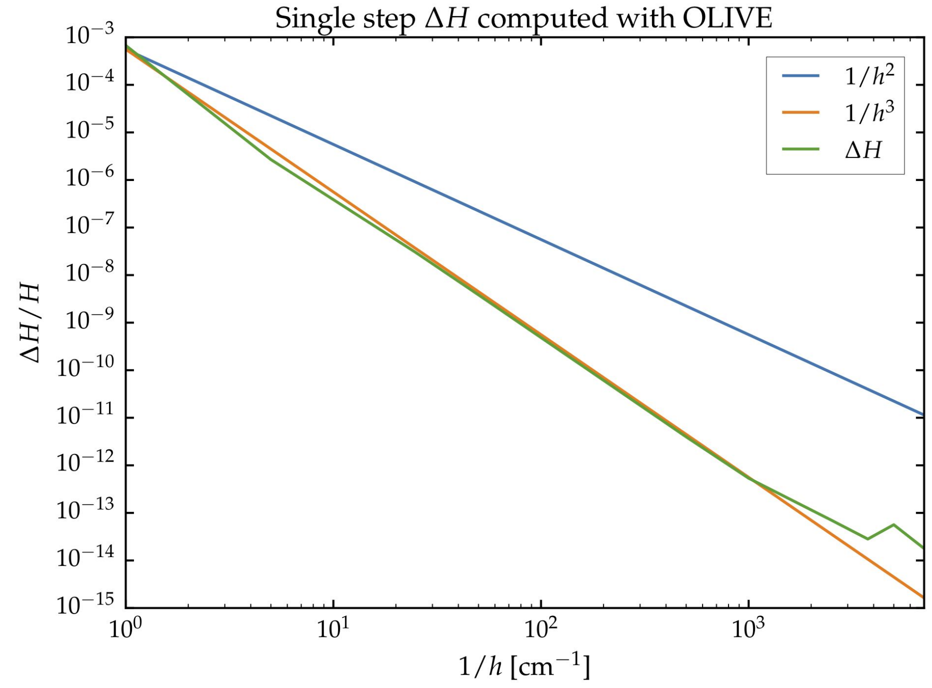
$$(\mathbf{A}, \phi)(\mathbf{x}_\perp, z - \beta c t)$$

F. E. Low, "A Lagrangian formulation of the Boltzmann-Vlasov equation for plasmas", *Proc. Royal Soc. A* (1958).

RadiaSoft has done work on transverse space charge, & long-range wakes due to beam loading in *rf* cavities

Long-range wake fields in rf cavities

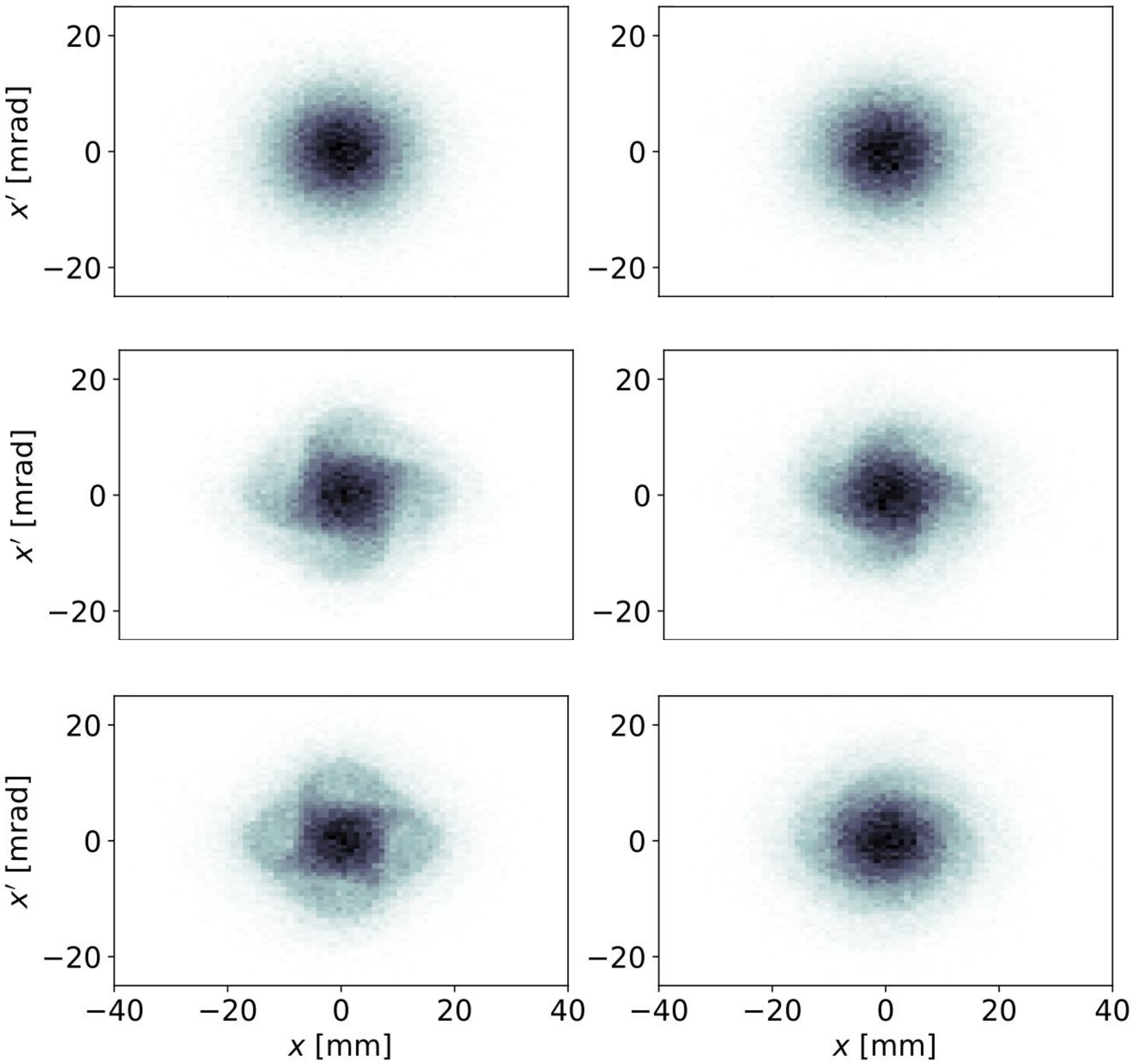




D. T. Abell, N. M. Cook, and S. D. Webb, “Symplectic modeling of beam loading in electromagnetic cavities”, Phys. Rev. Acc. Beams **20**, 052002 (2017).

Space-charge-driven octupole resonance in a FODO channel

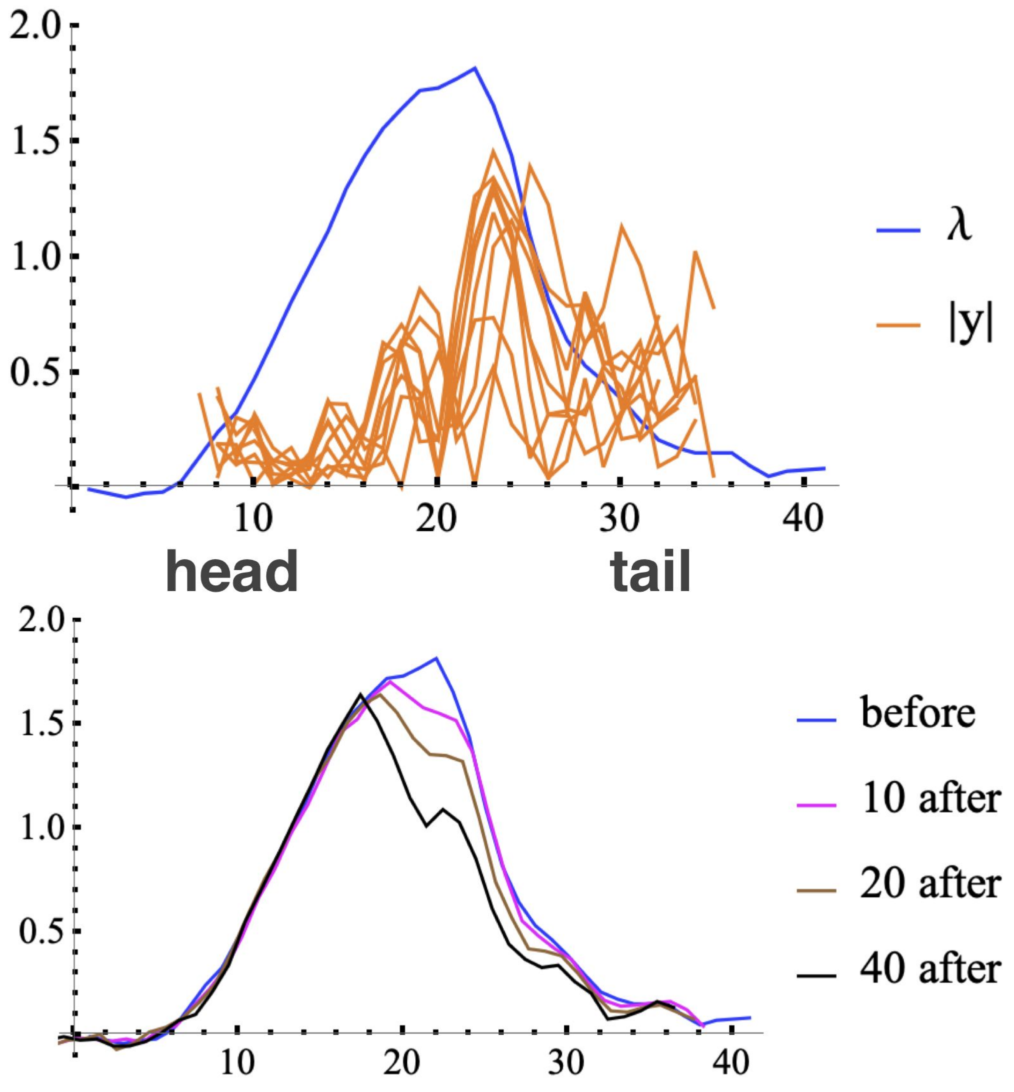
symplectic-spectral conventional PIC



J. Edelen, D. Abell, D. Bruhwiler, N. Cook, C. Hall, S. Swebb, “A Novel s-Based Symplectic Algorithm for Tracking with Space Charge”, Proc. of IPAC 2019, WEPTS068 (2019).

Near-Term RadiaSoft Goal:
a gridded symplectic electrostatic algorithm for space charge modeling

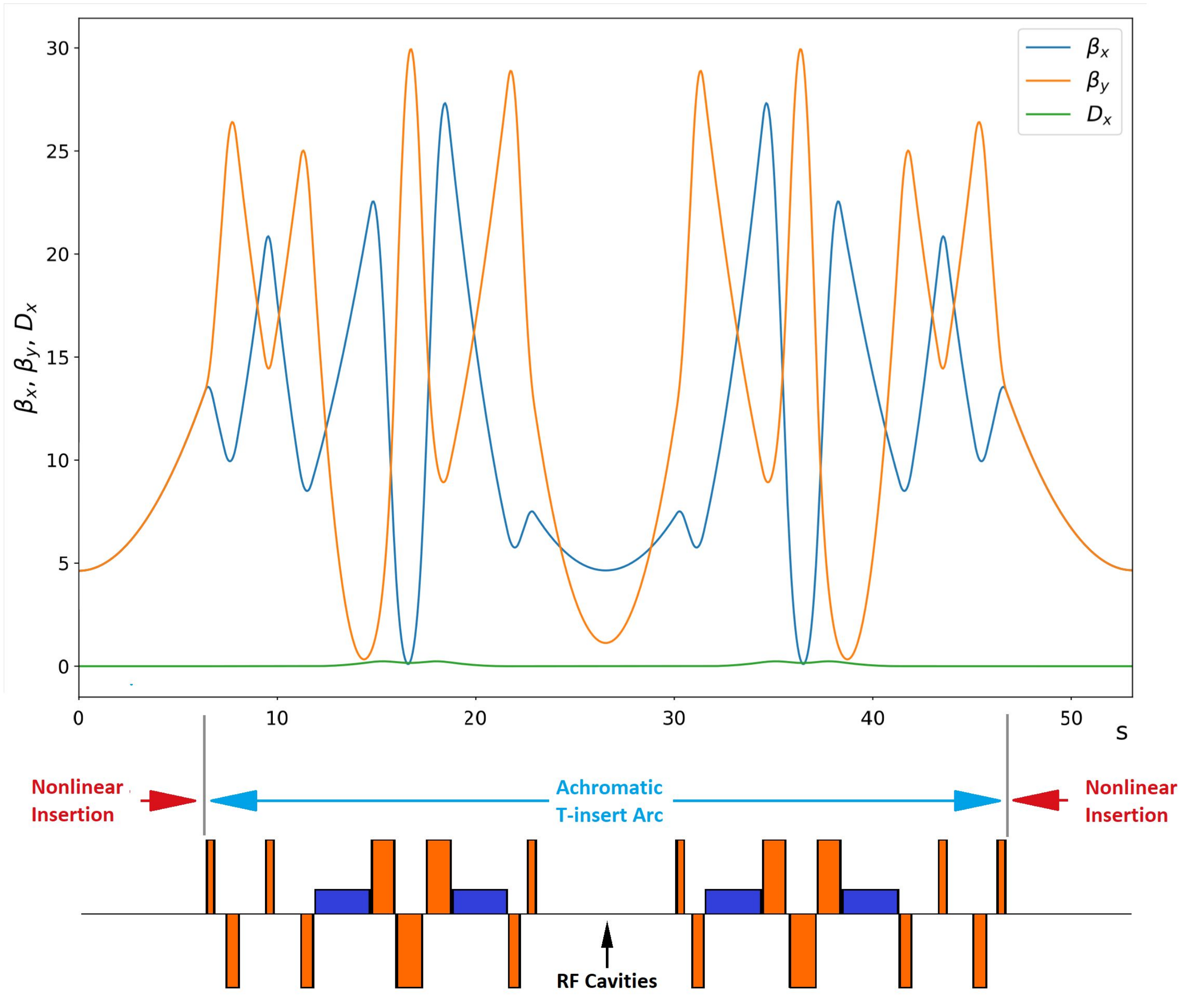
Convective head-tail observed at FNAL Booster



A. Burov, “Convective instabilities of bunched beams with space charge”, Phys. Rev. Accel. Beams **22**, 034202 (2019).

J. Eldred, “2019 Booster Collaboration Experiments”, Space Charge Workshop, Nov. 4-6 2019.

Integrable Rapid Cycling Synchrotron for PIP-III



J. Eldred and A. Valishev, “Simulation of Integrable Synchrotron with Space-Charge and Chromatic Tune-shift”, Proc. of 2018 IPAC, TUPAFA073 (2018).

Future Goals & Risks

Goal: Symplectic gridded space charge algorithm

Benefits to HEP: Ability to reliably model future Intensity Frontier accelerators

Risk: too computationally expensive compared to existing approaches; unable to reuse existing high performance linear algebra packages; difficult to implement with generality

Goal: Hamiltonian perturbation theory with space charge in rings

Benefits to HEP: Improved theoretical understanding of space charge in intense beams, i.e. dynamical definition of halo

Risk: calculations are horrible; difficult to interpret results; space charge is in a non-perturbative regime; poor convergence

Goal: Symplectic electromagnetic code

Benefits to HEP: Ability to model complex Hamiltonian dynamics in plasma accelerators

Risk: too computationally expensive compared to existing approaches; algorithms end up being implicit; algorithms do not scale well in parallel

Goal: Hamiltonian treatment of plasma kinetics in plasma accelerators

Benefits to HEP: General model for plasma accelerator properties, i.e. trapping, wake fields, witness bunch dynamics

Risk: calculations are horrible

Grand Challenges

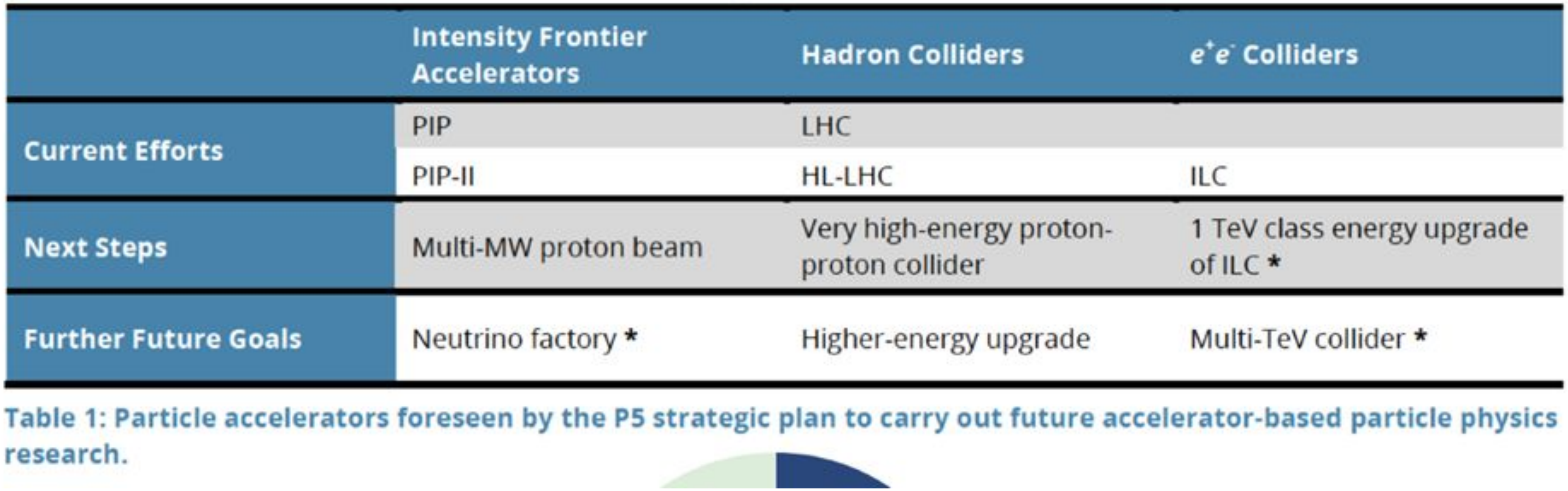
GC #1 Beam Intensity — "...A complete and robust understanding of these effects could help overcome the limits and increase beam intensities by orders of magnitude."

Conventional approach to describing beam instabilities relies on either (1) a perturbative Vlasov approach or (2) a reduced model (i.e. two-macroparticle head-tail model). Part of understanding instabilities is understanding their saturation -- linearized Vlasov cannot do this and reduced models may not accurately capture it.

GC #4 Beam Prediction — "Developing "virtual particle accelerators" will provide predictive tools that enable fast computer modeling of particle beams and accelerators at unprecedented levels of accuracy and completeness"

"virtual particle accelerators" will have to respect the Hamiltonian dynamics of the system i.e. symplectic space charge, reduced model symplectic representations for control system prediction (online modeling).

Connection with HEP facilities



	Intensity Frontier Accelerators	Hadron Colliders	e^+e^- Colliders
Current Efforts	PIP	LHC	
	PIP-II	HL-LHC	ILC
Next Steps	Multi-MW proton beam	Very high-energy proton-proton collider	1 TeV class energy upgrade of ILC *
Further Future Goals	Neutrino factory *	Higher-energy upgrade	Multi-TeV collider *

Table 1: Particle accelerators foreseen by the P5 strategic plan to carry out future accelerator-based particle physics research.

Intensity Frontier —

IOTA - nonlinear integrable optics; phase space dilution and instabilities; benchmark space charge simulations with experiment

PIP - benchmark simulations with experiment in the Fermilab booster (instabilities, emittance growth, beam loss)

PIP-II - use new techniques; understanding and codes to design the RCS; test key concepts in IOTA experiments

Energy Frontier / Lepton Colliders —

Argonne Wakefield Accelerator - benchmark space charge simulations with experiment (in a linac, as opposed to rings at Fermilab)

FACET-II - plasma accelerator beam dynamics; bubble shape beyond the Lu formula; wake function calculations; benchmark Hamiltonian vs conventional PIC codes vs experiment

Synergies

Nuclear Physics

Similar beam instabilities to Intensity Frontier -- impedances, space charge, etc.

Need to understand multi-mode long range wake instabilities in ERLs

Basic Energy Science

Plasma accelerator driven free-electron laser requires detailed understanding of beam dynamics in plasmas

New understanding of instabilities would aid design of improved FEL schemes

Nuclear Energy

Accelerator-driven systems will require detailed understanding of beam halo and other space charge effects

Fusion Energy Sciences

Hamiltonian theory of plasma instabilities would lend new understanding to basic plasma science

Improved modeling of space weather phenomena

Example Timeline

Theory

Well-posed problem of "single-turn map" including space charge

Hamiltonian perturbation theory for plasma collective effects — space charge

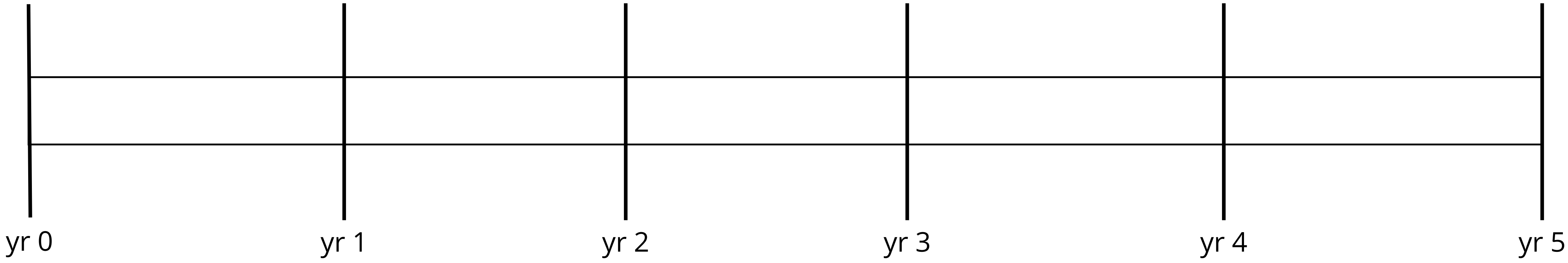
Hamiltonian picture for blowout plasma wave kinetics

Analysis of space charge nonlinear resonances in FODO channel with new formalism

General scaling laws for plasma accelerator bubbles from small number of parameters

Analytic calculation of plasma wave wake functions

Analytic analysis of integrable optics with space charge/other collective effects



Prototype gridded symplectic space charge algorithm

Production symplectic space charge model

Symplectic electromagnetic algorithm

Symplectic quasi-static algorithm

Production symplectic space charge model

Working symplectic electromagnetic algorithm

Production symplectic quasi-static algorithm

Benchmarking suite for multiple symplectic space charge codes

Benchmarking suite for symplectic electromagnetic codes

Adoption of symplectic self-consistent methods in existing codes

Computation

Collaborations

A brief and incomplete overview of people publishing work in this field

- B. Shadwick, A. Stamm, and E. Evstatiev, “Variational formulation of macro-particle plasma simulation algorithms”, Phys. Plasmas **21**, 5 (2014).
- J. Qiang, “Symplectic multi particle tracking model for self-consistent space-charge simulations”, Phys. Rev. Accel. Beams **20**, 014203 (2017).
- C. Mitchell and J. Qiang, “Analysis of Particle Noise in a Gridless Spectral Poisson Solver for Symplectic Multiparticle Tracking”, Proc. of IPAC 2019, WEPTS079 (2019).
- S. D. Webb, “A Spectral Canonical Electrostatic Algorithm”, Plasma Phys. Controlled Fusion **58**, 3 (2016).
- D. T. Abell, N. M. Cook, and S. D. Webb, “Symplectic modeling of beam loading in electromagnetic cavities”, Phys. Rev. Acc. Beams **20**, 052002 (2017).
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Synergistic and elsewhere

- M. Kraus, K. Kormann, P. J. Morrison, and E. Sonnendrücker, “GEMPIC: geometric electromagnetic particle-in-cell methods”, J. Plasma Phys. **83**, 4 (2017).
- J. Squire, H. Qin, and W. M. Tang, “Geometric integration of the Vlasov-Maxwell system with a variational particle-in-cell scheme”, Phys. Plasmas **19**, 084501 (2012).
- P. J. Morrison, “Hamiltonian and action principle formulations of plasma physics”, Phys. Plasmas **12**, 058102 (2005).