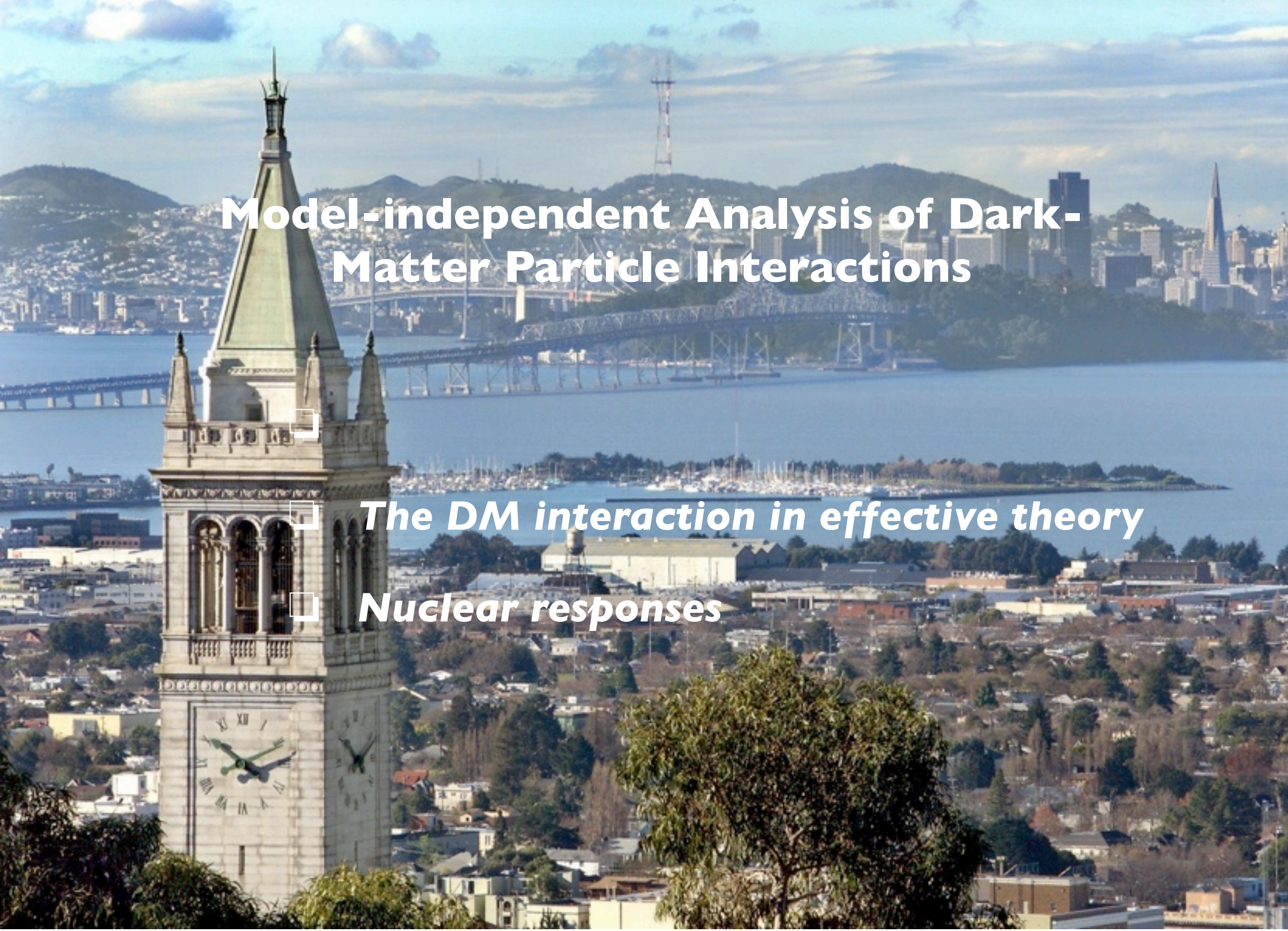

Model-independent Analysis of Dark-Matter Particle Interactions

- *The DM interaction in effective theory*
- *Nuclear responses*

Basic scales in WIMP dark matter scattering off nuclei

- if mass is on the weak scale, WIMP momentum transfers in elastic scattering can range to $q_{max} \sim 150$ MeV/c

- WIMP kinetic energy ~ 30 keV << typical nuclear excitation energies

- $R_{NUC} \sim 1.2\, A^{1/3}$ f □ $q_{max} R \sim 2.4$ □ 4.6 for F □ Xe

- the nuclear targets include a large range of masses, ground-state spins, and isospins

Nontrivial spins, momentum transfer ⇒ complexity. Despite this, cross sections are typically described as in a point-like system

$$\text{S.I.} \quad \Rightarrow \quad \langle g.s.| \sum_{i=1}^{A} (a_0^F + a_1^F \tau_3(i)) \, |g.s.\rangle$$

$$\text{S.D.} \quad \Rightarrow \quad \langle g.s.| \sum_{i=1}^{A} \vec{\sigma}(i) \, (a_0^{GT} + a_1^{GT} \tau_3(i)) \, |g.s.\rangle$$

with this providing the basis for most comparisons among experiments

Two issues are thus

- the use of a highly model-dependent, simplistic description of the low-energy WIMP-nucleus interaction

- absence of proper quantum mechanical treatment of the nuclear response

These are not technical shortcomings, but fundamental ones

- theory: lack of a basic understanding of what can and cannot be learned about the low-energy constants governing the WIMP interactions with nucleons

- experiment: lack of a realistic formalism for comparing experiments, and thus for assessing their consistency

Contrast with the efforts decades on weak interactions -- S, T, V, A, P leading to V-A

III. Effective theories: WIMP-nucleon and WIMP-nuclear*

EFT can address two basic questions

- how might low-energy DM particles interact with ordinary matter?
- to what extent can we distinguish among these possibilities, given only information on elastic scattering (ES) off nuclei?

For question #1, there are two possible approaches

- Top-down - study all ultraviolet theories. By dimensional analysis, no. of ultraviolet theories of DM $\gtrsim$ no. particle theorists
- Bottom-up: many theories are equivalent at low energies: we need only the most general effective theory*

  how are these ETs restricted by physics?
- Lorentz invariance: which simplifies to Galilean invariance
- other symmetries? we know of none: parity conserving or parity violating, time reversal invariant or non-invariant ?? ...
- quantum mechanics: instructs us to use Hermitian operators

*L Fitzpatrick, WH, A Katz, N Lubbers, and Y Xu, arXiv:1203.3542 and arXiv:1211:2818

Hermitian operator	$i\vec{q}$	$\vec{v}^{\perp}$	S_χ	S_N
parity	-1	-1	$+1$	$+1$
time reversal	$+1$	-1	-1	-1

where $$\vec{v}^{\perp} = \frac{1}{2}\left[(\vec{v}_{\chi,\mathrm{in}} - \vec{v}_{N,\mathrm{in}}) + (\vec{v}_{\chi,\mathrm{out}} - \vec{v}_{N,\mathrm{out}})\right]$$

$$\begin{aligned}
H_{ET} = \; & \left[a_1 + a_2\ \vec{v}^{\perp}\cdot\vec{v}^{\perp} + a_5\ i\vec{S}_\chi\cdot\left(\frac{\vec{q}}{m_N}\times\vec{v}^{\perp}\right)\right] + \vec{S}_N\cdot\left[a_3\ i\frac{\vec{q}}{m_N}\times\vec{v}^{\perp} + a_4\ \vec{S}_\chi + a_6\ \frac{\vec{q}}{m_N}\vec{S}_\chi\cdot\frac{\vec{q}}{m_N}\right] \\
+ \; & \left[a_8\ \vec{S}_\chi\cdot\vec{v}^{\perp}\right] + \vec{S}_N\cdot\left[a_7\ \vec{v}^{\perp} + a_9\ i\frac{\vec{q}}{m_N}\times\vec{S}_\chi\right] && \text{(parity odd)} \\
+ \; & \left[a_{11}\ i\vec{S}_\chi\cdot\frac{\vec{q}}{m_N}\right] + \vec{S}_N\cdot\left[a_{10}\ i\frac{\vec{q}}{m_N} + a_{12}\ \vec{v}^{\perp}\times\vec{S}_\chi\right] && \text{(time and parity odd)} \\
+ \; & \vec{S}_N\cdot\left[a_{13}\ i\frac{\vec{q}}{m_N}\vec{S}_\chi\cdot\vec{v}^{\perp} + a_{14}\ i\vec{v}^{\perp}\ \vec{S}_\chi\cdot\frac{\vec{q}}{m_N}\right] && \text{(time odd)}
\end{aligned}$$

14 distinct WIMP-nucleon interactions exist at NNLO -- lots of SI/SD interactions. Furthermore, nontrivial isospin: $a = a_0 + a_1\tau_3$

Hermitian operator	$i\vec{q}$	$\vec{v}^{\perp}$	S_χ	S_N
parity	-1	-1	$+1$	$+1$
time reversal	$+1$	-1	-1	-1

where $$\vec{v}^{\perp} = \frac{1}{2}\left[(\vec{v}_{\chi,\mathrm{in}} - \vec{v}_{N,\mathrm{in}}) + (\vec{v}_{\chi,\mathrm{out}} - \vec{v}_{N,\mathrm{out}})\right]$$

$$\begin{aligned}
H_{ET} = & \left[a_1 + \cancel{a_2}\, \vec{v}^{\perp}\cdot\vec{v}^{\perp} + a_5\, i\vec{S}_\chi\cdot\left(\frac{\vec{q}}{m_N}\times\vec{v}^{\perp}\right)\right] + \vec{S}_N\cdot\left[a_3\, i\frac{\vec{q}}{m_N}\times\vec{v}^{\perp} + a_4\, \vec{S}_\chi + a_6\, \frac{\vec{q}}{m_N}\vec{S}_\chi\cdot\frac{\vec{q}}{m_N}\right] \\
+ & \left[a_8\, \vec{S}_\chi\cdot\vec{v}^{\perp}\right] + \vec{S}_N\cdot\left[a_7\, \vec{v}^{\perp} + a_9\, i\frac{\vec{q}}{m_N}\times\vec{S}_\chi\right] \qquad \text{(parity odd)} \\
+ & \left[a_{11}\, i\vec{S}_\chi\cdot\frac{\vec{q}}{m_N}\right] + \vec{S}_N\cdot\left[a_{10}\, i\frac{\vec{q}}{m_N} + a_{12}\, \vec{v}^{\perp}\times\vec{S}_\chi\right] \qquad \text{(time and parity odd)} \\
+ & \vec{S}_N\cdot\left[a_{13}\, i\frac{\vec{q}}{m_N}\vec{S}_\chi\cdot\vec{v}^{\perp} + a_{14}\, i\vec{v}^{\perp}\, \vec{S}_\chi\cdot\frac{\vec{q}}{m_N}\right] \qquad \text{(time odd)}
\end{aligned}$$

one of these can be eliminated, as it does not correspond to the leading order reduction of any candidate ultraviolet theory

Hermitian operator	$i\vec{q}$	$\vec{v}^{\perp}$	S_χ	S_N
parity	-1	-1	$+1$	$+1$
time reversal	$+1$	-1	-1	-1

where $$\vec{v}^{\perp} = \frac{1}{2}\left[(\vec{v}_{\chi,\mathrm{in}} - \vec{v}_{N,\mathrm{in}}) + (\vec{v}_{\chi,\mathrm{out}} - \vec{v}_{N,\mathrm{out}})\right]$$

$$\begin{aligned}
H_{ET} = & \left[a_1 + \cancel{a_2}\, \vec{v}^{\perp}\cdot\vec{v}^{\perp} + a_5\, i\vec{S}_\chi\cdot\left(\frac{\vec{q}}{m_N}\times\vec{v}^{\perp}\right)\right] + \vec{S}_N\cdot\left[a_3\, i\frac{\vec{q}}{m_N}\times\vec{v}^{\perp} + a_4\, \vec{S}_\chi + a_6\, \frac{\vec{q}}{m_N}\vec{S}_\chi\cdot\frac{\vec{q}}{m_N}\right] \\
+ & \left[a_8\, \vec{S}_\chi\cdot\vec{v}^{\perp}\right] + \vec{S}_N\cdot\left[a_7\, \vec{v}^{\perp} + a_9\, i\frac{\vec{q}}{m_N}\times\vec{S}_\chi\right] \qquad \text{(parity odd)} \\
+ & \left[a_{11}\, i\vec{S}_\chi\cdot\frac{\vec{q}}{m_N}\right] + \vec{S}_N\cdot\left[a_{10}\, i\frac{\vec{q}}{m_N} + a_{12}\, \vec{v}^{\perp}\times\vec{S}_\chi\right] \qquad \text{(time and parity odd)} \\
+ & \vec{S}_N\cdot\left[a_{13}\, i\frac{\vec{q}}{m_N}\vec{S}_\chi\cdot\vec{v}^{\perp} + a_{14}\, i\vec{v}^{\perp}\, \vec{S}_\chi\cdot\frac{\vec{q}}{m_N}\right] \qquad \text{(time odd)} \\
- & \underline{\left((\vec{S}_\chi\times\vec{v}^{\perp})\cdot\frac{\vec{q}}{m_N}\right)(\vec{S}_N\cdot\frac{\vec{q}}{m_N})}
\end{aligned}$$

one must be added for the opposite reason (an orphaned UV theory)

The WIMP-nucleus Hamiltonian can be constructed

$$\sum_{\tau=0,1}\Bigg[l_0^\tau \sum_{i=1}^{A} e^{-i\vec{q}\cdot\vec{x}_i} + l_0^{A\tau}\sum_{i=1}^{A}\frac{1}{2M}\left(-\frac{1}{i}\overleftarrow{\nabla}_i\cdot\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\vec{\sigma}(i)\cdot\frac{1}{i}\overrightarrow{\nabla}_i\right)$$
$$+\ \vec{l}_5^\tau\cdot\sum_{i=1}^{A}\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i} + \vec{l}_M^\tau\cdot\sum_{i=1}^{A}\frac{1}{2M}\left(-\frac{1}{i}\overleftarrow{\nabla}_i e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\frac{1}{i}\overrightarrow{\nabla}_i\right)$$
$$+\ \vec{l}_E^\tau\cdot\sum_{i=1}^{A}\frac{1}{2M}\left(\overleftarrow{\nabla}_i\times\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\vec{\sigma}(i)\times\overrightarrow{\nabla}_i\right)\Bigg]_{int} t^\tau(i)$$

Follows from the one-body replacement $\vec{v}_N \to \sum_{i=1}^{A}\vec{v}_N(i)$

though for specific theories, one can include additional nuclear physics, such as exchange currents, meson loop corrections*

*e.g., chiral EFT: Cirigliano, Graesser, Ovanesyan arXiv:1205.2695
Menendez, Gazit, Schwenk arXiv:1208:1094

familiar interactions:

(generalized) vector charge

$$\sum_{\tau=0,1}\Bigg[l_0^\tau \boxed{\sum_{i=1}^{A} e^{-i\vec{q}\cdot\vec{x}_i}} + l_0^{A\tau}\sum_{i=1}^{A}\frac{1}{2M}\left(-\frac{1}{i}\overleftarrow{\nabla}_i\cdot\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\vec{\sigma}(i)\cdot\frac{1}{i}\overrightarrow{\nabla}_i\right)$$

$$+\ \vec{l}_5^\tau\cdot\sum_{i=1}^{A}\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i} + \vec{l}_M^\tau\cdot\sum_{i=1}^{A}\frac{1}{2M}\left(-\frac{1}{i}\overleftarrow{\nabla}_i e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\frac{1}{i}\overrightarrow{\nabla}_i\right)$$

$$+\ \vec{l}_E^\tau\cdot\sum_{i=1}^{A}\frac{1}{2M}\left(\overleftarrow{\nabla}_i\times\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\vec{\sigma}(i)\times\overrightarrow{\nabla}_i\right)\Bigg]_{int} t^\tau(i)$$

This is the vector charge density probed in elastic electron scattering or in coherent neutrino scattering

familiar interactions:

axial-vector charge

$$\sum_{\tau=0,1}\Bigg[l_0^\tau \sum_{i=1}^{A} e^{-i\vec{q}\cdot\vec{x}_i} + l_0^{A\tau} \sum_{i=1}^{A} \boxed{\frac{1}{2M}\left(-\frac{1}{i}\overleftarrow{\nabla}_i\cdot\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\vec{\sigma}(i)\cdot\frac{1}{i}\overrightarrow{\nabla}_i\right)}$$

$$+ \vec{l}_5^\tau\cdot\sum_{i=1}^{A}\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i} + \vec{l}_M^\tau\cdot\sum_{i=1}^{A}\frac{1}{2M}\left(-\frac{1}{i}\overleftarrow{\nabla}_i e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\frac{1}{i}\overrightarrow{\nabla}_i\right)$$

$$+ \vec{l}_E^\tau\cdot\sum_{i=1}^{A}\frac{1}{2M}\left(\overleftarrow{\nabla}_i\times\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\vec{\sigma}(i)\times\overrightarrow{\nabla}_i\right)\Bigg]_{int} t^\tau(i)$$

This is an operator density studied in beta decay, through only through inelastic $0^- \square\ 0^+$ transitions.

familiar interactions:

axial-vector spin current

$$\sum_{\tau=0,1}\Bigg[l_0^\tau \sum_{i=1}^{A} e^{-i\vec{q}\cdot\vec{x}_i} + l_0^{A\tau}\sum_{i=1}^{A}\frac{1}{2M}\left(-\frac{1}{i}\overleftarrow{\nabla}_i\cdot\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\vec{\sigma}(i)\cdot\frac{1}{i}\overrightarrow{\nabla}_i\right)$$

$$+\ \vec{l}_5^\tau\cdot\sum_{i=1}^{A}\boxed{\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i}} + \vec{l}_M^\tau\cdot\sum_{i=1}^{A}\frac{1}{2M}\left(-\frac{1}{i}\overleftarrow{\nabla}_i e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\frac{1}{i}\overrightarrow{\nabla}_i\right)$$

$$+\ \vec{l}_E^\tau\cdot\sum_{i=1}^{A}\frac{1}{2M}\left(\overleftarrow{\nabla}_i\times\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\vec{\sigma}(i)\times\overrightarrow{\nabla}_i\right)\Bigg]_{int} t^\tau(i)$$

This spin density dominates neutrino-nucleus inelastic scattering at solar and supernova neutrino energies. We know a lot about its elastic moments due to nuclear magnetic moments, etc.

familiar interactions:

vector convection current

$$\sum_{\tau=0,1}\Bigg[l_0^\tau \sum_{i=1}^{A} e^{-i\vec{q}\cdot\vec{x}_i} + l_0^{A\tau}\sum_{i=1}^{A}\frac{1}{2M}\left(-\frac{1}{i}\overleftarrow{\nabla}_i\cdot\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\vec{\sigma}(i)\cdot\frac{1}{i}\overrightarrow{\nabla}_i\right)$$

$$+\ \vec{l}_5^\tau\cdot\sum_{i=1}^{A}\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i} + \vec{l}_M^\tau\cdot\sum_{i=1}^{A}\boxed{\frac{1}{2M}\left(-\frac{1}{i}\overleftarrow{\nabla}_i e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\frac{1}{i}\overrightarrow{\nabla}_i\right)}$$

$$+\ \vec{l}_E^\tau\cdot\sum_{i=1}^{A}\frac{1}{2M}\left(\overleftarrow{\nabla}_i\times\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\vec{\sigma}(i)\times\overrightarrow{\nabla}_i\right)\Bigg]_{int} t^\tau(i)$$

This is the vector convection-current response familiar from inelastic electron and neutrino scattering; elastic response known from back-angle magnetic electron scattering and from atomic hyperfine interactions

familiar interactions:

$$\sum_{\tau=0,1}\Bigg[l_0^\tau \sum_{i=1}^{A} e^{-i\vec{q}\cdot\vec{x}_i} + l_0^{A\tau}\sum_{i=1}^{A}\frac{1}{2M}\left(-\frac{1}{i}\overleftarrow{\nabla}_i\cdot\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\vec{\sigma}(i)\cdot\frac{1}{i}\overrightarrow{\nabla}_i\right)$$

$$+\ \vec{l}_5^\tau\cdot\sum_{i=1}^{A}\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i} + \vec{l}_M^\tau\cdot\sum_{i=1}^{A}\frac{1}{2M}\left(-\frac{1}{i}\overleftarrow{\nabla}_i e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\frac{1}{i}\overrightarrow{\nabla}_i\right)$$

$$+\ \vec{l}_E^\tau\cdot\sum_{i=1}^{A}\boxed{\frac{1}{2M}\left(\overleftarrow{\nabla}_i\times\vec{\sigma}(i)e^{-i\vec{q}\cdot\vec{x}_i} + e^{-i\vec{q}\cdot\vec{x}_i}\vec{\sigma}(i)\times\overrightarrow{\nabla}_i\right)}\Bigg]_{int} t^\tau(i)$$

vector spin-velocity current

The most exotic of the contributing densities: appears at order $1/M^2$ in nonrelativistic reductions of weak operators

These are all the nucleon densities than can be constructed to first order in momenta

8 of the 14 EFT interactions involve derivative couplings. There is no way to treat such a DM operator properly in a point-nucleus limit

- Take, e.g., the Galilean-invariant interaction

$$\sum_{i=1}^{A} \vec{S}_N(i) \cdot (\vec{v}_\chi - \vec{v}_N(i))$$

- If one were to properly evaluate this for a point nucleus (rarely if ever done), one would get a spin-dependent contribution,

$$(\vec{v}_\chi - \vec{v}_T(i)) \cdot \sum_{i=1}^{A} \vec{S}_N(i) \qquad \sim 10^{-3}$$

- But in fact this is just one of A Galilean-invariant Jacobi momenta in the problem. The neglected part of the operator is

$$\frac{1}{A} \sum_{i>j=1}^{A} (\vec{S}_N(i) - \vec{S}_N(j)) \cdot (\vec{v}_N(i) - \vec{v}_N(j)) \quad \sim 10^{-1}$$

- Because this operator carries odd nuclear parity, it must be evaluated for a composite nucleus, that is, using the full operator

$$\vec{v}_N(i) \to \vec{v}_N(i) e^{i\vec{q}\cdot\vec{r}(i)}$$

- This does not weaken the operator as $\vec{q}\cdot\vec{r}(i) \sim 1$. With a bit of algebra operator such as the following can be identified

$$\frac{q}{m_N}\vec{l}(i) \qquad \sim 10^{-1}$$

- Such composite operators, not the point operators traditionally used in DM studies, *clearly* dominate the response for a large class of interactions as

$$\frac{q}{m_N} \gg v_\chi$$

yet have been universally neglected

One can reach the same conclusions using only symmetry arguments

The form of the DM-nucleus elastic scattering cross section can be determined from the underlying symmetry properties of the nuclear currents in combination with the good approximate P, CP of the nucleus

charges:

	even	odd
vector	C_0	C_1
axial	C_0^5	C_1^5

currents:

	even	odd	even	odd	even	odd
axial spin	L_0^5	L_1^5	$T_2^{5\mathrm{el}}$	$T_1^{5\mathrm{el}}$	$T_2^{5\mathrm{mag}}$	$T_1^{5\mathrm{mag}}$
vector velocity	L_0	L_1	T_2^{el}	T_1^{el}	T_2^{mag}	T_1^{mag}
vector spin – velocity	L_0	L_1	T_2^{el}	T_1^{el}	T_2^{mag}	T_1^{mag}

where we list only the leading multipoles in J above

Response constrained by good parity and time reversal of nuclear g.s.

	even	odd
vector	C_0	$\cancel{C_1}$
axial	$\cancel{C_0^5}$	C_1^5

	even	odd	even	odd	even	odd
axial spin	$\cancel{L_0^5}$	L_1^5	$\cancel{T_2^{5\mathrm{el}}}$	$T_1^{5\mathrm{el}}$	$T_2^{5\mathrm{mag}}$	$\cancel{T_1^{5\mathrm{mag}}}$
vector velocity	L_0	$\cancel{L_1}$	T_2^{el}	$\cancel{T_1^{\mathrm{el}}}$	$\cancel{T_2^{\mathrm{mag}}}$	T_1^{mag}
vector spin – velocity	L_0	$\cancel{L_1}$	T_2^{el}	$\cancel{T_1^{\mathrm{el}}}$	$\cancel{T_2^{\mathrm{mag}}}$	T_1^{mag}

Response constrained by good parity and time reversal of nuclear g.s.

	even	odd
vector	C_0	$\cancel{C_1}$
axial	$\cancel{C_0^5}$	$\cancel{C_1^5}$

	even	odd	even	odd	even	odd
axial spin	$\cancel{L_0^5}$	L_1^5	$\cancel{T_2^{5\mathrm{el}}}$	$T_1^{5\mathrm{el}}$	$\cancel{T_2^{5\mathrm{mag}}}$	$\cancel{T_1^{5\mathrm{mag}}}$
vector velocity	$\cancel{L_0}$	$\cancel{L_1}$	$\cancel{T_2^{\mathrm{el}}}$	$\cancel{T_1^{\mathrm{el}}}$	$\cancel{T_2^{\mathrm{mag}}}$	T_1^{mag}
vector spin – velocity	L_0	$\cancel{L_1}$	T_2^{el}	$\cancel{T_1^{\mathrm{el}}}$	$\cancel{T_2^{\mathrm{mag}}}$	$\cancel{T_1^{\mathrm{mag}}}$

Response constrained by good parity and time reversal of nuclear g.s.

	even	odd
vector	C_0	
axial		

	even	odd	even	odd	even	odd
axial spin		L_1^5		$T_1^{5\mathrm{el}}$		
vector velocity						T_1^{mag}
vector spin – velocity	L_0		T_2^{el}			

6 (not 2!) independent responses based on symmetry of 4-current densities

So we cut to the chase and do the ET → nucleus calculation, which yields a detailed result fully consistent with these symmetry considerations (a reasonable starting point yields a reasonable answer)

The results tell one

1) what in principal can be learned about WIMPS from ES
2) what ambiguities in the low-energy constants will remain even after a complete set of ES experiments is done

theorist's analog of the experimentalist's big magnet picture

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} \sim \frac{4\pi}{2J_i+1} \sum_{\tau=0,1} \sum_{\tau'=0,1} \Bigg\{ & R_C^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=0,2,\ldots}^{\infty} \langle J_i || \, M_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, M_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_L^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=0,2,\ldots}^{\infty} \langle J_i || \, \Phi''_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Phi''_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_{L/C}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=0,2,\ldots}^{\infty} \langle J_i || \, \Phi''_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, M_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_{T^{el}}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=2,4,\ldots}^{\infty} \langle J_i || \, \tilde{\Phi}'_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \tilde{\Phi}'_{J;\tau'}(q) \, || J_i \rangle \\
& + R_{L^5}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=1,3,\ldots}^{\infty} \langle J_i || \, \Sigma''_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Sigma''_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_{T^{mag}}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=1,3,\ldots}^{\infty} \langle J_i || \, \Delta_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Delta_{J;\tau'}(q) \, || J_i \rangle \\
& + R_{T^{el5}}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=1,3,\ldots}^{\infty} \langle J_i || \, \Sigma'_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Sigma'_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_{T^{mag}/T^{el5}}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=1,3,\ldots}^{\infty} \langle J_i || \, \Delta_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Sigma'_{J;\tau'}(q) \, || J_i \rangle \Bigg\}
\end{aligned}
$$

Response $\times \left[\frac{4\pi}{2J_i+1}\right]^{-1}$	Leading Multipole	Long-wavelength Limit	Response Type
$\sum_{J=0,2,\ldots}^{\infty} \lvert\langle J_i \lVert M_{JM} \rVert J_i\rangle\rvert^2$	$M_{00}(q\vec{x}_i)$	$\frac{1}{\sqrt{4\pi}}\boxed{1(i)}$	M_{JM} : Charge
$\sum_{J=1,3,\ldots}^{\infty} \lvert\langle J_i \lVert \Sigma''_{JM} \rVert J_i\rangle\rvert^2$	$\Sigma''_{1M}(q\vec{x}_i)$	$\frac{1}{2\sqrt{3\pi}}\boxed{\sigma_{1M}(i)}$	L^5_{JM} : Axial Longitudinal
$\sum_{J=1,3,\ldots}^{\infty} \lvert\langle J_i \lVert \Sigma'_{JM} \rVert J_i\rangle\rvert^2$	$\Sigma'_{1M}(q\vec{x}_i)$	$\frac{1}{\sqrt{6\pi}}\boxed{\sigma_{1M}(i)}$	$T^{\mathrm{el}5}_{JM}$: Axial Transverse Electric
$\sum_{J=1,3,\ldots}^{\infty} \lvert\langle J_i \lVert \frac{q}{m_N}\Delta_{JM} \rVert J_i\rangle\rvert^2$	$\frac{q}{m_N}\Delta_{1M}(q\vec{x}_i)$	$-\frac{q}{2m_N\sqrt{6\pi}}\boxed{\ell_{1M}(i)}$	T^{mag}_{JM} : Transverse Magnetic
$\sum_{J=0,2,\ldots}^{\infty} \lvert\langle J_i \lVert \frac{q}{m_N}\Phi''_{JM} \rVert J_i\rangle\rvert^2$	$\frac{q}{m_N}\Phi''_{00}(q\vec{x}_i)$	$-\frac{q}{3m_N\sqrt{4\pi}}\boxed{\vec{\sigma}(i)\cdot\vec{\ell}(i)}$	L_{JM} : Longitudinal
	$\frac{q}{m_N}\Phi''_{2M}(q\vec{x}_i)$	$-\frac{q}{m_N\sqrt{30\pi}}\boxed{[x_i \otimes (\vec{\sigma}(i) \times \frac{1}{i}\vec{\nabla})_1]_{2M}}$	
$\sum_{J=2,4,\ldots}^{\infty} \lvert\langle J_i \lVert \frac{q}{m_N}\tilde{\Phi}'_{JM} \rVert J_i\rangle\rvert^2$	$\frac{q}{m_N}\tilde{\Phi}'_{2M}(q\vec{x}_i)$	$-\frac{q}{m_N\sqrt{20\pi}}\boxed{[x_i \otimes (\vec{\sigma}(i) \times \frac{1}{i}\vec{\nabla})_1]_{2M}}$	T^{el}_{JM} : Transverse Electric

Two scalar (one scalar/tensor) , three vector, one tensor

Calculate in SM the responses for the key isotopes...

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} \sim \frac{4\pi}{2J_i+1} \sum_{\tau=0,1} \sum_{\tau'=0,1} \Bigg\{ & R_C^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=0,2,\ldots}^{\infty} \langle J_i || \, M_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, M_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_L^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=0,2,\ldots}^{\infty} \langle J_i || \, \Phi''_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Phi''_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_{L/C}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=0,2,\ldots}^{\infty} \langle J_i || \, \Phi''_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, M_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_{T^{el}}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=2,4,\ldots}^{\infty} \langle J_i || \, \tilde{\Phi}'_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \tilde{\Phi}'_{J;\tau'}(q) \, || J_i \rangle \\
& + R_{L^5}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=1,3,\ldots}^{\infty} \langle J_i || \, \Sigma''_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Sigma''_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_{T^{mag}}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=1,3,\ldots}^{\infty} \langle J_i || \, \Delta_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Delta_{J;\tau'}(q) \, || J_i \rangle \\
& + R_{T^{el5}}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=1,3,\ldots}^{\infty} \langle J_i || \, \Sigma'_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Sigma'_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_{T^{mag}/T^{el5}}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=1,3,\ldots}^{\infty} \langle J_i || \, \Delta_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Sigma'_{J;\tau'}(q) \, || J_i \rangle \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} \sim \frac{4\pi}{2J_i+1} \sum_{\tau=0,1} \sum_{\tau'=0,1} \Bigg\{ & R_C^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=0,2,\ldots}^{\infty} \langle J_i || \, M_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, M_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_L^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=0,2,\ldots}^{\infty} \langle J_i || \, \Phi''_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Phi''_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_{L/C}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=0,2,\ldots}^{\infty} \langle J_i || \, \Phi''_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, M_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_{T^{el}}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=2,4,\ldots}^{\infty} \langle J_i || \, \tilde{\Phi}'_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \tilde{\Phi}'_{J;\tau'}(q) \, || J_i \rangle \\
& + R_{L^5}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=1,3,\ldots}^{\infty} \langle J_i || \, \Sigma''_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Sigma''_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_{T^{mag}}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=1,3,\ldots}^{\infty} \langle J_i || \, \Delta_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Delta_{J;\tau'}(q) \, || J_i \rangle \\
& + R_{T^{el5}}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=1,3,\ldots}^{\infty} \langle J_i || \, \Sigma'_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Sigma'_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_{T^{mag}/T^{el5}}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=1,3,\ldots}^{\infty} \langle J_i || \, \Delta_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Sigma'_{J;\tau'}(q) \, || J_i \rangle \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} \sim \frac{4\pi}{2J_i+1} \sum_{\tau=0,1} \sum_{\tau'=0,1} \Bigg\{ & R_C^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=0,2,\ldots}^{\infty} \langle J_i || \, M_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, M_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_L^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=0,2,\ldots}^{\infty} \langle J_i || \, \Phi''_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Phi''_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_{L/C}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=0,2,\ldots}^{\infty} \langle J_i || \, \Phi''_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, M_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_{T^{el}}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=2,4,\ldots}^{\infty} \langle J_i || \, \tilde{\Phi}'_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \tilde{\Phi}'_{J;\tau'}(q) \, || J_i \rangle \\
& + R_{L^5}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=1,3,\ldots}^{\infty} \langle J_i || \, \Sigma''_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Sigma''_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_{T^{mag}}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=1,3,\ldots}^{\infty} \langle J_i || \, \Delta_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Delta_{J;\tau'}(q) \, || J_i \rangle \\
& + R_{T^{el5}}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=1,3,\ldots}^{\infty} \langle J_i || \, \Sigma'_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Sigma'_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_{T^{mag}/T^{el5}}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=1,3,\ldots}^{\infty} \langle J_i || \, \Delta_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Sigma'_{J;\tau'}(q) \, || J_i \rangle \Bigg\}
\end{aligned}
$$

we understand/control this

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} \sim \frac{4\pi}{2J_i+1} \sum_{\tau=0,1} \sum_{\tau'=0,1} \Bigg\{ & R_C^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=0,2,\ldots}^{\infty} \langle J_i || \, M_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, M_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_L^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=0,2,\ldots}^{\infty} \langle J_i || \, \Phi''_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Phi''_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_{L/C}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=0,2,\ldots}^{\infty} \langle J_i || \, \Phi''_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, M_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_{T^{el}}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=2,4,\ldots}^{\infty} \langle J_i || \, \tilde{\Phi}'_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \tilde{\Phi}'_{J;\tau'}(q) \, || J_i \rangle \\
& + R_{L^5}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=1,3,\ldots}^{\infty} \langle J_i || \, \Sigma''_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Sigma''_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_{T^{mag}}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=1,3,\ldots}^{\infty} \langle J_i || \, \Delta_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Delta_{J;\tau'}(q) \, || J_i \rangle \\
& + R_{T^{el5}}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=1,3,\ldots}^{\infty} \langle J_i || \, \Sigma'_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Sigma'_{J;\tau'}(q) \, || J_i \rangle \\
& + \frac{\vec{q}^{\,2}}{m_N^2} R_{T^{mag}/T^{el5}}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}) \sum_{J=1,3,\ldots}^{\infty} \langle J_i || \, \Delta_{J;\tau}(q) \, || J_i \rangle \langle J_i || \, \Sigma'_{J;\tau'}(q) \, || J_i \rangle \Bigg\}
\end{aligned}
$$

we hope to probe this

and this is precisely the WIMP physics we can learn

$$
\begin{aligned}
R_M^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= c_1^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left[\frac{\vec{q}^2}{m_N^2}\vec{v}_T^{\perp 2} c_5^\tau c_5^{\tau'} + \vec{v}_T^{\perp 2} c_8^\tau c_8^{\tau'} + \frac{\vec{q}^2}{m_N^2} c_{11}^\tau c_{11}^{\tau'}\right] \\
R_{\Phi''}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= \frac{\vec{q}^2}{4m_N^2} c_3^\tau c_3^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau - \frac{\vec{q}^2}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{\vec{q}^2}{m_N^2} c_{15}^{\tau'}\right) \\
R_{\Phi'' M}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= c_3^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left(c_{12}^\tau - \frac{\vec{q}^2}{m_N^2} c_{15}^\tau\right) c_{11}^{\tau'} \\
R_{\tilde{\Phi}'}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= \frac{j_\chi(j_\chi+1)}{12}\left[c_{12}^\tau c_{12}^{\tau'} + \frac{\vec{q}^2}{m_N^2} c_{13}^\tau c_{13}^{\tau'}\right] \\
R_{\Sigma''}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= \frac{\vec{q}^2}{4m_N^2} c_{10}^\tau c_{10}^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\Big[c_4^\tau c_4^{\tau'} + \\
&\quad \frac{\vec{q}^2}{m_N^2}(c_4^\tau c_6^{\tau'} + c_6^\tau c_4^{\tau'}) + \frac{\vec{q}^4}{m_N^4} c_6^\tau c_6^{\tau'} + \vec{v}_T^{\perp 2} c_{12}^\tau c_{12}^{\tau'} + \frac{\vec{q}^2}{m_N^2}\vec{v}_T^{\perp 2} c_{13}^\tau c_{13}^{\tau'}\Big] \\
R_{\Sigma'}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= \frac{1}{8}\left[\frac{\vec{q}^2}{m_N^2}\vec{v}_T^{\perp 2} c_3^\tau c_3^{\tau'} + \vec{v}_T^{\perp 2} c_7^\tau c_7^{\tau'}\right] + \frac{j_\chi(j_\chi+1)}{12}\Big[c_4^\tau c_4^{\tau'} + \\
&\quad \frac{\vec{q}^2}{m_N^2} c_9^\tau c_9^{\tau'} + \frac{\vec{v}_T^{\perp 2}}{2}\left(c_{12}^\tau - \frac{\vec{q}^2}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{\vec{q}^2}{m_N^2} c_{15}^{\tau\prime}\right) + \frac{\vec{q}^2}{2m_N^2}\vec{v}_T^{\perp 2} c_{14}^\tau c_{14}^{\tau'}\Big] \\
R_{\Delta}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= \frac{j_\chi(j_\chi+1)}{3}\left[\frac{\vec{q}^2}{m_N^2} c_5^\tau c_5^{\tau'} + c_8^\tau c_8^{\tau'}\right] \\
R_{\Delta\Sigma'}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= \frac{j_\chi(j_\chi+1)}{3}\left[c_5^\tau c_4^{\tau'} - c_8^\tau c_9^{\tau'}\right].
\end{aligned}
$$

Bottom lines:

DM practitioners purport to be studying unknown low-energy weak interactions, yet analyze experiments without the operators that we know are essential to describing *known* SM electroweak interactions

ES can in principle give us 8 constraints on DM interactions

To obtain those constraints one must turn the "nuclear physics knobs" to vary the contributing nuclear operators

The eight constraints are functions of the 2x14 parameters that govern the NNLO effective theory

So nuclear ES cannot fully determine the theory; conversely, the notion that one 3rd-generation experiment will suffice is silly

In the case of about half of the effective operators one can construct, it is impossible to represent the ES answer in terms of standard spin-independent/spin-dependent responses

a) the response magnitude will be in error by $\sim 10 \frac{\mu_T^2}{m_N^2} \sim 10^4$

b) the predicted multipolarity will be wrong

c) the predicted scaling of cross section with both WIMP and target mass will be wrong

Model-independent WIMP Scattering Responses and Event Rates: A Mathematica Package for Experimental Analysis

Nikhil Anand[1], A. Liam Fitzpatrick[2], W. C. Haxton[1]

August 27, 2013

The community's reliance on simplified descriptions of WIMP-nucleus interactions reflects the absence of analysis tools that integrate general theories of dark matter with standard treatments of nuclear response functions. To bridge this gap, we have constructed a public-domain Mathematica package for WIMP analyses based on our effective theory formulation. Script inputs are 1) the coefficients of the effective theory, through which one can characterize the low-energy consequences of arbitrary ultraviolet theories of WIMP interactions; and 2) one-body density matrices for commonly used targets, the most compact description of the relevant nuclear physics. The generality of the effective theory expansion guarantees that the script will remain relevant as new ultraviolet theories are explored; the use of density matrices to factor the nuclear physics from the particle physics will allow nuclear structure theorists to update the script as new calculations become available, independent of specific particle-physics contexts. The Mathematica package outputs the resulting response functions (and associated form factors) and also the differential event rate, once a galactic WIMP velocity profile is specified, and thus in its present form provides a complete framework for experimental analysis. The Mathematica script requires no *a priori* knowledge of the details of the non-relativistic effective field theory or nuclear physics, though the core concepts are reviewed here and in [1].

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