TAUP 2013
Asilomar, September 9-13, 2013

The Nuclear Matrix Elements for $0\nu\beta\beta$ decay: Current Status

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Frank Avignone:

Nuclear Matrix Elements are as important as DATA

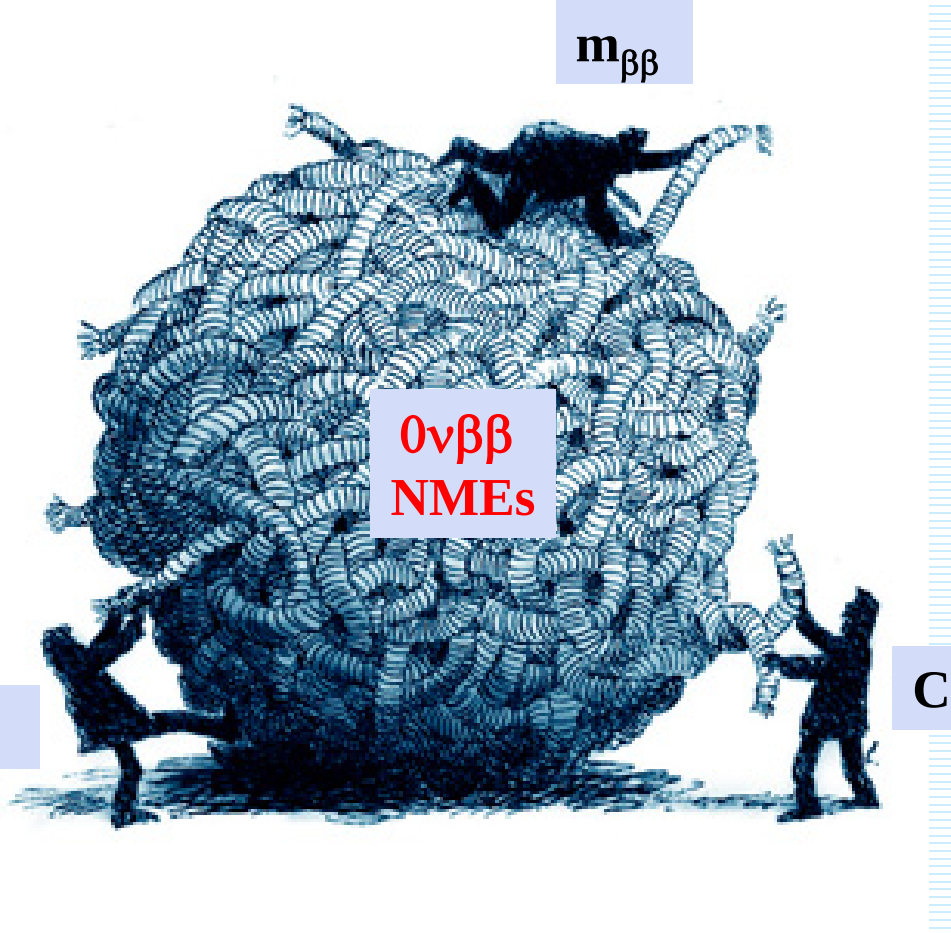
OUTLINE

$0\nu\beta\beta$

$0\nu\varepsilon\varepsilon$

$0\nu\varepsilon\beta$

$m_{\beta\beta}$



$0\nu\beta\beta$ NMEs

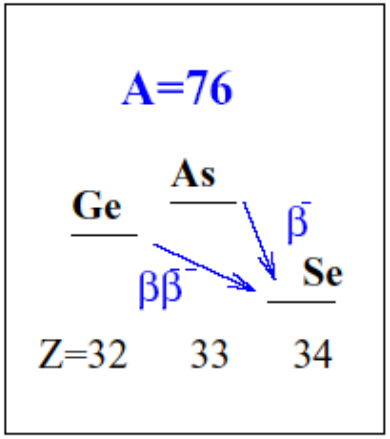
CP-phases

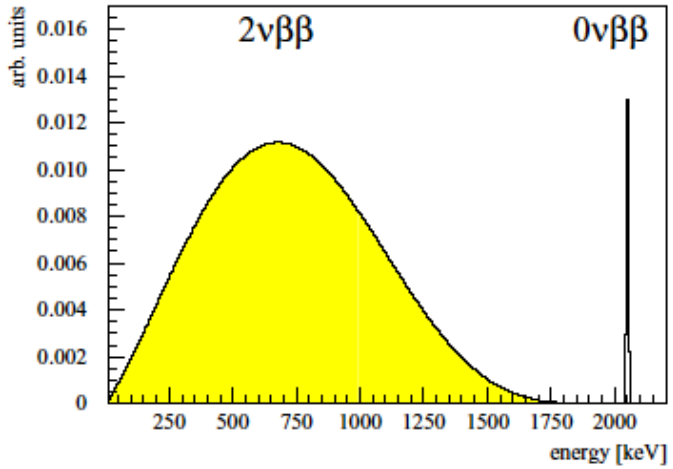
ν mass scale

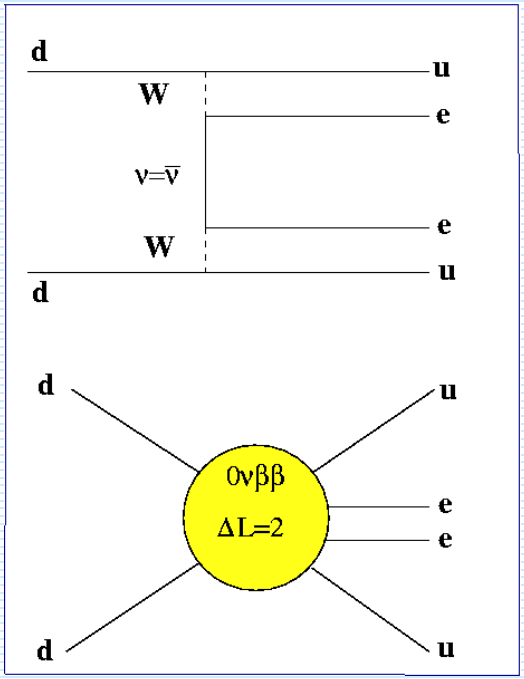
Nuclear structure

Neutrinoless Double-Beta Decay

$$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$$







The answer to the question whether neutrinos are their own antiparticles is of central importance, not only to our understanding of neutrinos, but also to our understanding of the origin of mass.

What is the nature of neutrinos?

ν $\Rightarrow$

GUT's

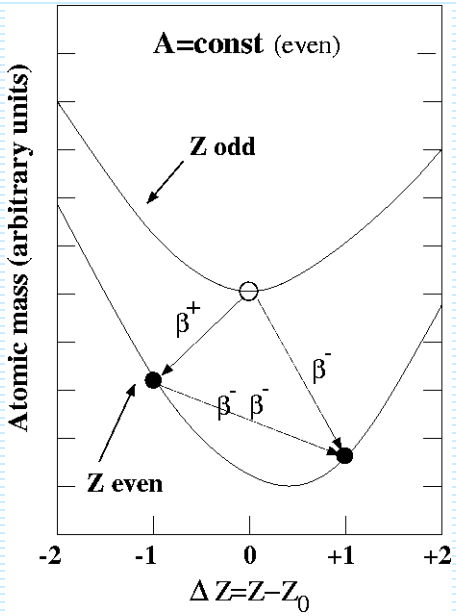
Only the 0νββ-decay can answer this fundamental question

Analogy with kaons: K_0 and $\overline{K_0}$

No NMEs needed!

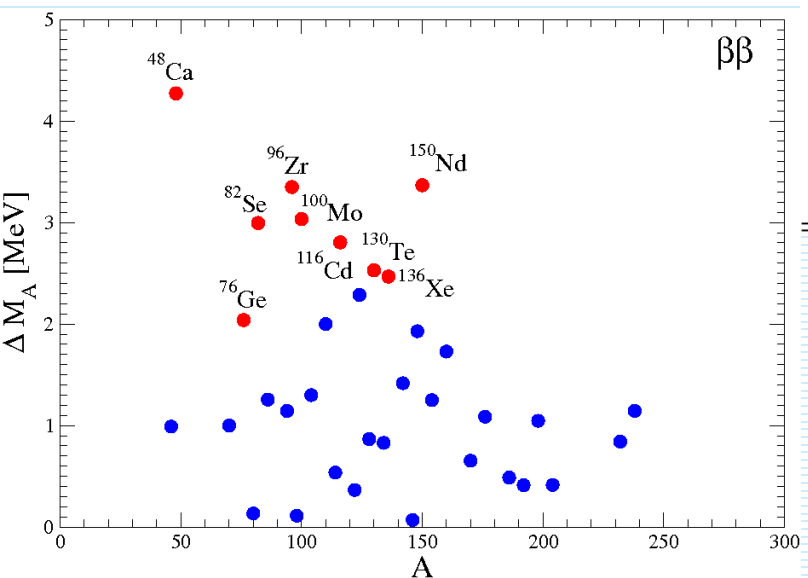
Analogy with π_0

The double beta decay process can be observed due to nuclear pairing interaction that favors energetically the even-even nuclei over the odd-odd nuclei



$$\left(T^{0\nu}_{1/2}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 g_A^4 \left|M^{0\nu}_{\nu}\right|^2 G^{0\nu}$$

transition	$G^{01}(E_0, Z)$ $\times 10^{14} y$	$Q_{\beta\beta}$ [MeV]	Abund. (%)	$\lvert M^{0\nu}\rvert^2$
$^{150}Nd \rightarrow {}^{150}Sm$	26.9	3.667	6	?
$^{48}Ca \rightarrow {}^{48}Ti$	8.04	4.271	0.2	?
$^{96}Zr \rightarrow {}^{96}Mo$	7.37	3.350	3	?
$^{116}Cd \rightarrow {}^{116}Sn$	6.24	2.802	7	?
$^{136}Xe \rightarrow {}^{136}Ba$	5.92	2.479	9	?
$^{100}Mo \rightarrow {}^{100}Ru$	5.74	3.034	10	?
$^{130}Te \rightarrow {}^{130}Xe$	5.55	2.533	34	?
$^{82}Se \rightarrow {}^{82}Kr$	3.53	2.995	9	?
$^{76}Ge \rightarrow {}^{76}Se$	0.79	2.040	8	?



The NMEs for 0νββ-decay must be evaluated using tools of nuclear theory

Neutrinoless double beta decay of ^{110}Pd

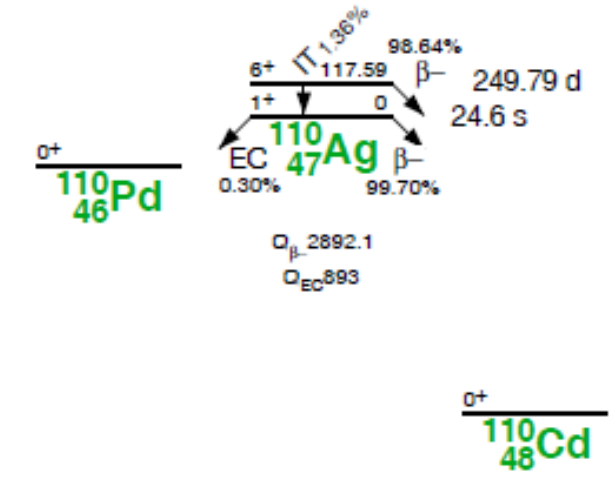
With its high natural abundance, the new results reveal ^{110}Pd to be an excellent candidate for double-β decay studies

Q-Value and Half-Lives for the Double-Beta-Decay Nuclide ^{110}Pd

D. Fink, et al.

Phys. Rev. Lett. 108 (2012) 062502.

	^{82}Se	^{110}Pd
Z	34	46
Abund. (%)	8.73	11.72
Q [keV]	2 995	2 017.8
$G^{0\nu}$ [10^{-15} yr^{-1}]	10.16	4.815
$0\nu\beta\beta$ NME	4.64	5.76
$T^{2\nu}_{1/2}$ [yr]	0.92 10^{20}	1.5(6) 10^{20}(SSD)



Effective Majorana neutrino mass

$$\left(T_{1/2}^{0\nu}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 g_A^4 \left|M_\nu^{0\nu}\right|^2 G^{0\nu}$$

$$\left(T^{0\nu}_{1/2}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 g_A^4 \left|M^{0\nu}_{\nu}\right|^2 G^{0\nu}$$

Absolute ν mass scale

Normal or inverted hierarchy of ν masses

CP-violating phases

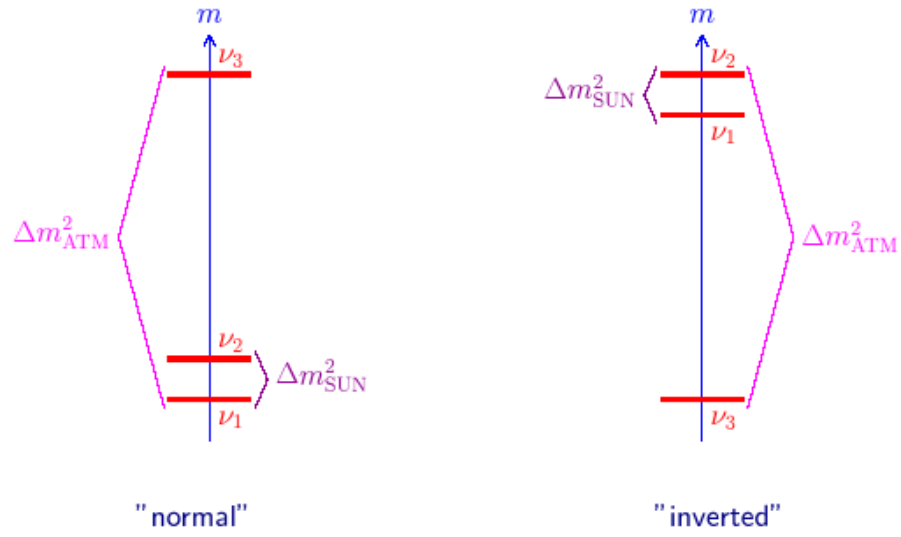
$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

An accurate knowledge of the nuclear matrix elements, which is not available at present, is however a pre-requisite for exploring neutrino properties.

Neutrinos mass spectrum

mass differences:
$|\Delta m^2_{sol}| = 7.65\ 10^{-5}\ eV^2$
$\Delta m^2_{atm} = 2.43\ 10^{-3}\ eV^2$

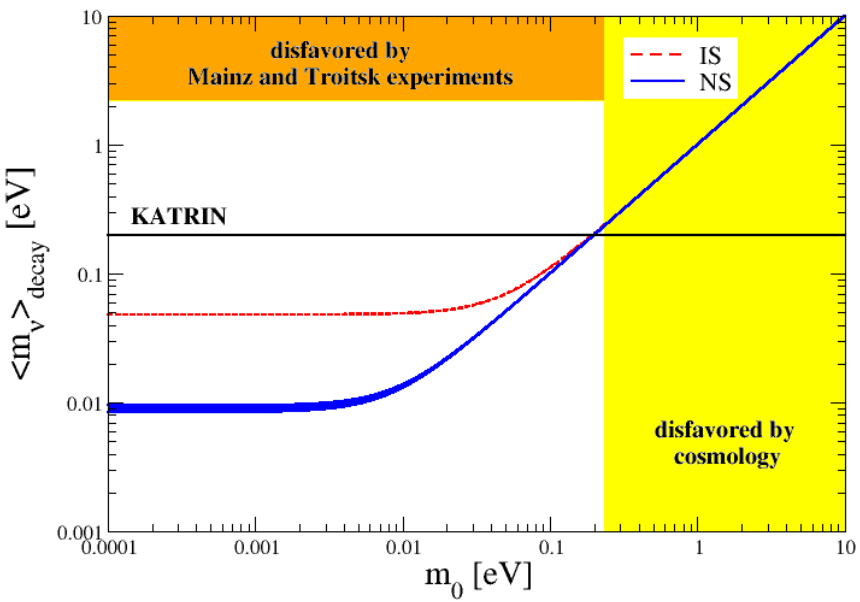


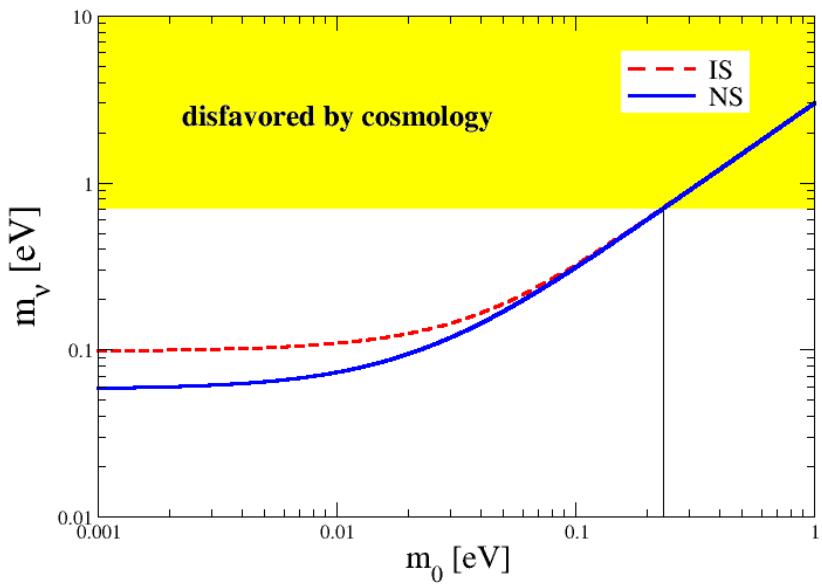
Tritium decay

$$m_\beta = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2\ m_i^2}$$

Cosmology

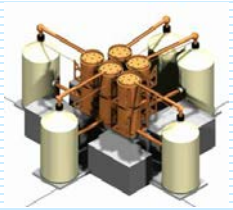
$$\sum_{i=1}^{3} m_i$$



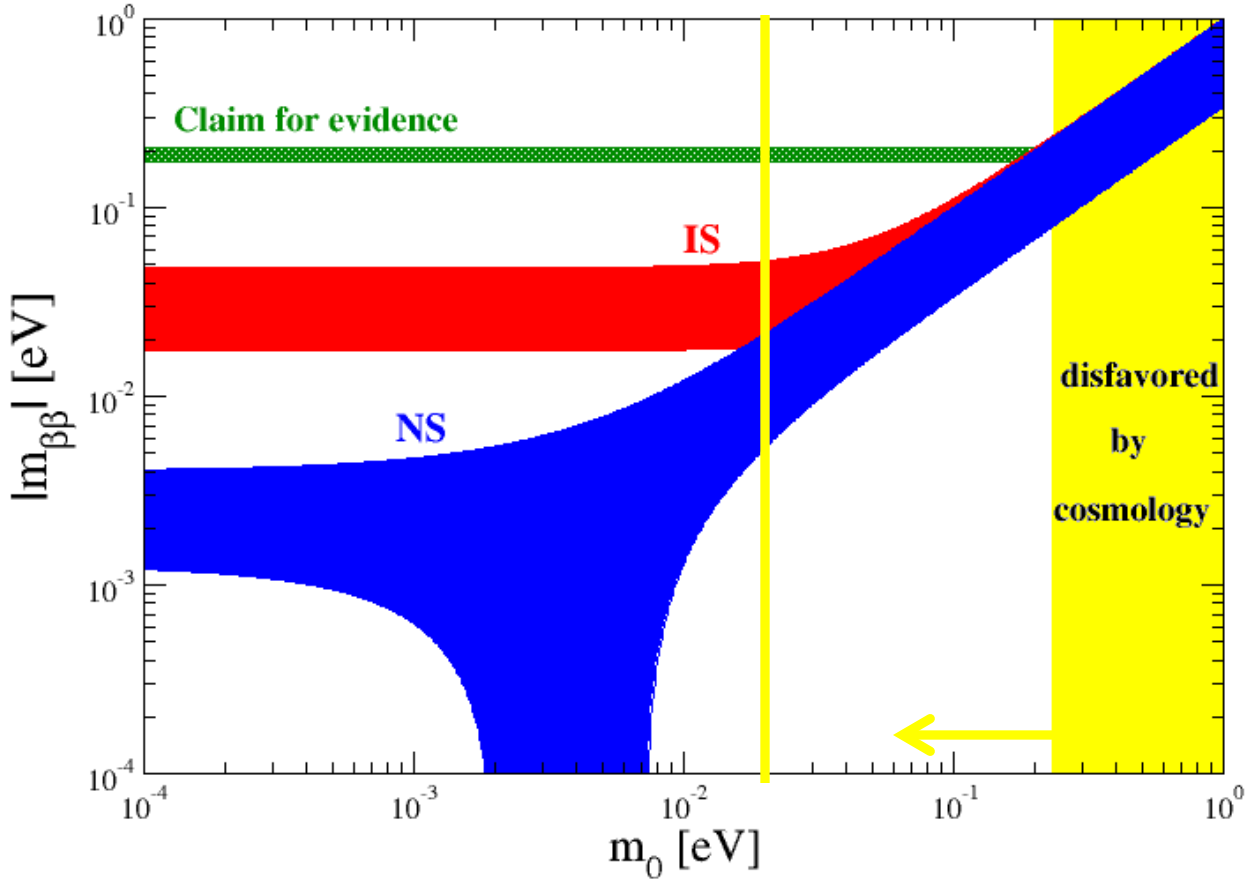


Daya Bay: $\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005$ (March 2012)

$$|m_{\beta\beta}{}^{(3\ \nu)}| = |c_{12}^2 c_{13}^2 e^{2i\alpha_1} m_1 + c_{13}^2 s_{12}^2 e^{2i\alpha_2} m_2 + s_{13}^2 m_3|$$



Issue: Lightest neutrino mass m_0



GUT's
(Rodejohann pres.)

On the possibility of measuring CP Majorana phases in the 0νββ-decay

F.Š., S. Bilenky, A. Faessler, Th. Gutsche, Phys. Rev. D 87, 073002 (2013)

Majorana phases

$$P = diag(e^{-i\alpha_1/2}, e^{-i\alpha_2/2}, e^{-i\alpha_3/2})$$
$$\alpha_3/2 = \delta$$

$$|m_{\beta\beta}| = |c_{12}^2 c_{13}^2 e^{i\alpha_1} m_1 + s_{12}^2 c_{13}^2 e^{i\alpha_2} m_2 + s_{13}^2 m_3|$$

Measured quantity

$$|m_{\beta\beta}|^2 = c_{12}^4 c_{13}^4 m_1^2 + s_{12}^4 c_{13}^4 m_2^2 + s_{13}^4 m_3^2 + 2c_{12}^2 s_{12}^2 c_{13}^4 m_1 m_2 \cos(\alpha_1 - \alpha_2) + 2c_{12}^2 c_{13}^2 s_{13}^2 m_1 m_3 \cos\alpha_1 + 2s_{12}^2 c_{13}^2 s_{13}^2 m_2 m_3 \cos\alpha_2 .$$

Normal hierarchy

$$m_1 \ll \sqrt{\Delta m^2_{\rm SUN}}$$
$$m_2 \simeq \sqrt{\Delta m^2_{\rm SUN}}$$
$$m_3 \simeq \sqrt{\Delta m^2_{\rm ATM}}$$

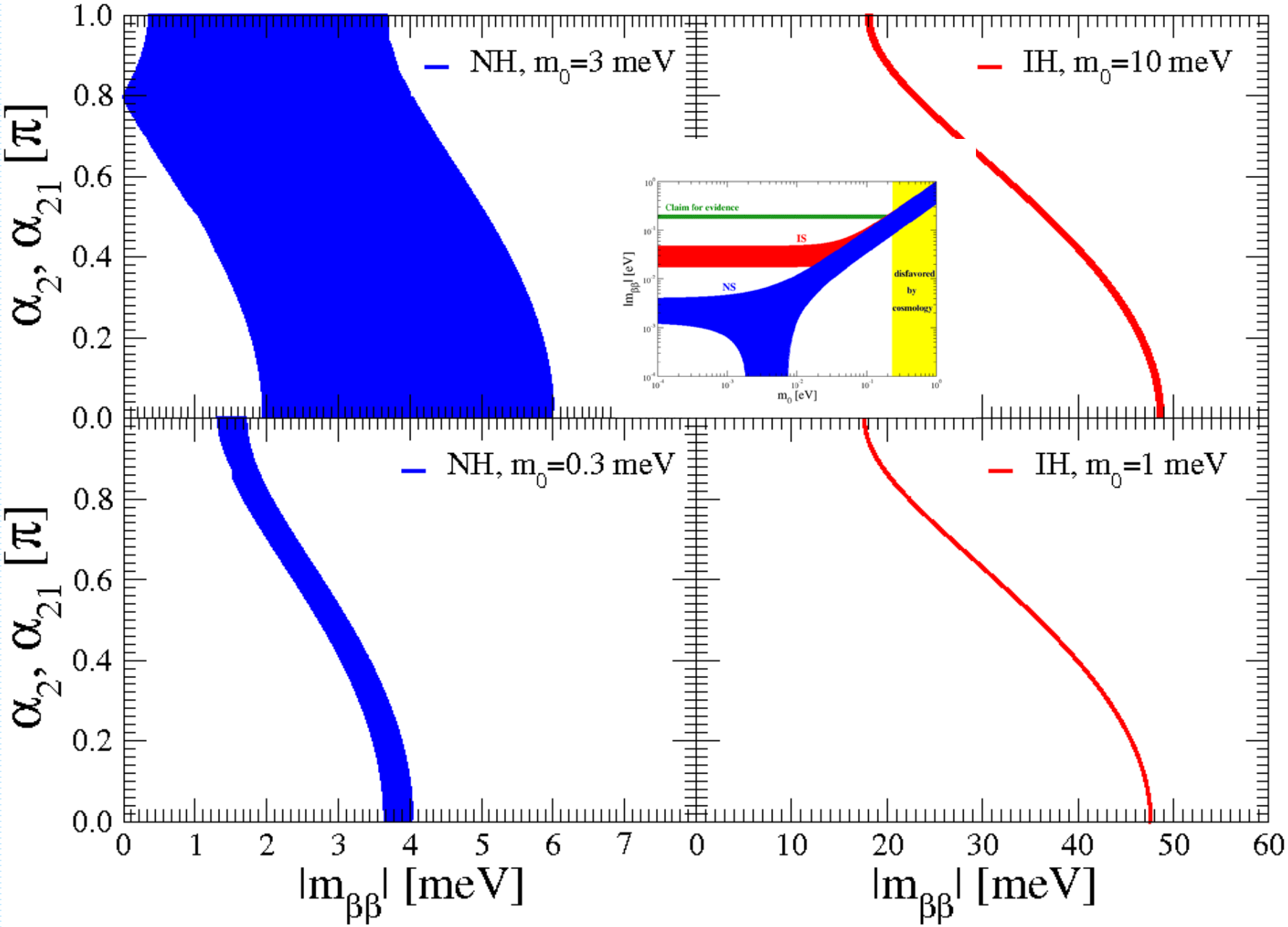
Inverted hierarchy

$$m_3 \ll \sqrt{\Delta m^2_{\rm ATM}}$$
$$m_1 \simeq m_2 \simeq \sqrt{\Delta m^2_{\rm ATM}}$$

Assuming lightest neutrino mass to be zero

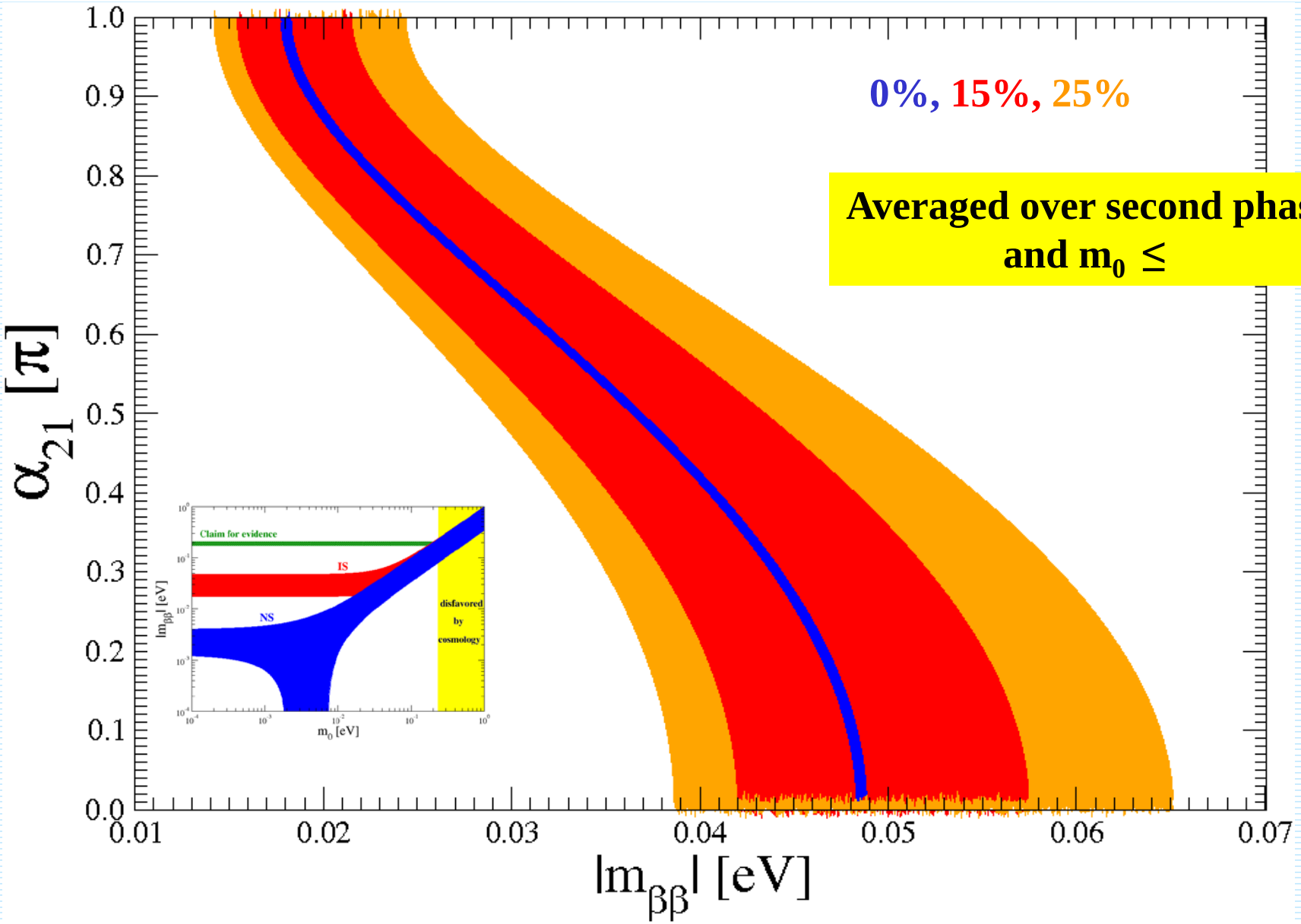
$$\cos\alpha_2 \simeq \frac{|m_{\beta\beta}|^2 - s_{12}^4 c_{13}^4 \Delta m^2_{\rm SUN} - s_{13}^4 \Delta m^2_{\rm ATM}}{2 s_{12}^2 c_{13}^2 s_{13}^2 \sqrt{\Delta m^2_{\rm SUN} \Delta m^2_{\rm ATM}}}$$

$$\cos\alpha_{12} = \frac{|m_{\beta\beta}|^2 - c_{13}^4 (1 - 2 s_{12}^2 c_{12}^2) \Delta m^2_{\rm ATM}}{2 c_{12}^2 s_{12}^2 c_{13}^4 \Delta m^2_{\rm ATM}}$$


NH, m_0=3 meV
IH, m_0=10 meV
NH, m_0=0.3 meV
IH, m_0=1 meV
α_2, α_{21} [π]
$|m_{\beta\beta}|$ [meV]
Claim for evidence
IS
NS
disfavored by cosmology
$|m_{\beta\beta}|$ [eV]
m_0 [eV]

$$|m_{\beta\beta}| = \frac{1}{\sqrt{T^{0\nu}_{1/2} G^{0\nu}(Q_{\beta\beta}, Z)}|M'^{0\nu}|}$$

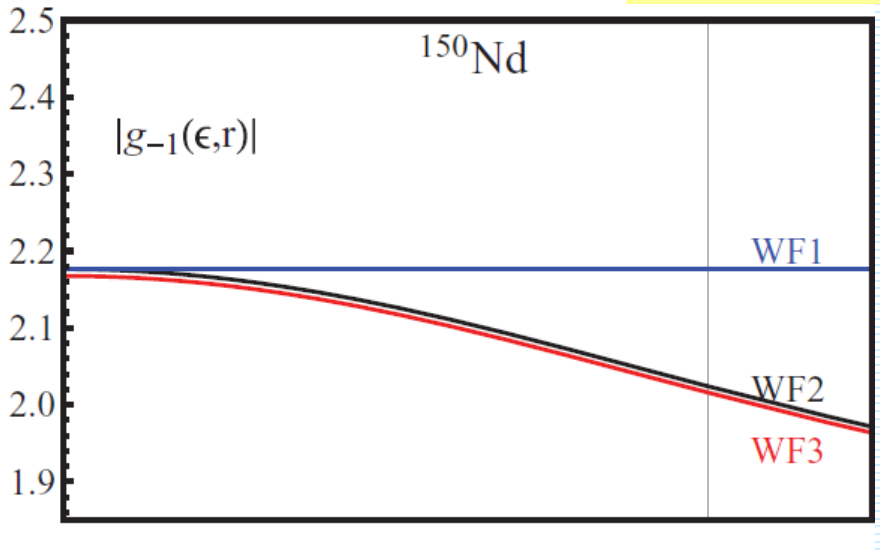
$$\frac{\sigma_{\beta\beta}}{|m_{\beta\beta}|^{obs}} = \sqrt{\frac{1}{4}\left(\frac{\sigma_{exp}}{T^{0\nu-obs}_{1/2}}\right)^2 + \left(\frac{\sigma_{th}}{|M'^{0\nu}|}\right)^2}$$

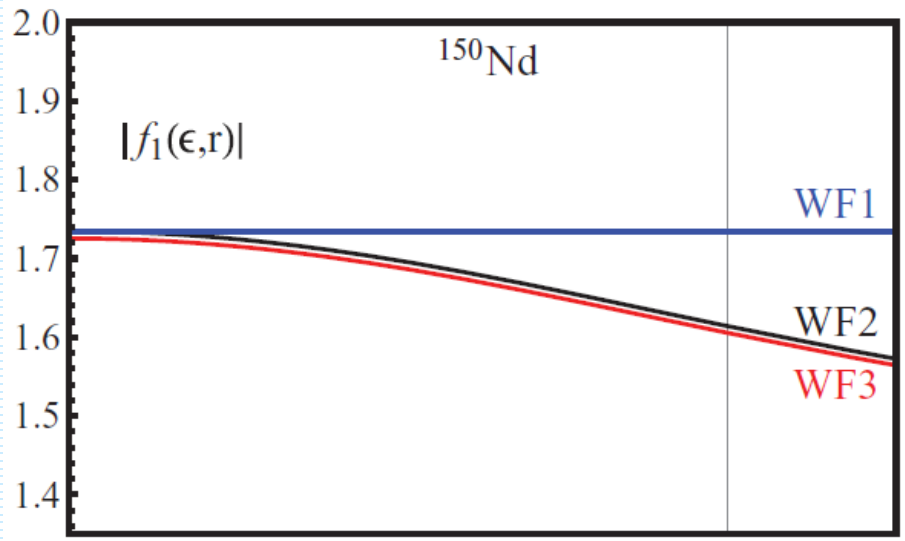


Phase-space factor

$$\left(T_{1/2}^{0\nu}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 g_A^4 \left|M_\nu^{0\nu}\right|^2 G^{0\nu}$$

Electron w.f. at r=R





Dirac equations:

$$\psi_{\epsilon\kappa\mu}(\mathbf{r}) = \begin{pmatrix} g_\kappa(\epsilon, r)\chi_\kappa^\mu \\ i f_\kappa(\epsilon, r)\chi_{-\kappa}^\mu, \end{pmatrix}$$

$$\frac{dg_\kappa(\epsilon, r)}{dr} = -\frac{\kappa}{r} g_\kappa(\epsilon, r) + \frac{\epsilon - V + m_e c^2}{c\hbar} f_\kappa(\epsilon, r)$$

$$\frac{df_\kappa(\epsilon, r)}{dr} = -\frac{\epsilon - V - m_e c^2}{c\hbar} g_\kappa(\epsilon, r) + \frac{\kappa}{r} f_\kappa(\epsilon, r)$$

$s_{1/2}$ electron wave state

$$e_s^{S_{1/2}}(\epsilon, \mathbf{r}) = \begin{pmatrix} g_{-1}(\epsilon, r)\chi_s \\ f_1(\epsilon, r)(\hat{\mathbf{p}} \cdot \vec{\sigma})\chi_s \end{pmatrix}$$

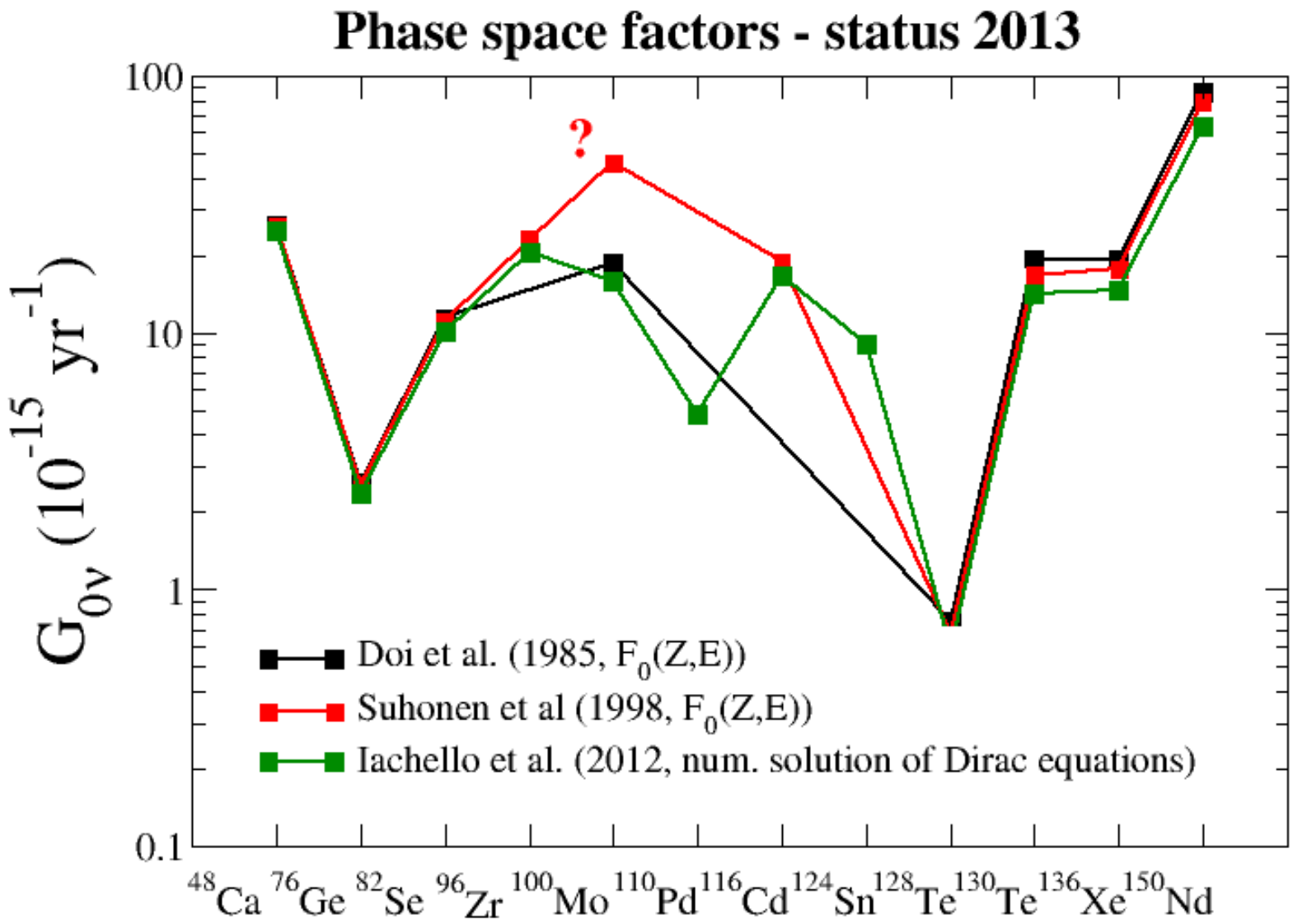
Finite nuclear size

WF1: approx. sol. inside nucl. (M. Rose approach)

WF2: numerical solution of Dirac eq.

WF3: numerical solution of Dirac eq. with consideration of electron screening

Nucleus is considered to be spherical



Nuclear Matrix Elements (NMEs)

$$\left(T_{1/2}^{0\nu}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 g_A^4 \left|M_\nu^{0\nu}\right|^2 G^{0\nu}$$

The $0\nu\beta\beta$-decay: A nuclear physics problem

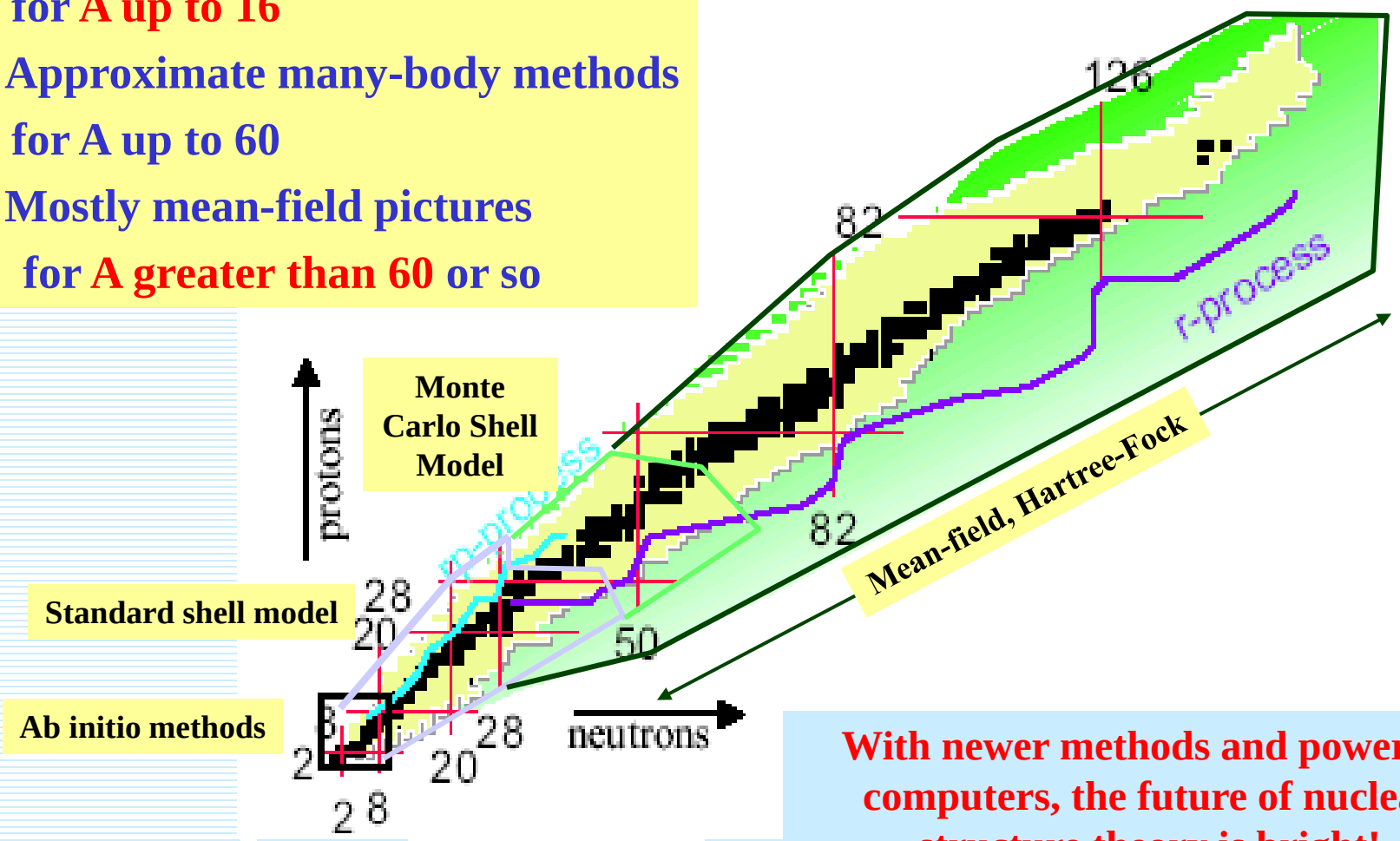
In double beta decay two neutrons bound in the ground state of an initial even-even nucleus are simultaneously transformed into two protons that are bound in the ground state or excited (0^+, 2^+) states of the final nucleus

It is necessary to evaluate, with a sufficient accuracy, wave functions of both nuclei, and evaluate the matrix element of the $0\nu\beta\beta$-decay operator connecting them

This can not be done exactly, some approximation and/or truncation is always needed. Moreover, there is no other analogues observable that can be used to judge directly the quality of the result.

Nuclear Structure

- Exact methods exist up to A=4
- Computationally exact methods for A up to 16
- Approximate many-body methods for A up to 60
- Mostly mean-field pictures for A greater than 60 or so



With newer methods and powerful computers, the future of nuclear structure theory is bright!

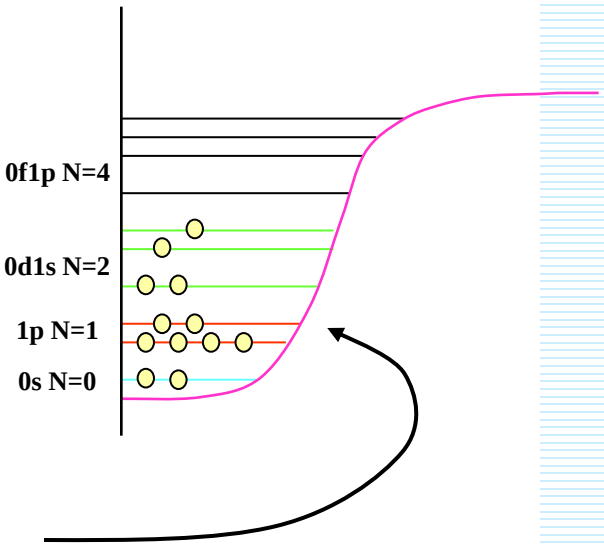
Many-body Hamiltonian

- **Start with the many-body Hamiltonian**

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} V_{NN}(r_i - r_j)$$

- **Introduce a mean-field *U* to yield basis**

$$H = \sum_i \left(\frac{p_i^2}{2m} + U(r_i) \right) + \underbrace{\sum_{i<j} V_{NN}(r_i - r_j) - \sum_i U(r_i)}_{\text{Residual interaction}}$$



The success of any nuclear structure calculation depends on the choice of the mean-field basis and the residual interaction!

- **The mean field determines the shell structure**
- **In effect, nuclear-structure calculations rely on perturbation theory**

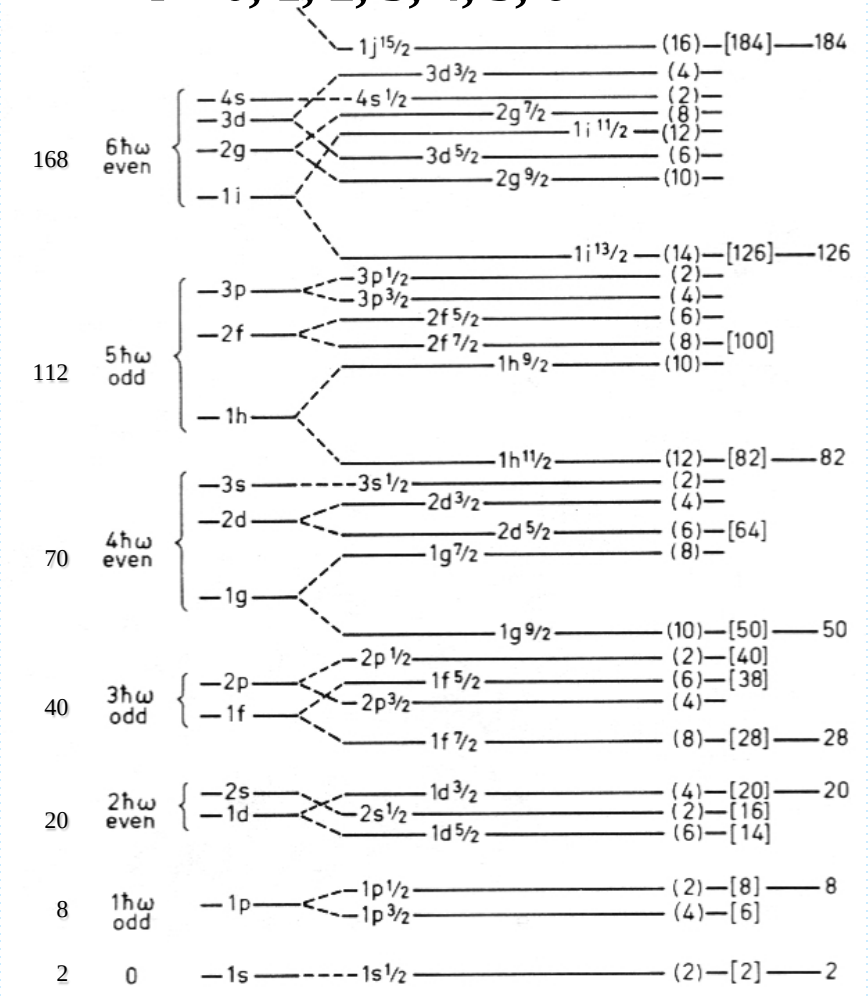
Shell structure of spherical nucleus

Goeppert-Mayer and Haxel, Jensen, and Suess proposed the independent-particle shell model to explain the magic numbers

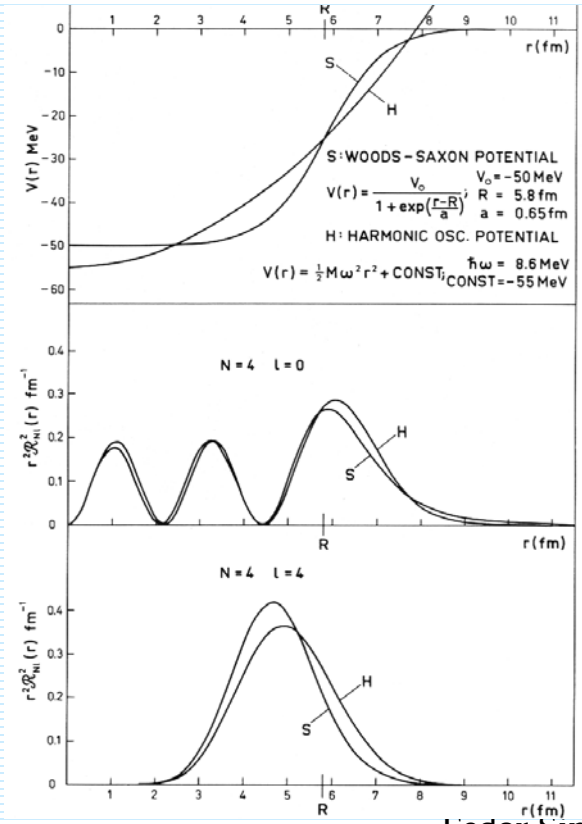
2, 8, 20, 28, 50, 82, 126, 184

s, p, d, f, g, h, i

l = 0, 1, 2, 3, 4, 5, 6



Harmonic oscillator with spin-orbit is a reasonable approximation to the nuclear mean field



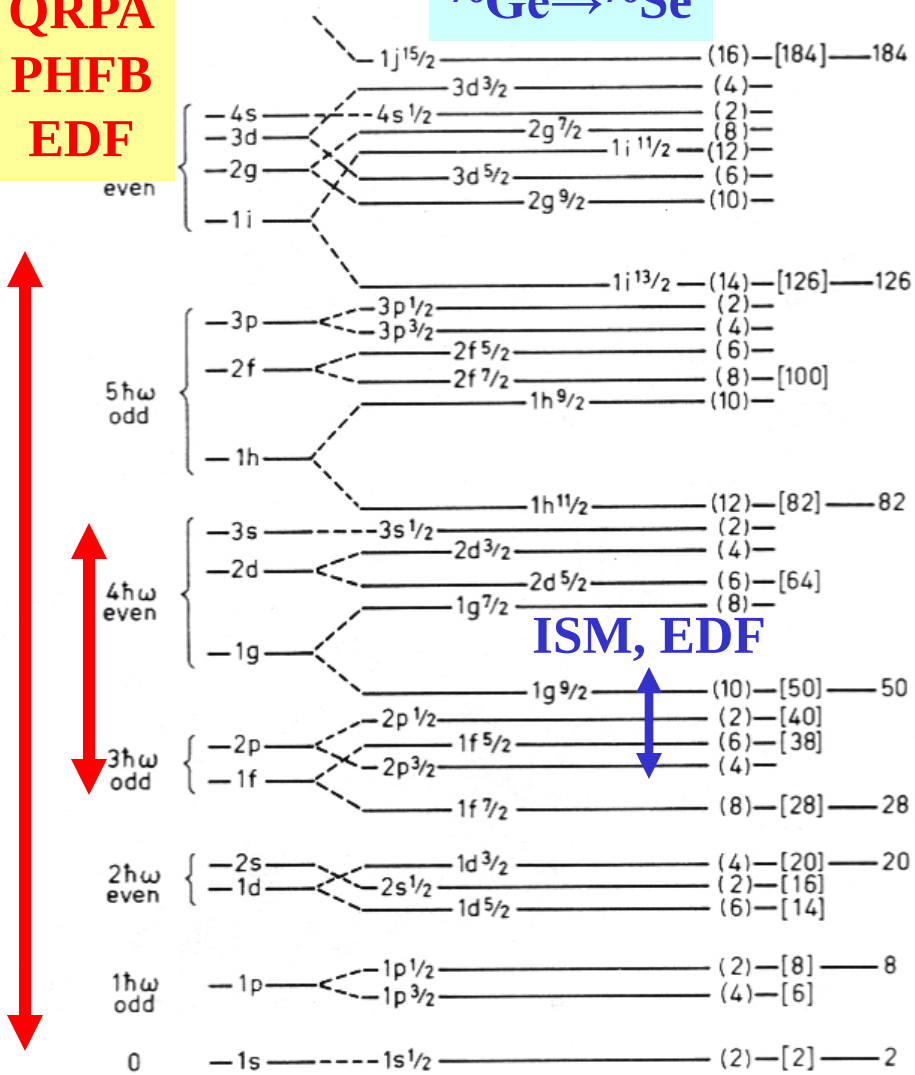
M.G. Mayer and J.H.D. Jensen, *Elementary Theory of Nuclear Shell Structure*, p. 58, Wiley, New York, 1955

Nuclear structure approaches

In QRPA a large valence space is used, but only a class of configurations is included. Describe collective states, but not details of dominantly few particle states. Relative simple, thus more 0nbb-decay calculations.

QRPA
PHFB
EDF

$^{76}Ge \rightarrow ^{76}Se$



In ISM a limited valence space is used but all configurations of valence nucleons are included. Describes well properties of low-lying nuclear states.

***IBM:** The low-lying states of the nucleus Are modeled in terms of L=0,2 bosons. They Interact through one-body and two-body Forces giving rise to bosonic wave functions*

***PHFB:** Mean field approach based on the Bogoliubov-Hartree-Fock method and Projection. States have good angular momentum due projection.*

***EDF:** Beyond mean field effects are incl. within generating coordinate method with particle and angular projections*

0νββ-decay matrix elements

$$M^{0\nu} = \frac{4\pi R}{g_A^2} \int \left(\frac{1}{(2\pi)^3} \int \frac{e^{-i\vec{q}.(\vec{x}_1-\vec{x}_2)}}{|q|} \right) \times$$
$$\sum_m \frac{< 0_f^+ | J_\alpha^\dagger(\vec{x}_1) | m >< m | J^{\alpha\dagger}(\vec{x}_2) | 0_i^+ >}{E_m - (E_i + E_f)/2 + |q|} d\vec{q} d\vec{x}_1 d\vec{x}_2$$

Weak hadron current

$$j^{\rho\dagger} = \overline{\Psi}\tau^+ \left[g_V(q^2)\gamma^\rho + i g_M(q^2) \frac{\sigma^{\rho\nu}}{2m_p} q_\nu - g_A(q^2)\gamma^\rho\gamma_5 - g_P(q^2) q^\rho \gamma_5 \right] \Psi,$$

Formfactor

$$g_V(\vec{q}^{\,2}) = g_V/(1 + \vec{q}^{\,2}/M_V^2)^2$$
$$g_A(\vec{q}^{\,2}) = g_A/(1 + \vec{q}^{\,2}/M_A^2)^2$$

Weak hadron current in a Breit frame

$$J^{\rho\dagger}(\vec{x}) = \sum_{n=1}^{A} \tau_n^+ [g^{\rho 0} J^0(\vec{q}^{\,2}) + \sum_k g^{\rho k} J_n^k(\vec{q}^{\,2})] \delta(\vec{x} - \vec{r}_n)$$
$$J^0(\vec{q}^{\,2}) = g_V(q^2)$$
$$\vec{J}_n(\vec{q}^{\,2}) = g_M(\vec{q}^{\,2}) i \frac{\vec{\sigma}_n \times \vec{q}}{2m_p} + g_A(\vec{q}^{\,2})\vec{\sigma} - g_P(\vec{q}^{\,2}) \frac{\vec{q}\,\vec{\sigma}_n \cdot \vec{q}}{2m_p}$$

One two-body operators

$$\langle p|O(1)|n\rangle\langle p'|O(2)|n'\rangle = \langle p,p'|O'(1,2)|n,n'\rangle$$

Integration over angular part of ν momentum

$$\int e^{i\vec{q}\cdot(\vec{r}_1-\vec{r}_2)}d\Omega_q = \int e^{i\vec{q}\cdot\vec{r}}d\Omega_q =$$

$$\sqrt{4\pi}\ 4\pi \sum_{lm} i^l j_l(qr)Y_{lm}(\Omega_r)\int Y^*_{lm}(\Omega_q)Y_{00}(\Omega_q)d\Omega_q = 4\pi j_0(qr)$$

Neutrino potential

$$\begin{aligned} O_F(r_{12},E^k_{J^\pi}) &= \tau^+(1)\tau^+(2)H_F(r_{12},E^k_{J^\pi})\,, \\ O_{GT}(r_{12},E^k_{J^\pi}) &= \tau^+(1)\tau^+(2)H_{GT}(r_{12},E^k_{J^\pi})\sigma_{12}\,, \\ O_T(r_{12},E^k_{J^\pi}) &= \tau^+(1)\tau^+(2)H_T(r_{12},E^k_{J^\pi})S_{12} \end{aligned}$$

$$H_K(r_{12},E^k_{J^\pi}) = \frac{2}{\pi g_A^2}R\int_0^\infty f_K(qr_{12})\,\frac{h_K(q^2)qdq}{q+E^k_{J^\pi}-(E_i+E_f)/2}$$

$$\begin{aligned} \sigma_{12} &= \vec{\sigma}_1\cdot\vec{\sigma}_2\,, \\ S_{12} &= 3(\vec{\sigma}_1\cdot\hat{r}_{12})(\vec{\sigma}_2\cdot\hat{r}_{12})-\sigma_{12} \end{aligned}$$

$$with\quad f_{F,GT}(qr_{12}) = j_0(qr_{12}),\qquad f_T(qr_{12}) = -j_2(qr_{12})$$

Nuclear matrix element

$$M^{0\nu} = -\frac{M_F}{g_A^2}+M_{GT}-M_T$$

$$M_K = \sum_{J^\pi,k_i,k_f,\mathcal{J}}\sum_{pnp'n'}(-1)^{j_n+j_{p'}+J+\mathcal{J}}\times$$

$$\sqrt{2\mathcal{J}+1}\begin{Bmatrix} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J}\end{Bmatrix}\times$$

$$\langle p(1),p'(2);\mathcal{J}\parallel \bar{f}(r_{12})O_K\bar{f}(r_{12})\parallel n(1),n'(2);\mathcal{J}\rangle\times$$

$$\langle 0^+_f||[\widetilde{c^+_{p'}\tilde{c}_{n'}}]_J||J^\pi k_f\rangle\langle J^\pi k_f|J^\pi k_i\rangle\langle J^\pi k_f i||[c^+_p\tilde{c}_n]_J||0^+_i\rangle$$

One two-body operators

$$\langle p|O(1)|n\rangle\langle p'|O(2)|n'\rangle = \langle p,p'|O'(1,2)|n,n'\rangle$$

Integration over angular part of ν momentum

$$\int e^{i\vec{q}\cdot(\vec{r}_1-\vec{r}_2)}d\Omega_q = \int e^{i\vec{q}\cdot\vec{r}}d\Omega_q =$$

$$\sqrt{4\pi}\ 4\pi \sum_{lm} i^l j_l(qr)Y_{lm}(\Omega_r)\int Y^*_{lm}(\Omega_q)Y_{00}(\Omega_q)d\Omega_q = 4\pi j_0(qr)$$

Neutrino potential

$$\begin{aligned} O_F(r_{12},E^k_{J^\pi}) &= \tau^+(1)\tau^+(2)H_F(r_{12},E^k_{J^\pi})\,, \\ O_{GT}(r_{12},E^k_{J^\pi}) &= \tau^+(1)\tau^+(2)H_{GT}(r_{12},E^k_{J^\pi})\sigma_{12}\,, \\ O_T(r_{12},E^k_{J^\pi}) &= \tau^+(1)\tau^+(2)H_T(r_{12},E^k_{J^\pi})S_{12} \end{aligned}$$

$$H_K(r_{12},E^k_{J^\pi}) = \frac{2}{\pi g_A^2}R\int_0^\infty f_K(qr_{12})\,\frac{h_K(q^2)qdq}{q+E^k_{J^\pi}-(E_i+E_f)/2}$$

$$\begin{aligned} \sigma_{12} &= \vec{\sigma}_1\cdot\vec{\sigma}_2\,, \\ S_{12} &= 3(\vec{\sigma}_1\cdot\hat{r}_{12})(\vec{\sigma}_2\cdot\hat{r}_{12})-\sigma_{12} \end{aligned}$$

$$with\quad f_{F,GT}(qr_{12}) = j_0(qr_{12}),\qquad f_T(qr_{12}) = -j_2(qr_{12})$$

Calculation of two-body matrix elements

From j-j to LS coupling

$$\mathcal{M}^{2body} = \langle a(1), b(2); J' | O(1,2) | c(1) d(2); J' \rangle$$

$$|n_c l_c j_c, n_d l_d j_d; J' M' \rangle = \sum_{SL} \hat{S}^2 \hat{L}^2 \hat{j}_c \hat{j}_d \begin{Bmatrix} 1/2 & l_c & j_c \\ 1/2 & l_d & j_d \\ S & L & J' \end{Bmatrix} |n_c l_c, n_d l_d, SL; J' M' \rangle$$

Moshinsky transformation to relative coordinates

$$|n_c l_c n_d l_d; L M_L \rangle = \sum_{\substack{nl \\ \mathcal{N}\mathcal{L}}} \langle nl, \mathcal{N}\mathcal{L}, L | n_c l_c, n_d l_d, L \rangle |nl, \mathcal{N}\mathcal{L}; L M_L \rangle$$

Two-body m.e.

$$\begin{aligned} \mathcal{M}^{2body}_{F,GT} &= \hat{J}' \sum_{SL} \hat{S} \hat{L} \hat{j}_a \hat{j}_b \hat{j}_c \hat{j}_d \begin{Bmatrix} 1/2 & l_c & j_c \\ 1/2 & l_d & j_d \\ S & L & J' \end{Bmatrix} \begin{Bmatrix} 1/2 & l_a & j_a \\ 1/2 & l_b & j_b \\ S & L & J' \end{Bmatrix} \\ &\times \sum_{\substack{nl \\ \mathcal{N}\mathcal{L}}} \sum_{\substack{n'l' \\ \mathcal{N}'\mathcal{L}'}} \langle nl, \mathcal{N}\mathcal{L}, L | n_c l_c, n_d l_d, L \rangle \langle n'l', \mathcal{N}'\mathcal{L}', L | n_a l_a, n_b l_b, L \rangle \\ &\times \langle n'l', \mathcal{N}'\mathcal{L}'; L || j_0(q|\vec{r}_{i,j}|) || nl, \mathcal{N}\mathcal{L}; L \rangle \langle s_a s_b; S || \begin{pmatrix} 1 \\ \vec{\sigma}_1 \cdot \vec{\sigma}_2 \end{pmatrix} || s_c s_d; S \rangle \end{aligned}$$

$$\langle n'l', \mathcal{N}'\mathcal{L}'; L || j_0(q|\vec{r}_{i,j}|) || nl, \mathcal{N}\mathcal{L}; L \rangle = \delta_{ll'} \delta_{\mathcal{N}\mathcal{N}'} \delta_{\mathcal{L}\mathcal{L}'} \langle n'l | j_0(q|\vec{r}_{i,j}|) | nl \rangle$$

$$\begin{aligned} \langle s_a s_b; S || \vec{\sigma}_1 \cdot \vec{\sigma}_2 || s_c s_d; S \rangle &= \hat{S}(\delta_{S1} - 3\delta_{S0}), \\ \langle s_a s_b; S || 1 || s_c s_d; S \rangle &= \hat{S}(\delta_{S1} + \delta_{S0}) \end{aligned}$$

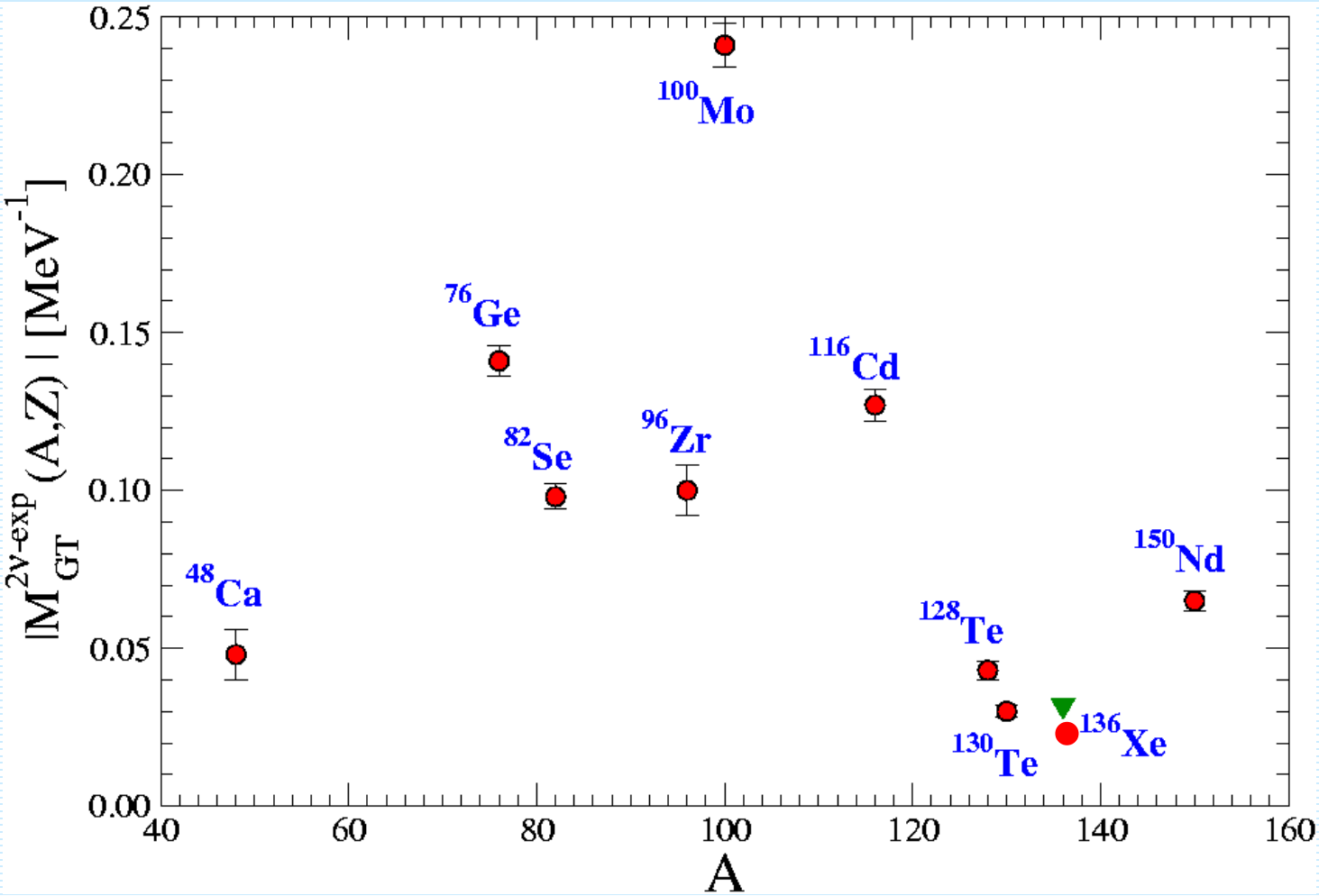
QRPA and isospin symmetry restoration

F.Š., V. Rodin, A. Faessler, and P. Vogel

PRC 87, 045501 (2013)

2νββ-decay NMEs

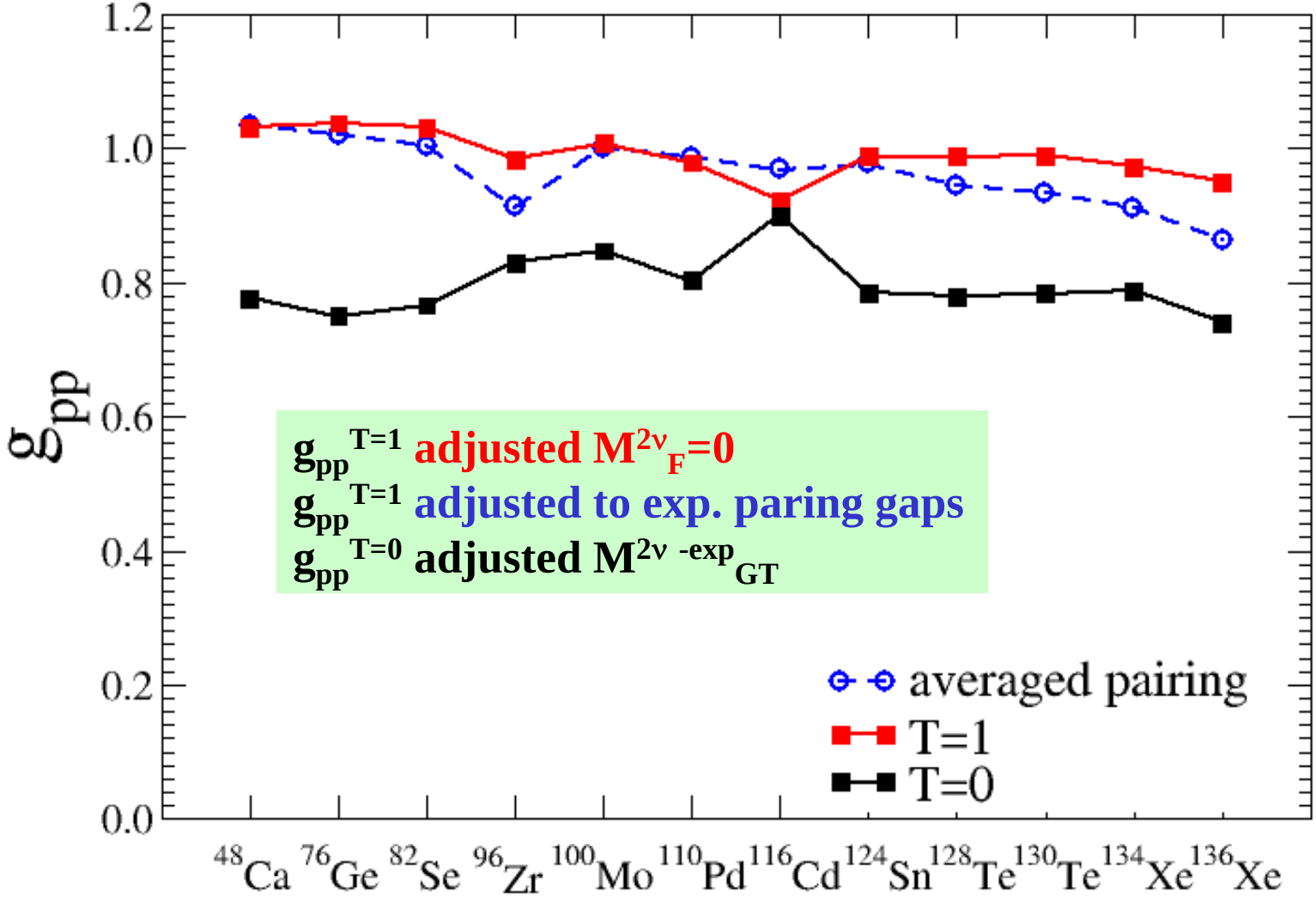
$$\frac{1}{T_{1/2}^{2\nu-exp}} = G^{2\nu}(E_0, Z)\ g_A^4\ |M_{GT}^{2\nu}|^2$$

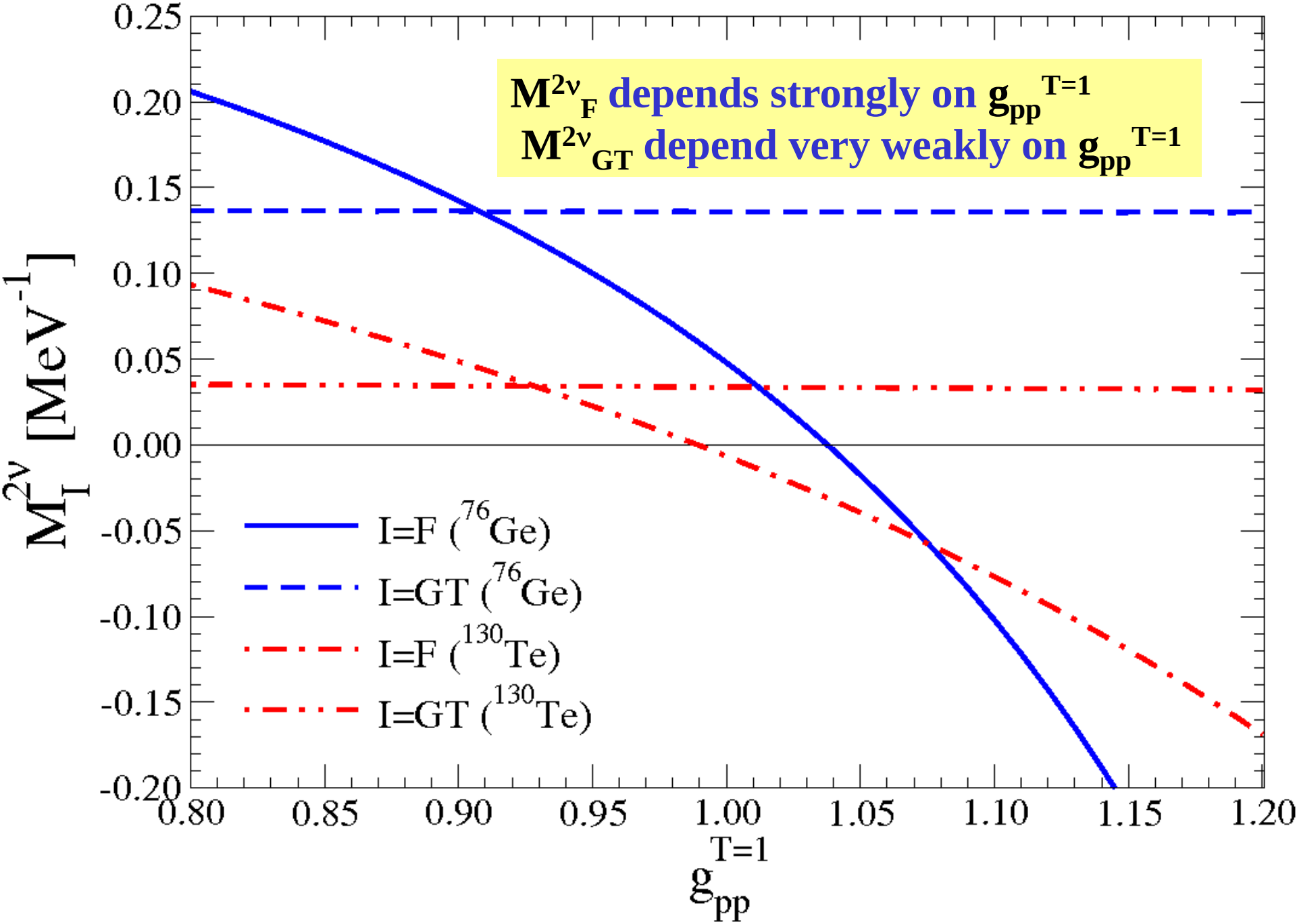


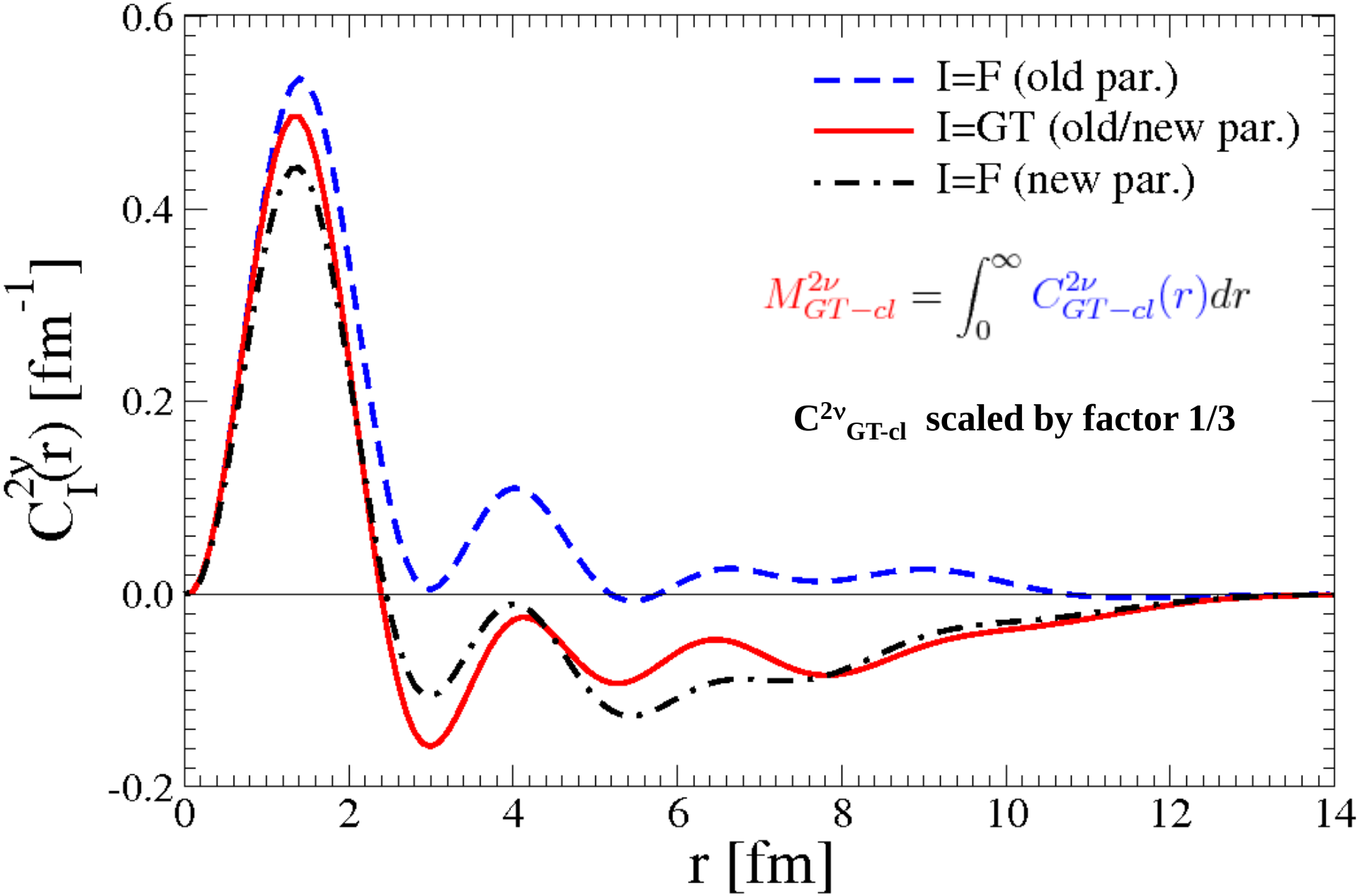
g_{pp} adjusted to 2νββ-decay half-life

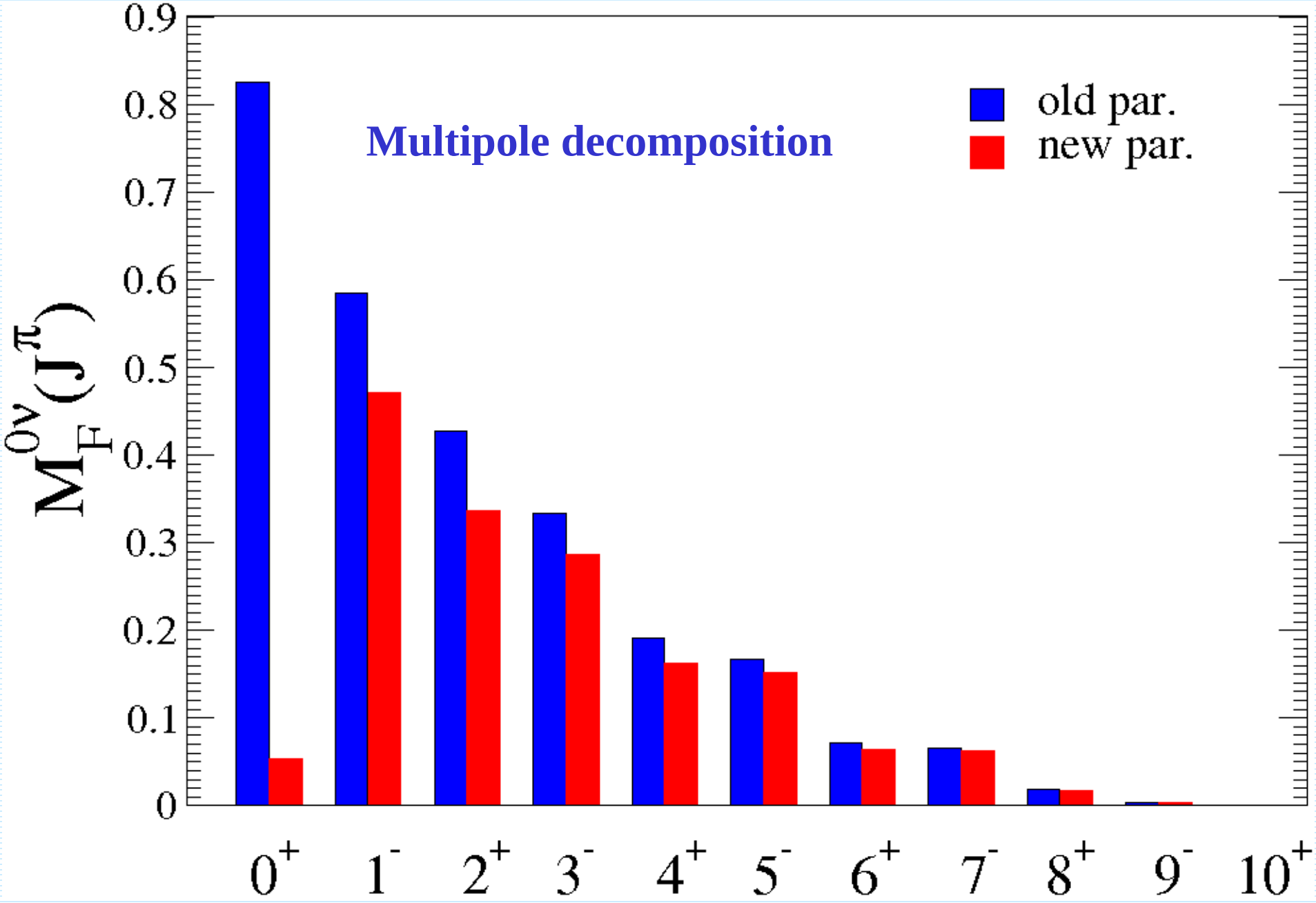
Separation of g_{pp} into $g_{pp}^{T=0}$ and $g_{pp}^{T=1}$

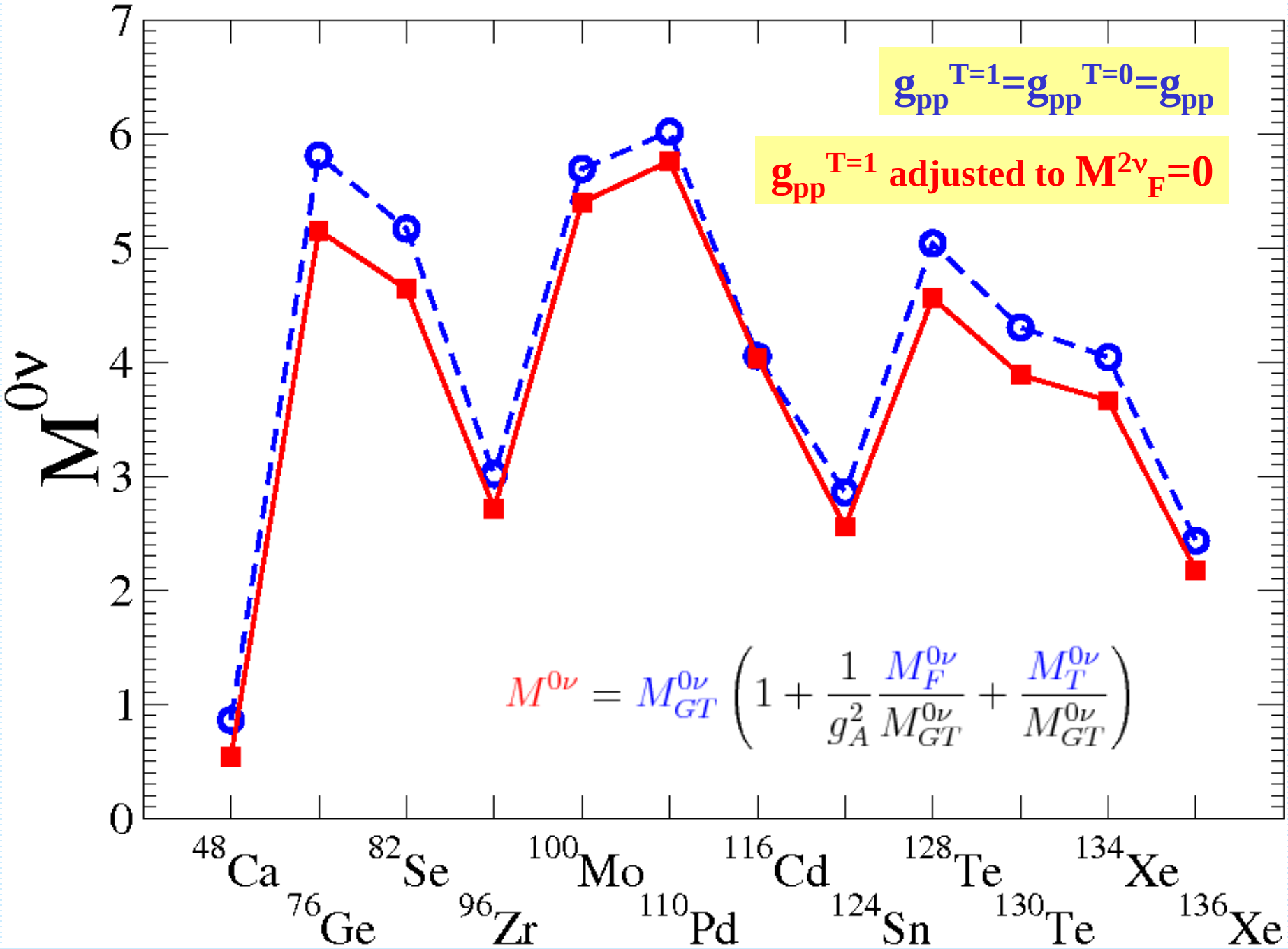
Close values ■ and ■ => no new parameter



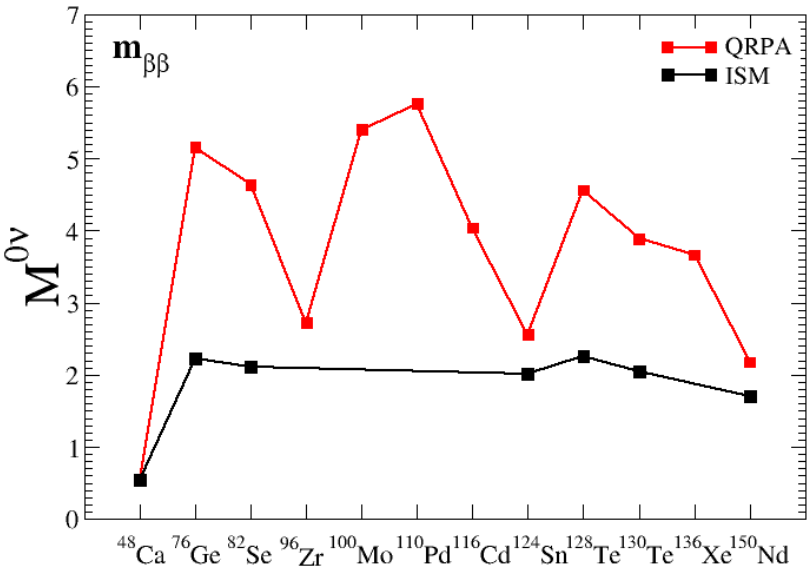

$M^{2\nu}_F$ depends strongly on $g_{pp}^{T=1}$
$M^{2\nu}_{GT}$ depend very weakly on $g_{pp}^{T=1}$
$M^{2\nu}_I$ [MeV^{-1}]
$g_{pp}^{T=1}$
I=F (^{76}Ge)
I=GT (^{76}Ge)
I=F (^{130}Te)
I=GT (^{130}Te)
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
0.80
0.85
0.90
0.95
1.00
1.05
1.10
1.15
1.20


I=F (old par.)
I=GT (old/new par.)
I=F (new par.)
$M^{2\nu}_{GT-cl} = \int_0^\infty C^{2\nu}_{GT-cl}(r)dr$
$C^{2\nu}_{GT-cl}$ scaled by factor 1/3
$C^{2\nu}_{I}(r)$ [fm^{-1}]
r [fm]
0.6
0.4
0.2
0.0
-0.2
0
2
4
6
8
10
12
14

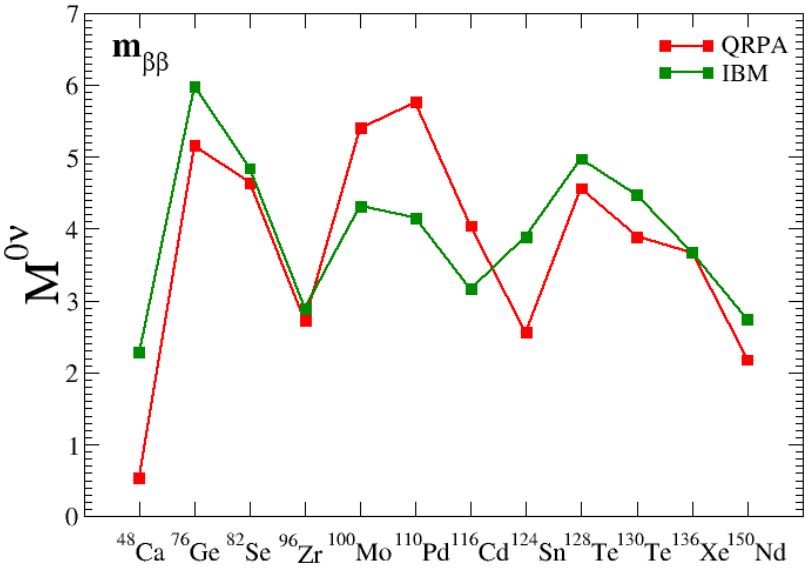

Multipole decomposition
old par.
new par.
$M_F^{0\nu}(J^\pi)$
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0^+
1^-
2^+
3^-
4^+
5^-
6^+
7^-
8^+
9^-
10^+


$g_{pp}^{T=1} = g_{pp}^{T=0} = g_{pp}$
$g_{pp}^{T=1}$ adjusted to $M^{2\nu}_F = 0$
$M^{0\nu}$
$M^{0\nu} = M^{0\nu}_{GT}\left(1 + \frac{1}{g_A^2}\frac{M^{0\nu}_F}{M^{0\nu}_{GT}} + \frac{M^{0\nu}_T}{M^{0\nu}_{GT}}\right)$
0
1
2
3
4
5
6
7
^{48}Ca
^{76}Ge
^{82}Se
^{96}Zr
^{100}Mo
^{110}Pd
^{116}Cd
^{124}Sn
^{128}Te
^{130}Te
^{134}Xe
^{136}Xe

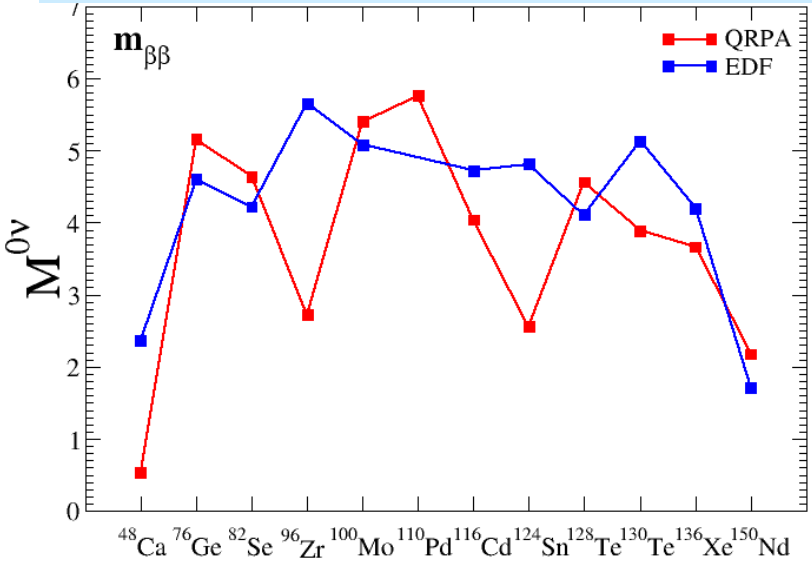
ISM: Menendez et al. NPA 818 (2009) 139



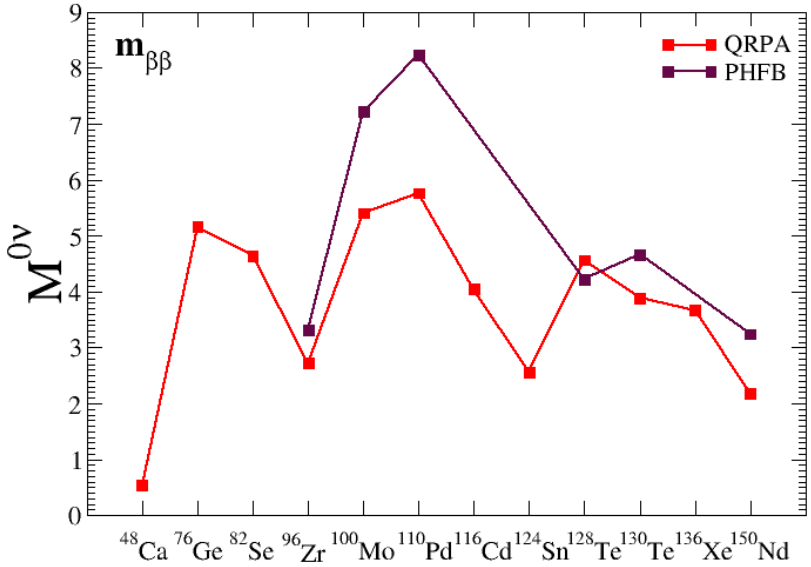
IBM: Barea, Kotila, Iachello, PRC (2013) 014315



EDF: Rodrigez, Martinez-Pinedo, PRL (2010) 105



PHFB: K. Rath et al., PRC 85 (2012) 014308



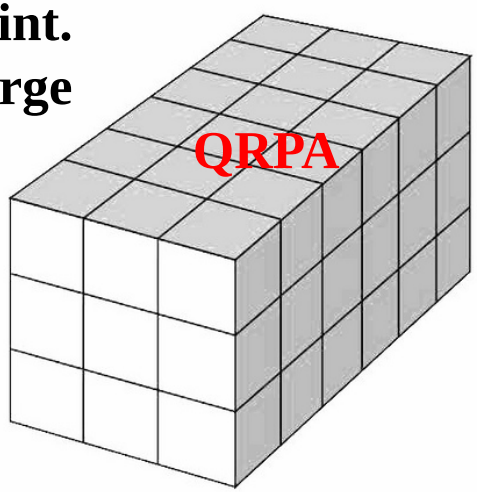
***Differences**: mean field; residual int.; size of the m.s.; many-body appr.*

QRPA uncertainties and their correlations in the analysis of 0νββ decay

A. Faessler, G.L. Fogli, E. Lisi, V. Rodin, A. M. Rotunno,F.Š.,
PRD 87, 053002 (2013)

Effects of isospin restoration not included yet.

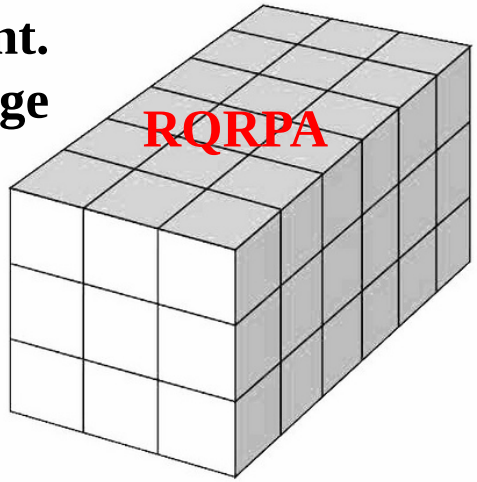
s.ms.: small
int.
large



src: Argonne
CD-Bonn

g_A=1.0,1.25

s.ms.: small
int.
large

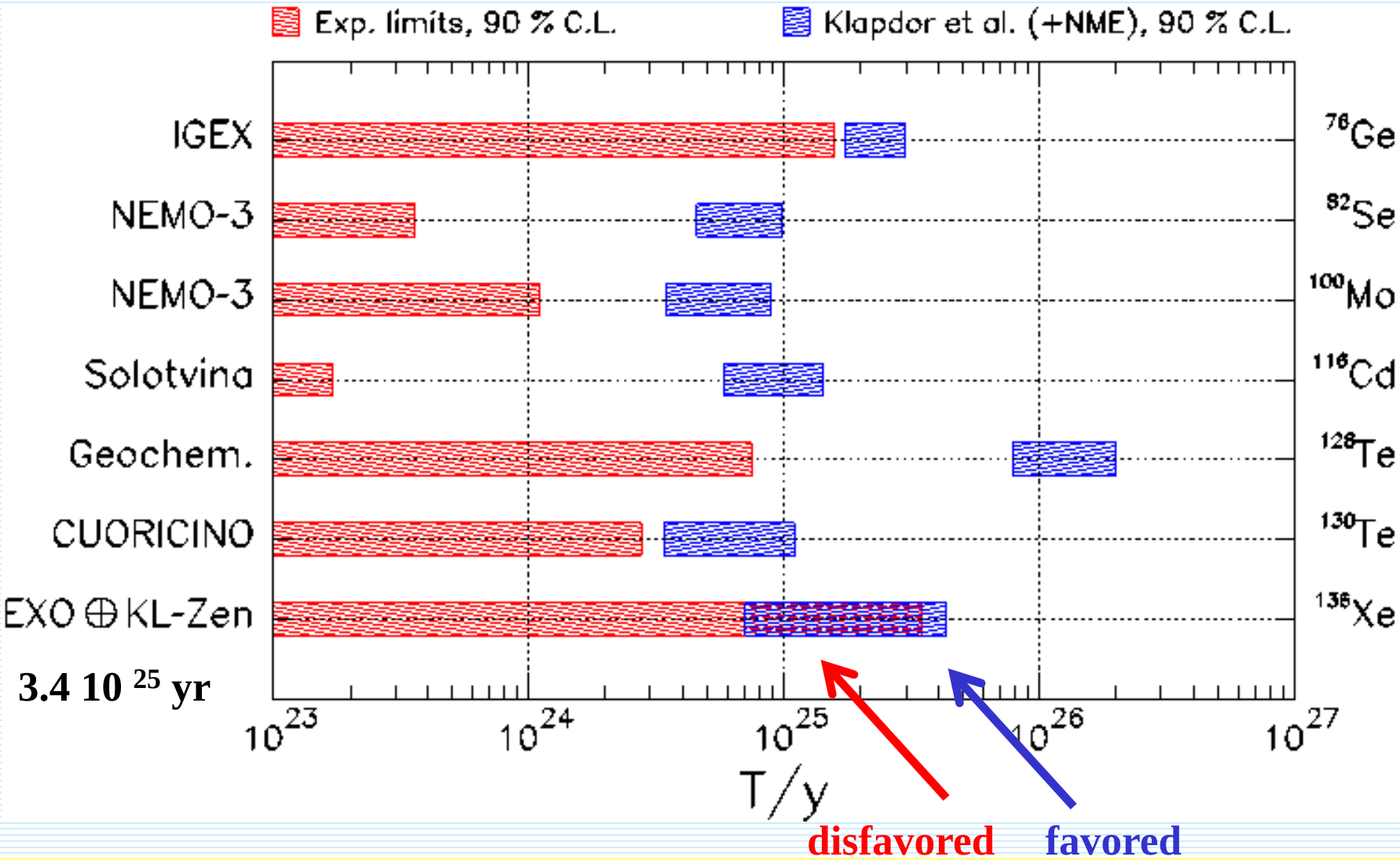


src: Argonne
CD-Bonn

g_A=1.0,1.25

For each nucleus 2x2x2x3=24 NMEs

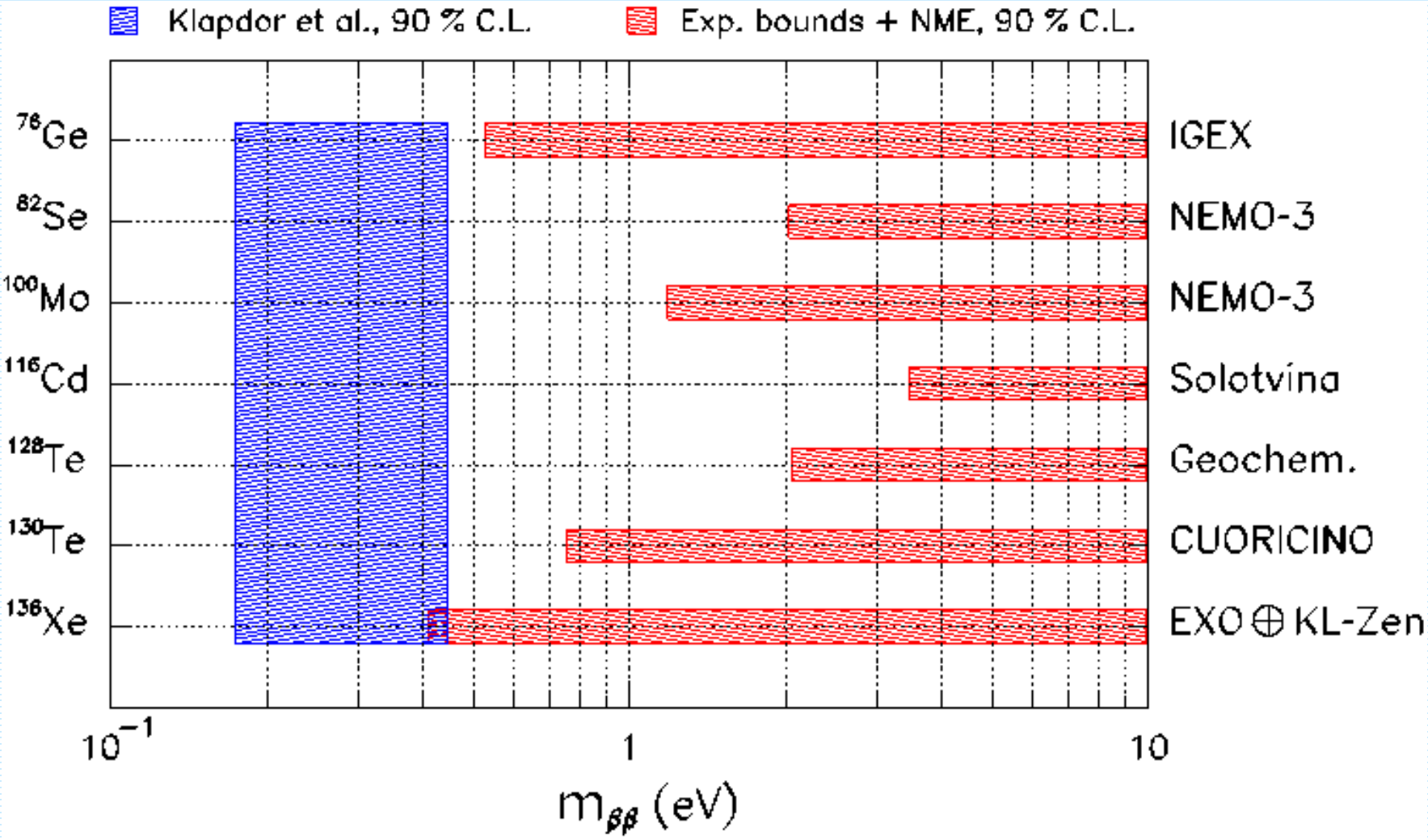
Range of half-lives preferred at 90% C.L. by the 0νββ claim of evidence compared with the 90% exclusion limits placed by other experiments.



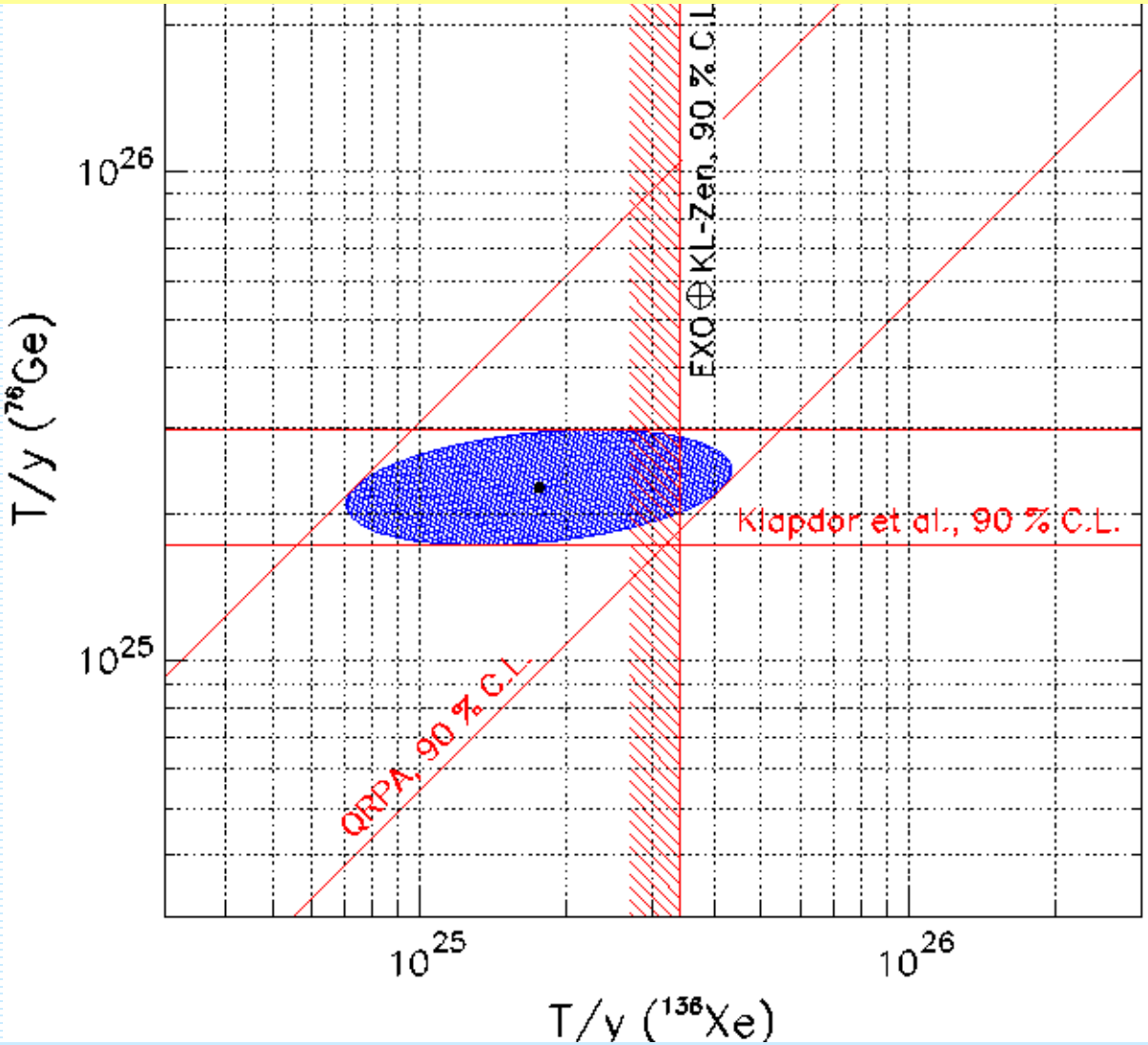
The comparison involves the NME and their errors as well as their correlations

Range of $m_{\beta\beta}$ allowed by the $0\nu\beta\beta$ claim of evidence compared with the limits placed by other experiments (All at 90% C.L.).

Before GERDA data have appeared:



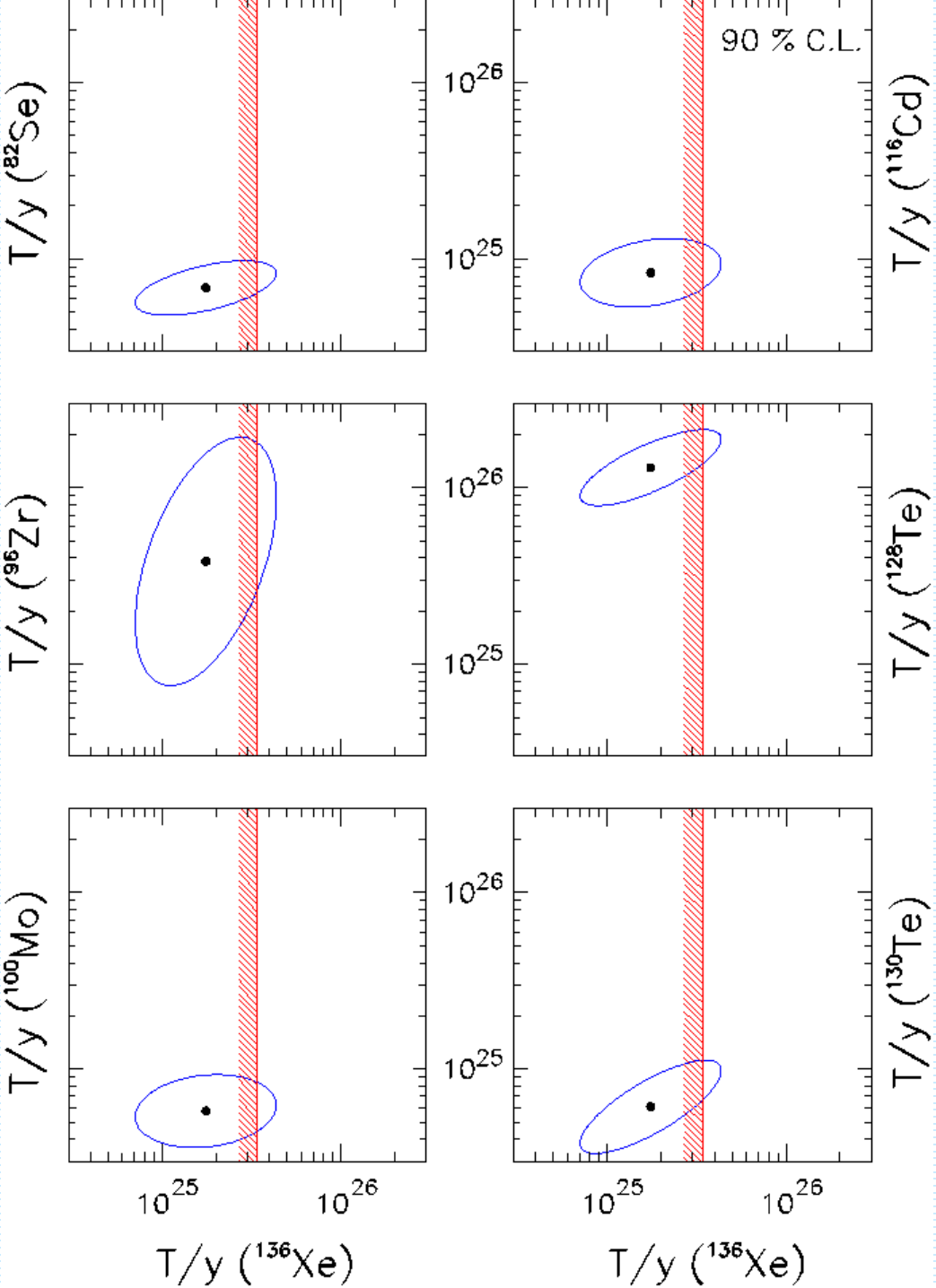
Theoretical and expperimental constraints in the plane charted by the 0νββ half-lives of ^{76}Ge and ^{136}Xe.



Horizontal band:range prefered by claim. Slanted band: constraint place by our QRPA estimates. The combination provides the shaded ellipse, whose projection on the abscissa gives the range preferred at 90% C.L. for the ^{136}Xe half-life.

Allowed regions (ellipses) as derived from Klapdor's claim in the plane charted by the half-lives of ^{136}Xe and each of six nuclei

A large fraction of each ellipse Is excluded by the combined EXO & KL-Zen results. (All bounds are at 90% C.L. on one variable)

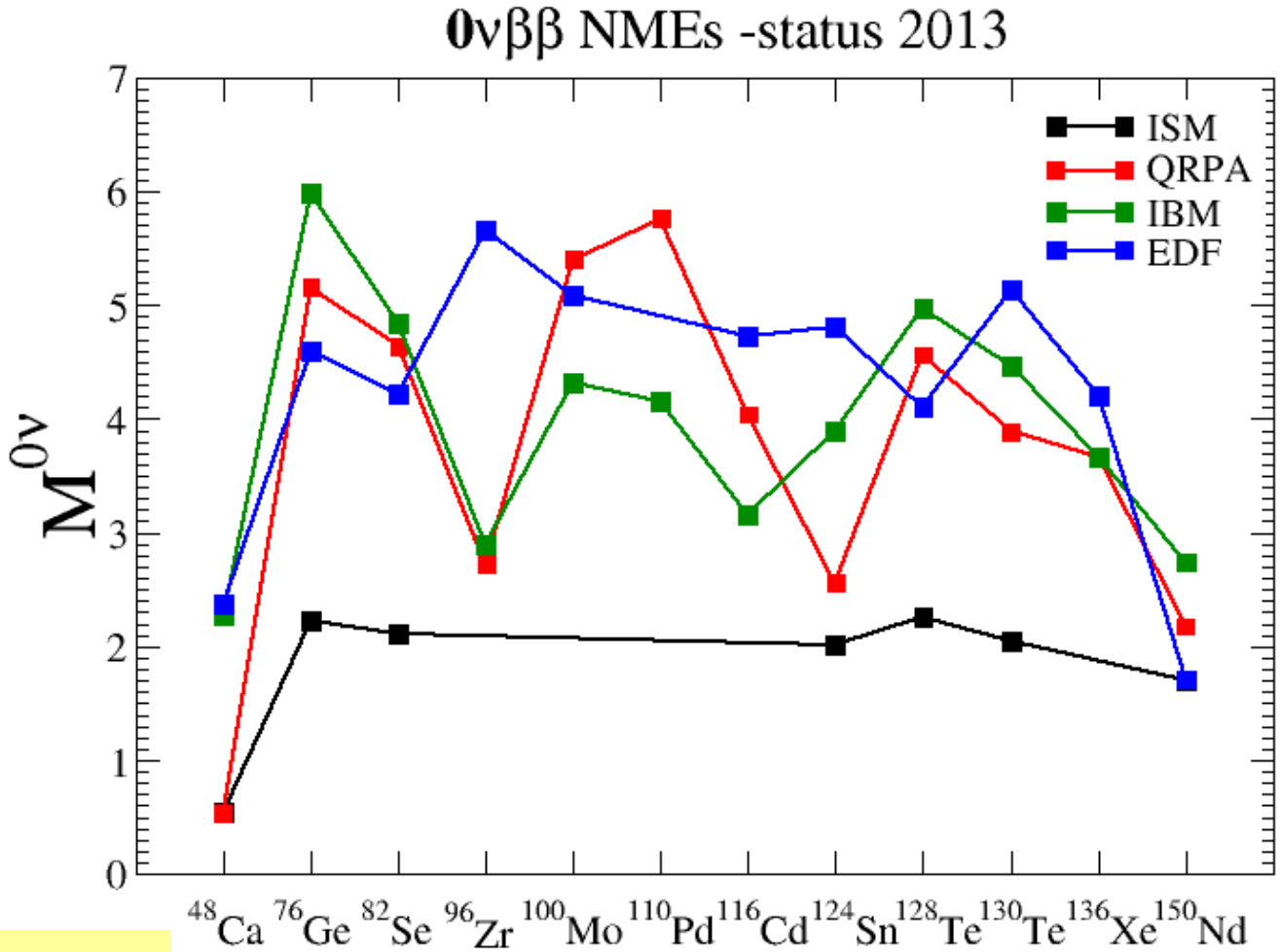


Some notes about the $0\nu\beta\beta$-decay NMEs

The 0νββ-decay NMEs (Status:2013)

Nobody is perfect:

g_A=1.25(7), CCm or UCOM s.r.c., r_0=1.20 fm



Differences:

i) mean field;

ii) residual int.;

iii) size of the m.s.

iv) many-body appr.

LSSM (small m.s., negative parity states)

PHFB (GT force neglected)

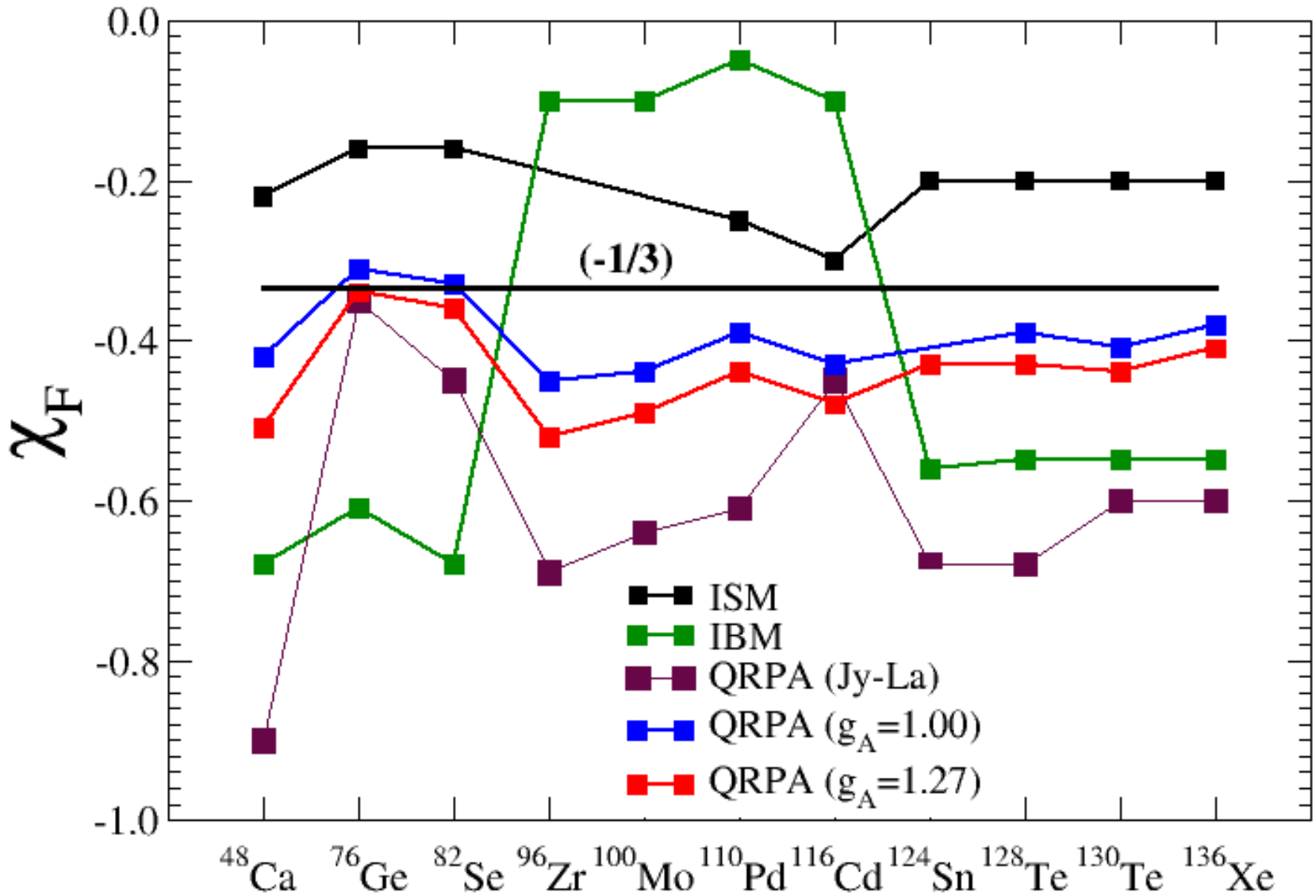
IBM (Hamiltonian truncated)

(R)QRPA (g.s. correlations not accurate enough)

$\chi_F = M^{0\nu}_F / M^{0\nu}_{GT} \approx -1/3$

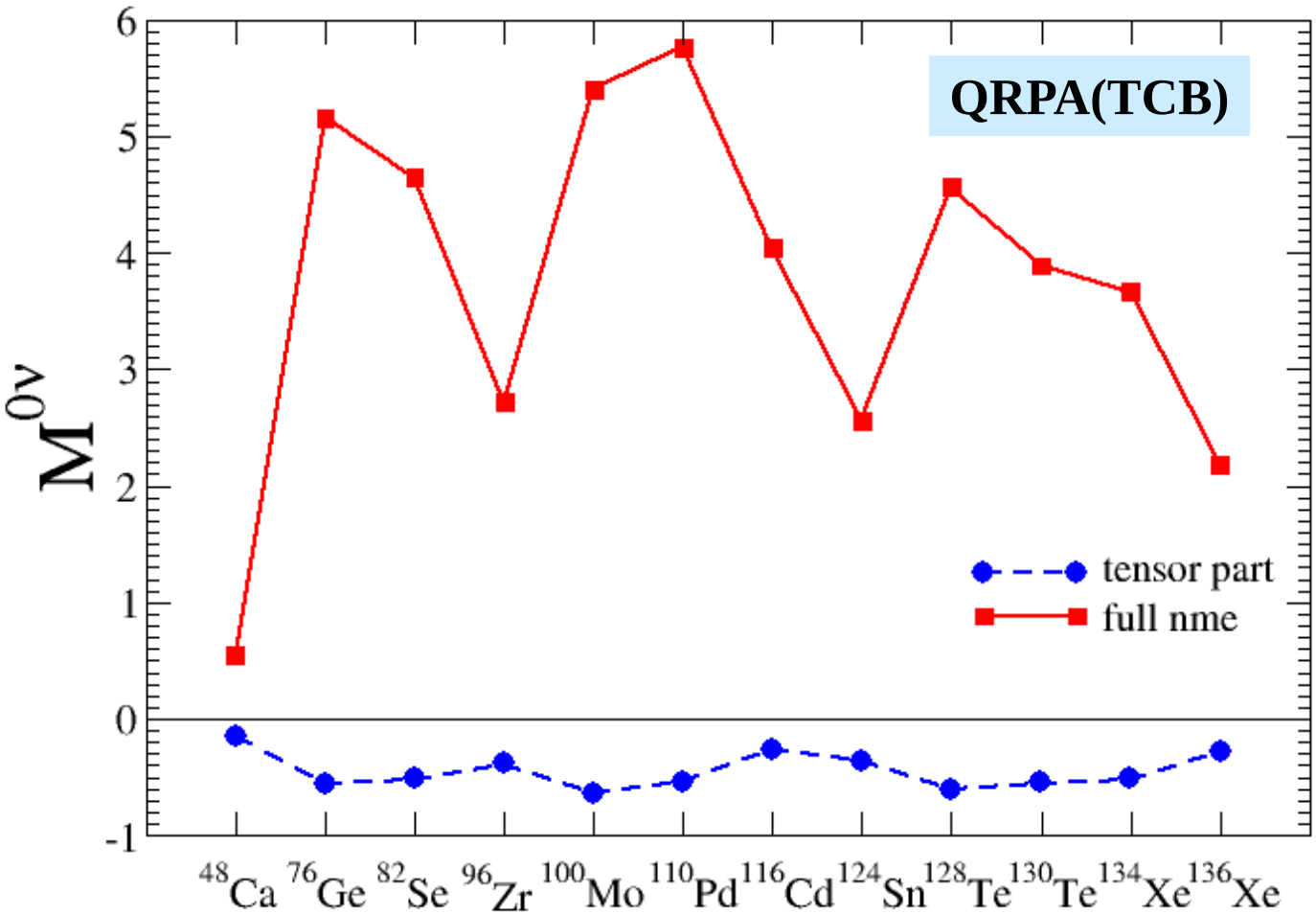
Fermi : $1 = \Omega(S=0) + \Omega(S=1)$

Gamow-Teller: $\sigma.\sigma = -3\,\Omega(S=0) + \Omega(S=1)$



Tensor part of the $0\nu\beta\beta$ NME

(some disagreement)



ISM: effect is small, QRPA(J): negligable; PHFB, EDF: not calculated; QRPA(TBC), IBM: up to 10%

Quenching of g_A and two-body currents

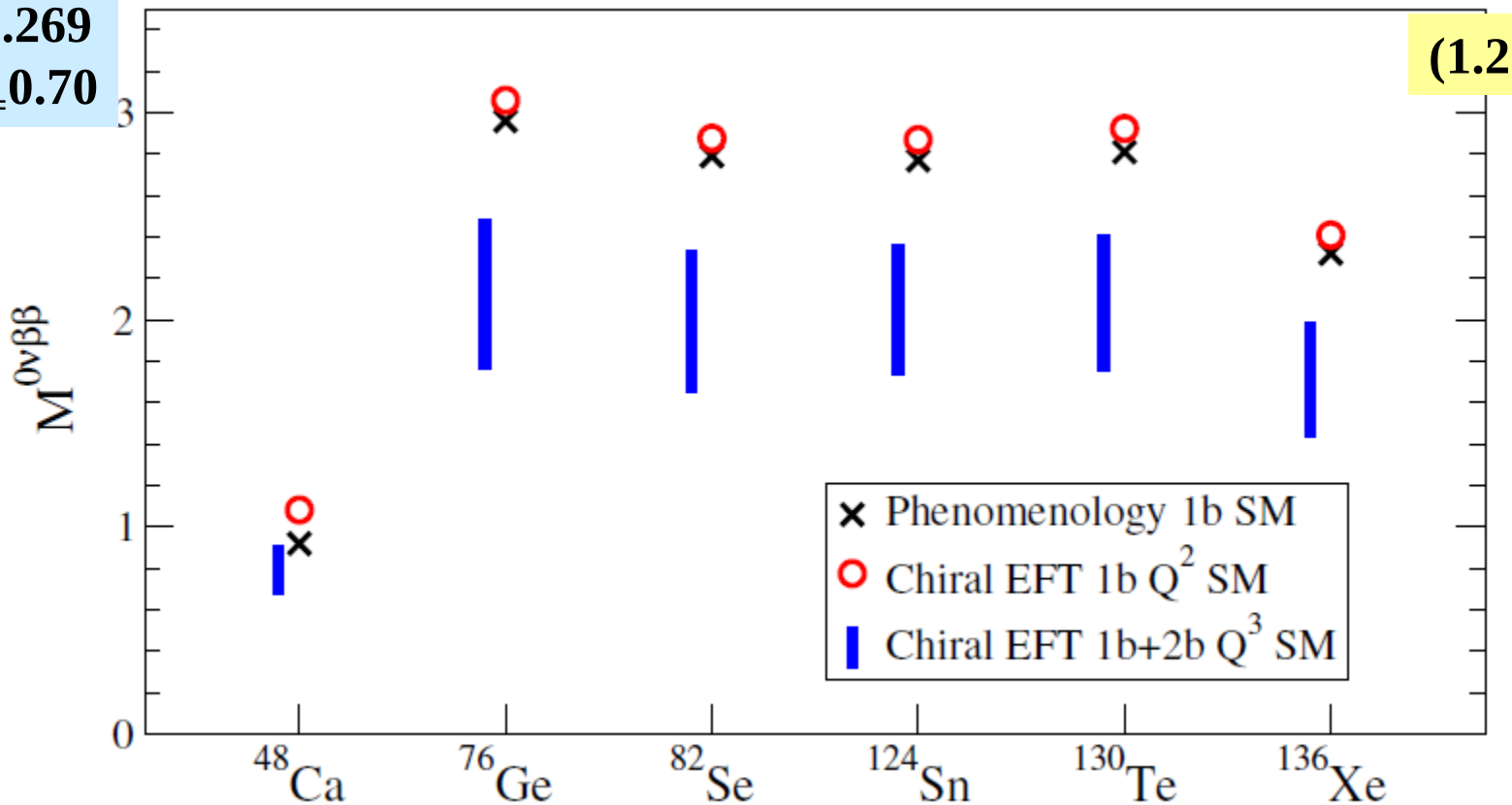
Menendez, Gazit, Schwenk, PRL 107 (2011) 062501; MEDEX13 contribution

$$\mathbf{J}^{\rm eff}_{i,2b} = -g_A \boldsymbol{\sigma}_i \tau_i^- \frac{\rho}{F_\pi^2}\left[\frac{2}{3}c_3\frac{p^2}{4m_\pi^2+p^2} + I(\rho,P)\left(\frac{1}{3}(2c_4-c_3)+\frac{1}{6m}\right)\right] = -g_A\,\delta(p)\,\boldsymbol{\sigma}_i\tau_i^-$$

The 0νββ operator calculated within effective field theory. Corrections appear as 2-body current predicted by EFT. The 2-body current contributions are related To the quenching of Gamow-Teller transitions found in nuclear structure calc.

$g_{A=}$1.269
$g^{eff}_{A=}$0.70

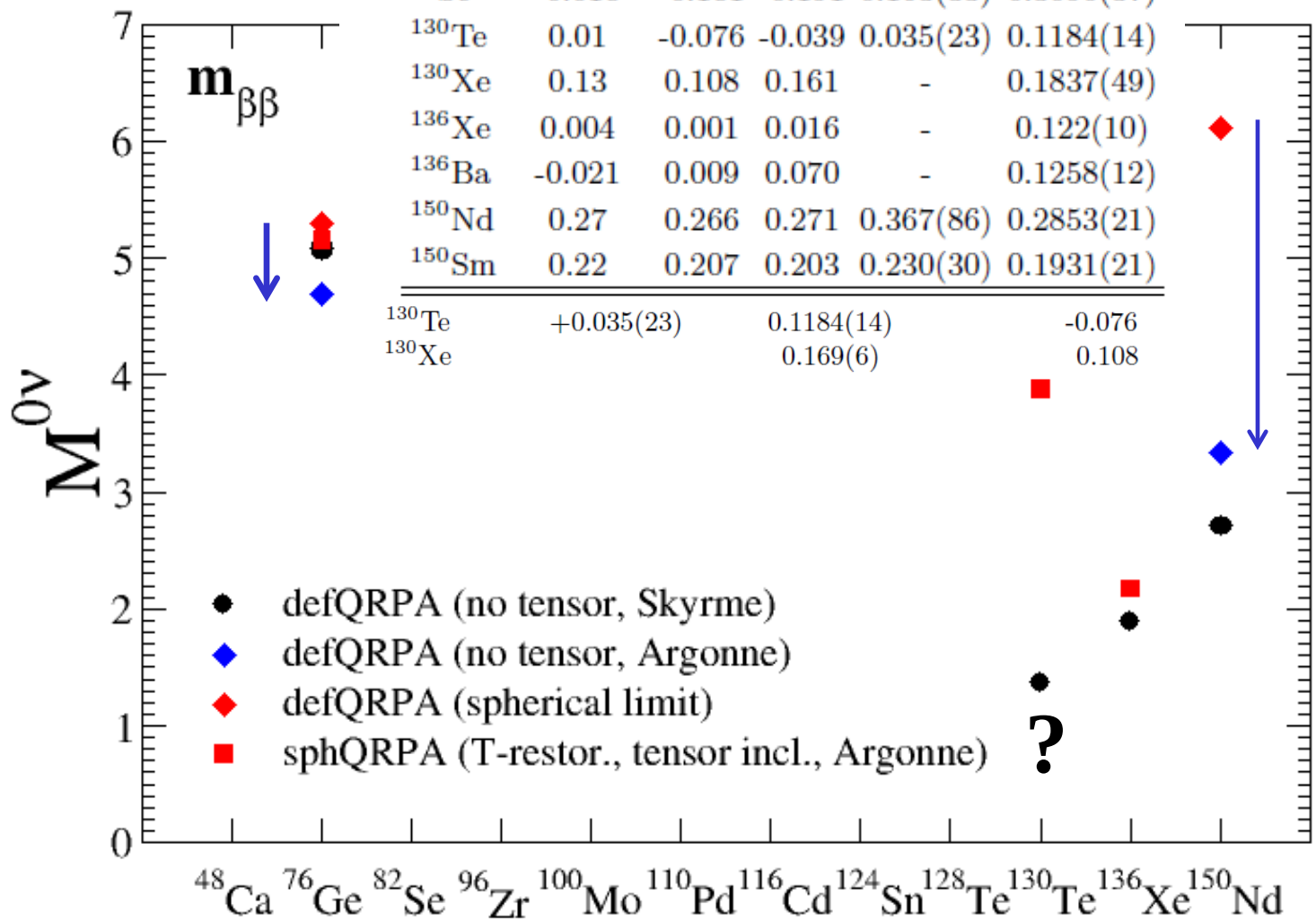
$(1.269)^4$=2.6



Deformed QRPA

		Ref. [24]		Exp.	
	this work	Sk3	SG2	Ref. [25]	Ref. [26]
^{76}Ge	0.184[a]	0.161	0.157	0.095(30)	0.2623(9)
^{76}Se	-0.018	-0.181	-0.191	0.163(33)	0.3090(37)
^{130}Te	0.01	-0.076	-0.039	0.035(23)	0.1184(14)
^{130}Xe	0.13	0.108	0.161	-	0.1837(49)
^{136}Xe	0.004	0.001	0.016	-	0.122(10)
^{136}Ba	-0.021	0.009	0.070	-	0.1258(12)
^{150}Nd	0.27	0.266	0.271	0.367(86)	0.2853(21)
^{150}Sm	0.22	0.207	0.203	0.230(30)	0.1931(21)

^{130}Te	+0.035(23)	0.1184(14)	-0.076
^{130}Xe		0.169(6)	0.108



Skyrme int: Mustonen, Engel, arXiv:1301.6997 [nucl-th]
Argonn int: Fang, Faessler, Rodin, F.Š. , PRC 83 (2011) 034320

On the relation between $0\nu\beta\beta$-decay and $2\nu\beta\beta$-decay (GT) NMEs

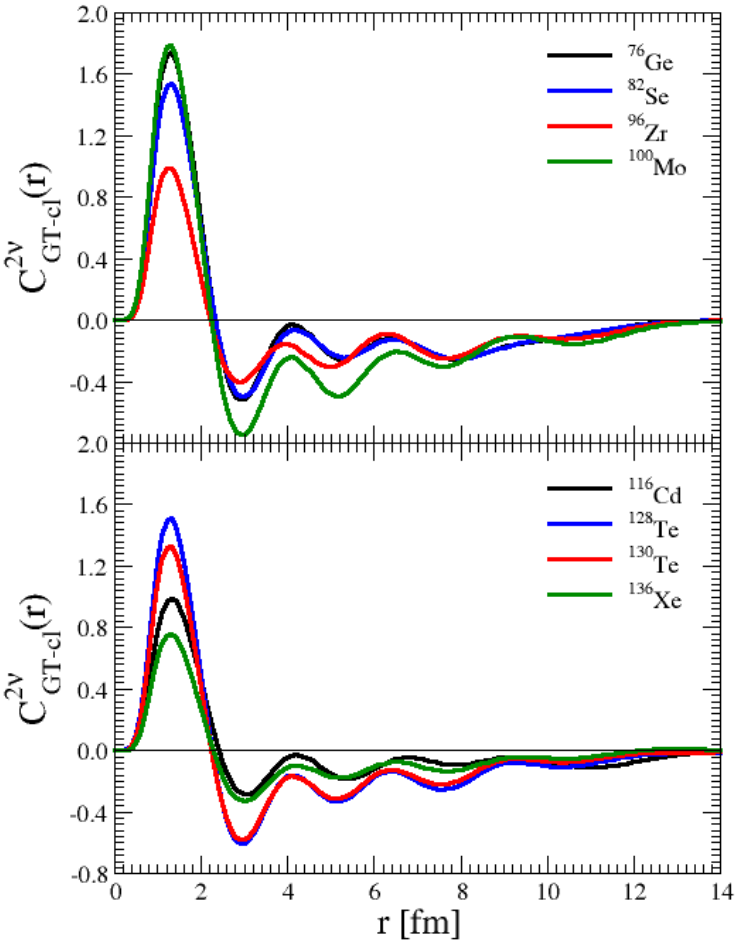
F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

$$M^{0\nu} = M^{0\nu}_{GT}\left(1 + \frac{1}{g_A^2}\frac{M^{0\nu}_F}{M^{0\nu}_{GT}} + \frac{M^{0\nu}_T}{M^{0\nu}_{GT}}\right)$$

Going to relative coordinates:

$$M^{2\nu}_{GT-cl} = \int_0^\infty C^{2\nu}_{GT-cl}(r)dr$$

***r*- relative distance of two nucleons**



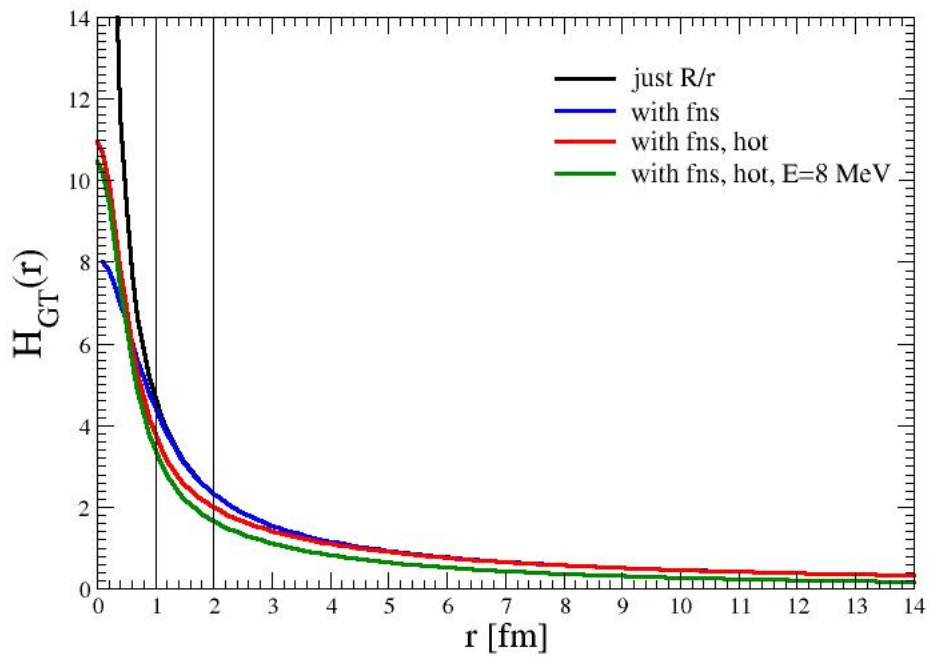
A connection between closure $2\nu\beta\beta$ and $0\nu\beta\beta$ GT NMEs

F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

$$M^{0\nu}_{GT} = \int_0^\infty H^{0\nu}_{GT}(r) C^{2\nu}_{GT-cl}(r)dr$$

Neutrino potential

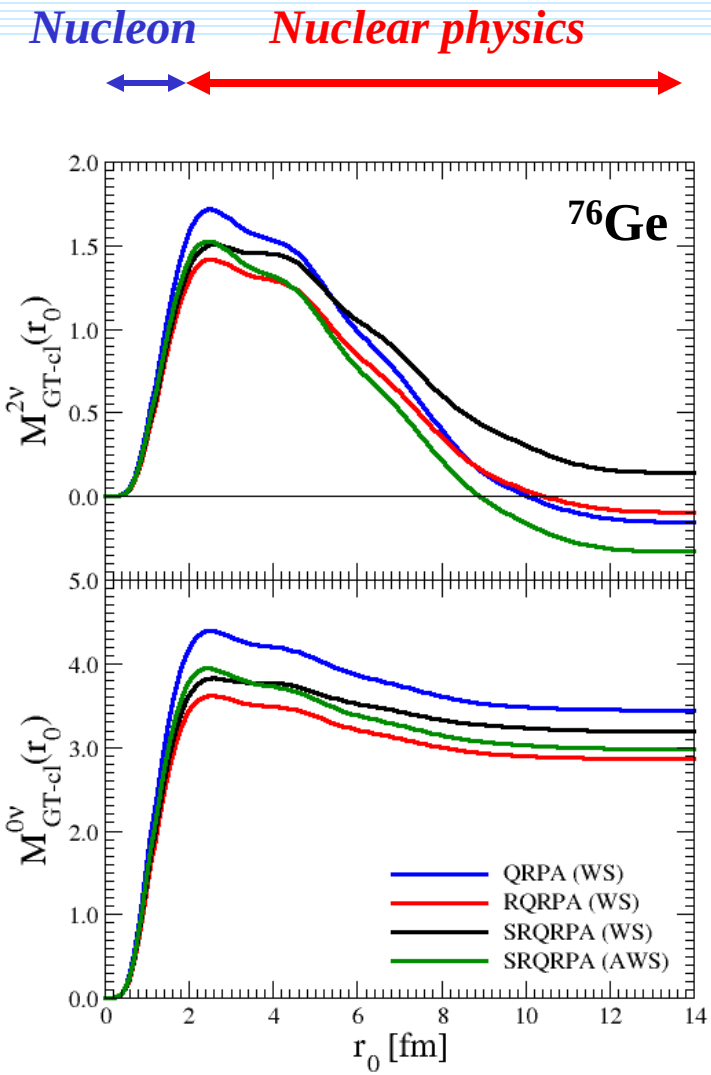
$$H(r) = R\frac{2}{\pi}\int_0^\infty j_0(qr)\frac{q}{q+\overline{E}} f^2_{FNS}(q^2) g_{HOT}(q^2)dr$$



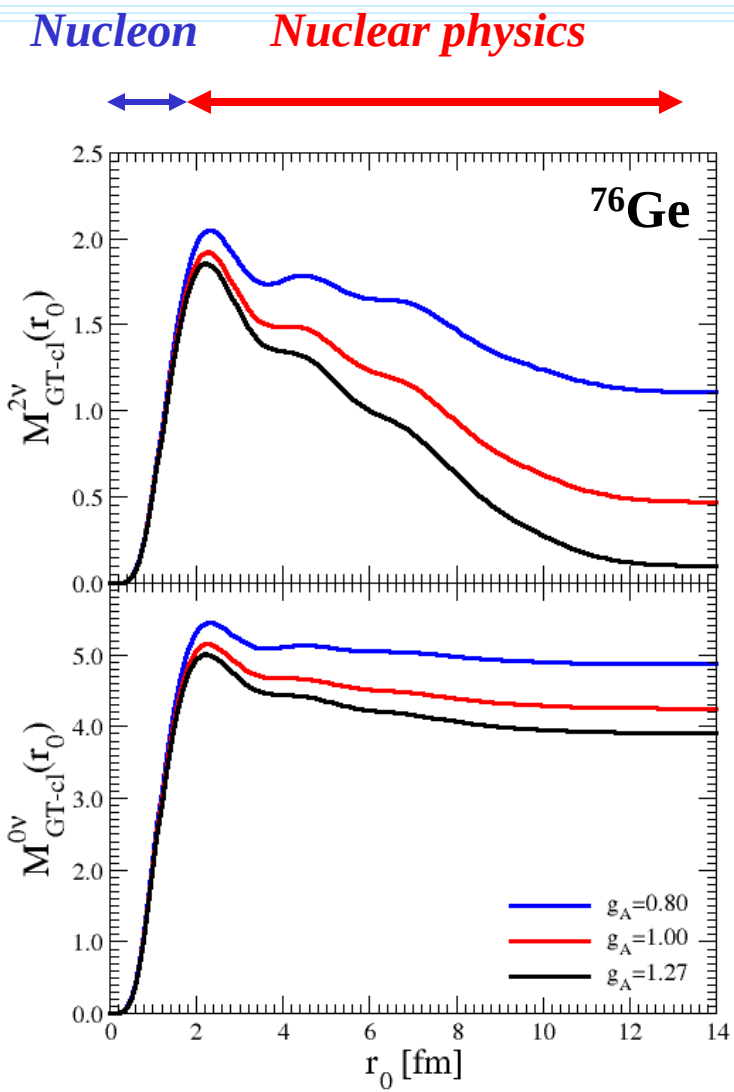
Neutrino potential prefer short distances

$M^{0\nu}_{GT}$ depends weakly on g_A/g_{pp} and QRPA approach unlike $M^{2\nu}_{GT}$

$$M^{0\nu}_{GT}(r_0) = \int_0^{r_0} H^{0\nu}_{GT}(r) C^{2\nu}_{GT-cl}(r) dr$$



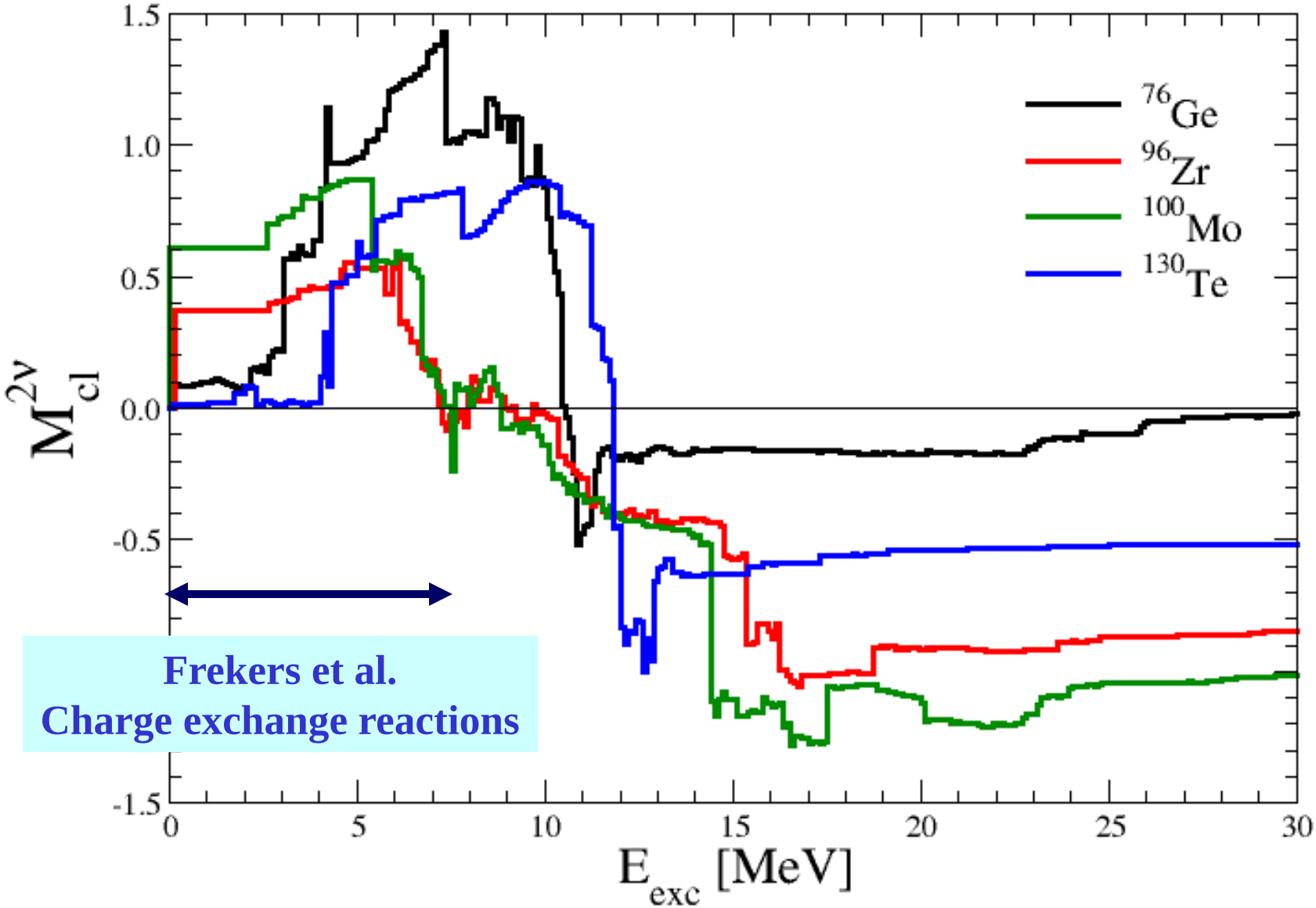
F.Š.,



Different QRPA-like approches

Dependence on axial-vector coupling

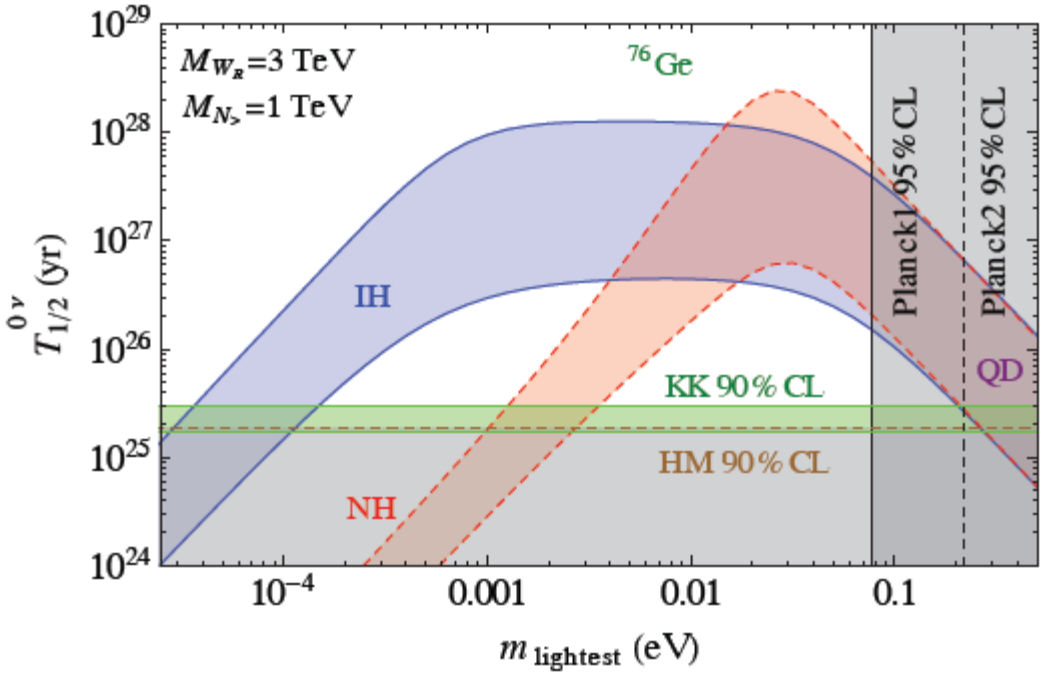
There is no proportionality between 0νββ-decay and 2νββ-decay NMEs!!!



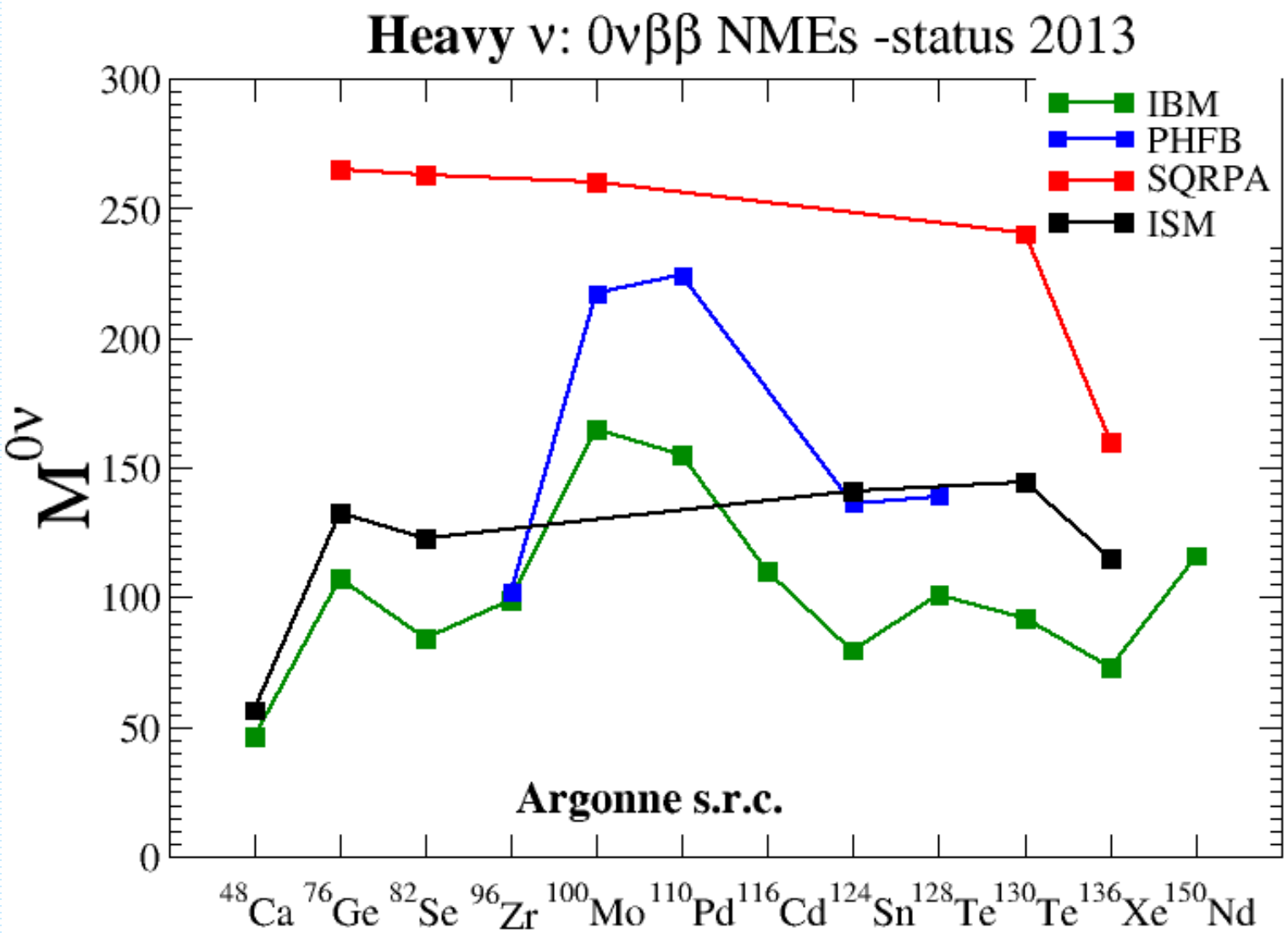
Frekers et al.
Charge exchange reactions

Heavy ν
0νββ-decay NMEs

LHC
and L-R symmetric models



Presentation of W. Rodejohann



PHFB: **K. Rath et al., PRC 85 (2012) 014308**
IBM: **Barea, Kotila, Iachello, PRC (2013) 014315**

SQRPA: **Vergados, Ejiri, F. Š., RPP 75 (2012) 106301**
ISM: Menendez, privite communications

Co-existence of few mechanisms of the $0\nu\beta\beta$-decay

It may happen that in year 201? (or 2???) the $0\nu\beta\beta$-decay will be detected for 2-3 or more isotopes …

(If there will be enough money for enrichment of isotopes!?)

Probing the see-saw mechanism

Bilenky, Faessler, Potzel, F.Š, Eur. Phys. J. C 71 (2011) 1754

There exist heavy Majorana neutral leptons N_i (singlet of SU(2)xU(1) group)

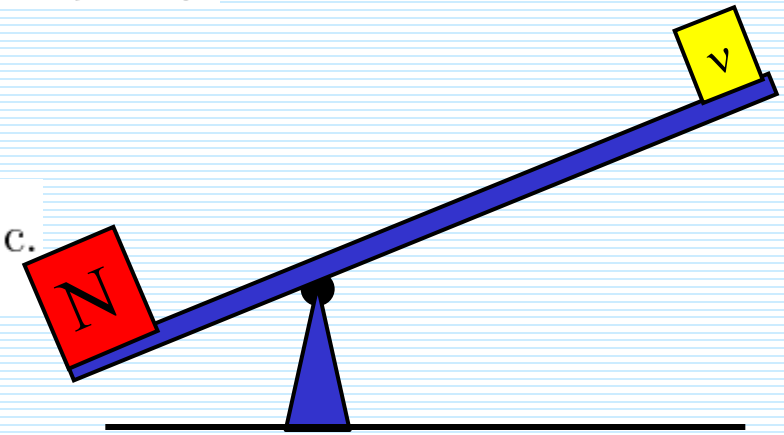
$$\mathcal{L} = -\sqrt{2}\sum_{i,l} Y_{li}\overline{L}_{lL}N_{iR}\tilde{H} + \text{h.c.}.$$

$$L_{lL} = \begin{pmatrix} \nu_{lL} \\ l_L \end{pmatrix}$$

$$N_i = N_i^c = C\bar{N}_i^T$$

Effective interaction for processes with virtual N_i at electroweak scale

$$\mathcal{L}_{\text{eff}} = -\frac{1}{\Lambda}\sum_{l',l,i}\overline{L_{l'L}}\tilde{H}\sum_i (Y_{l'i}\frac{\Lambda}{M_i}Y_{li})C\tilde{H}^T(\overline{L_{lL}})^T + \text{h.c.}$$

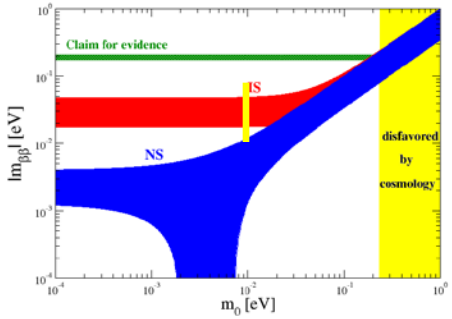


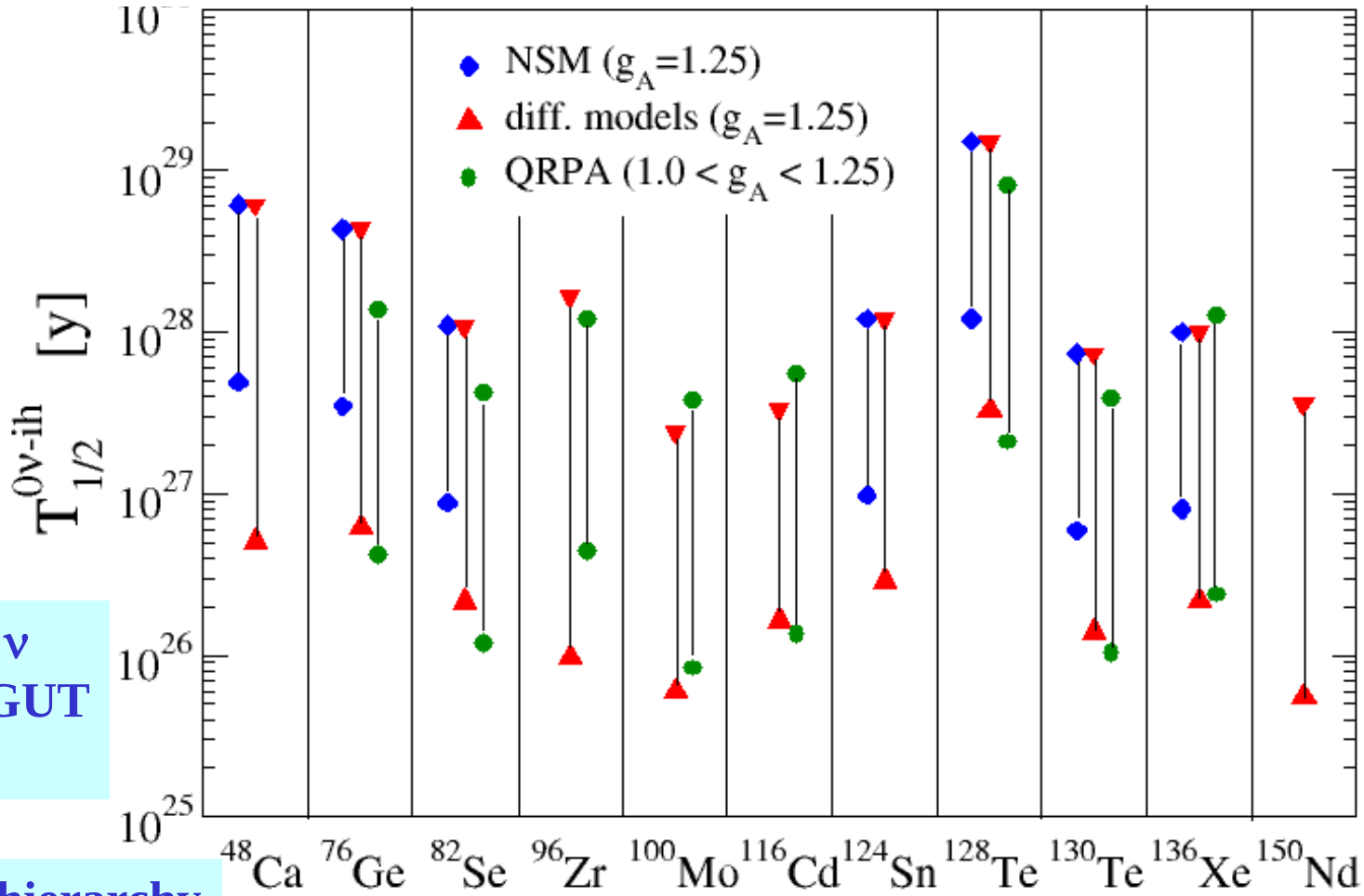
After spontaneous violation of the electroweak symmetry the left-handed Majorana mass term is generated

$$M^L = Y\frac{v^2}{M}Y^T = U\ m\ U^T$$

$$\begin{aligned}\mathcal{L}^{\text{M}} &= -\frac{1}{2}\sum_{l',l}\overline{\nu}_{l'L}\ M^L_{l'l}\ (\nu_{lL})^c + \text{h.c.} \\ &= -\frac{1}{2}\sum_i m_i\bar{\nu}_i\nu_i,\end{aligned}$$

Probing the standard see-saw mechanism





Heavy ν Mass at GUT scale

Inverted hierarchy

Co-existence of 2, 3 or more interferring mechanisms of 0νββ-decay

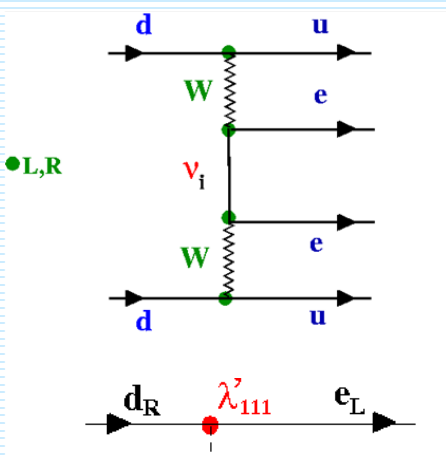
It is well-known that there exist many mechanisms that may contribute to the 0νββ. Let consider 3 mechanisms: i) light ν-mass mechanism, ii) heavy ν-mass mechanism, iii) R-parity breaking SUSY mechanism with gluino exchange and CP conservation

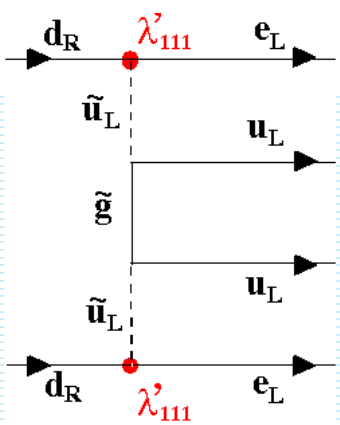
$$\frac{1}{T^{0\nu}_{1/2}} = G^{0\nu}(E_0, Z)\left|\frac{m_{\beta\beta}}{m_e} M^{0\nu}_{\nu} + \eta^L_N M^{0\nu}_N + \eta_{\lambda'_{111}} M^{0\nu}_{\lambda'_{111}} \ldots\right|^2$$

$$m_{\beta\beta} = \sum_k \left(U^L_{ek}\right)^2 \xi_k m_k$$

$$\eta^L_N = \sum_k^{heavy} U^2_{ek} \frac{m_p}{M_k}$$

$$\eta_{\lambda'_{111}} = \frac{\pi\alpha_s}{6} \frac{\lambda'^2_{111}}{G_F^2 m^4_{\tilde{d}_R}} \frac{m_p}{m_{\tilde{g}}} \left[1 + \left(\frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}}\right)^2\right]^2 .$$





Claim of evidence:

Klapdor-Kleingrothaus, Krivosheina, Mod. Phys. A 21, 1547 (2009)

$$T^{0\nu}_{1/2}(^{76}Ge) = 2.23^{+0.44}_{-0.31} \times 10^{25}\ \mathrm{y}$$

$$T^{0\nu}_{1/2}(^{100}\mathrm{Mo}) \geq 5.8 \times 10^{23}\ \mathrm{y}$$

$$T^{0\nu}_{1/2}(^{130}\mathrm{Te}) \geq 3.0 \times 10^{24}\ \mathrm{y}$$

$\xi_{Te} < 1.2$

$\xi_{Mo} < 2.6$

We introduce

$$\xi = \frac{|M^\nu_1|\sqrt{T_1\ G_1}}{|M^\nu_2|\sqrt{T_2\ G_2}}$$

ξ=0, non-observation ($T_2 \to \infty$)

ξ=1, solution for single active mech. is reproduced

4 sets of two linear eq.

$$\frac{\pm 1}{\sqrt{T_1\ G_1}} = \frac{m_{\beta\beta}}{m_e} M_1^\nu + \eta M_1^\eta$$

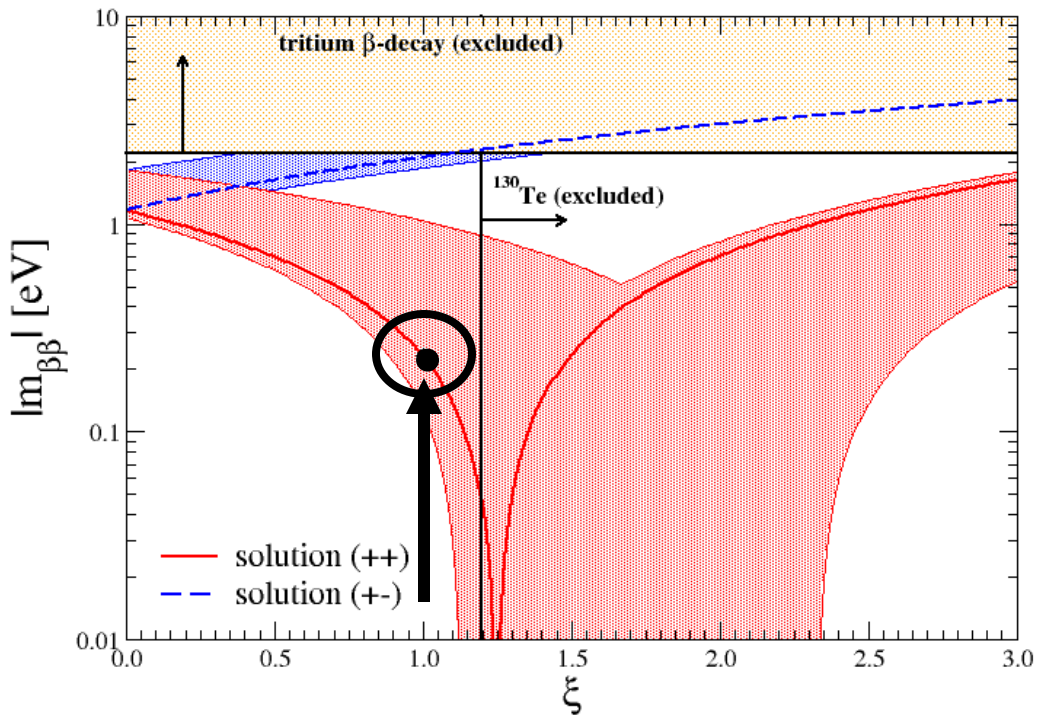
$$\frac{\pm 1}{\sqrt{T_2\ G_2}} = \frac{m_{\beta\beta}}{m_e} M_2^\nu + \eta M_2^\eta$$

2 different solutions ***CP-conservation assumed***

$$|m_{\beta\beta}| = \left| \frac{m_e}{M_1^\nu \sqrt{T_1\ G_1}} \frac{M_1^\nu\ M_2^\eta}{(M_1^\nu M_2^\eta - M_2^\nu M_1^\eta)} \pm \frac{m_e}{M_2^\nu \sqrt{T_2\ G_2}} \frac{M_2^\nu\ M_1^\eta}{(M_1^\nu M_2^\eta - M_2^\nu M_1^\eta)} \right|$$

2 active mechanisms of the 0νββ-decay: Light and heavy ν-mass mechanism

F.Š., J.D. Vergados, A. Faessler, PRD 82, 113015 (2010)



Non-observation of the 0νββ-decay for some isotopes might be in agreement with non- zero $m_{\beta\beta}$

Non-observation for ^{130}Te

- ***Single solution for light ν-mass mech.***

Resonant Neutrinoless Double-Electron Capture

$(A,Z) \rightarrow (A,Z-2)^{**}$

Winter, Phys. Rev. 100 (1955) 142; Bernabeu, de Rujula, Jarlskog PRC 15 (1993) 223

Additional modes of the 0νECEC-decay:

$$e_b + e_b + (A,Z) \rightarrow (A,Z-2) + \gamma$$
$$+ 2\gamma$$
$$+ e^+e^-$$
$$+ M$$

Resonance enhancement of neutrinoless double electron capture

M.I. Krivoruchenko, F. Šimkovic, D. Frekers, and A. Faessler,
Nucl. Phys. A 859, 140-171 (2011)

- **New physical phenomenon, oscillations of atoms, was proposed. A connection to process of resonant neutrinoless double electron capture (0νεε) established.**
- **The process of the 0νεε has been revisited for those cases where the two participating atoms are nearly degenerate in mass. New 0νεε transitions with parity violation to ground and excited states of final atom/nucleus were found. Selection rules for the 0νεε transitions were established. The explicit form of corresponding NMEs was derived.**
- **Available data of atomic masses, as well as nuclear and atomic excitations were used to select the most likely candidates for resonant 0νεε transitions. Assuming an effective Majorana neutrino mass of 1 eV, some half-lives has been predicted to be as low as 10^{22} years in the unitary limit. According to obtained estimates, in the case of ^{152}Gd the sensitivity can be comparable to the favored 0νββ decays of nuclei.**
- **More accurate atomic mass measurements in the context of the 0νεε were initialized, which have been partially accomplished using the modern high-precision ion traps. In addition, new 0νεε experiments were initialized (TGV, R. Bernabei group at Gran Sasso, Muenster-Bratislava)**

Nuclear matrix elements for $0\nu\varepsilon\varepsilon$

Ground state to ground state nuclear transitions

Initial (final)	β_{Q_p}	$\beta_{B(E2)}$	$\langle BCS_i \| BCS_f \rangle$
^{152}Gd (^{152}Sm)	(+0.29)	0.212 (0.306)	0.44
^{164}Er (^{164}Dy)	0.36 (+0.32)	0.333 (0.348)	0.73
^{180}W (^{180}Hf)	0.27 (+0.27)	0.252 (0.273)	0.75

Deformed QRPA

Nucleus	$M^{2\nu}_{GT}$ $[MeV^{-1}]$	$M^{0\nu}$ sph. QRPA	def. QRPA ($\beta_2 = 0$)	def. QRPA
^{152}Gd	0.10	7.59	7.50	3.23
	0.00	7.21		2.67
^{164}Er	0.10	6.12	7.20	2.64
	0.00	5.94		2.27
^{180}W	0.10	5.79	6.22	2.05
	0.00	5.56		1.79

Fang et al., PRC 85, 035503 (2012)

EDF

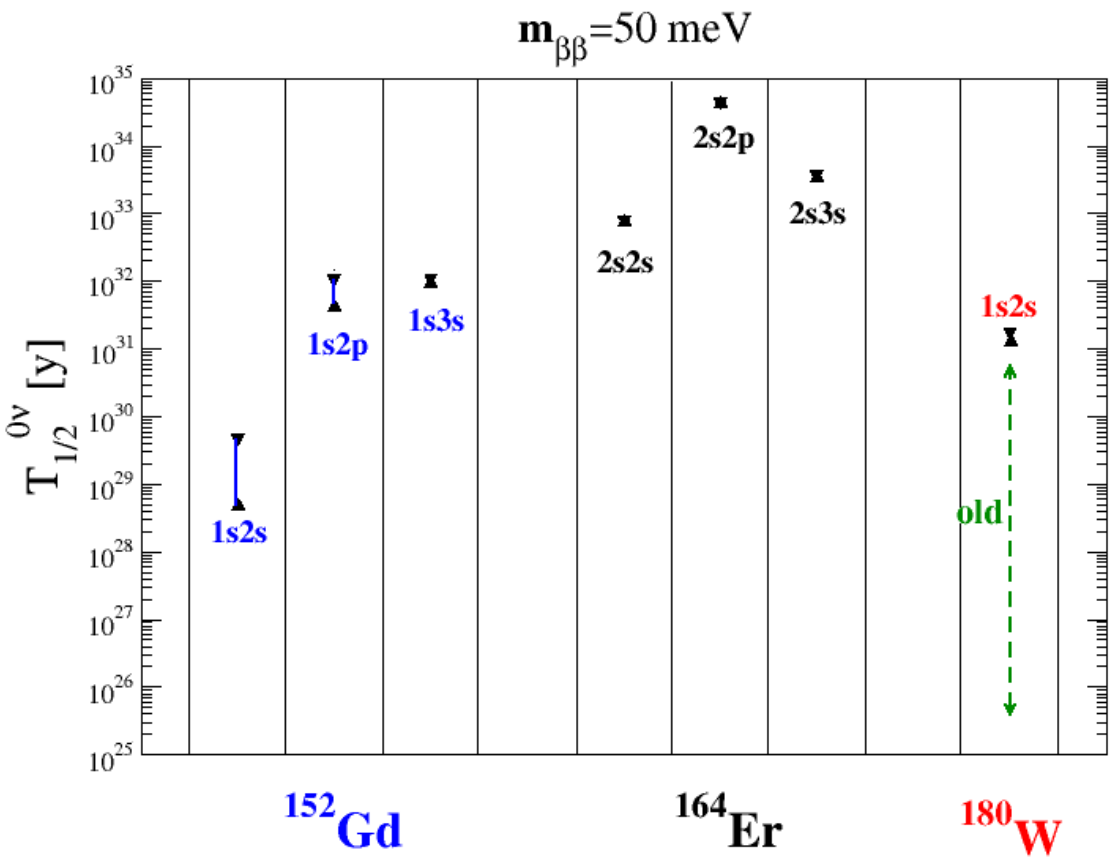
	$M^{0\nu}$
^{152}Gd	0.89, 1.07
^{164}Er	0.64, 0.50
^{180}W	0.58, 0.38

Rodrigez, Martinez-Pinedo, PRC 85, 044310 (2012)

Suppression of the NME depends not only on the relative deformation but also their absolute values

$0\nu\varepsilon\varepsilon$ half-lives

$m_{\beta\beta}$=50 meV



Nucleus	$(n2jl)_a$	$(n2jl)_b$	E_a	E_b	E_C	Γ_{ab} (keV)	Δ (keV)	$T^{\min}_{1/2}$ (y)	$T^{\max}_{1/2}$ (y)
^{152}Gd	110	210	46.83	7.74	0.34	2.3×10^{-2}	-0.83 ± 0.18	4.7×10^{28}	4.8×10^{29}
	110	211	46.83	7.31	0.32	2.3×10^{-2}	-1.27 ± 0.18	4.2×10^{31}	1.1×10^{32}
	110	310	46.83	1.72	0.11	3.2×10^{-2}	-7.07 ± 0.18	9.4×10^{31}	1.1×10^{32}
^{164}Er	210	210	9.05	9.05	0.22	8.6×10^{-3}	-6.82 ± 0.12	7.5×10^{32}	8.4×10^{32}
	210	211	9.05	8.58	0.23	8.3×10^{-3}	-7.28 ± 0.12	4.2×10^{34}	4.6×10^{34}
	210	310	9.05	2.05	0.11	1.8×10^{-2}	-13.92 ± 0.12	3.5×10^{33}	3.9×10^{33}
^{180}W	110	110	63.35	63.35	1.26	7.2×10^{-2}	-11.24 ± 0.27	1.3×10^{31}	1.8×10^{31}

A comparison

$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$

$e^- + e^- + (A,Z) \rightarrow (A,Z-2)^{**}$

Perturbation theory

$$\frac{1}{T^{0\nu}_{1/2}} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 G^{01}(E_0, Z) \left|M^{0\nu}\right|^2$$

Breit-Wigner form

$$\Gamma^{0\nu ECEC}(J^\pi) = \frac{|V_{\alpha\beta}(J^\pi)|^2}{(M_i - M_f)^2 + \Gamma^2_{\alpha\beta}/4}\,\Gamma_{\alpha\beta}$$

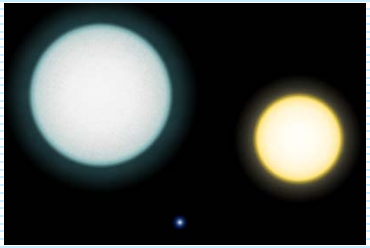
- **2νββ-decay background can be a problem**
- **Uncertainty in NMEs factor ~2, 3**
- **$0^+ \rightarrow 0^+, 2^+$ transitions**
- **Large Q-value**
- **^{76}Ge, ^{82}Se, ^{100}Mo, ^{130}Te, ^{136}Xe ...**
- **Many exp. in construction, potential for observation in the case of inverted hierarchy (2020)**

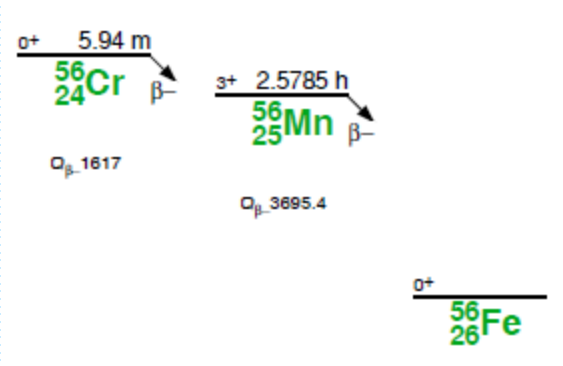
- **2νεε-decay strongly suppressed**
- **NMEs need to be calculated**
- **$0^+ \rightarrow 0^+, 0^-, 1^+, 1^-$ transitions**
- **Small Q-value**
- **Q-value needs to be measured at least with 100 eV accuracy**
- **^{152}Gd, looking for additional**
- **small experiments yet**

Universe as a laboratory to study LN violation

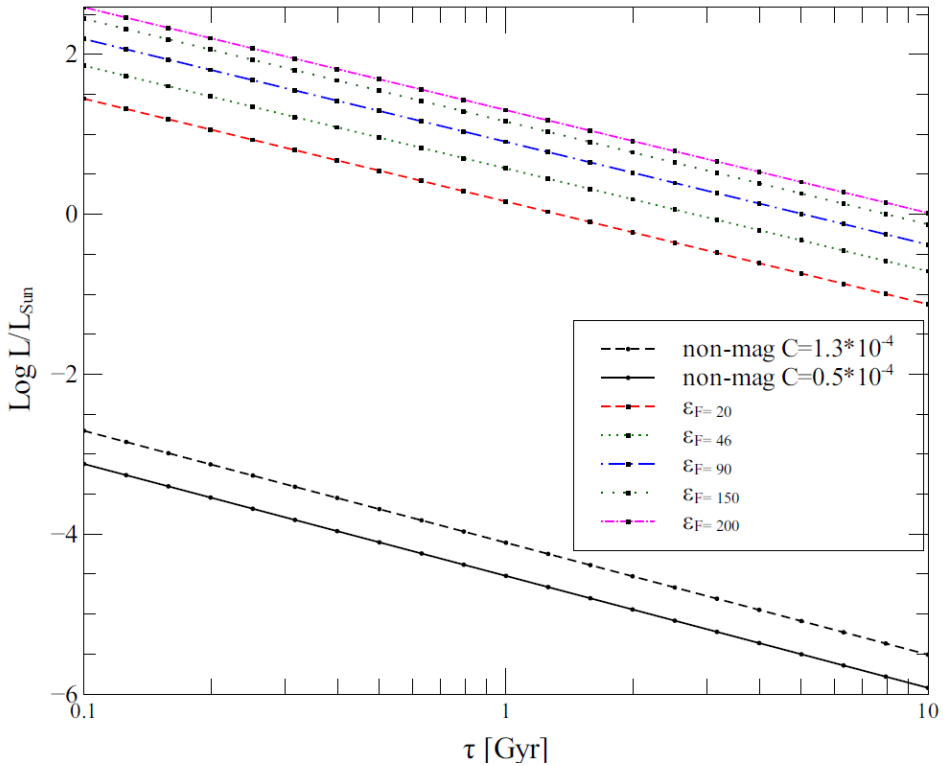
Belyaev, Ricci, Simkovic, Truhlik, arXiv: 1212.3155, Truhlik, MEDEX13 presentation

Cooling of strongly magnetized iron White dwarfs





$$e^- + {}^{56}Fe \rightarrow {}^{56}Cr + e^+$$

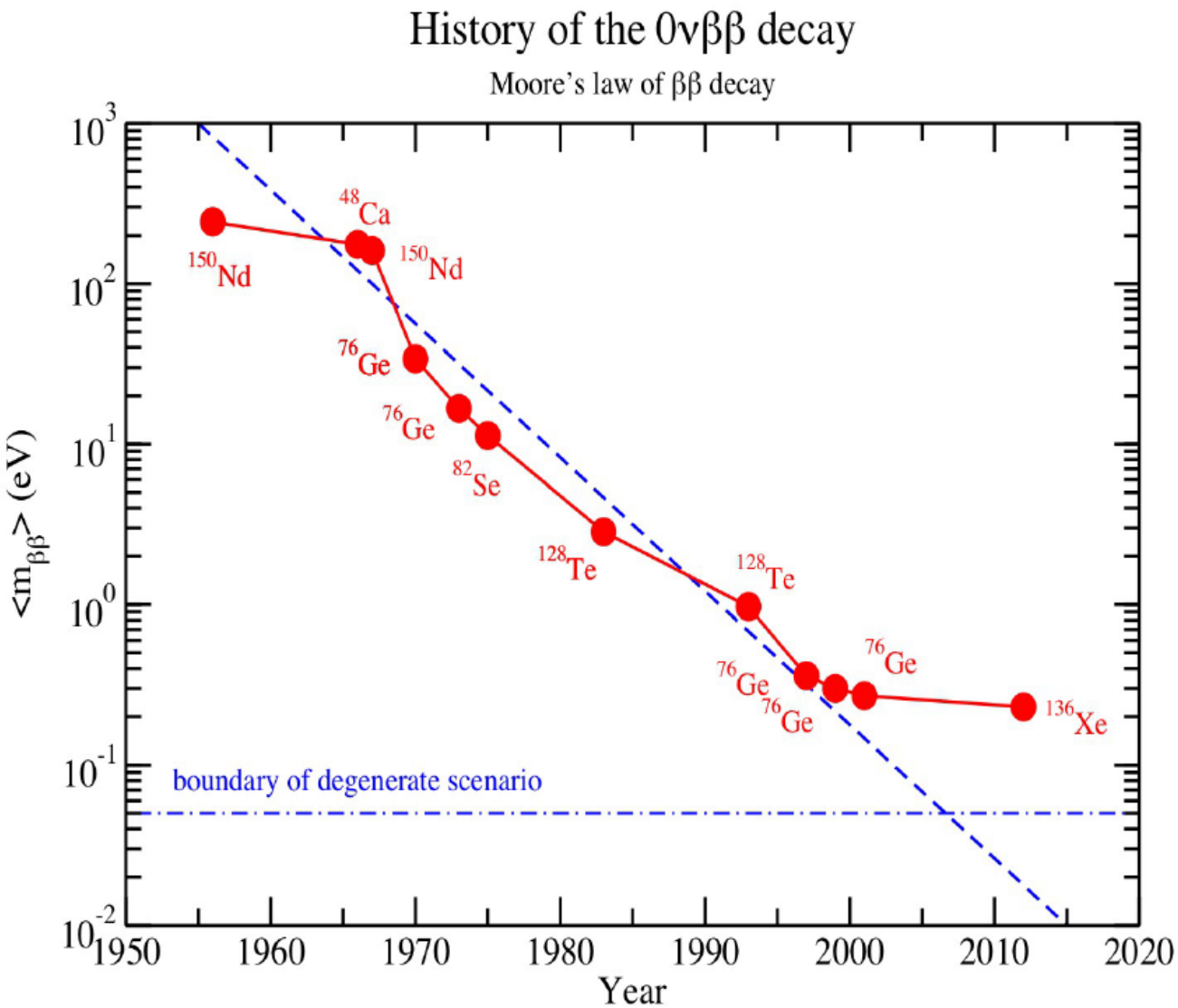


OUTLOOK

(slide of P. Vogel at Indian summer school, Prague, 2012)

Historically, there are > 100 experimental limits on $T_{1/2}$ of the $0\nu\beta\beta$ decay.

However, during the last decade the complexity and cost of such experiments increased dramatically The constant slope is no longer maintained.



Actually, when NMEs will be needed?

IOP PUBLISHING REPORTS ON PROGRESS IN PHYSICS

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Theory of neutrinoless double-beta decay

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