Global analysis of the Sivers functions at NLO+NNLL in QCD (2009.10710)

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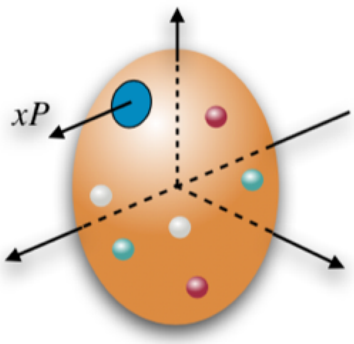
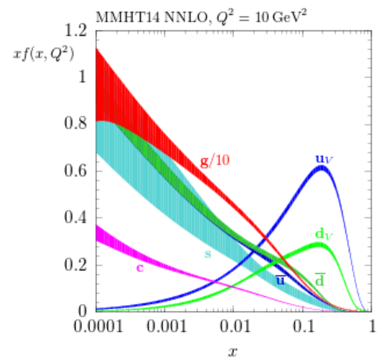
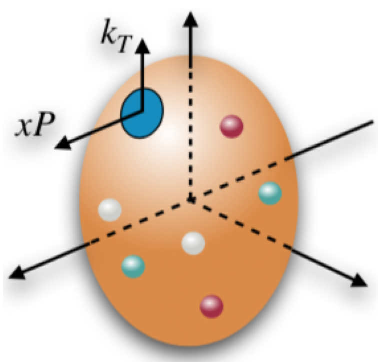
Motivation for Transverse Momentum Distributions (TMDs)





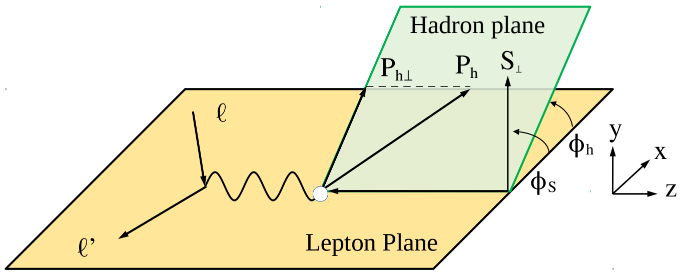
PDFs, momentum densities along collinear direction.

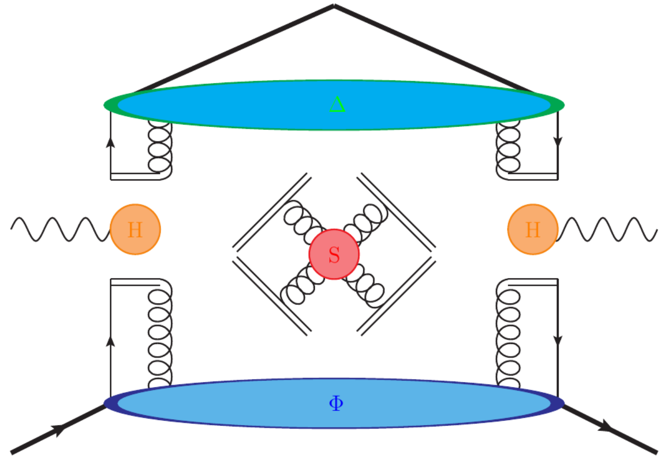


Goals of TMDs

- Momentum: Provide a 3-dimensional picture of the motion of the partons in a hadron.
- Correlation: Are there correlations between the motion of the partons and the hadron's spin?

Semi-Inclusive DIS for TMDs





$$\frac{d\sigma(\boldsymbol{S}_\perp)}{dx_B\, dy\, dz_h\, d^2P_{h\perp}} \sim \sigma_0\, H(Q^2)\, \Phi(x, k_\perp, P, \boldsymbol{S}_\perp)\, \Delta\, (z, k'_\perp, P_h) S(\lambda_\perp)$$

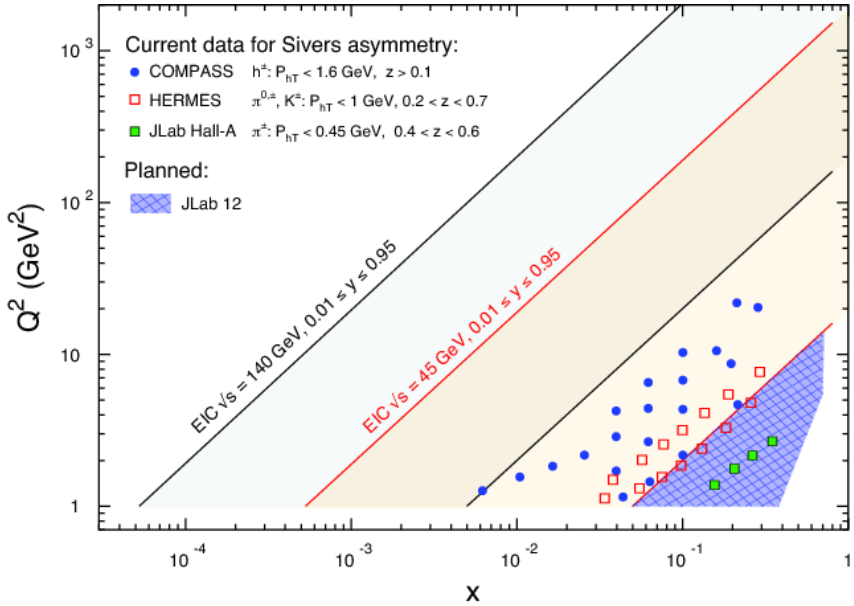
Quark Pol (columns); Proton Pol (rows)

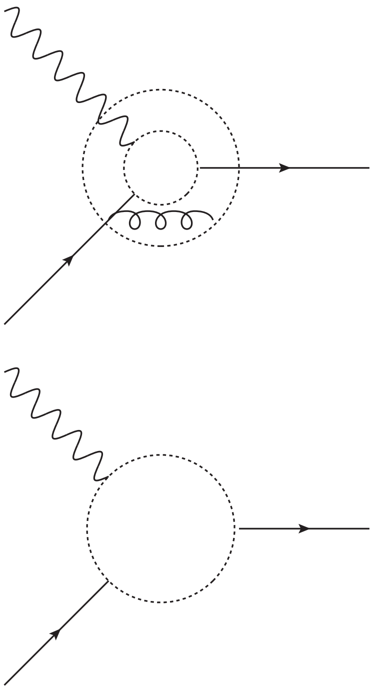
Φ	U	L	T
U	f		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

Quark Pol (columns); Hadron Pol (rows)

Δ	U	L	T
U	D		$H_1^\perp$
L		G_1	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	H_1 $H_{1T}^\perp$

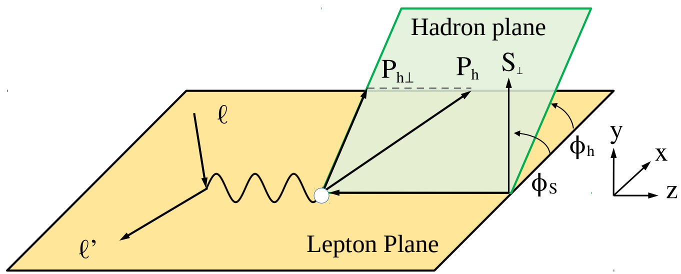
QCD evolution

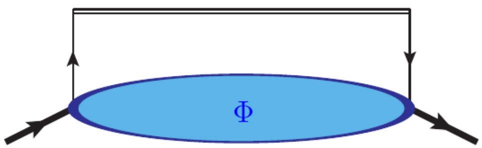




- DGLAP evolution provides information on how the x_B dependence of the distributions evolve in Q.
- TMD evolution provides information on how the $k_\perp$ dependence of the distributions evolve in Q.

The Sivers asymmetry





$$= f_{q/p}(x_B, k_\perp) - \sin(\phi_s - \phi_k) \frac{k_\perp}{M} f_{1T,q/p}^\perp(x_B, k_\perp)$$

$$A_{\mathrm{UT}}^{\sin(\phi_h - \phi_s)} = \frac{d\sigma(\boldsymbol{S}_\perp) - d\sigma(-\boldsymbol{S}_\perp)}{d\sigma(\boldsymbol{S}_\perp) + d\sigma(-\boldsymbol{S}_\perp)} = \frac{F_{UT}^{\sin(\phi_h - \phi_s)}}{F_{UU}}$$

$$\frac{F_{UT}^{\sin(\phi_h - \phi_s)}}{F_{UU}} \sim \frac{f_{1T,q/p}^\perp(x_B, k_\perp) D_{h/q}(z_h, k'_\perp)}{f_{q/p}(x_B, k_\perp) D_{h/q}(z_h, k'_\perp)} .$$

Structure functions

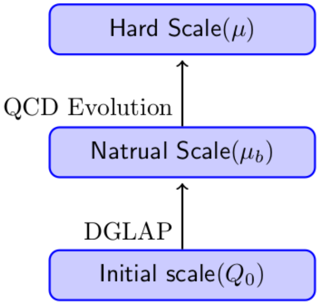
Unpolarized structure function in the TMD formalism

$$F_{UU}\left(x_B, z_h, P_{h\perp}, Q\right) = \int_0^\infty \frac{bdb}{2\pi} J_0\left(\frac{b\,P_{h\perp}}{z_h}\right) \widetilde{F}_{UU}\left(x_B, z_h, b, Q\right)$$

$$\widetilde{F}_{UU}\left(x_B, z_h, b, Q\right) \equiv H\left(Q^2\right) \sum_q e_q^2 C_{q\leftarrow i} \otimes f_{i/p}\left(x_B, \mu_{b*}\right) \hat{C}_{j\leftarrow q} \otimes D_{h/j}\left(z_h, \mu_{b*}\right) \times \exp\left(S_{pert} + S_{NP}\right)$$

$$\mu_{b*} \sim c_0/b$$

- DGLAP evolution from initial scale to μ_{b*}.
- DGLAP evolution from initial scale to μ_{b*}.
- Perturbative TMD evolution from μ_{b*} to Q.
- Non-perturbative TMD evolution from initial scale to Q.

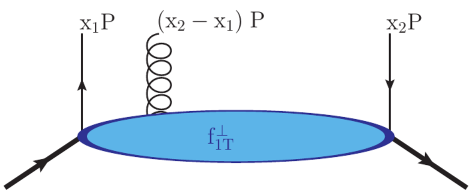


Structure functions

Polarized structure function in the TMD formalism

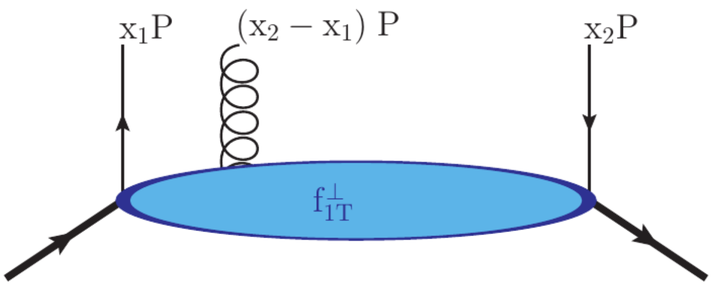
$$F_{UT}^{\sin(\phi_h-\phi_s)}(x_B, z_h, P_{h\perp}, Q) = \int_0^\infty \frac{b^2 db}{4\pi} J_1\left(\frac{b P_{h\perp}}{z_h}\right) \widetilde{F}_{UT}^{\sin(\phi_h-\phi_s)}(x_B, z_h, b, Q)$$

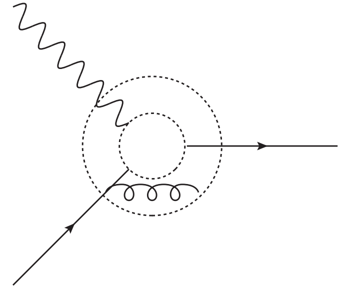
$$\widetilde{F}_{UT}^{\sin(\phi_h-\phi_s)}(x_B, z_h, b, Q) \equiv H\left(Q^2\right) \sum_q e_q^2 \bar{C}_{q\leftarrow i} \otimes T_{F,i/p}(x_B, \mu_{b_*})$$
$$\times \hat{C}_{j\leftarrow q} \otimes D_{h/j}(z_h, \mu_{b_*}) \exp\left(S_{pert} + S_{NP}^{\perp}\right)$$



- DGLAP evolution for Qiu-Sterman function is not well-understood.
- This evolution is treated phenomenologically in one of two schemes.

DGLAP evolution of the Qiu-Sterman function





- Scheme 1.) Zhong-bo and Jianwei showed in 2009 that at large x_B, the evolution can be treated as a parallel evolution.

$$\frac{\partial T_{F,q/p}(x,x,\mu)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \left[P^{\mathrm{T}}_{q\leftarrow q} \otimes T_{F,q/p}\right](x,\mu)$$

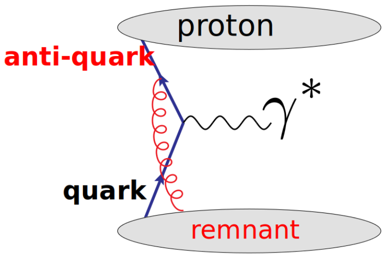
$$P^{\mathrm{T}}_{q\leftarrow q}(x) = P_{q\leftarrow q}(x) - \eta\, \delta(1-x)\,, \qquad \eta = N_c$$

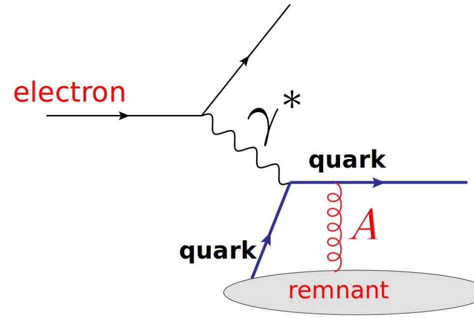
- Scheme 2.) For phenomenological applications, it is common to treat the evolution to be the same as for the collinear PDF. Can model this behavior with $\eta = 0$. (Anselmino et al 2012)

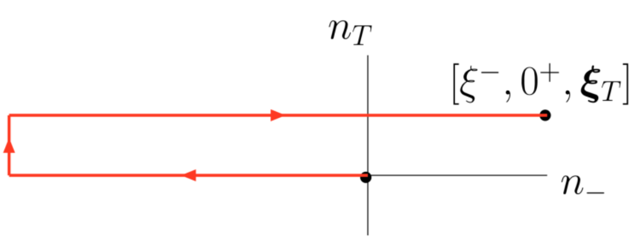
Fit to low energy data

Collaboration	Scattering event	Number of points	Year
COMPASS	$e + P \to e + h^+$	34	2017
	$e + P \to e + h^-$	31	2017
	$e + D \to e + \pi^+$	12	2008
	$e + D \to e + \pi^+$	12	2008
	$e + D \to e + K^+$	13	2008
	$e + D \to e + K^0$	7	2008
	$e + D \to e + K^-$	11	2008
	$\pi^- + P \to \gamma^*$	15	2017
hermes	$e + P \to e + K^-$	14	2009
	$e + P \to e + K^+$	14	2009
	$e + P \to e + \pi^0$	13	2009
	$e + P \to e + \pi^+$	14	2009
	$e + P \to e + \pi^-$	14	2009
	$e + P \to e + \pi^-$	14	2009
Jefferson Lab	$e + P \to e + \pi^+$	4	2011
	$e + P \to e + \pi^-$	4	2011
RHIC	~~$P + P \to W/Z$~~	~~17~~	2015

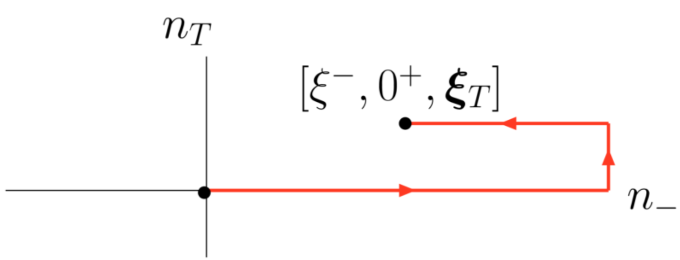
Signs change of the Sivers function







DY
past-pointing



SIDIS
future-pointing

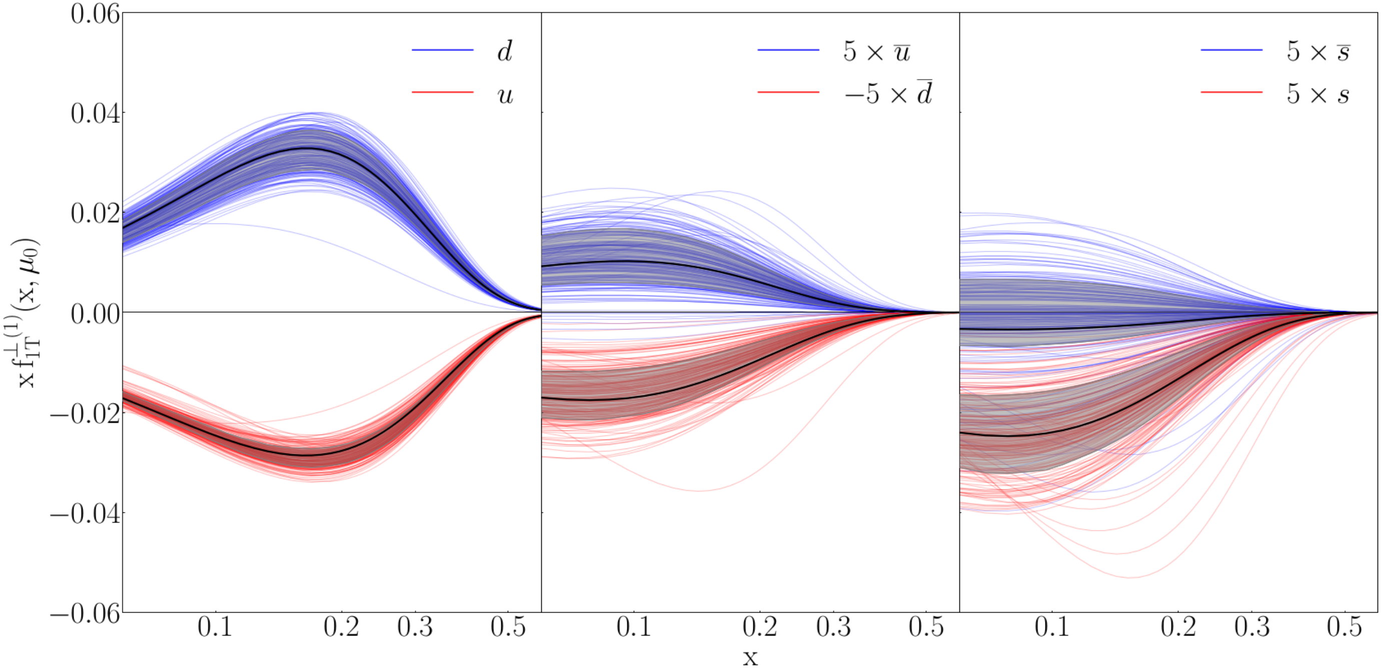
Possible sign change between Sivers function in SIDIS and Drell-Yan.

$$f_{1T,q/p}^{\perp q,\mathrm{DY}}(x, k_\perp^2) = -f_{1T,q/p}^{\perp q,\mathrm{SIDIS}}(x, k_\perp^2)$$

Collins 02, Boer-Mulders-Pijlman 03, Collins-Metz 04, Kang-Qiu 09

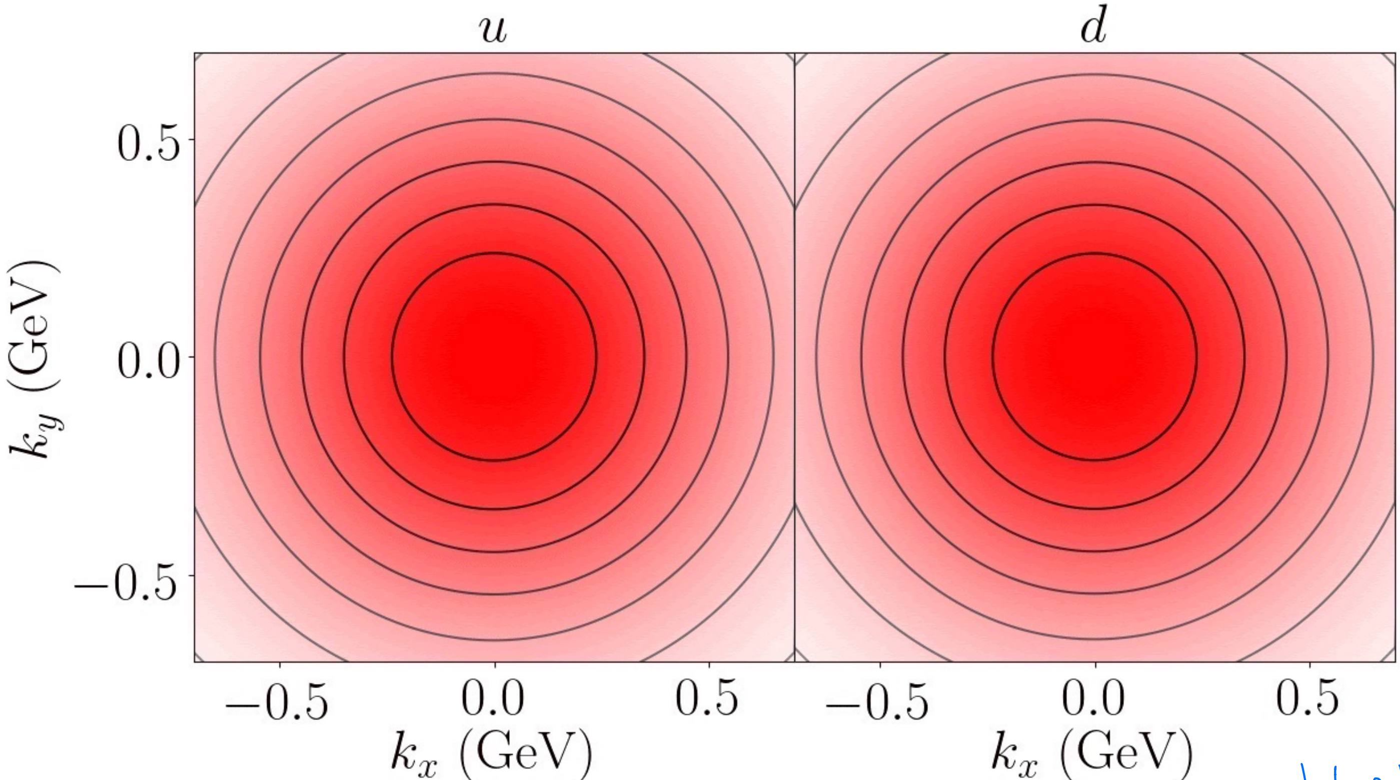
Fit results without RHIC data

$\chi^2/\text{d.o.f} = 1.032$ for 12 parameters and 226 points.



Unpolarized TMD PDF

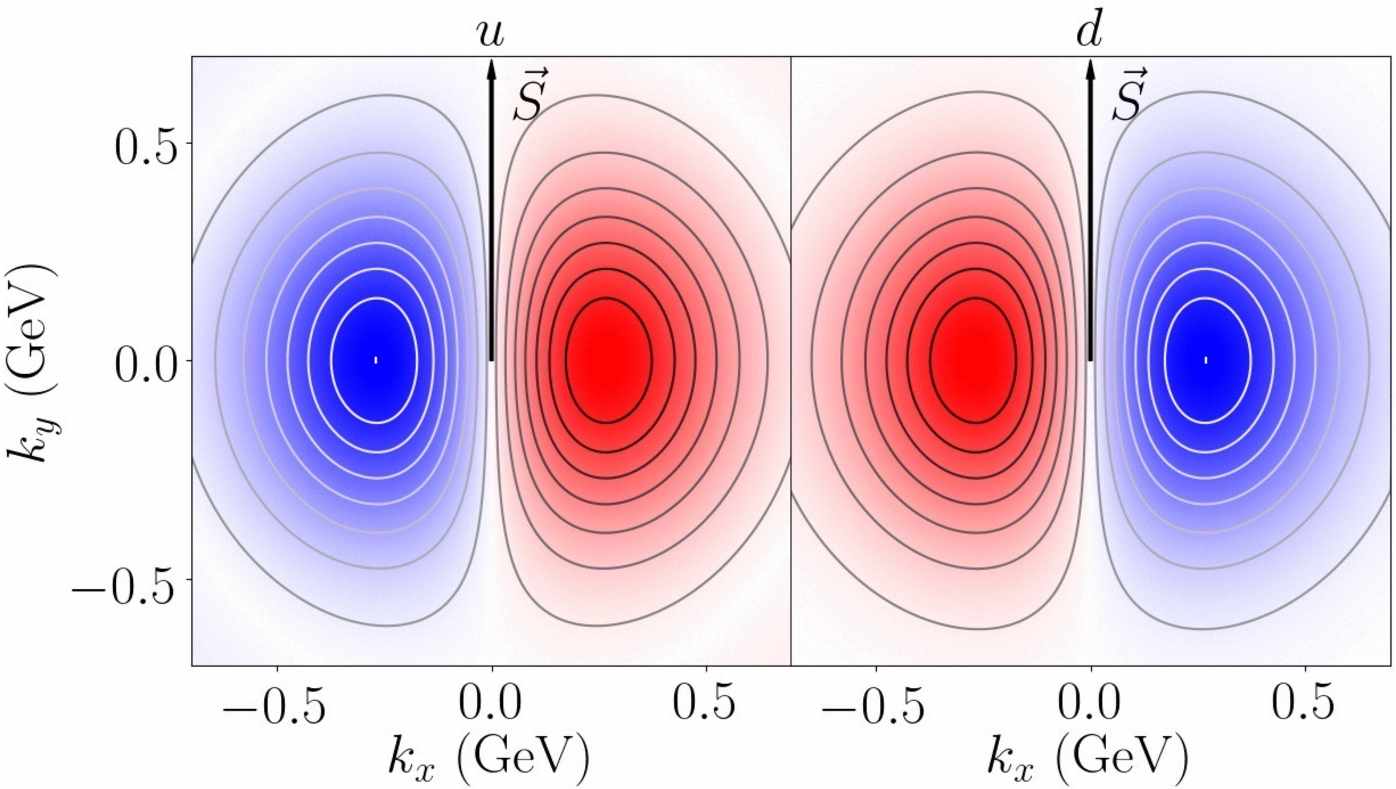
$f_{q/p}(x_B, k_\perp) \qquad x_B = 0.2 \qquad Q^2 \in [2, 100]\,\mathrm{GeV}^2$



1 GeV

Sivers function

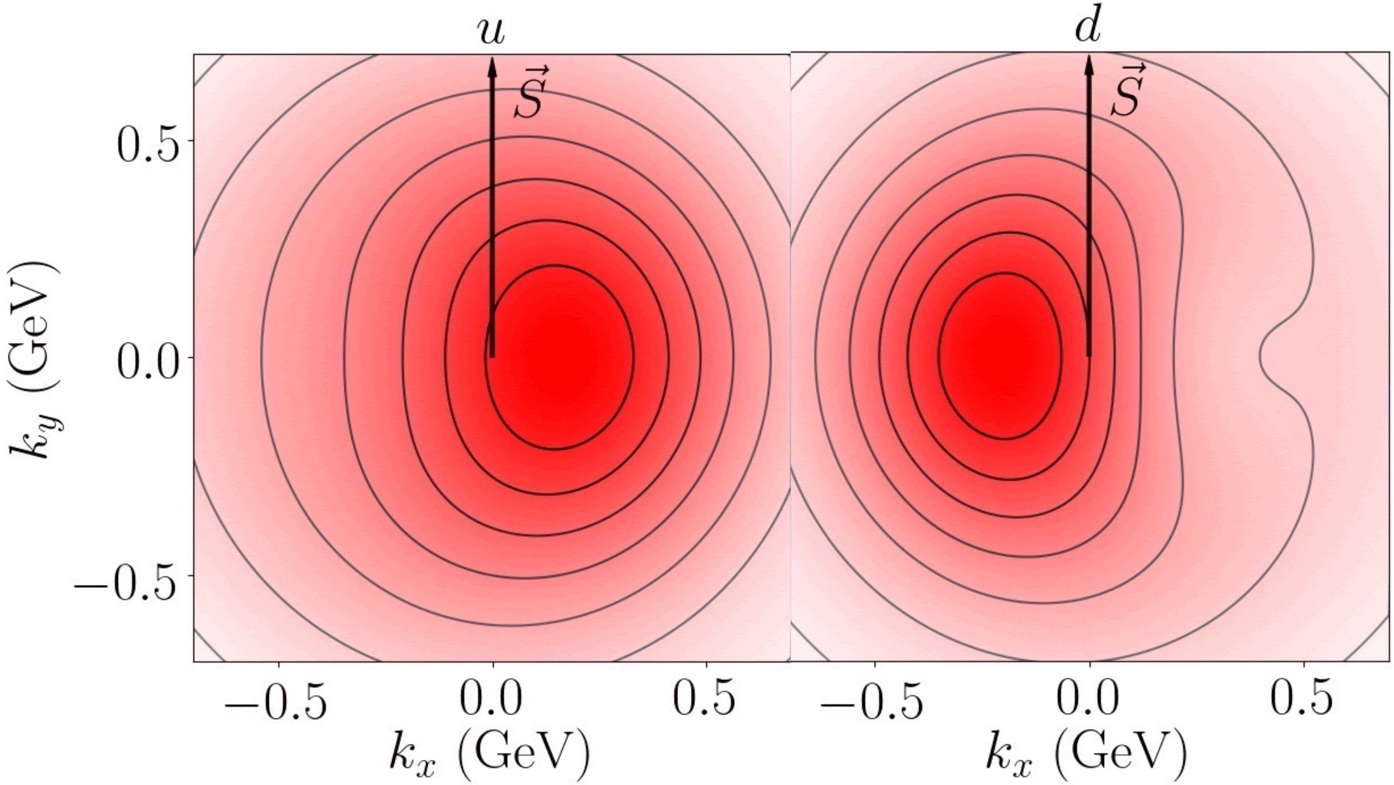
$$-\sin(\phi_s - \phi_k)\frac{k_\perp}{M} f^{\perp}_{1T,q/p}(x_B, k_\perp) \qquad x_B = 0.2 \qquad Q^2 \in [2, 100]\,\mathrm{GeV}^2$$



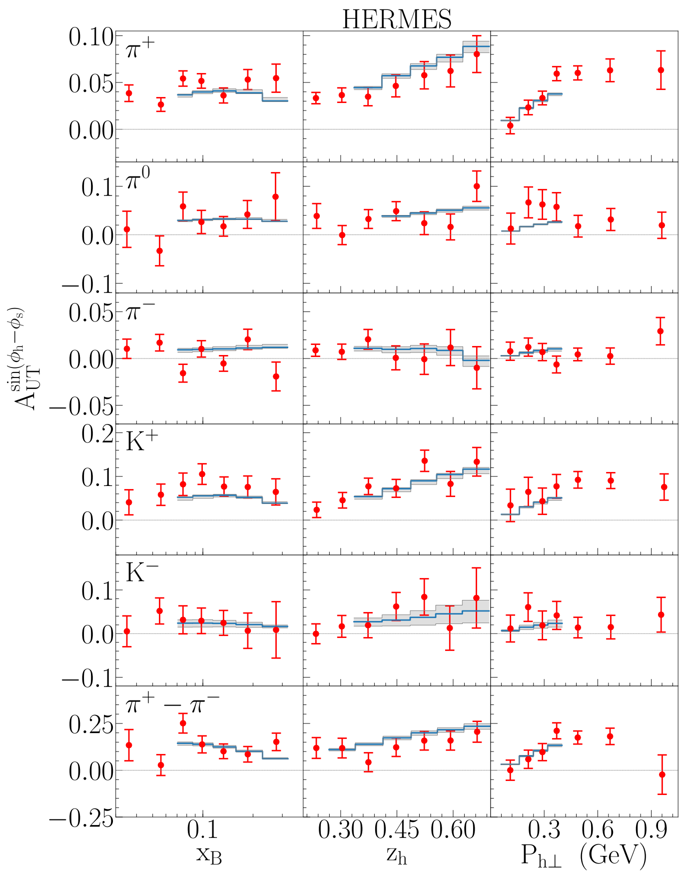
Spin-dependent TMD PDF

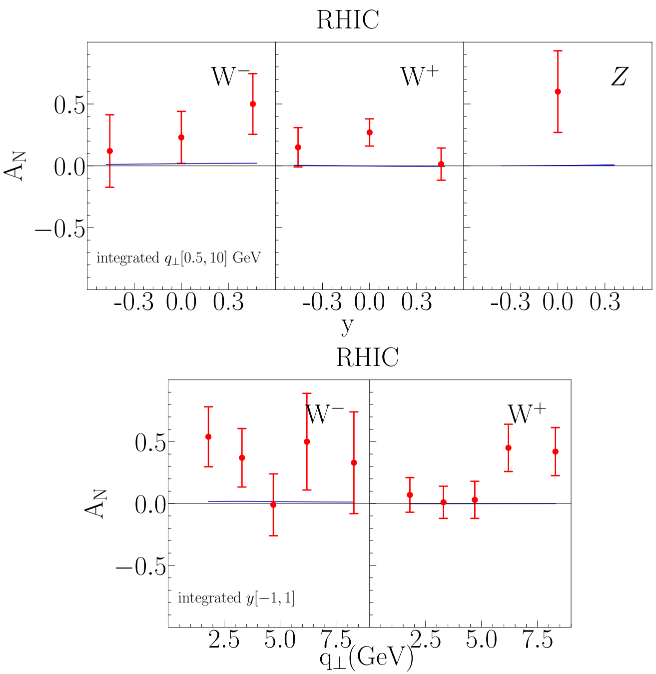
$$f_{q/p}(x_B, k_\perp) - \sin(\phi_s - \phi_k)\frac{k_\perp}{M} f_{1T,q/p}^{\perp}(x_B, k_\perp)$$

$$x_B = 0.2 \qquad Q^2 \in [2, 100]\,\text{GeV}^2$$



Description of the data (RHIC not included in fit)



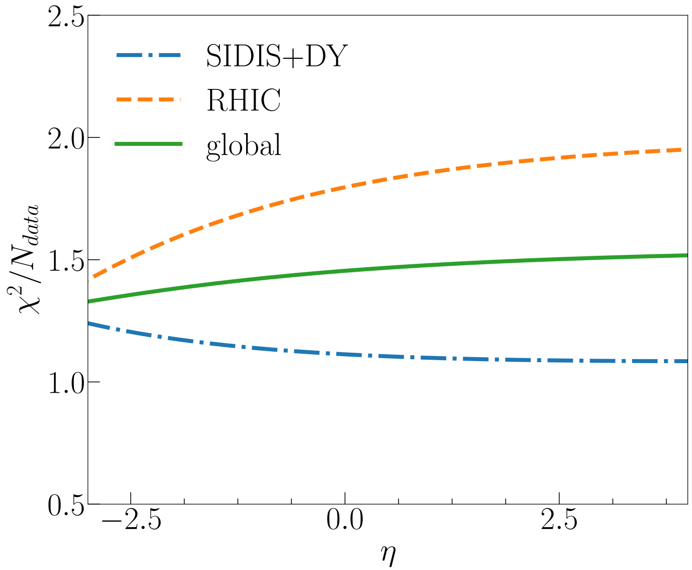


Dependence on the DGLAP evolution

$$P^{\mathrm{T}}_{q\leftarrow q}(x) = P_{q\leftarrow q}(x) - \eta\,\delta(1-x)\,,$$

$$\Delta\mathcal{T} = \frac{1}{N_{\mathrm{set}}}\sum_{i=1}^{N_{\mathrm{set}}}\left|\frac{\mathrm{Scheme}_1(\eta = N_C) - \mathrm{Scheme}_2(\eta = 0)}{\mathrm{Scheme}_1(\eta = N_C)}\right| \times 100$$

- For SIDIS and COMPASS DY data $\Delta\mathcal{T} \sim 1\%$.
- For RHIC data $\Delta\mathcal{T} \sim 50\%$.



Global weighted fit ($\omega = 226/17$)

Collaboration	Scattering event	Number of points	Year
COMPASS	$e + P \to e + h^+$	34	2017
	$e + P \to e + h^-$	31	2017
	$e + D \to e + \pi^+$	12	2008
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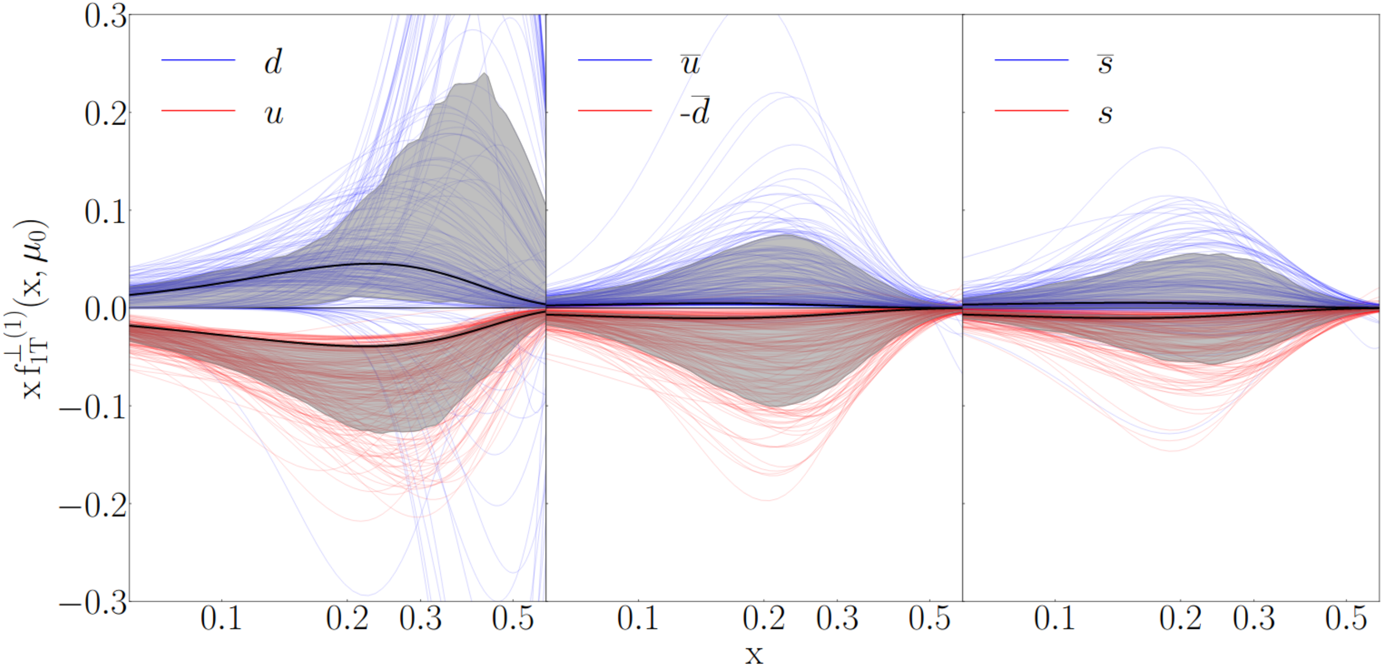
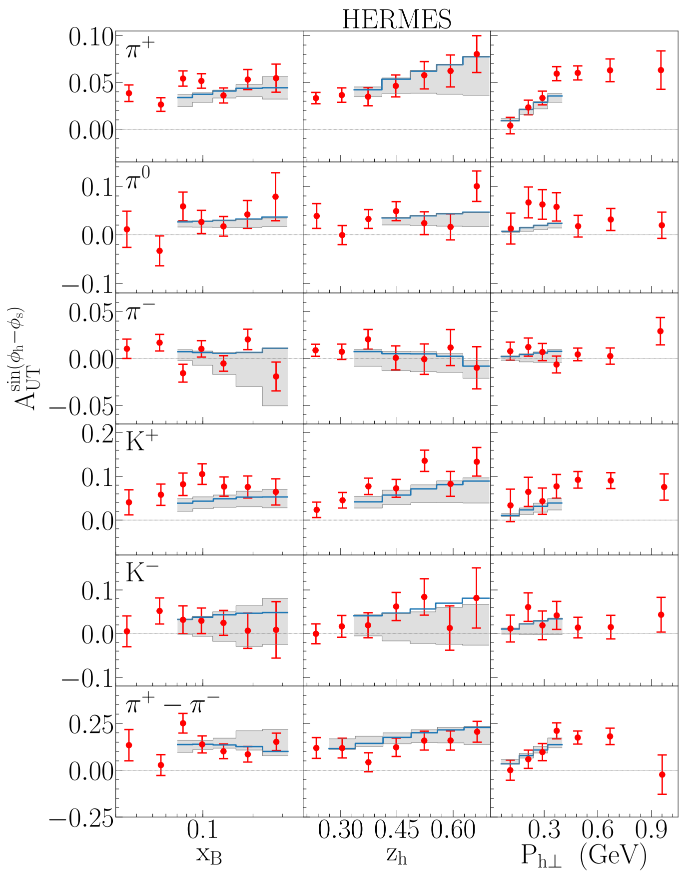
	$e + P \to e + K^+$	14	2009
	$e + P \to e + \pi^0$	13	2009
	$e + P \to e + \pi^+$	14	2009
	$e + P \to e + \pi^-$	14	2009
	$e + P \to e + \pi^-$	14	2009
Jefferson Lab	$e + P \to e + \pi^+$	4	2011
	$e + P \to e + \pi^-$	4	2011
RHIC	$P + P \to W/Z$	$17\times\omega$	2015

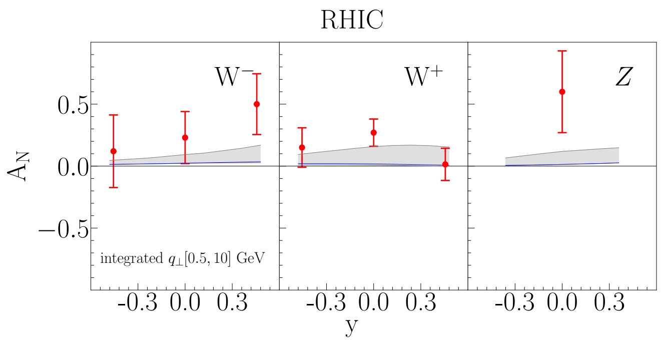
Fit results with RHIC data

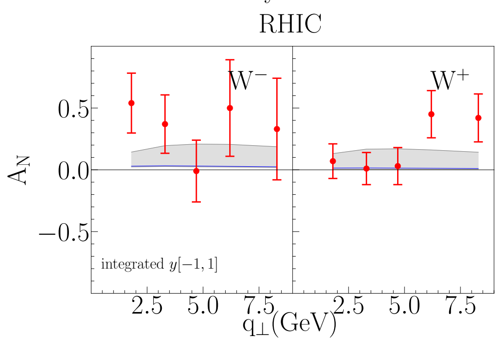
χ^2/d.o.f = 1.482 for 12 parameters and 243 points.



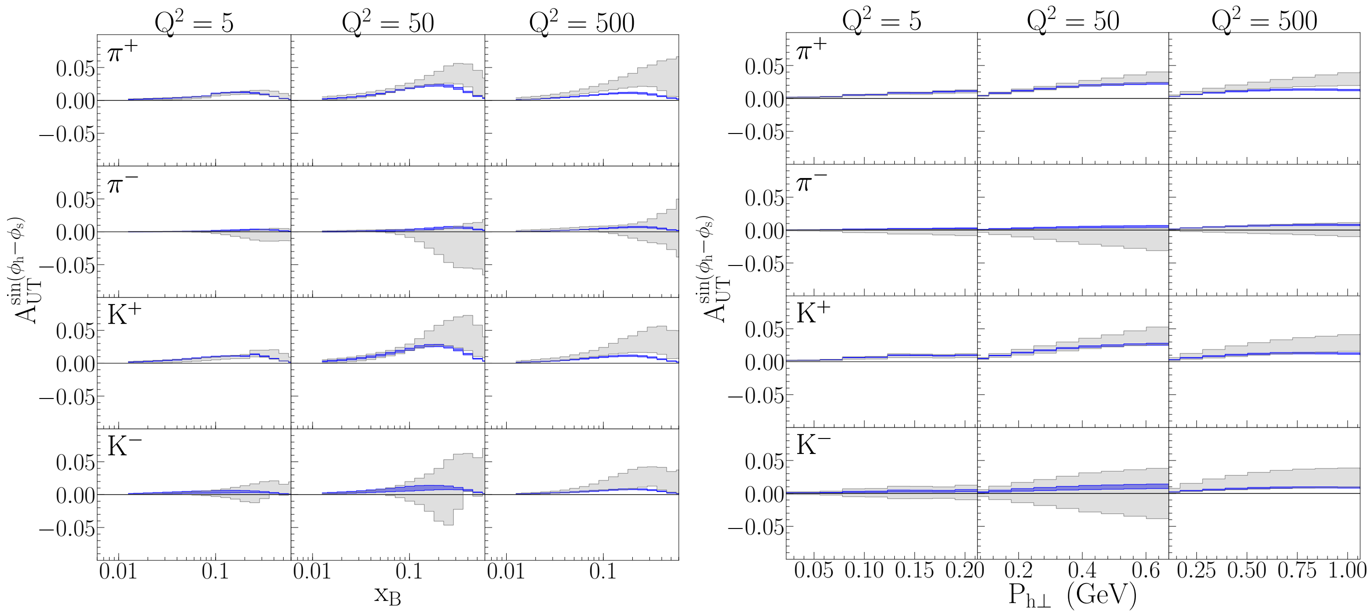
Description of the data (with RHIC data included)







Prediction at the EIC/Conclusion



- Additional data at large Q is needed to reduce the error at large Q
- The same effects should occur for all $k_\perp$-odd TMDs.