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Neutron Energy Effects on Asteroid Deflection

Workshop for Applied Nuclear Data Activities (WANDA 2021)

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Overview

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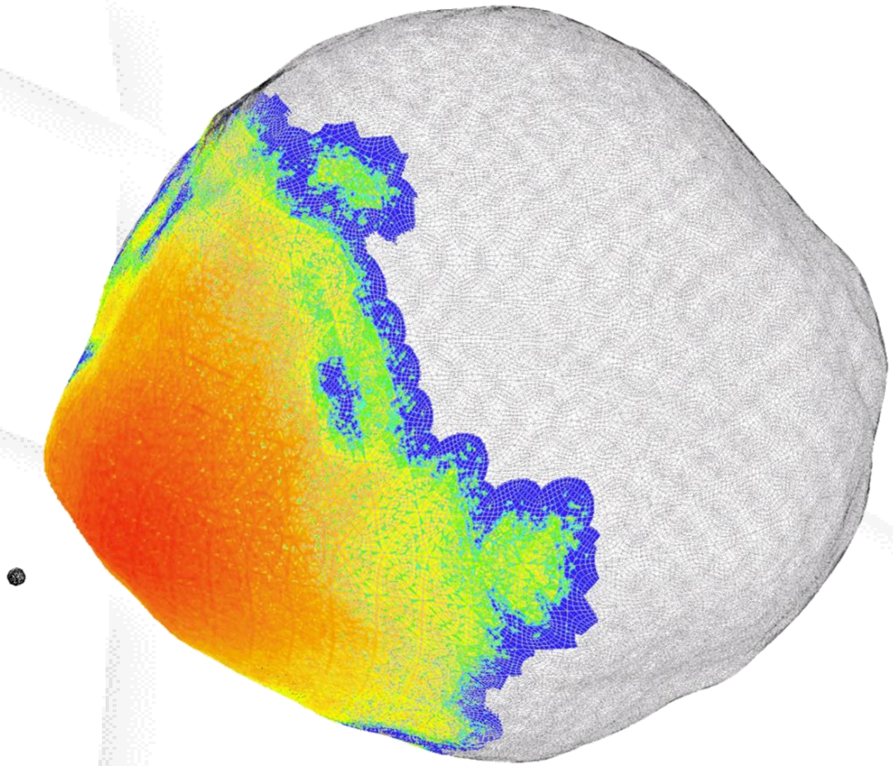
4. Summary & Conclusions

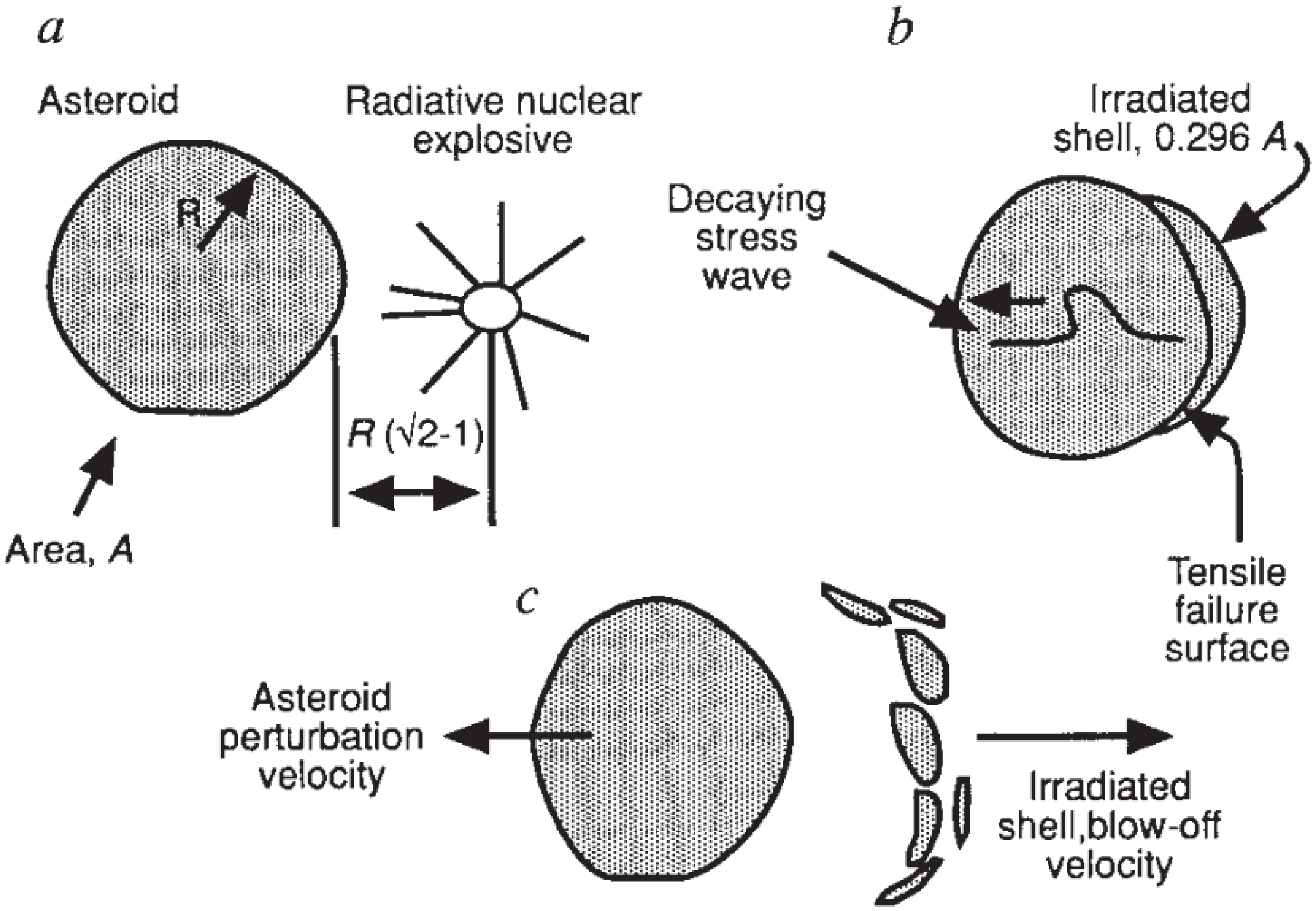


https://www.llnl.gov/news/nuclear-impulse-could-deflect-massive-asteroid

1. Background & Motivation

How does asteroid deflection via a stand-off nuclear detonation work?

Thomas Ahrens & Alan Harris. Deflection and fragmentation of near-Earth asteroids. *Nature*, 1992.



Newton's Law of Conservation of Momentum.

Nuclear deflection is an established concept. But, open-question: which neutron energy is best?

- **Problem:** Does the neutron energy affect asteroid deflection?
- **Hypothesis:** Affirmative.
- **Why?** Neutrons of different energies can interact very differently when they traverse the same material, which can change:
  - energy deposition profiles
  - energy coupling efficiencies

Why does this matter? This type of research could help determine which type of device outputs are most effective for deflecting asteroids, and whether altering the neutron energy spectrum would ever be worthwhile.

Specifications of the sources and the target considered in this work.

- **Sources:**
  - Neutron energies – 14.1 MeV (fusion) & 1 MeV (fission)
  - Neutron yield – 50 kt
  - Stand-off distance ~ 62 m from asteroid
- **Target:**
  - 300 m diameter asteroid, perfectly spherical
  - SiO_2 @ 2.65 g/cc, with 30% porosity (1.855 g/cc bulk density)

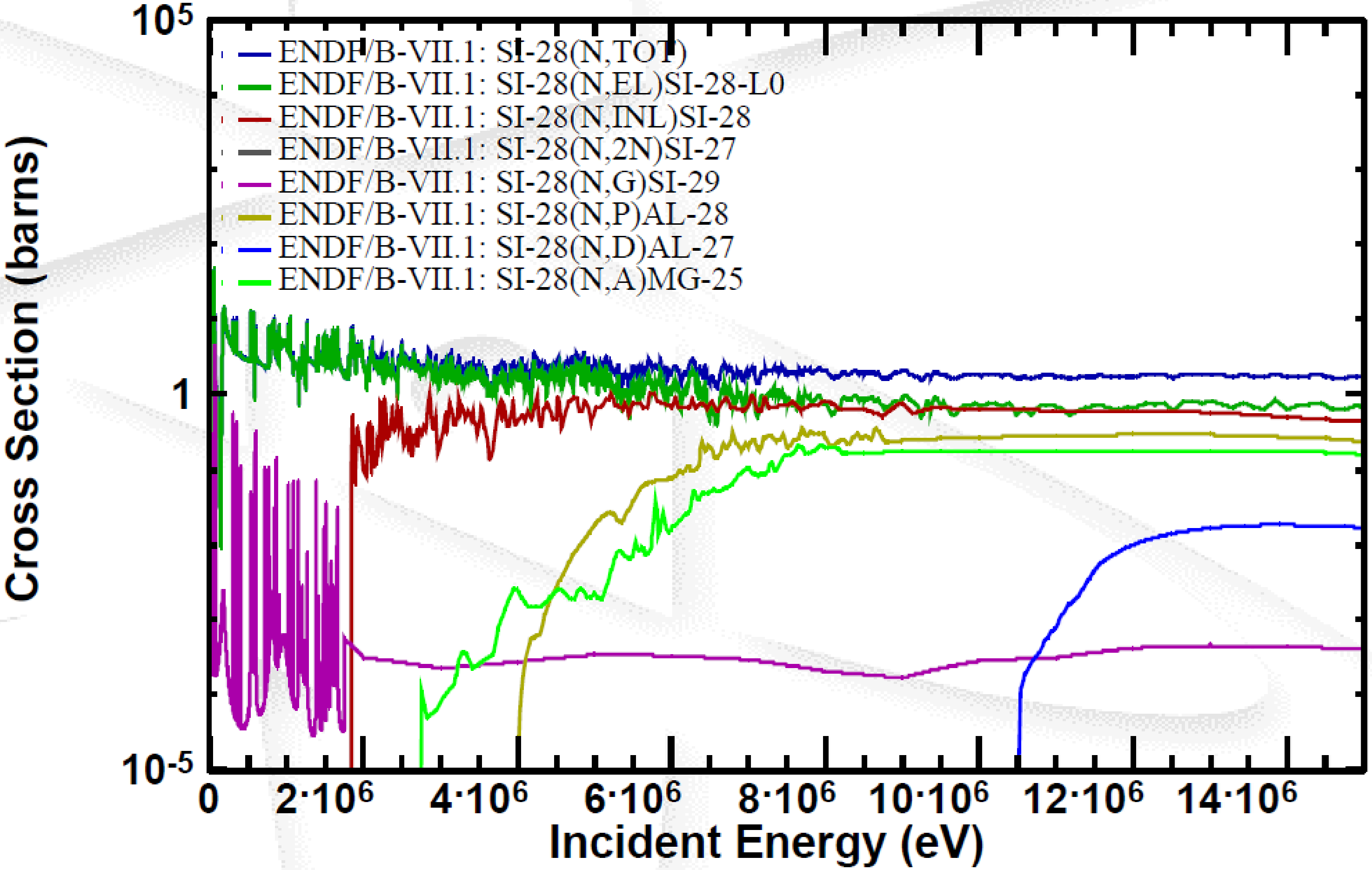
Phase I, Neutron Energy Deposition.
Sources were simulated in MCNP6.2,
a Monte Carlo radiation transport code.

Phase II, Asteroid Deflective Response.
Target was simulated in ALE3D,
a hydrodynamic material response code.

2. Role of Nuclear Data

Nuclear data is the very foundation of this work.

- For a given problem geometry and material composition, **nuclear cross sections are how bulk neutron interactions are mapped to energy deposition profiles.**



changing the neutron energy changes:

1. cross section magnitude
2. mean-free-path
3. spatial extent/distribution
4. energy deposition profiles

changing the neutron energy changes:

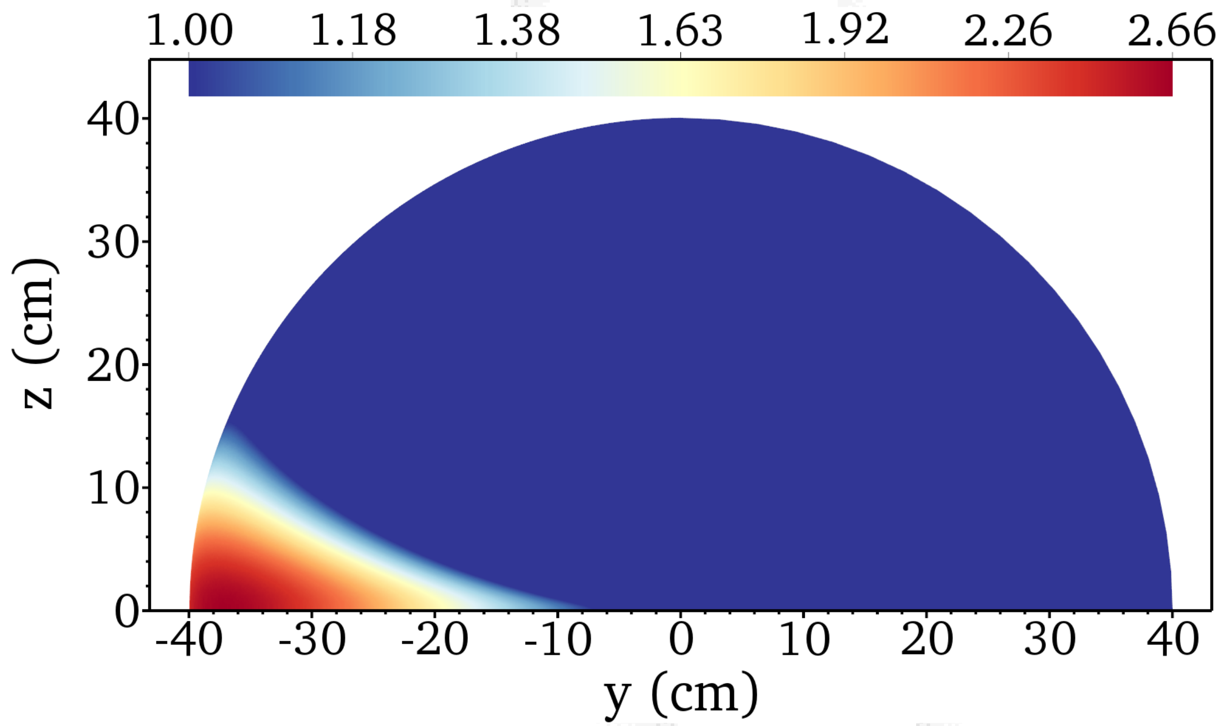
1. open/closed reaction channels
2. endothermic/exothermic reactions
3. energy coupling efficiencies

Nuclear data *ultimately determines the end results of asteroid deflection*.

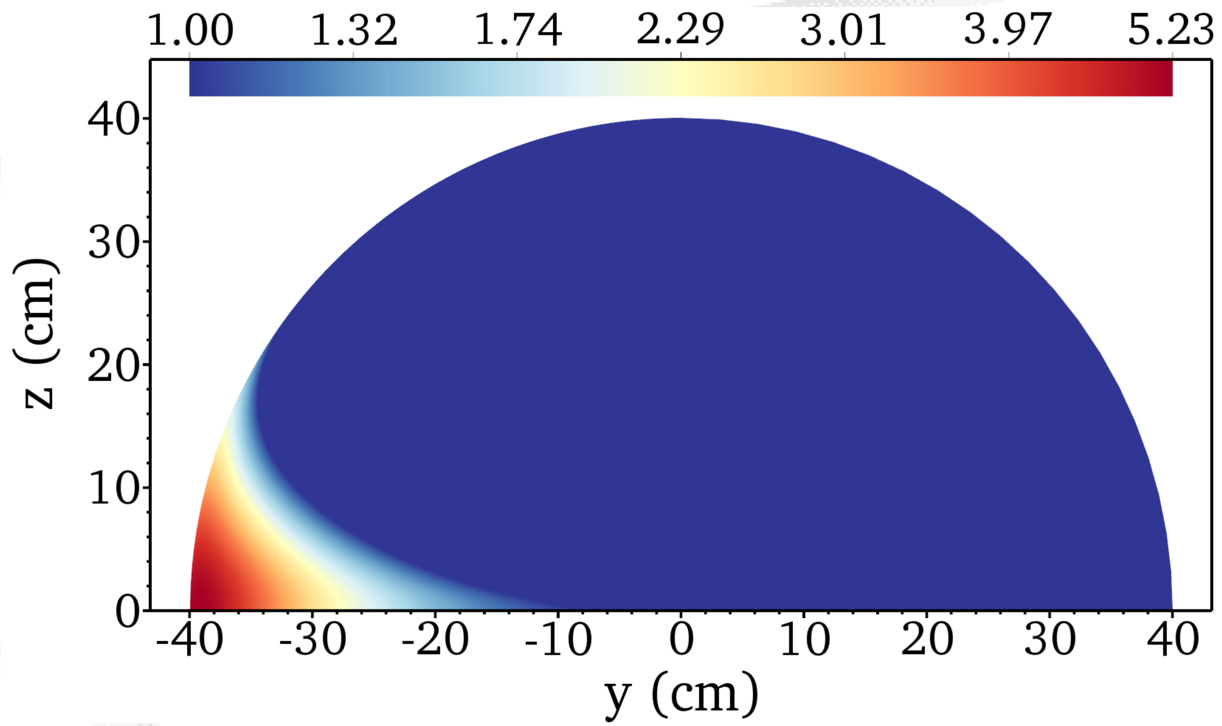
3. Energy Deposition & Deflective Response

MCNP

Input: nuclear cross section data
Output: energy deposition



14.1 MeV neutrons
max melt depth ~33 cm
max melt angle ~22.5°
peak intensity ~2.66X melt threshold
coupling efficiency ~69%



1 MeV neutrons
max melt depth ~31 cm
max melt angle ~35°
peak intensity ~5.23X melt threshold
coupling efficiency ~109%

ALE3D

Input: energy deposition
Output: deflection velocity change

- **For a 50 kt neutron yield, the deflection velocity change, δV, is 61% higher for 1 MeV neutrons vs 14.1 MeV neutrons.**

Y_n	E_n	src-n	$E_{dep,tot}^{ALE3D}$	δV
50 kt	14.1 MeV	$9.31469 \cdot 10^{25}$	5.0364 kt	6.19 ± 0.06 cm/s
50 kt	1 MeV	$1.26157 \cdot 10^{27}$	7.9785 kt	9.99 ± 0.12 cm/s

In some respects, for certain deflection scenarios, devices with greater 1 MeV neutron outputs can be more effective at deflecting asteroids than 14.1 MeV neutrons.

4. Summary & Conclusions

All of my results depend on the underlying nuclear data that I accessed, and how I accessed it.

- I used ENDF/B-VII.1 cross section libraries via MCNP6.2.
- Preliminary work with Mercury (discontinued due to lack of time): energy deposition trends were in agreement, but some unresolved quantitative differences vs. MCNP.
- ENDF/B-VII is not the newest U.S. library (ENDF/B-VIII.0), nor did I get to consider/compare to JENDL, KAERI, etc. libraries.
- MCNP's energy deposition (radiation) w/high-fidelity nuclear data was only loosely coupled to ALE3D's deflective response (hydrodynamics) – time-dependence of neutron energy deposition warrants further investigation.

Accuracy/precision of cross sections is paramount to correct energy deposition: any changes to cross section data could have significant effects on the end results.

Acknowledgements.

- LLNL Planetary Defense Group Collaborators
  - Dr. Megan Bruck Syal
  - Dr. Joseph Wasem
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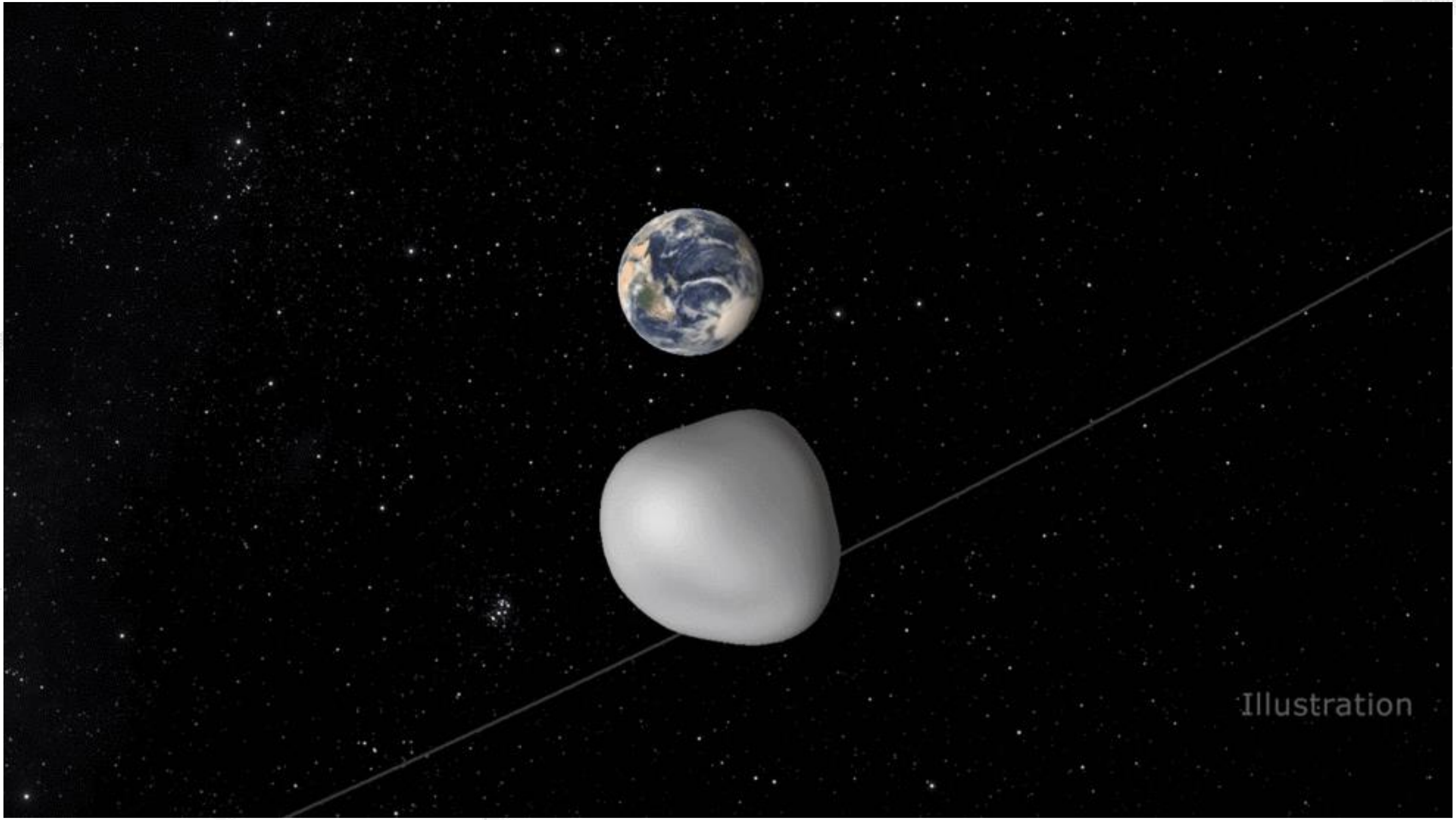
  - Dr. Darren Holland, thesis research advisor
  - Maj James Bevins, Ph.D.
  - Dr. John McClory
- NNSA, under grant NA000103
- SMART Scholarship, with AFTAC as sponsoring facility

Questions?



NASA/JPL-Caltech: https://www.nasa.gov/feature/jpl/asteroid-flyby-will-benefit-nasa-detection-and-tracking-network

Summary & Conclusions.

- **Problem:** Does the neutron energy affect asteroid deflection?
- **Hypothesis:** Affirmative. Confirmed.
- **Why?** Because changing the neutron energy means changing the:
  - energy deposition profiles
  - energy coupling efficiencies

Why does this matter? This type of research could help determine which type of device outputs are most effective for deflecting asteroids, and whether altering the neutron energy spectrum would ever be worthwhile.

Backup Slides

Two ways how changing the neutron interactions amounts to changing the energy deposition.

- First: changing the neutron energy = changing total cross-section magnitude = changing the mean-free-path = changing the spatial extent/distribution of energy.

E_{src}	Nuclide	σ	mfp
14.1 MeV	Si-28 O-16	1.81 b 1.59 b	10.8 cm
1 MeV	Si-28 O-16	4.68 b 8.15 b	2.6 cm

On average, 14.1 MeV neutrons are more penetrative than 1 MeV neutrons.

Two ways how changing the neutron interactions amounts to changing the energy deposition. (Cont.)

- Second: changing the neutron energy = opening or closing reaction channels = changing the energy coupling.

$$Q = \left(m_{initial} - m_{final}\right) c^2$$

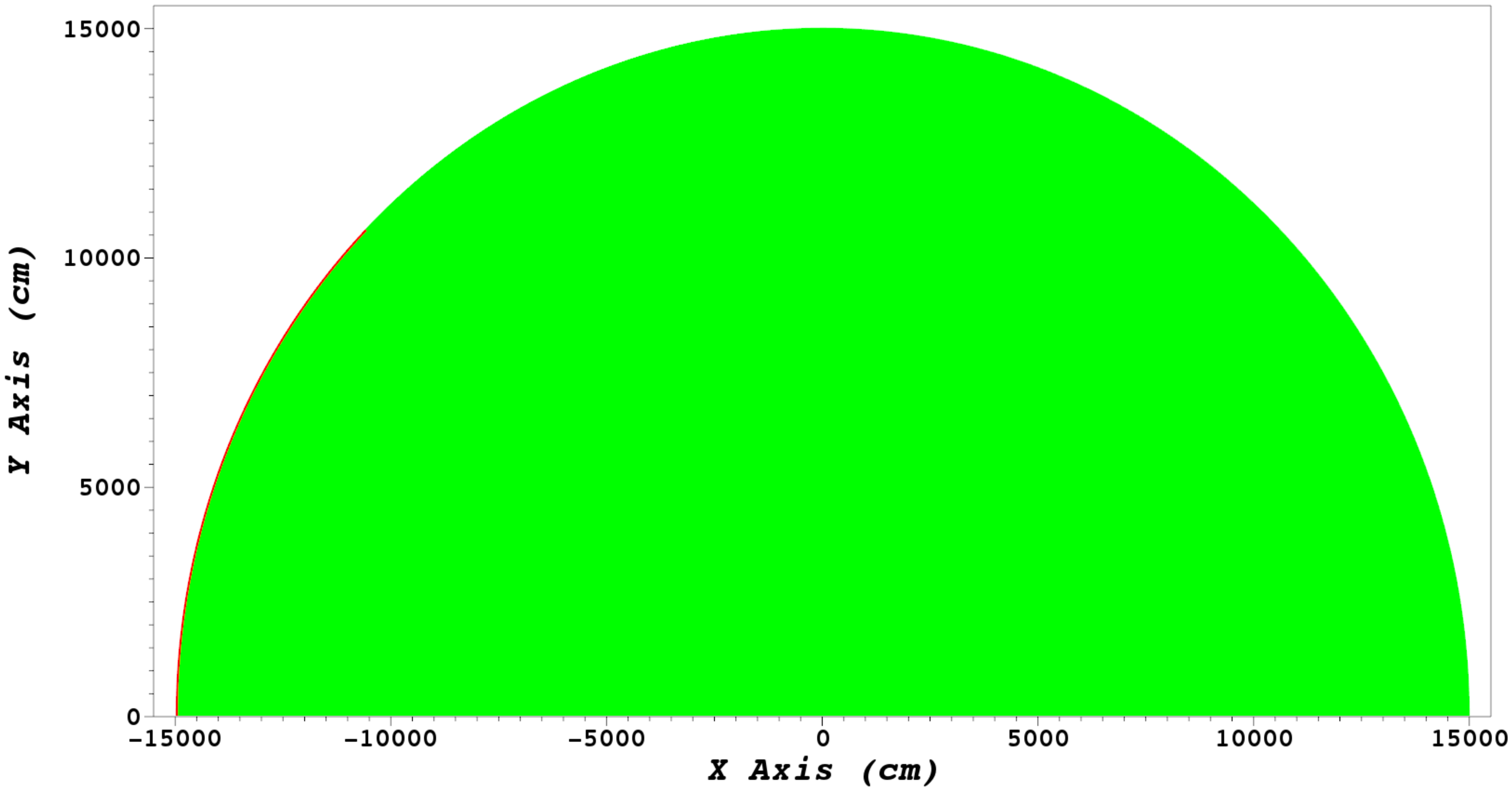
Isotope	Capture Reaction						E_1^*
	(n,γ)	(n,p)	(n,d)	(n,t)	(n,α)	(n,2n)	
^{28}Si	8.474	-3.466	-9.698	-16.743	-2.749	-17.799	-1.7790
^{16}O	4.143	-9.669	-10.527	-15.391	-2.355	-16.651	-6.0494

Mass destroyed,
Energy created

Energy destroyed,
Mass created

Exothermic (+Q) reaction channels are a bonus for energy coupling, while endothermic (-Q) reactions draw a coupling penalty.

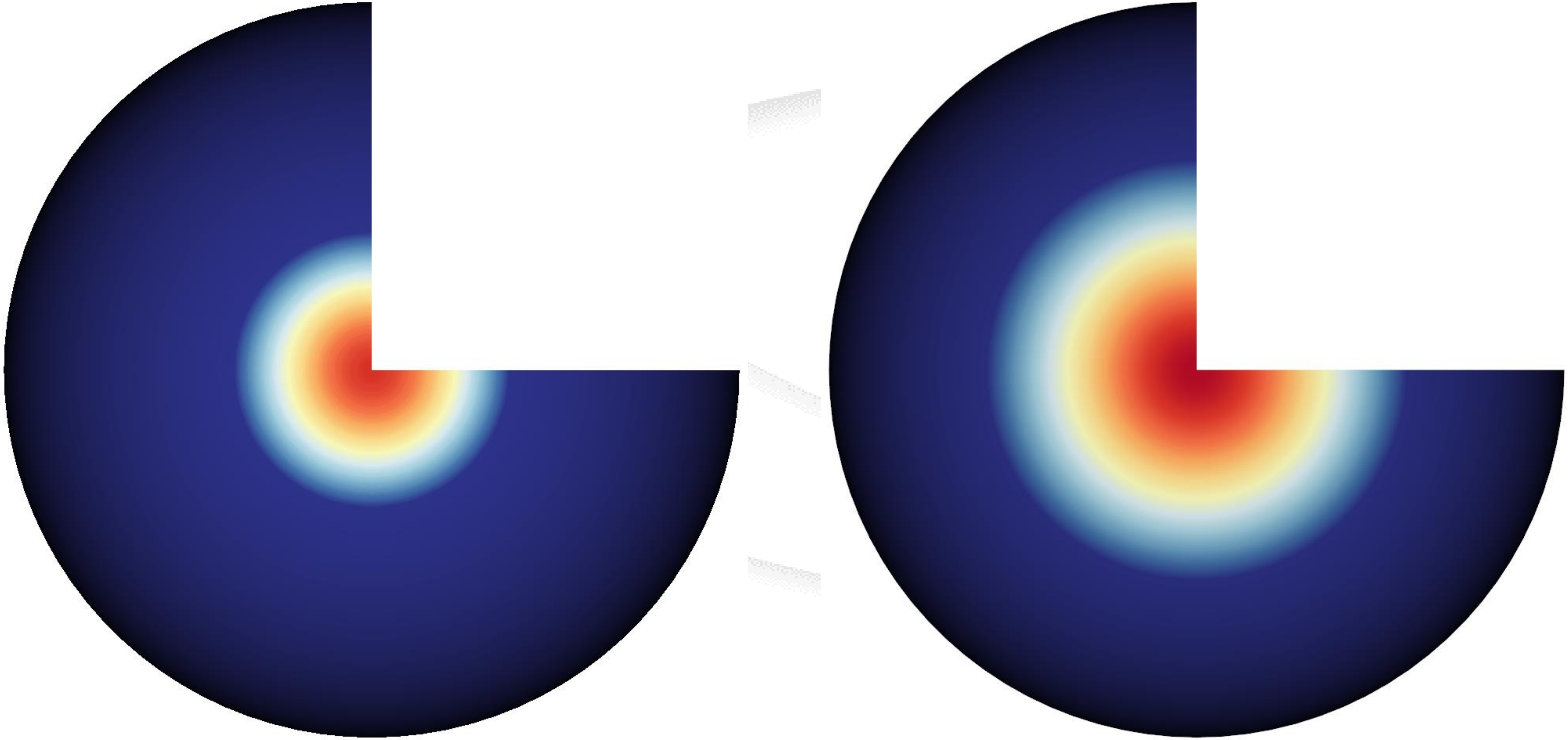
The region where some material is melted is very thin (in depth) and very long (in angle).



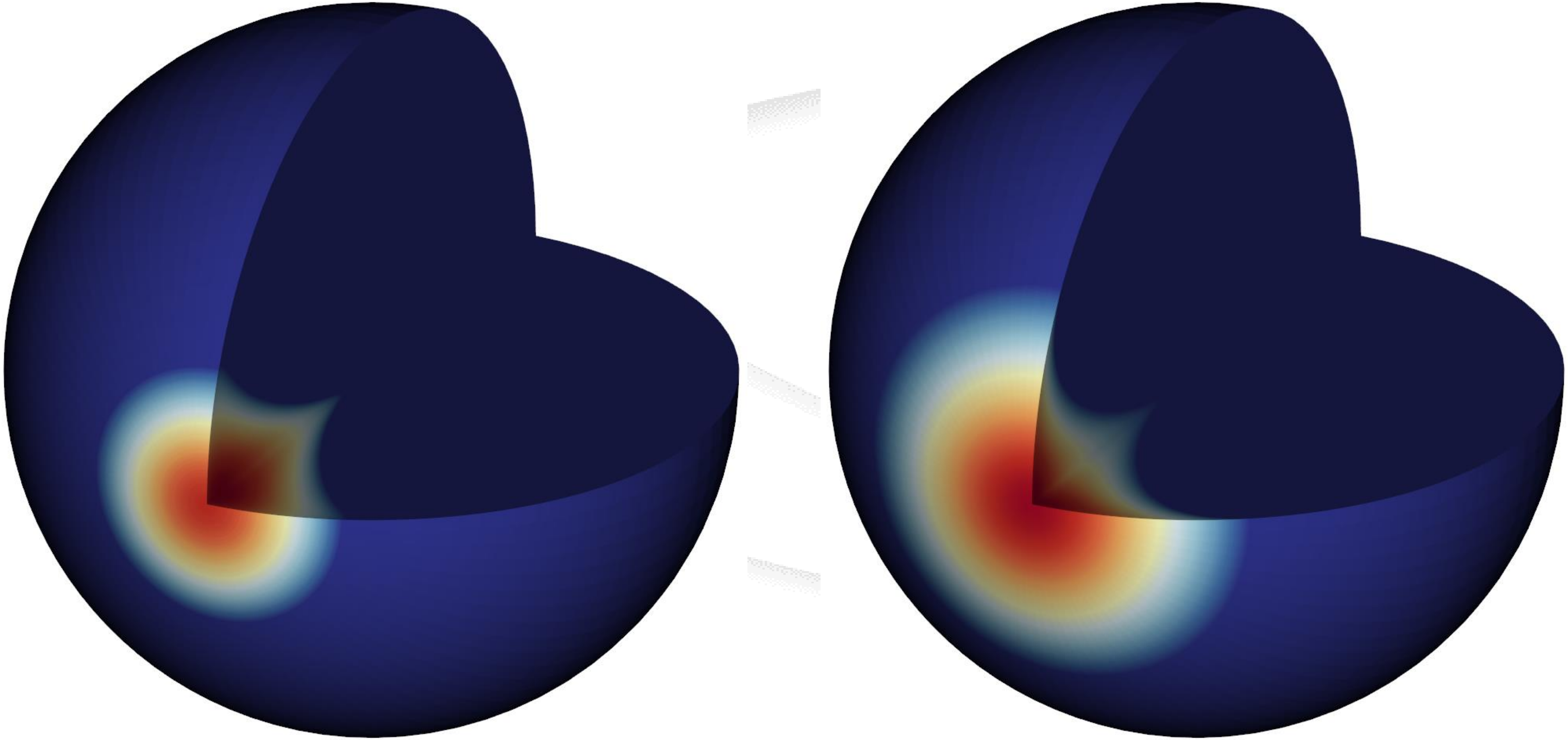
melt-depths ~ 19 cm up to 200 cm
arc-length ~ 11,781 cm

massive disparity

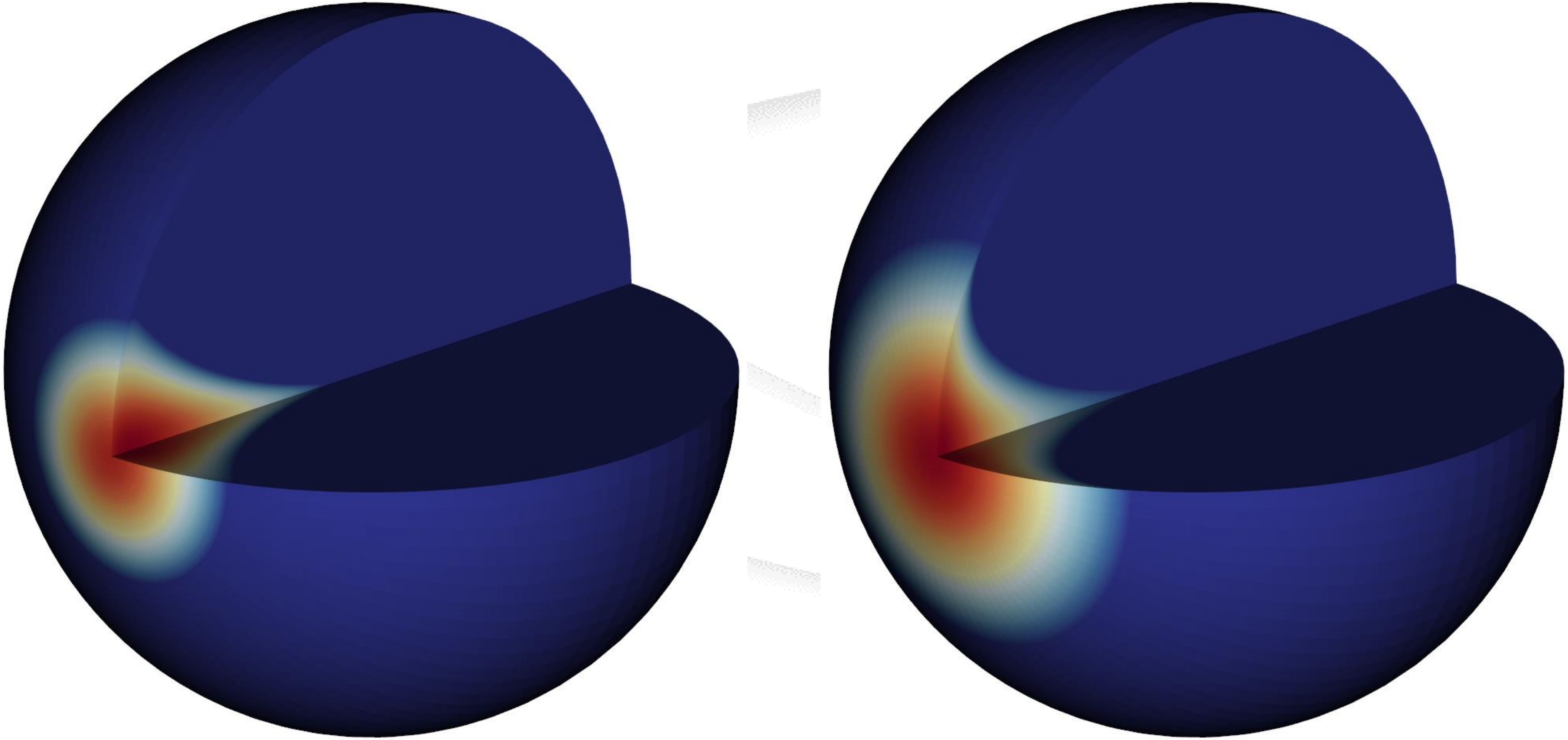
Energy deposition heatmap resulting from 50 kt’s worth of 14.1 MeV neutrons (left) and 1 MeV neutrons (right).



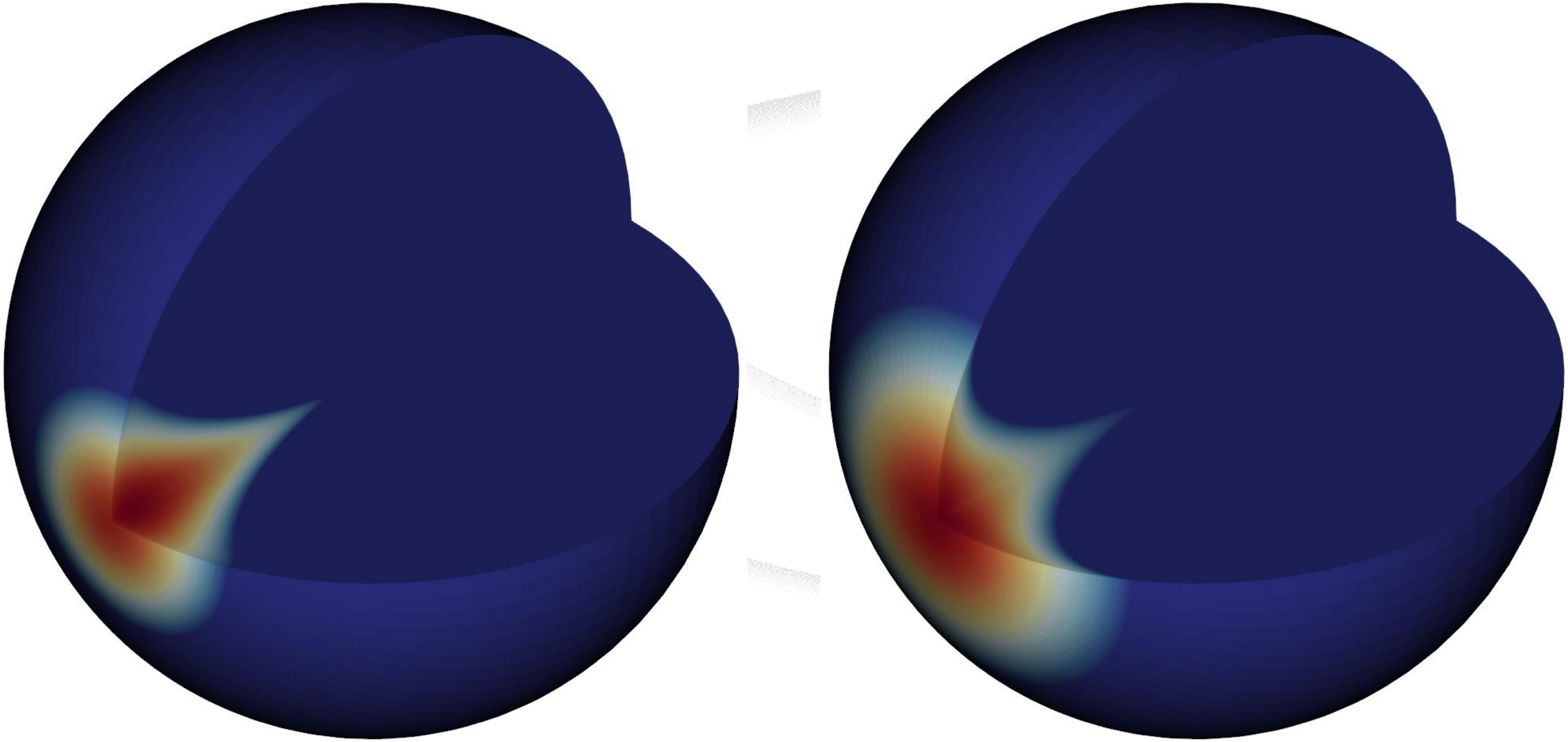
Energy deposition heatmap resulting from 50 kt's worth of 14.1 MeV neutrons (left) and 1 MeV neutrons (right).



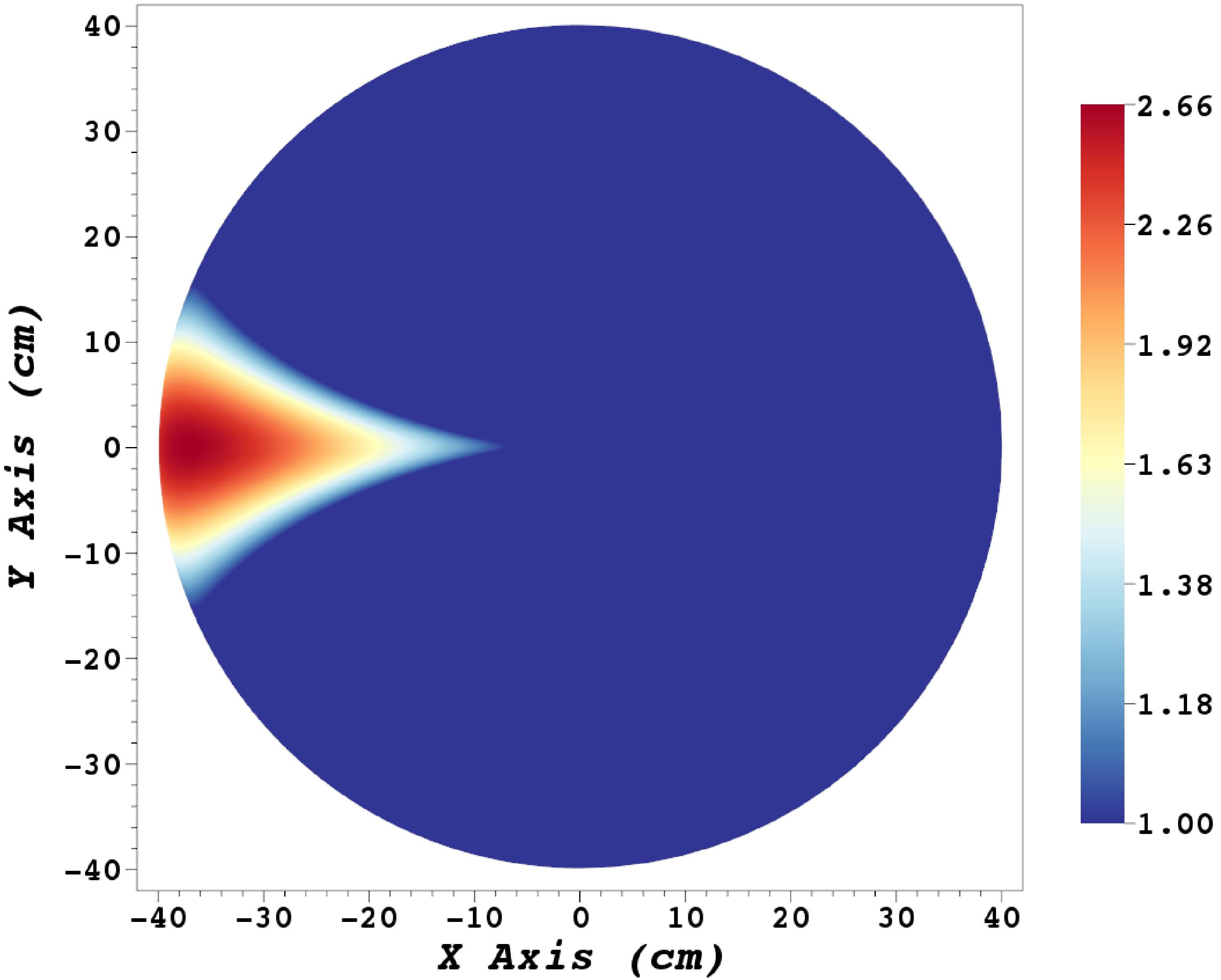
Energy deposition heatmap resulting from 50 kt's worth of 14.1 MeV neutrons (left) and 1 MeV neutrons (right).



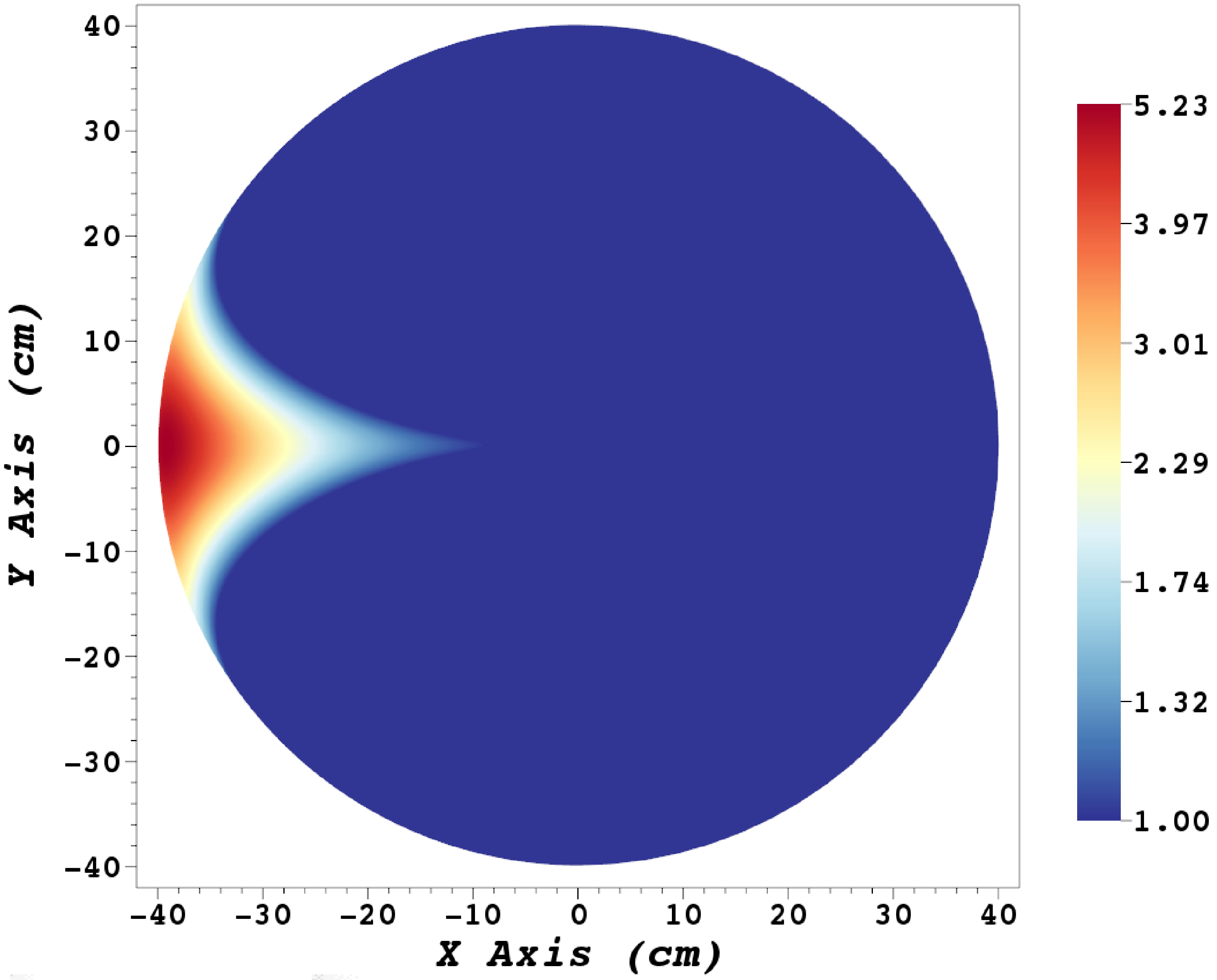
Energy deposition heatmap resulting from 50 kt's worth of 14.1 MeV neutrons (left) and 1 MeV neutrons (right).



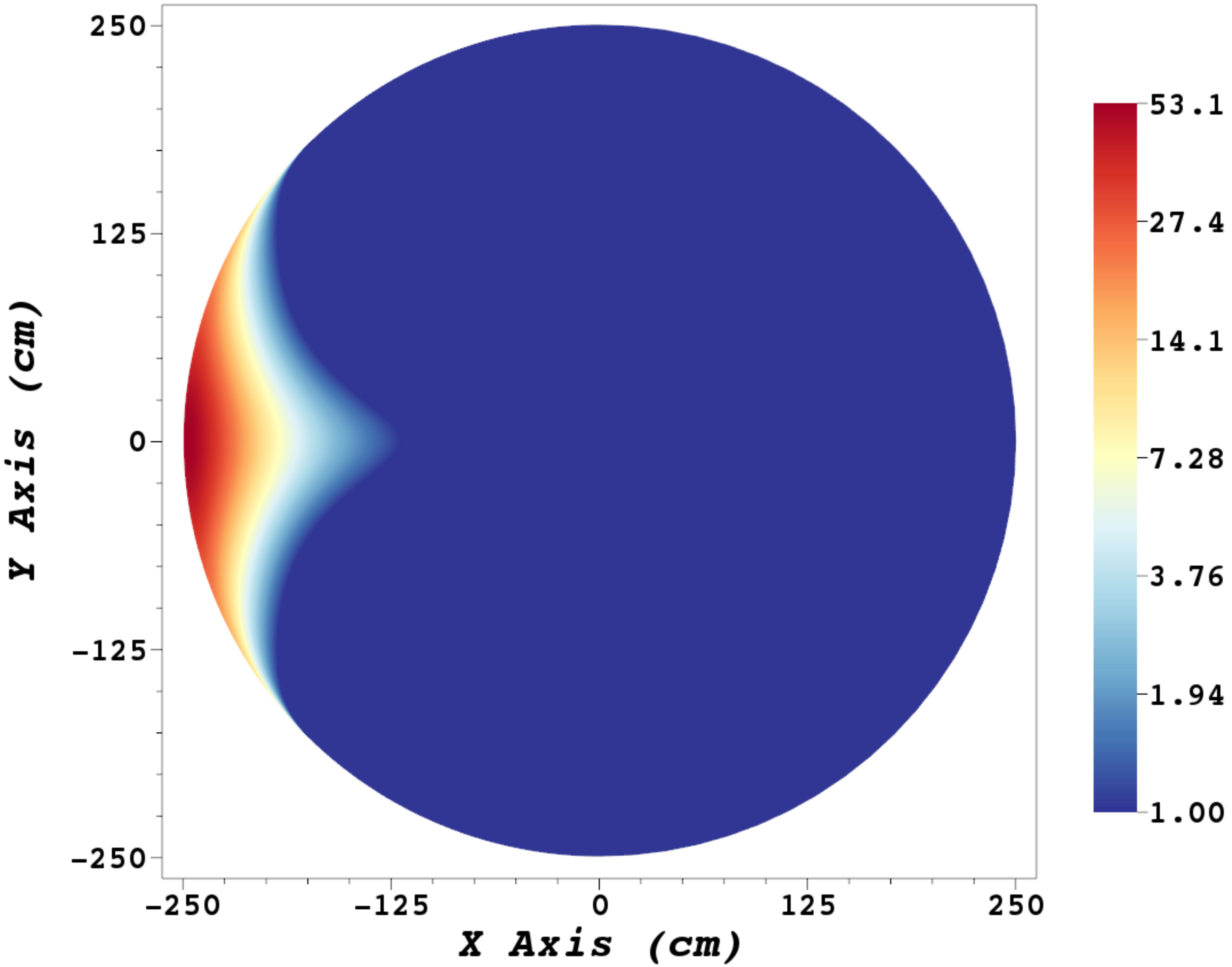
Energy deposition heatmap resulting from 50 kt's worth of 14.1 MeV neutrons.



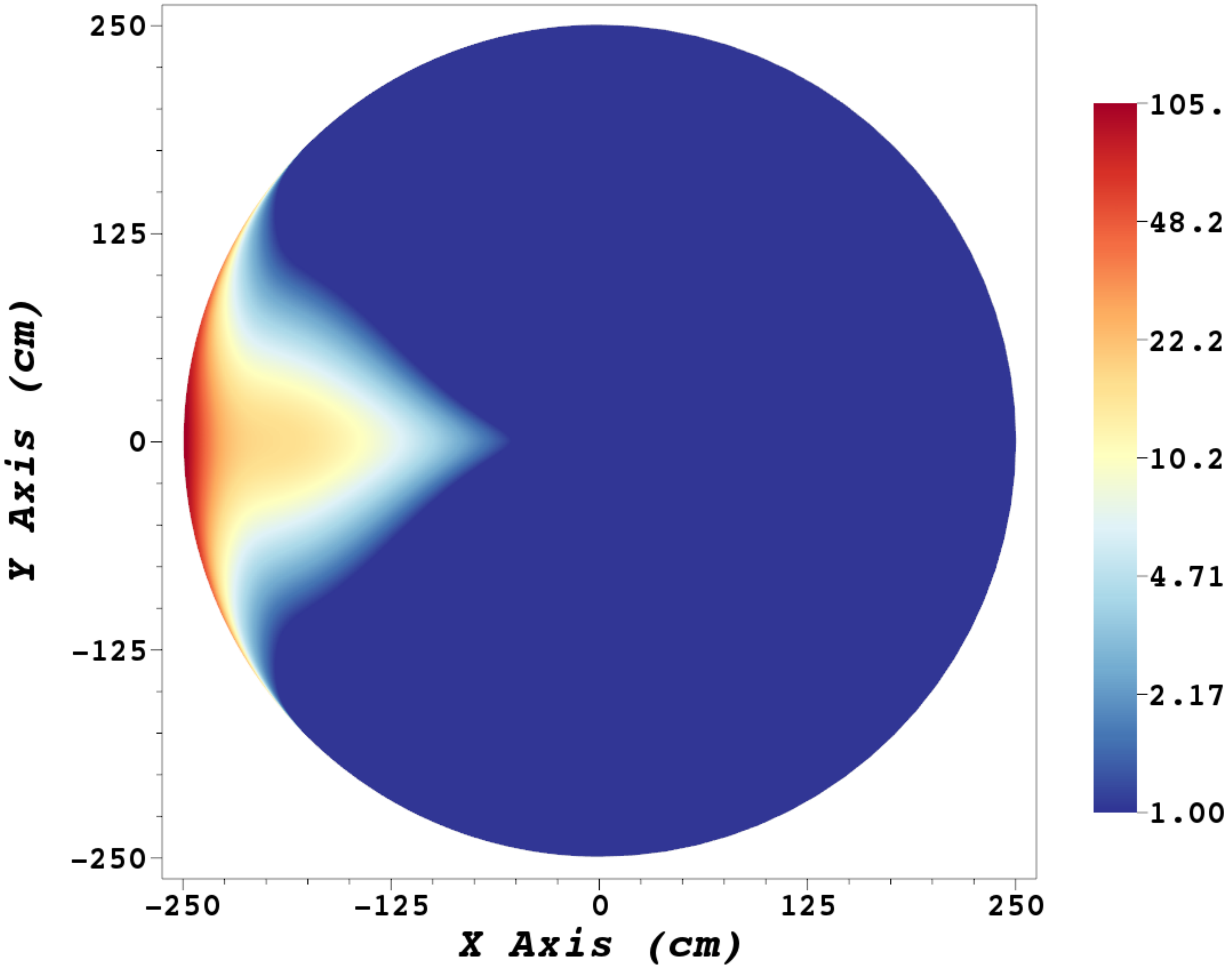
Energy deposition heatmap resulting from 50 kt's worth of 1 MeV neutrons.



Energy deposition heatmap resulting from 1 Mt's worth of 14.1 MeV neutrons.



Energy deposition heatmap resulting from 1 Mt's worth of 1 MeV neutrons.

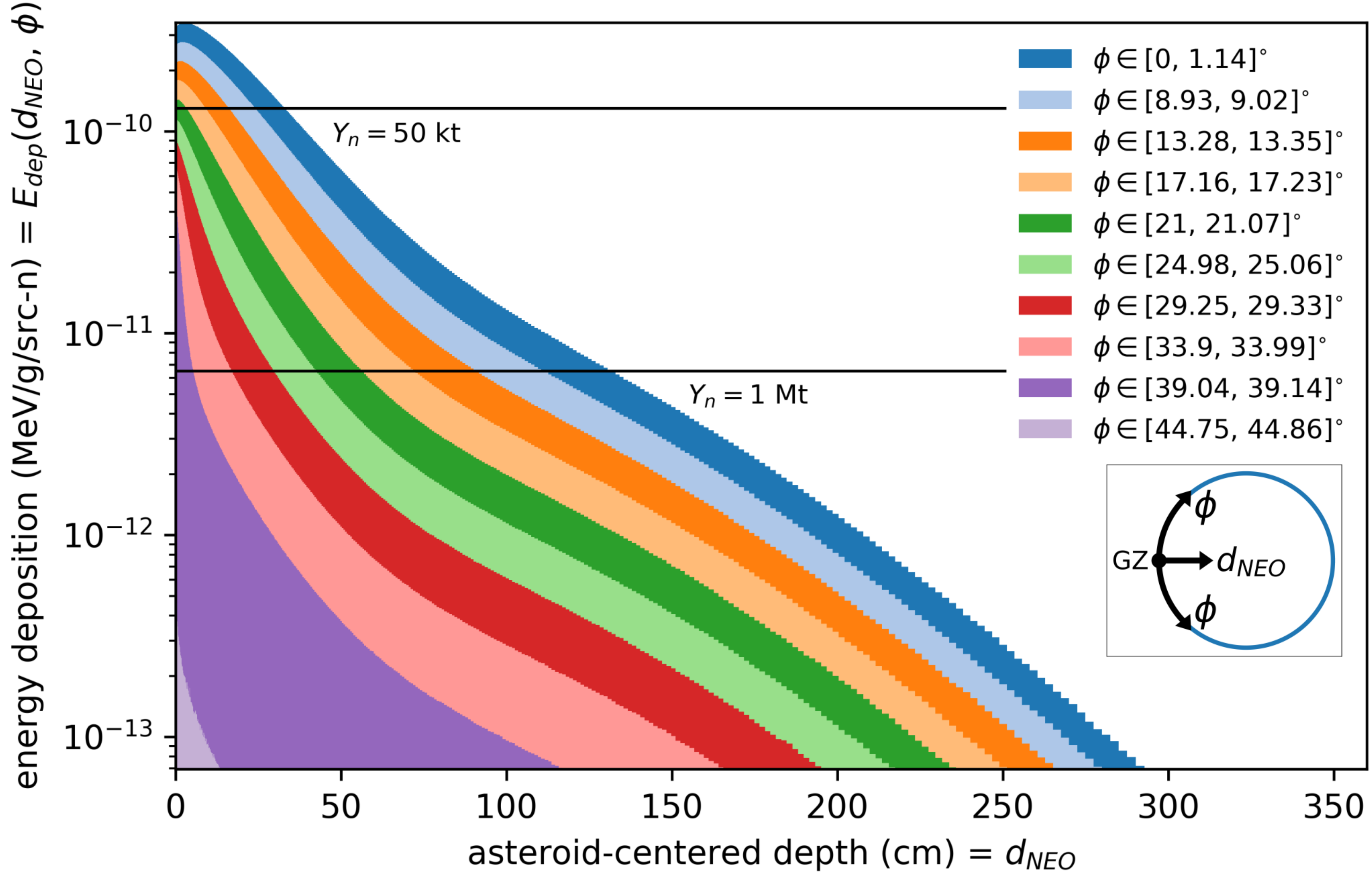


A nuclear device is the most efficient technology for asteroid deflection.

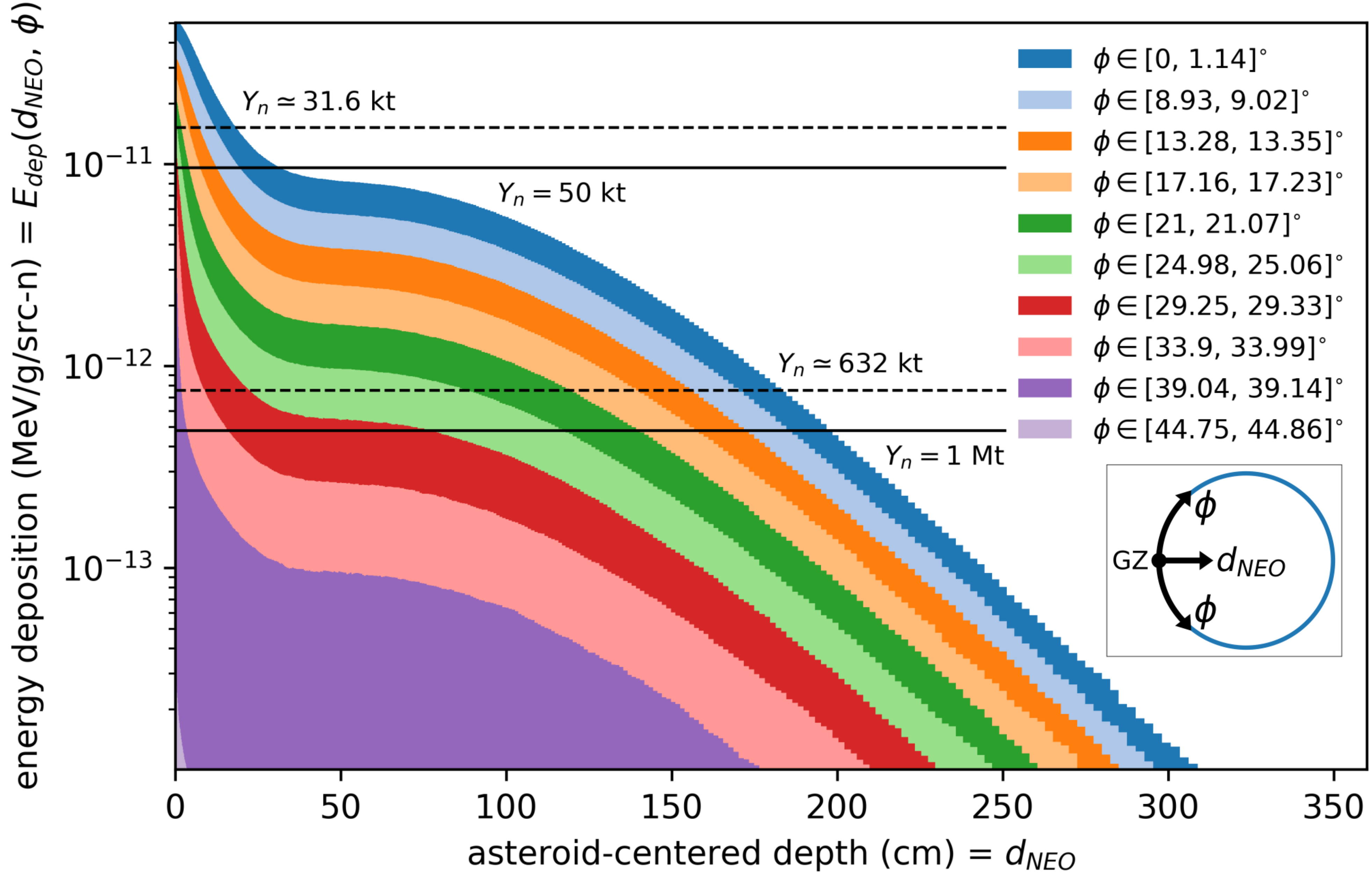
- Nuclear standoff explosions are "<u>10-100 times more effective than non-nuclear alternatives</u>"
  - NASA study
- Nuclear energy densities (energy/mass) are *millions of times greater* than chemical bonds
  - mass payload considerations are vital for space travel, delivery
- Nuclear deflection could mitigate an asteroid threat **within a few years** for objects a few hundred meters in size
  - other mitigation technologies **require decades or more** of warning time

If NASA announced tomorrow an asteroid was going to hit in 5 years, a nuclear device would likely be the most effective choice of combat.

14.1 MeV neutron energy deposition profiles.



1 MeV neutron energy deposition profiles.



The 1 MeV profiles have different spatial features than the 14.1 MeV profiles.

Notable historical asteroid impacts, recent and long-ago.

- In 1908, in Tunguska, Siberia:
  - 60 m asteroid
  - airburst, 10-20 Mt
  - 2,000 km^2 of forest and 80 million trees destroyed
- In 2013, in Chelyabinsk, Russia:
  - 19 m asteroid
  - airburst, 400-600 kt
  - injured 1,500 people, damaged 7,000 buildings
- 65 million years ago, Yucatan region of Mexico:
  - 10 km asteroid
  - direct impact with ground (Chicxulub crater)
  - K-T dinosaur extinction, 70% of all species eliminated

Asteroids “small” and large can cause devastation.

Summary table of all yield & neutron configurations.

Y_n	E_n	src-n	$E_{dep,tot}^{ALE3D}$	δV
50 kt	14.1 MeV	$9.31469 \cdot 10^{25}$	5.0364 kt	6.19 ± 0.06 cm/s
50 kt	1 MeV	$1.26157 \cdot 10^{27}$	7.9785 kt	9.99 ± 0.12 cm/s
31.5913 kt	1 MeV	$7.97093 \cdot 10^{26}$	5.0410 kt	6.02 ± 0.08 cm/s
1 Mt	14.1 MeV	$1.86294 \cdot 10^{27}$	100.68 kt	98.09 ± 0.41 cm/s
1 Mt	1 MeV	$2.52314 \cdot 10^{28}$	158.75 kt	166.9 ± 0.50 cm/s
631.825 kt	1 MeV	$1.59419 \cdot 10^{28}$	100.30 kt	114.7 ± 0.34 cm/s

Equal 50 kt detonation yields.

Y_n	E_n	src-n	$E_{dep,tot}^{ALE3D}$	δV
50 kt	14.1 MeV	$9.31469 \cdot 10^{25}$	5.0364 kt	6.19 ± 0.06 cm/s
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1 MeV δV is 61% greater than 14.1 MeV δV.
1 MeV E_{dep} is 58% higher.

Equal 1 Mt detonation yields.

Y_n	E_n	src-n	$E_{dep,tot}^{ALE3D}$	δV
50 kt	14.1 MeV	$9.31469 \cdot 10^{25}$	5.0364 kt	6.19 ± 0.06 cm/s
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1 MeV δV is 70% greater than 14.1 MeV δV.
1 MeV E_{dep} is 58% higher.

Equal ~5 kt energy depositions.

Y_n	E_n	src-n	$E_{dep,tot}^{ALE3D}$	δV
50 kt	14.1 MeV	$9.31469 \cdot 10^{25}$	5.0364 kt	6.19 ± 0.06 cm/s
50 kt	1 MeV	$1.26157 \cdot 10^{27}$	7.9785 kt	9.99 ± 0.12 cm/s
31.5913 kt	1 MeV	$7.97093 \cdot 10^{26}$	5.0410 kt	6.02 ± 0.08 cm/s
1 Mt	14.1 MeV	$1.86294 \cdot 10^{27}$	100.68 kt	98.09 ± 0.41 cm/s
1 Mt	1 MeV	$2.52314 \cdot 10^{28}$	158.75 kt	166.9 ± 0.50 cm/s
631.825 kt	1 MeV	$1.59419 \cdot 10^{28}$	100.30 kt	114.7 ± 0.34 cm/s

14.1 MeV δV is 3±2% greater than 1 MeV δV.

Equal ~100 kt energy depositions.

Y_n	E_n	src-n	$E_{dep,tot}^{ALE3D}$	δV
50 kt	14.1 MeV	$9.31469 \cdot 10^{25}$	5.0364 kt	6.19 ± 0.06 cm/s
50 kt	1 MeV	$1.26157 \cdot 10^{27}$	7.9785 kt	9.99 ± 0.12 cm/s
31.5913 kt	1 MeV	$7.97093 \cdot 10^{26}$	5.0410 kt	6.02 ± 0.08 cm/s
1 Mt	14.1 MeV	$1.86294 \cdot 10^{27}$	100.68 kt	98.09 ± 0.41 cm/s
1 Mt	1 MeV	$2.52314 \cdot 10^{28}$	158.75 kt	166.9 ± 0.50 cm/s
631.825 kt	1 MeV	$1.59419 \cdot 10^{28}$	100.30 kt	114.7 ± 0.34 cm/s

1 MeV δV is 17% greater than 14.1 MeV δV.

Does the neutron energy affect asteroid deflection? Yes.

- **Why? Because different neutron energies result in different energy deposition profiles & different energy coupling efficiencies.**
- The significance of these differences is sensitive to the yield.
- The energy coupling appears more important than the energy deposition profile, especially at low yields and shallow melt-depths.
- 1 MeV neutrons are equal-or-better than 14.1 MeV neutrons in terms of coupling and profiles, but they require many more source neutrons to compensate for the MeV/src-n reduction.

Asteroid impacts are rare, but potentially devasting.

- 1+ kilometer asteroid impacts:
  - once every 500,000 years
  - ~25% of the world's <u>global</u> population would perish
  - individual's annual chance of death: 5×10^{-5} %
    - comparable to risk of dying from an airplane crash
- 100 meter asteroid impacts:
  - once every 300 years
  - damage more confined to <u>region</u> of impact (cities, states)
  - individual's annual chance of death: 3.3×10^{-6} %

The magnitude of the damage that could result from these fairly-rare, one-off impact events makes the planetary defense mission a prudent pursuit.

How does changing the neutron interactions change the energy deposition? (cont.)

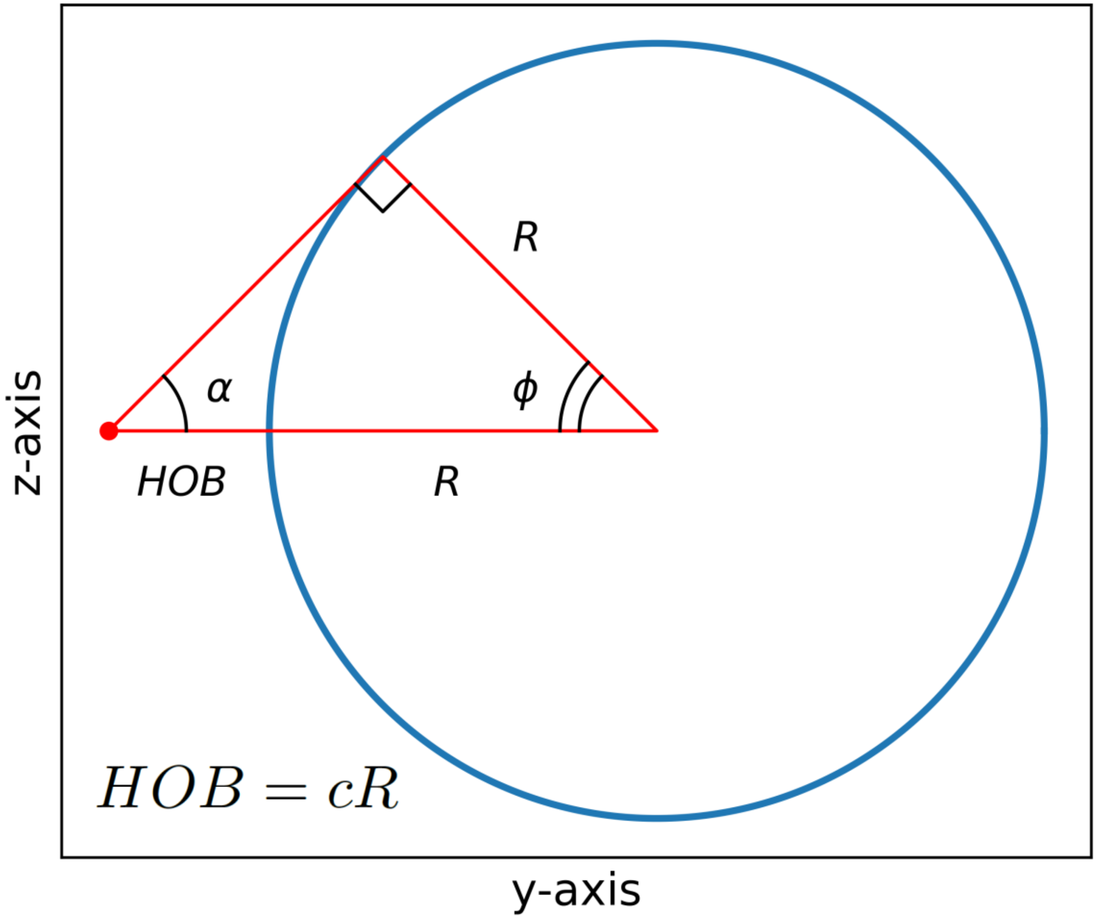
Isotope	Capture Reaction						E_1^*
	(n,γ)	(n,p)	(n,d)	(n,t)	(n,α)	(n,2n)	
^{28}Si	$\boxed{8.474}$	-3.466	-9.698	-16.743	$\boxed{-2.749}$	-17.799	-1.7790
^{16}O	4.143	-9.669	-10.527	-15.391	-2.355	-16.651	-6.0494

- Energy deposition = transferring the energy from radiation (neutrons) to the asteroid particle population (nuclei)
- Consider 14.1 MeV n's being absorbed by ^{28}Si:
  - (n,γ) = ^{29}Si nuclei keeps all 14.1 MeV, *and* an extra 8.474 MeV is shared between ^{29}Si and a γ. E_{dep} = 14.1 + 8.474*
  - (n,α) = ^{29}Si nuclei initially has all 14.1 MeV, *but* it quickly loses 2.749 MeV because it chose to emit an α. E_{dep} = 14.1 – 2.749

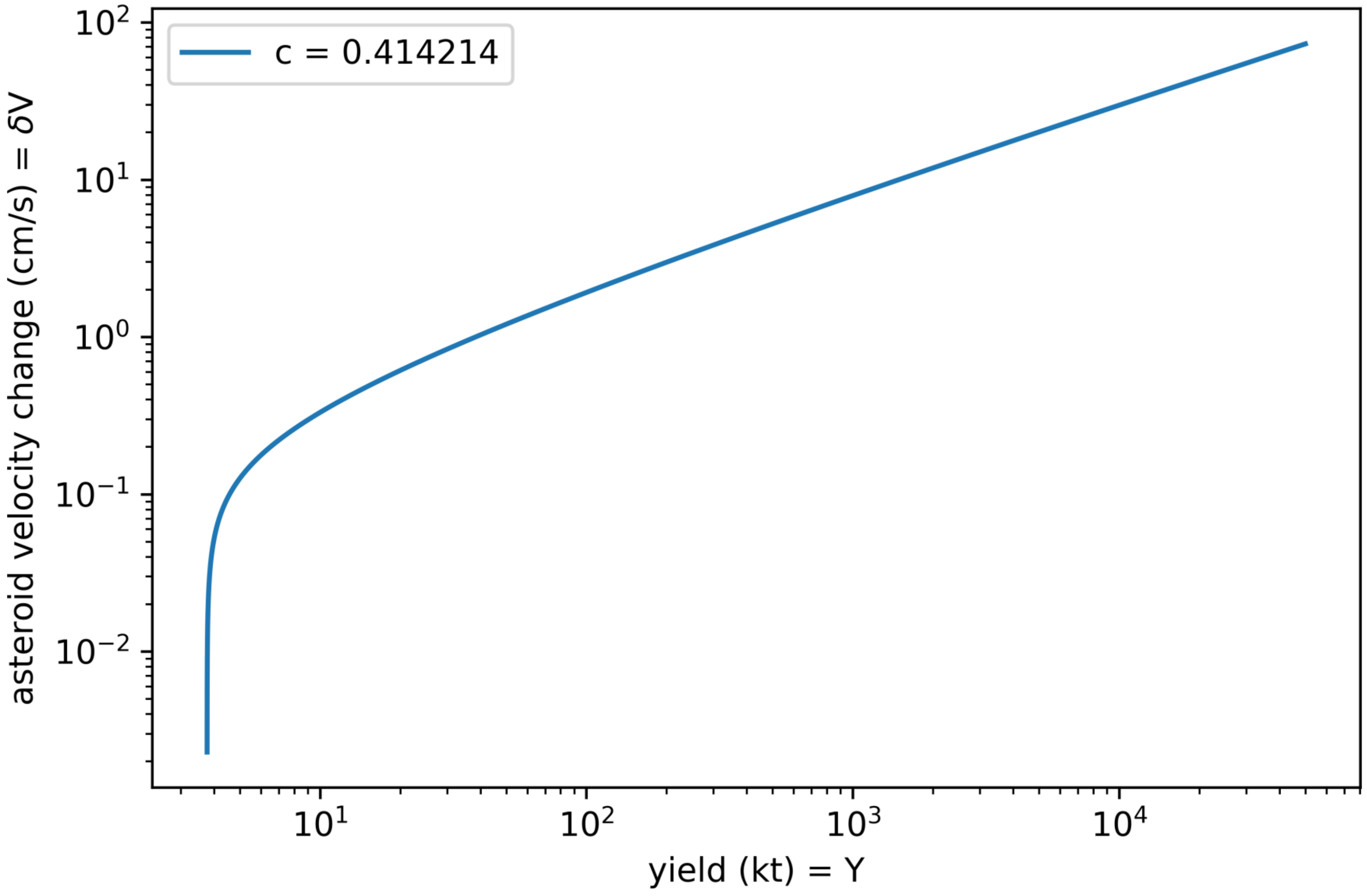
Exothermic (+Q) reaction channels are a bonus for energy coupling, while endothermic (-Q) reactions draw a coupling penalty.

Stand-off distance (HOB) selection.

- Hammerling & Remo: HOB ~ 0.414 × R
  - geometrical optimal HOB; $\alpha = \phi = 45°$
  - maximizes the sum of (fraction of asteroid surface area irradiated) + (fraction of nuclear energy incident on the asteroid)

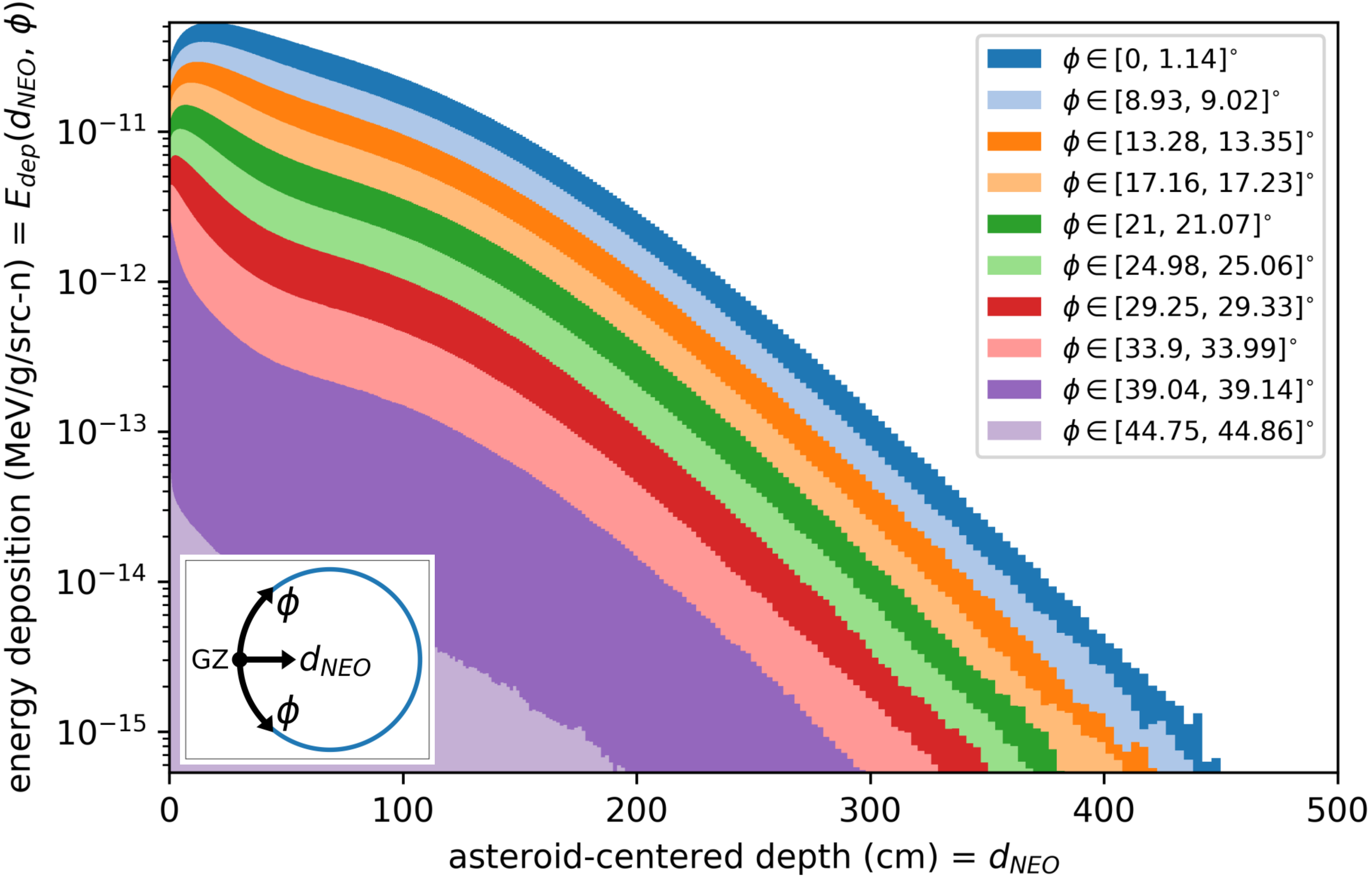


Joe Wasem's δV vs. Y analytical equation.



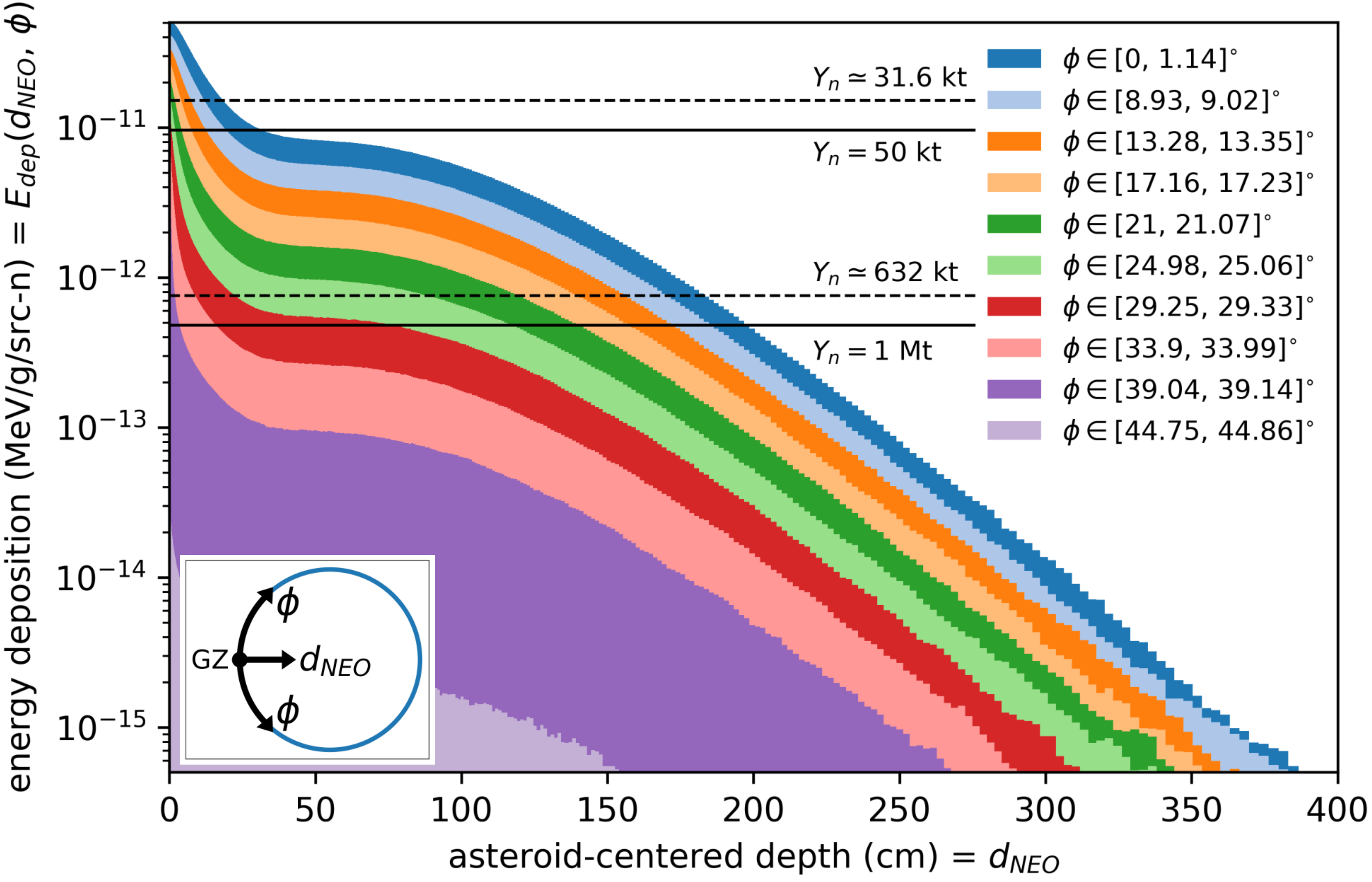
Away from the threshold (need to melt), $\delta V \sim Y^{2/3}$, roughly.

~2.45 MeV neutron energy deposition profiles.



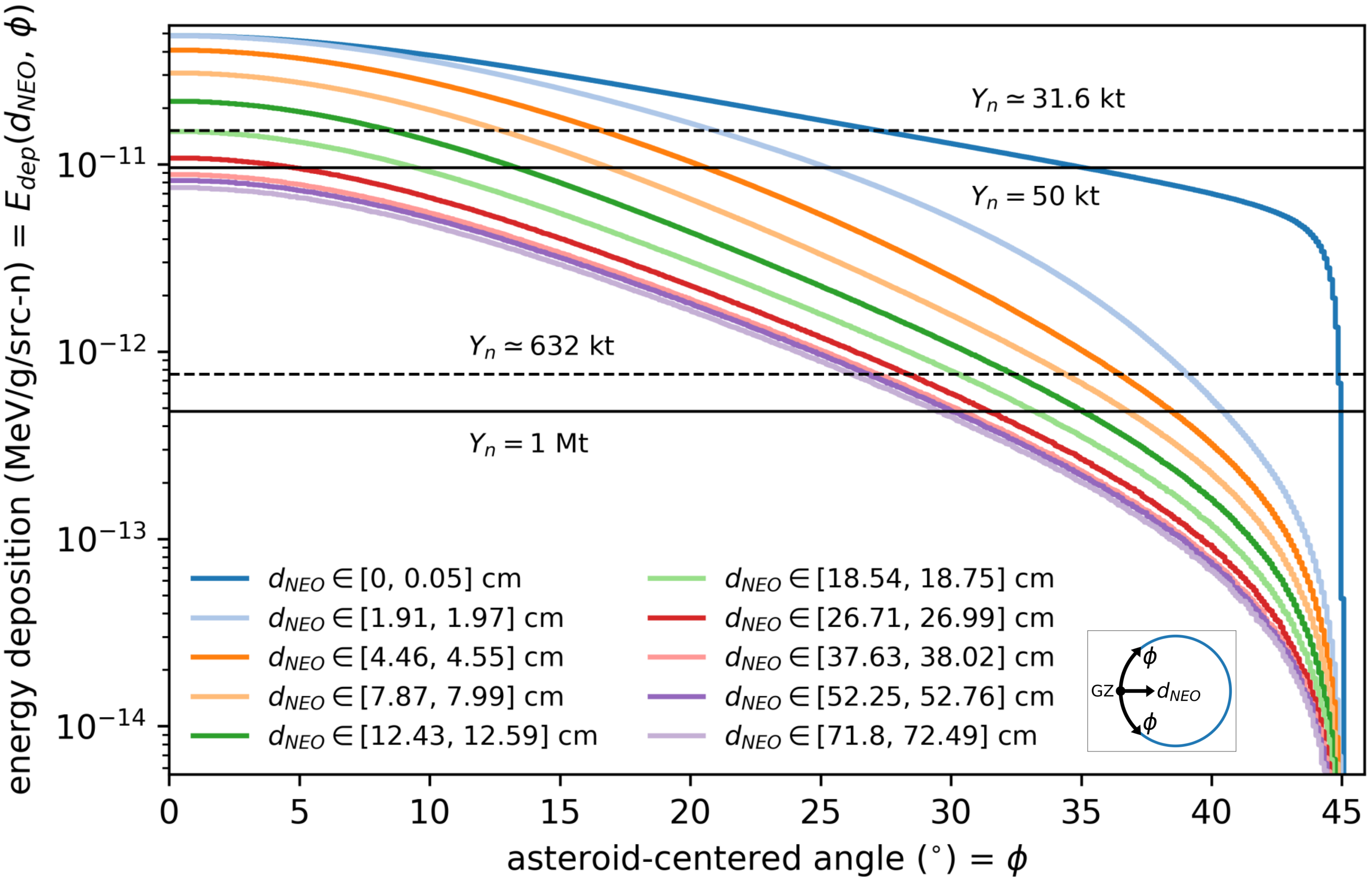
Actually, these profiles are from an average/midpoint energy of 2.346 MeV.

1 MeV neutron energy deposition profiles.



The “pause” region exists at ~40-80 cm depths; to melt, need higher yields.

1 MeV neutron energy deposition profiles.

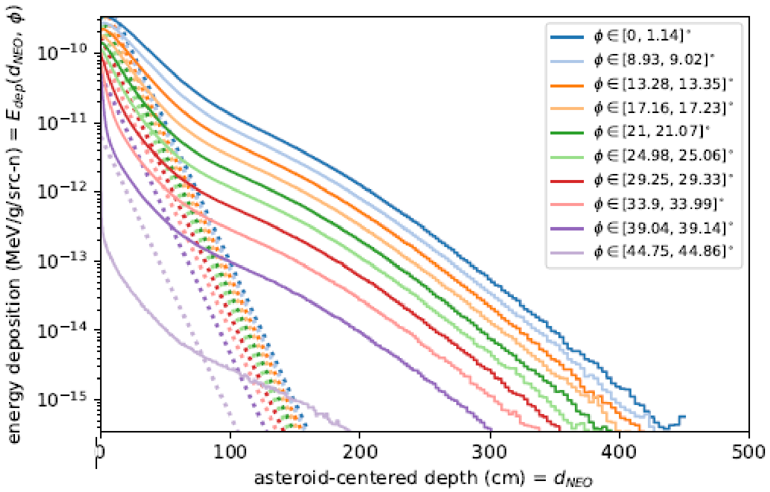


Clumping together of the 3 pink-purple colors is the “pause” region.

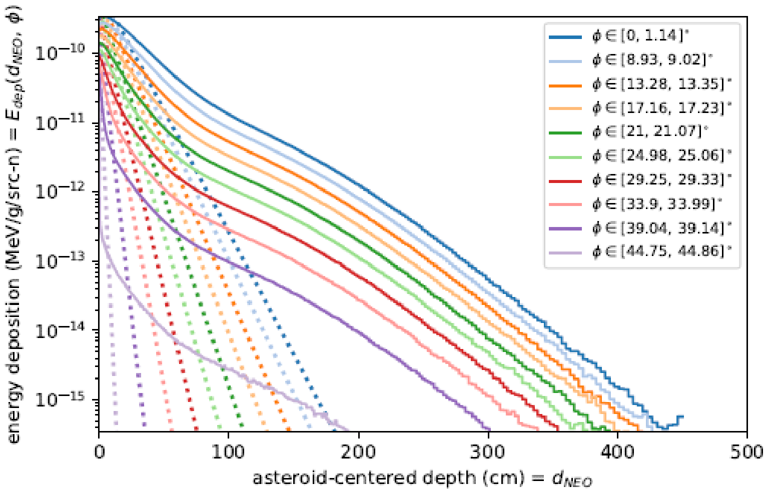
Group #	E_{low}	E_{high}	$E_{mid} = Y_n$	Y_{int}	E_{dep}	η_{rel}	η_{abs}
0	$1.6905 \cdot 10^{1}$	$1.9640 \cdot 10^{1}$	$1.8273 \cdot 10^{1}$	$2.6759 \cdot 10^{0}$	$1.6663 \cdot 10^{0}$	$6.2271 \cdot 10^{-1}$	$9.1194 \cdot 10^{-2}$
1	$1.4918 \cdot 10^{1}$	$1.6905 \cdot 10^{1}$	$1.5912 \cdot 10^{1}$	$2.3302 \cdot 10^{0}$	$1.5208 \cdot 10^{0}$	$6.5264 \cdot 10^{-1}$	$9.5576 \cdot 10^{-2}$
2	$1.4191 \cdot 10^{1}$	$1.4918 \cdot 10^{1}$	$1.4555 \cdot 10^{1}$	$2.1315 \cdot 10^{0}$	$1.4447 \cdot 10^{0}$	$6.7780 \cdot 10^{-1}$	$9.9262 \cdot 10^{-2}$
3	$1.3840 \cdot 10^{1}$	$1.4191 \cdot 10^{1}$	$1.4015 \cdot 10^{1}$	$2.0525 \cdot 10^{0}$	$1.4130 \cdot 10^{0}$	$6.8844 \cdot 10^{-1}$	$1.0082 \cdot 10^{-1}$
4	$1.2523 \cdot 10^{1}$	$1.3840 \cdot 10^{1}$	$1.3181 \cdot 10^{1}$	$1.9304 \cdot 10^{0}$	$1.3696 \cdot 10^{0}$	$7.0952 \cdot 10^{-1}$	$1.0391 \cdot 10^{-1}$
5	$1.2214 \cdot 10^{1}$	$1.2523 \cdot 10^{1}$	$1.2369 \cdot 10^{1}$	$1.8113 \cdot 10^{0}$	$1.3026 \cdot 10^{0}$	$7.1917 \cdot 10^{-1}$	$1.0532 \cdot 10^{-1}$
6	$1.1052 \cdot 10^{1}$	$1.2214 \cdot 10^{1}$	$1.1633 \cdot 10^{1}$	$1.7036 \cdot 10^{0}$	$1.2040 \cdot 10^{0}$	$7.0676 \cdot 10^{-1}$	$1.0350 \cdot 10^{-1}$
7	$1.0000 \cdot 10^{1}$	$1.1052 \cdot 10^{1}$	$1.0526 \cdot 10^{1}$	$1.5415 \cdot 10^{0}$	$1.1329 \cdot 10^{0}$	$7.3491 \cdot 10^{-1}$	$1.0763 \cdot 10^{-1}$
8	$9.0484 \cdot 10^{0}$	$1.0000 \cdot 10^{1}$	$9.5242 \cdot 10^{0}$	$1.3948 \cdot 10^{0}$	$1.0486 \cdot 10^{0}$	$7.5181 \cdot 10^{-1}$	$1.1010 \cdot 10^{-1}$
9	$8.1873 \cdot 10^{0}$	$9.0484 \cdot 10^{0}$	$8.6179 \cdot 10^{0}$	$1.2621 \cdot 10^{0}$	$9.4784 \cdot 10^{-1}$	$7.5103 \cdot 10^{-1}$	$1.0999 \cdot 10^{-1}$
10	$7.4082 \cdot 10^{0}$	$8.1873 \cdot 10^{0}$	$7.7978 \cdot 10^{0}$	$1.1420 \cdot 10^{0}$	$8.6316 \cdot 10^{-1}$	$7.5586 \cdot 10^{-1}$	$1.1069 \cdot 10^{-1}$
11	$6.3763 \cdot 10^{0}$	$7.4082 \cdot 10^{0}$	$6.8922 \cdot 10^{0}$	$1.0093 \cdot 10^{0}$	$7.8290 \cdot 10^{-1}$	$7.7565 \cdot 10^{-1}$	$1.1359 \cdot 10^{-1}$
12	$4.9659 \cdot 10^{0}$	$6.3763 \cdot 10^{0}$	$5.6711 \cdot 10^{0}$	$8.3051 \cdot 10^{-1}$	$6.9965 \cdot 10^{-1}$	$8.4243 \cdot 10^{-1}$	$1.2337 \cdot 10^{-1}$
13	$4.7237 \cdot 10^{0}$	$4.9659 \cdot 10^{0}$	$4.8448 \cdot 10^{0}$	$7.0950 \cdot 10^{-1}$	$6.2448 \cdot 10^{-1}$	$8.8017 \cdot 10^{-1}$	$1.2890 \cdot 10^{-1}$
14	$4.0657 \cdot 10^{0}$	$4.7237 \cdot 10^{0}$	$4.3947 \cdot 10^{0}$	$6.4359 \cdot 10^{-1}$	$5.8679 \cdot 10^{-1}$	$9.1174 \cdot 10^{-1}$	$1.3352 \cdot 10^{-1}$
15	$3.0119 \cdot 10^{0}$	$4.0657 \cdot 10^{0}$	$3.5388 \cdot 10^{0}$	$5.1825 \cdot 10^{-1}$	$4.6507 \cdot 10^{-1}$	$8.9739 \cdot 10^{-1}$	$1.3142 \cdot 10^{-1}$
16	$2.3852 \cdot 10^{0}$	$3.0119 \cdot 10^{0}$	$2.6986 \cdot 10^{0}$	$3.9519 \cdot 10^{-1}$	$4.7157 \cdot 10^{-1}$	$1.1933 \cdot 10^{0}$	$1.7475 \cdot 10^{-1}$
17	$2.3069 \cdot 10^{0}$	$2.3852 \cdot 10^{0}$	$2.3460 \cdot 10^{0}$	$3.4357 \cdot 10^{-1}$	$5.3797 \cdot 10^{-1}$	$1.5658 \cdot 10^{0}$	$2.2931 \cdot 10^{-1}$
18	$1.8268 \cdot 10^{0}$	$2.3069 \cdot 10^{0}$	$2.0669 \cdot 10^{0}$	$3.0268 \cdot 10^{-1}$	$3.6753 \cdot 10^{-1}$	$1.2143 \cdot 10^{0}$	$1.7782 \cdot 10^{-1}$
19	$1.4227 \cdot 10^{0}$	$1.8268 \cdot 10^{0}$	$1.6248 \cdot 10^{0}$	$2.3794 \cdot 10^{-1}$	$2.9624 \cdot 10^{-1}$	$1.2450 \cdot 10^{0}$	$1.8233 \cdot 10^{-1}$
20	$1.1080 \cdot 10^{0}$	$1.4227 \cdot 10^{0}$	$1.2654 \cdot 10^{0}$	$1.8531 \cdot 10^{-1}$	$2.4287 \cdot 10^{-1}$	$1.3106 \cdot 10^{0}$	$1.9194 \cdot 10^{-1}$
21	$9.6164 \cdot 10^{-1}$	$1.1080 \cdot 10^{0}$	$1.0348 \cdot 10^{0}$	$1.5155 \cdot 10^{-1}$	$1.6513 \cdot 10^{-1}$	$1.0896 \cdot 10^{0}$	$1.5957 \cdot 10^{-1}$
22	$8.2085 \cdot 10^{-1}$	$9.6164 \cdot 10^{-1}$	$8.9125 \cdot 10^{-1}$	$1.3052 \cdot 10^{-1}$	$1.8618 \cdot 10^{-1}$	$1.4265 \cdot 10^{0}$	$2.0890 \cdot 10^{-1}$
23	$7.4274 \cdot 10^{-1}$	$8.2085 \cdot 10^{-1}$	$7.8180 \cdot 10^{-1}$	$1.1449 \cdot 10^{-1}$	$1.9029 \cdot 10^{-1}$	$1.6620 \cdot 10^{0}$	$2.4340 \cdot 10^{-1}$
24	$6.3928 \cdot 10^{-1}$	$7.4274 \cdot 10^{-1}$	$6.9101 \cdot 10^{-1}$	$1.0120 \cdot 10^{-1}$	$1.9980 \cdot 10^{-1}$	$1.9744 \cdot 10^{0}$	$2.8914 \cdot 10^{-1}$
25	$5.5023 \cdot 10^{-1}$	$6.3928 \cdot 10^{-1}$	$5.9475 \cdot 10^{-1}$	$8.7100 \cdot 10^{-2}$	$1.7870 \cdot 10^{-1}$	$2.0517 \cdot 10^{0}$	$3.0046 \cdot 10^{-1}$
26	$3.6883 \cdot 10^{-1}$	$5.5023 \cdot 10^{-1}$	$4.5953 \cdot 10^{-1}$	$6.7297 \cdot 10^{-2}$	$1.2905 \cdot 10^{-1}$	$1.9176 \cdot 10^{0}$	$2.8082 \cdot 10^{-1}$
27	$2.4724 \cdot 10^{-1}$	$3.6883 \cdot 10^{-1}$	$3.0804 \cdot 10^{-1}$	$4.5111 \cdot 10^{-2}$	$1.2075 \cdot 10^{-1}$	$2.6768 \cdot 10^{0}$	$3.9201 \cdot 10^{-1}$
28	$1.5764 \cdot 10^{-1}$	$2.4724 \cdot 10^{-1}$	$2.0244 \cdot 10^{-1}$	$2.9647 \cdot 10^{-2}$	$1.0630 \cdot 10^{-1}$	$3.5857 \cdot 10^{0}$	$5.2511 \cdot 10^{-1}$
29	$1.1109 \cdot 10^{-1}$	$1.5764 \cdot 10^{-1}$	$1.3437 \cdot 10^{-1}$	$1.9677 \cdot 10^{-2}$	$1.5085 \cdot 10^{-1}$	$7.6661 \cdot 10^{0}$	$1.1227 \cdot 10^{0}$
30	$5.2475 \cdot 10^{-2}$	$1.1109 \cdot 10^{-1}$	$8.1782 \cdot 10^{-2}$	$1.1977 \cdot 10^{-2}$	$1.3375 \cdot 10^{-1}$	$1.1167 \cdot 10^{1}$	$1.6354 \cdot 10^{0}$
31	$3.4307 \cdot 10^{-2}$	$5.2475 \cdot 10^{-2}$	$4.3391 \cdot 10^{-2}$	$6.3545 \cdot 10^{-3}$	$1.3519 \cdot 10^{-1}$	$2.1275 \cdot 10^{1}$	$3.1157 \cdot 10^{0}$
32	$2.4788 \cdot 10^{-2}$	$3.4307 \cdot 10^{-2}$	$2.9547 \cdot 10^{-2}$	$4.3271 \cdot 10^{-3}$	$1.3086 \cdot 10^{-1}$	$3.0241 \cdot 10^{1}$	$4.4287 \cdot 10^{0}$
33	$2.1875 \cdot 10^{-2}$	$2.4788 \cdot 10^{-2}$	$2.3331 \cdot 10^{-2}$	$3.4168 \cdot 10^{-3}$	$1.3013 \cdot 10^{-1}$	$3.8085 \cdot 10^{1}$	$5.5774 \cdot 10^{0}$
34	$1.0333 \cdot 10^{-2}$	$2.1875 \cdot 10^{-2}$	$1.6104 \cdot 10^{-2}$	$2.3584 \cdot 10^{-3}$	$1.3004 \cdot 10^{-1}$	$5.5140 \cdot 10^{1}$	$8.0751 \cdot 10^{0}$
35	$3.3546 \cdot 10^{-3}$	$1.0333 \cdot 10^{-2}$	$6.8438 \cdot 10^{-3}$	$1.0023 \cdot 10^{-3}$	$1.3182 \cdot 10^{-1}$	$1.3152 \cdot 10^{2}$	$1.9261 \cdot 10^{1}$
36	$1.2341 \cdot 10^{-3}$	$3.3546 \cdot 10^{-3}$	$2.2944 \cdot 10^{-3}$	$3.3600 \cdot 10^{-4}$	$1.3594 \cdot 10^{-1}$	$4.0459 \cdot 10^{2}$	$5.9251 \cdot 10^{1}$
37	$5.8295 \cdot 10^{-4}$	$1.2341 \cdot 10^{-3}$	$9.0852 \cdot 10^{-4}$	$1.3305 \cdot 10^{-4}$	$1.4043 \cdot 10^{-1}$	$1.0555 \cdot 10^{3}$	$1.5457 \cdot 10^{2}$
38	$2.7536 \cdot 10^{-4}$	$5.8295 \cdot 10^{-4}$	$4.2916 \cdot 10^{-4}$	$6.2848 \cdot 10^{-5}$	$1.4472 \cdot 10^{-1}$	$2.3027 \cdot 10^{3}$	$3.3722 \cdot 10^{2}$
39	$1.0130 \cdot 10^{-4}$	$2.7536 \cdot 10^{-4}$	$1.8833 \cdot 10^{-4}$	$2.7580 \cdot 10^{-5}$	$1.5013 \cdot 10^{-1}$	$5.4432 \cdot 10^{3}$	$7.9714 \cdot 10^{2}$
40	$2.9023 \cdot 10^{-5}$	$1.0130 \cdot 10^{-4}$	$6.5162 \cdot 10^{-5}$	$9.5427 \cdot 10^{-6}$	$1.5813 \cdot 10^{-1}$	$1.6570 \cdot 10^{4}$	$2.4267 \cdot 10^{3}$
41	$1.0677 \cdot 10^{-5}$	$2.9023 \cdot 10^{-5}$	$1.9850 \cdot 10^{-5}$	$2.9070 \cdot 10^{-6}$	$1.6830 \cdot 10^{-1}$	$5.7896 \cdot 10^{4}$	$8.4787 \cdot 10^{3}$
42	$3.0590 \cdot 10^{-6}$	$1.0677 \cdot 10^{-5}$	$6.8680 \cdot 10^{-6}$	$1.0058 \cdot 10^{-6}$	$1.7947 \cdot 10^{-1}$	$1.7844 \cdot 10^{5}$	$2.6131 \cdot 10^{4}$
43	$1.1253 \cdot 10^{-6}$	$3.0590 \cdot 10^{-6}$	$2.0921 \cdot 10^{-6}$	$3.0639 \cdot 10^{-7}$	$1.9367 \cdot 10^{-1}$	$6.3210 \cdot 10^{5}$	$9.2568 \cdot 10^{4}$
44	$4.1399 \cdot 10^{-7}$	$1.1253 \cdot 10^{-6}$	$7.6964 \cdot 10^{-7}$	$1.1271 \cdot 10^{-7}$	$2.0772 \cdot 10^{-1}$	$1.8429 \cdot 10^{6}$	$2.6989 \cdot 10^{5}$
45	$1.0000 \cdot 10^{-11}$	$4.1399 \cdot 10^{-7}$	$2.0700 \cdot 10^{-7}$	$3.0314 \cdot 10^{-8}$	$2.3166 \cdot 10^{-1}$	$7.6419 \cdot 10^{6}$	$1.1191 \cdot 10^{6}$

DPLUS 46-group neutron sources.

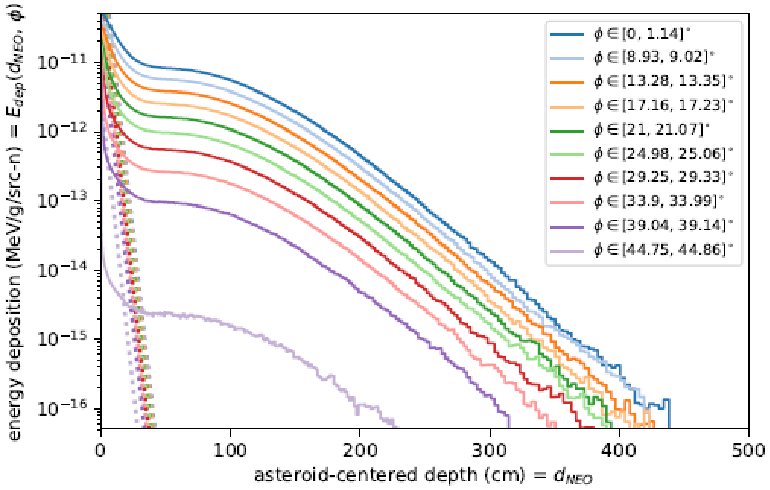
Comparison to analytical approximation.



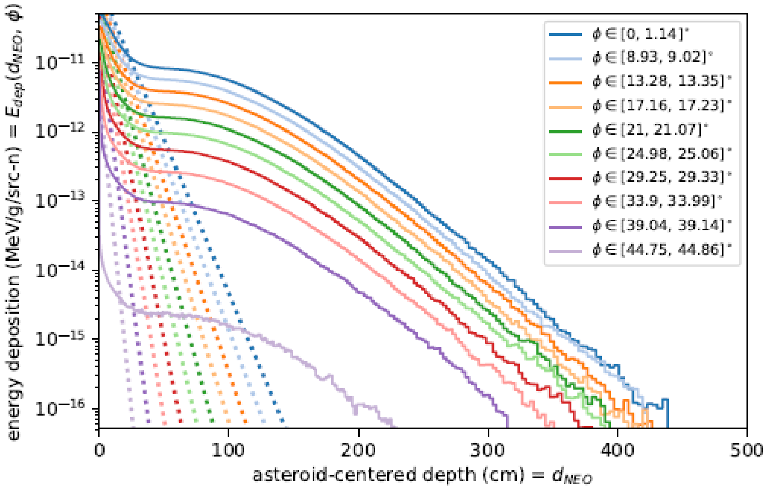
(a) 14.1 MeV source with the mean-free-path λ_d definition.



(b) 14.1 MeV source with the $(1 - 1/e)$ λ_d definition.



(c) 1 MeV source with the mean-free-path λ_d definition.



(d) 1 MeV source with the $(1-1/e)$ λ_d definition.