Fermilab U.S. DEPARTMENT OF ENERGY Office of Science

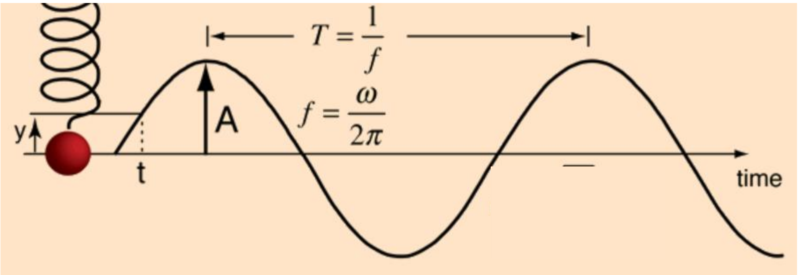
Vibrations in (superconducting) coils

Stoyan Stoynev /FNAL/

MDP meeting

6 January 2021

Preface 1: Periodic motion



$$y = A\sin\omega t$$

$$v = \omega A\cos\omega t$$ (vibration velocity, time derivative of y)

$$a = -\omega^2 A\sin\omega t = -\omega^2 y$$ (vibration acceleration, double time derivative of y)

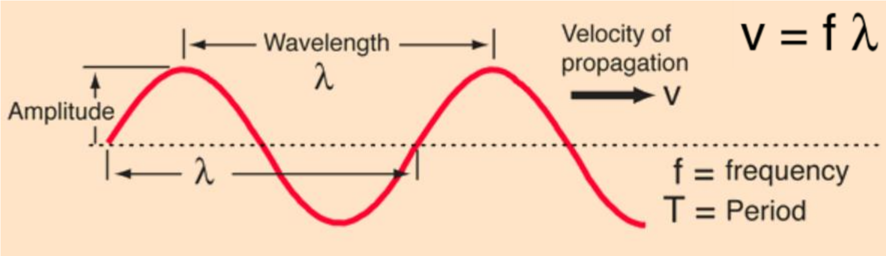
Relation between quantities at extremes:

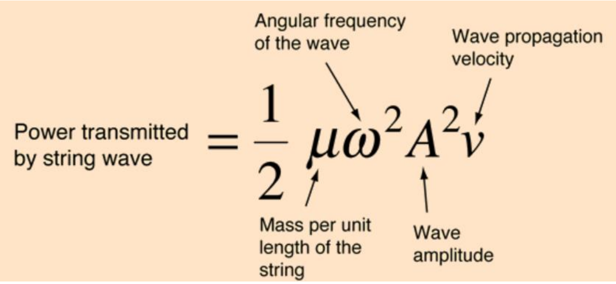
$$x_{max} = A$$
$$v_{max} = A\omega$$
$$a_{max} = A\omega^2$$

(x≡y)

http://hyperphysics.phy-astr.gsu.edu

Preface 1: Wave power relations





- Let's assume the cable of a coil can serve as an acoustic transmission line (at least initially), that is it does not "radiate" energy
- Let's see what amplitudes we can achieve along this "string" with some set of parameters

$$A = \frac{\sqrt{\frac{2P}{v\mu}}}{w}$$

$v \sim 4 \times 10^3$ m/s

$\mu \sim 0.5$ kg/m

P (W)	w (1/s)	A (m)	Aw (m/s)	λ (m)
10	10^5	**10^{-6}**	0.1	0.25
100	10^5	**3.1×10^{-6}**	sqrt(0.1)	0.25
1000	10^5	**10^{-5}**	1	0.25
10	5×10^5	**2×10^{-7}**	0.1	5×10^{-2}
100	5×10^5	**6.2×10^{-7}**	sqrt(0.1)	5×10^{-2}
1000	5×10^5	**2×10^{-6}**	1	5×10^{-2}
1000	5×10^6	**2×10^{-7}**	1	5×10^{-3}

(P/μ remains the same for a single "string" or a bunch of "strings" if all of them are "powered")

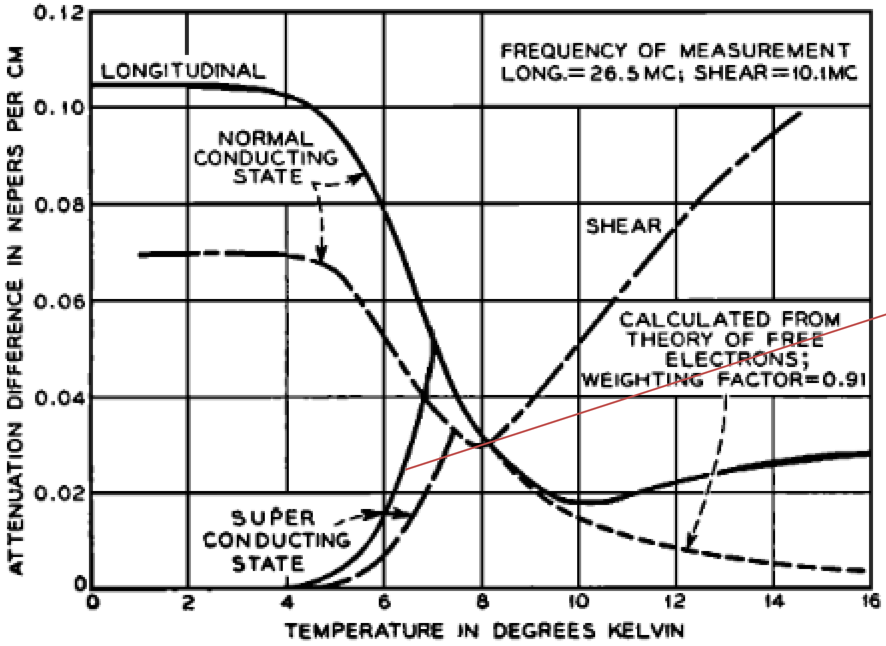
http://hyperphysics.phy-astr.gsu.edu

Ultrasonic driven periodic displacements of 1-10 microns are "easy".

Preface 1: More realistic conditions

- Coils are not strings, there is more complicated energy transfer through the bulk and interfaces
  - This doesn’t invalidate all string projections but there are apparent limitations
- Attenuation – it is complicated

Falls ~ linearly with frequency, falls with temperature except for very low temperatures, superconductors become “transparent” at low temperatures, magnetic field decreases attenuation, multi-material entities (like Nb_3Sn+Cu) have features,…

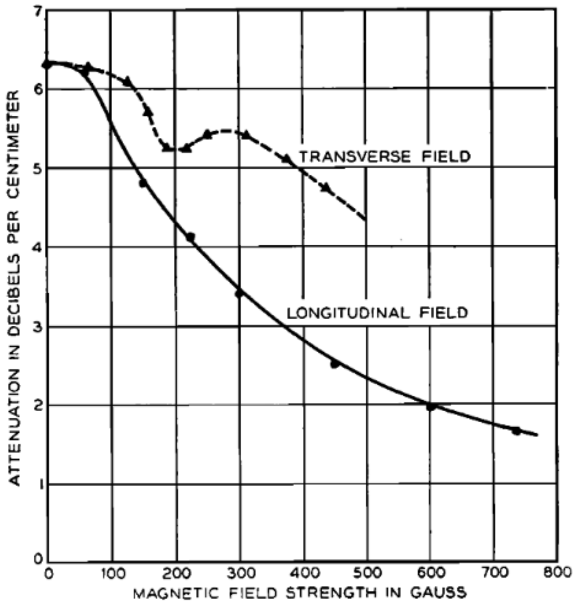


$$\frac{\alpha_{sc}}{\alpha} \sim \frac{u}{e^{v\,Tc/T}+1}$$

(away from T_c), u, v are ~ 1-10

α is attenuation coefficient

$$A = A_o \exp(-\alpha x)$$

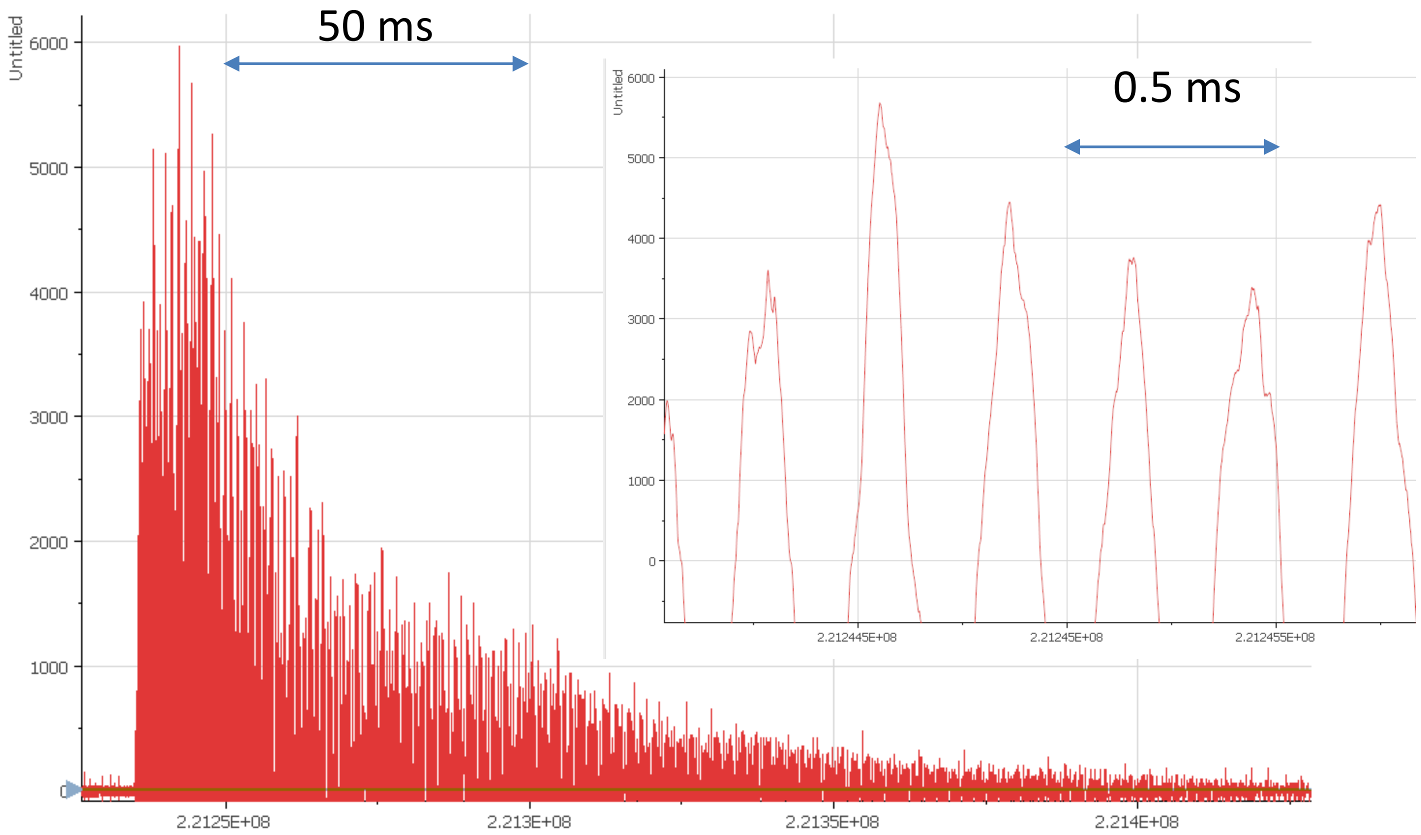


THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA VOLUME 28, NUMBER 5 SEPTEMBER, 1956

Ultrasonic Attenuation at Low Temperatures for Metals in the Normal and Superconducting States

W. P. Mason and H. E. Bömmel
Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey

Preface 1: Acoustic attenuation in SC magnets



In "15 T" magnet tests we see the amplitude drops by ~1/3 in ~30 ms. The periodic pattern is associated to the length of the magnet (1 m -> ~0.3 ms period). So it drops by 1/3 for 100 m (~ 4 kHz) which is ~0.1 dB/m.

At room temperature I find this formula:
$\alpha \approx$ Cd x f, Cd ~ 30 dB/m/MHz (steel/Cu; Al is ~10)
and thus $\alpha \approx 30 \times 4 \times 10^{-3} = 0.12$ dB/m.
This is the same order of magnitude as above (though different temperatures).

In first approximation we can accept an ultrasonic wave will not get attenuated too fast in the coil (it takes at least tens of periods to drop significantly).

applied sciences

MDPI

Review

A Comprehensive Report on Ultrasonic Attenuation of Engineering Materials, Including Metals, Ceramics, Polymers, Fiber-Reinforced Composites, Wood, and Rocks

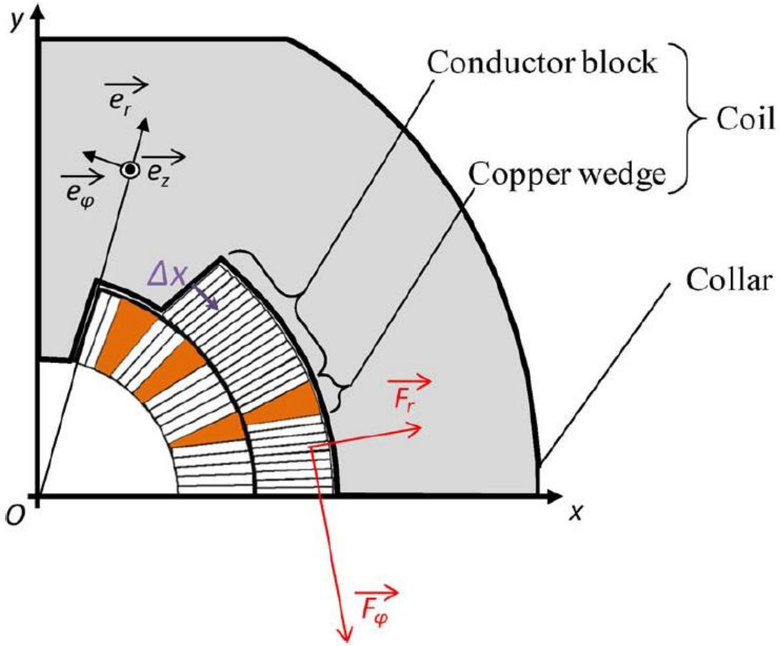
Kanji Ono

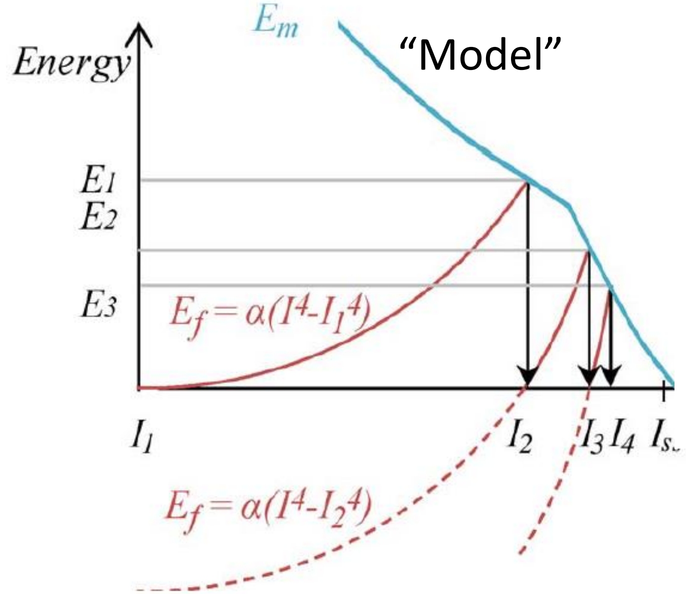
Department of Materials Science and Engineering, University of California, Los Angeles (UCLA), Los Angeles, CA 90095, USA; ono@ucla.edu; Tel.: +1-310-825-5534

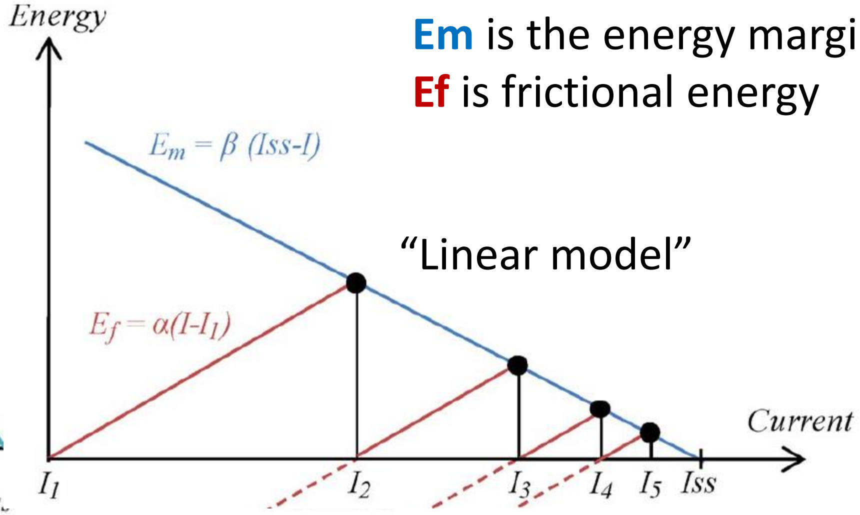
Received: 24 February 2020; Accepted: 18 March 2020; Published: 25 March 2020

check for updates

Preface 2: Slip-Stick in (LHC) magnets







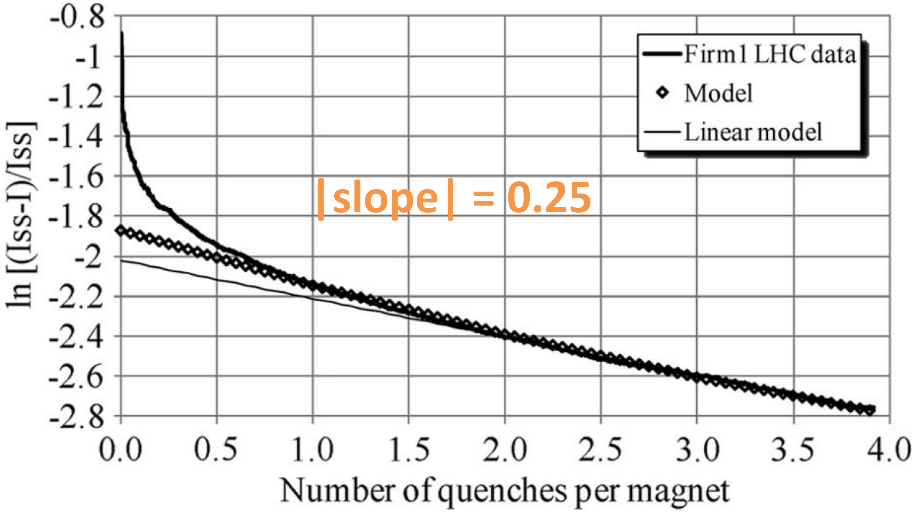
Em is the energy margin

Ef is frictional energy

$$I_{ss} - I_{n+1} = (I_{ss} - I_1)\left(1 + \frac{\beta}{\alpha}\right)^{-n}$$

$$ln\left(1 - \frac{I_{n+1}}{I_{ss}}\right) = ln\left(1 - \frac{I_1}{I_{ss}}\right) - n\, ln(1 + \beta/\alpha)$$

|slope|



IEEE TRANSACTIONS ON APPLIED SUPERCONDUCTIVITY, VOL. 21, NO. 5, OCTOBER 2011

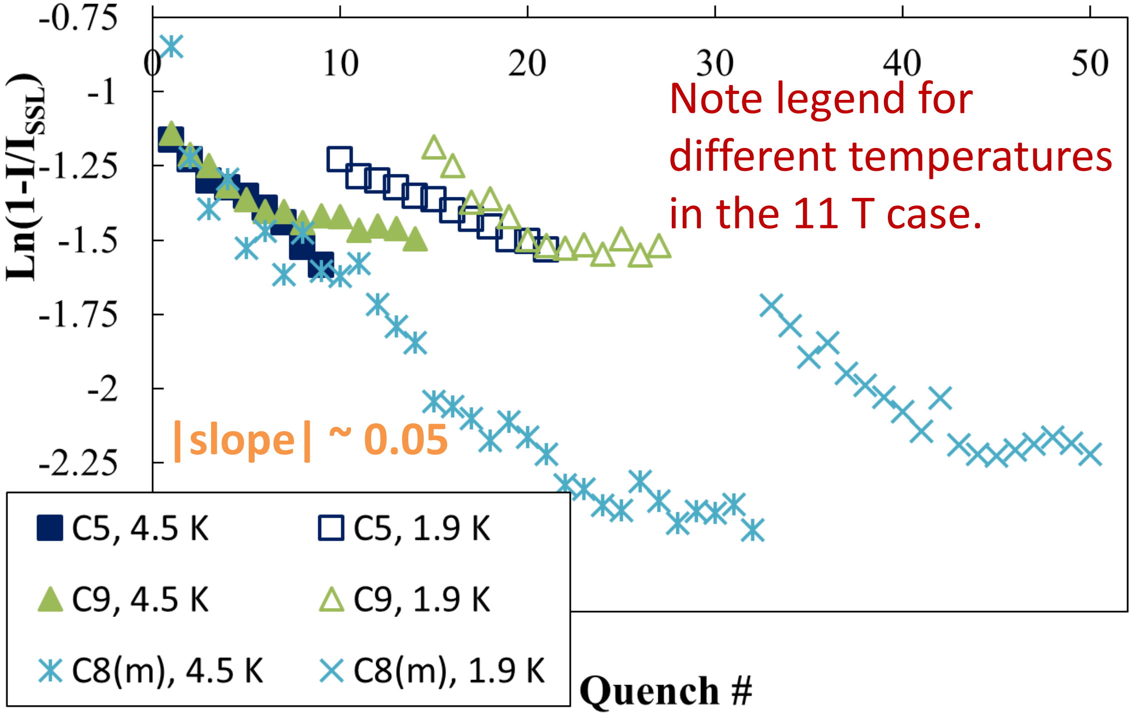
Slip–Stick Mechanism in Training the Superconducting Magnets in the Large Hadron Collider

Pier Paolo Granieri, Clément Lorin, and Ezio Todesco

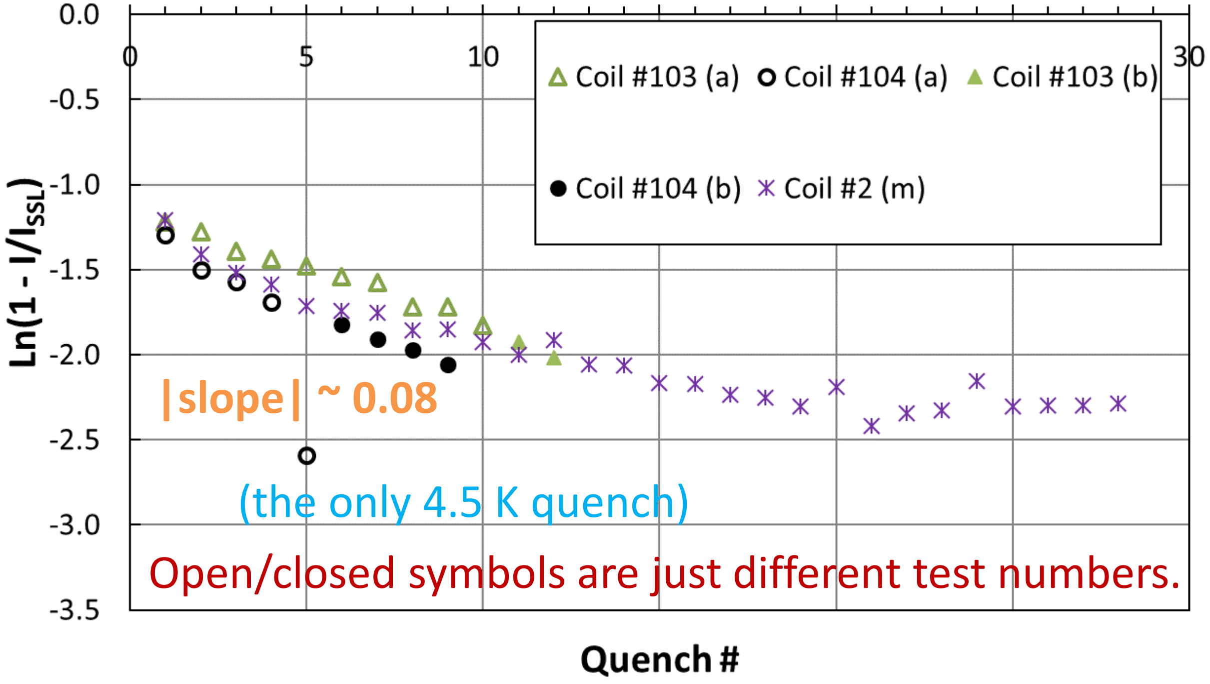
Preface 2: Analogous 11 T and LARP s-quad curves

(FNAL data)

11 T short coils



LARP short coils



The flattening of curves mean coils reaching a training limit (at certain conditions)

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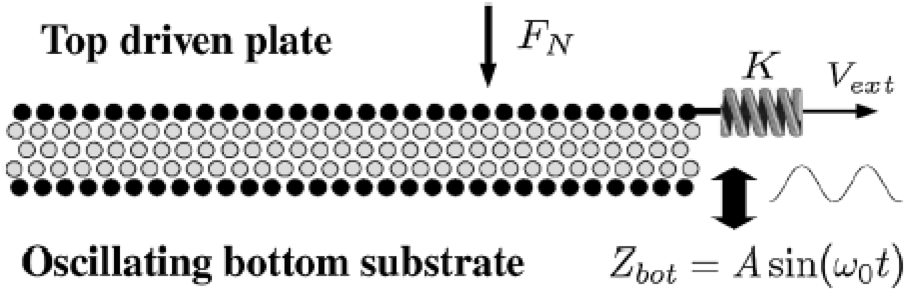
Vibration, friction, magnetic field

Non-rigid interface

Rigid surfaces confining non-rigid material
(you can think metal and insulation material)



PRL **103,** 085502 (2009) PHYSICAL REVIEW LETTERS

Suppression of Friction by Mechanical Vibrations

Rosario Capozza,[1] Andrea Vanossi,[2,1] Alessandro Vezzani,[1,3] and Stefano Zapperi[1,4]

- In first approximation there are three forces acting in vertical direction:
  - ✓ Applied normal force, F_N
  - ✓ Inertial force (from vibrations), F_{in}
  - ✓ Damping force (energy dissipation), F_{damp}

Let's $M = M_p + M_{\rm top}$ be the total mass of the non-rigid and the top rigid material.
$F_{\rm in} = M\ddot{Z}_{\rm bot}$ and then the maximal force (of oscillation) is MAw_0^2. The damping force is proportional to velocity (of oscillation) $F_{\rm damp} = M_p \eta \dot{Z}_{\rm bot}$ (η is damping coefficient) and then its maximum is $M_p\eta A w_0$.
At frequency w_1 the inertial force overcomes the other forces and friction (proportional to the normal down-force) vanishes.

$$MA\omega_1^2 = F_N + M_p \eta A \omega_1.$$

Using dimensionless variables $\tilde{f} \equiv \frac{F_N}{MA\eta^2}$ $\tilde{m} \equiv \frac{M_p}{M}$ $\tilde{\omega} \equiv \frac{\omega}{\eta}.$

one gets $$\tilde{\omega}_1 = \frac{1}{2}(\tilde{m} + \sqrt{\tilde{m}^2 + 4\tilde{f}})$$

Friction above oscillation frequency w_1 is suppressed.

Non-rigid interface (2)

PRL **103,** 085502 (2009) PHYSICAL REVIEW LETTERS

Suppression of Friction by Mechanical Vibrations

Rosario Capozza,[1] Andrea Vanossi,[2,1] Alessandro Vezzani,[1,3] and Stefano Zapperi[1,4]

- During oscillations particles are subject to forces
- There is a characteristic time Δt for their action expressed in the impulse formula $F\Delta t = m\Delta v$
- In our case:

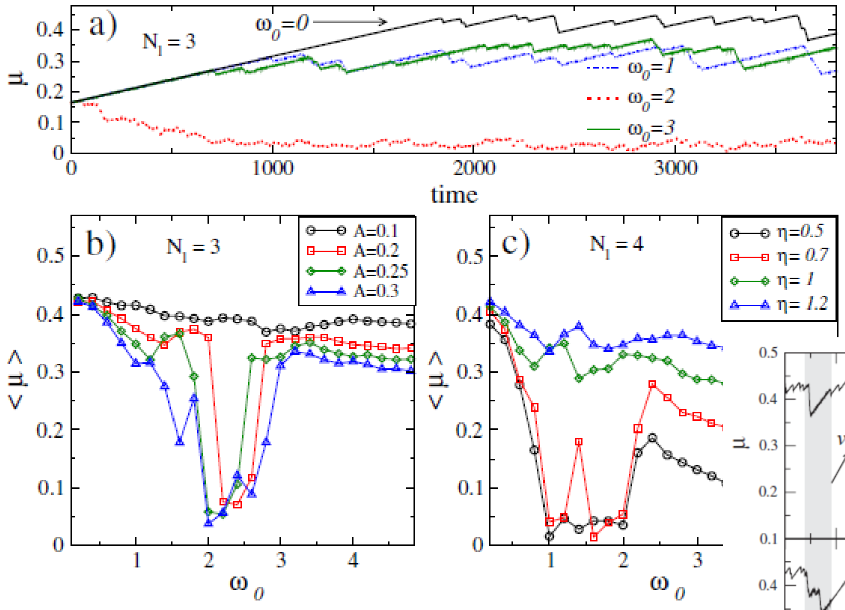
$$\Delta t \simeq \dot{Z}_{\rm bot} M/F_N \simeq A\omega_0 M/F_N$$

- If the period of oscillation is smaller than this characteristic time $(2\pi/w_0 < \Delta t)$ there is no effective transfer of oscillation energy between particles.
- Explicitly:

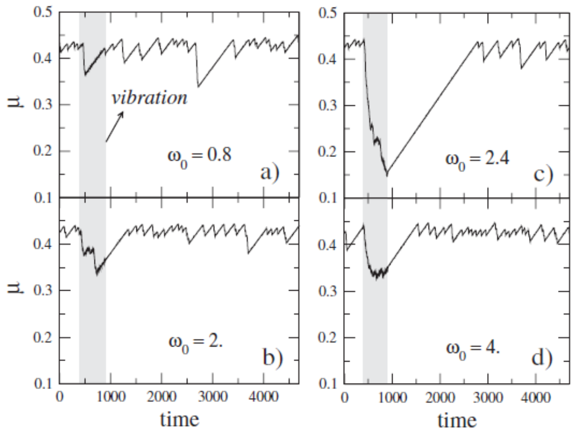
$$\tilde{\omega}_2 = \sqrt{2\pi \tilde{f}}$$

- Above w_2 friction is no longer effectively suppressed
- Thus, friction is suppressed if $w_1 < w_0 < w_2$

Friction coefficient vs time and time averaged coefficient vs w_0 in numerical simulations



Effect of a short time vibration pulse on the stick-slip behavior



Numerical simulations confirm the findings as well as later experimental reports (other authors).

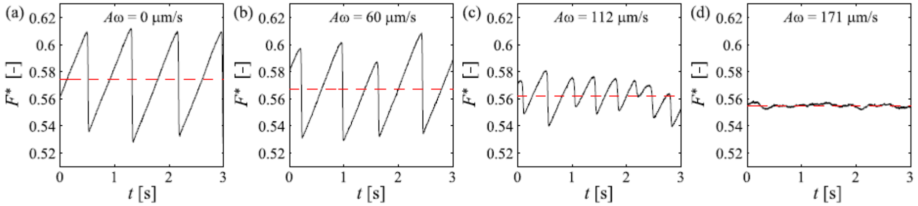
Granular surfaces

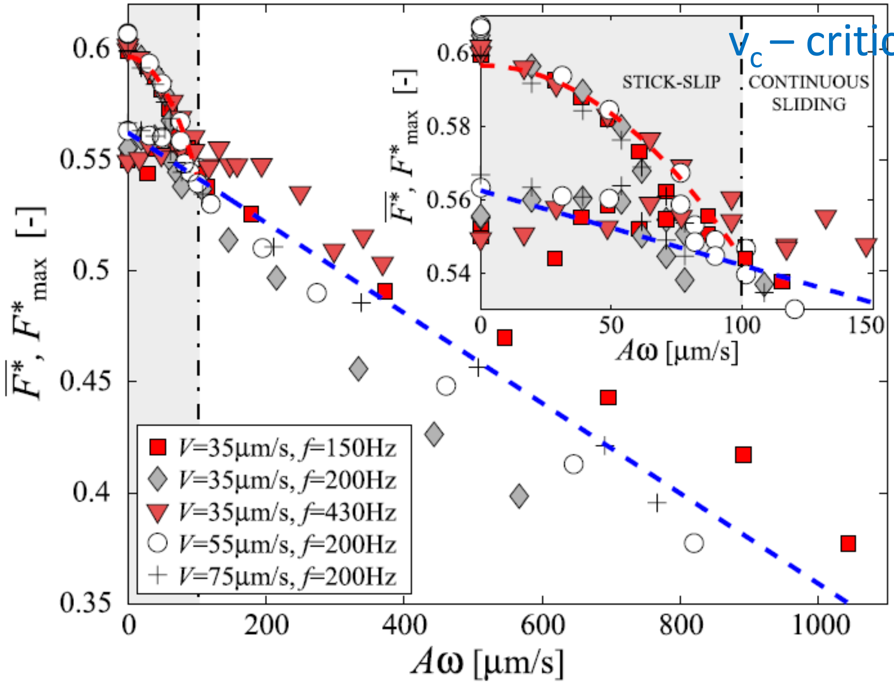
www.nature.com/scientificreports

Granular friction: Triggering large events with small vibrations

Henri Lastakowski, Jean-Christophe Géminard & Valérie Vidal

Normalized tangential force (friction coefficient)
vs time at different vibration velocities and vs vibration velocity





v_c – critical velocity

Beyond v_c slip-stick transitions to continuous sliding.
The velocity dependence is direct
(no hidden dependences to other parameters).

F* is the driving force normalized to pressing (normal) force or effectively – the friction coefficient.

Experiments confirm those findings.

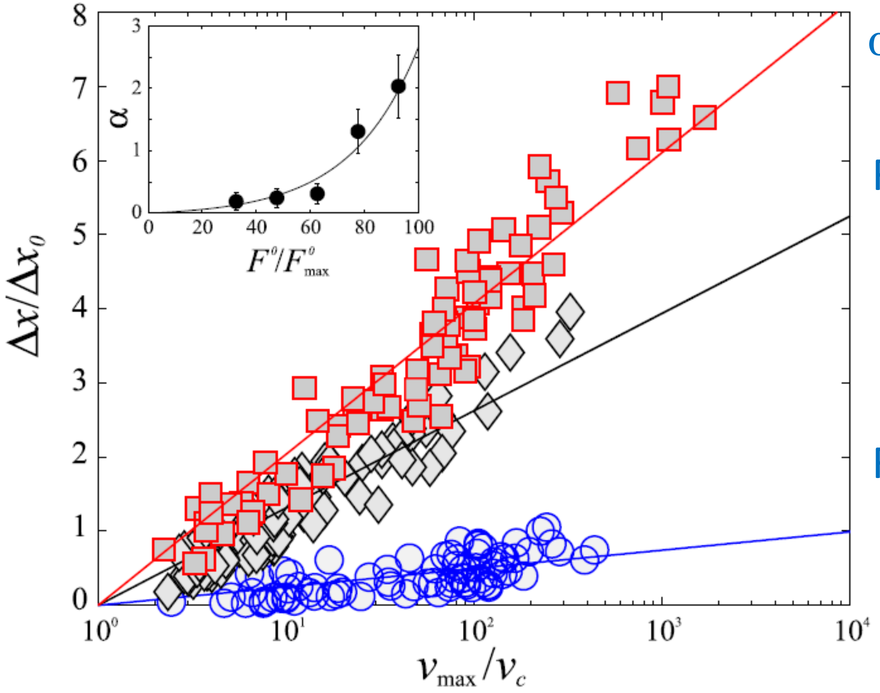
Granular surfaces (2)

www.nature.com/scientificreports

Granular friction: Triggering large events with small vibrations

Henri Lastakowski, Jean-Christophe Géminard & Valérie Vidal

Displacement after single mechanical disturbance (from static case)
Normalization is to the case with no disturbance (x_0)
and critical velocity v_c



α here is the slope of $\Delta x/\Delta x_0$

F^0/F^0_{max} on the plot is
(0.85,1) - red
(0.7,0.85) - black
(0.45,0.55) – blue

F^0_{max} is when slip occurs without vibrations

As before, the vibrational velocity (Aw) is the parameter of interest. Higher Aw higher the released energy (Δx) from a slip.

If a barrier with size ξ_c needs to be overcome to “unstick” movement:

$$(\rho d^3 g)\,\xi \sim \rho d^3 (A\omega)_c^2$$

Potential energy — Kinetic (inertial) energy

$$\xi_c \sim (A\omega)_c^2/g$$

g is characteristic acceleration of the normal force

Indeed, critical velocity was found to correlate well with surface granularity independently on vibration type (authors point out that high pressure may require high A explicitly).

Summary 1

- Studies show that vibrations in any direction with respect to friction force have similar characteristics although quantitatively there are some (small) differences

- In sliding over non-rigid material one can expect a window of vibration frequencies (w_1, w_2) where friction is suppressed

$$w_1 < w < w_2$$

- Granular "imperfections" induce a characteristic critical (friction) velocity related to their size

$$\xi_c \sim (A\omega)_c^2/g$$

- Other authors point out that rearrangements in the frictional media (which affects friction) can be induced when the mechanical wavelength is of the order of the rearrangement scale

Context: coils

- Coils can be subjected up to 200 MPa due to Lorentz force and pre-stress
  - Let's take 100 MPa normal to the cable wide side (2 cm cable width, for instance)
  - Let's use 5 cm as a characteristic test scale over the cable length, then F_N = 100 kN
  - The coil mass (all turns) over that area is M~ 5 kg

Just for convenience (can work with pressure alone)

Damping force: $M_p \eta A w_0$
(non-rigid interface)

Aw is <1 (in SI) for us, Mp (cable insulation for the above M) is a very small number, the damping constant could not be large, or we wouldn't hear anything during magnet quenching;
at the end, this term is negligible with respect to large forces in the magnet

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w_1 and w_2:
(non-rigid interface)

$w_1 = \mathrm{sqrt}[F_N/(MA)]$ and $w_2 = \mathrm{sqrt}(2\pi)\ w_1$
If we set A = 3 microns, $w_1 \sim 10^5$ Hz (same as $f_1 \sim 17$ kHz),
which we can achieve with ~100 W of power according to an earlier table.
$w_2 \sim 2.5 \times 10^5$ Hz

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g, ξ_c :
(rigid granular barriers)

$g = F_N/M \sim 20\ \mathrm{km/s^2}$ $\quad \xi_c \sim (A\omega)_c/g$
Within reasonable constraints we can achieve at most Aw = 1 m/s.
Thus, we can "resolve" barriers at scale $\xi_c < M/F_N < 50$ microns

...and higher Aw higher the released energy at slip

λ: Realistically our wavelengths now are > 1 cm, can't "rearrange" friction media

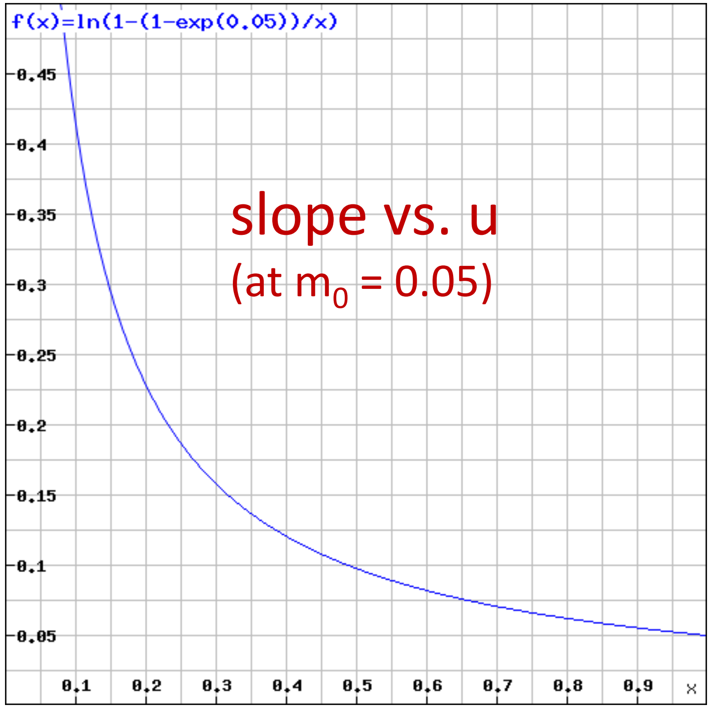
Coil training with lower friction

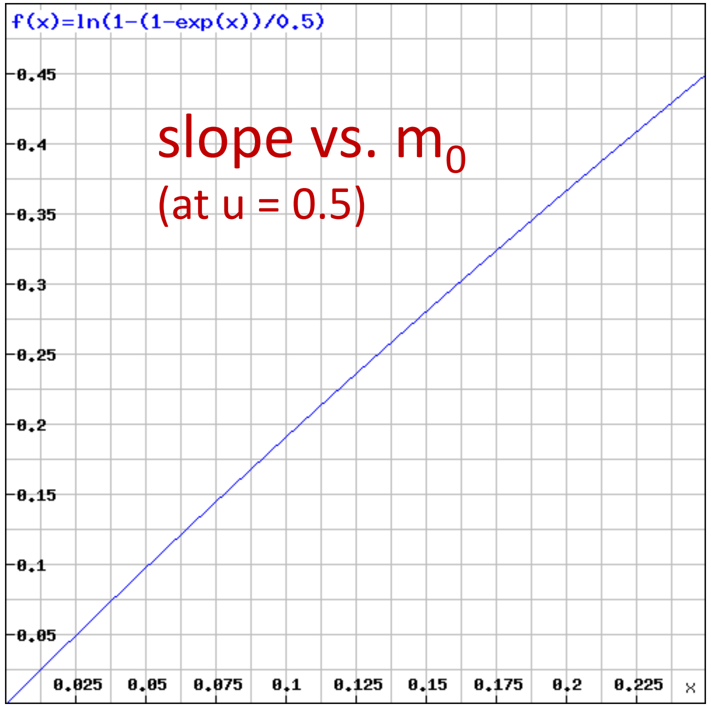
$$f(n) \equiv ln\left(1-\frac{I_{n+1}}{I_{ss}}\right) = ln\left(1-\frac{I_1}{I_{ss}}\right) - n\, \boxed{ln(1+\beta/\alpha)}$$

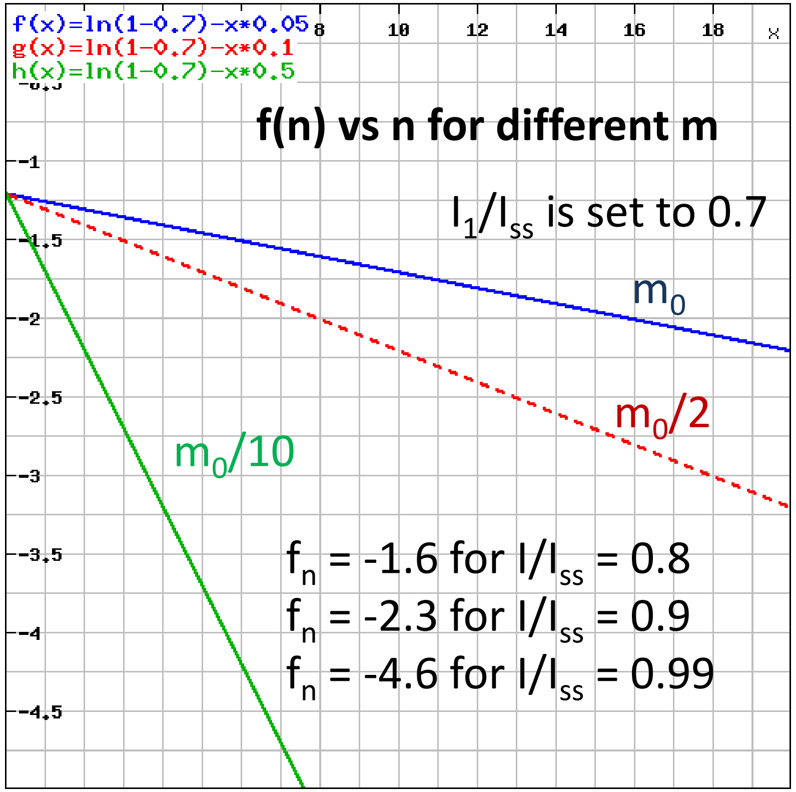
|slope|

α is proportional to the static friction coefficient μ_s; let's denote by $u\alpha$ any change in the parameter caused by change in static friction ($u\alpha \sim u\mu_s$) and the initial/nominal slope at $u=1$ by m_0.

$$slope \equiv m = ln\left(1-\left(\frac{1-e^{m_0}}{u}\right)\right)$$







(0,0)

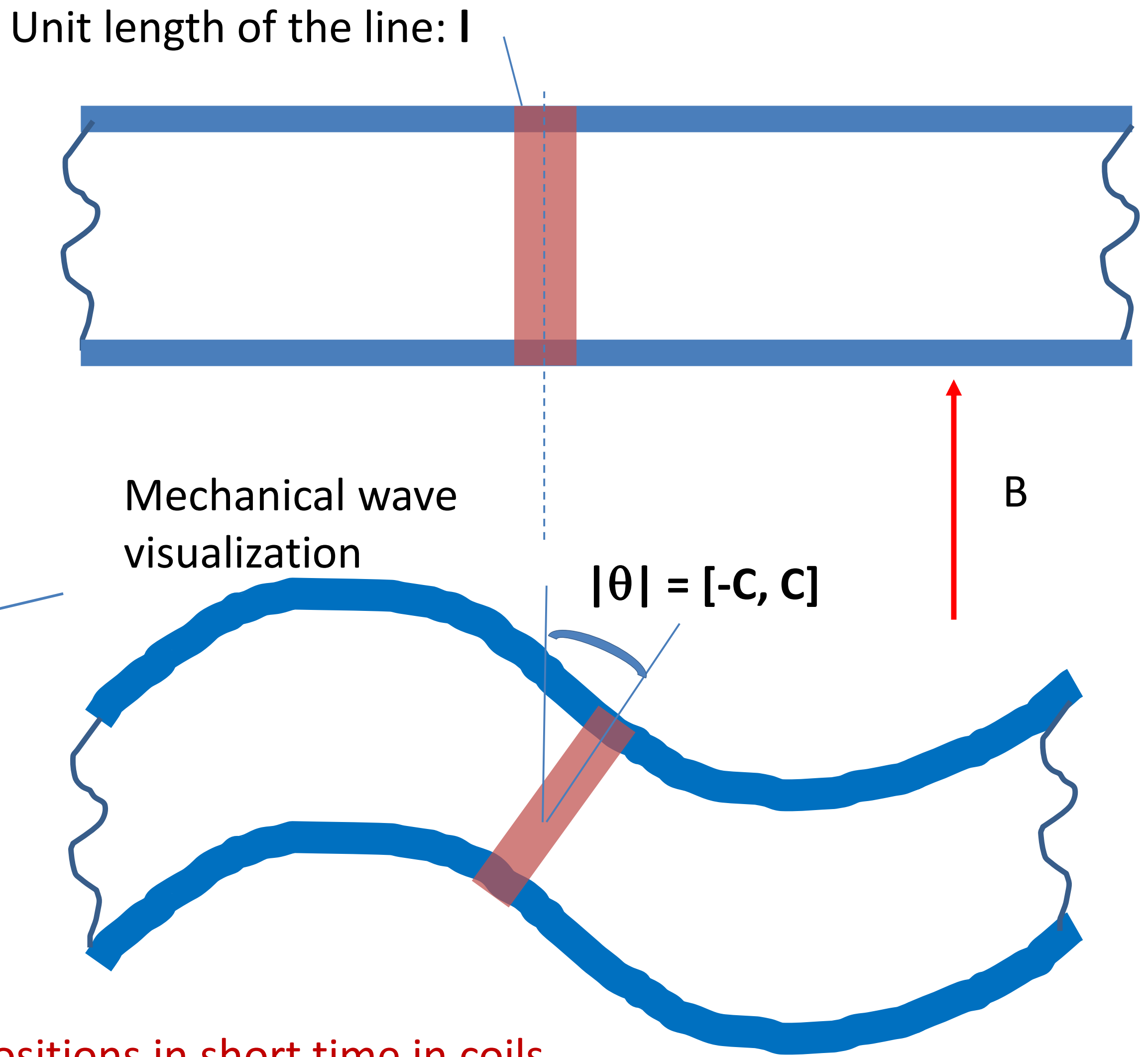
https://rechneronline.de/function-graphs/

Vibrations in magnetic field

- If we have a non-superconducting closed-circuit area with varying magnetic flux density, we'll have energy losses

- One way to have that in a coil is by vibrations though they have to have specific characteristics to fulfil the conditions above

- The power losses need to be compensated by the vibration energy source

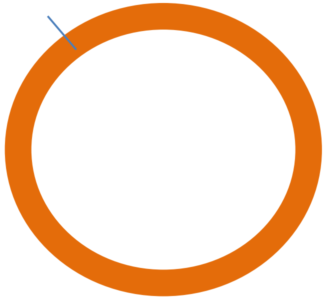


This is not the only way to describe the process, the important part is that vibrations of the proper kind will force repeatable flux changes through a surface.

Now, I want to check if it is possible at all to have sizable energy depositions in short time in coils.

Vibrations in magnet cables

Copper line cross-section



Let's take two scales: strand and filament

Strand:

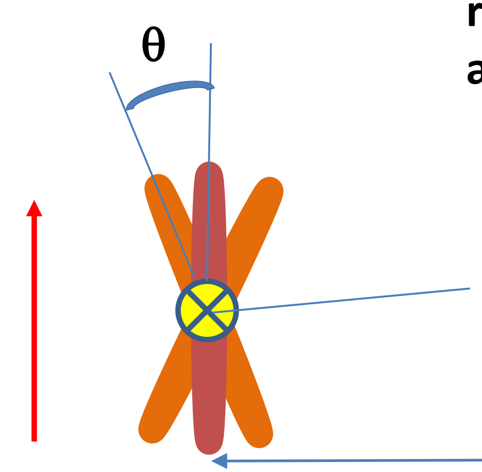
diameter **d** = 2**r** ~ 1 mm

average copper tube thickness **a** ~ 0.07 mm

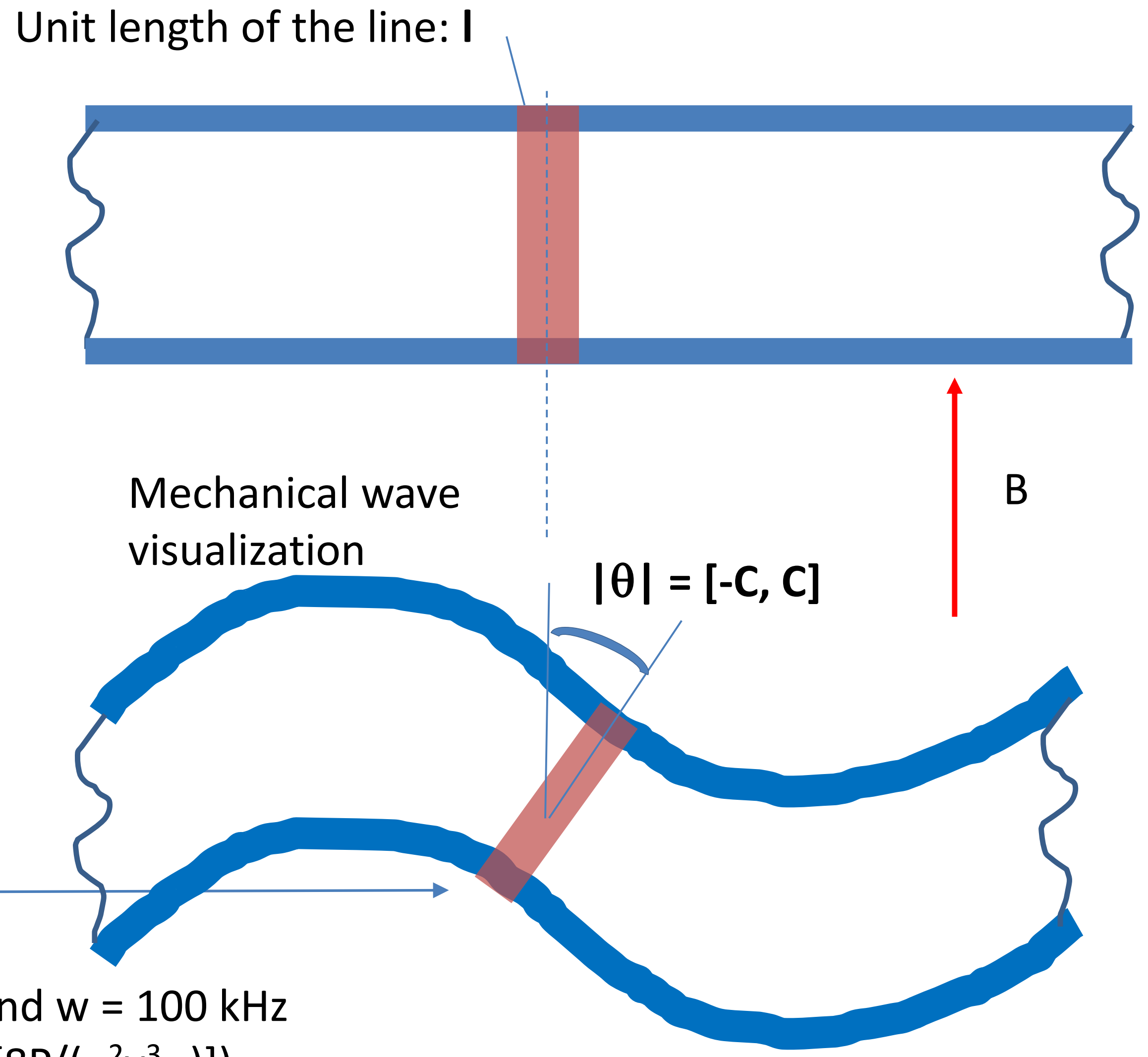
Filament:

r = 25 x 10^{-6} m (Cu matrix cell)

a = 3 x 10^{-6} m



Axis of rotation (vibration) with frequency **w** (~ 2π 20 kHz)

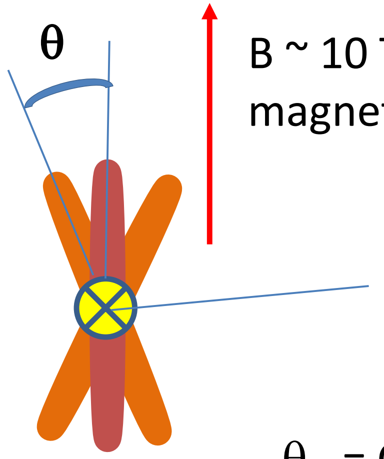


B ~ 10 T magnetic flux density

θ = C sin(wt)

If the amplitude A of vibrations is 10 microns and w = 100 kHz then C ~ 150 μrad ~π/20000 (~ A/(λ/4) = sqrt[8P/($\pi^2 v^3 \mu$)])

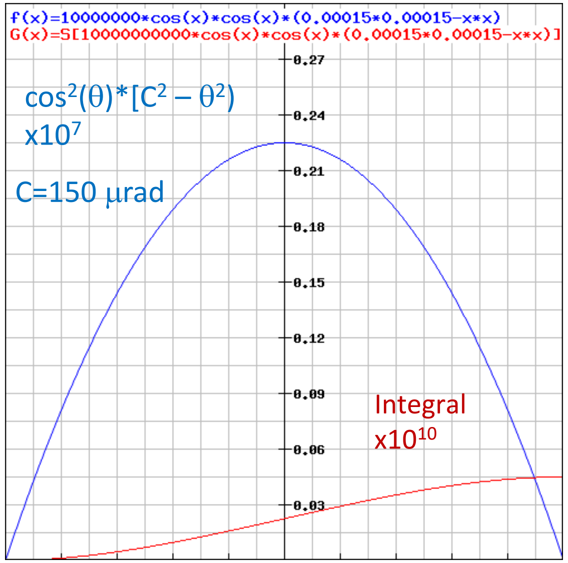
Power losses



B ~ 10 T
magnetic flux density

Axis of rotation (vibration) with frequency **w** (~ 2π 20 kHz)

$\theta = C \sin(wt)$



https://rechneronline.de/function-graphs/

magnetic flux through this ring-like area

$$\Phi = B\, \pi r^2 \sin(\theta) = B\, \pi r^2 \sin(C \sin(wt))$$

induced emf (electro-motive force) due to those vibrations

$$\varepsilon = \partial\Phi/\partial t = B\, \pi r^2\, w \cos(\theta)\, \mathrm{sqrt}[C^2-\theta^2]$$

Ignoring signs, self-inductance

resistance of the ring

$$R = \rho\, 2\pi r/(la) \equiv 2\pi r/(la\, \sigma)$$

Copper resistivity

Copper conductivity

energy per unit time (and per unit **l**):

$$E/t = P = emf^2/R = ½\, \sigma\, \pi\, l\, a\, r^3\, B^2\, w^2 \cos^2(\theta)\, [C^2 - \theta^2]$$

Over one period: $\langle ang \rangle \equiv \langle \cos^2(\theta)*[C^2 - \theta^2] \rangle|_{C=150\ \mu rad} = 15 \times 10^{-9}$ and $l = \lambda = 2\pi v/w$

$$\langle \cos^2(\theta)*[C^2 - \theta^2] \rangle|_{C=10\ \mu rad} = 14 \times 10^{-15}$$

Power dissipation at different scales

$<P> = \pi^2 \sigma \ a \ v <ang> B^2 \ w \ r^3$ (average power)

	strand	filament
	$r = 0.5 \times 10^{-3}$ m	$r = 25 \times 10^{-6}$ m
	$a = 7 \times 10^{-5}$ m	$a = 3 \times 10^{-6}$ m

$\sigma = 1 \times 10^{10}$ S/m at 2 K (RRR ~ 150) B = 10 T $v = 4 \times 10^3$ m/s $w = 10^5$ Hz (λ ~ 25 cm)

$a_s \times r_s^{\,3}/(a_f \times r_f^{\,3}) = 1.7 \times 10^{11}$

strand

$<P> = \pi^2 \quad \sigma \quad a \quad v \quad <ang> \quad B^2 \quad w \quad r^3 \sim$

$\sim 9.5 \times 10^{10} \times 7\times10^{-5} \times 4\times10^3 \times 15\times10^{-9} \times 10^2 \times 10^5 \times 125\times10^{-6} \sim$

$\sim 5 \times 10^5$ W

One can vibrate strands in that manner but at much lower amplitude (with C=10 μrad, which is A~ 0.7 microns, it goes 6 orders of magnitude down)

filament

(those are over 1 λ)

$<P> = \pi^2 \quad \sigma \quad a \quad v \quad <ang> \quad B^2 \quad w \quad r^3 \sim$

$\sim 9.5 \times 10^{10} \times 3\times10^{-6} \times 4\times10^3 \times 15\times10^{-9} \times 10^2 \times 10^5 \times 15.6 \times10^{-15} \sim$

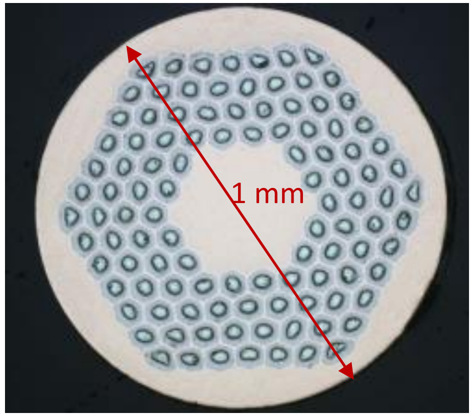
$\sim 3 \times 10^{-6}$ W

then ~150 filaments (or like-sized closed-circuit structures) per 40 strands don't need much power to vibrate with the assumed amplitude: total is ~20 mW (ignoring any other constraining force).

Also interesting to consider – effect of emf induced in the superconductor on transport properties.

“Optimal” power dissipation

Looking at what energy can reasonably be dissipated in a coil (no other losses).



108/127 RRP wire

/Let’s clarify again, $\langle P \rangle$ is for one period, integrated over one wavelength. /
At $w=10^5$, one wavelength is ~25 cm, T ~ 60 μs; a short coil is ~125 m, so 500 λ.
If we have 0.25 kW discharged in 30 ms this is 0.5 J/60 μs or 0.5 J per 1 λ. Having 40 strands: P_{strand} is 12.5 mJ/ 1 λ (or 0.5 mJ per cm of strand)

Finding a hypothetical “tube” radius where 1/10 of the power (2 mJ per cm of cable in 30 ms) would be dissipated:

$r^3 = \langle P \rangle (\pi^2 \sigma\ a\ v \langle ang \rangle B^2 w)^{-1}$

Keeping all parameters the same, including $a = 3 \times 10^{-6}$ m (filament parameter)

$r = \text{cubic root}\ (\ P \times (\pi^2\ \sigma\ a\ v\ \langle ang \rangle\ B^2\ w)^{-1})\sim$

$\sim\ \text{cubic root}\ (1.25\times10^{-3} \times 9.5^{-1} \times 10^{-10} \times 3^{-1}\times10^{6} \times 4^{-1}\times10^{-3} \times 15^{-1}\times10^{9} \times 10^{-2} \times 10^{-5})\sim$

$\sim 0.2 \times 10^{-3}$ m (this is 2/5 the strand radius)

How filaments or parts of strands could move/vibrate within the strand is hard to model (for me).
However, we can expect significant energy loss **IF** we manage to supply proper wave excitations wrt magnetic field lines.

Summary 2

- Vibrations in coils (conductor or else) has the potential to affect training performance
  - There is no direct studies on coils yet
  - Projected wave parameters are in an accessible range
  - We don’t have proper expertise but on the other hand there is no necessarily fine tuning of parameters

- Vibrations also have the potential to induce energy loss in magnet conductor in magnetic field
  - Wave parameters are in an accessible range
  - Generally, this requires much more demanding wave parameters tuning
  - On the other hand, lack of additional energy losses benefits friction-reduction

Possible path forward

- **<u>Dedicated</u>** research by <u>focused</u> individual(s)
  - Postdoc (or may be a PhD student)
  - Development of simulations/software framework
  - Dedicated tests at small-scale (benchmarking)
  - Larger scale test plans
- Dedicated material support
  - Specialized ultrasonic machine(s)
  - Likely close coordination with manufacturers
  - Integration with other developments

US Magnet Development Program (MDP) Goals:

GOAL 1:

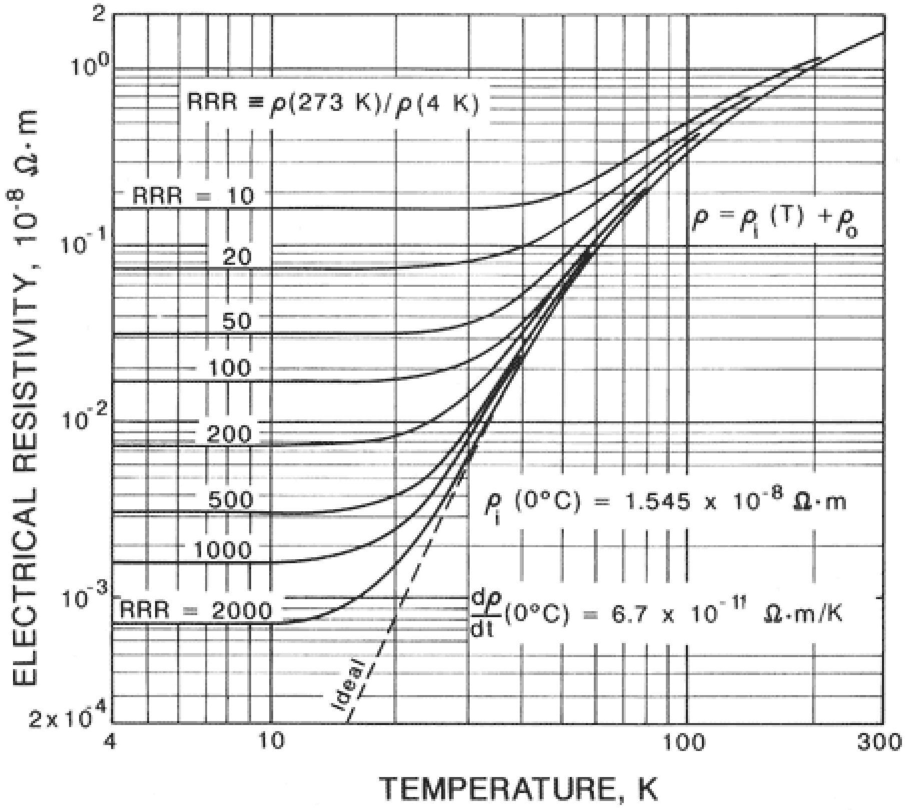
Explore the performance limits of Nb_3Sn accelerator magnets with a focus on minimizing the required operating margin and significantly reducing or eliminating training.

What resources and distribution we want to provide for reaching Goal 1?

There are no projections yet any of those will be supported. Then we can invest in small auxiliary experiments, extending run plans in other tests. We could hope that those small-scale tests will give us more insights or results making attraction of funds more likely.

There are no projections those will get enough "critical mass" either. So "we" can decide to work along those lines or dedicated individuals may have to decide how dedicated they can afford to be.

Spare



https://www.copper.org/resources/properties/cryogenic/images/Electrical-Resistivity.gif

Note that for rotating frame in magnetic filed and using the same formula so far

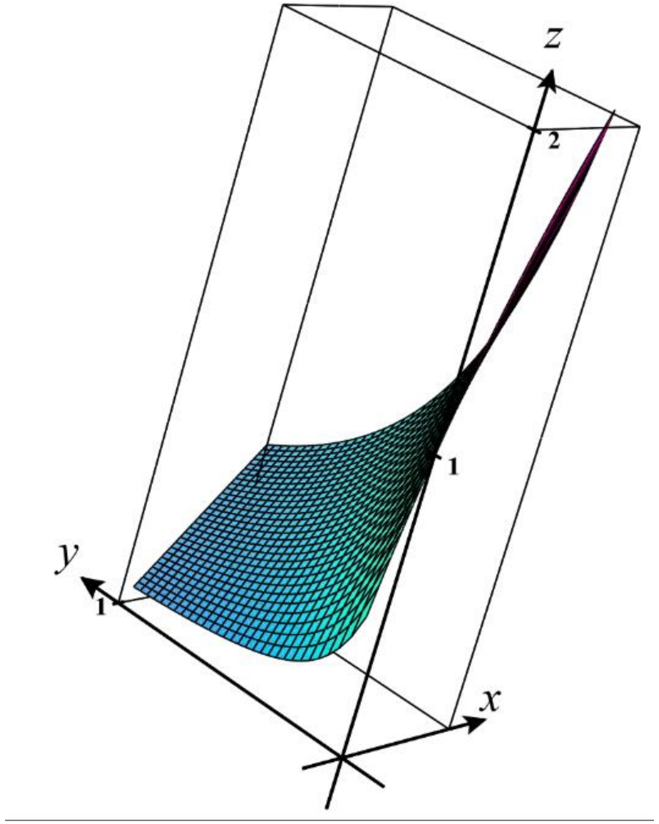
$\langle \cos^2(\theta)*[C^2 - \theta^2] \rangle|_{C=\pi} \approx 1/2$

Thus

$\langle \varepsilon^2 \rangle = \frac{1}{2} (B \pi r^2 w)^2$

which is the known solution

Spare



z=ln(1-(1-exp(x))/y)

https://www.monroecc.edu/faculty/paulseeburger/calcnsf/CalcPlot3D/