

Pion Condensation in the Early Universe

Volodymyr Vovchenko

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In collaboration with:

B. Brandt, G. Endrödi (Bielefeld U.)

F. Cuteri, F. Hajkarim, J. Schaffner-Bielich (Frankfurt U.)

based on [Phys. Rev. Lett. 126, 012701 \(2021\)](#)



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Bose-Einstein condensation

$$n(E_p) = \frac{1}{e^{(E_p - \mu)/T} - 1}$$

S.N. Bose, A. Einstein, 1924

$$\text{BEC at } T < T_c, \quad T_c \approx 3.31 \frac{n^{2/3}}{m}, \quad \frac{n_0}{n} = 1 - \left(\frac{T}{T_c} \right)^{3/2} \quad \text{for non-rel. Bose gas}$$

Velocity-distribution data for a gas of rubidium atoms confirming the discovery of BEC in 1995

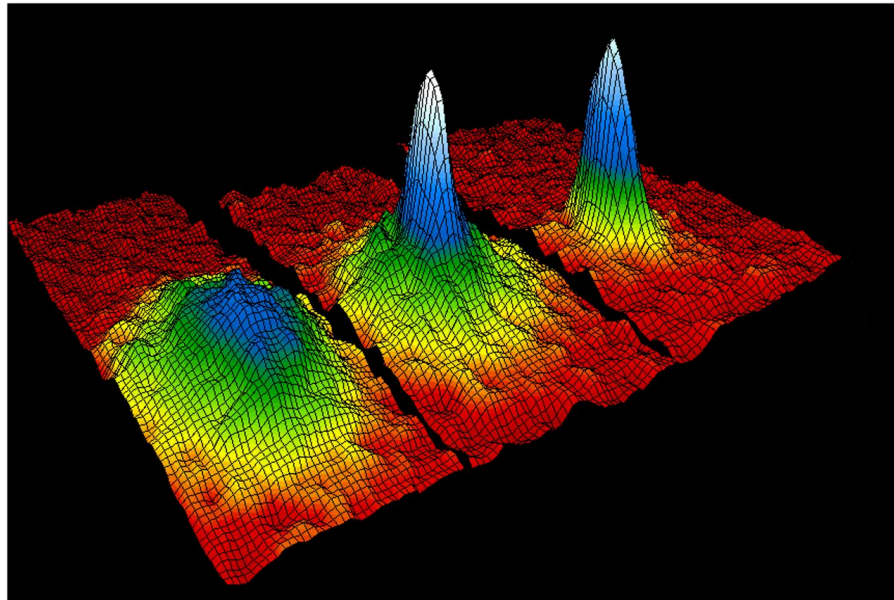


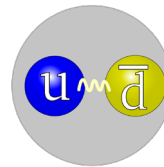
Image courtesy of NIST/JILA/CU-Boulder

Pion condensation

The relevant QCD degrees of freedom at low energies are **pions**

$$\mu_{\pi^\pm} = \pm \mu_I, \quad n_I = (n_u - n_d)/2$$

isospin



- chiral perturbation theory ($T=0$) [D.T. Son, M. Stephanov, PRL '01]

- vacuum at $\mu_{\pi^\pm} < m_\pi$
- pion BEC** at $\mu_{\pi^\pm} \geq m_\pi$ (2nd order phase transition)

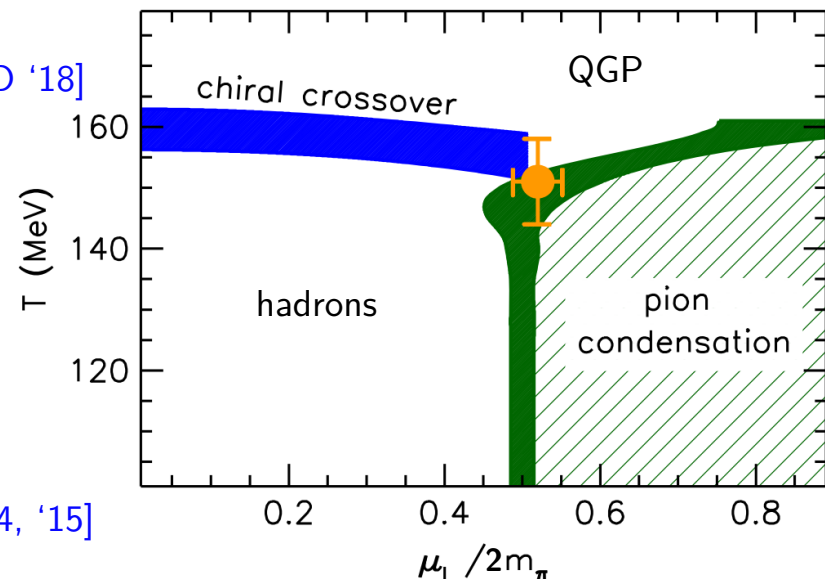
- Lattice QCD [Brandt, Endrodi, Schmalzbauer, PRD '18]

- no sign problem at finite μ_I
- physical quark masses achieved
- consistent with χ PT predictions

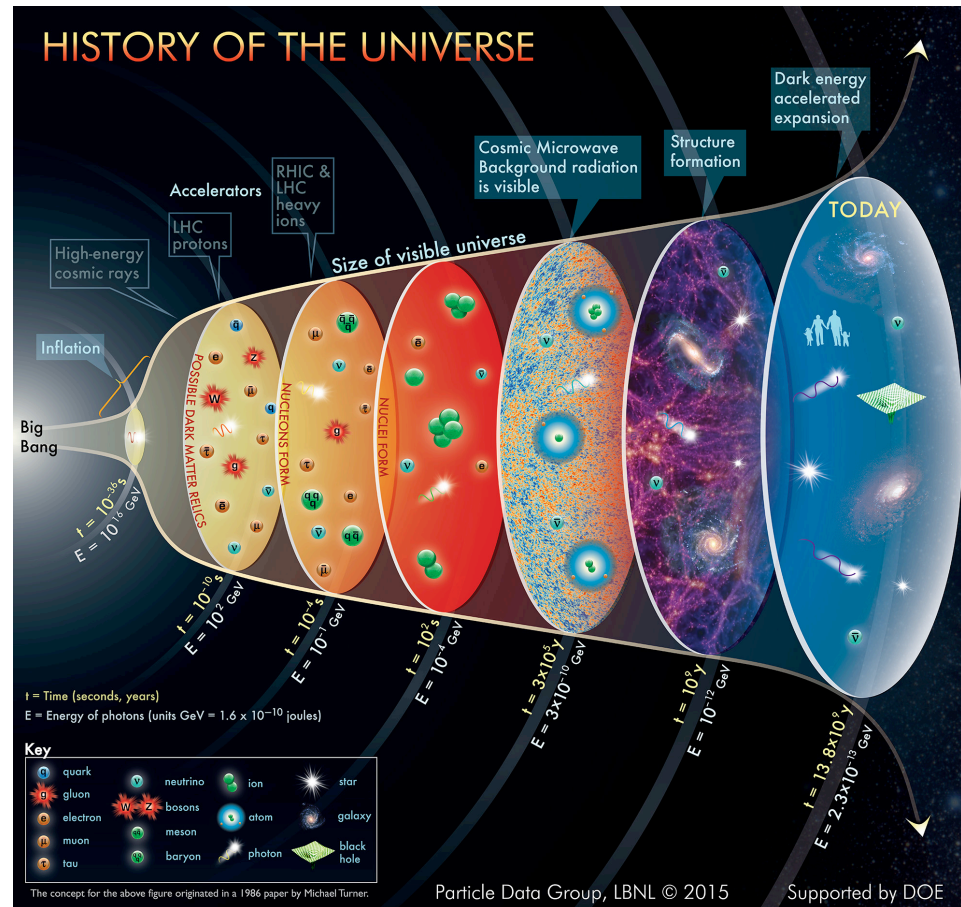
- Heavy-ion collisions(?)

- Low-pT pion enhancement

[Begun, Florkowski, Rybczynski, PRC '14, '15]



Early Universe



QCD epoch: $\sim 10 \text{ MeV} < T < \sim 100 \text{ GeV}$

$\sim 10^{-11} \text{ s} < t < 1 \text{ s}$

Cosmic trajectories

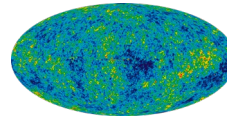
- conservation equations for isentropic expansion

$$\frac{n_B}{s} = b, \quad \frac{n_Q}{s} = 0, \quad \frac{n_{L_\alpha}}{s} = l_\alpha \quad (\alpha \in \{e, \mu, \tau\})$$

- trajectory is a line in 6-dim space of temperature and chemical potentials

$$T, \quad \mu_B, \quad \mu_Q, \quad \mu_{L_\alpha}$$

- empirical constraints (CMB anisotropies)



$$b = (8.60 \pm 0.06) \cdot 10^{-11}$$

[Planck collab., 1502.01589]

$$|l_e + l_\mu + l_\tau| < 0.012$$

[Oldengott, Schwarz, 1706.01705]

- equation of state (QCD epoch)

$$p \approx p_{\text{QCD}} + p_{\text{leptons}} + p_{\text{photons}}$$

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- empirical constraints (CMB anisotropies) Pion condensation may occur if $|\mu_Q| > m_\pi$ at $T < 160$ MeV

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Pion condensation may occur if $|\mu_Q| > m_\pi$ at $T < 160$ MeV

As pointed out in [M. Wygas et al., PRL '18]

Modeling the cosmic equation of state

$$p \approx p_{\text{QCD}} + p_{\text{leptons}} + p_{\text{photons}}$$

- leptons

$$p_{\text{leptons}}(T, \mu_Q, \mu_{L_\alpha}) = \sum_{\alpha \in \{e, \mu, \tau\}} [p_\alpha^{\text{id}}(T, \mu_Q, \mu_{L_\alpha}) + p_{\nu_\alpha}^{\text{id}}(T, \mu_{L_\alpha})] + \text{antiparticles}$$

- photons

$$p_\gamma(T) = \frac{\pi^2}{45} T^4$$

- QCD?

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- QCD?

The typical model of choice for hadronic matter is hadron resonance gas (HRG)

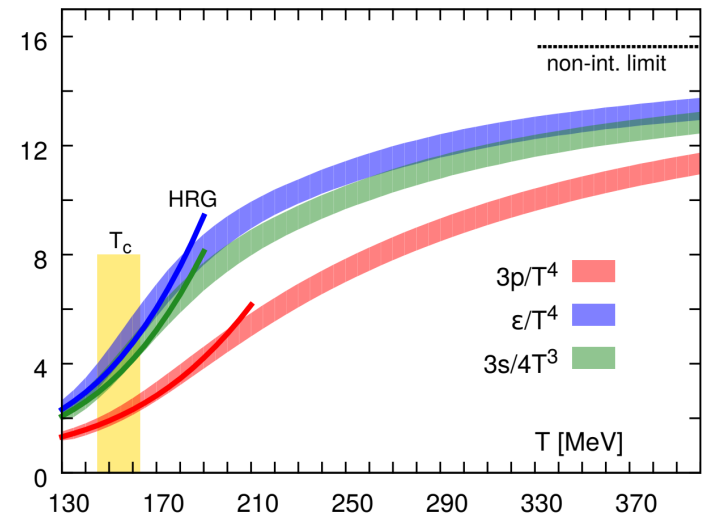


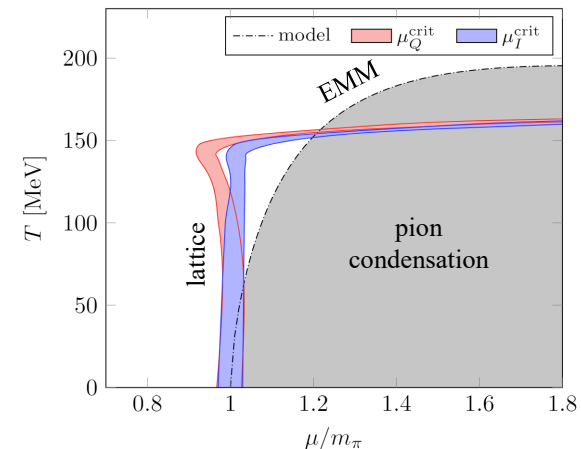
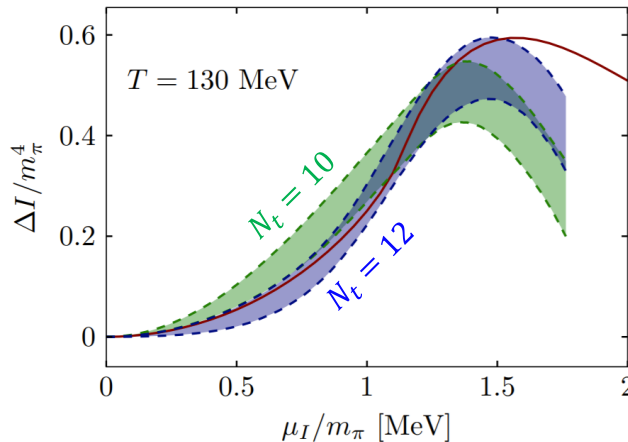
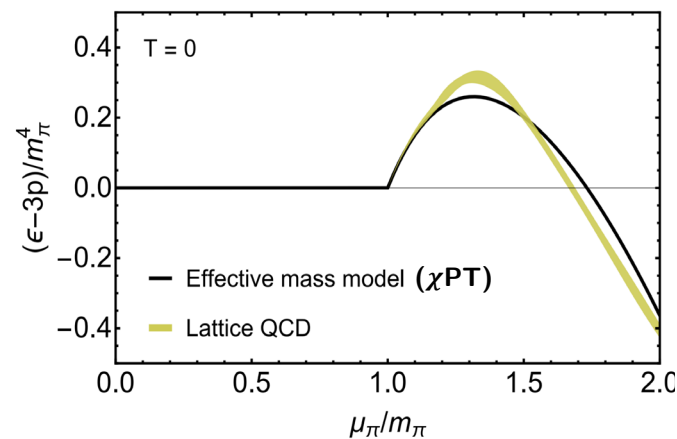
Figure from HotQCD coll., PRD '14

Strategy: Implement pion-pion interactions into the HRG model to account for the pion-condensed phase

HRG model with pion interactions

$$p_{\text{QCD}}(T, \mu_B, \mu_Q) \approx \sum_{\substack{i \in \pi^\pm, \pi^0 \\ \text{interacting pions}}} p_i^{\text{EMM}}(T, \mu_i) + \sum_j p_j^{\text{id}}(T, \mu_j) \quad \text{free hadrons and resonances}$$

- Pion interactions via effective mass $m_i^*(T, \mu_Q)$ matched to χ PT at $T = 0$
- Validate the model using lattice QCD data for $\Delta I = I(T, \mu_I) - I(T, 0)$, $I \equiv \varepsilon - 3p$

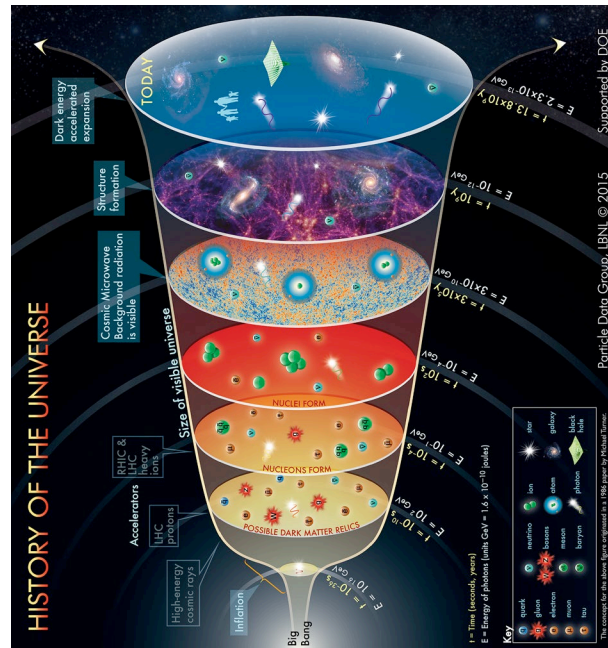


- Validity range of the model:

$$T \lesssim 160 \text{ MeV}, \quad \mu_I \lesssim 1.5 m_\pi$$

Calculating the cosmic trajectories

Early universe



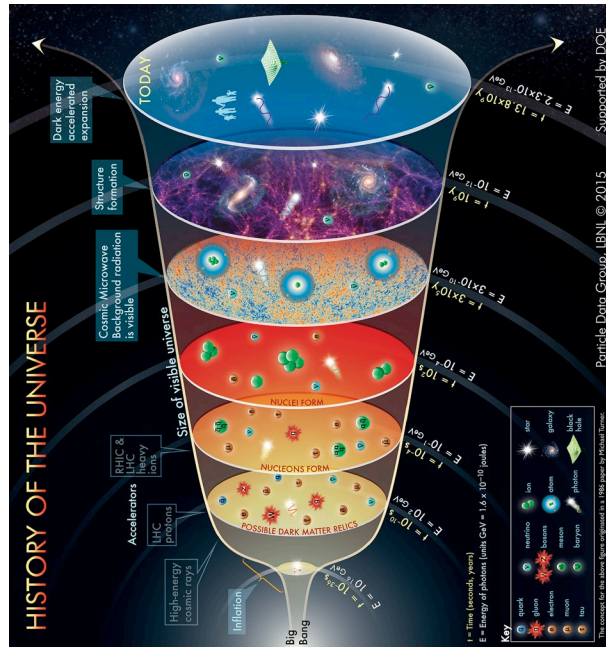
PDG

$$\frac{n_B}{s} = b, \quad \frac{n_Q}{s} = 0, \quad \frac{n_{L\alpha}}{s} = l_\alpha$$

$$p \approx p_{\text{QCD}} + p_{\text{leptons}} + p_{\text{photons}}$$

Calculating the cosmic trajectories

Early universe

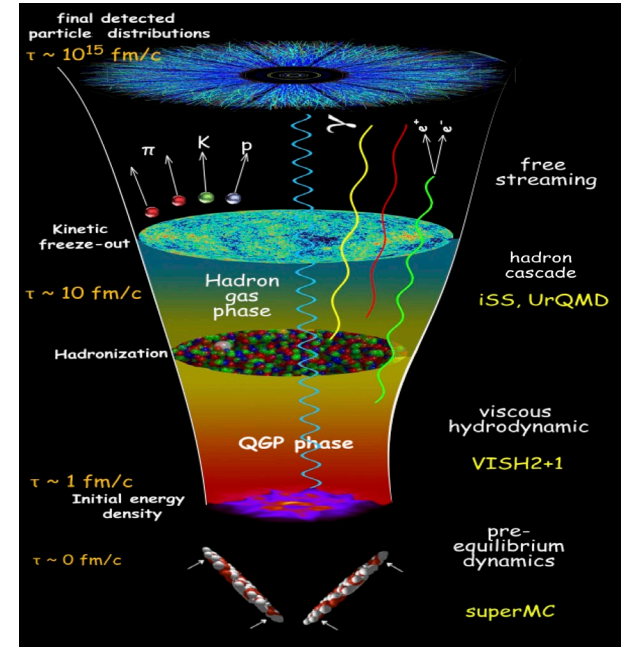


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Heavy-ion collision



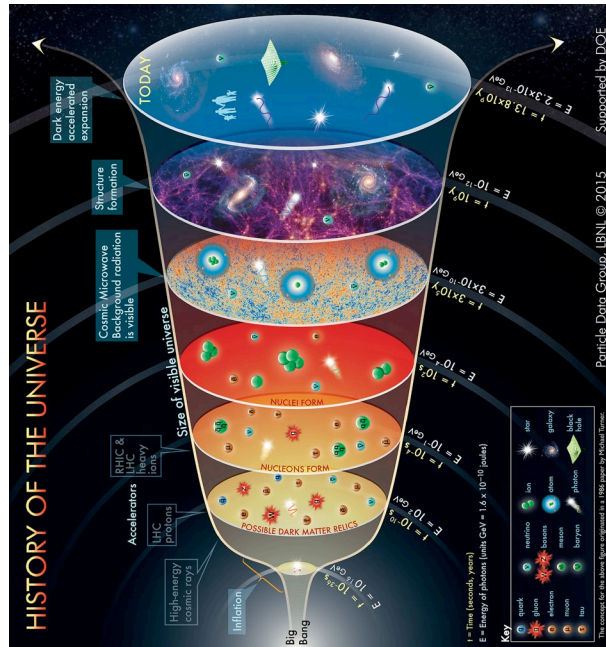
C. Shen, Ohio State U.

$$\frac{n_B}{s} = b, \quad \frac{n_Q}{s} = q, \quad \frac{n_S}{s} = 0$$

$$p = p_{\text{QCD}}$$

Calculating the cosmic trajectories

Early universe

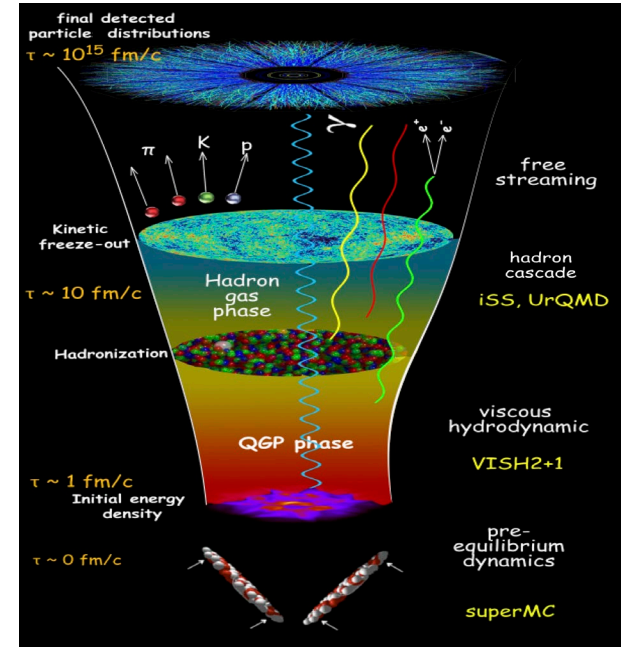


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Cosmic trajectories implemented within (extended) **Thermal-FIST** package

[V.V., H. Stoecker, *Computer Physics Communications* **244**, 295 (2019); on [github](#)]

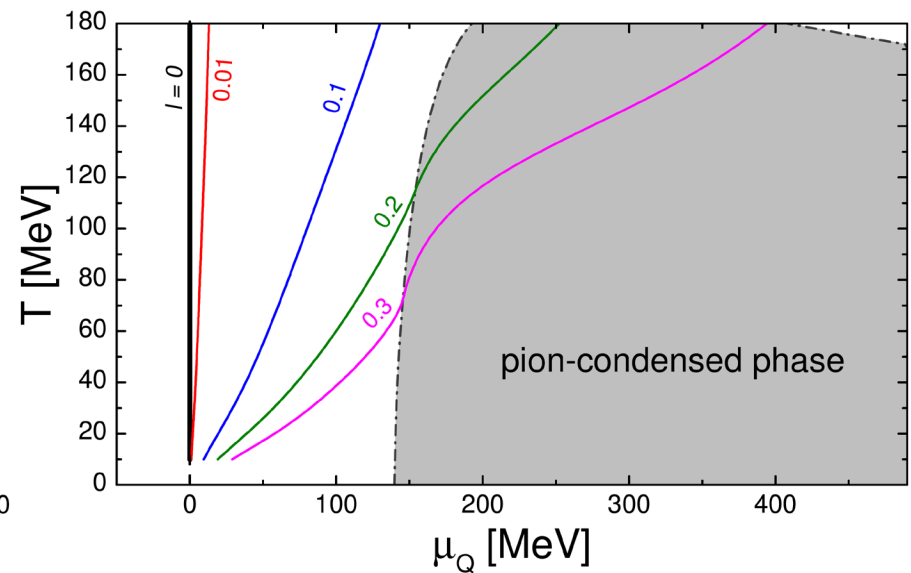
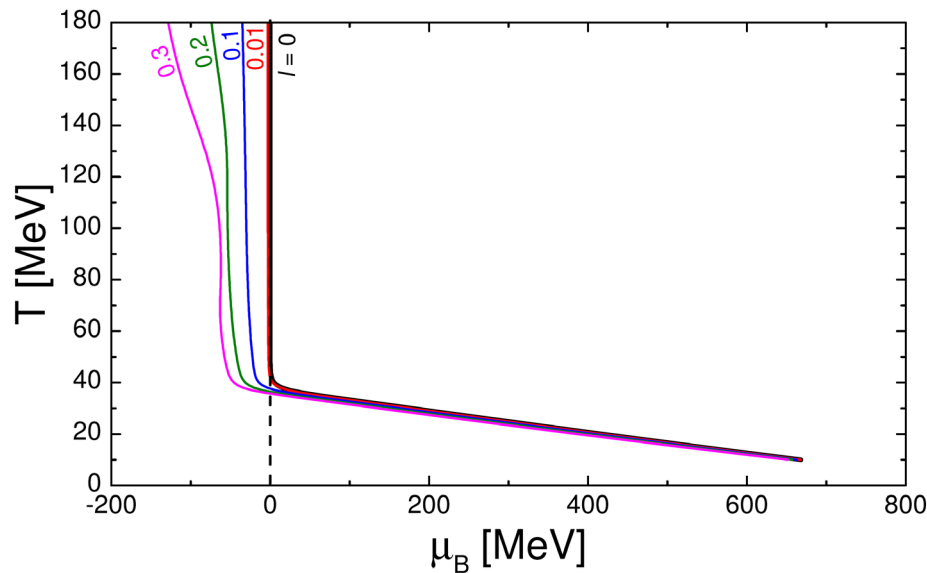


Using a *heavy-ion* tool in *cosmology*

Trajectories: Lepton-flavor symmetric case

Fix $b = 8.6 \cdot 10^{-11}$ and do a parametric scan in lepton asymmetries

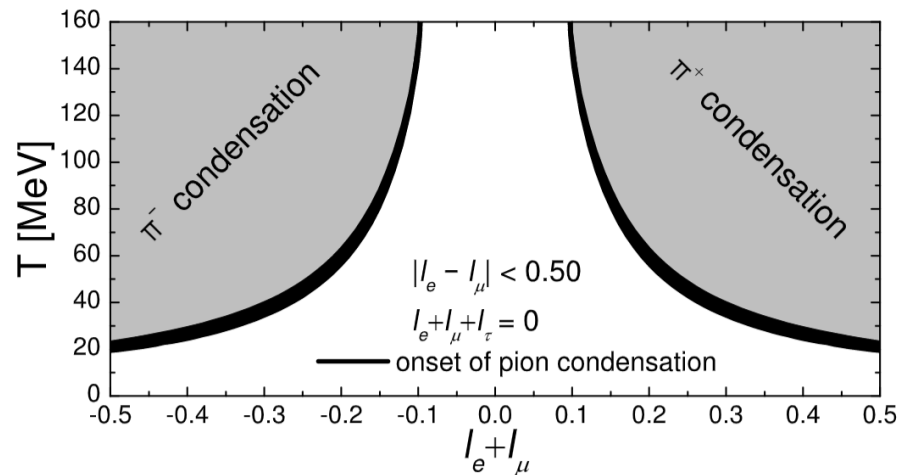
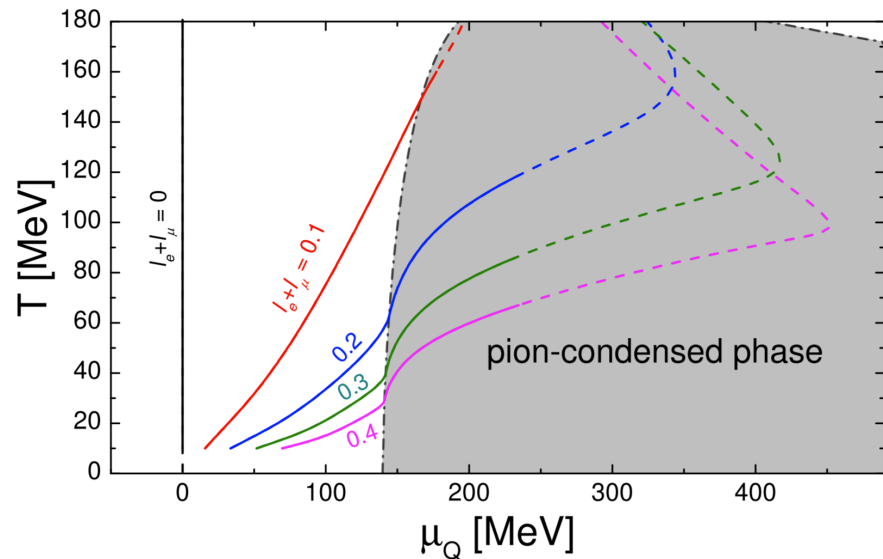
First consider $l_e = l_\mu = l_\tau = l/3$



- Pion condensation in the symmetric scenario occurs if $|l| > \sim 0.15$
- However, this violates the empirical constraint $|l| < 0.012$

Outside pion-condensed region reproduces HRG model results of [M. Wygas et al., PRL '18; 2009.00036]

Trajectories: Lepton-flavor asymmetric case

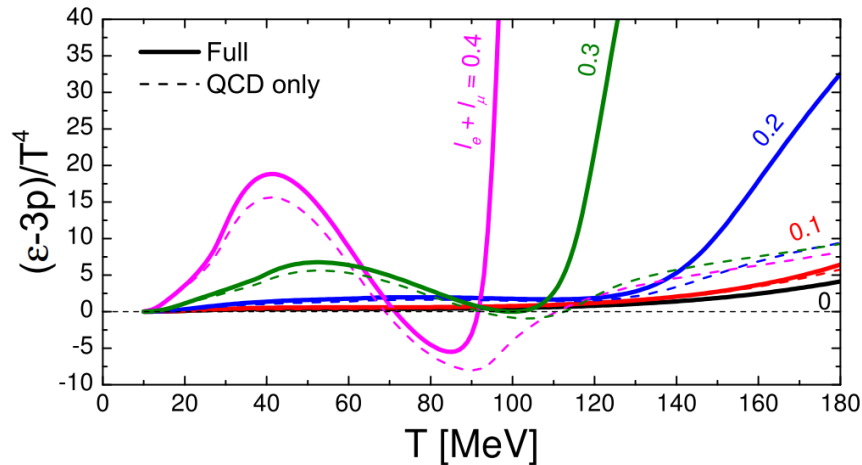


- Individual lepton flavor asymmetries are much less constrained
- Set total lepton asymmetry to zero but vary individual flavor ones
 $l_e + l_\mu + l_\tau = 0$ but $l_e \neq l_\mu \neq l_\tau$
- 2D scan in $(l_e + l_\mu, l_e - l_\mu)$

Pion condensation occurs if

$$|l_e + l_\mu| \gtrsim 0.1$$

Implications of pion condensation

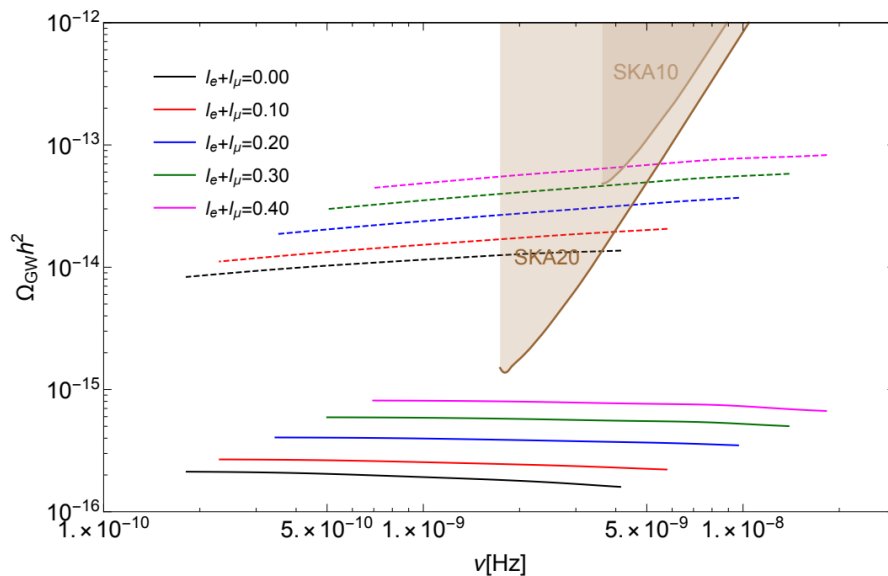


Strong effect on the pre-BBN equation of state

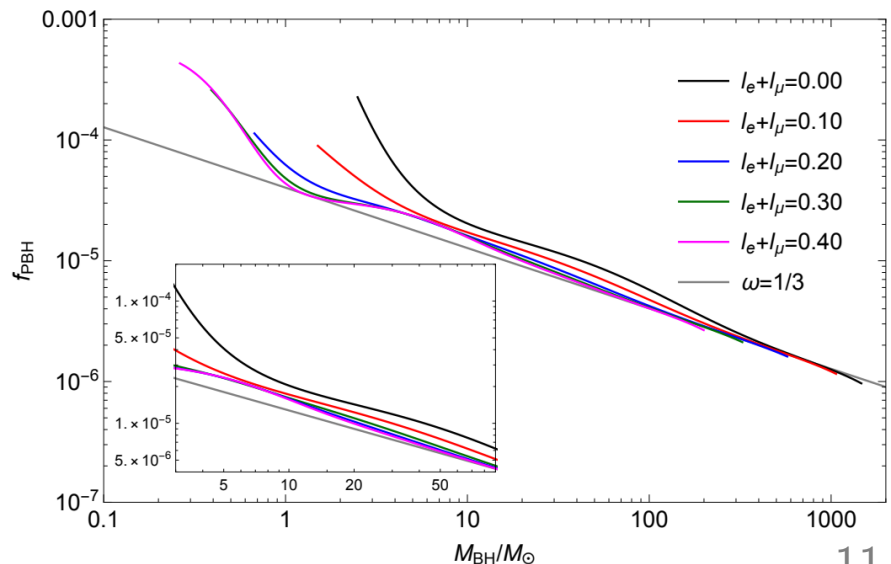
$$s(T)[a(T)]^3 = \text{const} \quad \text{scale factor}$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \epsilon \quad \text{Hubble rate}$$

Primordial gravitational waves



Primordial black holes



Production mechanism for pion stars

- Pion stars are gravitationally bound objects whose main constituent is the Bose-Einstein condensate of charged pions

[Carignano et al., 1610.06097; Brandt et al., 1802.06685; Andersen, Kneschke, 1807.08951]

- Pion condensation serves as a primordial production mechanism

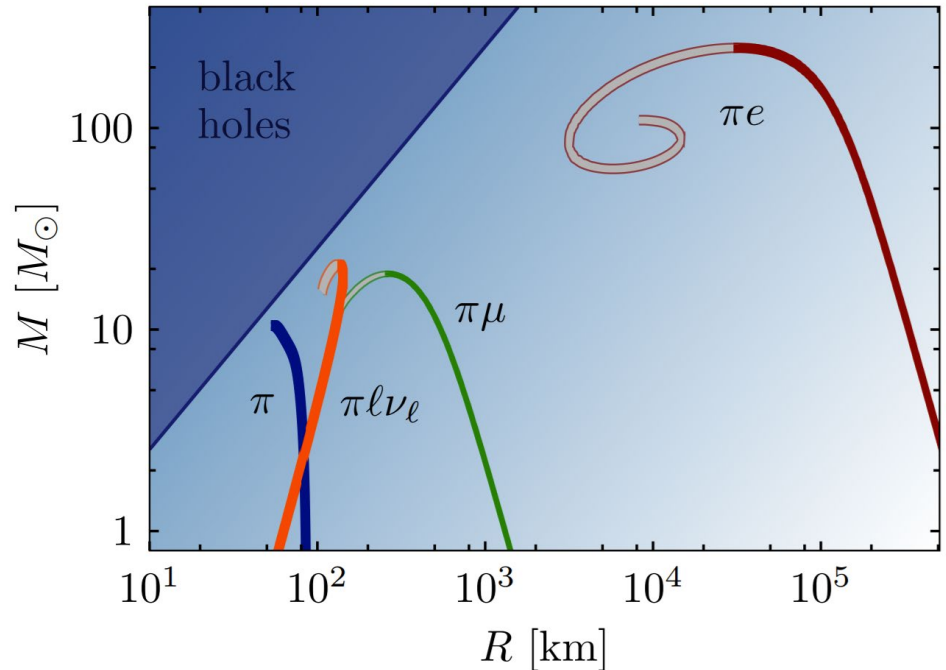


Figure from Brandt et al., 1802.06685

- Stabilized by neutrinos
- If pion stars decay around the time of big bang nucleosynthesis, the produced high energy leptons can influence the primordially produced nuclei

Summary

- The early universe passes through a pion-condensed phase if electron and muon lepton asymmetry is sufficiently large:

$$|l_e + l_\mu| > 0.1$$

- **Implications:**

- Large effect on the pre-BBN equation of state
- Enhanced relic density of primordial gravitational waves (relative to amplitude at $l_e + l_\mu = 0$)
- Pion condensation as a source of primordial black holes heavier than M_\odot
- Possible formation and decay of pion stars, effect on big bang nucleosynthesis

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Thank you!