TMD's at High Density: Quantum Evolution

Part 2 of 2

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with Daniel Pitonyak

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BROOKHAVEN NATIONAL LABORATORY

OHIO STATE

1511.06737
1505.01176

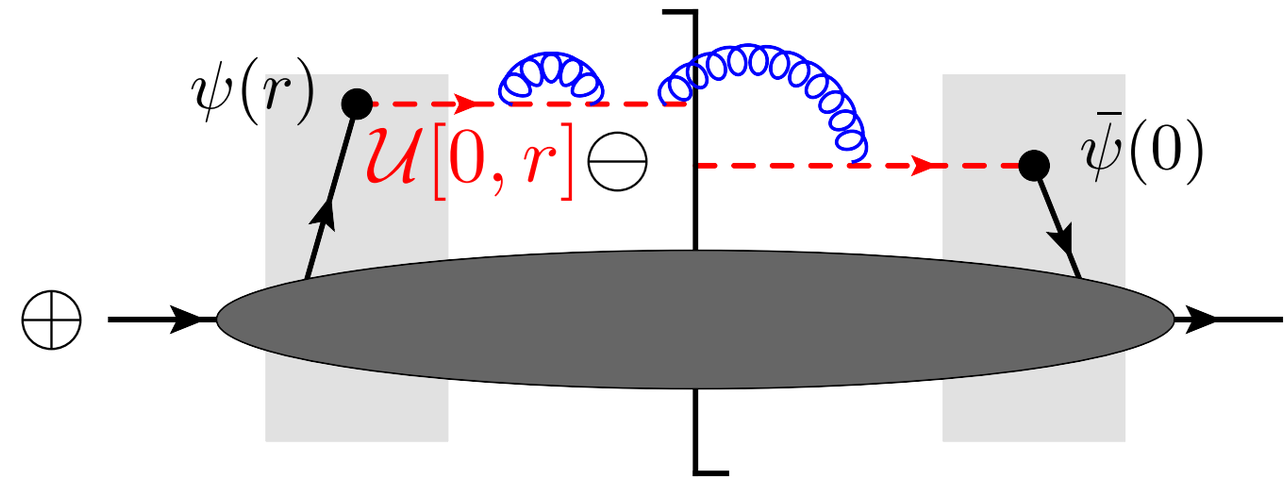
EIC Users' Group Meeting

Friday Jan. 8, 2016

Overview

Large-x Evolution (CSS)

$$\alpha_s \ln^2 \frac{Q^2}{\mu^2} \sim 1$$



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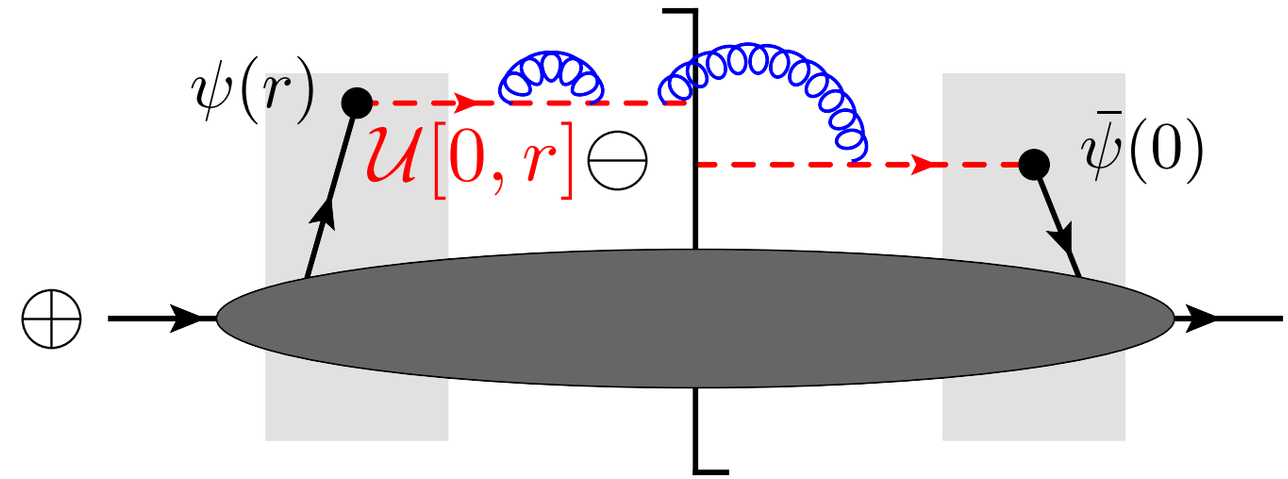

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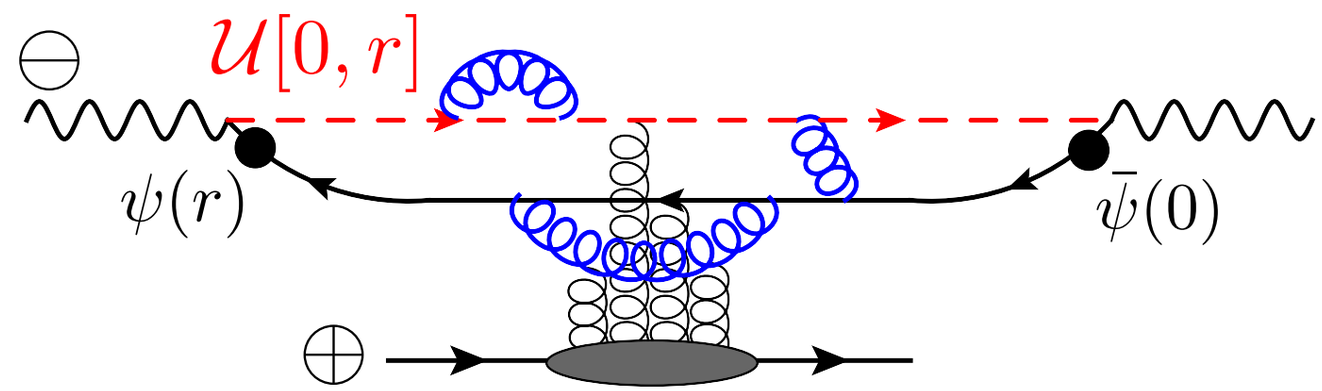

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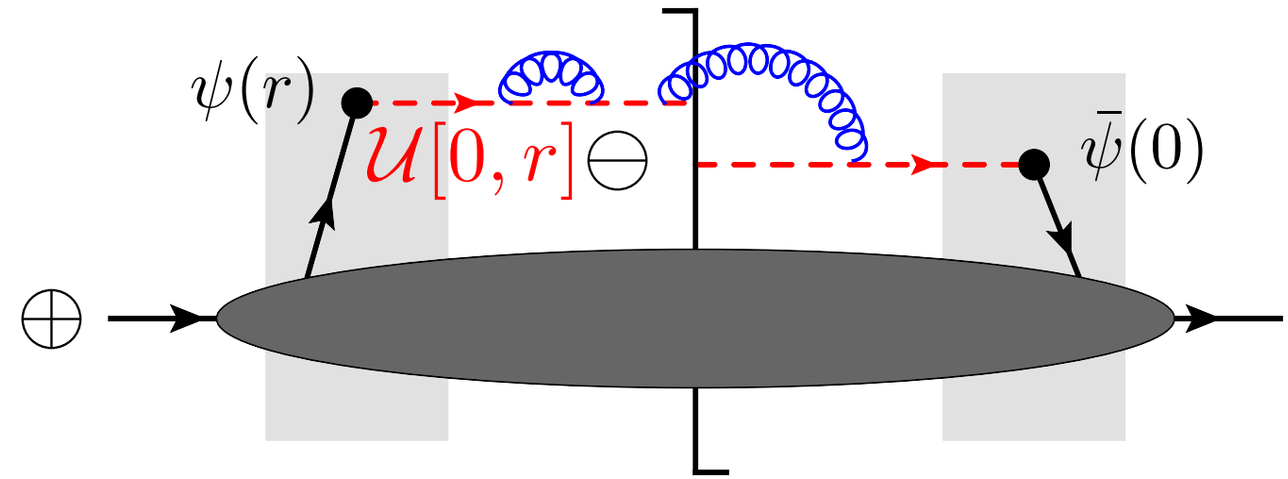

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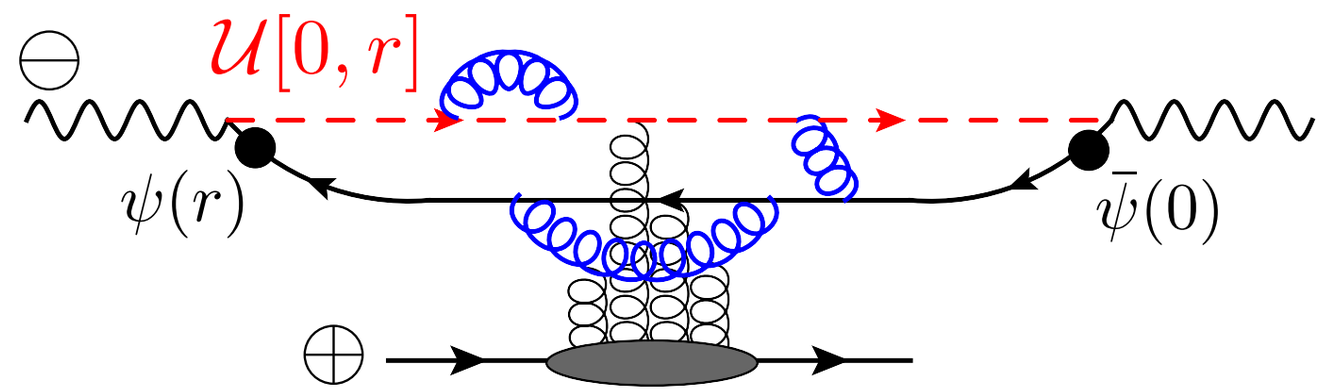

Small-x Helicity Evolution (BER)

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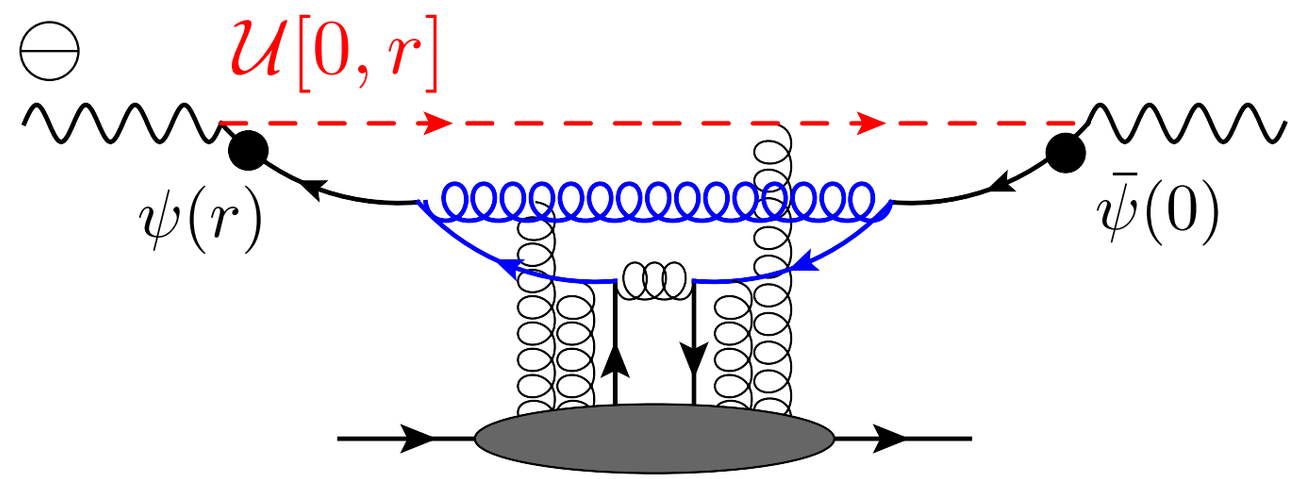
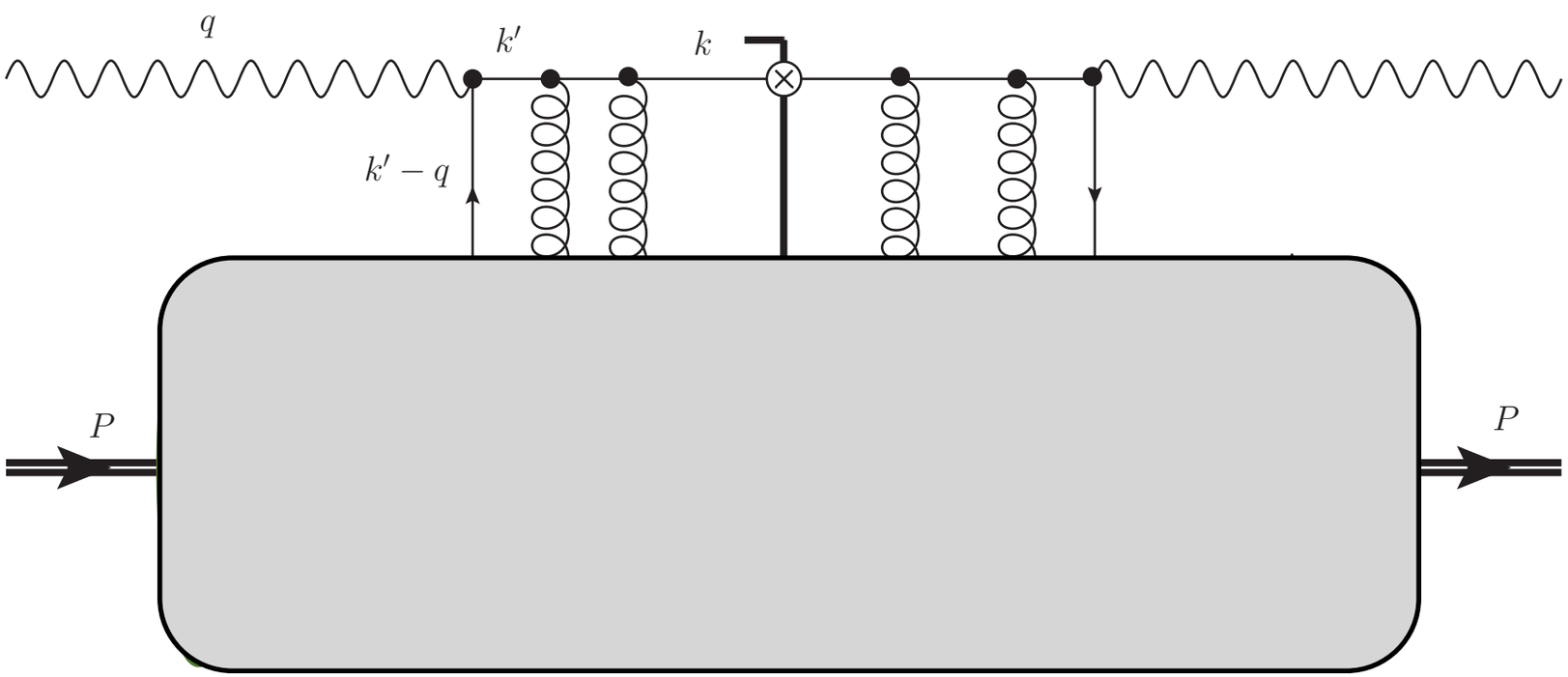


Initial Conditions: Classical Gluon Fields



$$\phi_{\alpha\beta}(x, \vec{k}_\perp) = \int \frac{d^{2-}r}{(2\pi)^3}\, e^{ik\cdot r} \langle h(p,S)|\, \underline{\bar{\psi}_\beta(0)}\, \mathcal{U}[0,r]\, \underline{\psi_\alpha(r)}\, |h(p,S)\rangle$$

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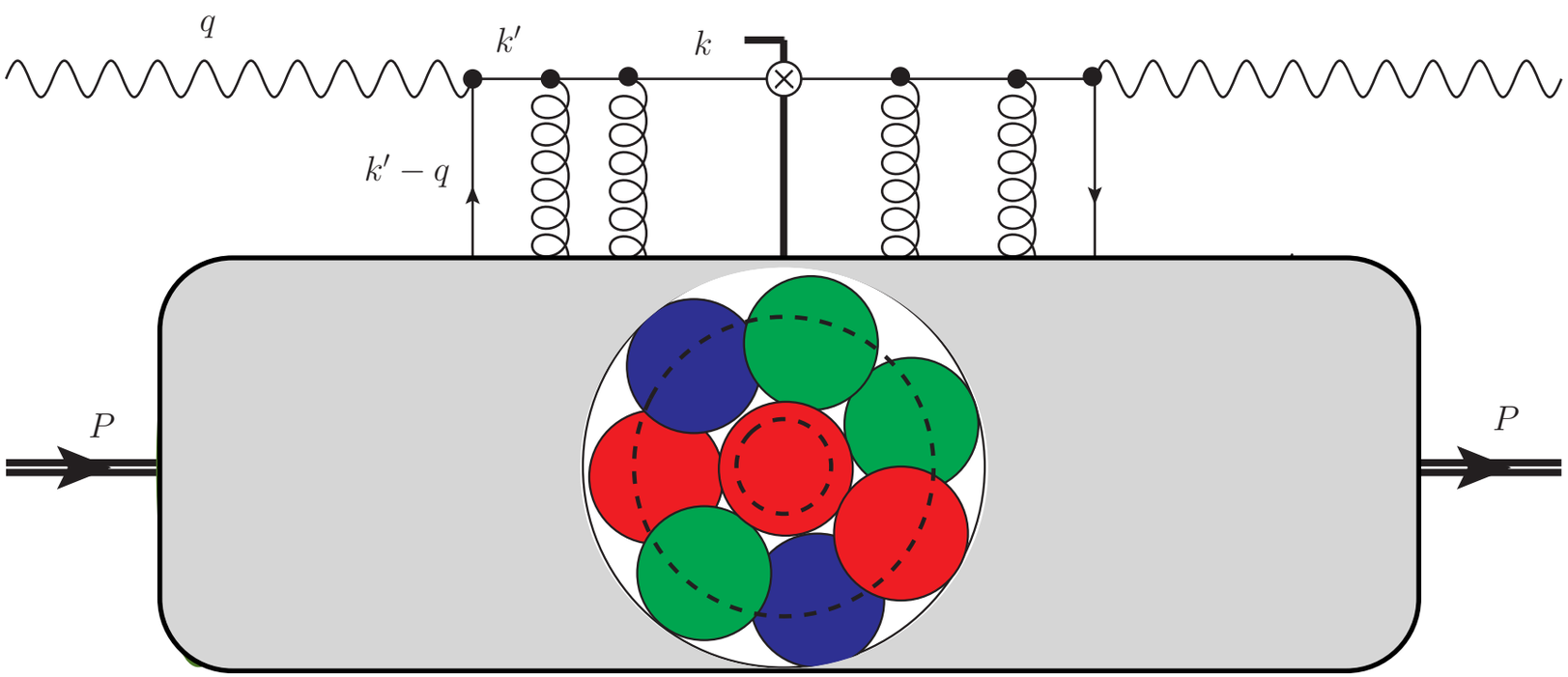

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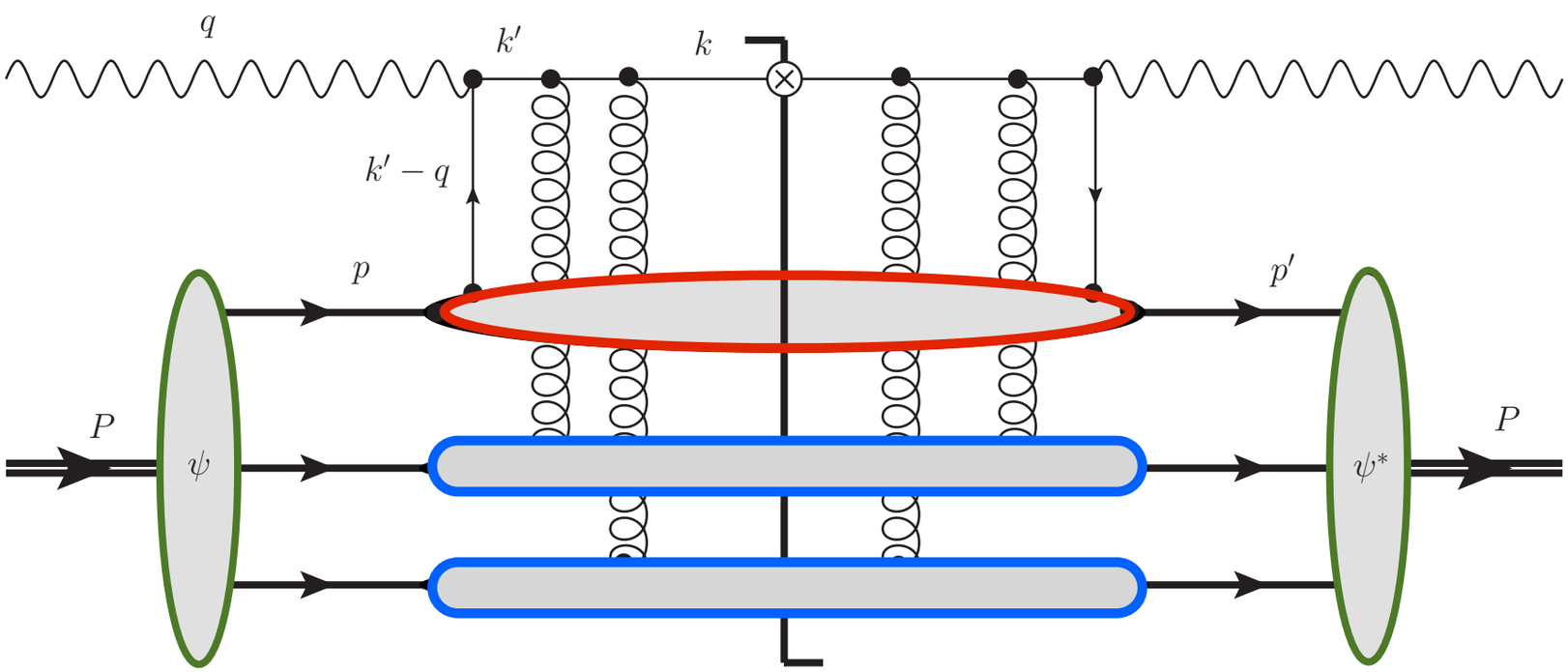

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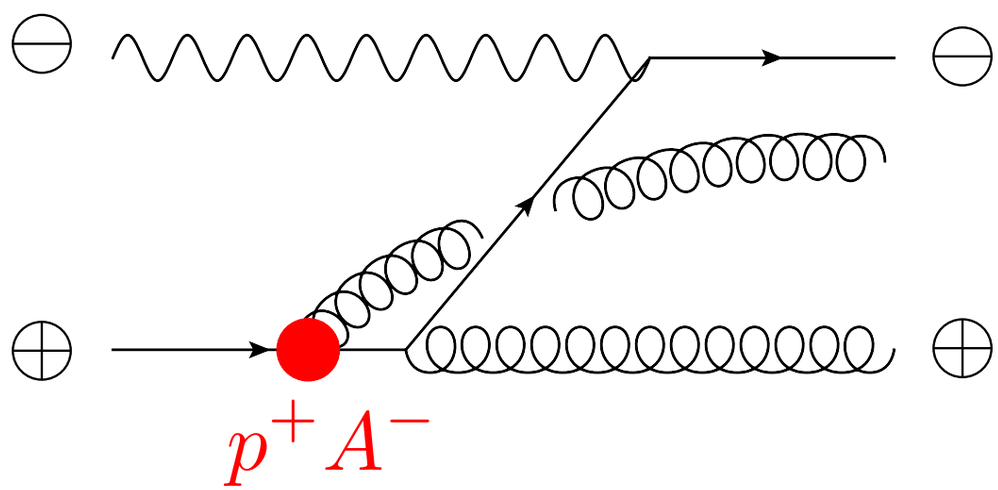
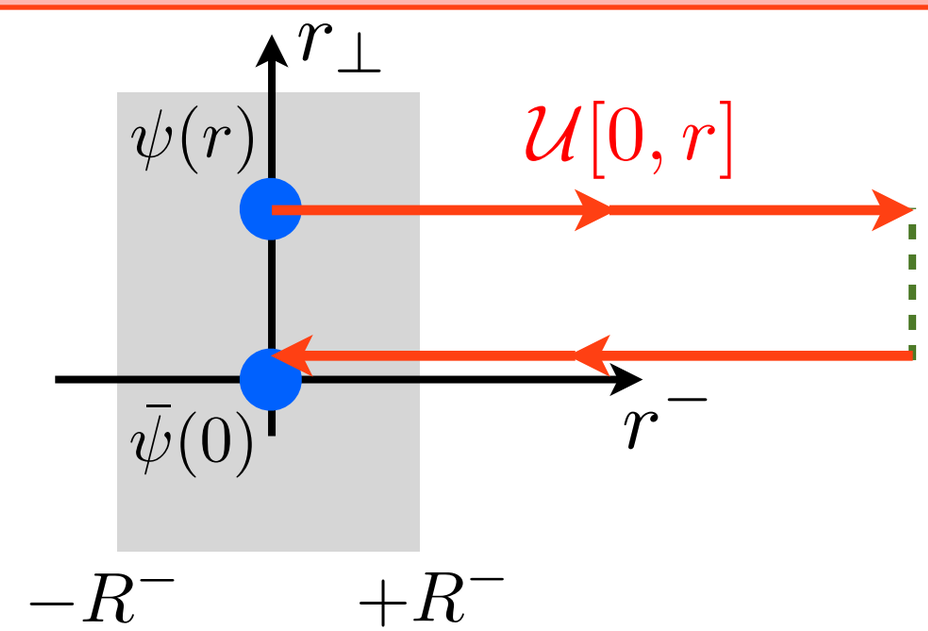
- A quark TMD is a matrix element of quark fields, dressed by a staple-shaped gauge link.
- At high density, the IR gauge fields are screened by rescattering.
- Leads to a "quasi-classical factorization" in which the gauge link can be calculated perturbatively (classical Yang-Mills fields)

Quantum Evolution in the Light-Cone Gauge





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DGLAP: $\alpha_s \ln \frac{Q^2}{\mu^2} \sim 1$

BFKL: $\alpha_s \ln \frac{s}{\mu^2} \sim 1$

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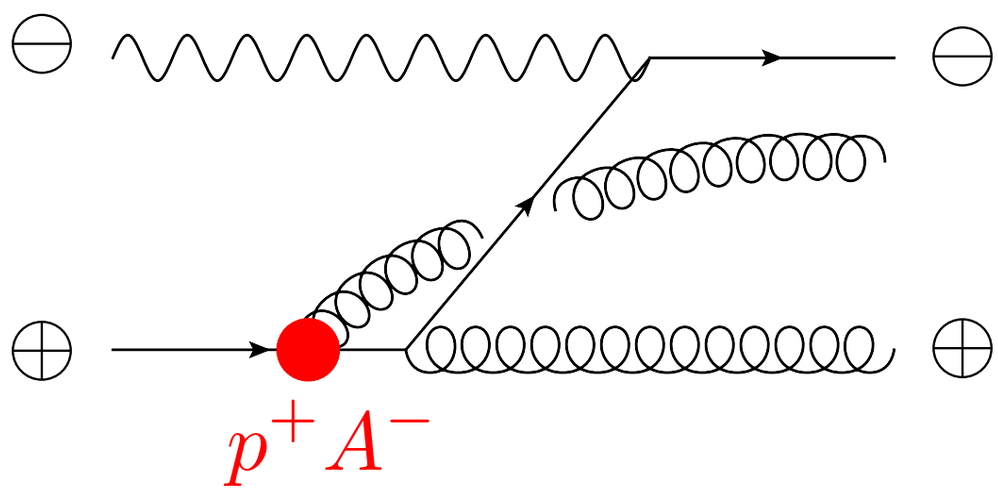

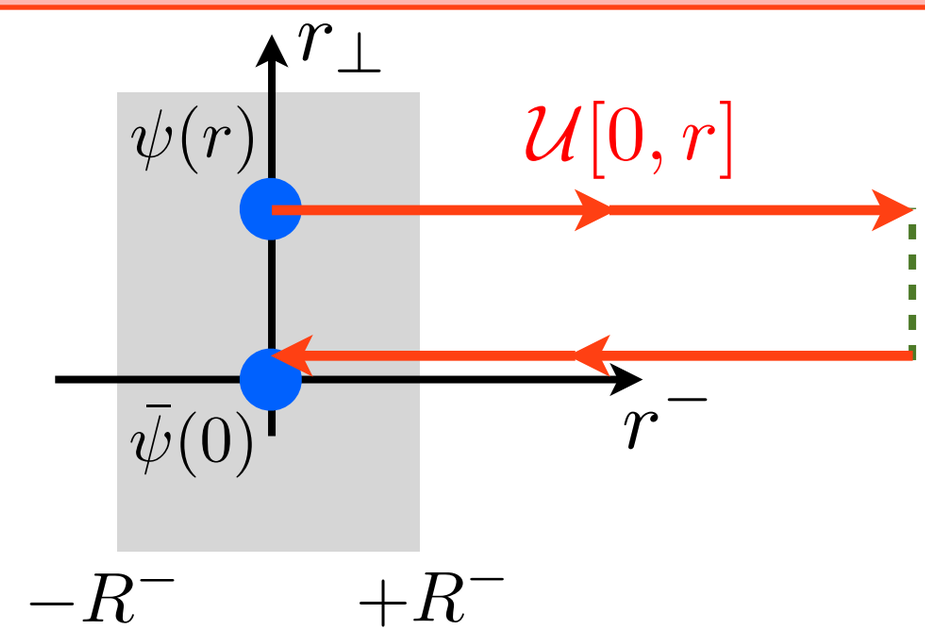

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  - Suppress eikonal radiation from $\oplus$ moving target if $A^- = 0$.
  - Evolution takes place completely within the $\ominus$ moving projectile.

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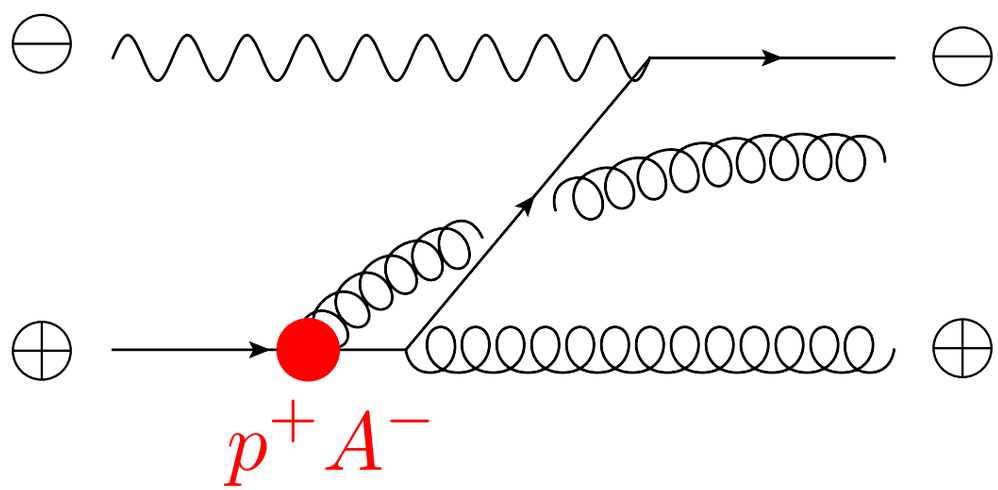

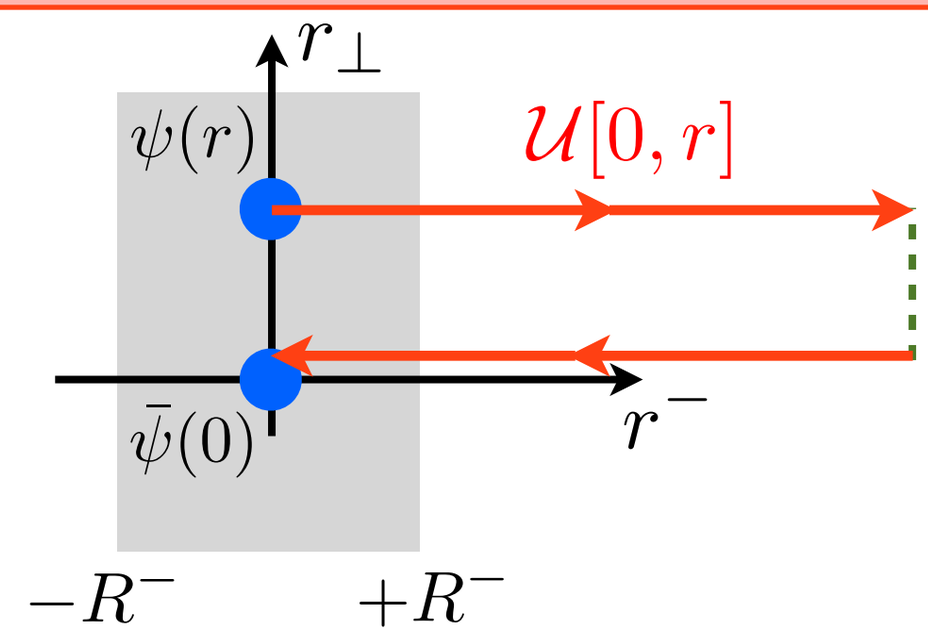

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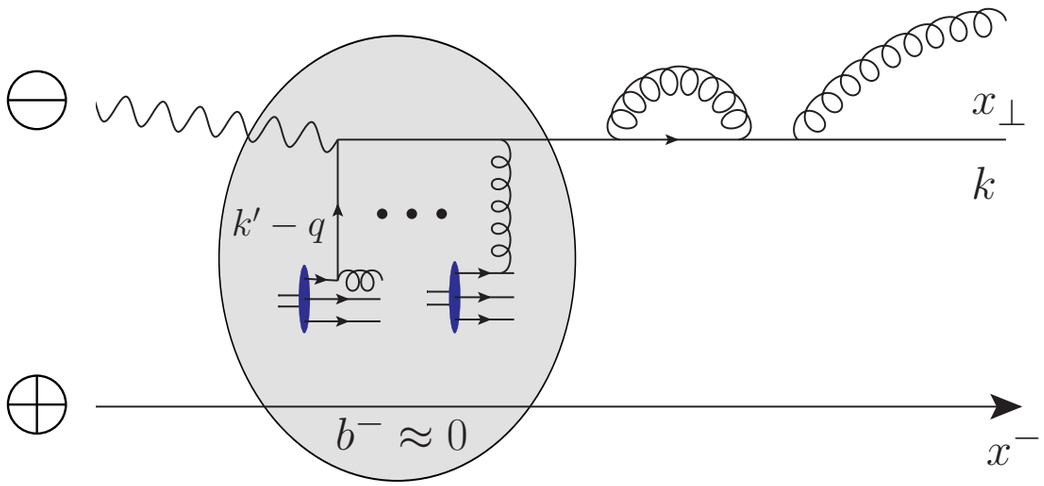
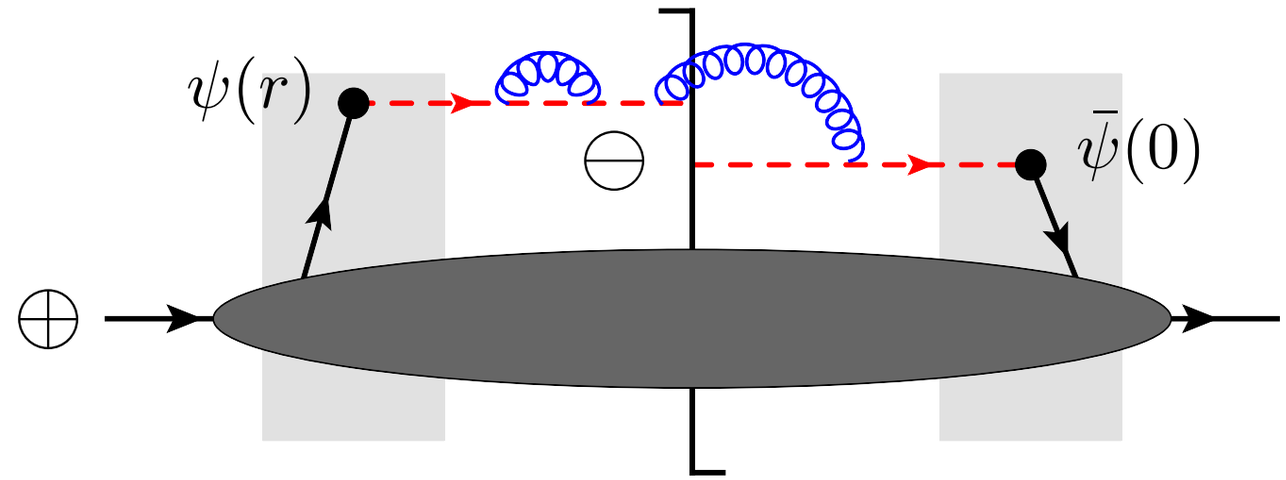
- Transverse part of the TMD gauge link vanishes in this gauge.

Evolution in Q^2 at Large x





$$\langle h(p,S)|\, \underline{\bar{\psi}_\beta(0)}\, \mathcal{U}[0,r]\, \underline{\psi_\alpha(r)}\, |h(p,S)\rangle$$

- At large x, SIDIS proceeds by quark "knockout"
  - TMD extracts a quark collinear to the target
  - Does not radiate in the $A^- = 0$ light-cone gauge.

$$s \sim Q^2 \gg k_T^2$$
$$x = \frac{Q^2}{s+Q^2} \sim \mathcal{O}(1)$$

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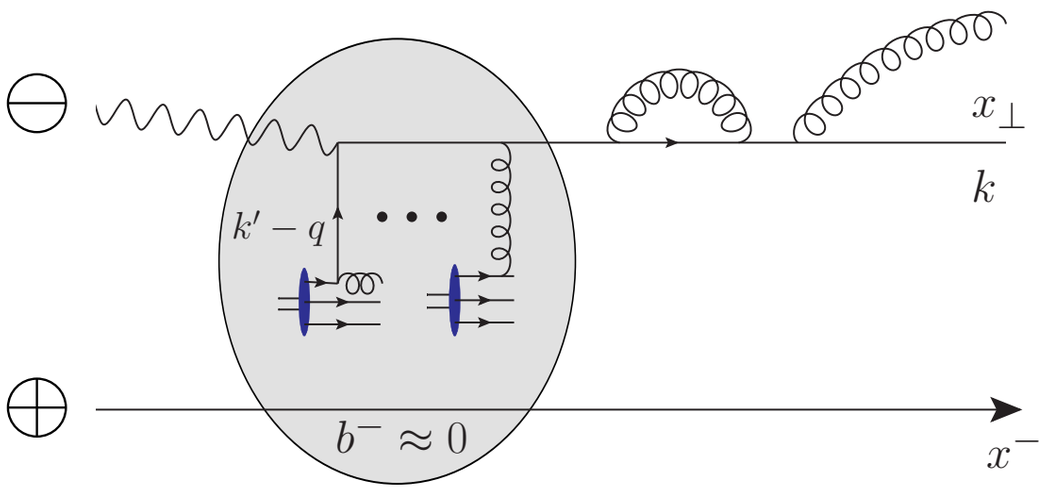

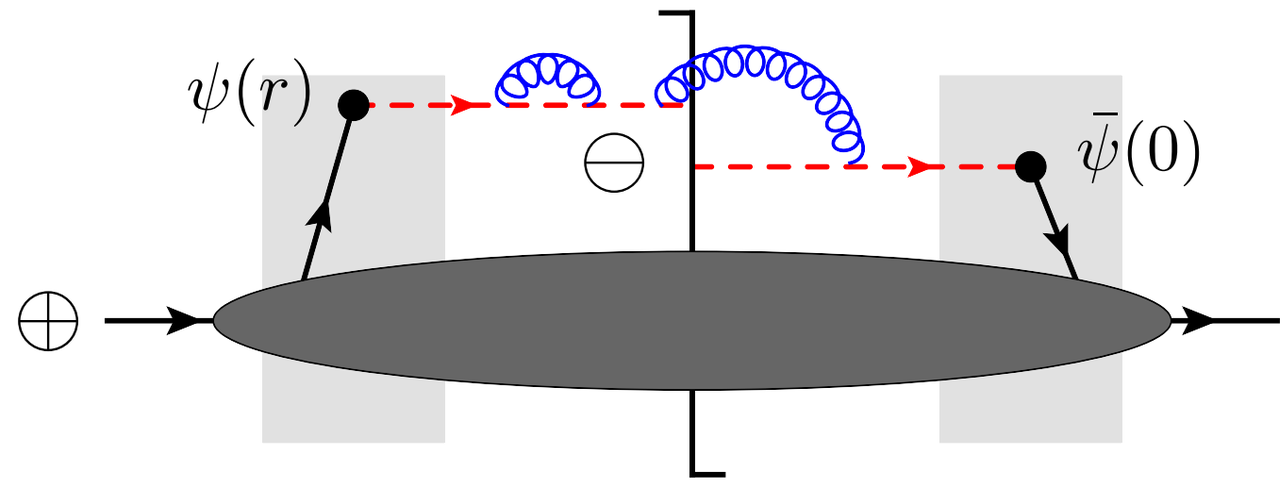

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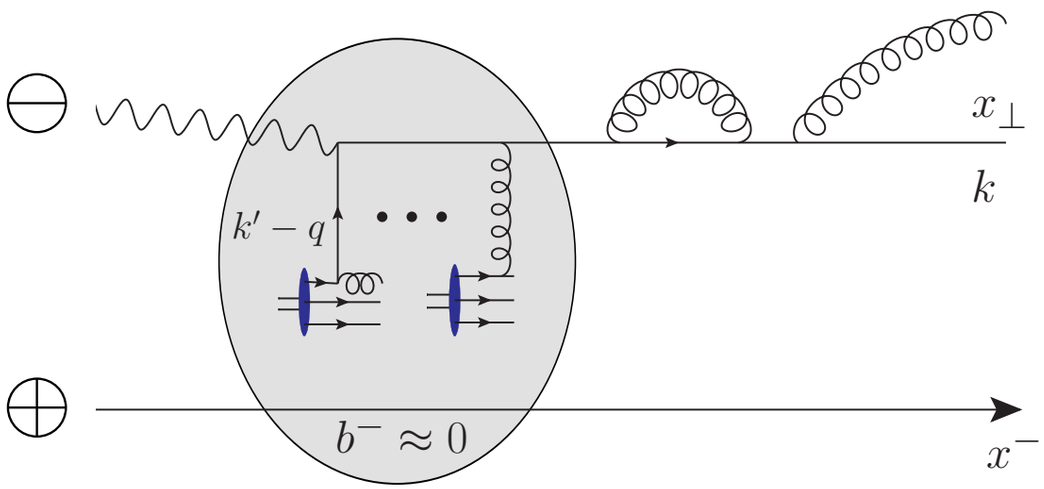

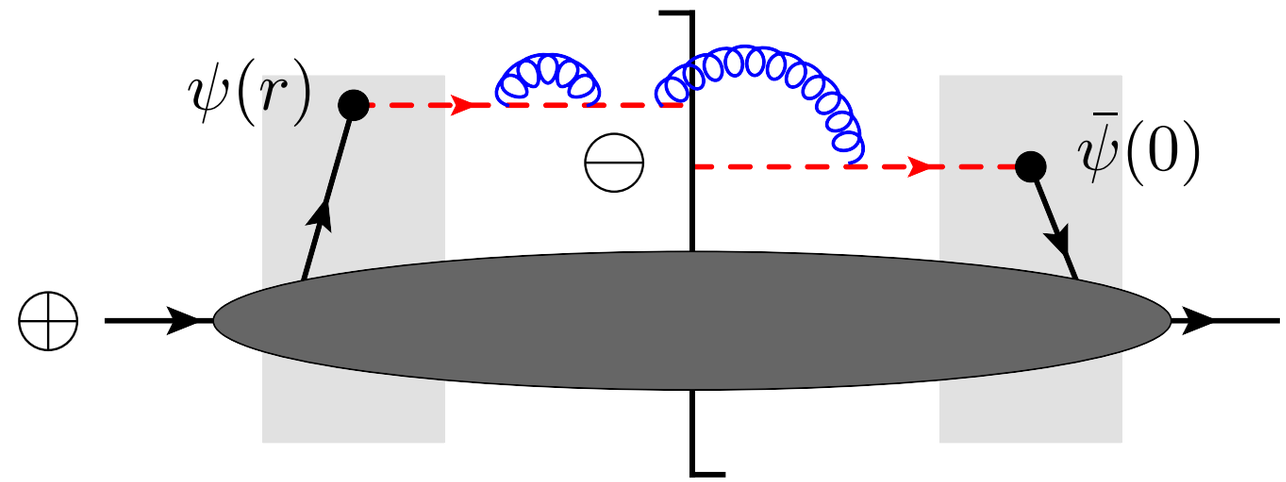

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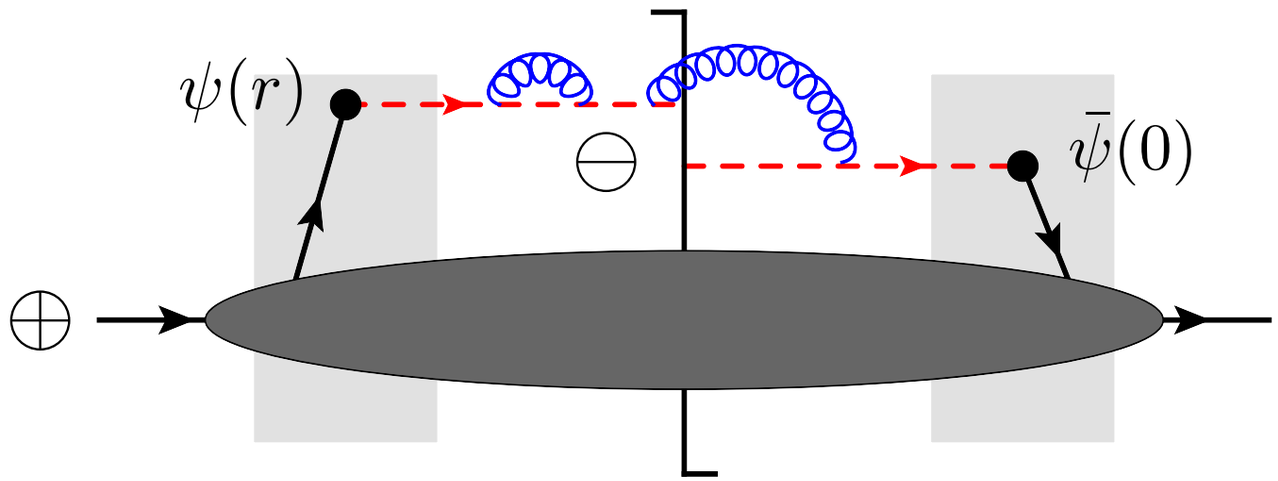
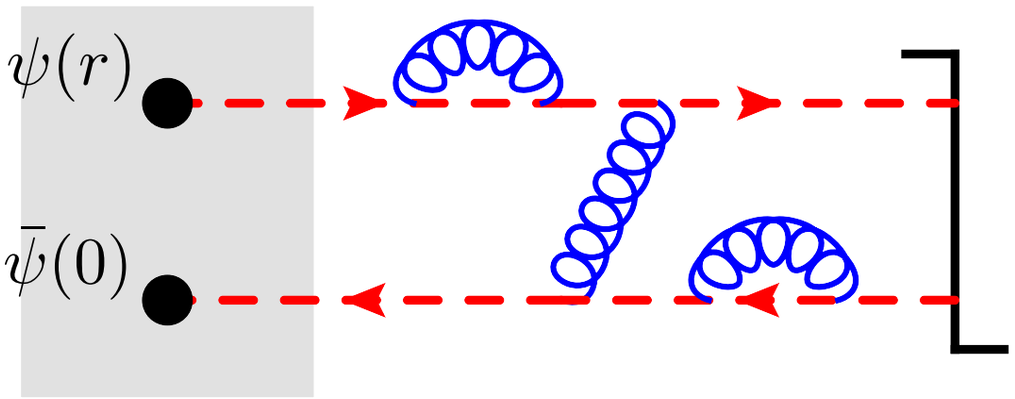
$$\alpha_s \ln^2 \frac{Q^2}{\mu^2}$$

- In LC gauge, TMD evolution occurs exclusively within the gauge link.

Large x: CSS Evolution

Gauge Link → Dipole Scattering Amplitude





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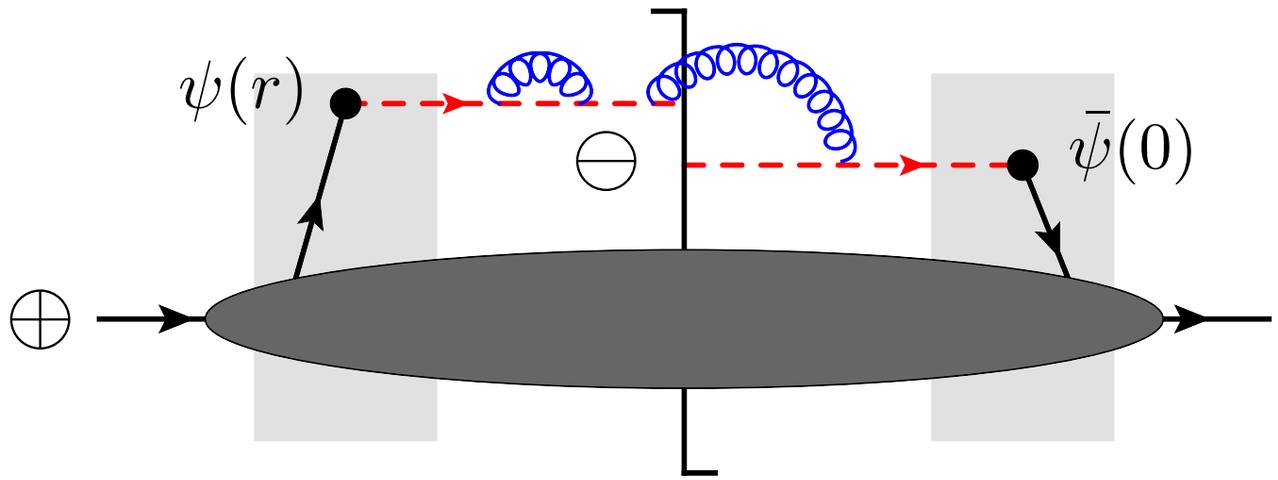

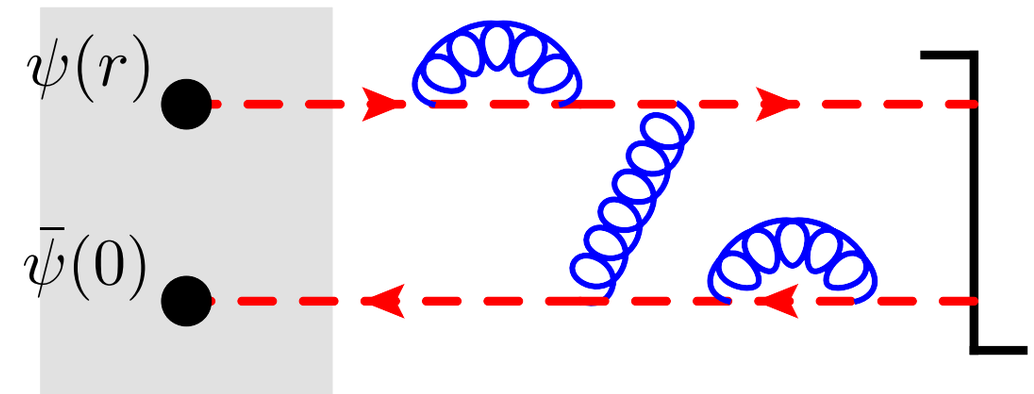

- Dipole evolution well known:
  - BFKL "virtual" emissions.

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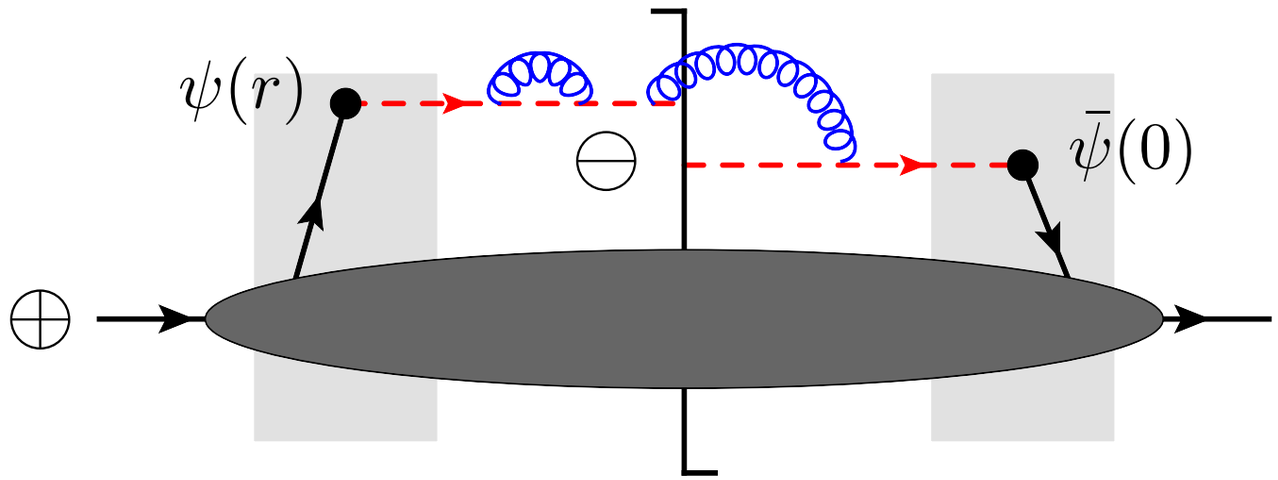

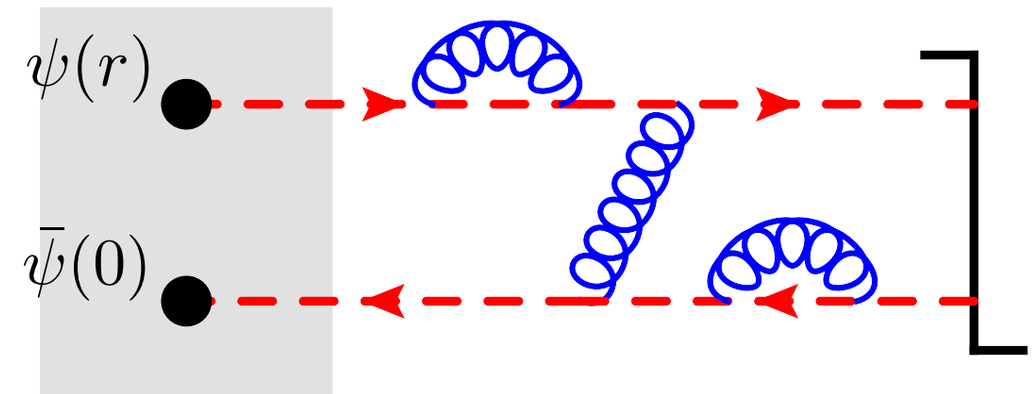

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- Leads to Sudakov double-logarithms (CSS evolution)

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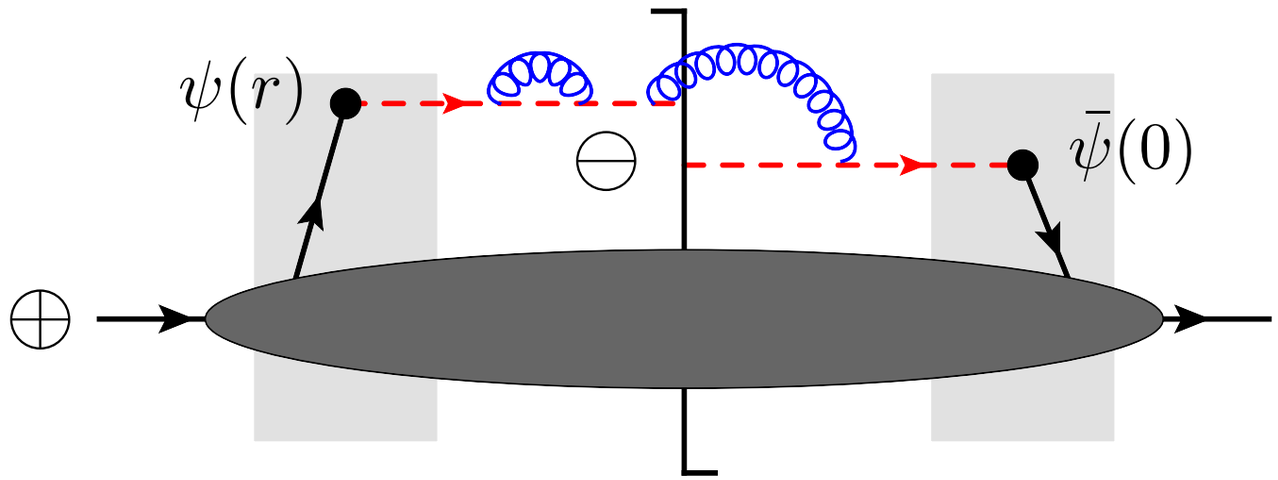

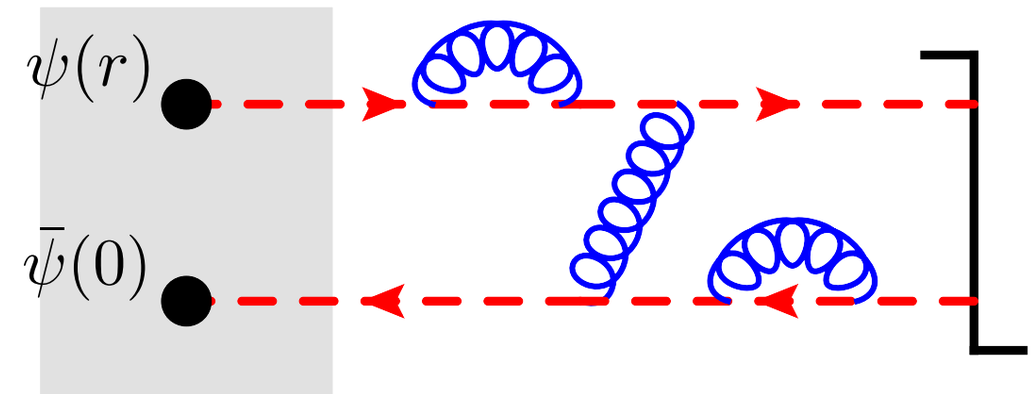

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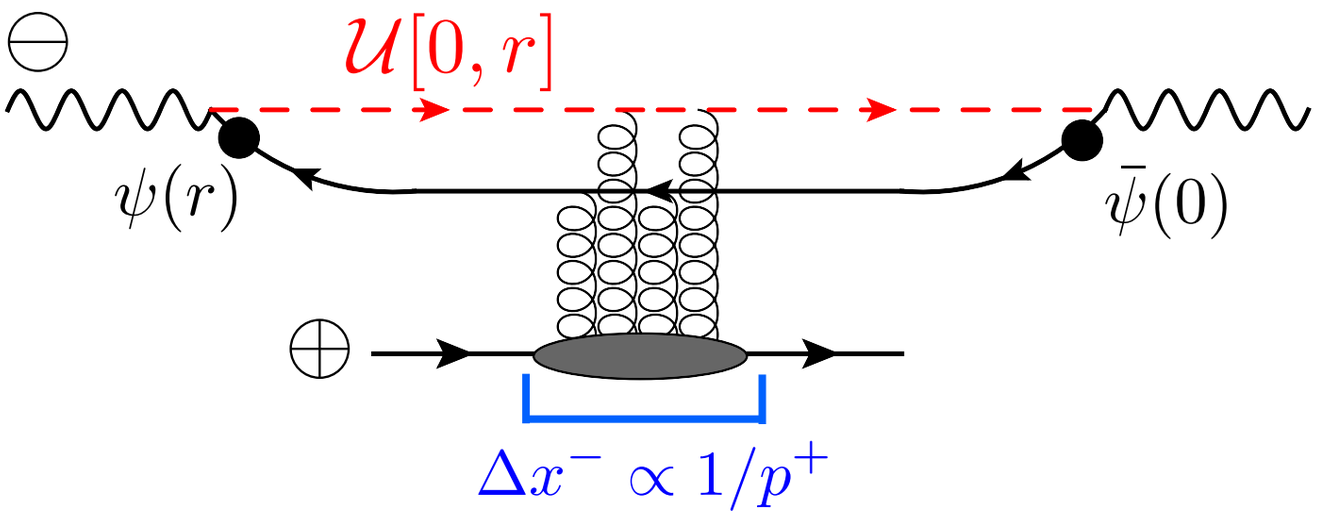
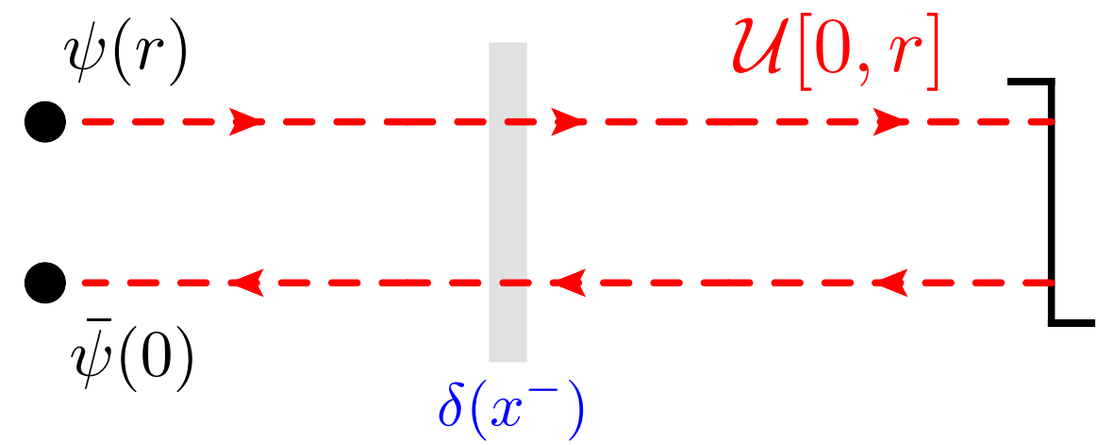
- Nonperturbative IR physics stays regulated by high density!

$$\Phi^A(Q^2) \sim \underline{\phi^N(Q_0^2)} \otimes S^{[\infty^-,b^-]}_{(r_T,b_T)}(Q^2)$$

TMD's at Small x





- At small x, the target is contracted to a "shockwave."
  - Dominated by dipole splitting
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$$x = \frac{Q^2}{s} \ll 1$$

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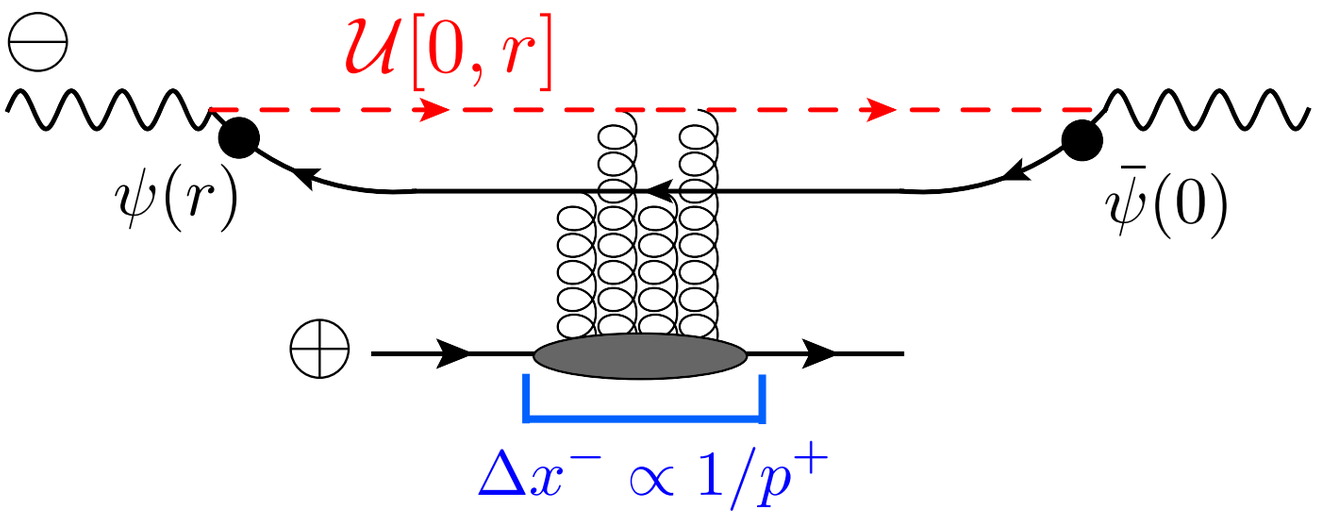

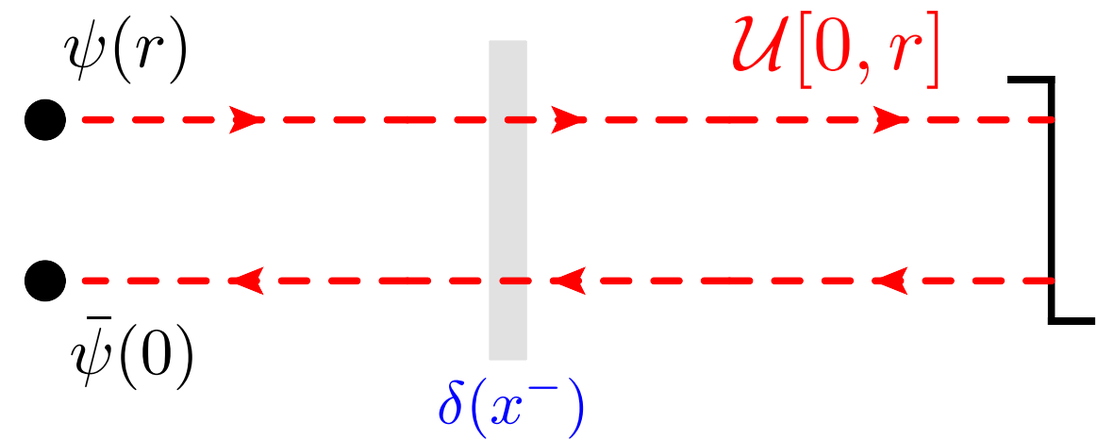

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- TMD operator creates an antiquark rather than annihilates a quark.

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Quark transport is x-suppressed!

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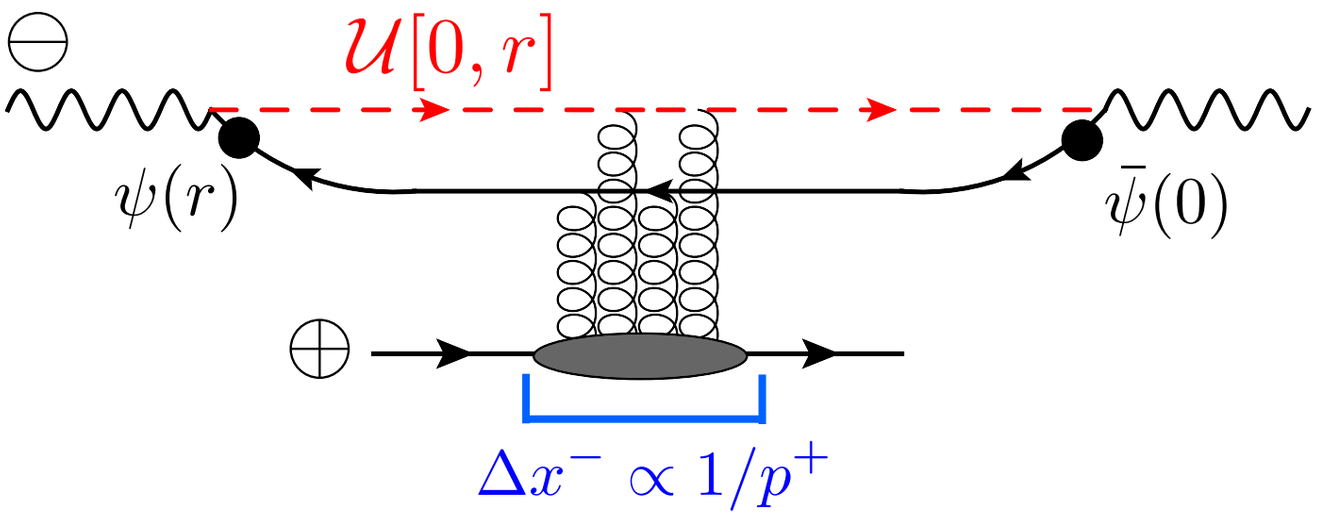

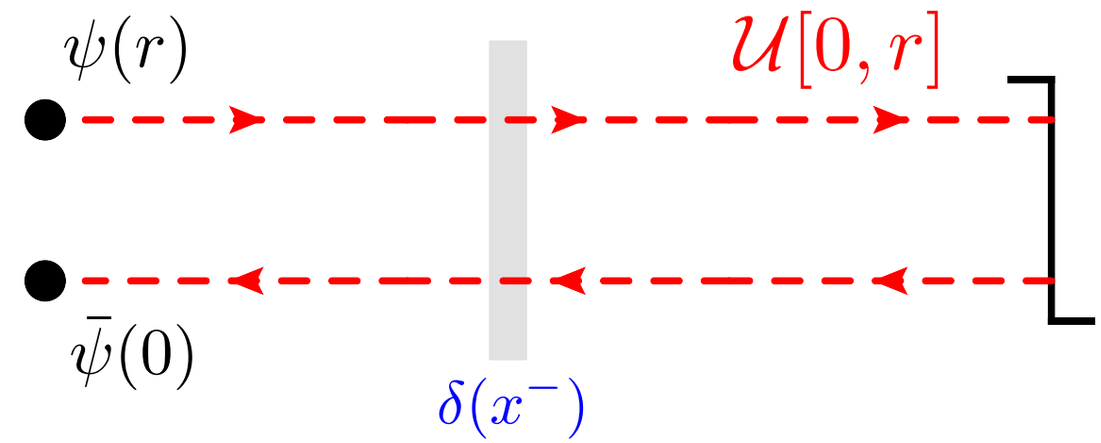

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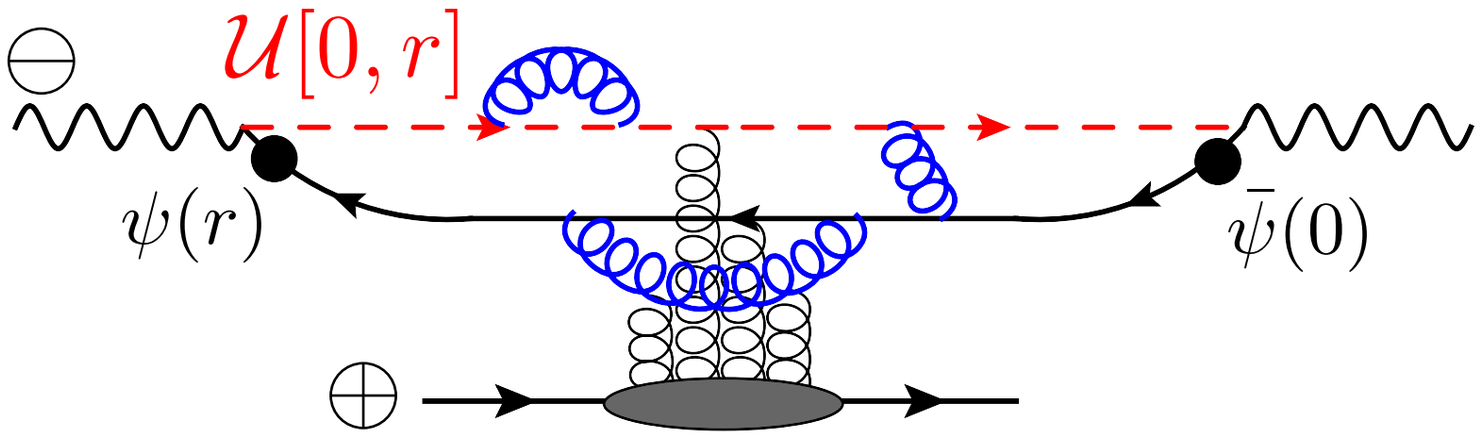
- TMD operator creates an antiquark rather than annihilates a quark.

Quark transport is x-suppressed!

- Gauge link is now fully infinite over entire target.
  - Infinite dipoles from quark / antiquark Wilson lines.

$$S_{xy} = \frac{1}{N_c}\mathrm{Tr}\left[V_x V_y^\dagger\right]$$

Unpolarized Quarks: BK / BFKL Evolution



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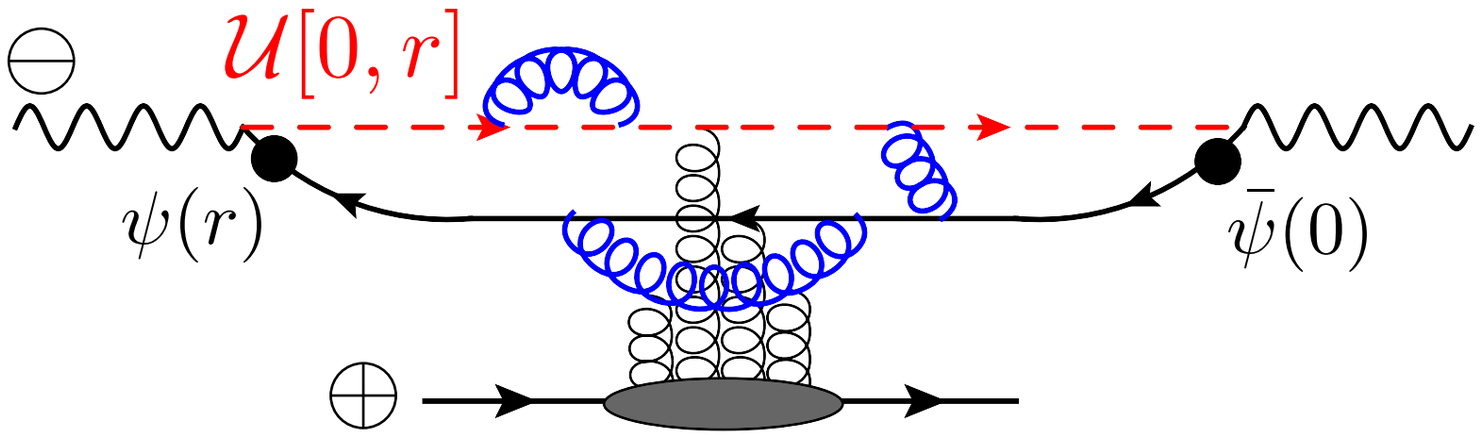

$$\frac{\partial}{\partial \ln s}\langle S_{xy}\rangle_{(s)} = \bar{\alpha}_s \int d^2 z \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2} \left[\langle S_{xz} S_{zy}\rangle_{(s)} - \langle S_{xy}\rangle_{(s)}\right]$$

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- Formulated as the dipole amplitude evolution.
  - Resums leading logarithms of energy.

$$\alpha_s \ln \frac{s}{Q^2}$$

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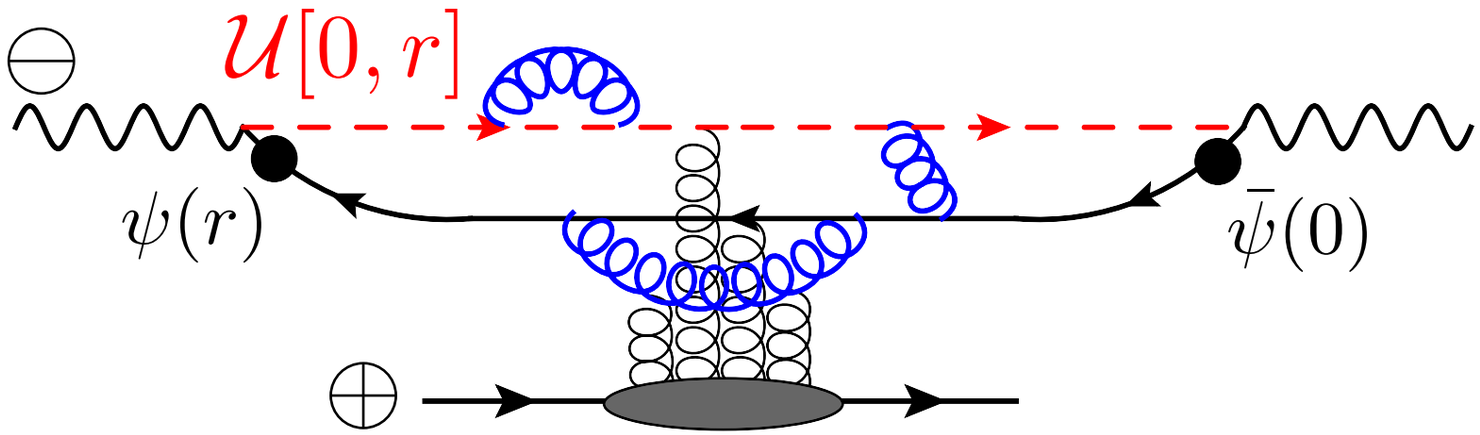

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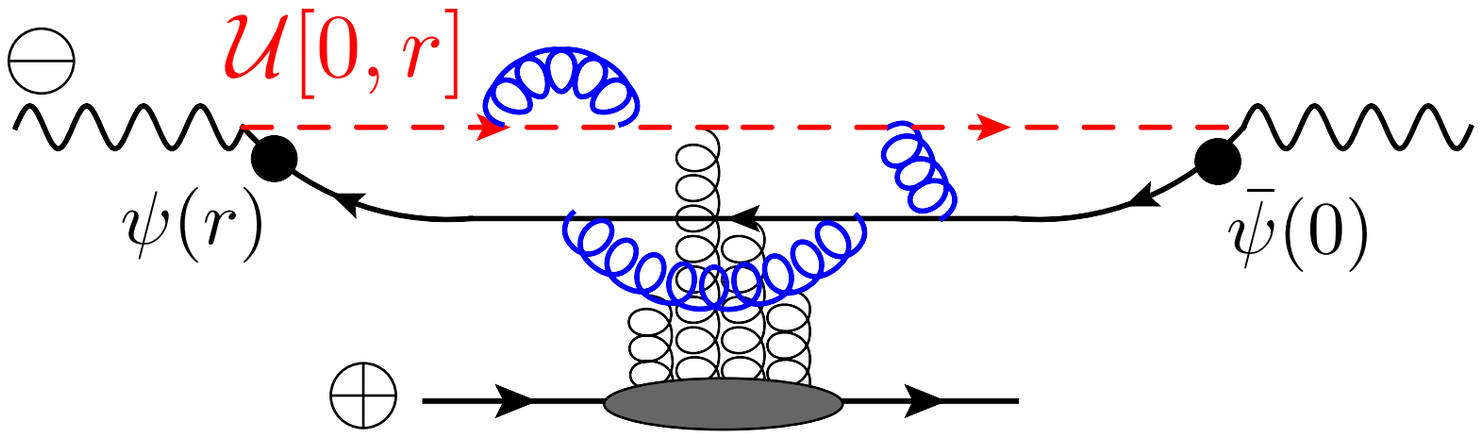
- Nonlinear rescattering leads to an operator hierarchy.

Unpolarized Quarks: BK / BFKL Evolution



$$\frac{\partial}{\partial \ln s}\langle S_{xy}\rangle_{(s)} = \bar{\alpha}_s \int d^2z \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2}\left[\underline{\langle S_{xz}\rangle_{(s)}\langle S_{zy}\rangle_{(s)}} - \langle S_{xy}\rangle_{(s)}\right]$$

- Evolution now occurs in the both the gauge link (quark Wilson line) and the matrix element (antiquark Wilson line).
- Formulated as the dipole amplitude evolution.
  - Resums leading logarithms of energy.

$$\alpha_s \ln \frac{s}{Q^2}$$

- Nonlinear rescattering leads to an operator hierarchy.
  - Evolution closes in the large N_c limit (BK).

$$Q_s^2(x) \sim \left(\frac{A}{x}\right)^{1/3}$$

Polarization at Small x: A New Ballgame

- At small x, the leading-order quark structure is spin independent.
  - Small-x evolution: energy dependence of total cross-section.

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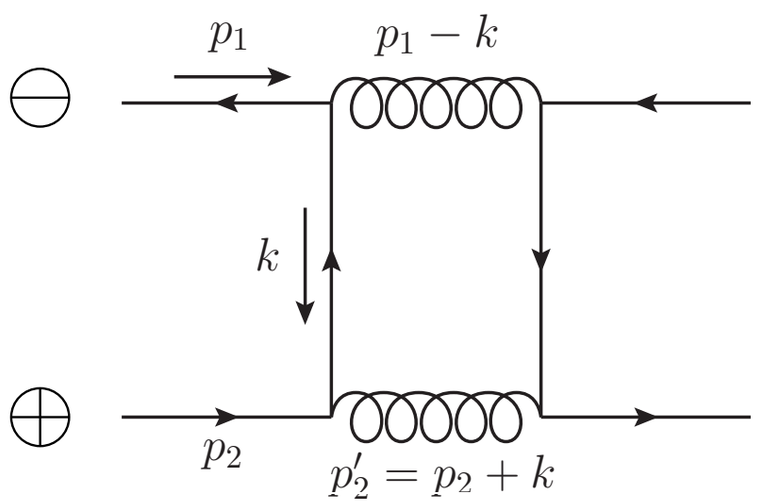

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  - Longitudinal spin asymmetry

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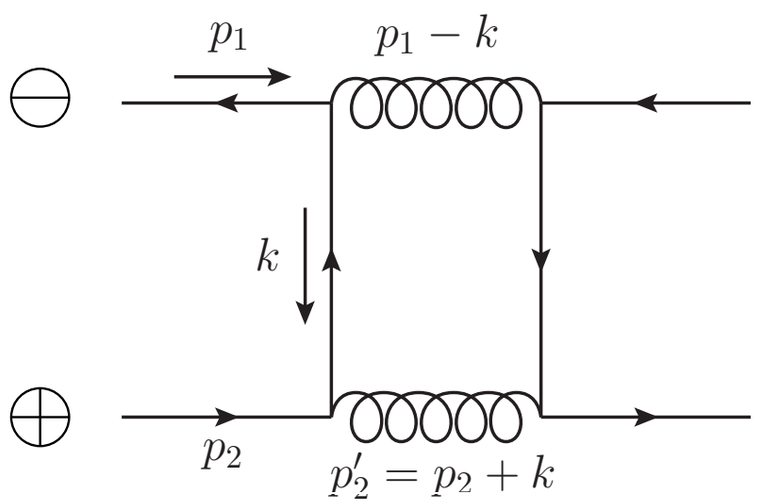

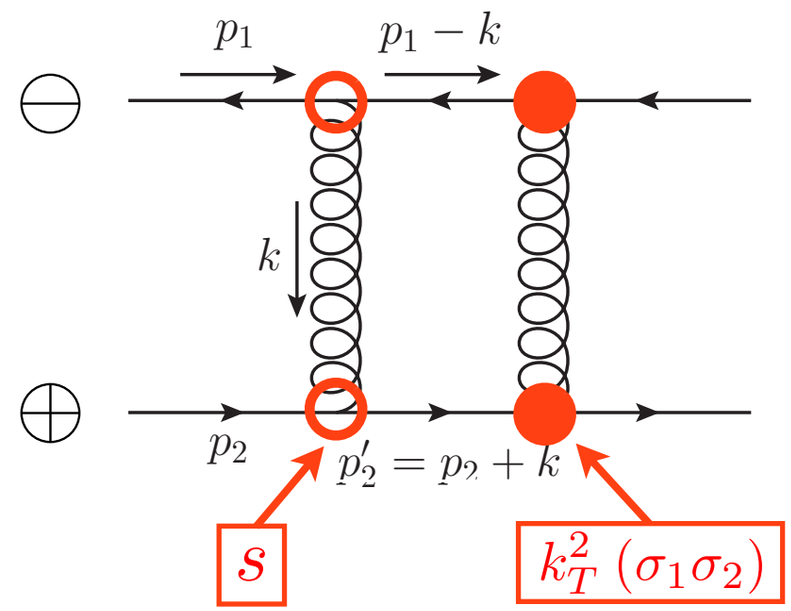

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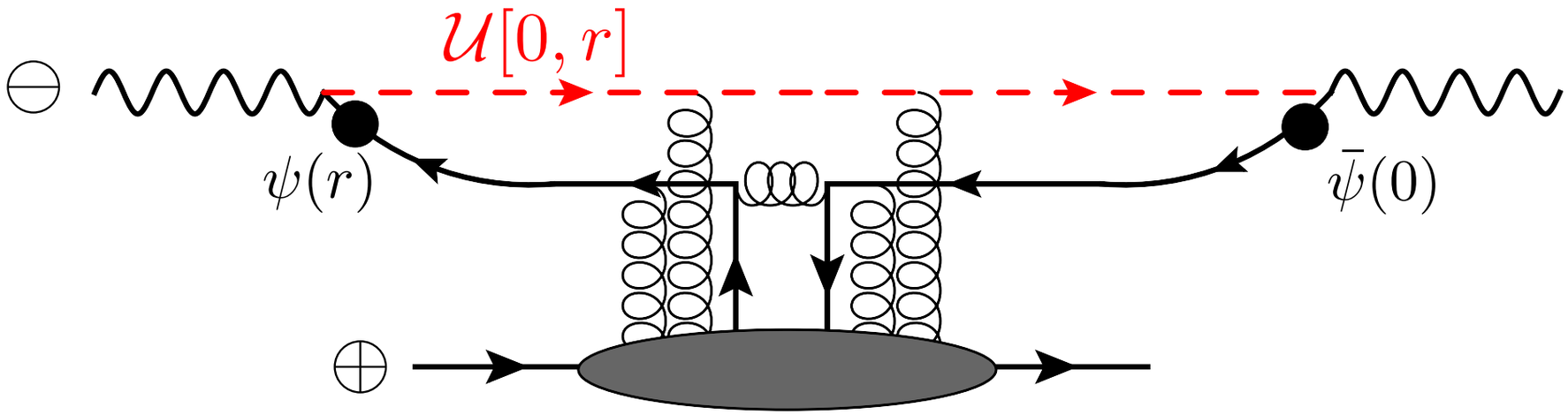
- Gluon exchange: leading-order is spin independent.
  - Sub-leading: Energy suppressed and polarized!

$$\frac{d\Delta\sigma_G}{d^2k} = +\frac{\alpha_s^2 C_F}{N_c}\ \frac{1}{s}\frac{1}{k_T^2}$$

The Polarized Dipole



- Generalize dipole DoF to include spin dependence.
  - Each spin exchange is energy suppressed.
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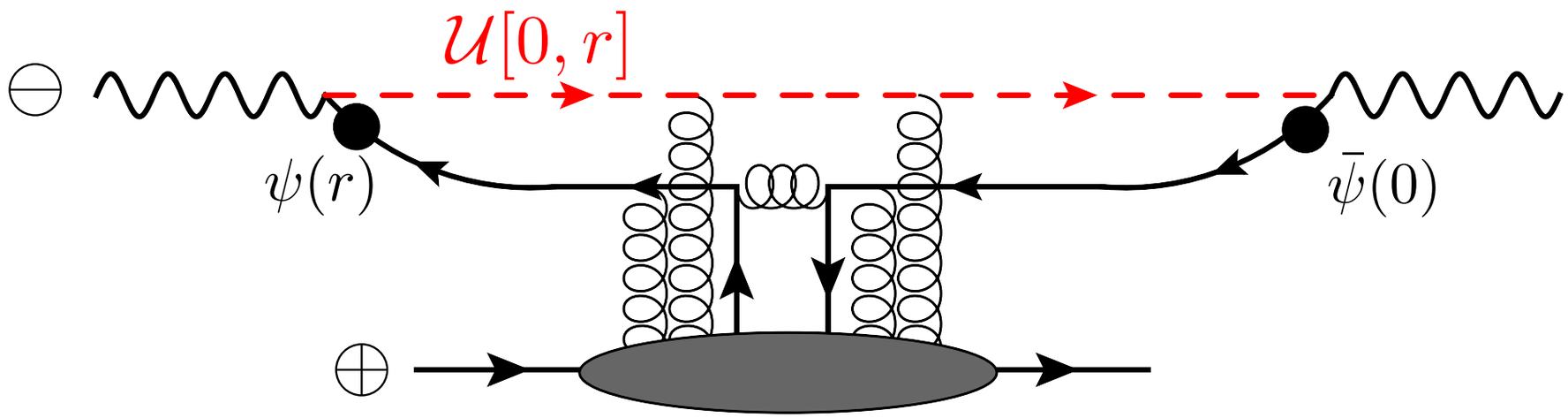

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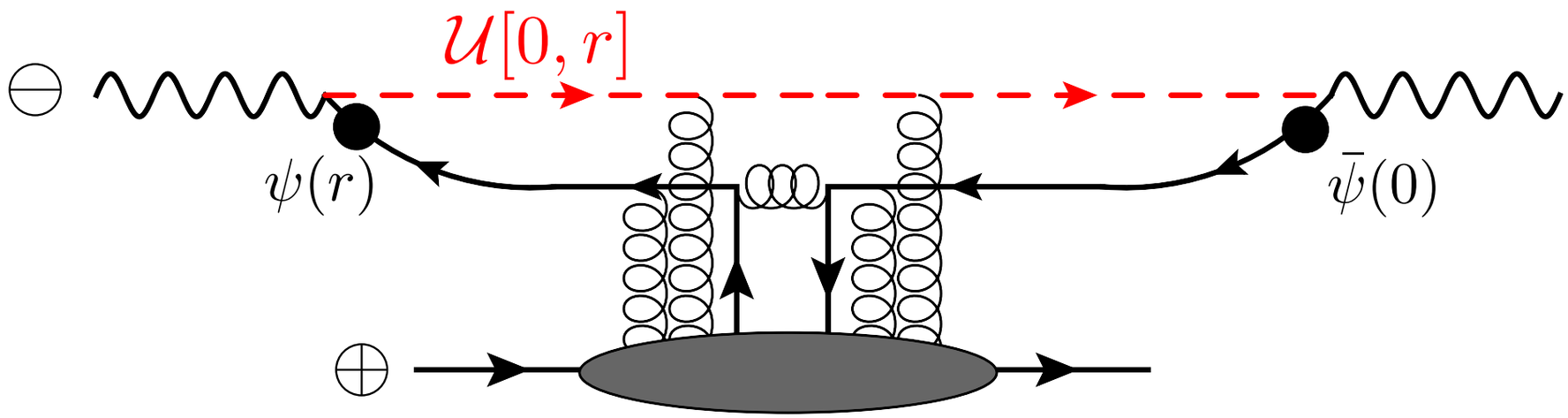

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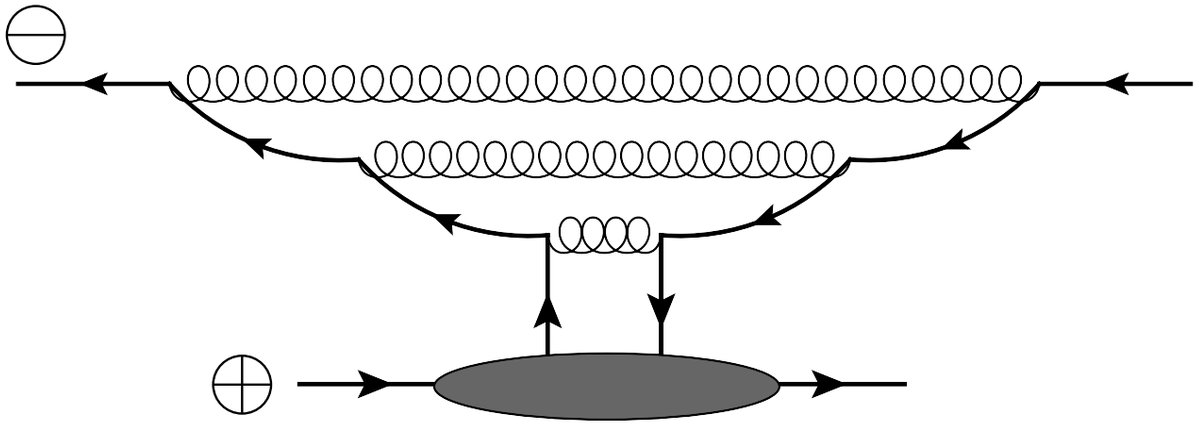
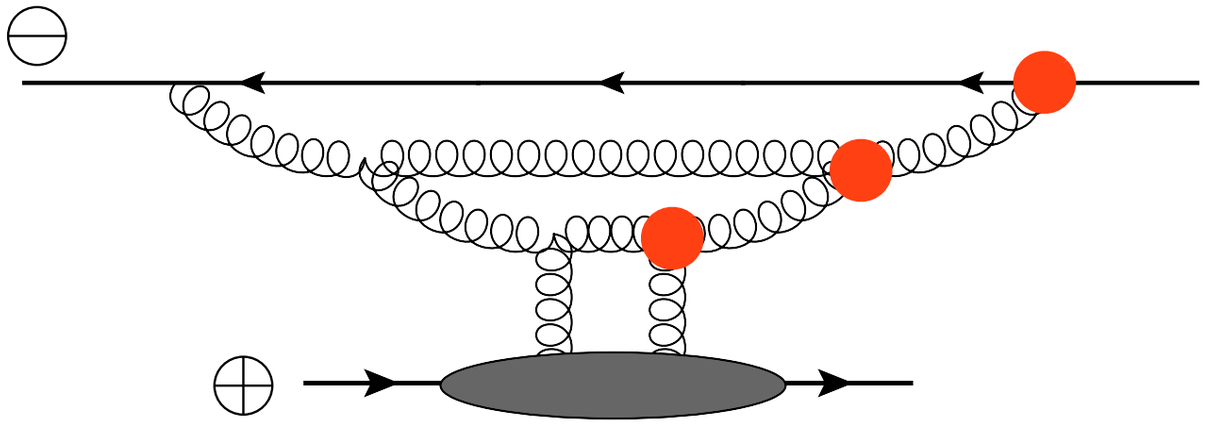
- Building block: the "Polarized Dipole Operator"

$$G_{xy} \equiv \frac{1}{2N_c}\mathrm{Tr}\left[V_x V_y^\dagger(\sigma) + V_y(\sigma)V_x^\dagger\right]$$

Evolution: Transporting Spin to Small x





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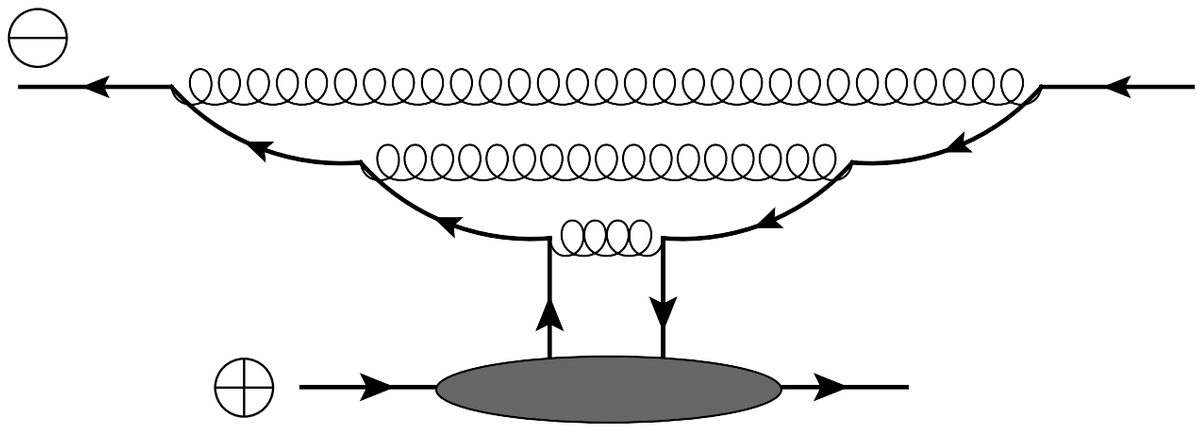

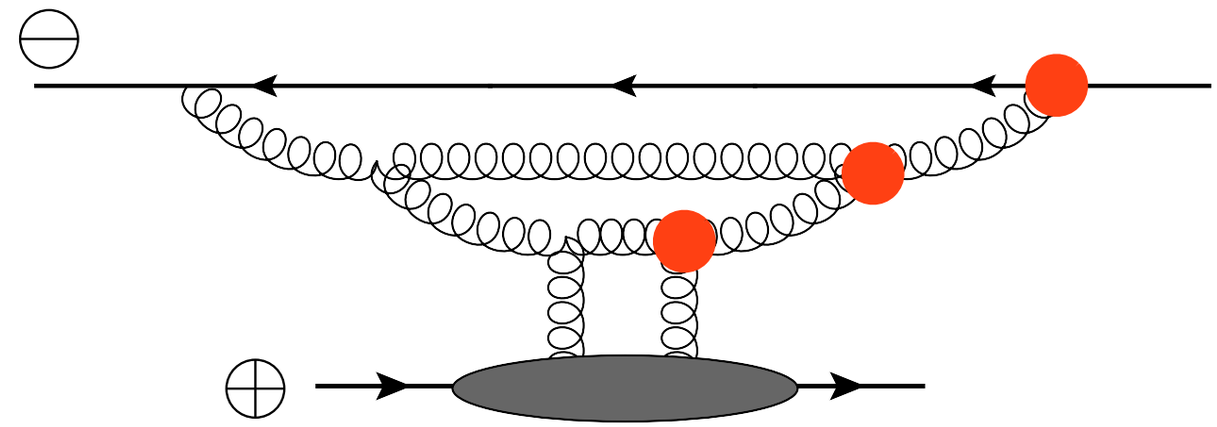

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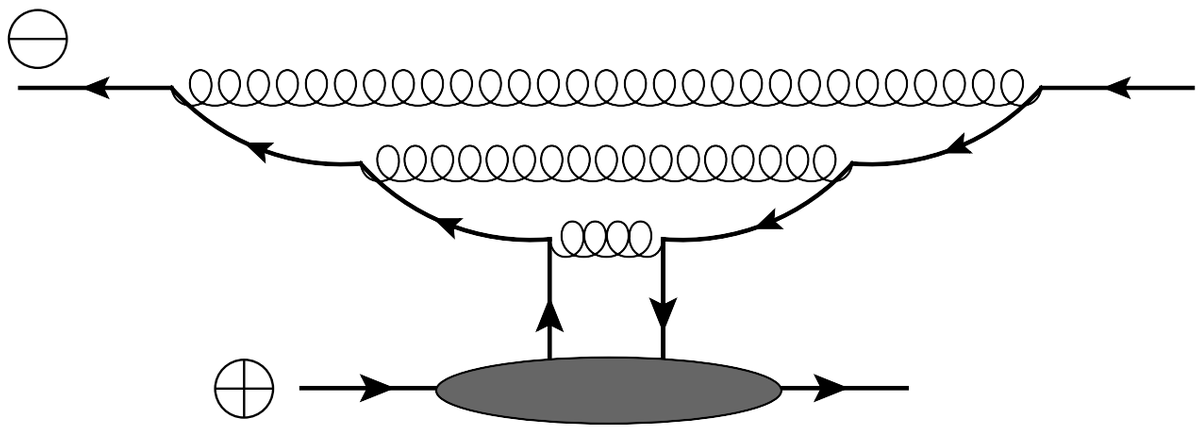

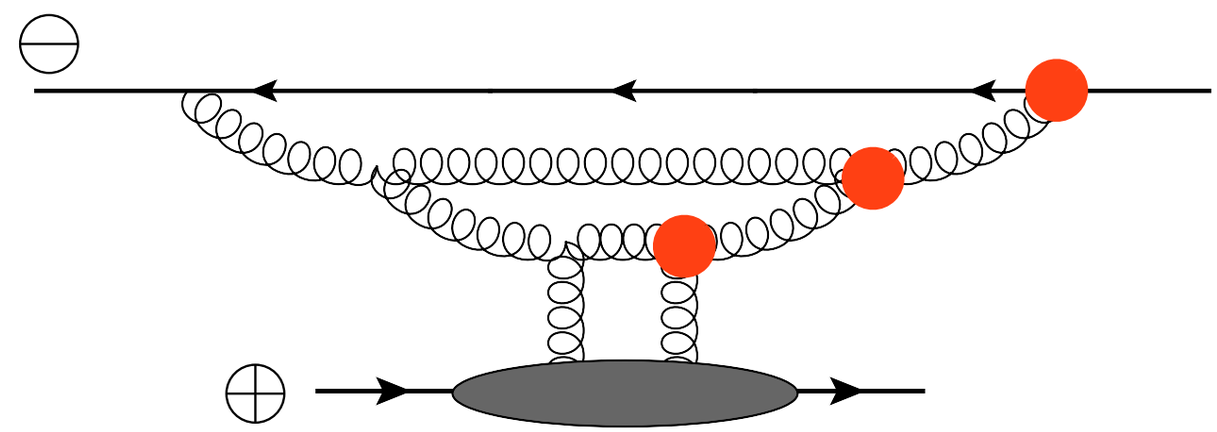

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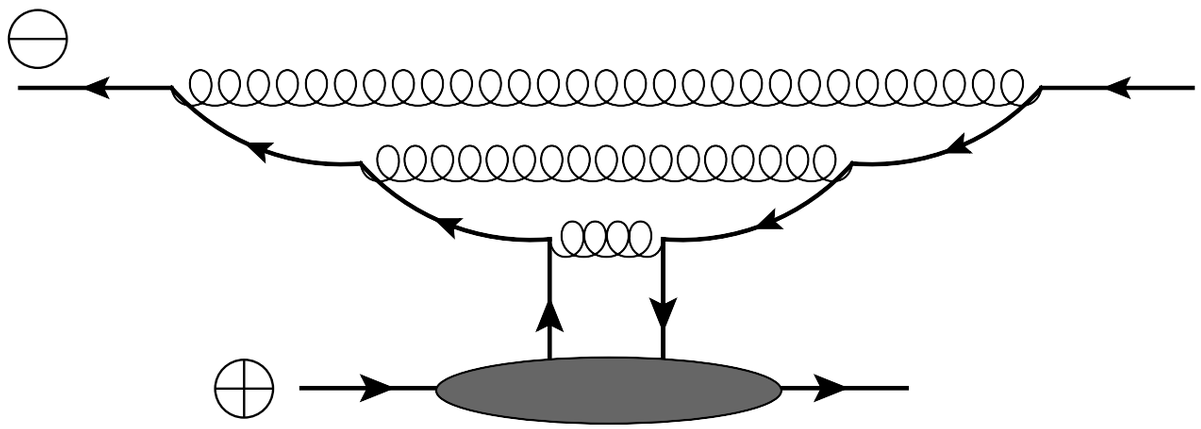

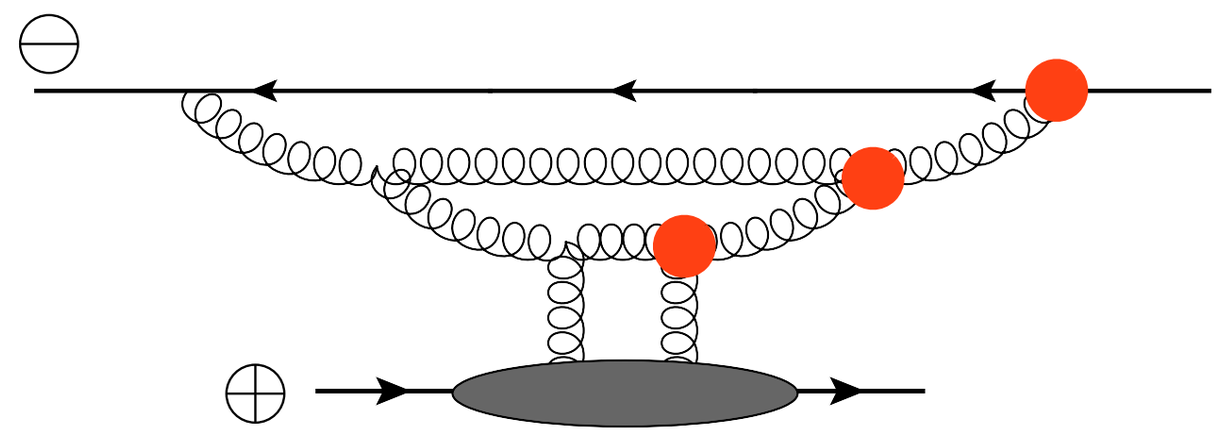

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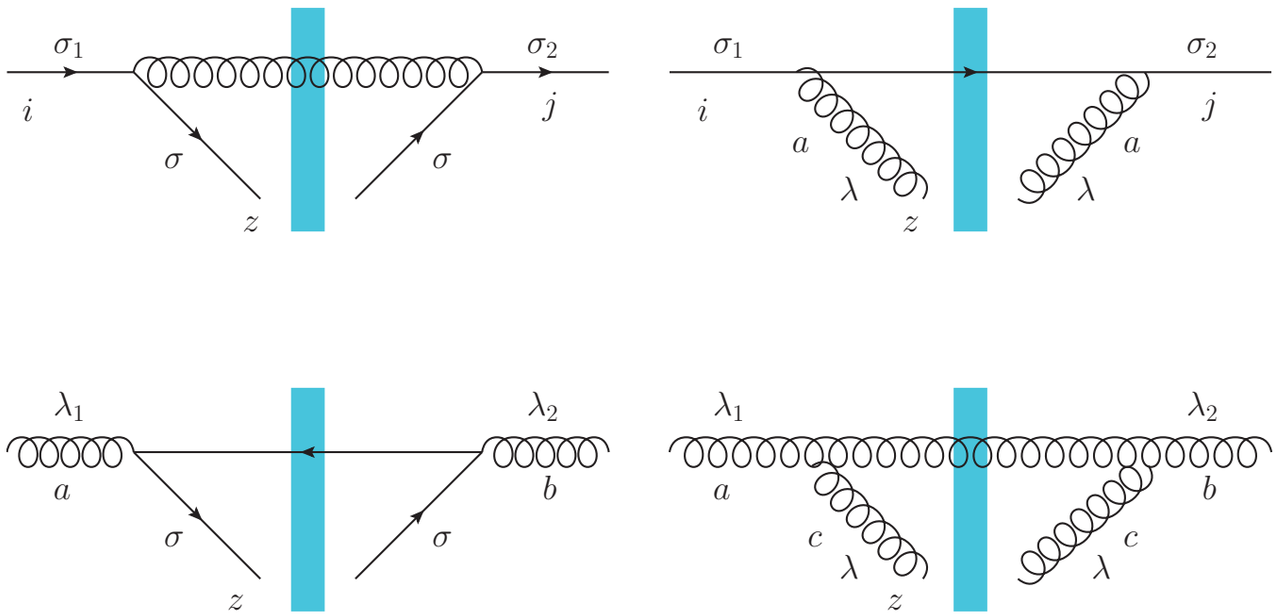
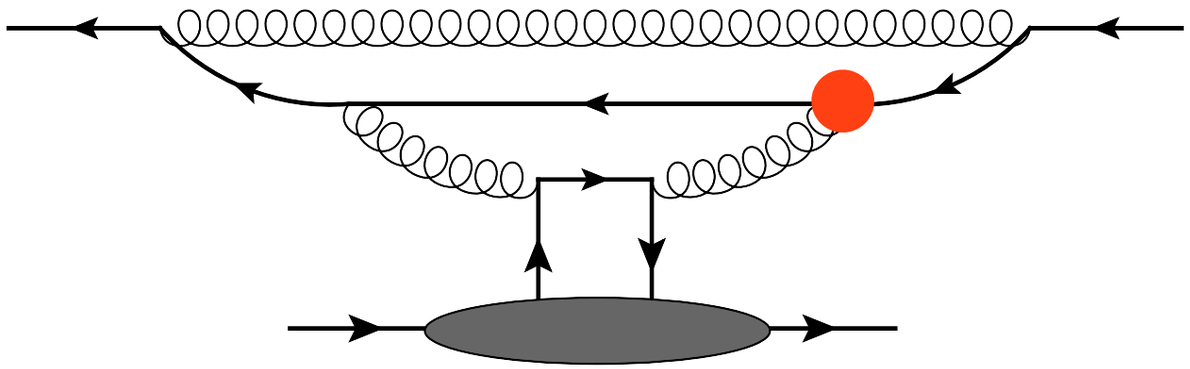
- Leads to double logarithms of energy
  - Could it off-set the initial energy suppression?

$$\alpha_s \ln^2 \frac{s}{Q^2}$$

Ladder Approximation





- Ladder evolution: quark, gluon splitting
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$$\frac{\alpha_s}{2\pi}\int\frac{dz}{z}\int\frac{dk_T^2}{k_T^2}\begin{pmatrix} C_F & 2C_F \\ -N_f & 4N_c \end{pmatrix}$$

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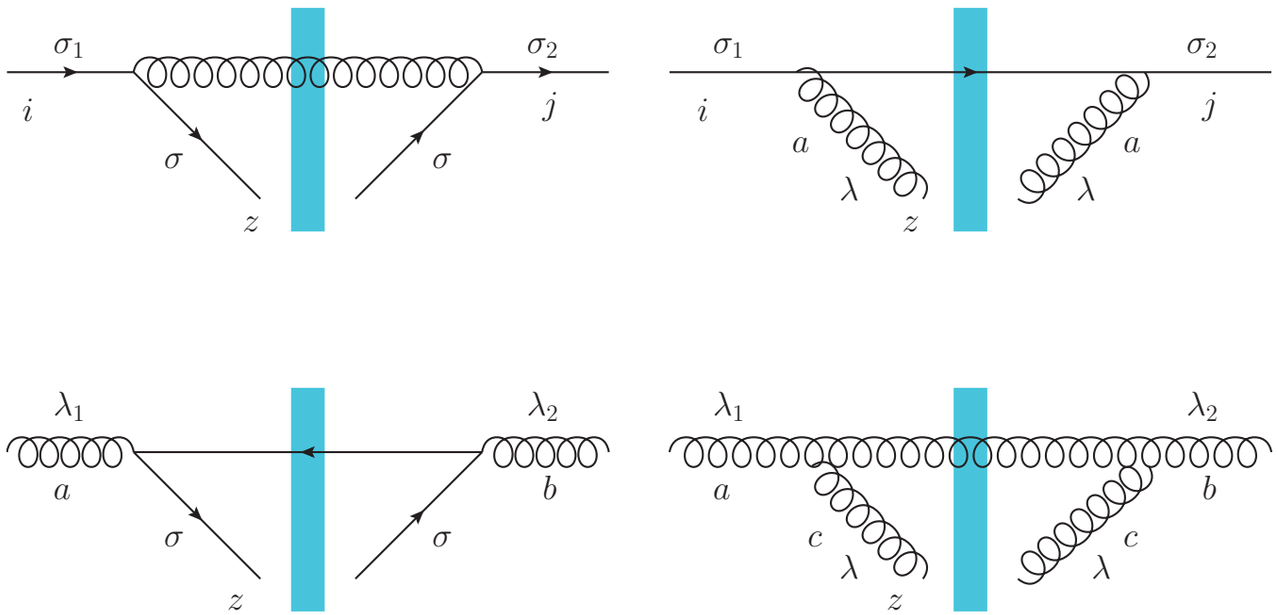

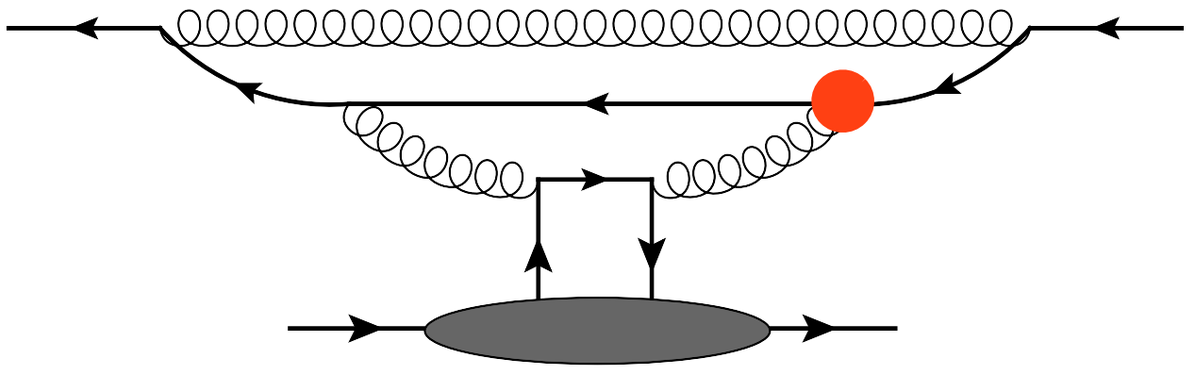

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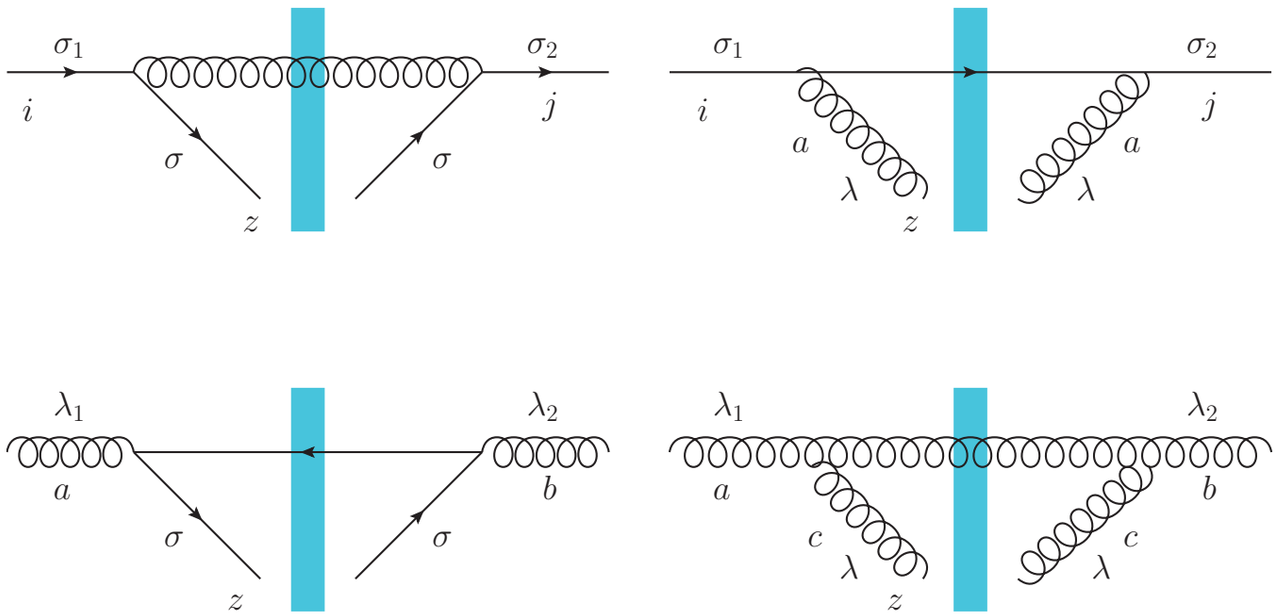

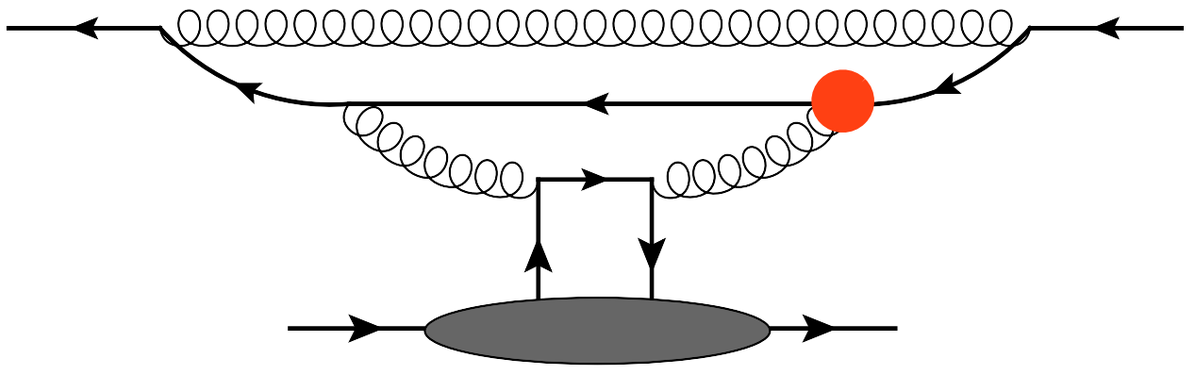

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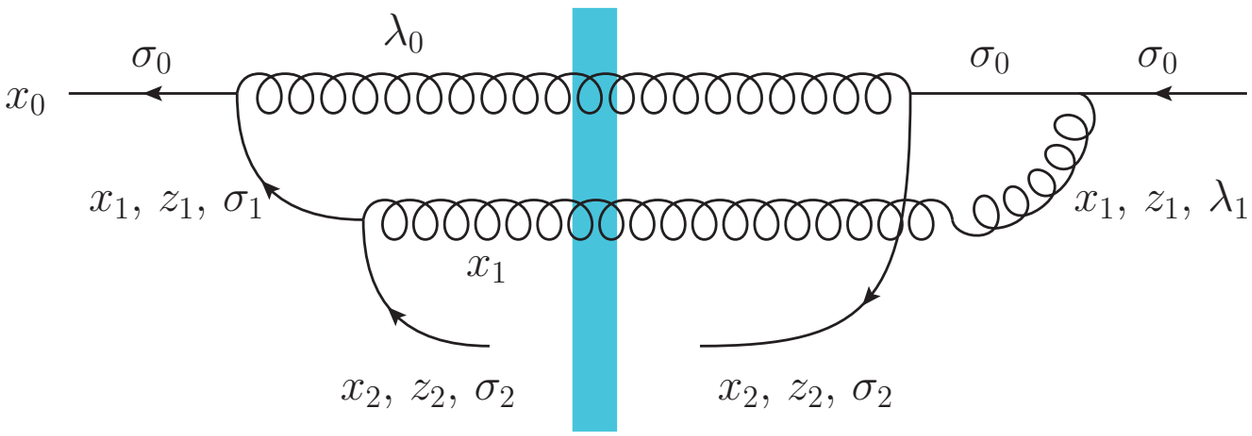
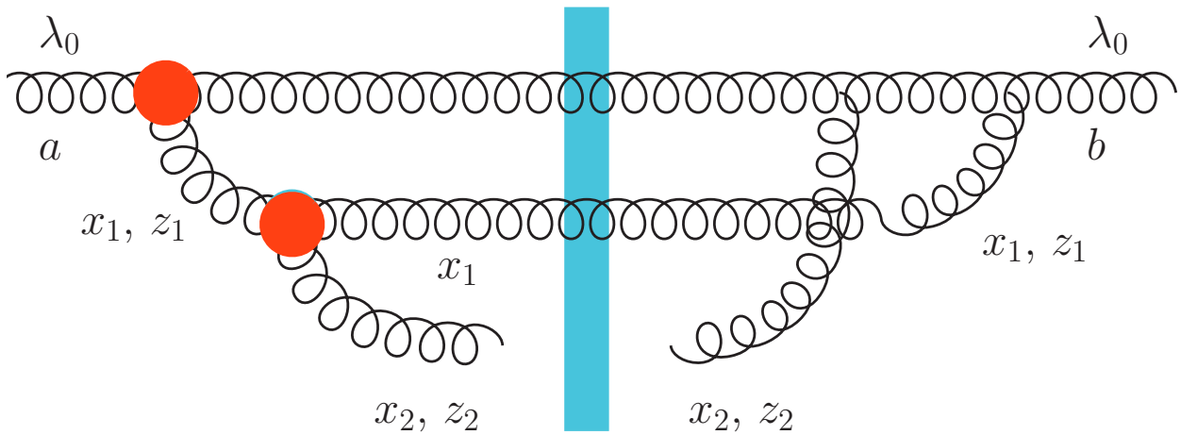
BK: $\sim \left(\frac{s}{Q^2}\right)^{0.3}$

- Fast growth at small x! (Faster than BK...)
  - Large contribution to the proton spin?

The Plot Thickens: Non-Ladder Graphs





- "Non-ladder graphs" can also generate double logs.
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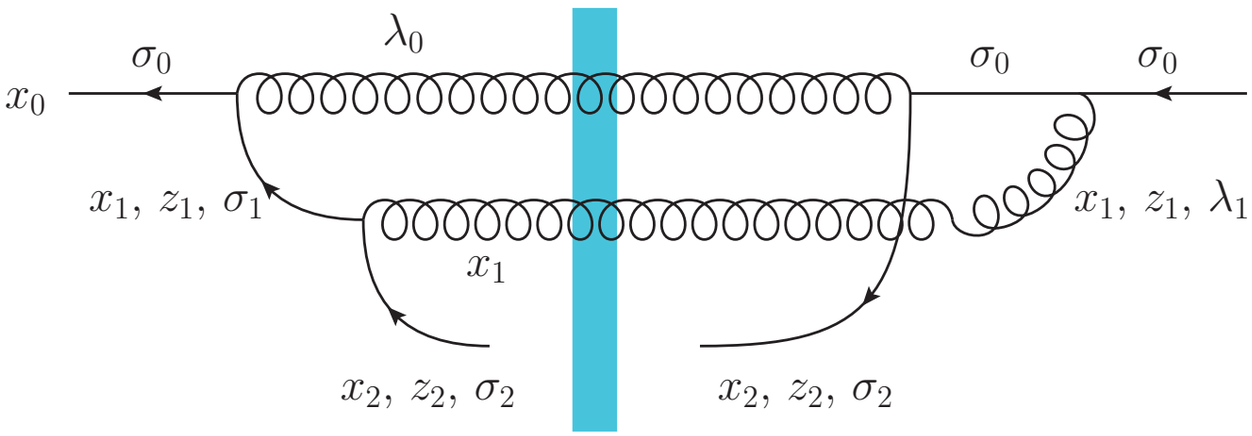

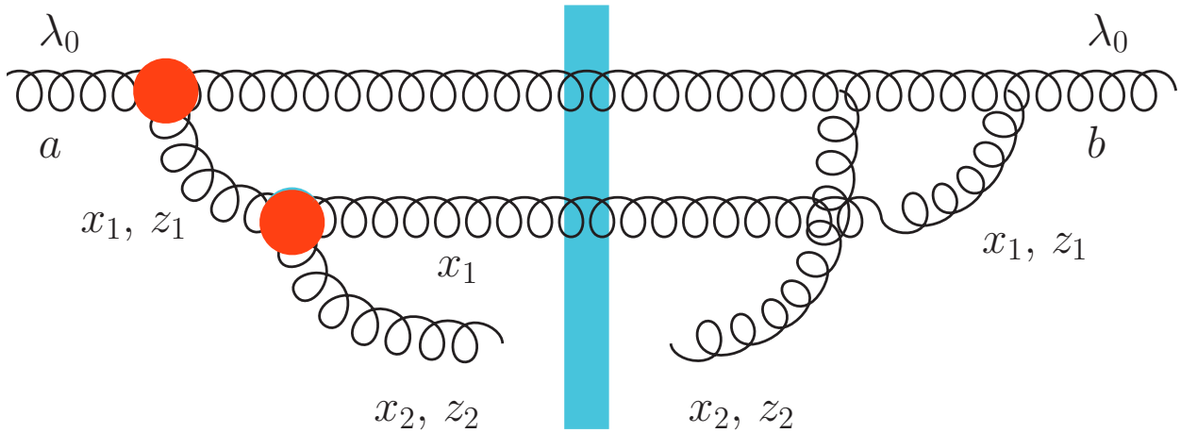

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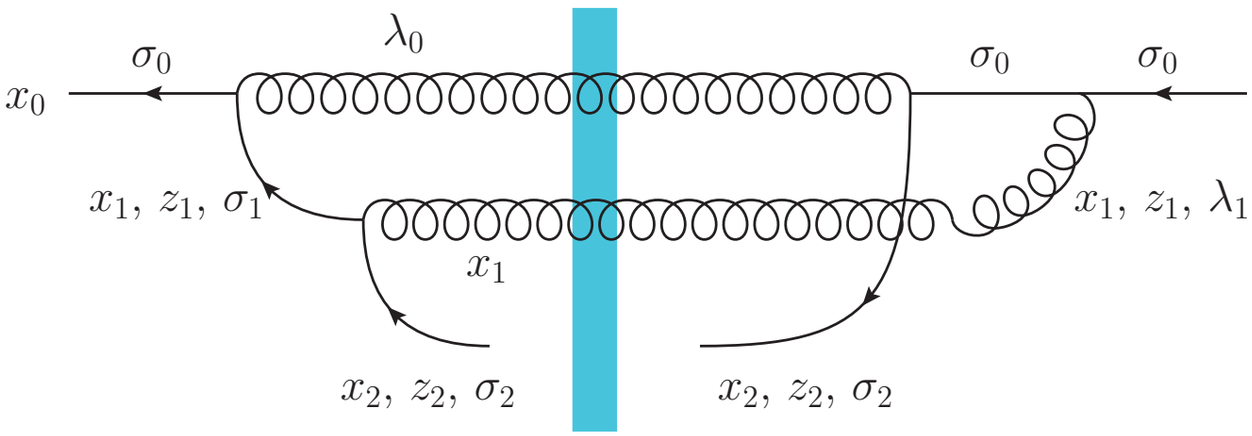

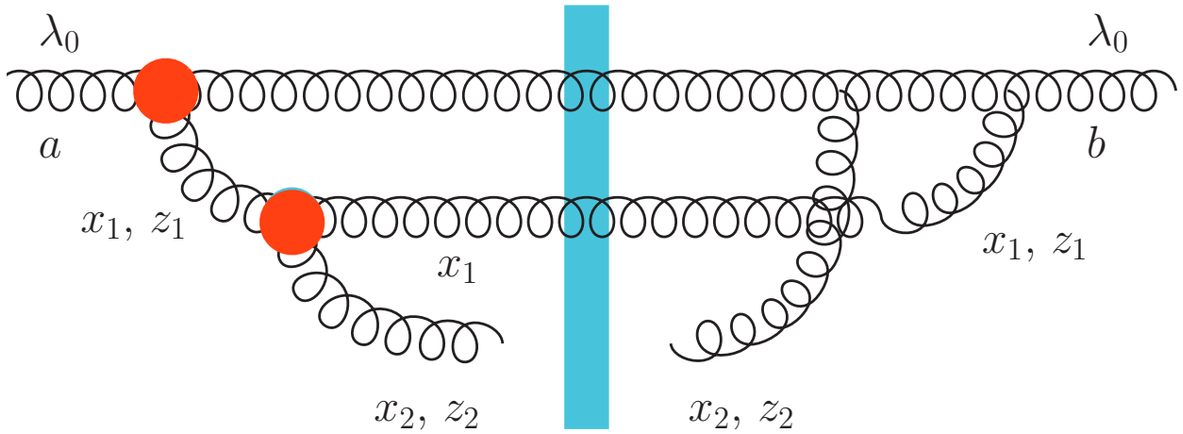

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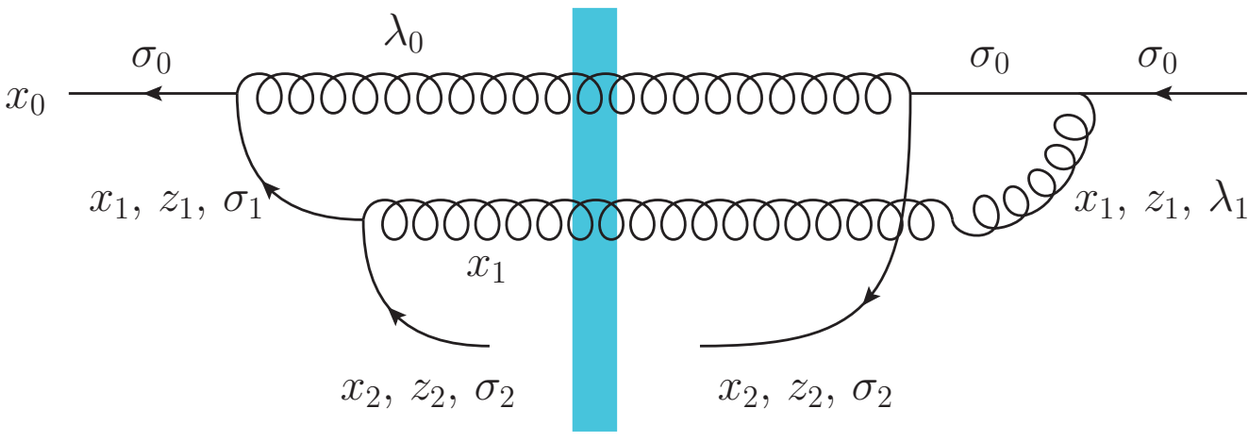

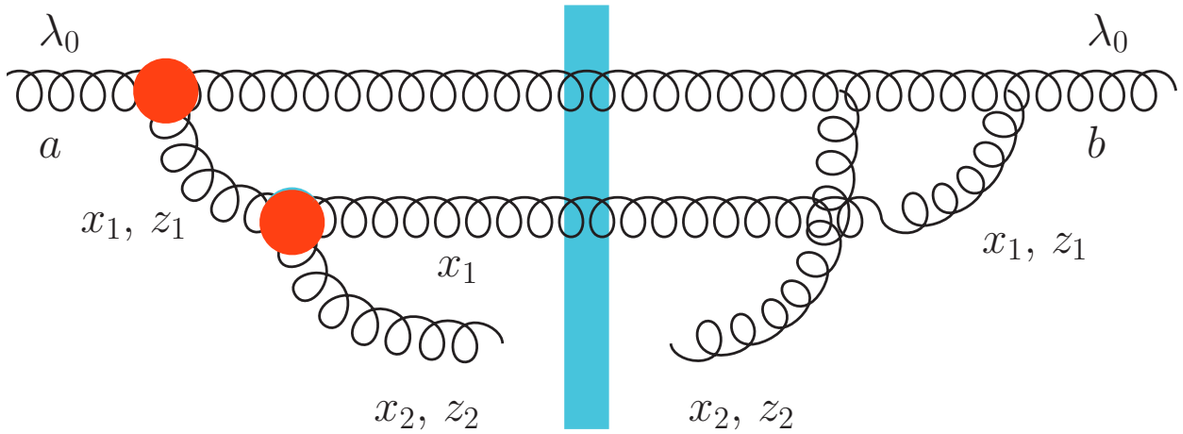

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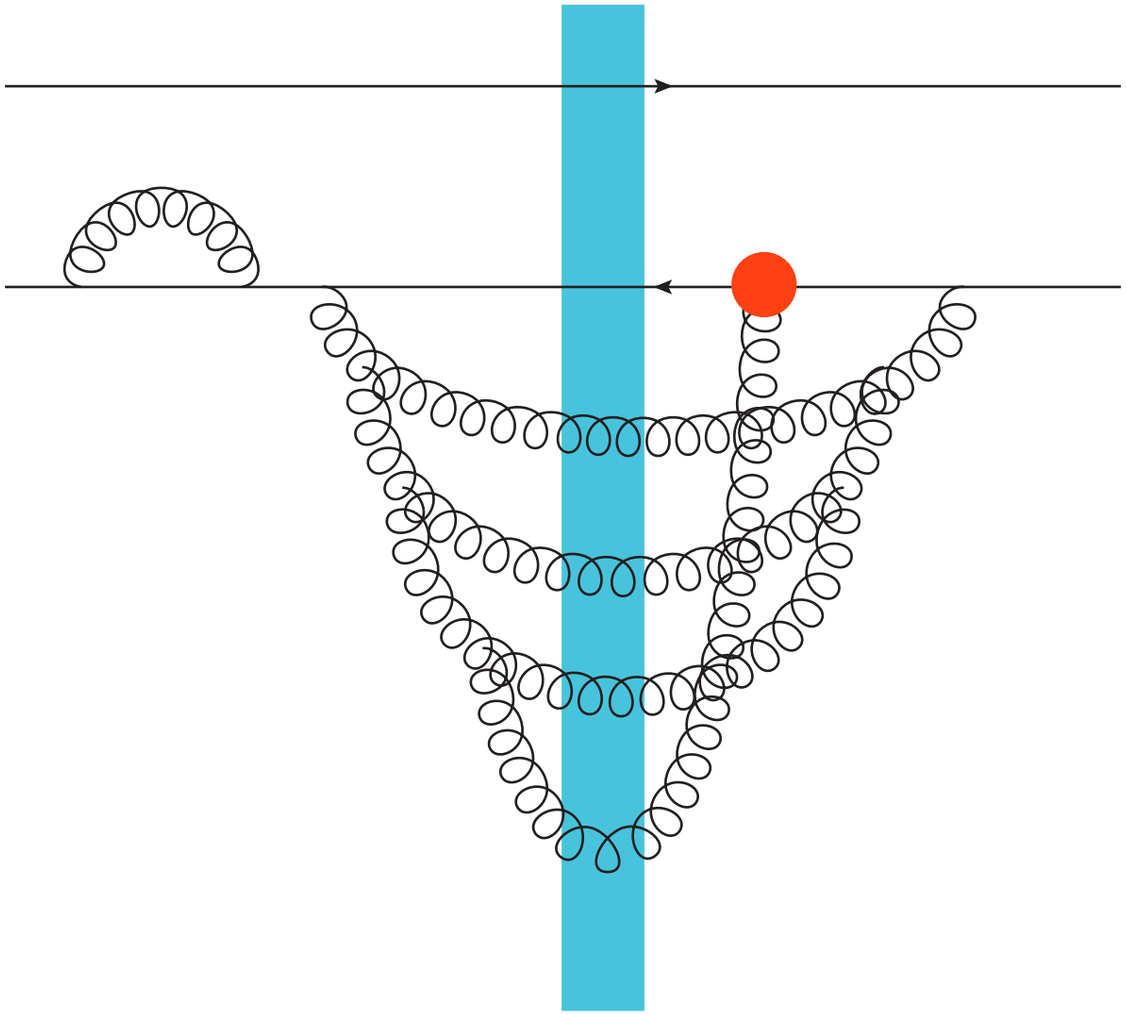
- Ladder truncation is NOT a justifiable approximation!

A Tangled Mess

- Non-ladder graphs can stack in entangled ways which are still DLA
  - Polarization can "jump a rung" of evolution and still be transmitted to the target.
  - Very difficult to resum.



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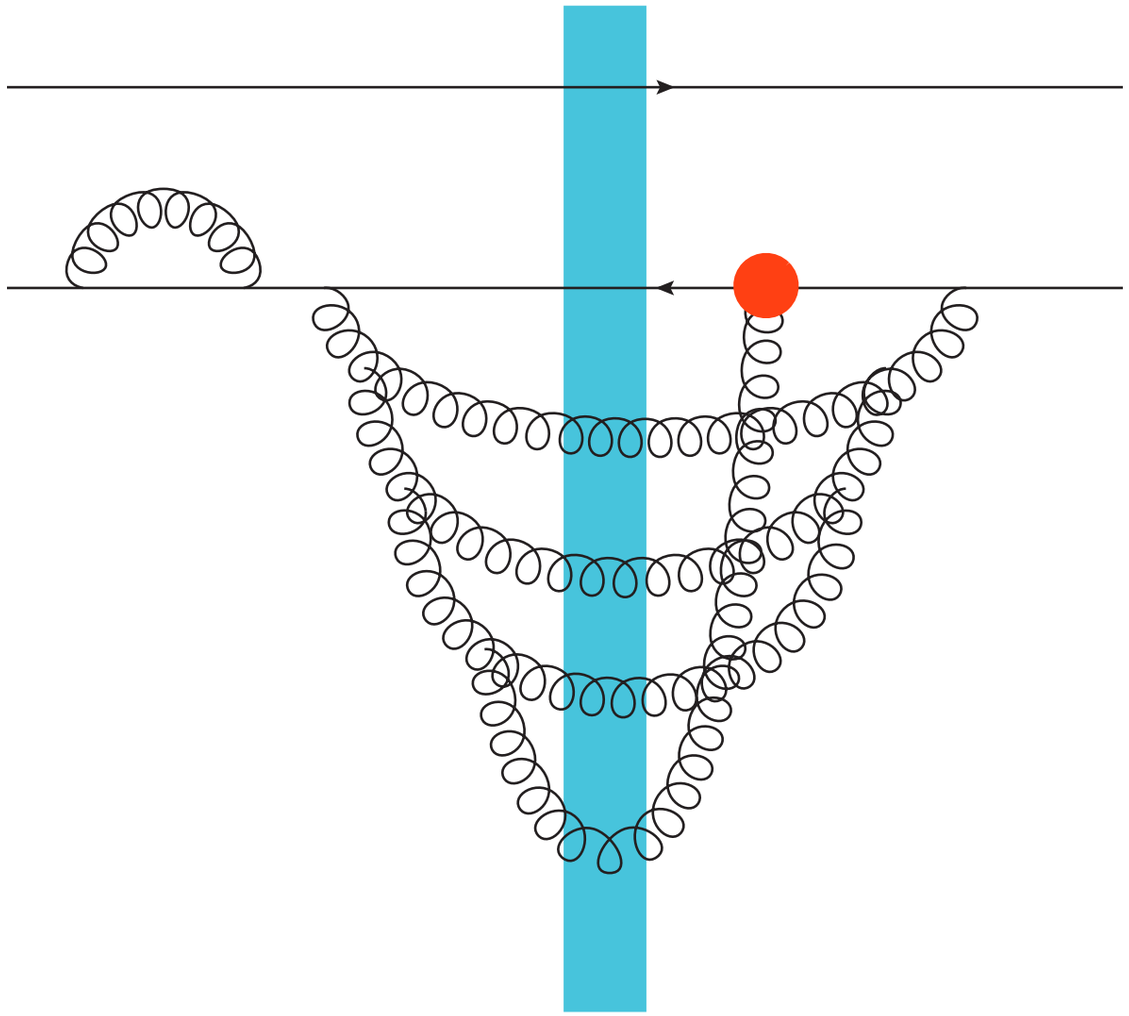
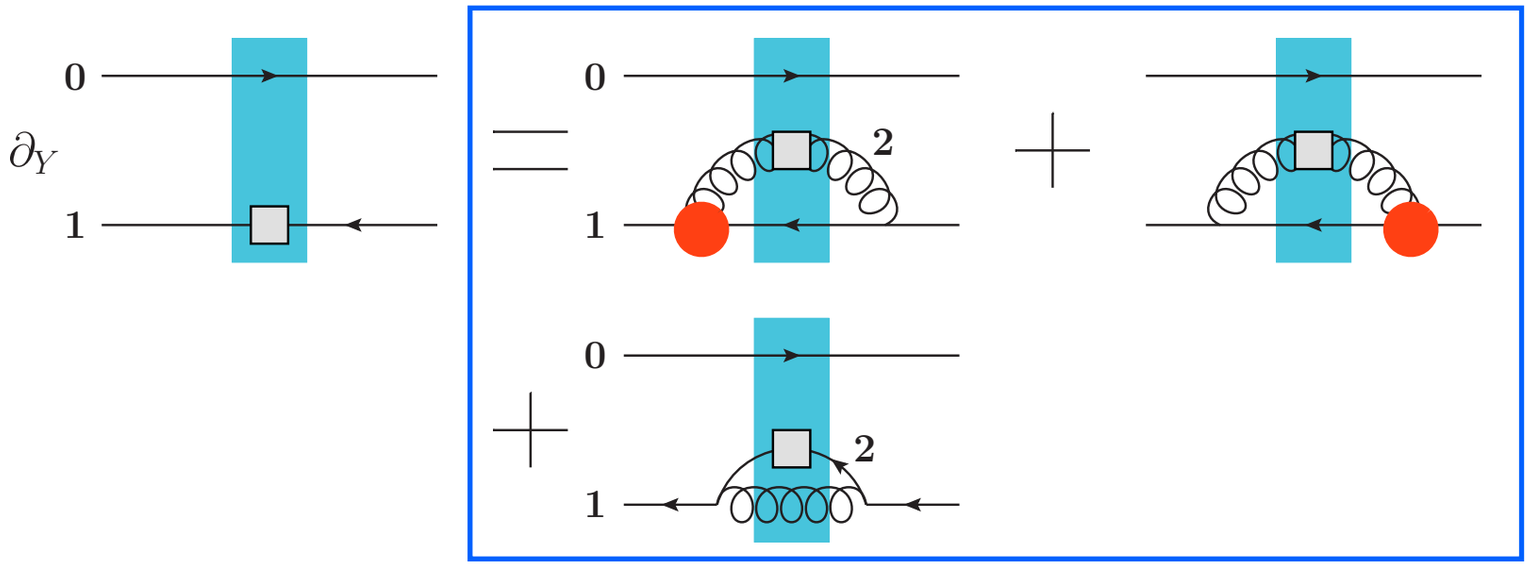


- Even allows for eikonal, unpolarized evolution in an intermediate step
  - Transition to color-octet evolution

Polarized Dipole Evolution

Ladder:



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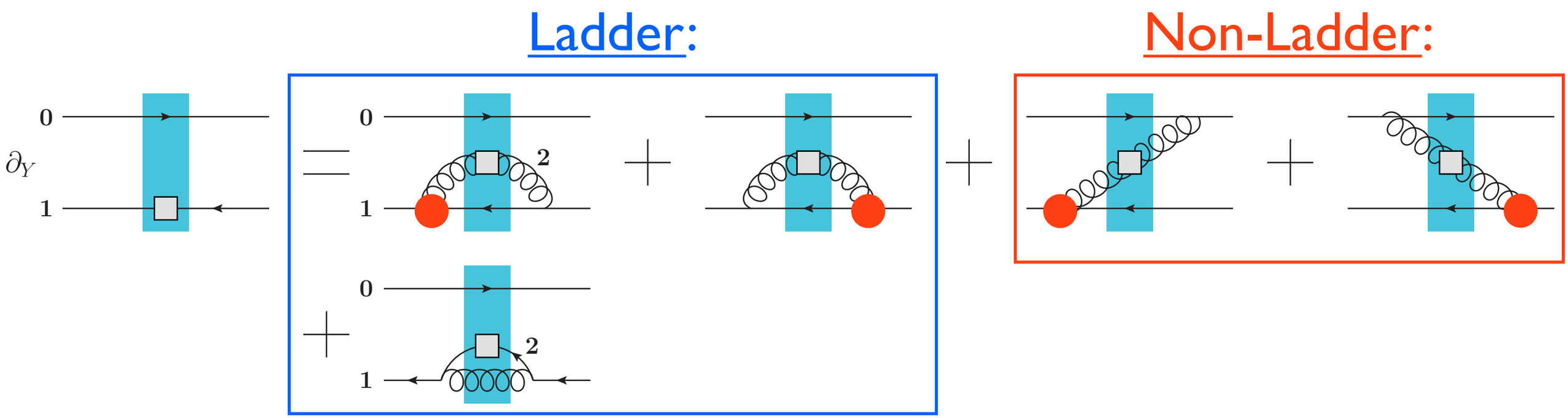

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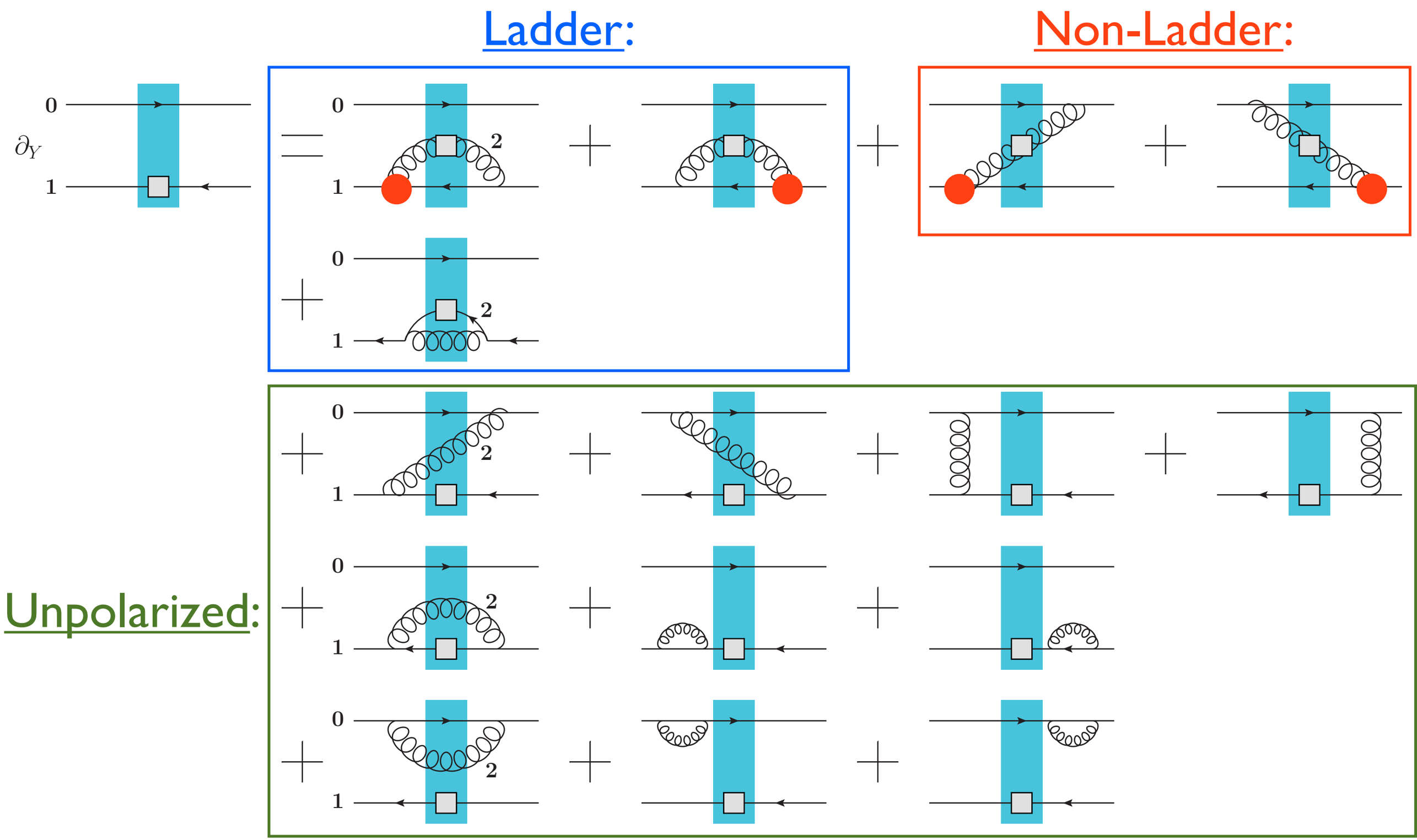
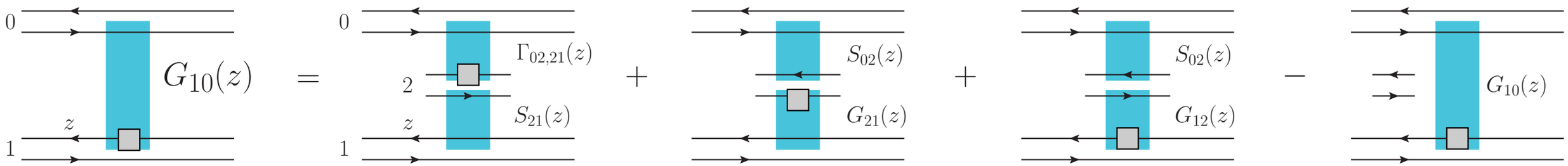


The Large - N_c Limit



- The operator evolution yields an infinite hierarchy of operators.
  - In the formal large N_c limit, the system closes (like BK)
  - Neglects quarks

Not well-justified!

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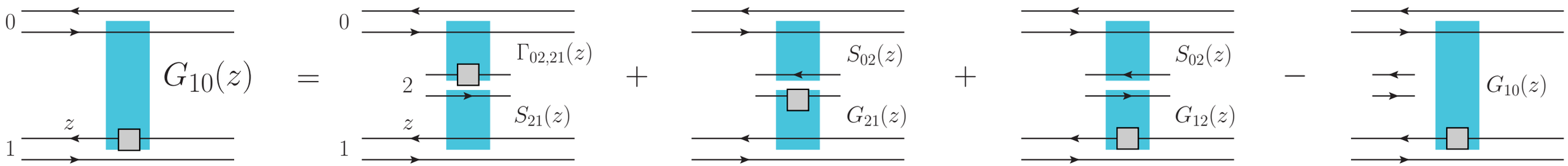

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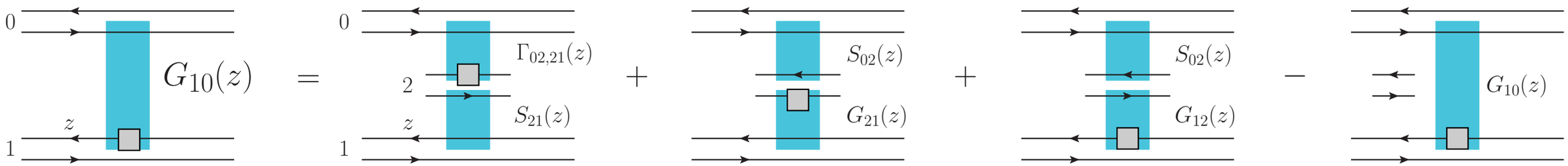

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Not well-justified!

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- However, dipoles are not independent.
  - Phase space depends on the "neighbor dipole."

$$Q^2 \ll \frac{k_{1T}^2}{z_1} \ll \frac{k_{2T}^2}{z_2} \ll \cdots$$

More Realistic: Large N_c , N_f

- To keep the quark contributions, must take both N_c, N_f large.

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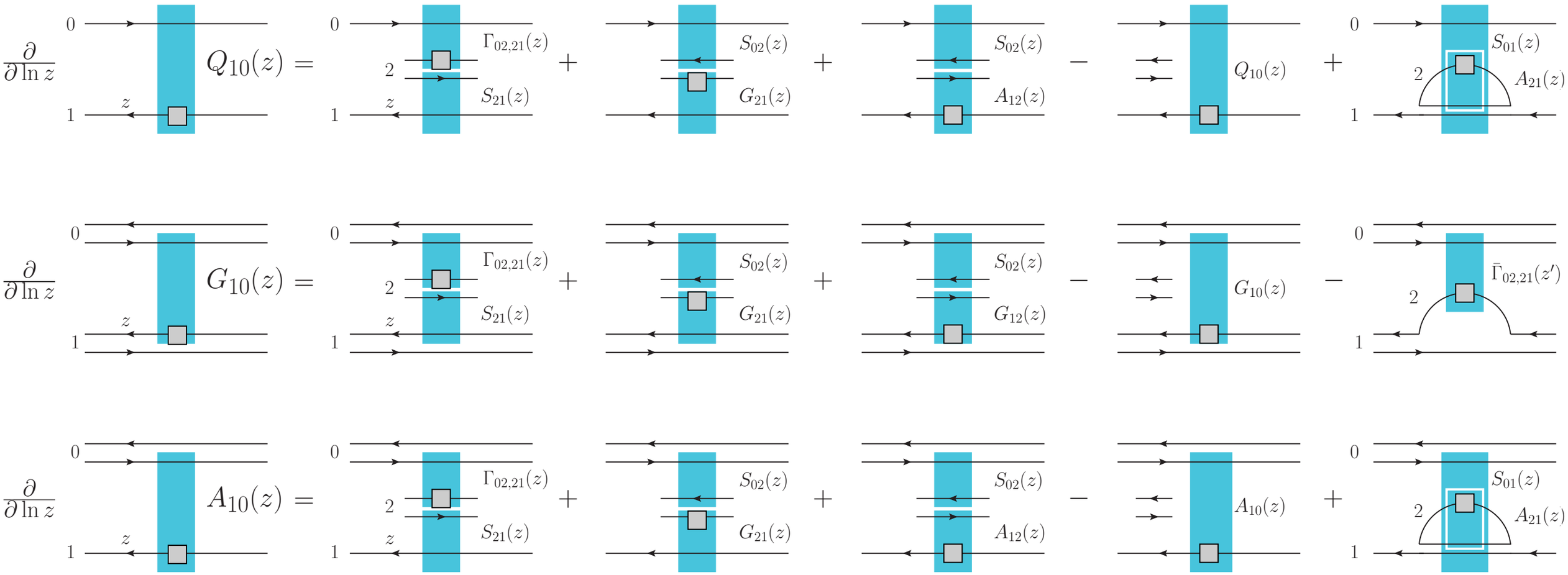

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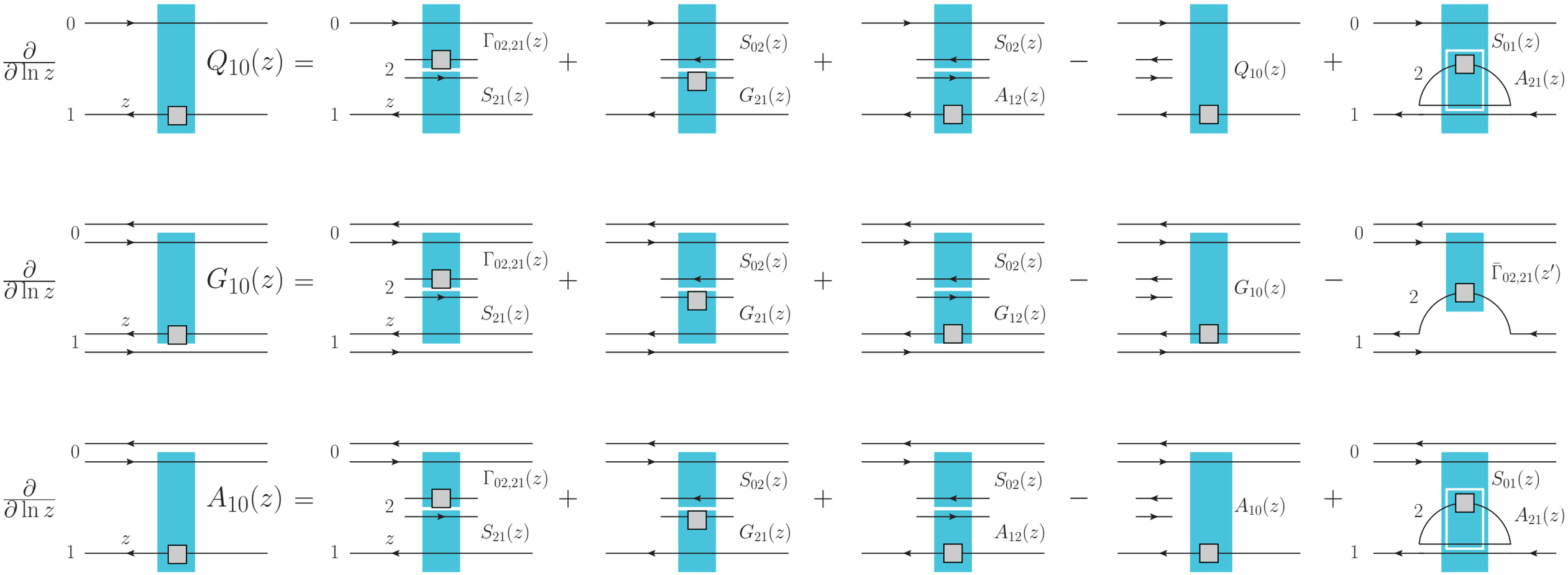

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- System closes, but even more complicated.
- Must distinguish between "true quark" dipoles and "false gluon" dipoles.

Outlook: Pressing Questions

- Can we solve the helicity evolution in ANY systematic approximation?
  - Large N_c ,N_f ? Only large N_c ?
  - Does the growth persist at small x?

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- What about other polarization observables like transversity?

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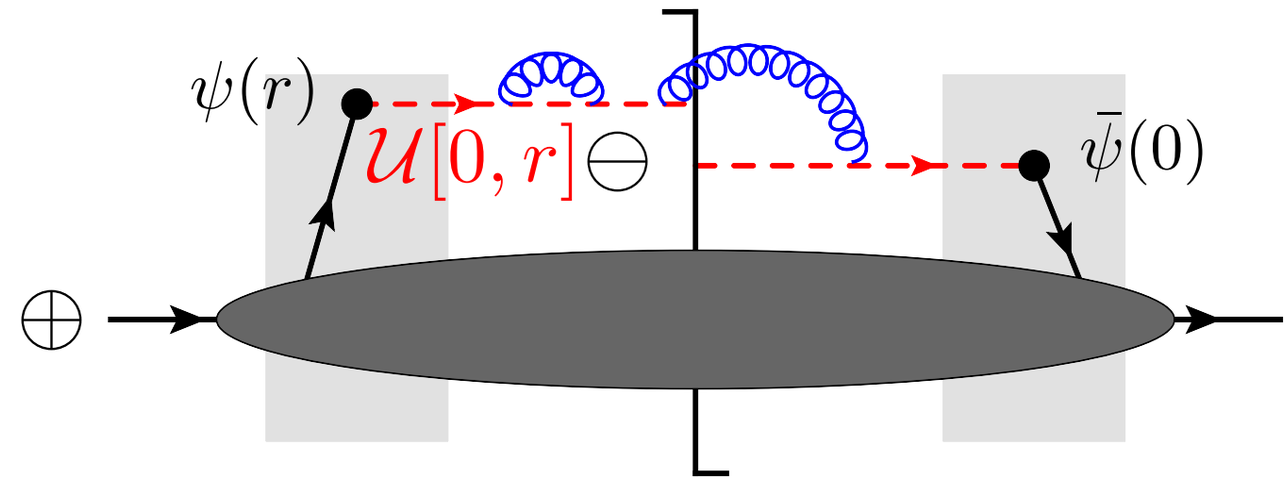
Large-x Evolution (CSS)

$$\alpha_s \ln^2 \frac{Q^2}{\mu^2} \sim 1$$



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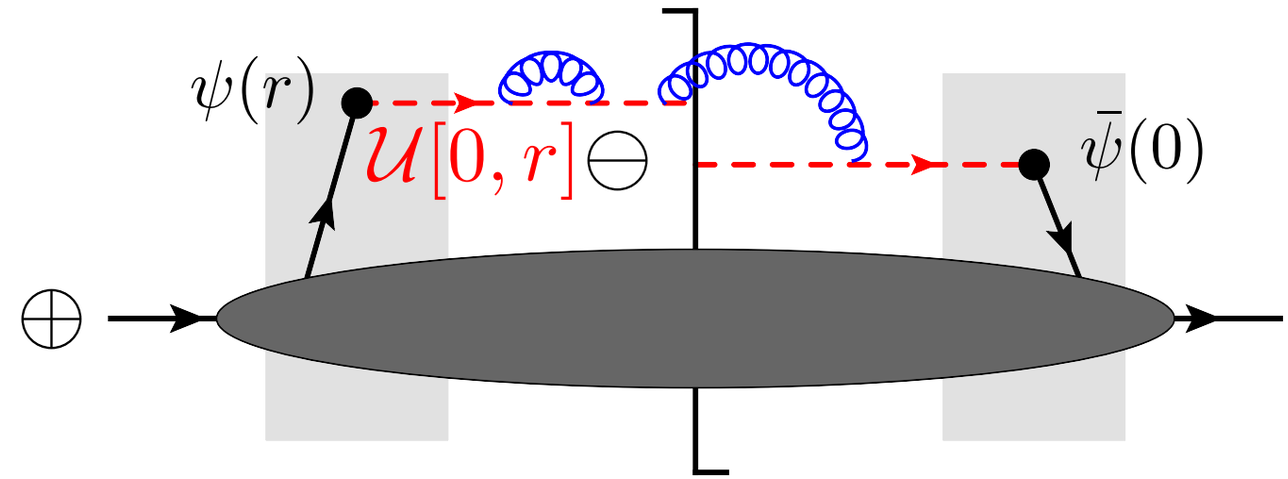

Unpolarized Small-x Evolution (BK)

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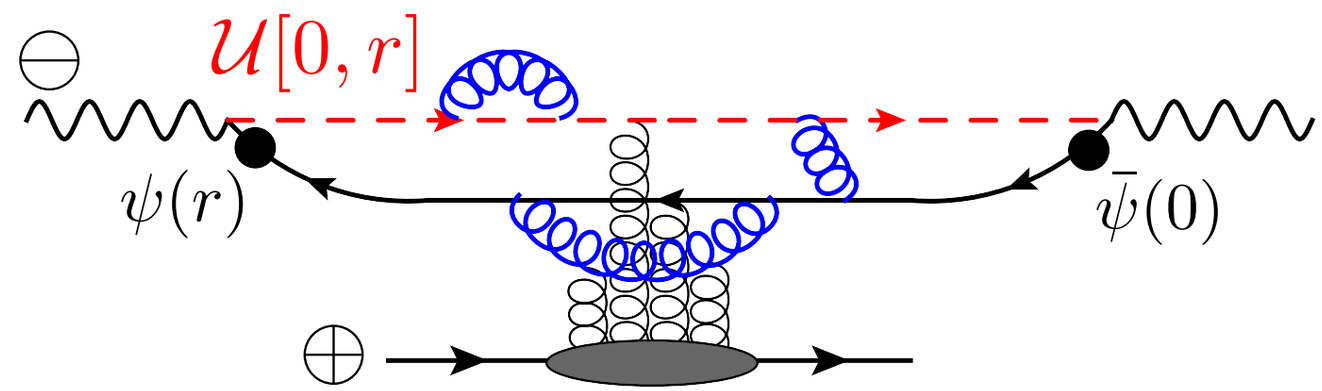

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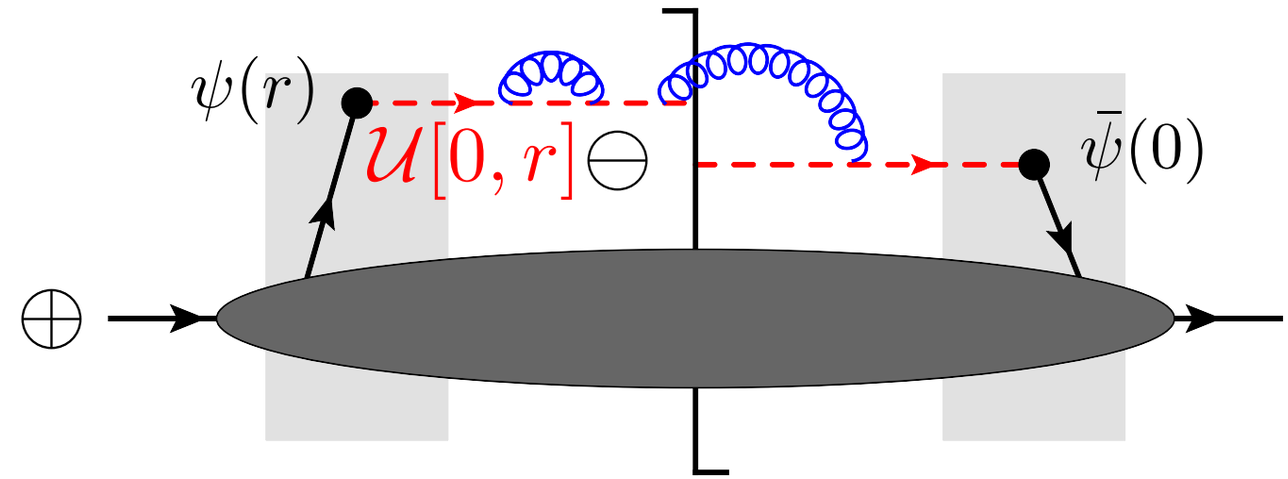

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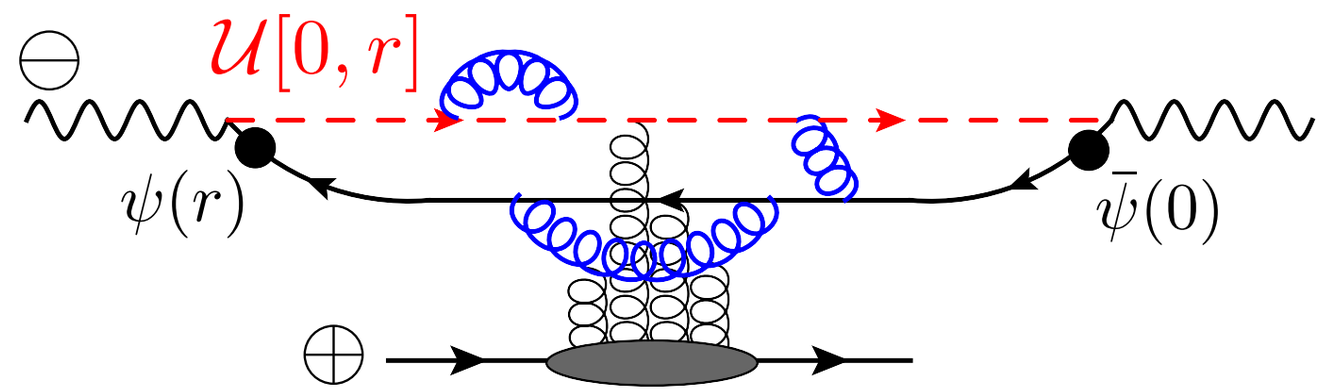
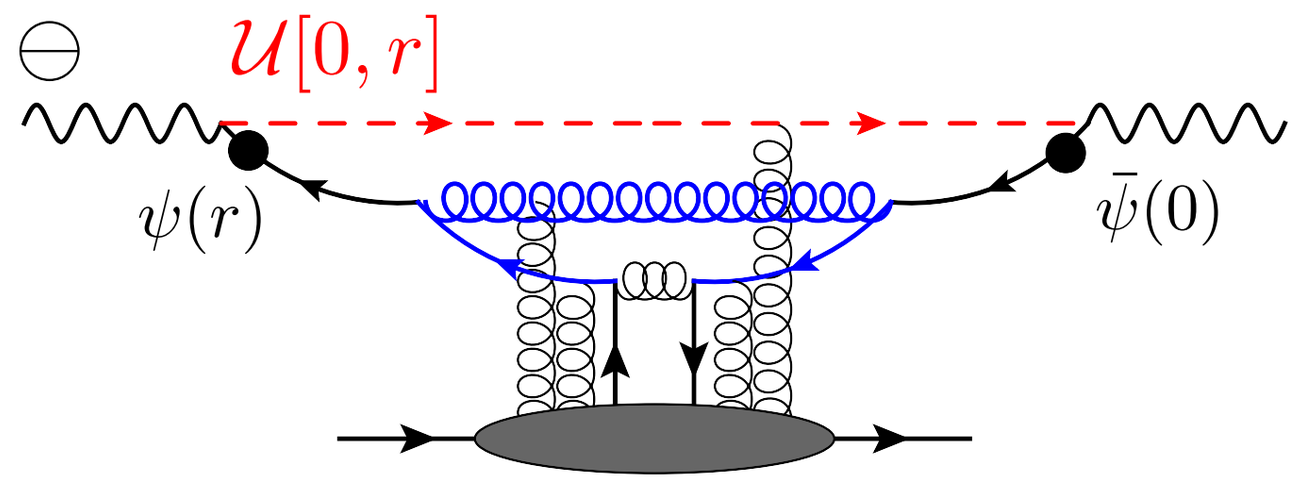


Small-x Helicity Evolution (BER)

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