SIDIS NLO Theory

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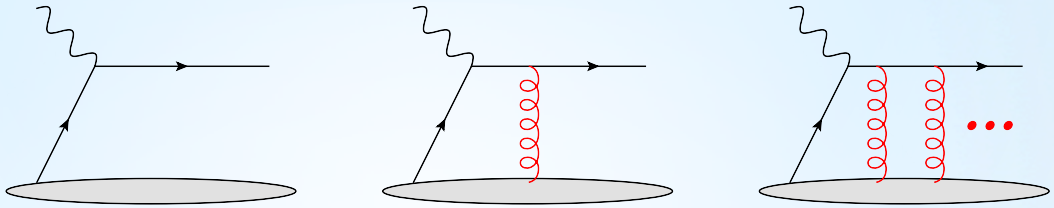
EST. 1943

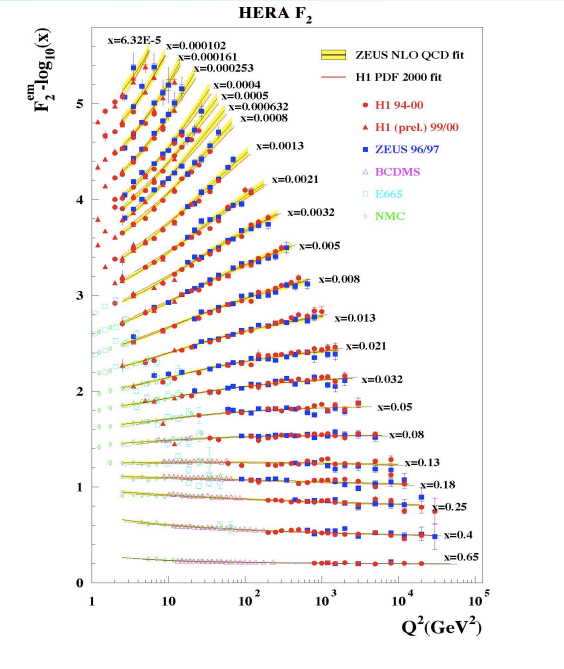
Success of QCD factorization (single scattering)

- Recall: Structure function in Deep Inelastic Scattering

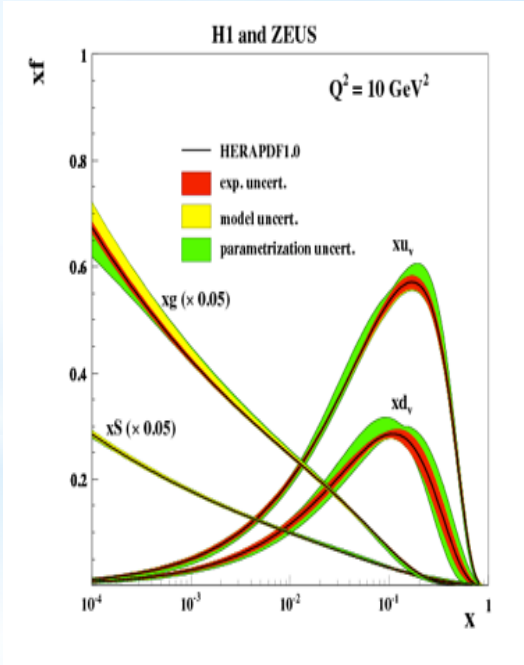


single scattering

multiple scatterings



$$F(x) = \sum_q \int_x^1 d\xi C_q(x/\xi, Q^2) f_q(\xi, Q^2)$$



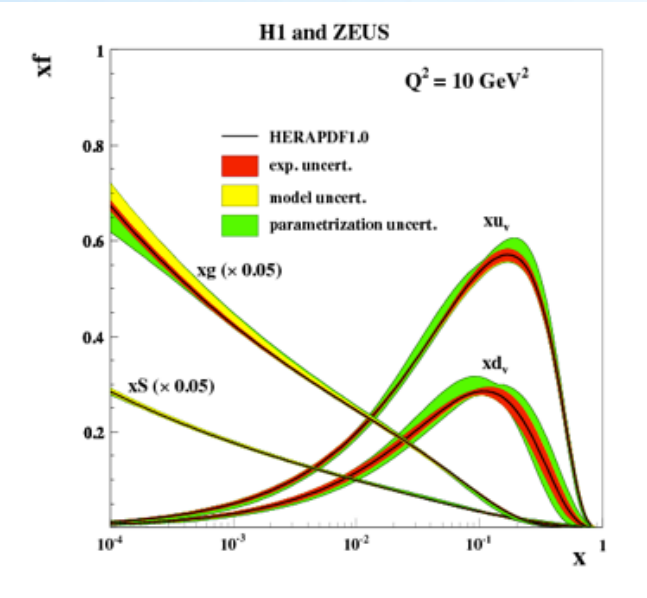
Structure function

PDFs

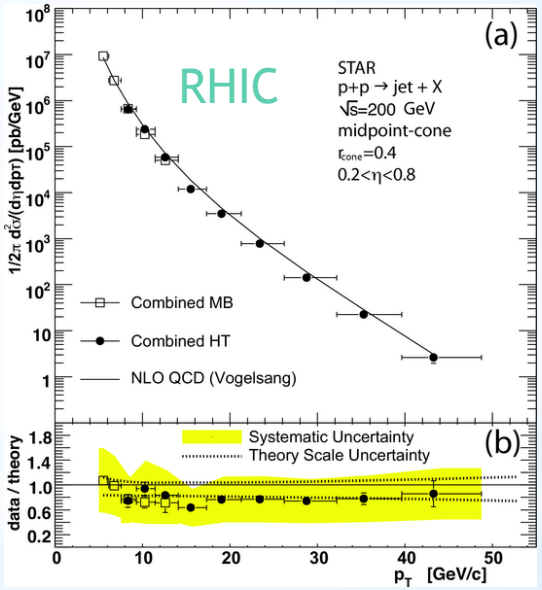
Success of QCD factorization (single scattering)

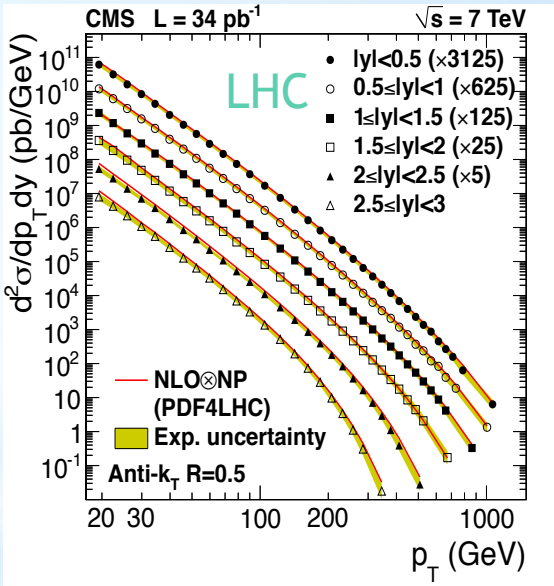
- Prediction: single inclusive jet production in pp collisions

$$\frac{d\sigma}{dp_T} = \sum_{n=2}^{\infty} \int dx_a \int dx_b f_a(x_a,\mu) f_b(x_b,\mu) \int d\mathcal{P}S^n \frac{d\hat{\sigma}}{d\mathcal{P}S^n} \mathcal{S}_n(p_1,\ldots,p_n)$$



PDFs

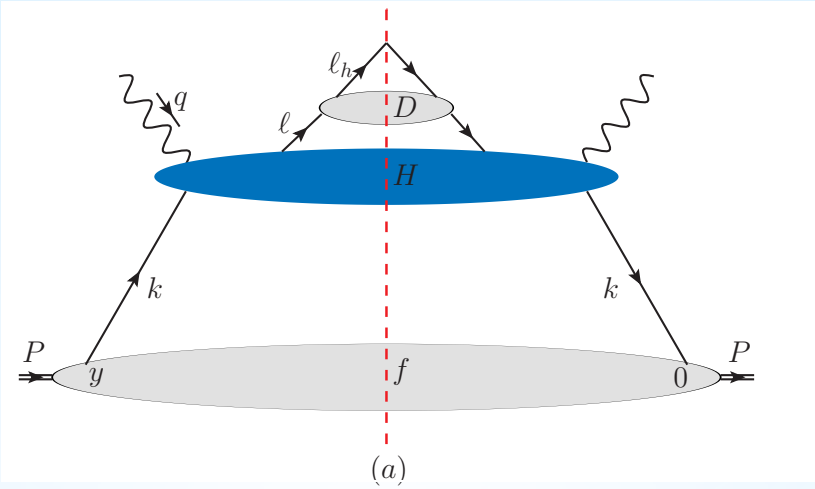




cross sections

Prediction power of pQCD!

Semi-Inclusive Deep Inelastic Scattering



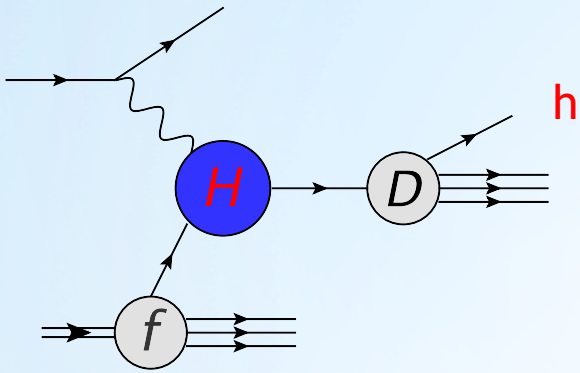
- Complements to DIS

  Flavor separation of partonic contribution

- Opportunity to study hadronization

  Probe of nuclear medium properties

SIDIS at leading twist

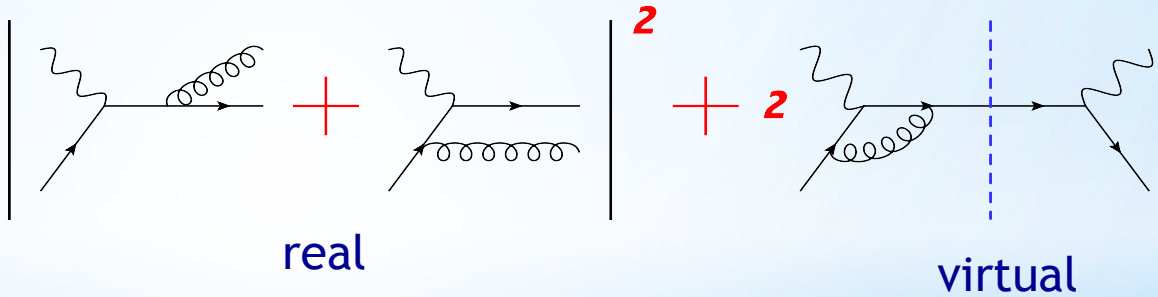


$$e(L_1) + A(P) \to e(L_2) + h(\ell_h) + X$$

$$x_B = \frac{Q^2}{2p\cdot q}, \qquad y = \frac{p\cdot q}{p\cdot L_1}, \qquad z_h = \frac{p\cdot \ell_h}{p\cdot q},$$

$$\hat{x} = \frac{x_B}{x}, \qquad \hat{z} = \frac{z_h}{z}, \qquad z = \frac{\ell_h}{\ell}$$

- NLO - SIDIS:



Four possible regions of gluon momentum:

1. collinear to initial quark (PDF)
2. collinear to final quark (FF)
3. soft (IR singularities cancel)
4. hard (NLO correction)

- # Redefinition of nonperturbative function

Dimensional regularization: $n = 4 - 2\epsilon$

$\overline{MS}$ - scheme: $\frac{1}{\hat{\epsilon}} = \frac{1}{\epsilon} + \ln(4\pi) - \gamma_E$

FF: $$D_q(z_h, \mu^2) = D_q^0(z_h) + \frac{\alpha_s}{2\pi} \int \frac{dz}{z} \left(-\frac{1}{\hat{\epsilon}}\right) P_{qq}(\hat{z}) D_q(z)$$

PDF: $$f_q(x_B, \mu^2) = f_q^0(x_B) + \frac{\alpha_s}{2\pi} \int \frac{dx}{x} \left(-\frac{1}{\hat{\epsilon}}\right) P_{qq}(\hat{x}) f_q(x)$$

- # DGLAP evolution

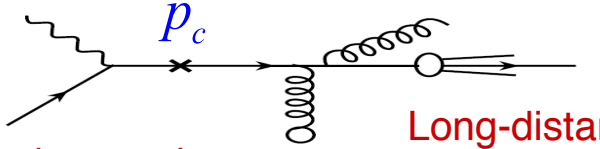
$$\mu^2 \frac{d}{d\mu^2} \begin{bmatrix} f_q(x_B, \mu^2) \\ f_g(x_B, \mu^2) \end{bmatrix} = \frac{\alpha_s}{2\pi} \int_{x_B}^1 \frac{dx}{x} \begin{bmatrix} P_{qq}(\hat{x}) & P_{qg}(\hat{x}) \\ P_{gq}(\hat{x}) & P_{gg}(\hat{x}) \end{bmatrix} \begin{bmatrix} f_q\left(x, \mu^2\right) \\ f_g\left(x, \mu^2\right) \end{bmatrix} \longrightarrow \text{PDF}$$

$$\mu^2 \frac{d}{d\mu^2} \begin{bmatrix} D_q(z_h, \mu^2) \\ D_g(z_h, \mu^2) \end{bmatrix} = \frac{\alpha_s}{2\pi} \int_{z_h}^1 \frac{dz}{z} \begin{bmatrix} P_{qq}(\hat{z}) & P_{qg}(\hat{z}) \\ P_{gq}(\hat{z}) & P_{gg}(\hat{z}) \end{bmatrix} \begin{bmatrix} D_q\left(z, \mu^2\right) \\ D_g\left(z, \mu^2\right) \end{bmatrix} \longrightarrow \text{FF}$$

SIDIS: probe of nuclear medium effect

- **Multiple scattering with/out radiation:**

Guo & Wang PRL 2000, …
Wang & Wang, PRL 2002, …



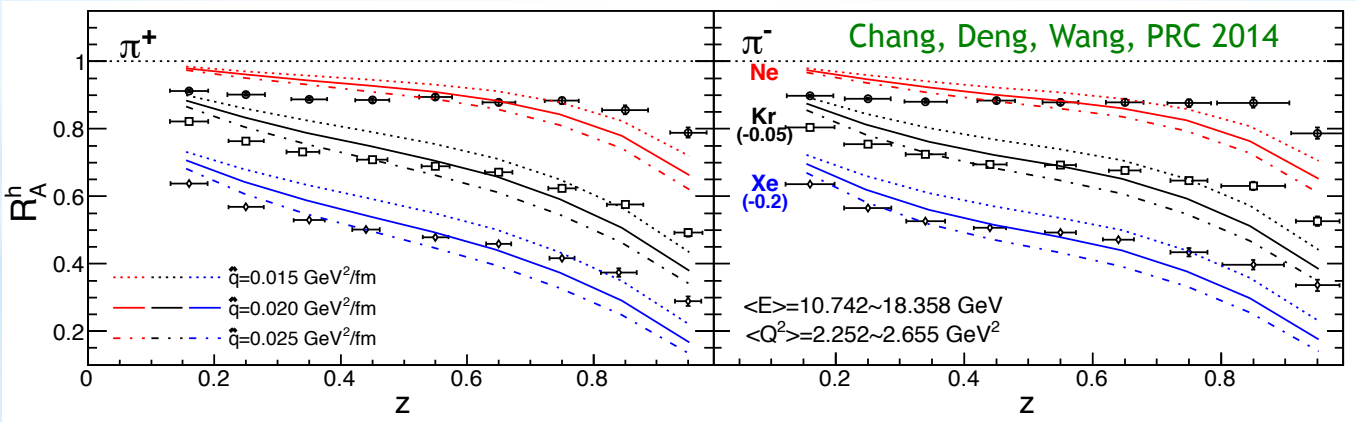
Long-distance fragmentation

Short-distance hard scattering

$p_c^2 \ll O(Q^2)$ → Induced radiation modifies the fragmentation

→ First scattering determines the parton production rate

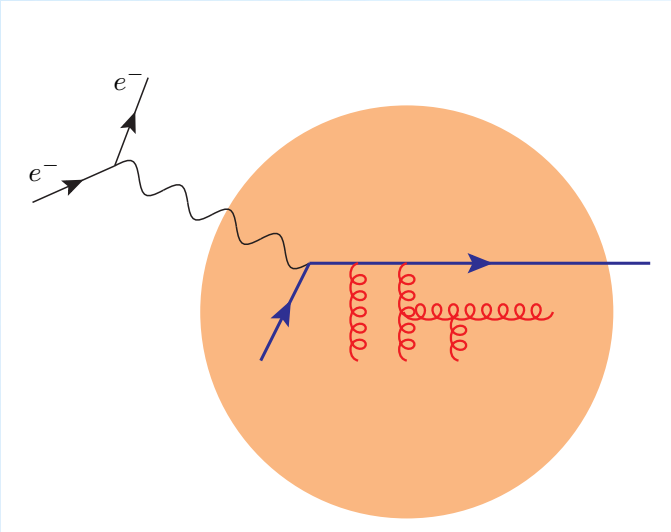
$\ln(Q^2/p_c^2)$ → Modifies *DGLAP evolution of the Fragmentation function – "energy loss"*

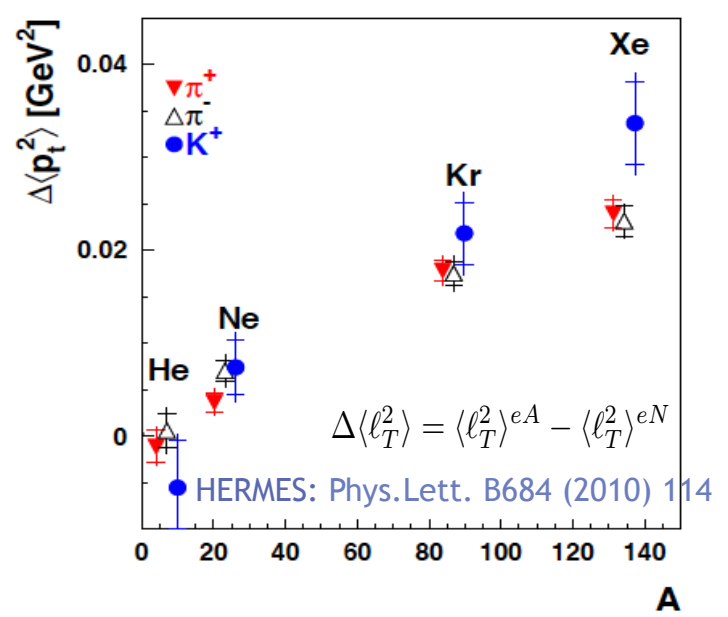


Energy loss in cold nuclear matter

Transverse momentum broadening

- Transverse momentum broadening (TMB)





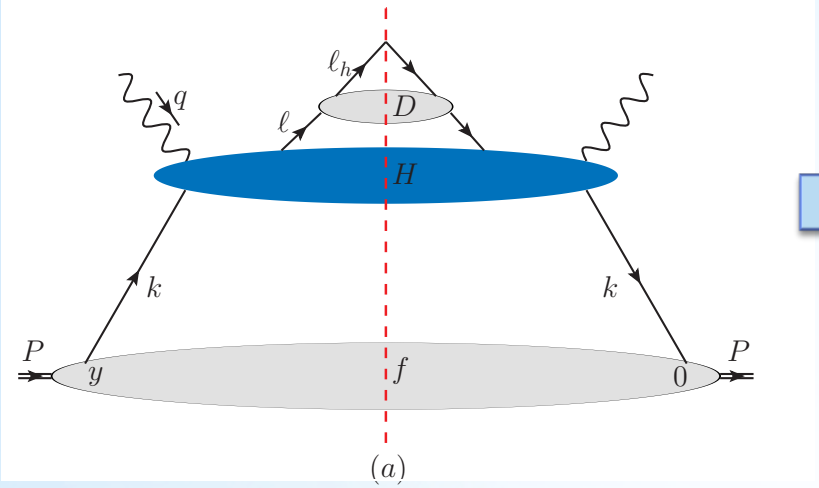
$$\Delta\langle \ell_T^2\rangle = \langle \ell_T^2\rangle^{eA} - \langle \ell_T^2\rangle^{eN} \approx \frac{\int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eA}^D}{dQ^2 d\ell_T^2}}{\frac{d\sigma_{eA}}{dQ^2}}$$

Double scattering - leading contribution to TMB

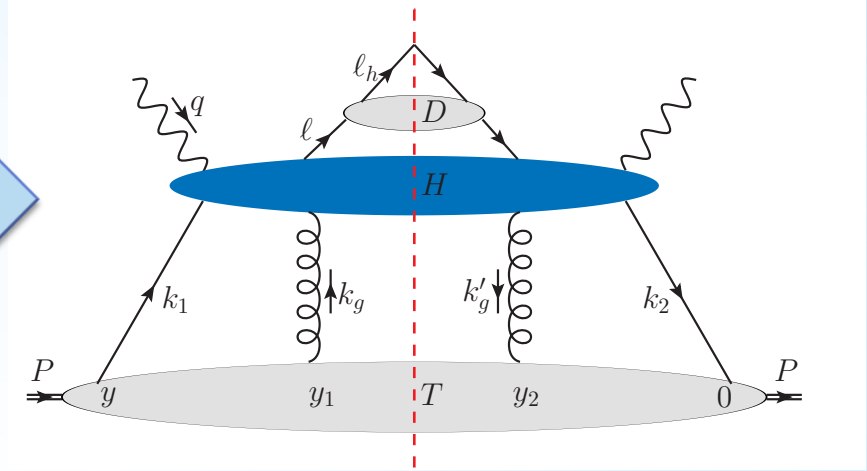
Easy to measure, perturbatively calculable

Double scattering in SIDIS

- Final state multiple scattering



Single scattering



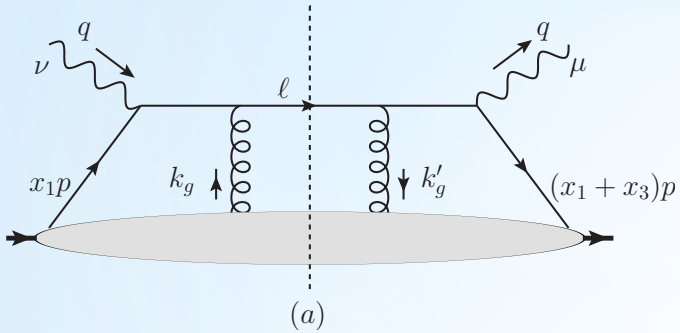
Double scattering

- Twist-4 factorization formalism Qiu, Sterman 1990s

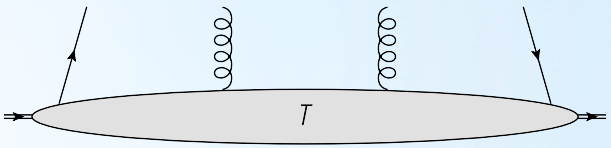
$$\sigma^D \sim T_{qg} \otimes H_4 \otimes D_h$$

Double scattering leading order

- Leading order contribution



T-4 q-g correlation



- TMB LO Guo, 1998; Guo, Qiu 2000

$$\Delta\langle \ell_{hT}^2 \rangle = \left(\frac{4\pi^2 \alpha_s}{N_c} z_h^2 \right) \frac{\sum_q e_q^2 T_{qg}(x_B, 0, 0) D_{h/q}(z_h)}{\sum_q e_q^2 f_{q/A}(x_B) D_{h/q}(z_h)}$$

- T-4 q-g correlation

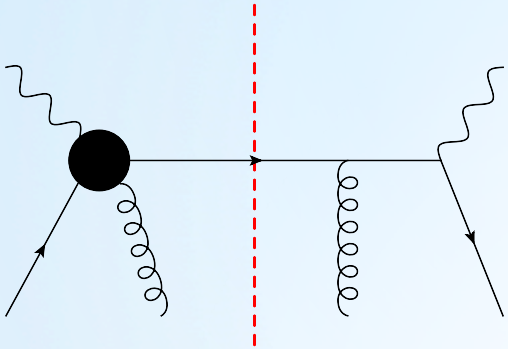
$$T_{qg}(x_1, x_2, x_3) = \int \frac{dy^-}{2\pi} e^{ix_1 p^+ y^-} \int \frac{dy_1^- dy_2^-}{4\pi} e^{ix_2 p^+ (y_1^- - y_2^-)} e^{ix_3 p^+ y_2^-}$$
$$\times \langle A | \bar{\psi}_q(0) \gamma^+ F_\sigma^+(y_2^-) F^{\sigma +}(y_1^-) \psi_q(y^-) | A \rangle \theta(y_2^-) \theta(y_1^- - y^-)$$

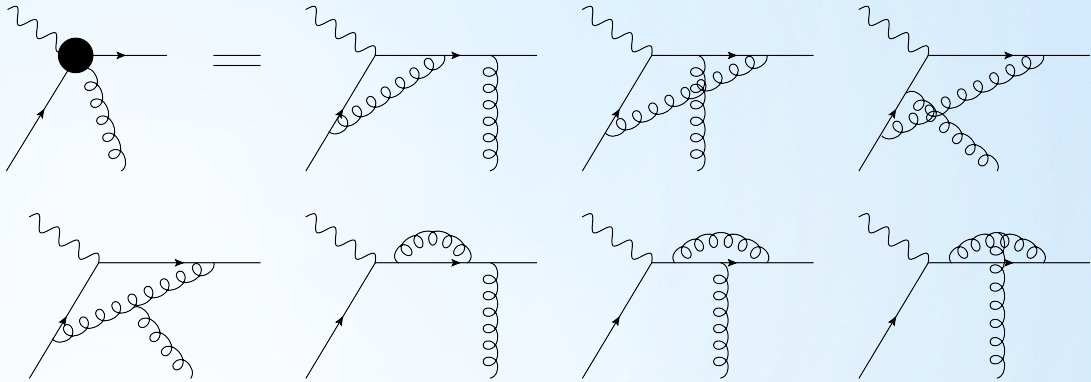
Provide a way to measure the T-4 quark-gluon correlation function.

Double scattering NLO

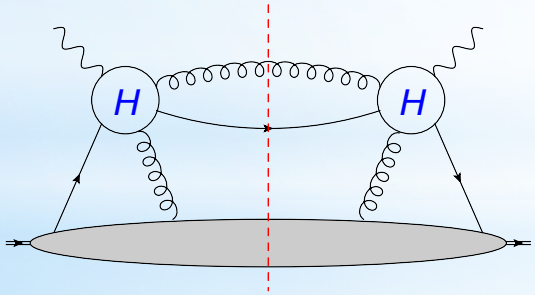
- Virtual (7 diagrams)

Kang, Wang, Wang, Xing, PRL 112, 102001 (2014)

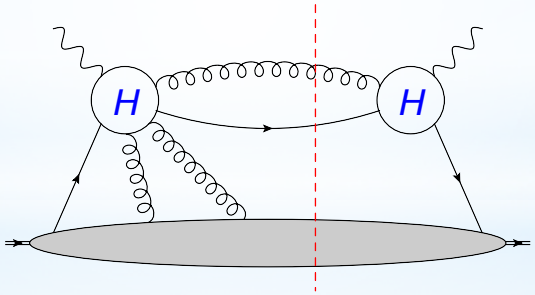




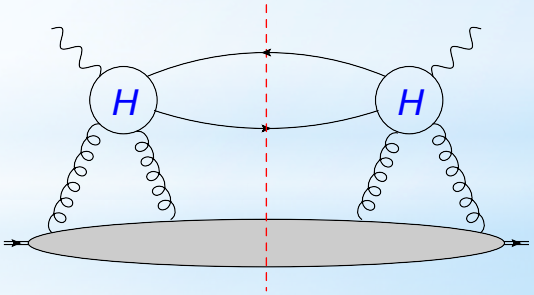
- Real (69 diagrams)



q-g double scattering



q-g single-triple interference

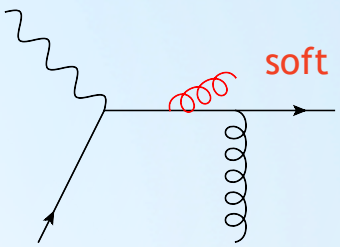


g-g double scattering

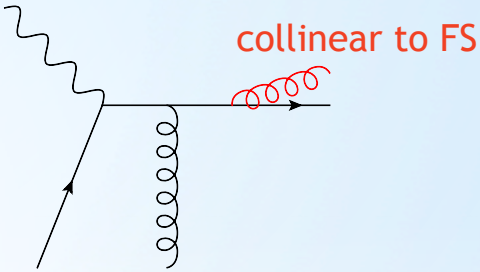
Phase space identification and renormalization

- Soft divergence: p_g --> 0

Real + virtual = 0



- collinear divergence I: p_g // p_q



$$-\frac{1}{\epsilon}\delta(1-\hat{x})T_{qg}(x,0,0)P_{qq}(\hat{z})$$

$$\Big\downarrow \overline{MS} \qquad n = 4 - 2\epsilon$$

$$D_{h/q}(z_h,\mu_f^2) = D_{h/q}(z_h) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu^2}{\mu_f^2}\right)\int_{z_h}^{1}\frac{dz}{z}P_{qq}(\hat{z})D_{h/q}(z)$$

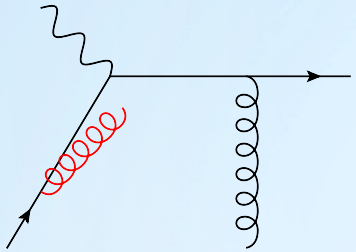
Redefinition of FF

$$\Big\downarrow$$

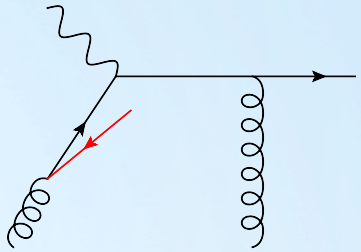
$$\mu^2\frac{\partial D_{h/q}(z_h,\mu^2)}{\partial\mu^2} = \frac{\alpha_s}{2\pi}\int\frac{dz}{z}P_{qq}(\hat{z})D_{h/q}(z,\mu^2)$$

DGLAP of FF

- collinear divergence II: p_g // k_q



$$-\frac{1}{\epsilon}\delta(1-\hat{z})\mathcal{P}_{qg\to qg}(\hat{x})\otimes T_{qg}D_{h/q}(z)$$



$$-\frac{1}{\epsilon}\delta(1-\hat{z})P_{qg}(\hat{x})T_{gg}(x,0,0)D_{h/q}(z)$$

New splitting kernel

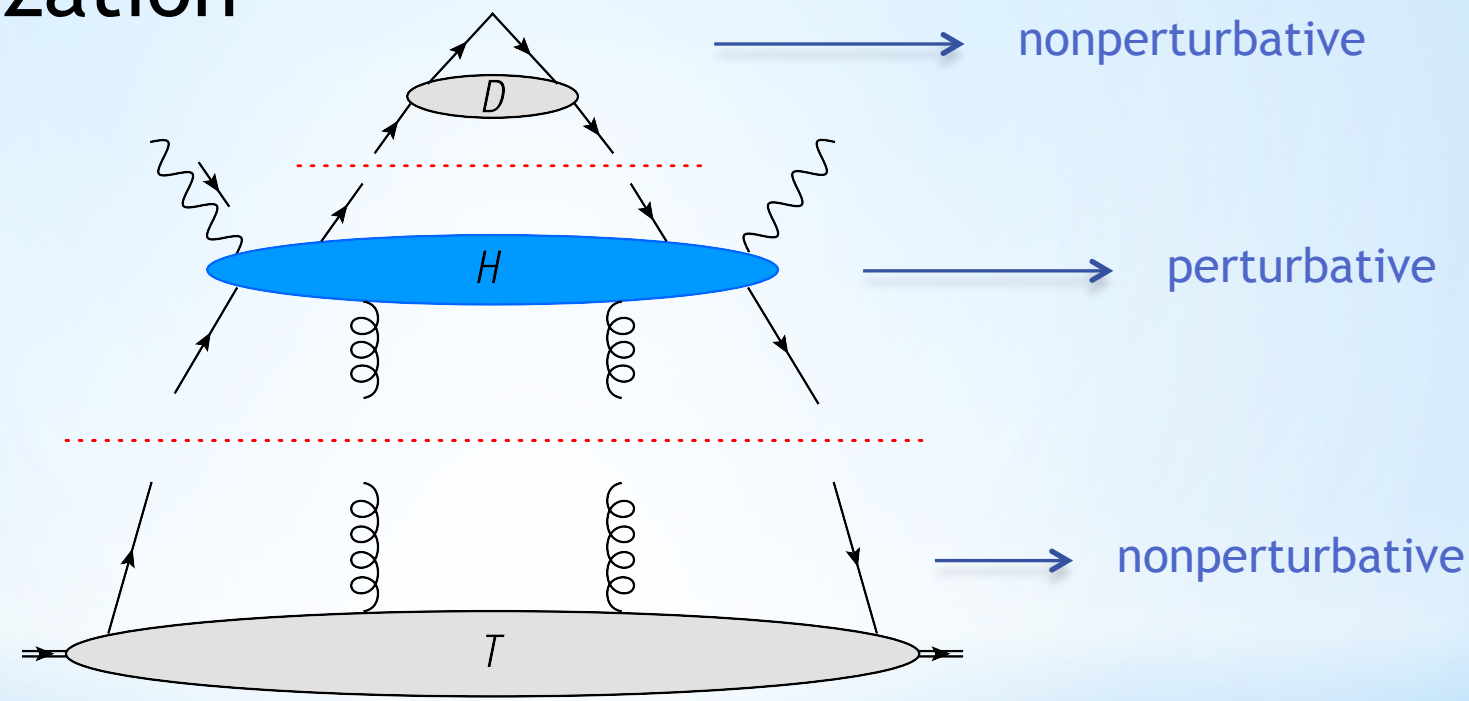
$$\mathcal{P}_{qg\to qg}\otimes T_{qg}$$
$$\equiv P_{qq}(\hat{x})T_{qg}(x,0,0)+\frac{C_A}{2}\left\{\frac{4}{(1-\hat{x})_+}T_{qg}(x_B,x-x_B,0)\right.$$
$$\left.-\frac{1+\hat{x}}{(1-\hat{x})_+}\left[T_{qg}(x,0,x_B-x)+T_{qg}(x_B,x-x_B,x-x_B)\right]\right\}$$
$$+2C_A\delta(1-\hat{x})T_{qg}(x,0,0)$$

$$T_{qg}(x_B,0,0,\mu_f^2)=T_{qg}(x_B,0,0)-\frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}}+\ln\frac{\mu^2}{\mu_f^2}\right)\int_{x_B}^{1}\frac{dx}{x}\left[\mathcal{P}_{qg\to qg}\otimes T_{qg}+P_{qg}(\hat{x})T_{gg}(x,0,0)\right]$$

Redefinition of T-4 quark-gluon correlation function

Full NLO at twist-4 in SIDIS

- Factorization

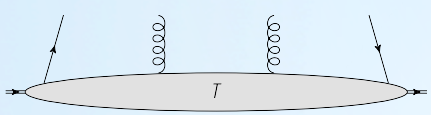


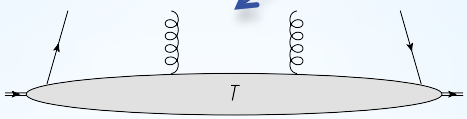
$$\frac{d\langle \ell_{hT}^2 \sigma\rangle}{dz_h} \propto D_{q/h}(z,\mu^2) \otimes H^{LO}(x,z) \otimes T_{qg}(x,0,0,\mu^2) + \frac{\alpha_s}{2\pi} D_{q/h}(z,\mu^2) \otimes H^{NLO}(x,z,\mu^2) \otimes T_{qg(gg)}(x,0,0,\mu^2)$$

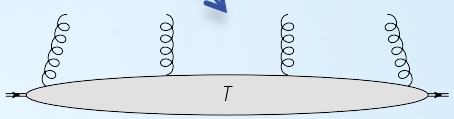
Multiple scattering hard part coefficients and medium properties can be factorized!!!

- Evolution equation for q-g correlation function - NEW

$$\mu_f^2 \frac{\partial}{\partial \mu_f^2} T_{qg}(x_B, 0, 0, \mu_f^2) = \frac{\alpha_s}{2\pi} \int_{x_B}^{1} \frac{dx}{x} \left[\mathcal{P}_{qg\to qg} \otimes T_{qg} + P_{qg}(\hat{x}) T_{gg}(x, 0, 0, \mu_f^2) \right]$$

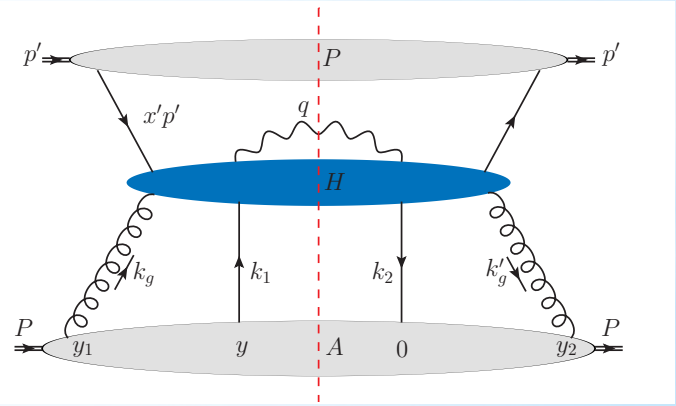






- SIDIS VS DY: Final state vs initial state

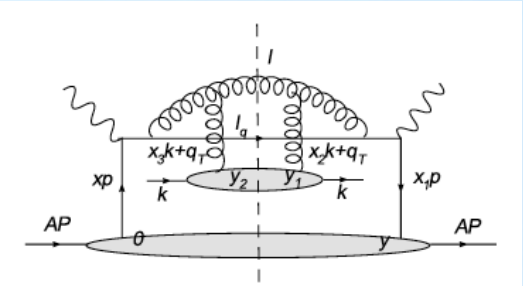
Same renormalization result for q-g correlation function at NLO , indicating qhat is universal, independent of the hard process.



Discussion - Evolution of jet transport parameter

- Related to jet transport parameter

J. Casalderrey-Solana and X.-N. Wang (2008)



$$T_{qg}(x_B,0,0,\mu_f^2) \approx \frac{N_c}{4\pi^2\alpha_s} f_{q/A}(x_B,\mu_f^2)\int dy^-\,\hat{q}_q(\mu_f^2,y^-)$$

- Evolution equation of jet transport parameter

$$\mu_f^2\frac{\partial}{\partial\mu_f^2}T_{qg}(x_B,0,0,\mu_f^2) = \frac{\alpha_s}{2\pi}\int_{x_B}^1\frac{dx}{x}\left[\mathcal{P}_{qg\to qg}\otimes T_{qg} + P_{qg}(\hat{x})T_{gg}(x,0,0,\mu_f^2)\right]$$

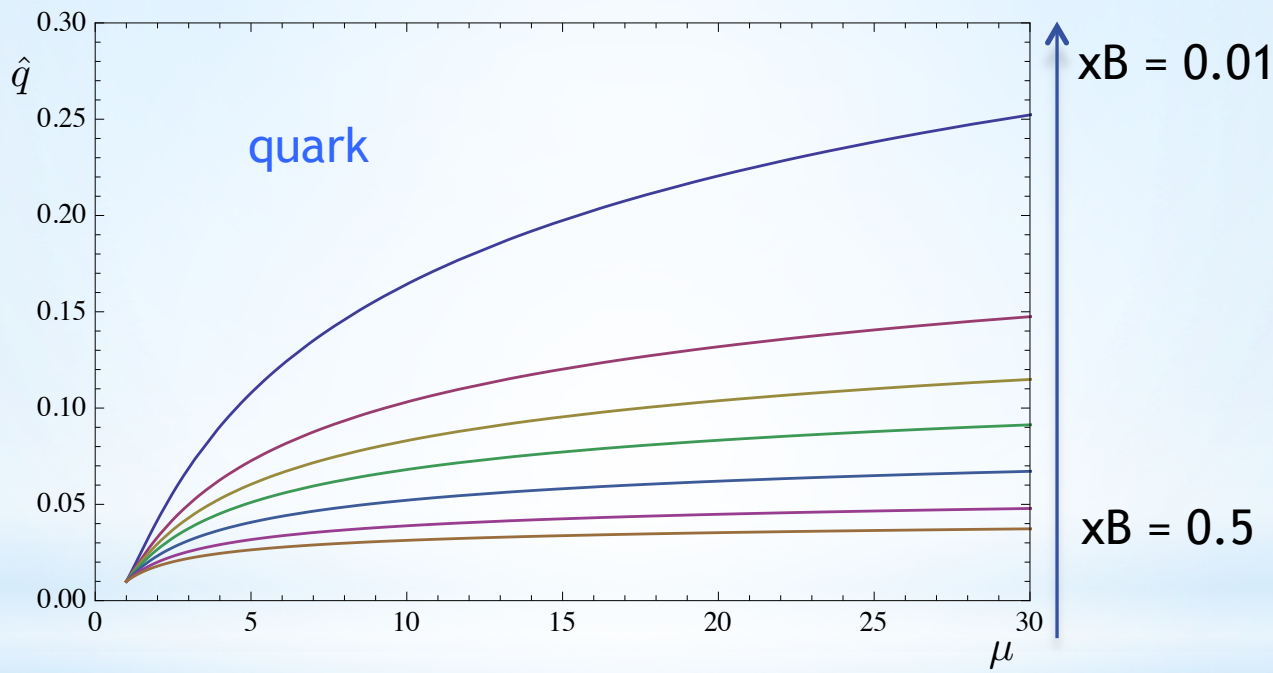
$$\mathcal{P}_{qg\to qg}\otimes T_{qg} \equiv P_{qq}(\hat{x})T_{qg}(x,0,0) + \frac{C_A}{2}\left\{\frac{4}{(1-\hat{x})_+}T_{qg}(x_B,x-x_B,0) - \frac{1+\hat{x}}{(1-\hat{x})_+}\left[T_{qg}(x,0,x_B-x) + T_{qg}(x_B,x-x_B,x-x_B)\right]\right\} + 2C_A\delta(1-\hat{x})T_{qg}(x,0,0)$$

1. Soft radiation limit ($\hat{x}\to 1$)

$$\mu^2\frac{\partial\hat{q}_q(\mu^2)}{\partial\mu^2} = \frac{\alpha_s}{2\pi}2C_A\hat{q}_q(\mu^2)$$

2. Evolution of qhat in intermediate-x

$$\frac{\partial \hat{q}(\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi} C_A \left[\ln(1/x_B) + 2\right] \hat{q}(\mu^2) \Rightarrow \hat{q}(\mu^2) = \hat{q}(\mu_0^2) Exp\left[\frac{\alpha_s}{2\pi} C_A \left[\ln(1/x_B) + 2\right] \ln(\mu^2/\mu_0^2)\right]$$

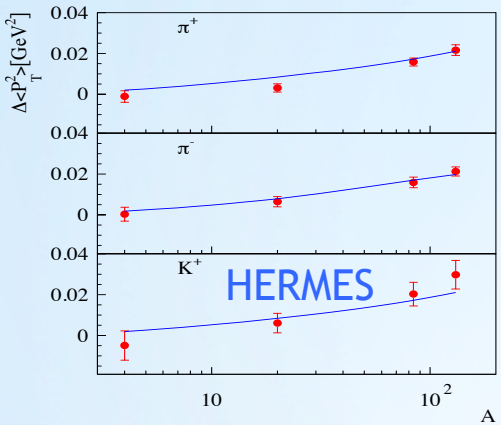


Energy dependence → consistent with earlier expectation $x_B \sim 1/E$

J. Casalderrey-Solana and X.-N. Wang (2008)

Different channels to probe T-4 parton-parton correlation function

SIDIS



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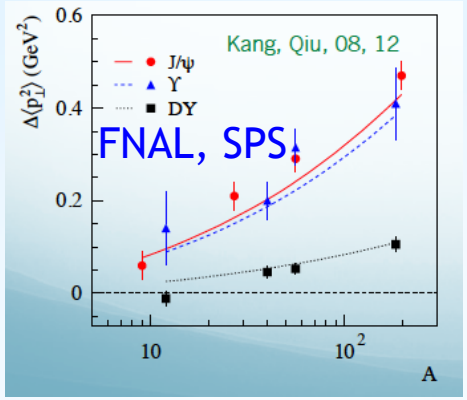
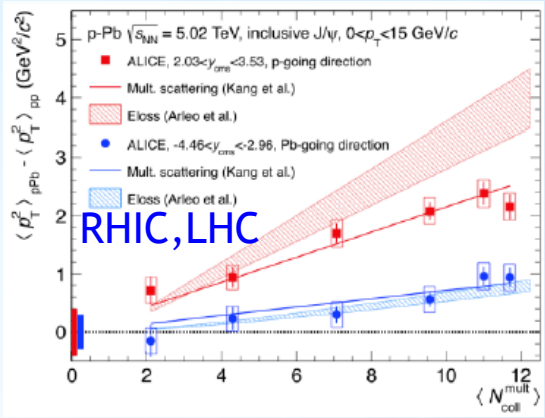


J/Psi



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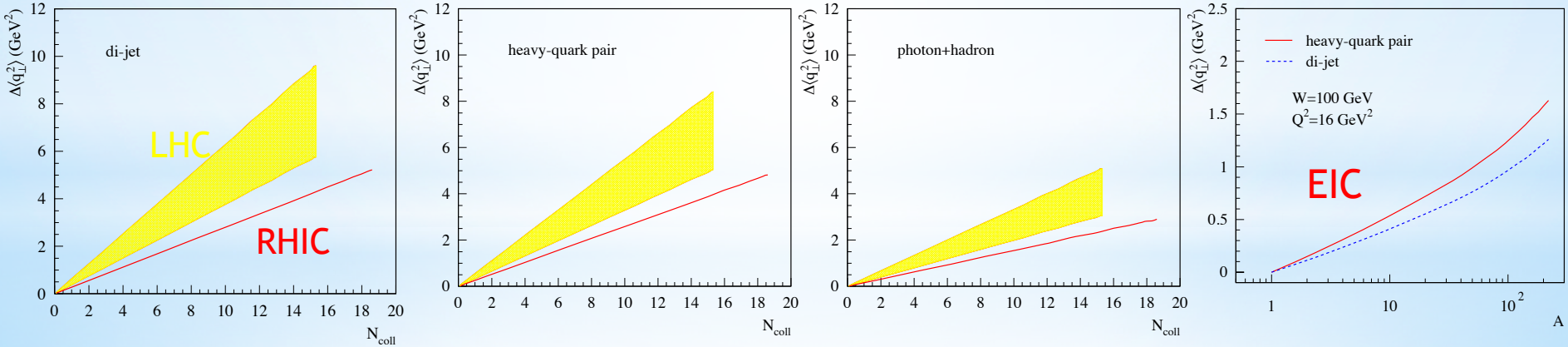

Transverse momentum imbalance for back-to-back particle in pA and eA collisions
Xing et al, PRD 86, 094010 (2012)

Summary

- Verify QCD factorization beyond leading twist
- Identify QCD evolution of nuclear q-g correlation function
- Probe QCD dynamics for multiple scattering