Why does a Quantum Field Theory for Intense Laser Matter Interactions matter?

Bella-i Workshop, Berkeley, Jan. 22, 2016

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The University of Texas at Austin

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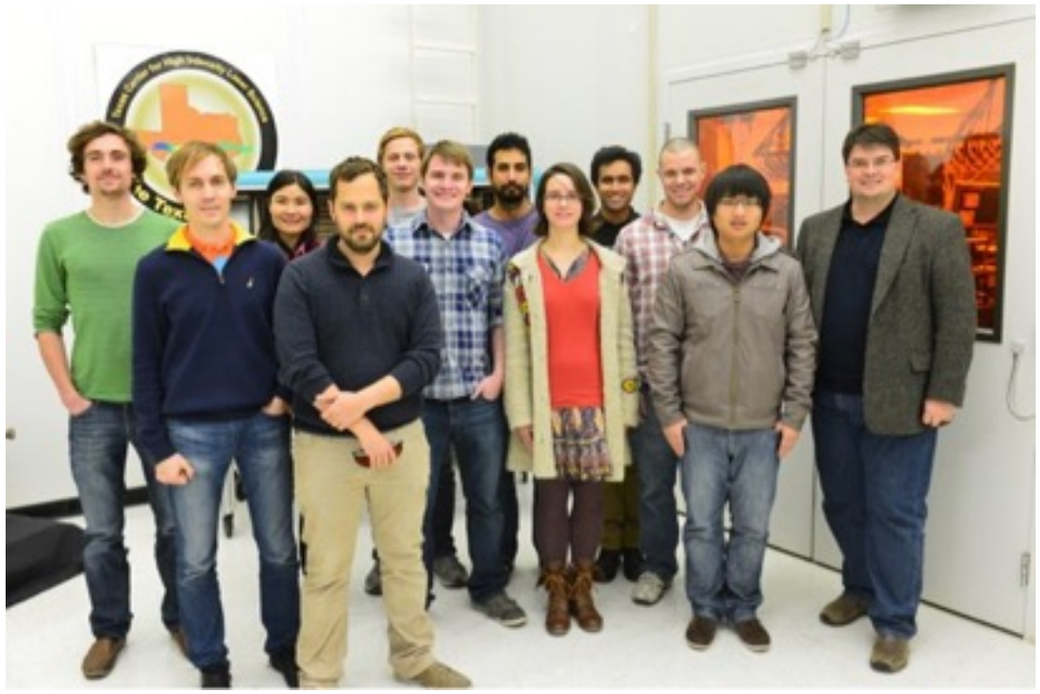
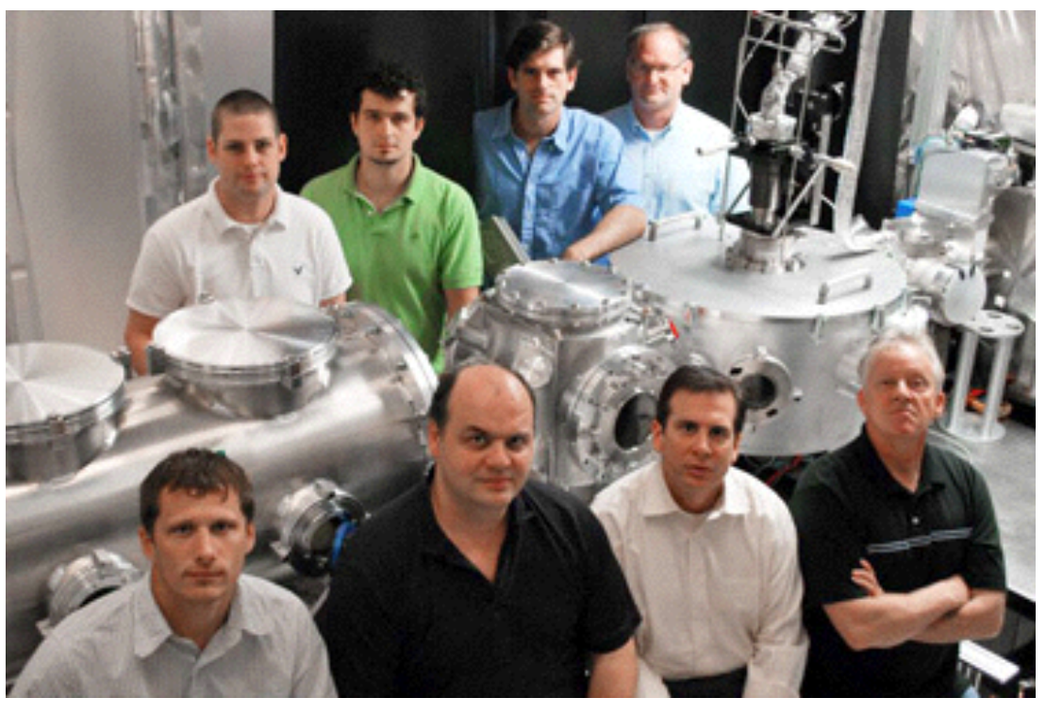
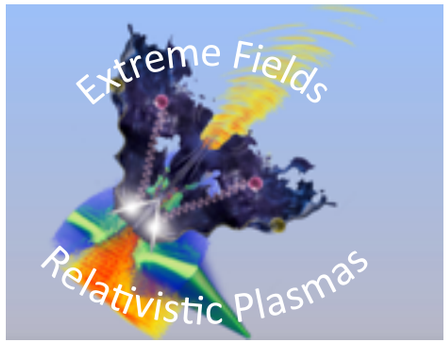
Extreme Fields and Relativistic Plasmas Group established September 2012







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Chunhua Wang (now Corning Inc.)

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Toma Toncian (now CHEDS)

Outline

- TPW & Bella-i: Synergies
- Texas PetaWatt Laser(TPW) Upgrades
  - Contrast
  - Beam quality/intensity
- Relativistic Plasmas
- Ion Acceleration
- nonlinear QED & Quantum Plasma Dynamics

Extreme Fields and Relativistic Plasmas Group - Sponsors & Collaborators

Sponsors

- NNSA
  - CHEDS
  - LANL
- AFOSR
- DTRA (pending)
- DARPA
- NIH
- UT

Collaborators

- UT
  - CHEDS
  - Condensed Matter Group (Li, Tsoi)
  - IFS
- Academic
  - National:
    - OSU, UCSD, Stanford, Loma Linda, UNR
  - International
    - LMU Munich (Germany), TU Darmstadt (Germany), QU Belfast (UK), Univ. Warwick (UK), Peking University (China)
- National Labs
  - LANL, LLNL, NRL, SLAC
  - GSI (Germany), SIOM (China)
- Industry
  - National Energetic
  - Radiabeam Technologies

High Intensity Lasers at UT Austin
3 other TW-lasers besides TPW

Texas PetaWatt Laser (TPW)

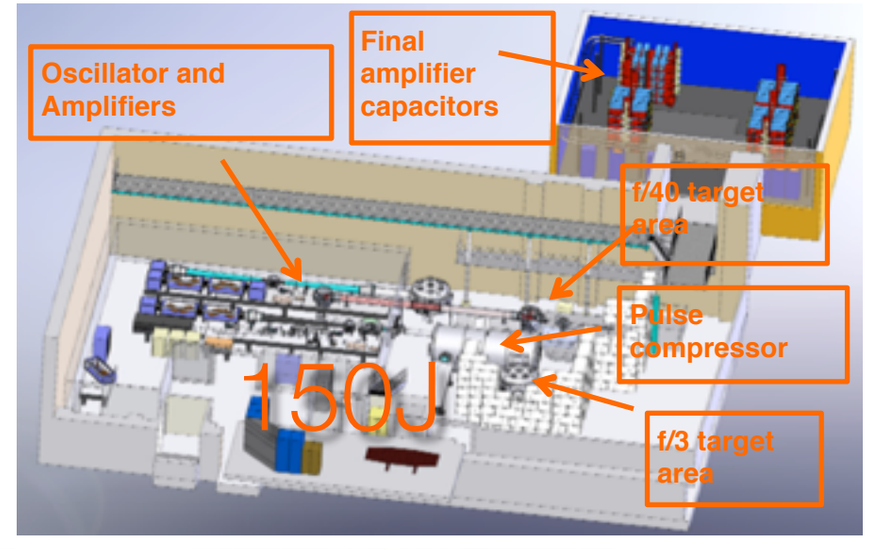
OPCPA-Glass, 1.1 PW = 1.1 x10^{15} W

2 Target Areas: F/3, F/40 (F/1, F/8)

>10^{21} W/cm^2 (>3x10^{22} W/cm^2 w F/1)

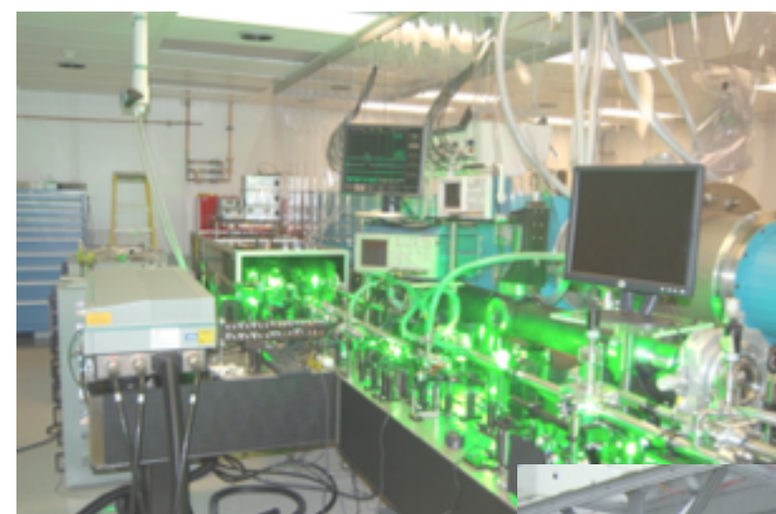
160 J, 140 fs

1 shot/hr

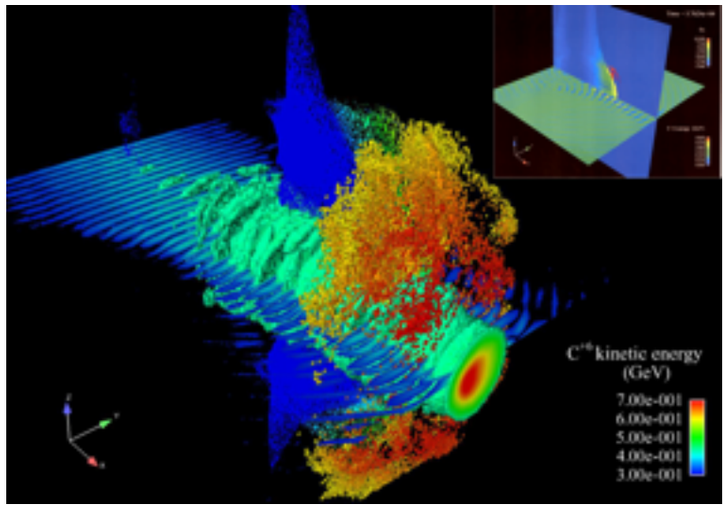


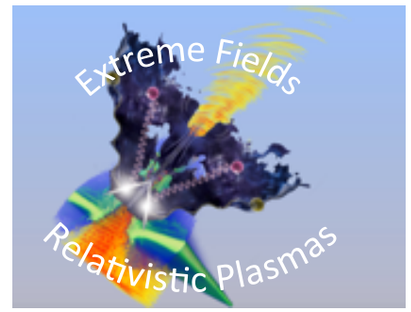
Bella-i:

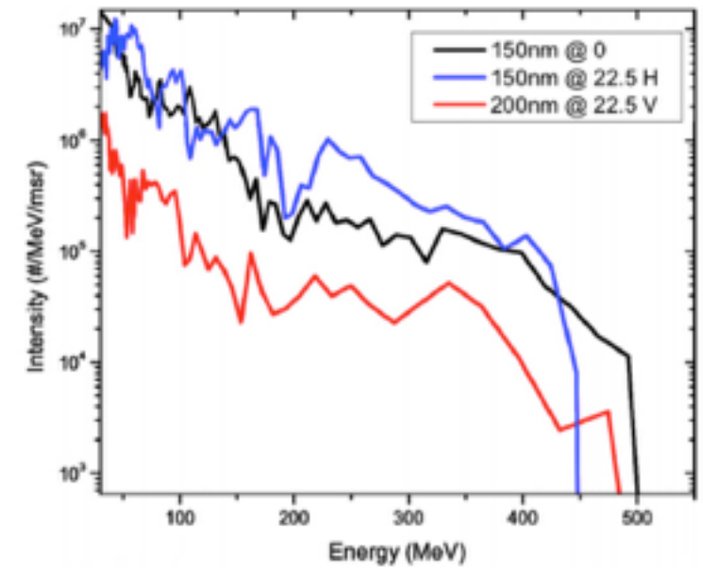
- same power
- similar focusing arrangements, intensities
- less energy
- higher repetition rate



- TPW can shoot at 40J
- Bella-i can shoot at 150s
- ⇒ overlapping parameter scans possible







Contrast Matters: Transferring Research from Trident to the Texas Petawatt Laser

Scaling:

TPW has 2x the energy and 1/4 the pulse duration

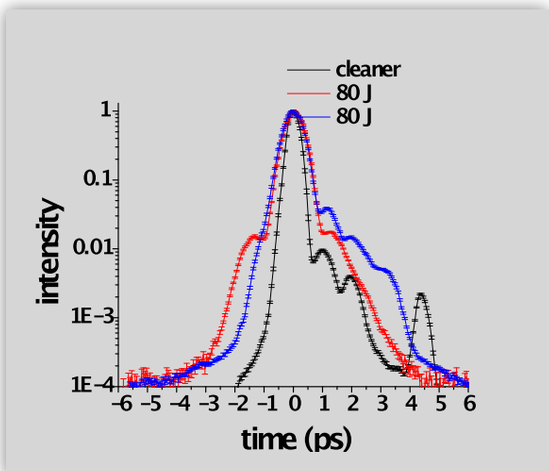
8x increase in intensity

expect even better performance!

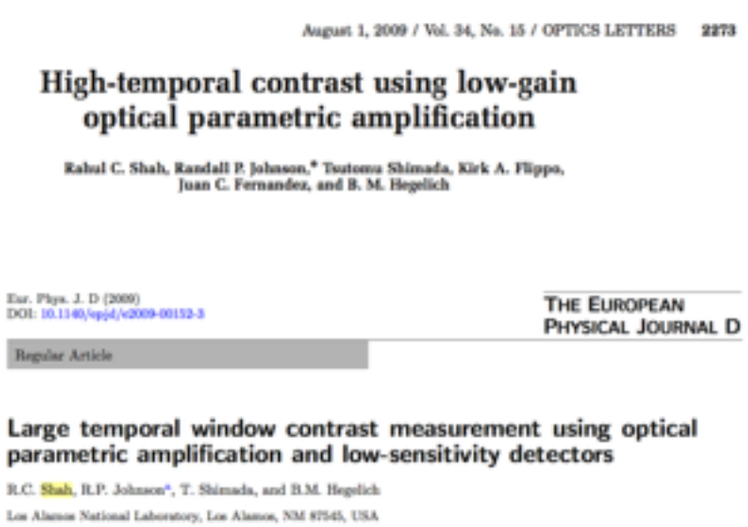
Influence of contrast on laser-plasma interaction

100nm Diamond:

- 22156 “good” contrast
- 22160 “bad” contrast



Single shot 3rd order cross correlator


August 1, 2009 / Vol. 34, No. 15 / OPTICS LETTERS 2273

High-temporal contrast using low-gain optical parametric amplification

Rahul C. Shah, Randall P. Johnson,* Tsutomu Shimada, Kirk A. Flippo, Juan C. Fernandez, and B. M. Hegelich

Eur. Phys. J. D (2009)
DOI: 10.1140/epjd/e2009-00152-3

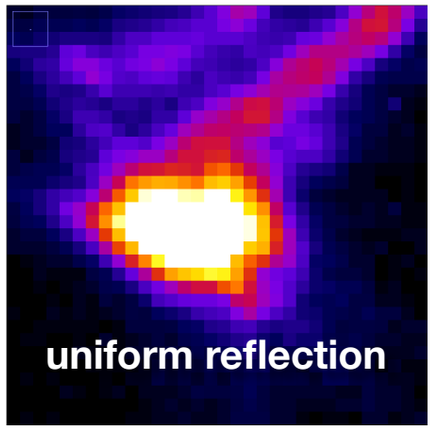
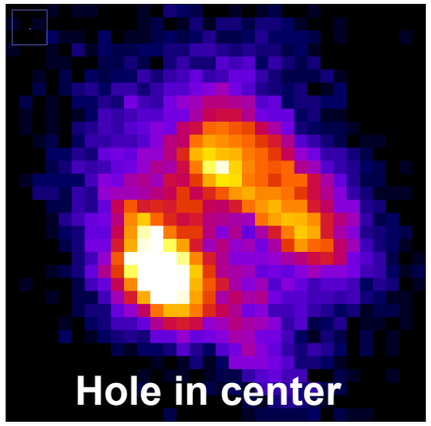
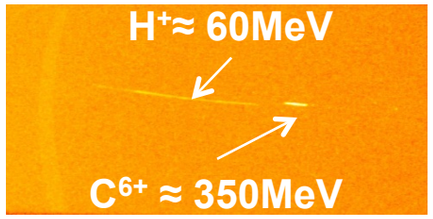
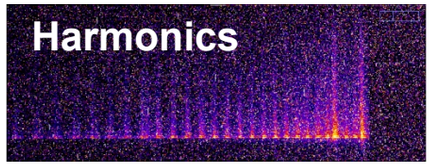
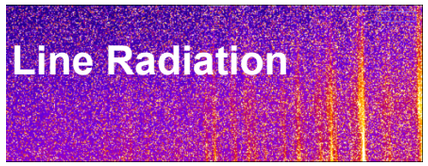
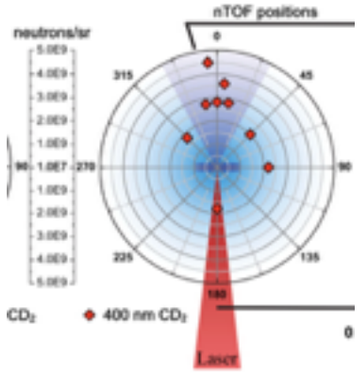
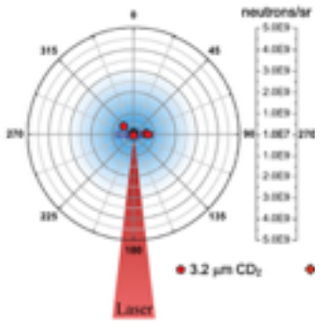
THE EUROPEAN PHYSICAL JOURNAL D

Regular Article

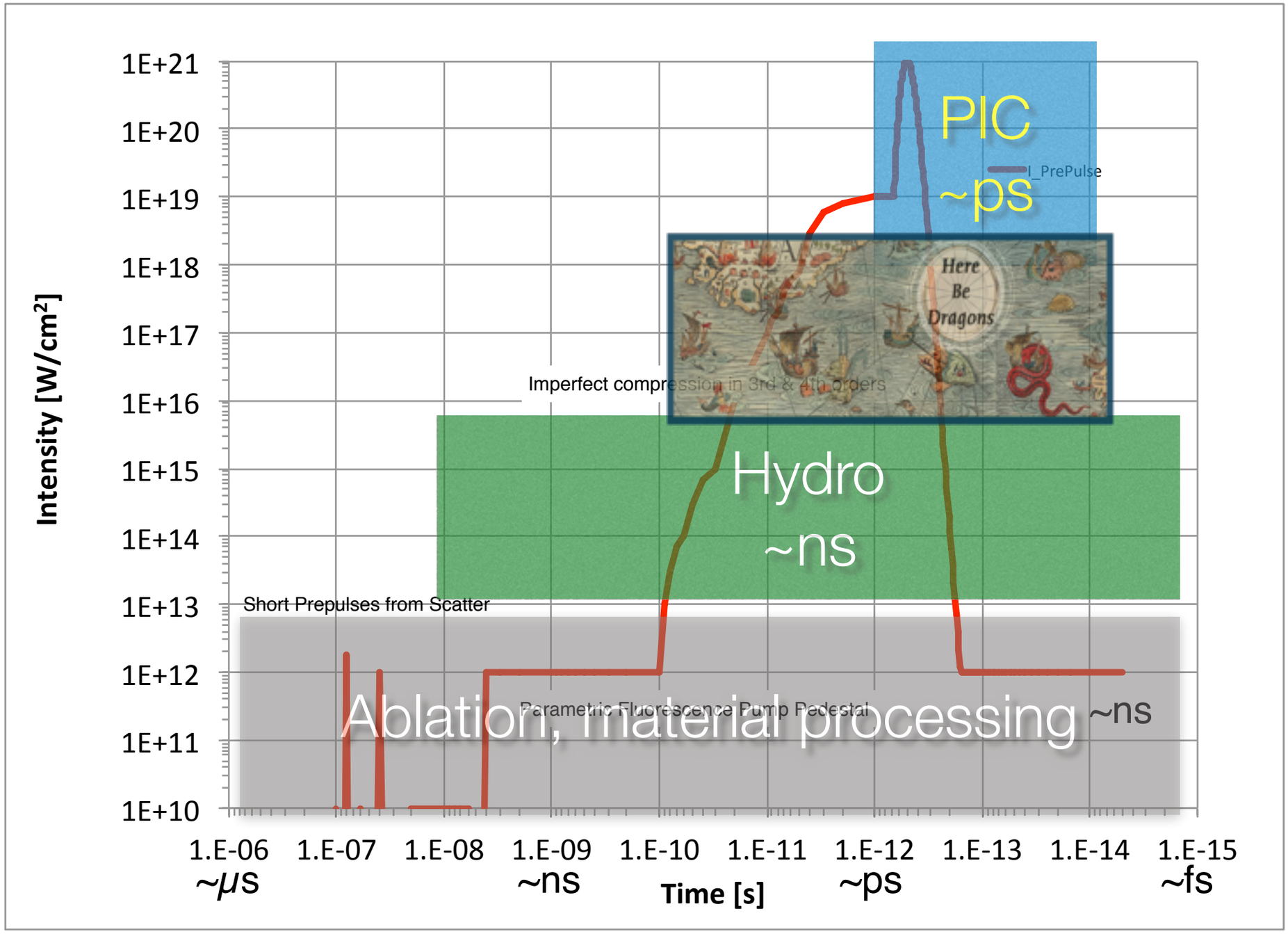
Large temporal window contrast measurement using optical parametric amplification and low-sensitivity detectors

R.C. Shah, R.P. Johnson*, T. Shimada, and B.M. Hegelich

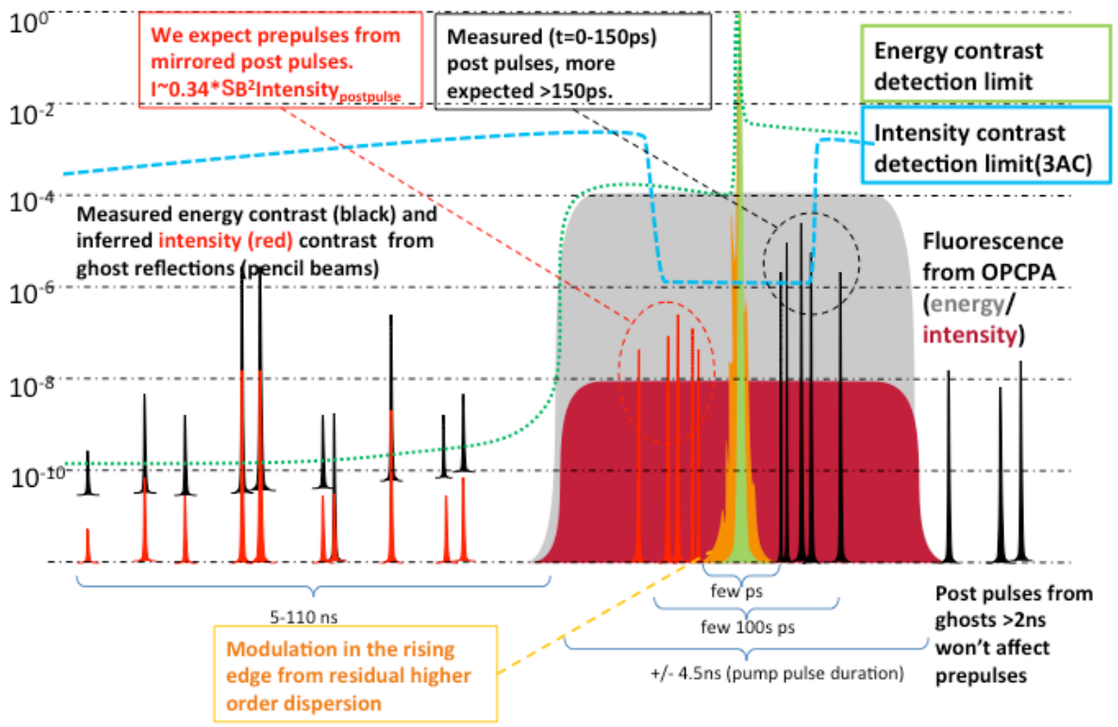
Los Alamos National Laboratory, Los Alamos, NM 87545, USA

Good Contrast		Bad Contrast
Shot 22156		Shot 22160
	3ω back scatter	
	ions	
	coherent x-rays	
	neutrons	

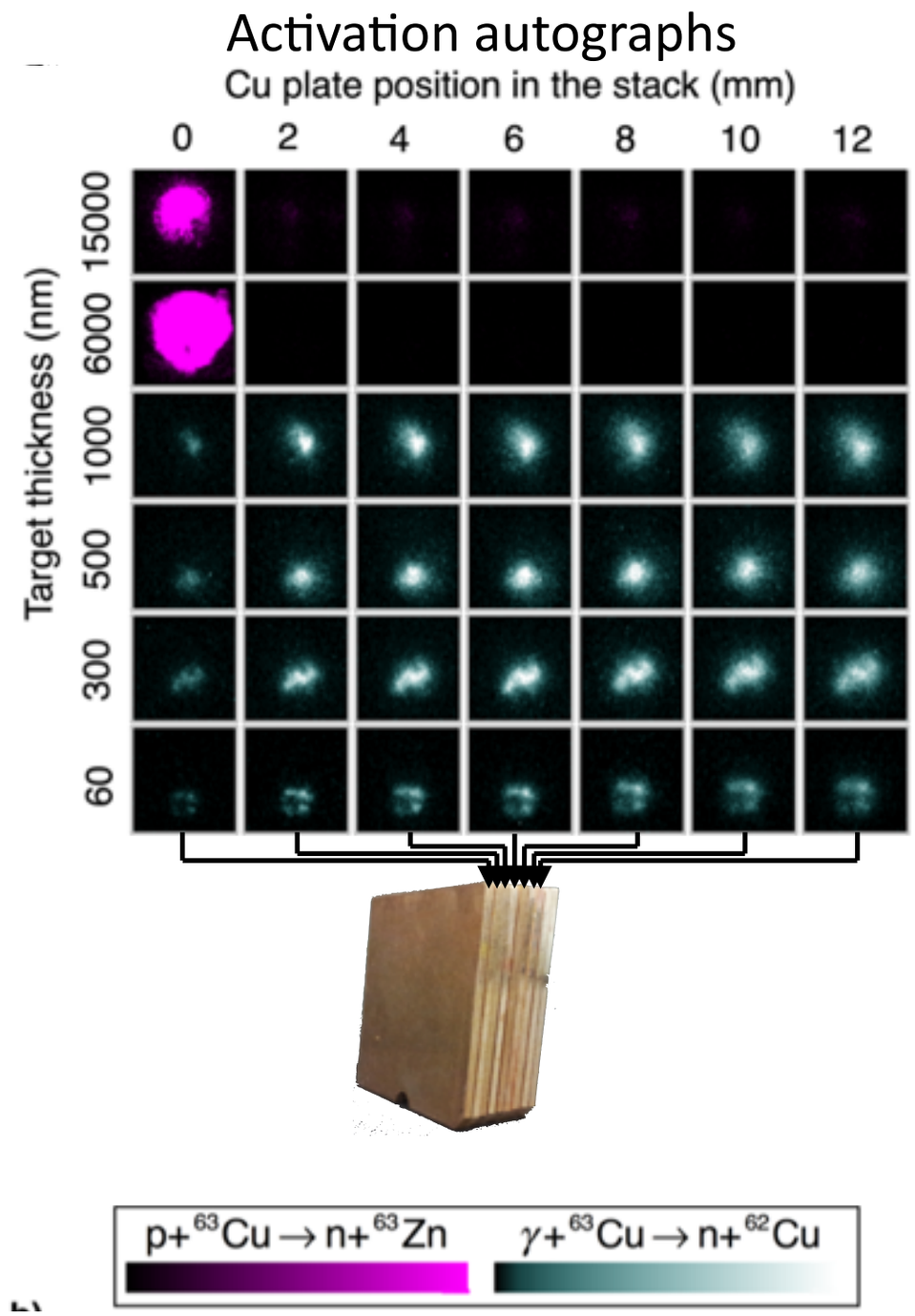
Simplified High Dynamic Range Pulse II



In 2013 we characterized and tested TPW contrast to sobering effect



Before upgrade



Contrast Measurements

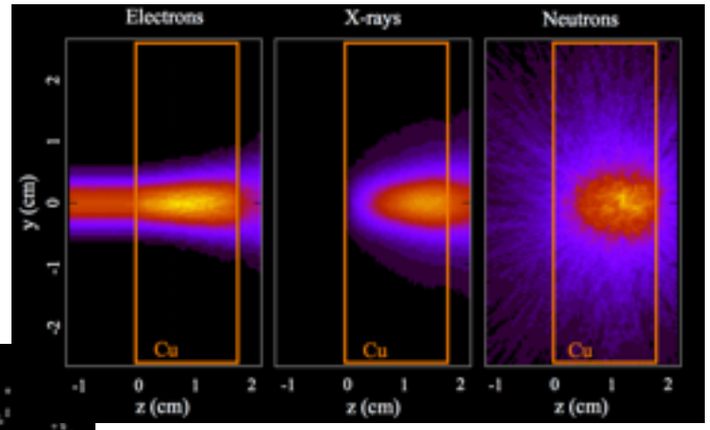
- numerous prepuces up to 110s before main pulse
- strong fluorescence pedestal

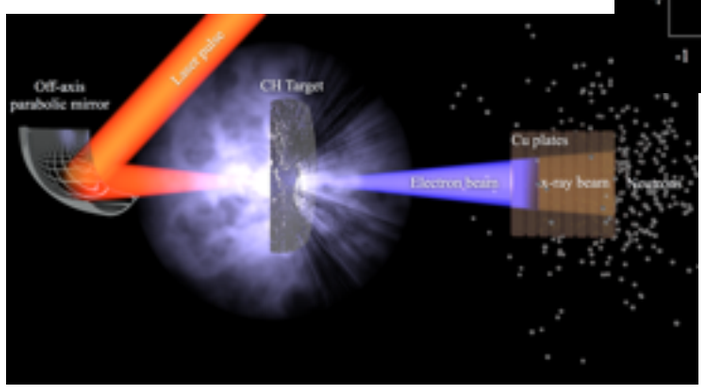
Experiment

- Targets below 6μm expanded and decompressed before main pulse arrival
- no relativistic transparent plasma
- no ion acceleration

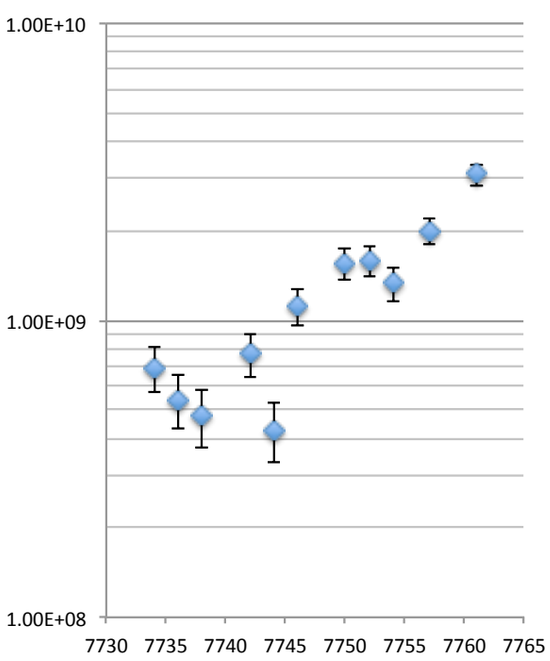
Turning a plight into a virtue: *Laser-electron-neutron source* on TPW achieves flux record

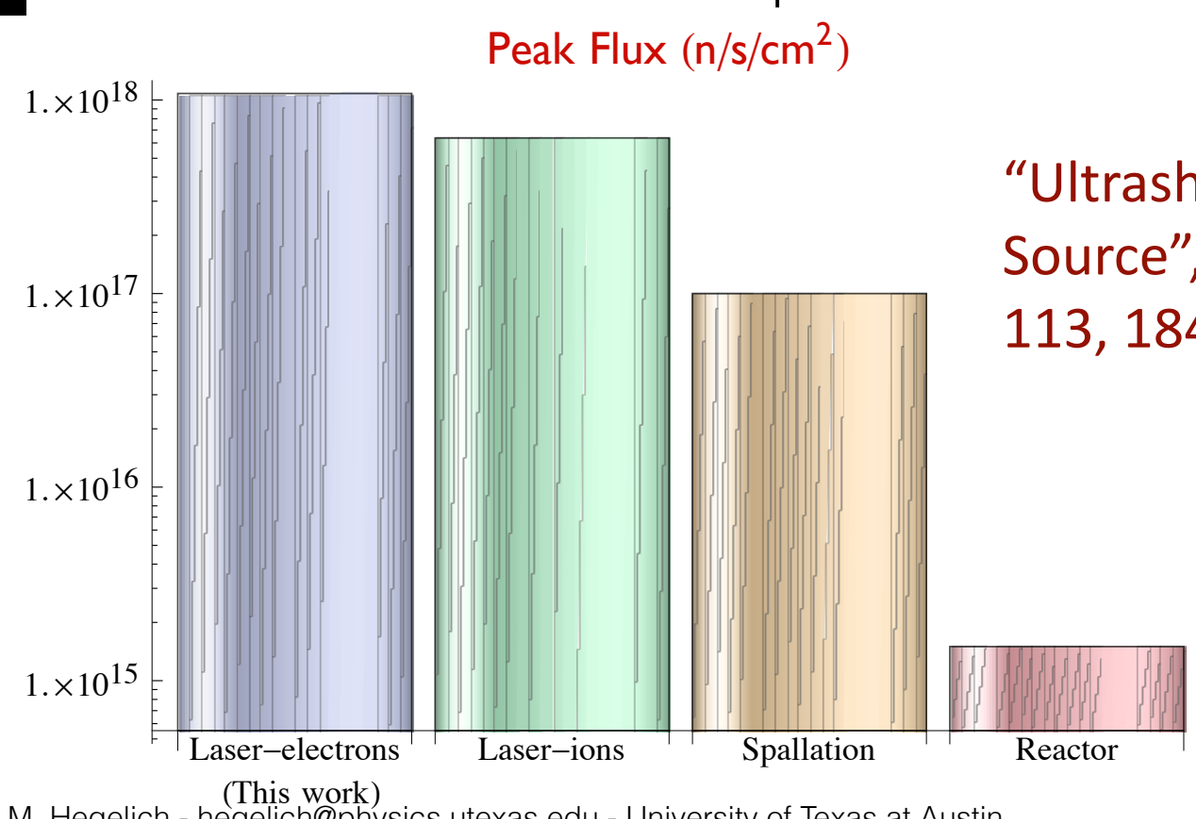
I. Pomerantz





- Laser accelerates electrons from plasma.
- Electrons convert to gamma rays via bremsstrahlung in Cu plates.
- Gamma rates create neutrons in Cu plates.

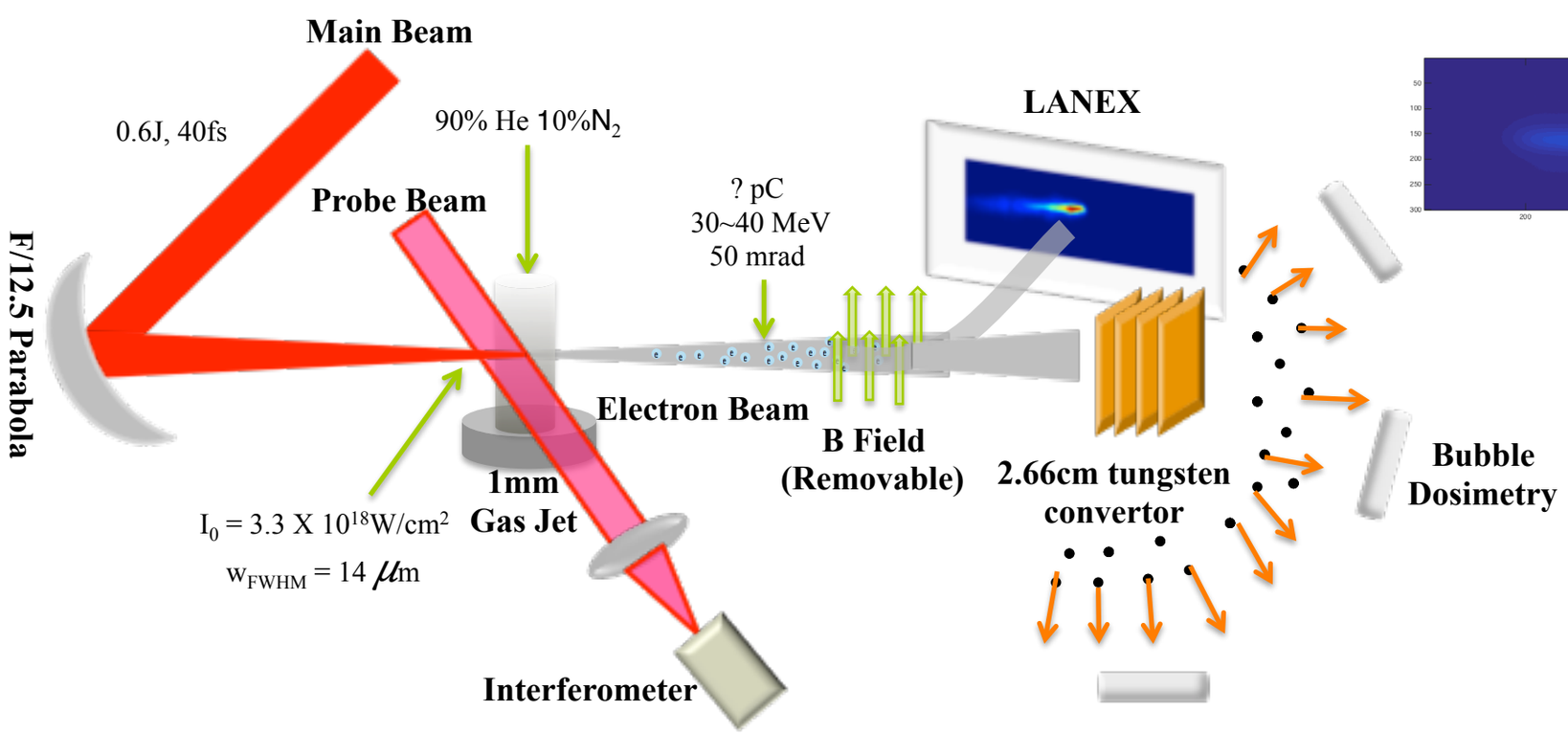


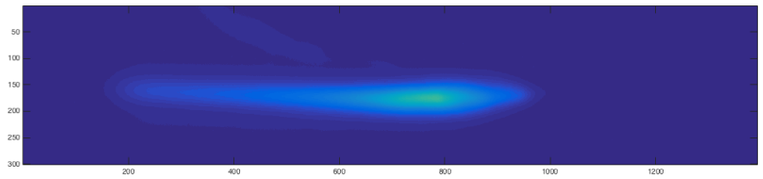


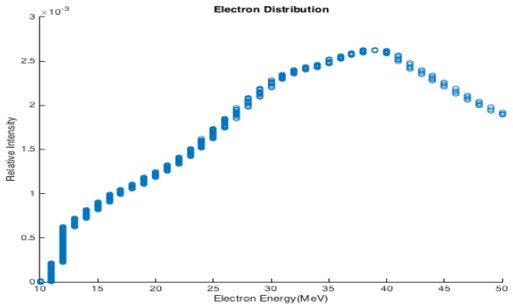
"Ultrashort Pulsed Neutron Source", ***Phys. Rev. Letters*** 113, 184801 (2014)

Miniature neutron source on UT³
10^6- 10^7 neutrons per shot - not yet optimized

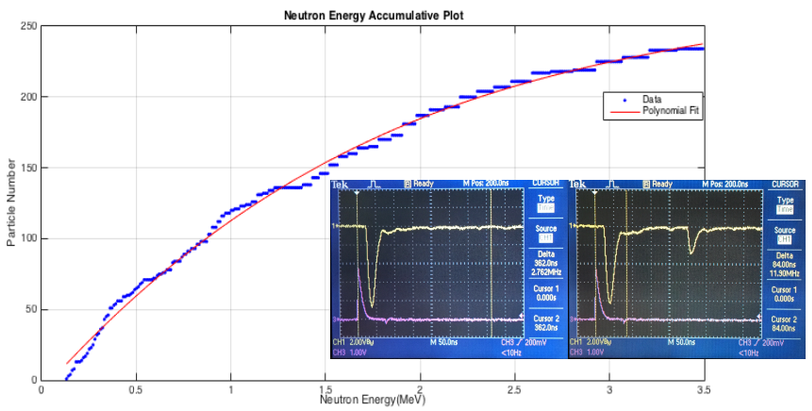
X. Jiao
grad student

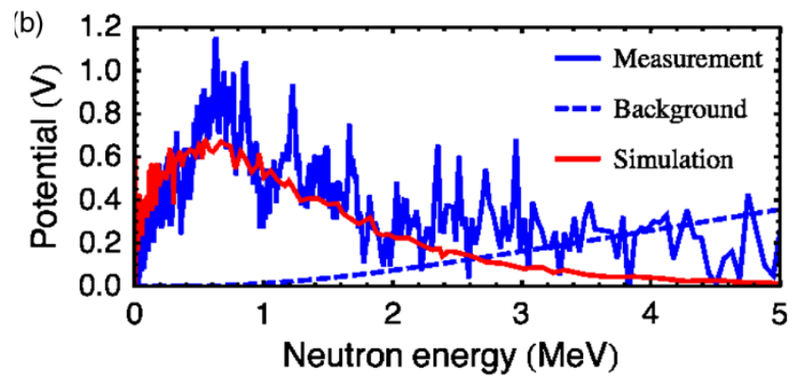






TOF neutron detectors





Bubble counters

BD number	Sensitivity (bubbles/mRem)	Experiment Data (per 200 shots)	Radioactivity (mRem/200shot)	Neutron Number (per shot)
BD2	35	14	546	
BD3	34	10	540	
BD4	32	8	457	
BD5	30	19	493	
BD6	29	56	1804	

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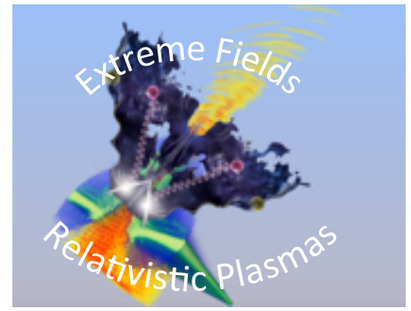
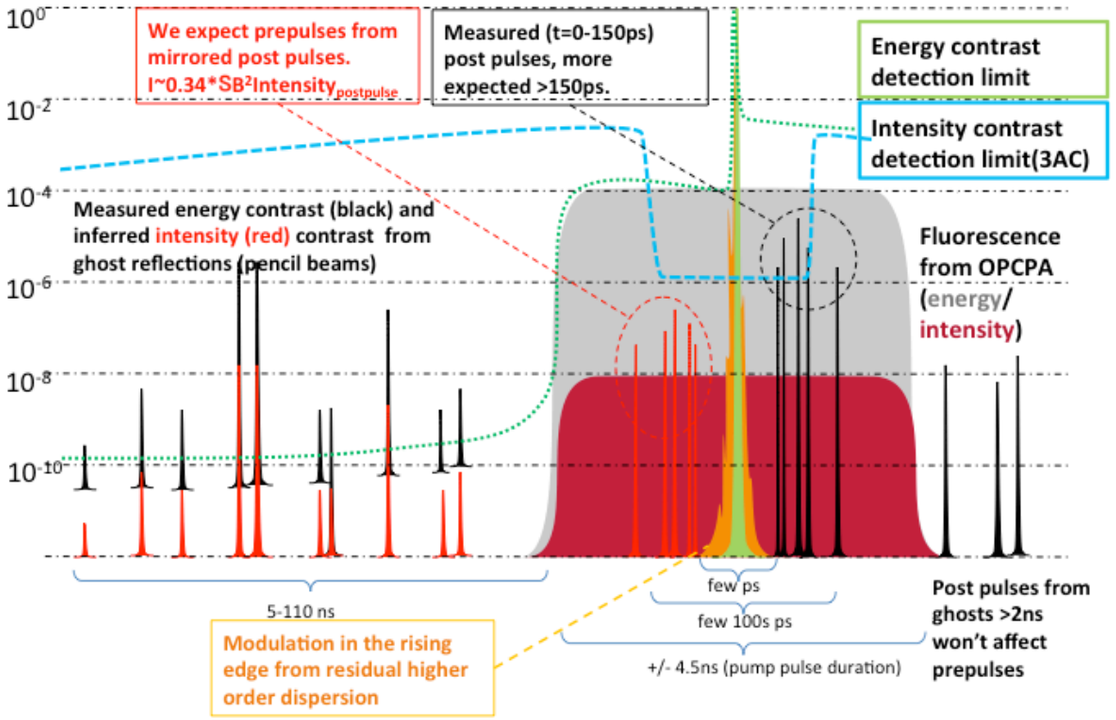


Transferring Research from Trident to the Texas Petawatt Laser

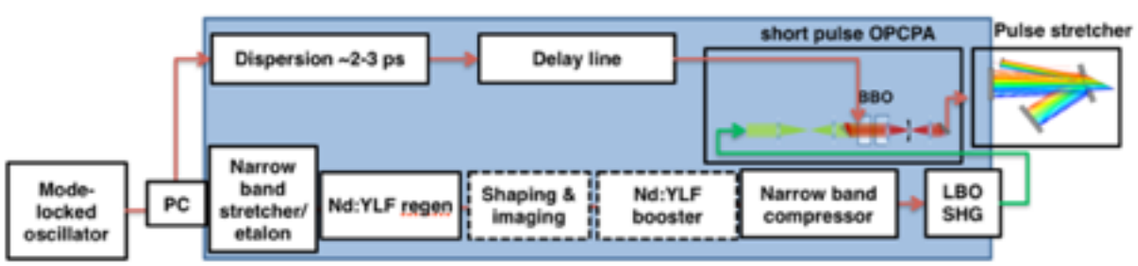
TPW contrast has been upgraded

B. M. Hegelich - hegelich@physics.utexas.edu - University of Texas at Austin

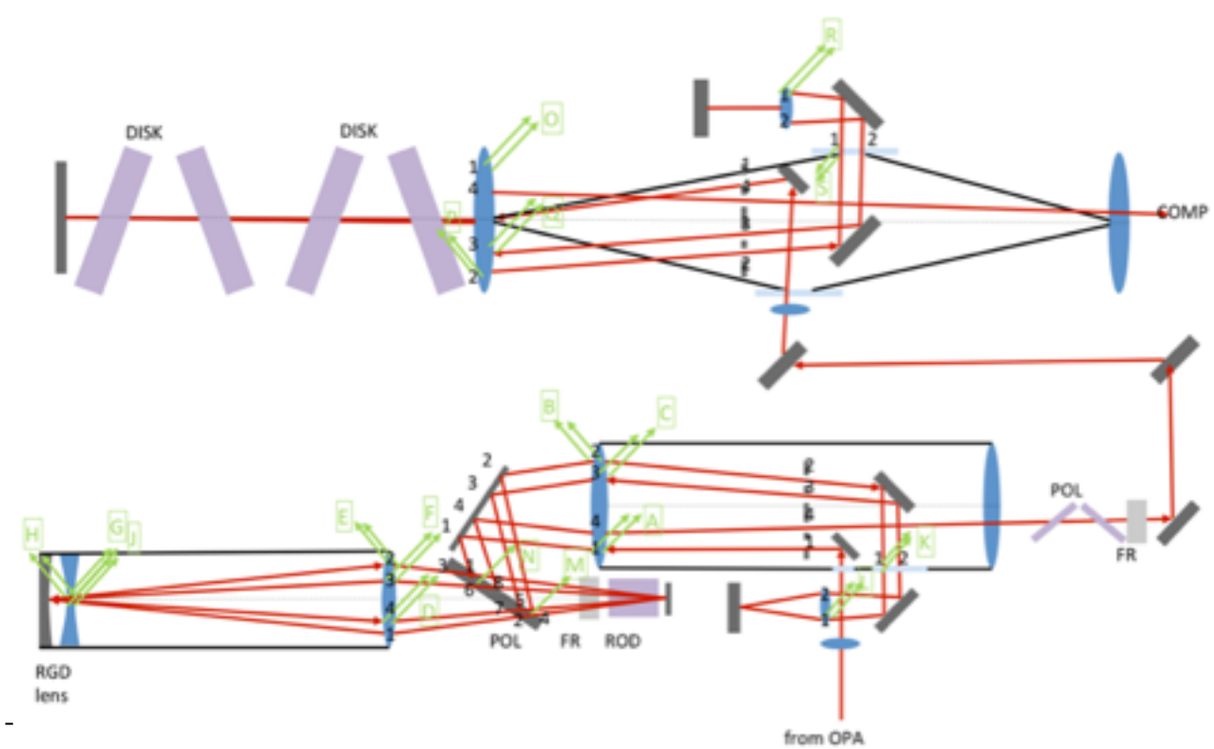
In 2013 we characterized TPW contrast, identified sources of prepulses and designed solutions.



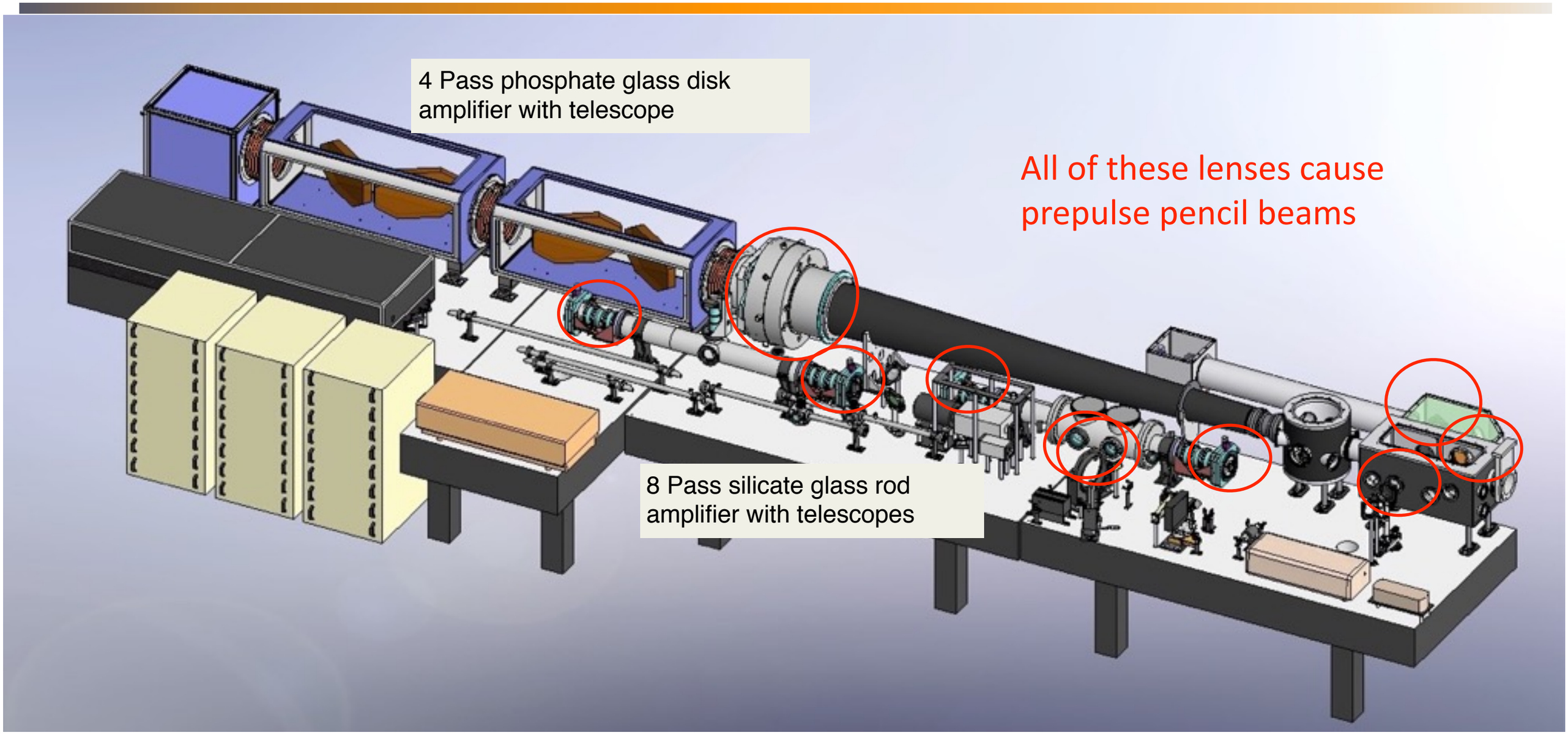
- Prepulses at 10-100ns from multi-pass architecture
  - ⇒ reflective telescopes
- Fluerescence pedestal from OPCPA pump and postpulses
  - ⇒ shortpulse OPA before stretcher



A ps pump laser seeded by the same oscillator reduce the parasitic fluorescence in short pulse OPA stage.



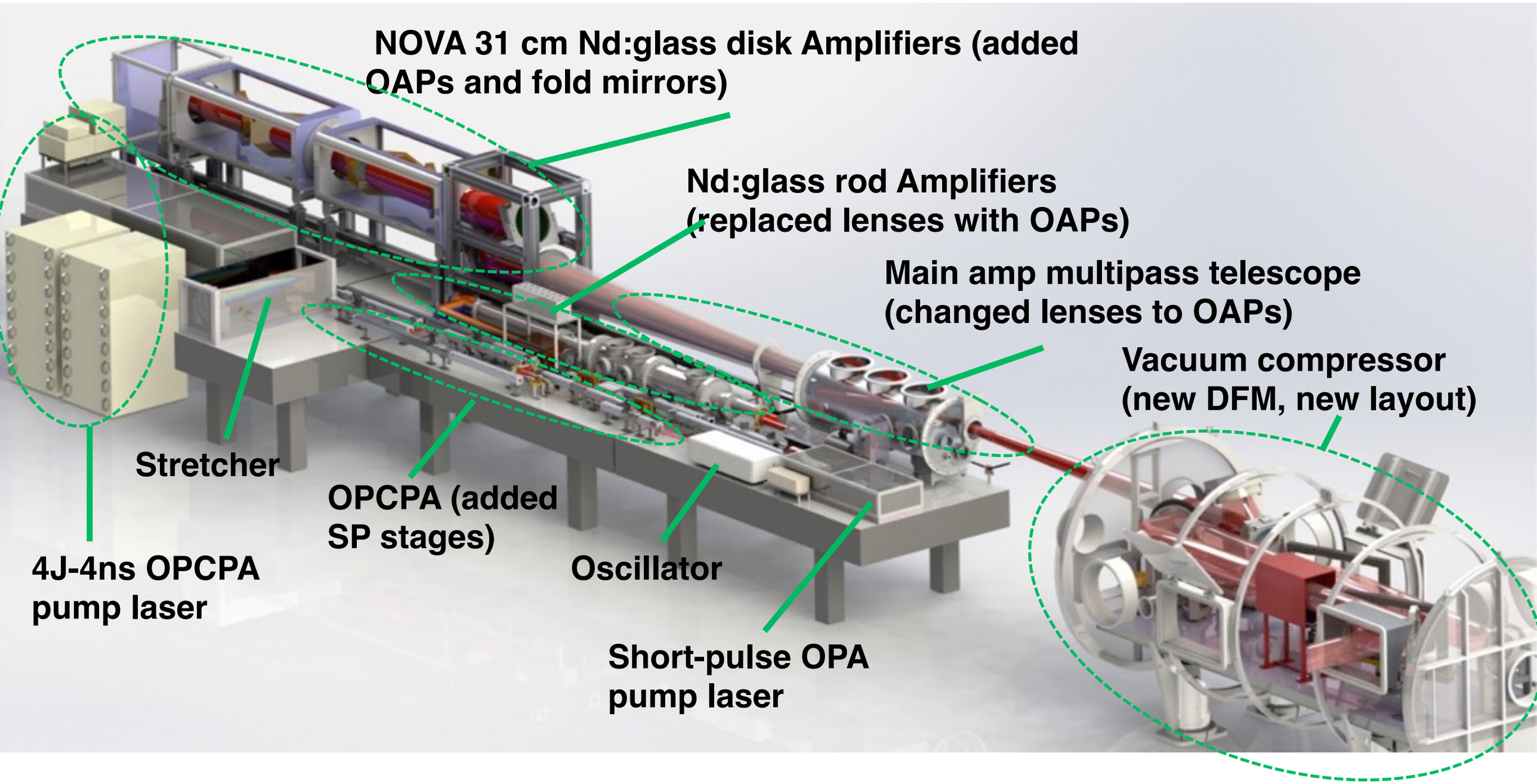
Lenses in a multipass geometry cause prepulses



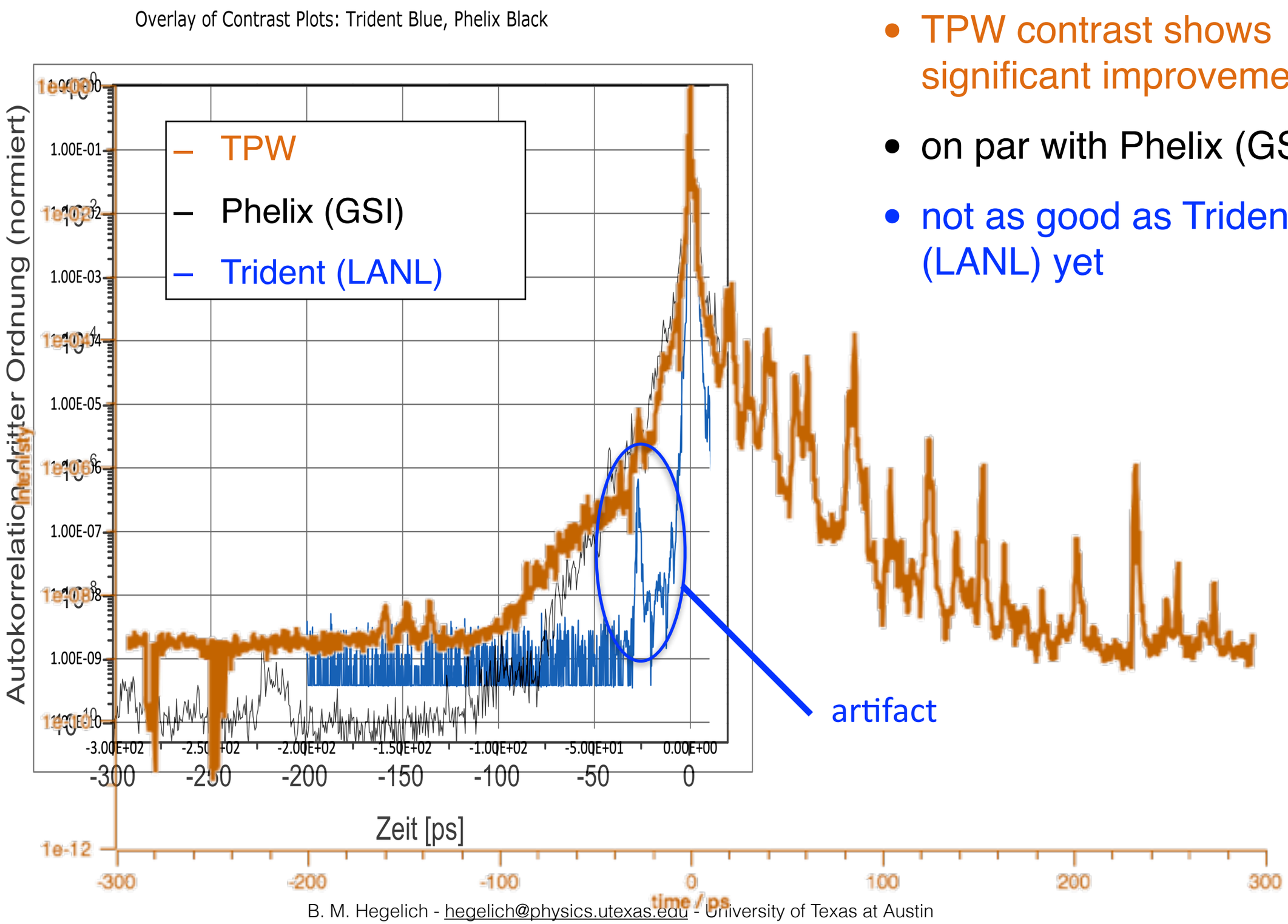
- **Tilting or wedging lenses would cause excessive wavefront and chromatic errors**

➢ **Replace all major lenses with Off-Axis-Parabolas combined with fold mirrors and windows**
  - **Significant side benefit: OAPs have no chromatics errors and no group delay distortion**
  - **Significant drawback: Every single part of the laser moved.**

We have completely rebuilt the laser

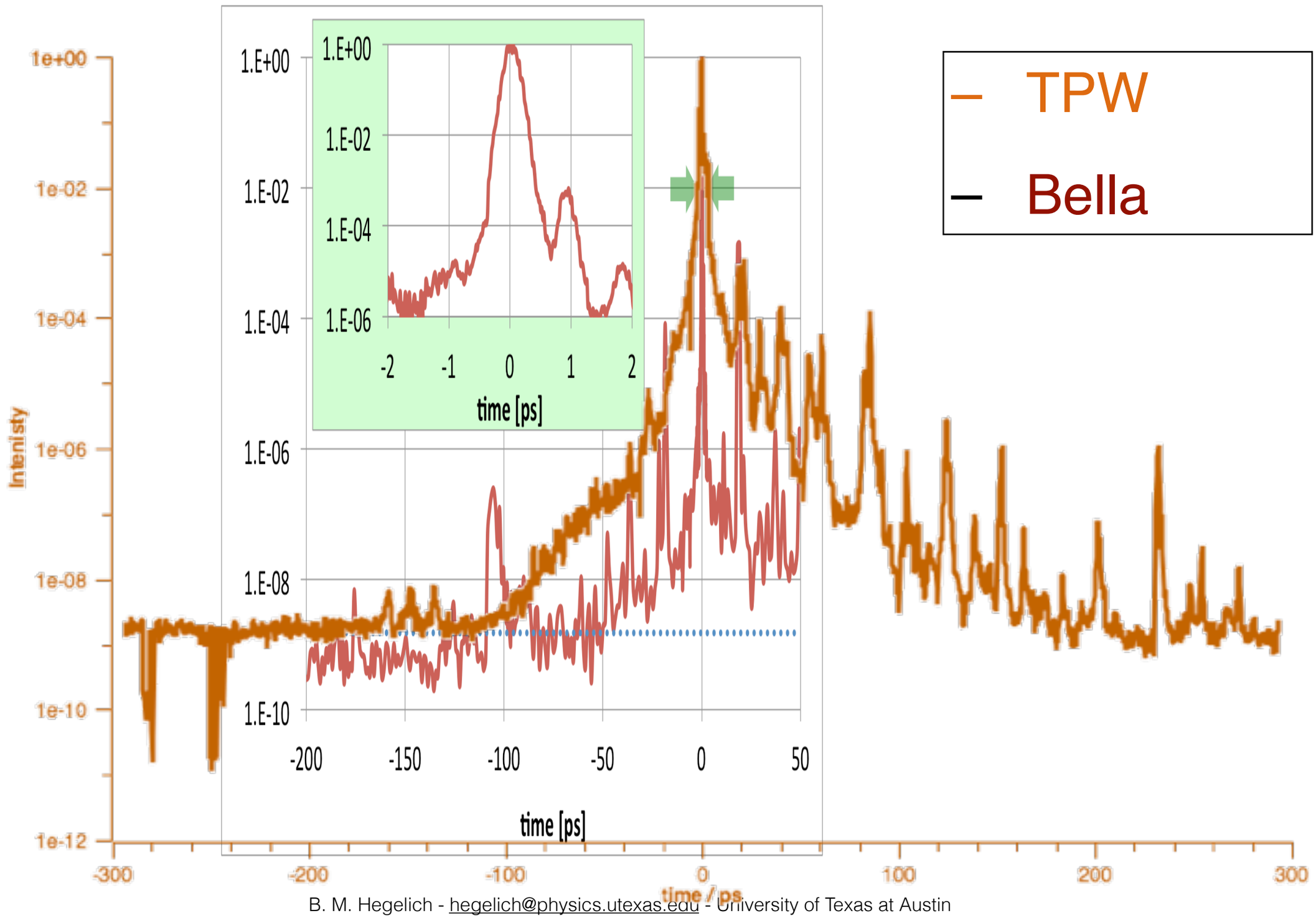


The intensity contrast reaches 10^4 at 10ps and 10^9 at 100ps

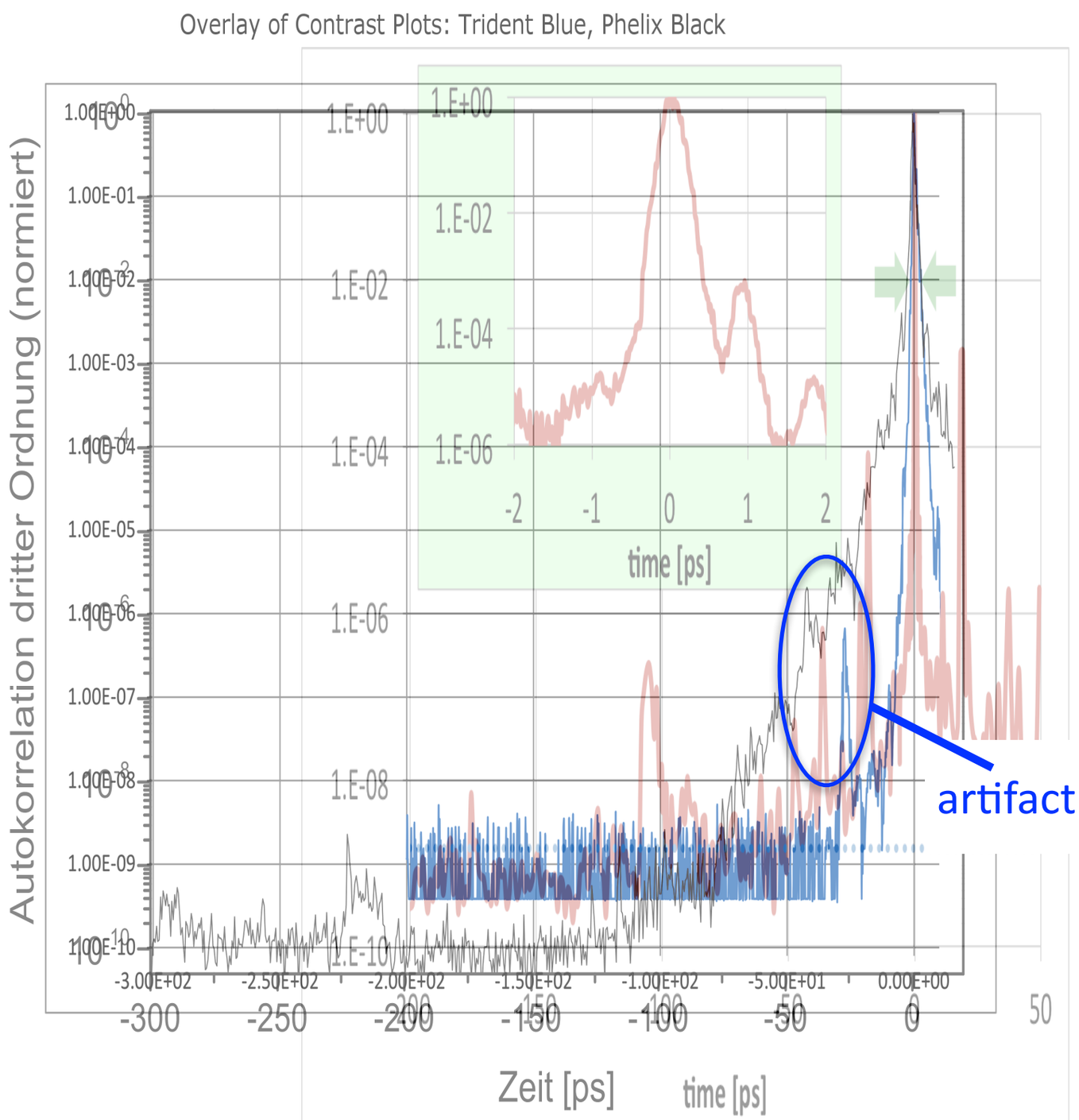


- TPW contrast shows significant improvement
- on par with Phelix (GSI)
- not as good as Trident (LANL) yet

Contrast Comparison TPW - Bella



Contrast Comparison Trident - Bella

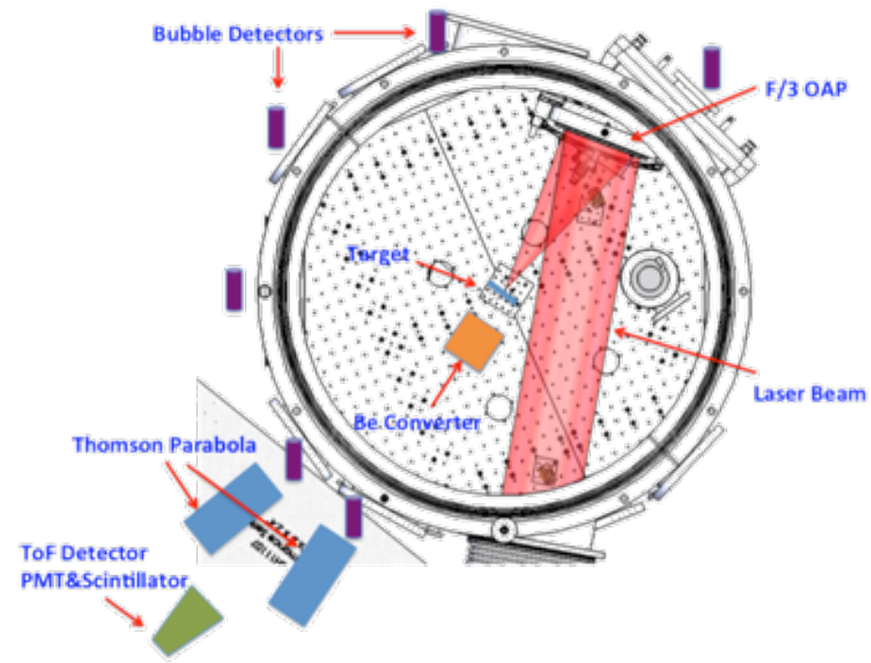


- TPW contrast shows significant improvement
- on par with Phelix (GSI)
- not as good as Trident (LANL) yet

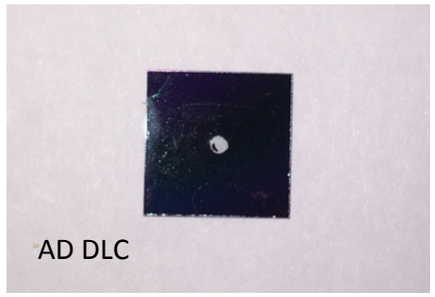
We conducted a first experiment with upgraded TPW looking at ion acceleration from thin foils

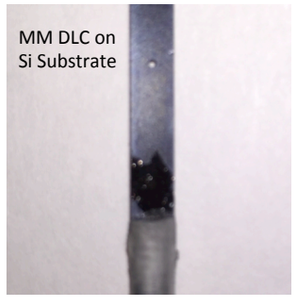
- Laser: E ~ 110J, t: 140fs - 1ps, r~4μm
- Shots without plasma mirrors and with one plasma mirror

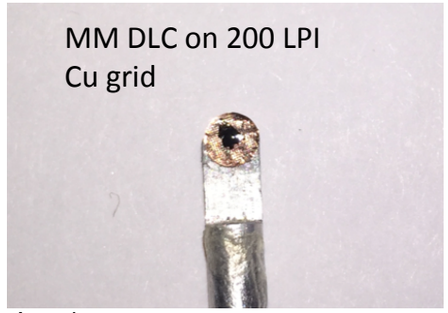
See poster by E. McCary, Thu., 1:05pm

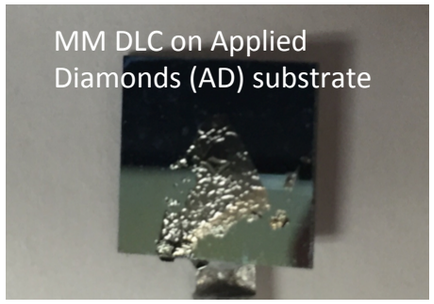


- Diamond-like Carbon foils (DLC)
  - Applied Diamonds (AD) DLC: High diamond content, transparent, brittle, only mounted AD substrates
  - Micromatter DLC: Less transparent and more flexible, higher graphite content, can be mounted on any substrate
- CH foils (300-1000nm)
- CD_2 foils (500-900nm, TU Darmstadt)
- Au-foils (2μm)
- Substrates:
  - Silicon substrates with holes of of 1 mm
  - AD substrates with holes of 5 mm
  - Cu grids with 100 lines per inch (25 um spacing)
  - Cu Grids with 200 lines per inch (12 um spacing)



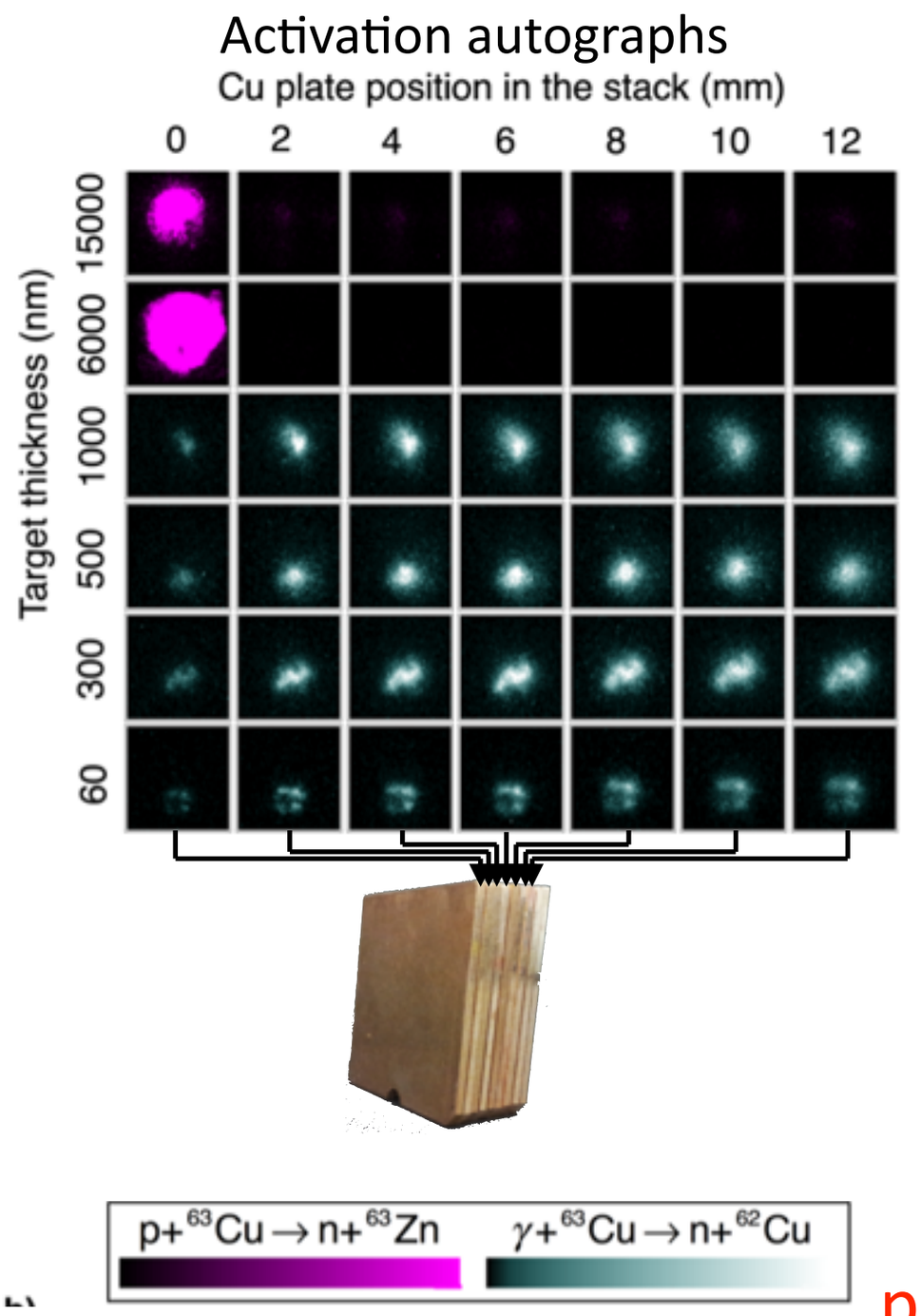




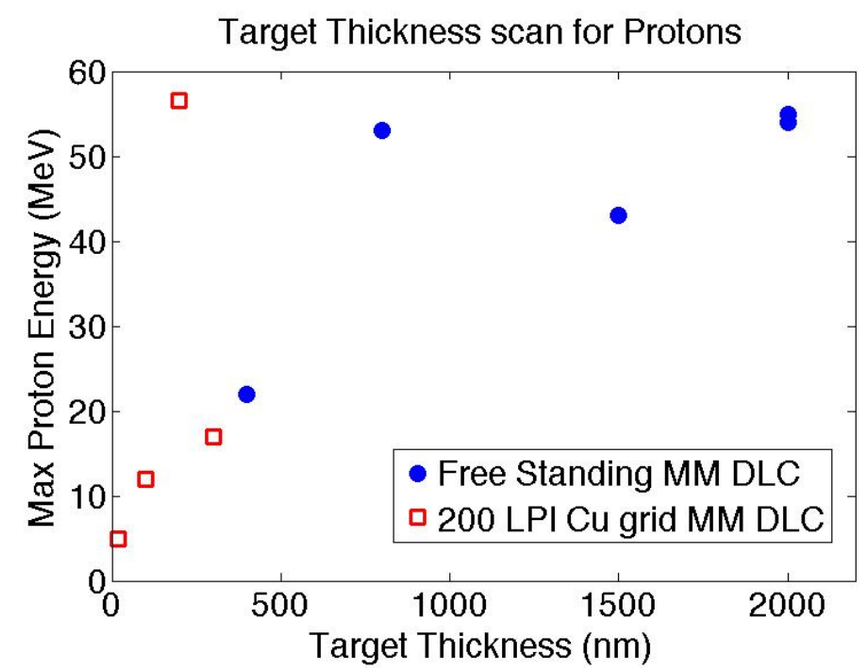


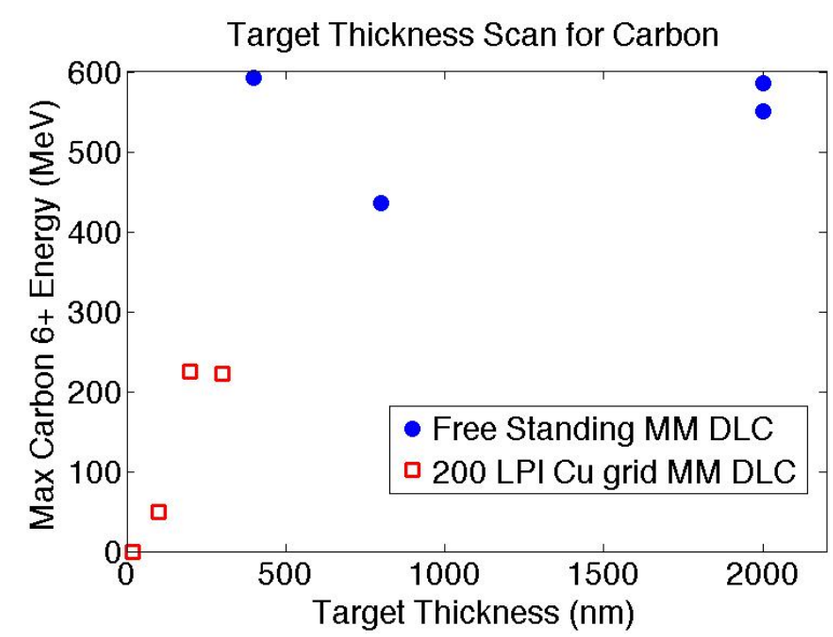
We can now shoot targets as thin as 20nm without plasma mirrors

Before upgrade



After upgrade

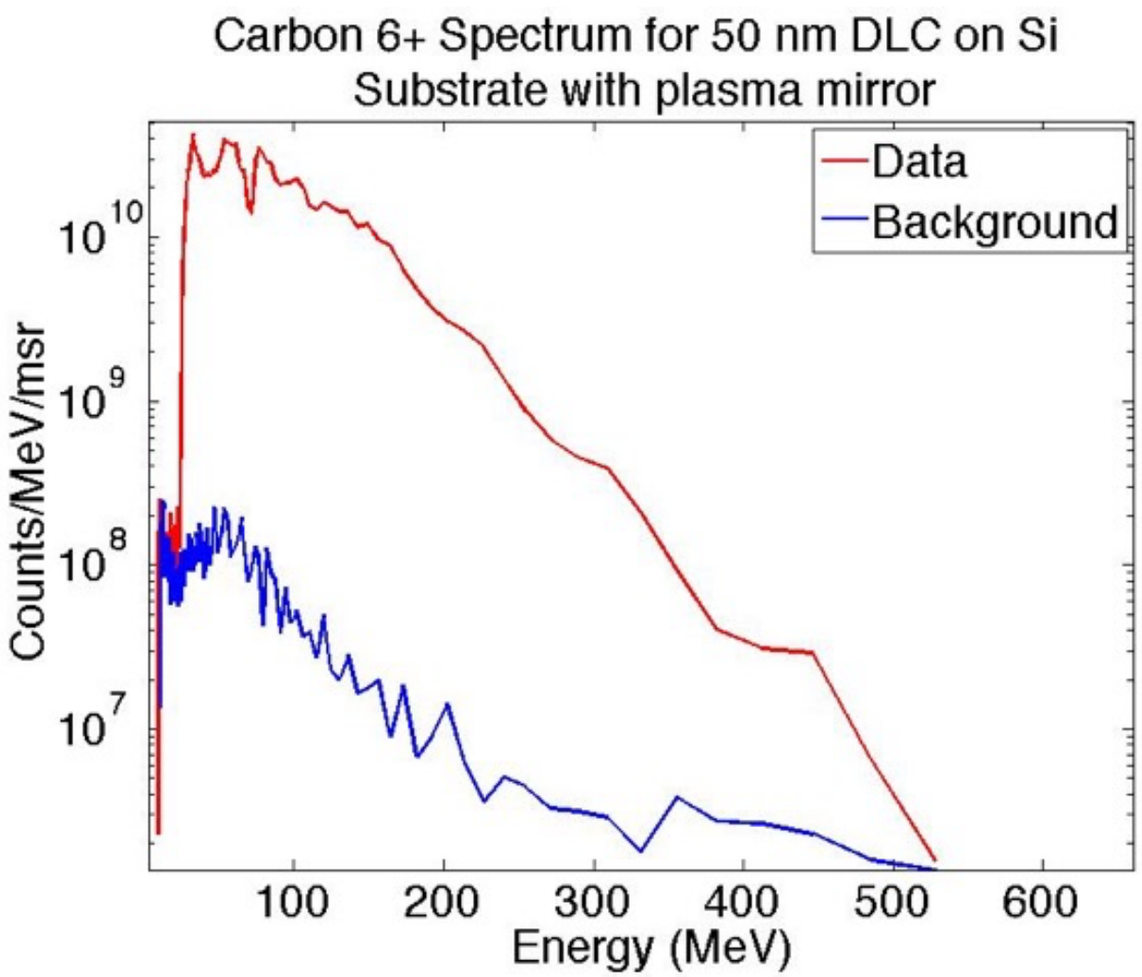




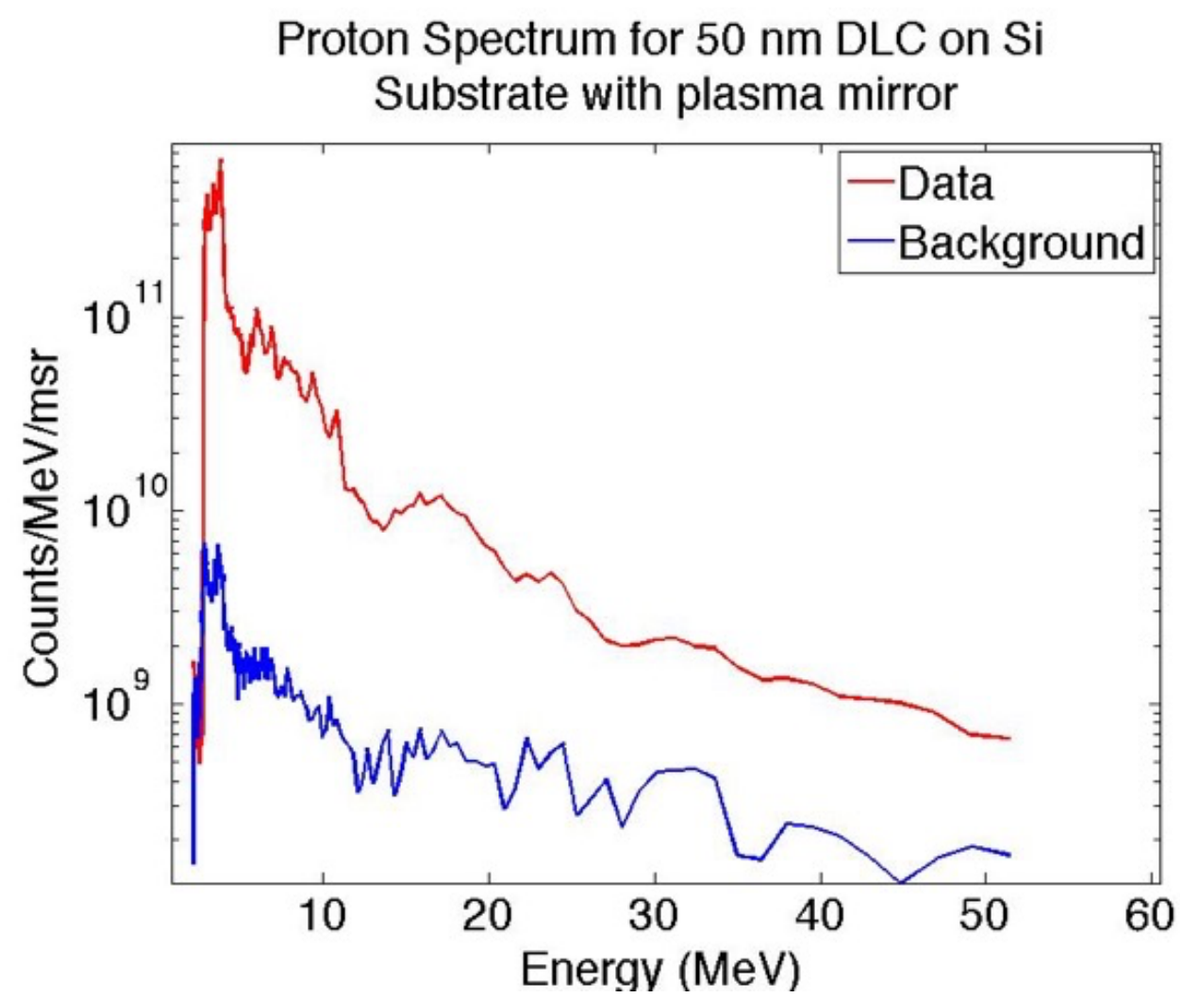
preliminary

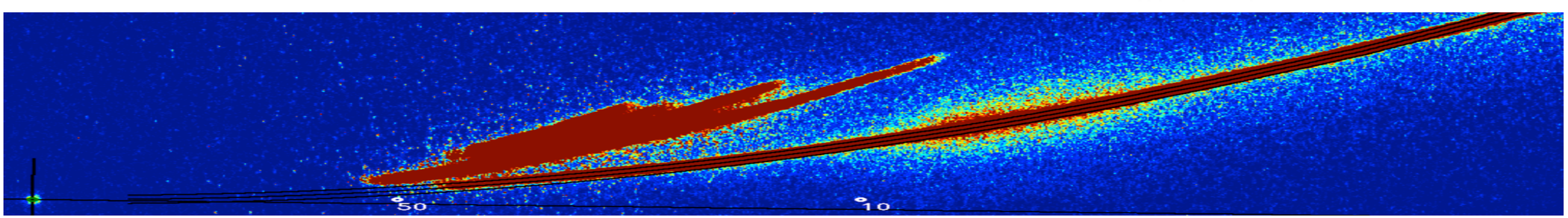
Carbon ions from 50nm target show same energy/nucleon than protons, indicative of BOA

Carbon



Protons





preliminary

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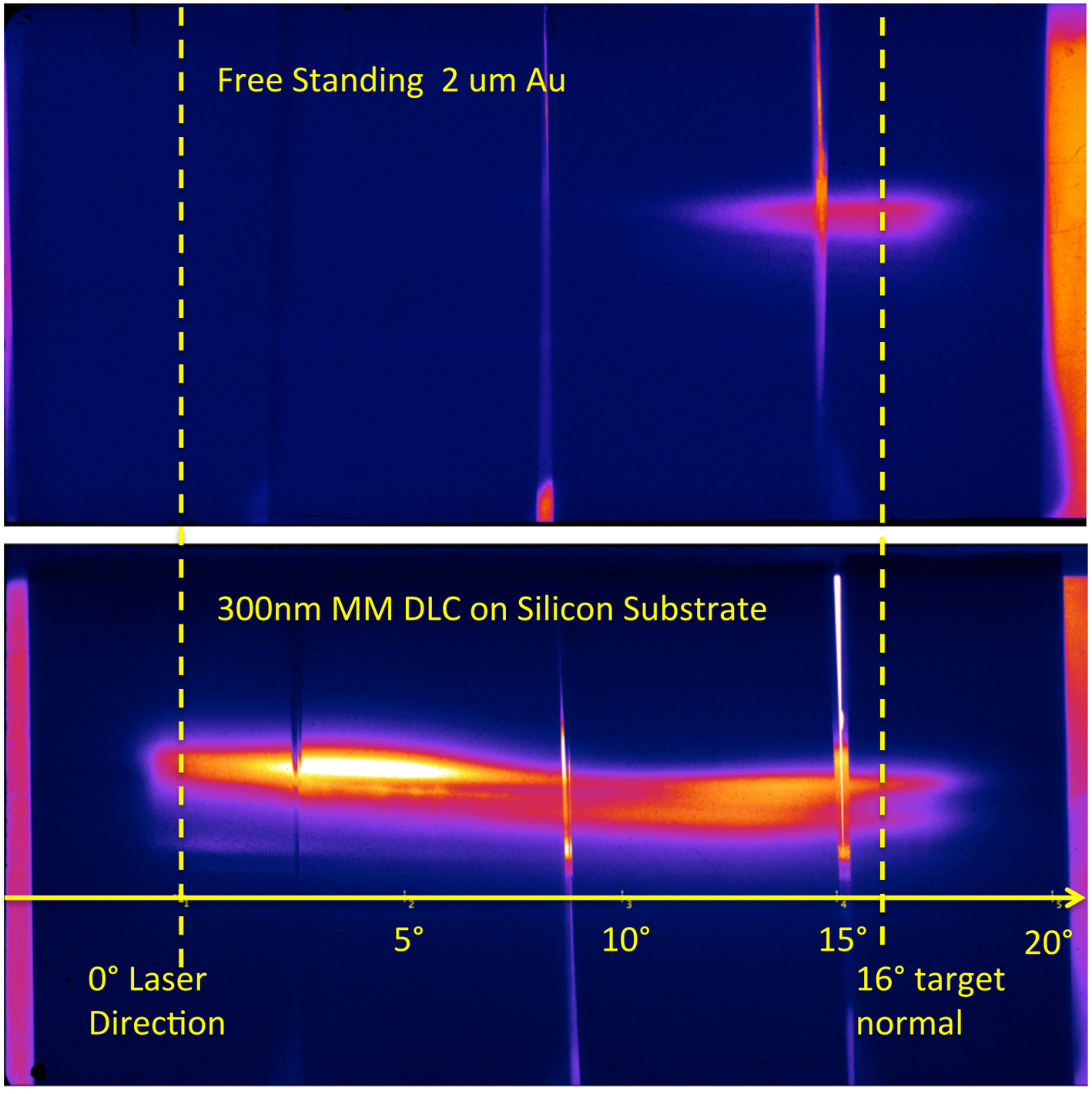
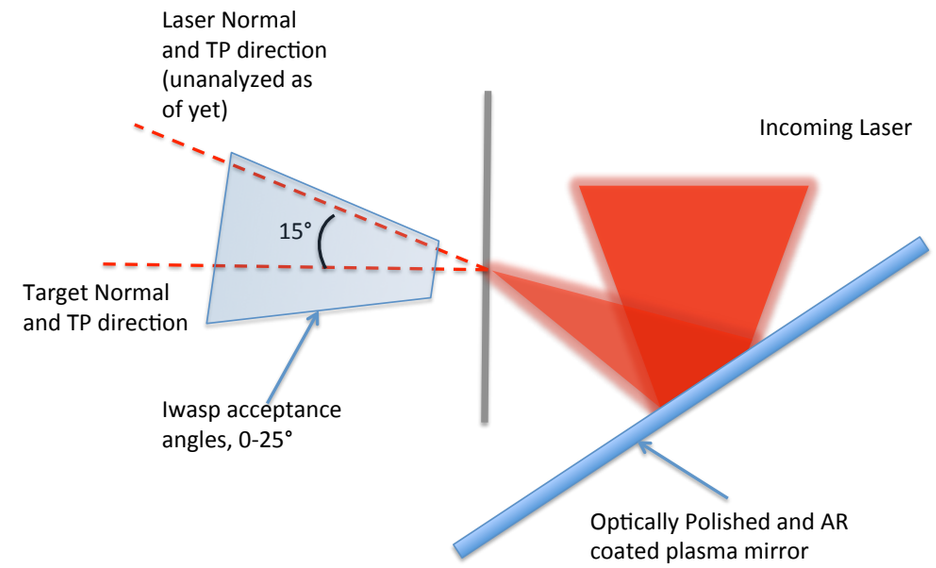
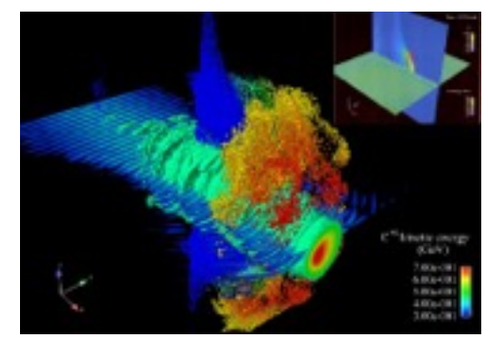
ion Wide Angle Spectrometer shows BOA typical lobes in plane normal to polarization

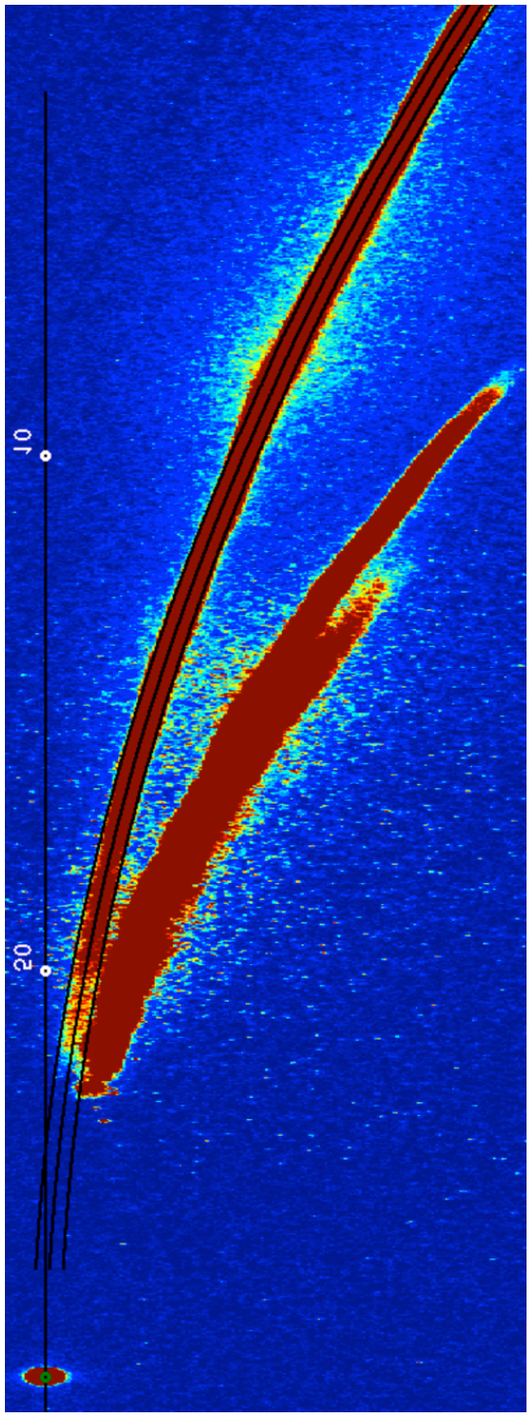


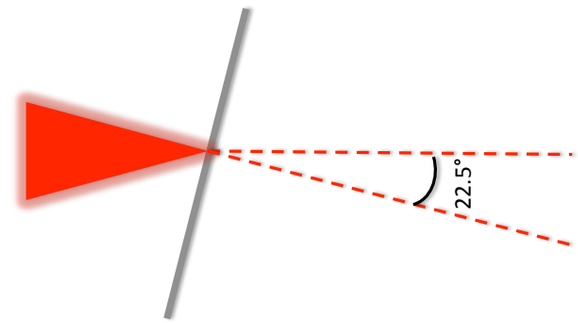


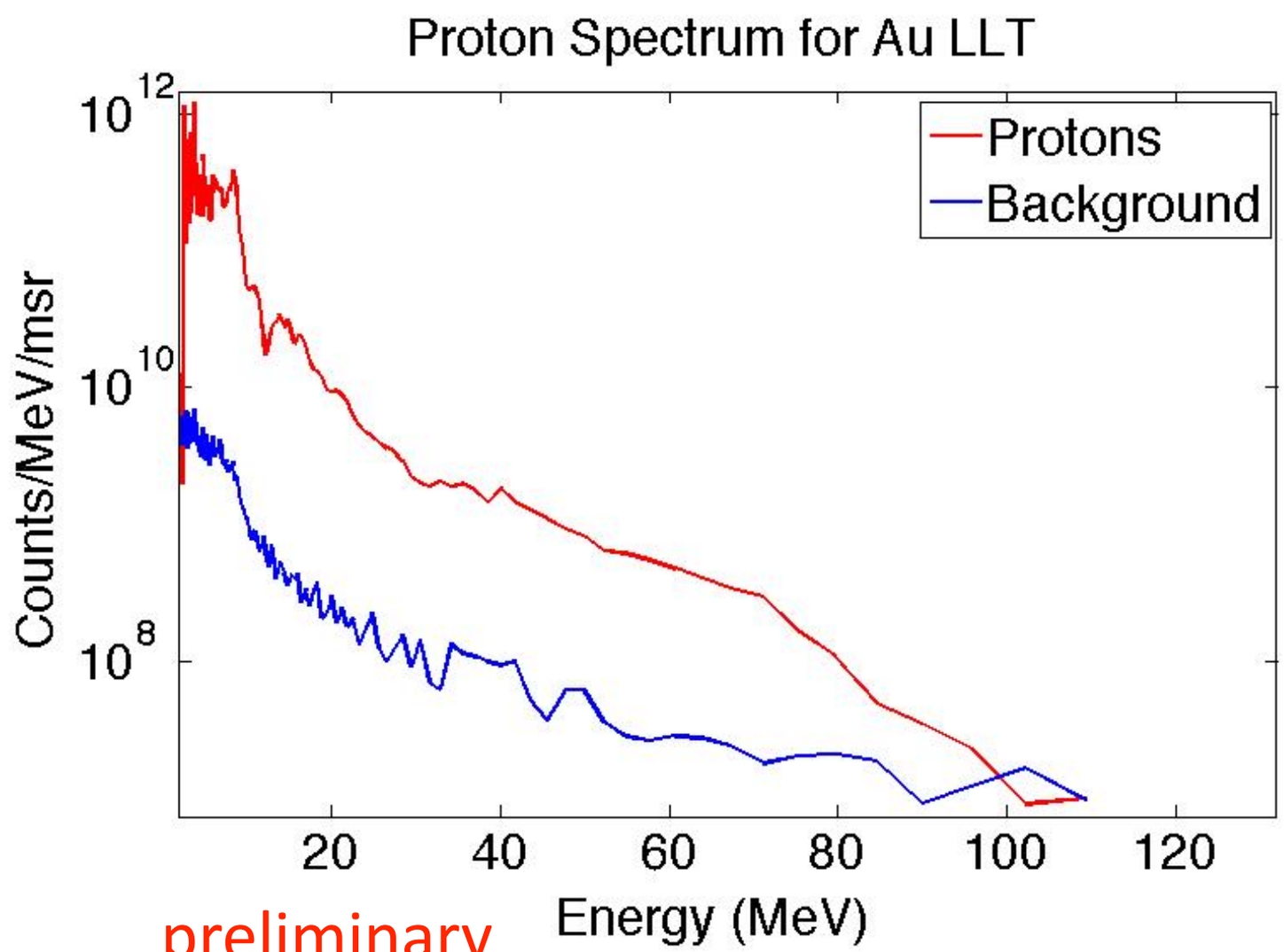


- The Au target saw mainly protons with energies up to 60 MeV, DLC saw Protons up to 60 but carbon up to 500 MeV
- Image Plates had two layers of CR-39 acting as a filter to eliminate low energy ions.

100 MeV TNSA protons from 2μm Gold foil

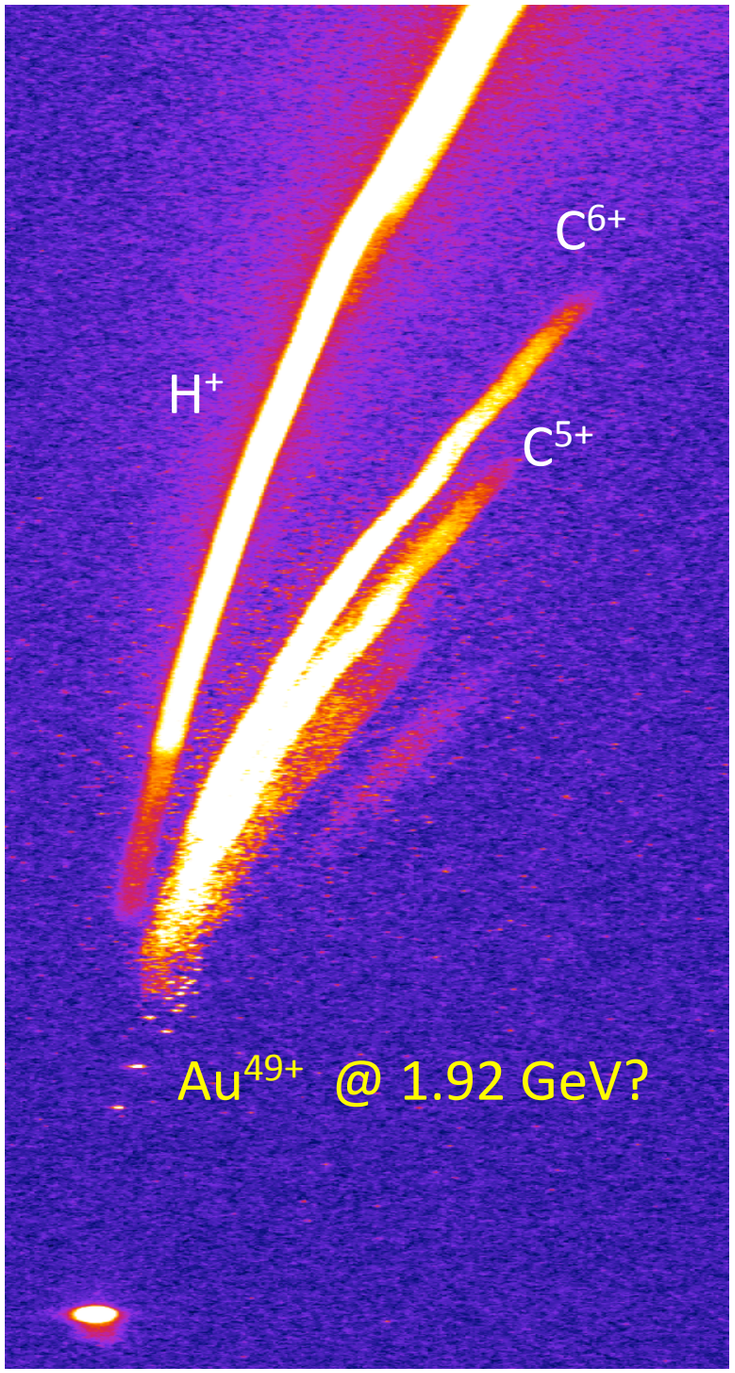


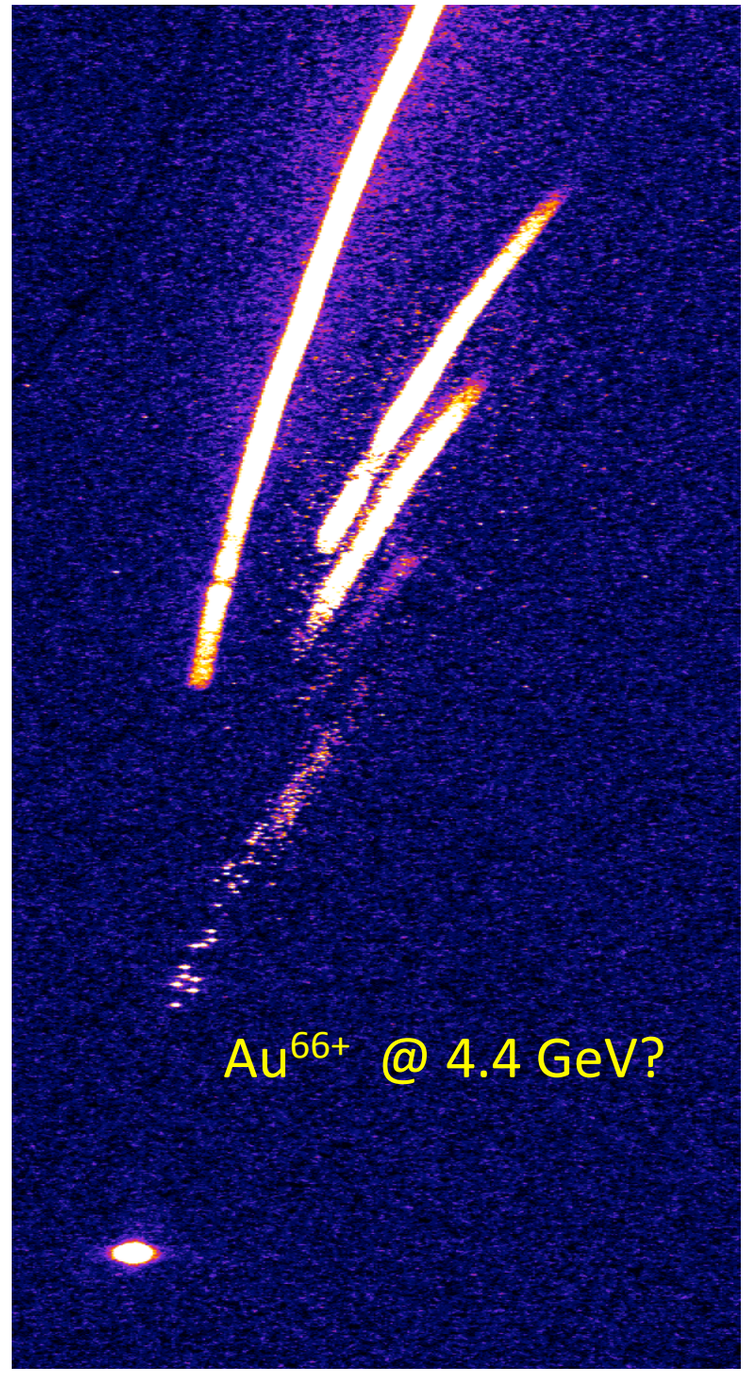




preliminary

Gold ion acceleration at TPW





- Au shot 1:
  - q/m ~ 0.25, i.e.. possibly Au^{49+}
  - E_{max} ~1.92 GeV
- Au shot 2:
  - q/m ~ 0.33, i.e. possibly Au^{66+}
  - E_{max} ~4.46 GeV 4.4 GeV Au.
- More data required:
  - thinner targets
  - CR-39 for track size identification
  - heating to remove protons

preliminary

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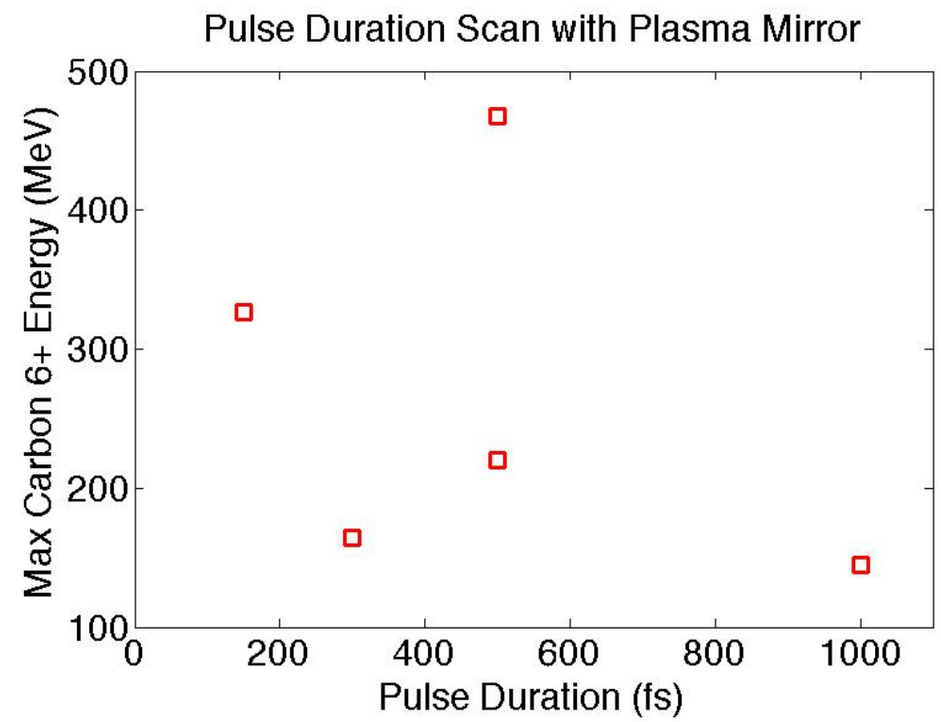
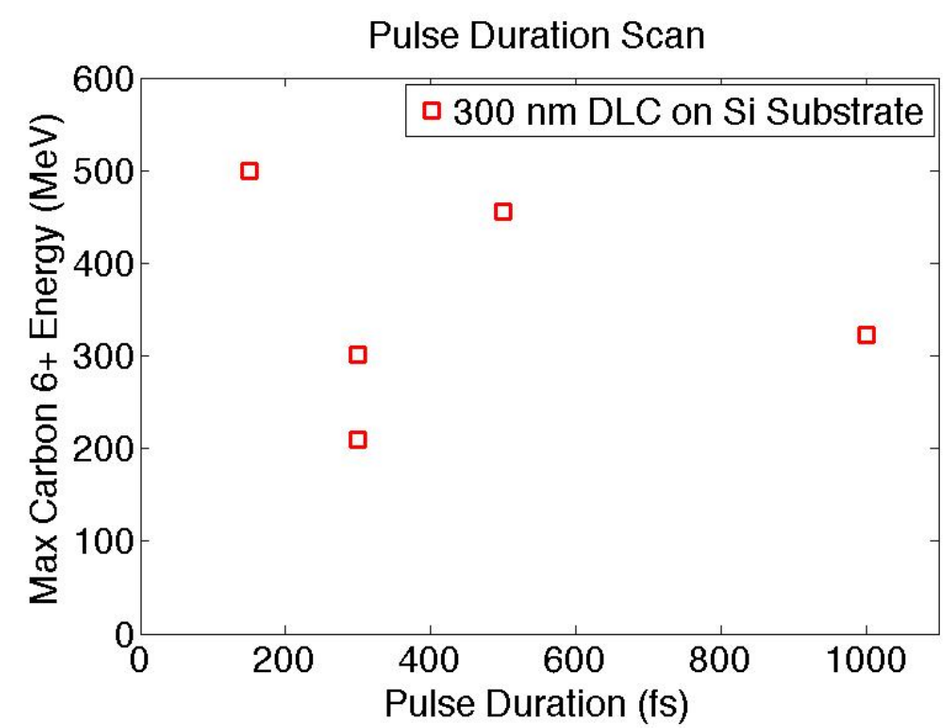
Frontiers in EFRP

- Statistics Frontier: higher average power
  - ELI, Apollon, … (Europe): 0.1 -10Hz @ ~PW
  - Split disk amplifiers at UT: TPW @ 0.1 Hz?
  - UT (Hegelich): kHz, kW mid-IR laser @ 10-100 TW
  - Bella-i: PW @ 1Hz
- Intensity Frontier - higher peak power
  - ~5×10^{22} W/cm^2 on TPW with F/1 OAP (March 2016)
  - new multi-PW lasers: ELI (Europe), SIOM (China), PEARL+ (Russia): 10^{23} - 10^{24} W/cm^2
  - 75PW, OPAL (LLE, Rochester): 10^{23} - 10^{24} W/cm^2?

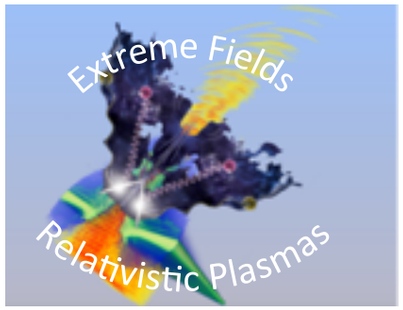
Statistics Challenge: Current high energy lasers are single shot ⇒ sparse datasets

- Current glass lasers, e.g. TPW fire <1 shot/hr
- single shift: ~5 shots per day
- typical experiment: 50 - 100 shots
- typically multiple, independent relevant parameters:
  - laser pulse:
    - duration, energy, diameter, contrast, shape, wavelength,
    - …
  - target:
    - material, thickness, shape, surface quality, holder
    - …
- TPW vastly overbooked
  - next opportunity for further investigation in ~1 year
- *dire need for more shots!*





Frontiers in EFRP



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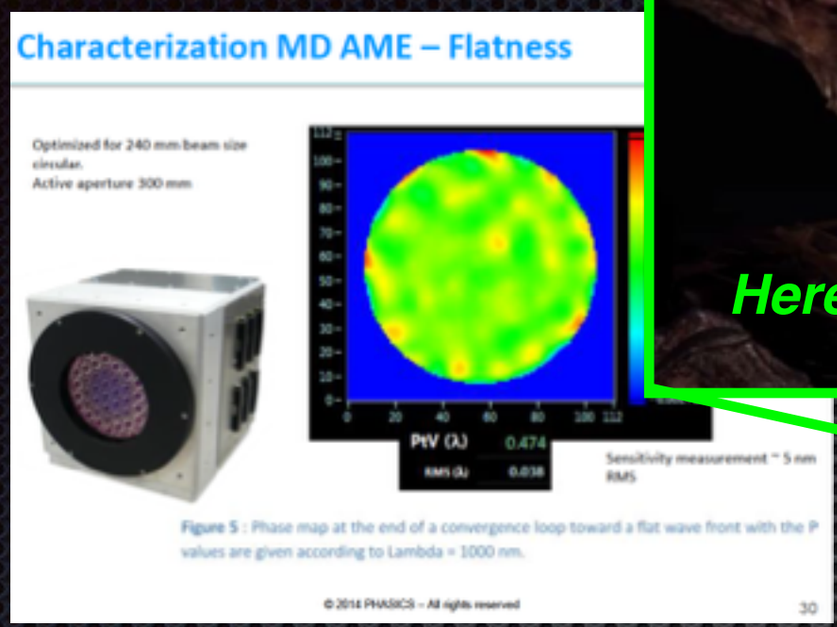
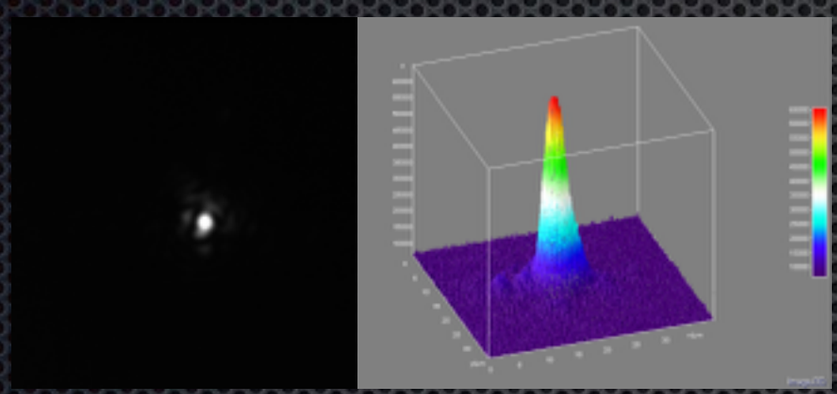
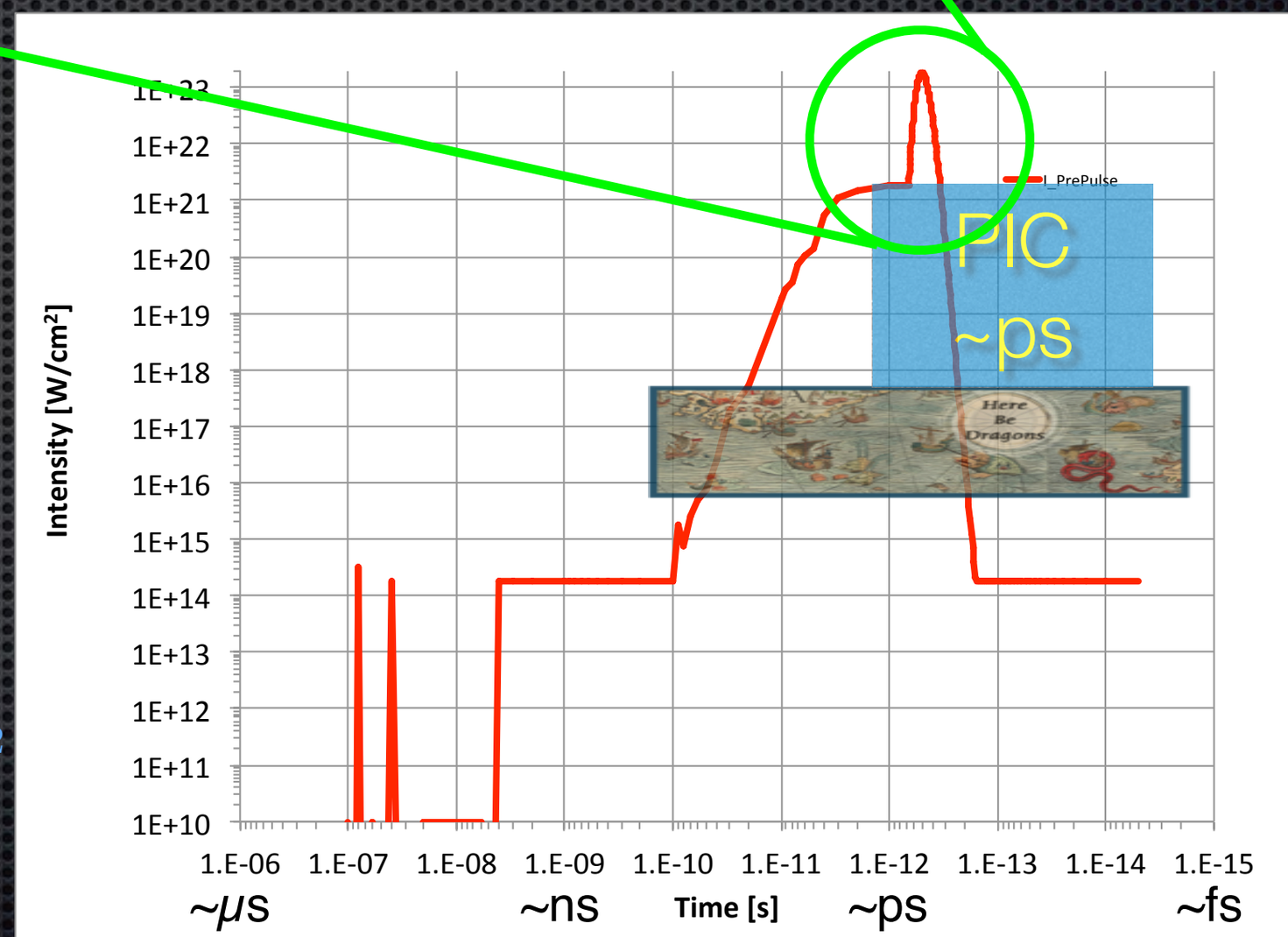
  - new multi-PW lasers: ELI (Europe), SIOM (China), PEARL+ (Russia): 10^{23} - 10^{24} W/cm^2
  - 75PW, OPAL (LLE, Rochester): 10^{23} - 10^{24} W/cm^2?
- Statistics Frontier: higher average power
  - ELI, Apollon, … (Europe): 0.1 -10Hz @ ~PW
  - Split disk amplifiers at UT: TPW @ 0.1 Hz?
  - UT (Hegelich): kHz, kW mid-IR laser @ 10-100 TW

TPW Intensity Upgrade will reach >10^{22} W/cm^2



- focused with F/1 optic:
  - $I_{FWHM} = 1.6 \times 10^{22}$ W/cm^2 (a_0=150)
  - $I_{Peak} = 1.8 \times 10^{23}$ W/cm^2 (a_0=350)





$$I_{Peak} = \frac{P_0 A}{\lambda^2 f^2} = 1.8 \times 10^{23} W/cm^2$$

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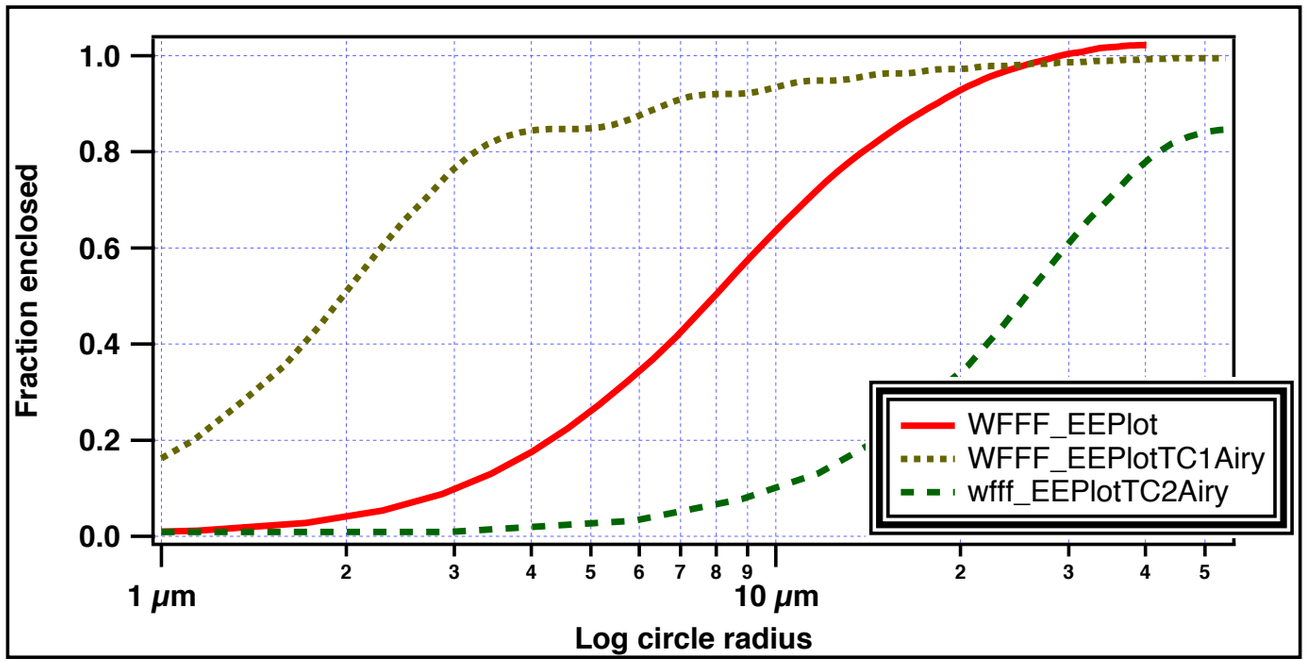
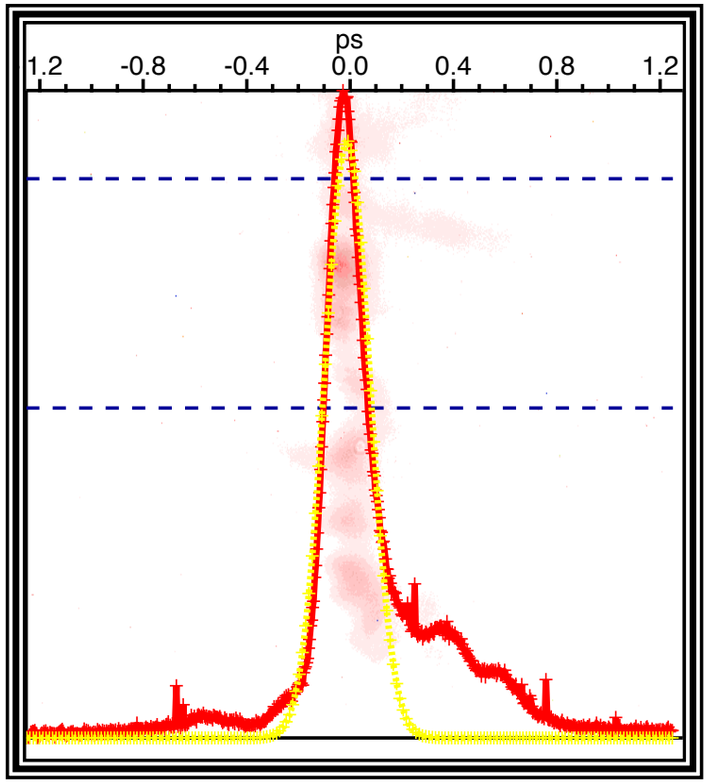
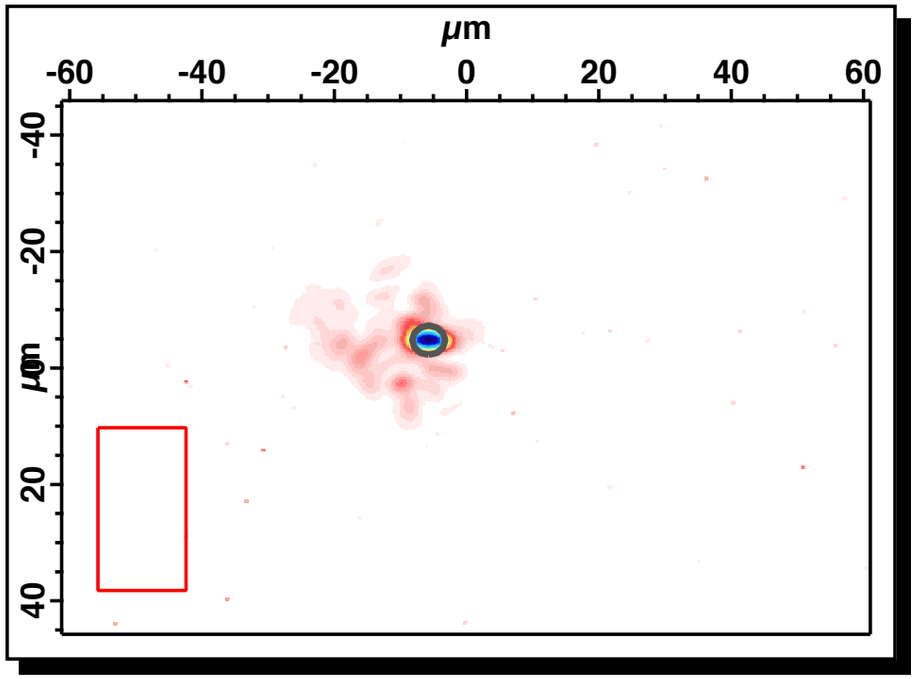
Typical focus in 2013: Strehl ~0.4, 50% energy in 8μm radius

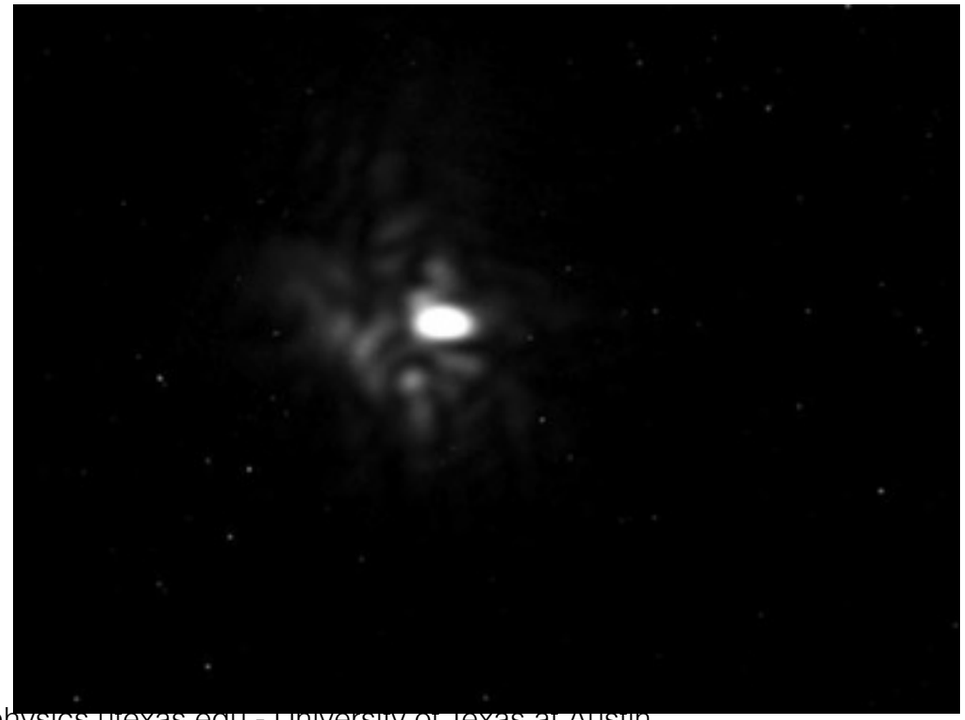
SHOT# 006737 2013-07-23

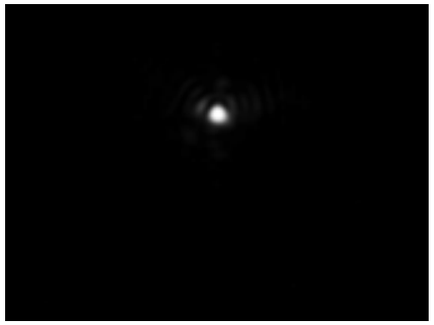
16:53:05

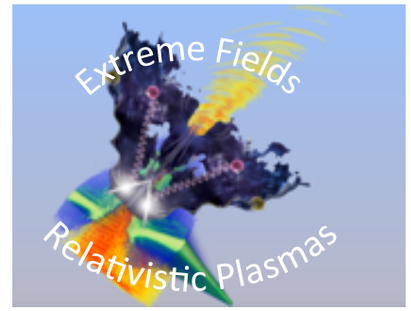












Why Extreme Fields?

Why does a Quantum Field Theory for Intense Laser Matter Interactions matter?

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Is there a limit to acceleration?

- Lorentz:
  - Trajectory of a charge in an external field:

$$m\ddot{x}^{\alpha} = eF^{\alpha\beta}\dot{x}_{\beta}$$

- Lamor:
  - Accelerating charge radiates and looses energy

$$\frac{dE_R}{dt} = \frac{2}{3}e^2\ddot{x}^2$$

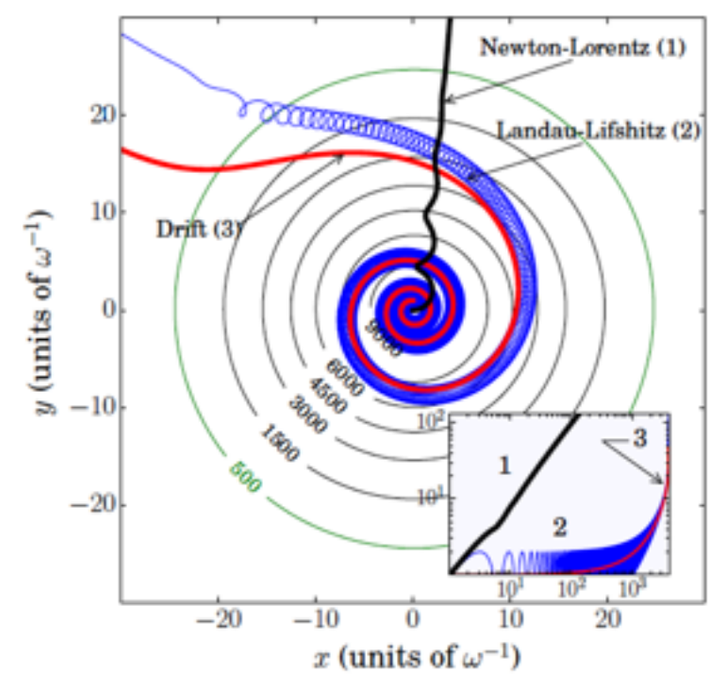
- Lorentz-Force does not include that energy loss
  - Additiniol term: Abraham-Lorentz force

$$F_{rad} = \frac{q^2}{6\pi\epsilon_0 c^3}\dddot{x}^{\beta}$$

- Dirac:
  - renormalized the mass in the equation of motion with the Lorentz-Abraham-Dirac (LAD) equation:*

$$m\ddot{x}^{\alpha} = eF^{\alpha\beta}\dot{x}_{\beta} + m\tau_0\left(\delta^{\alpha}_{\beta} + \dot{x}^{\alpha}\dot{x}_{\beta}\right)\dddot{x}^{\beta}$$

  - continuous theory work for the last 100 years
  - no experiments



Fedotov et al., arXiv 1408.0363v1 (2014)

“Radiation Reactions” are one of the oldest and most fundamental issues in electrodynamics*

* Paul A.M. Dirac, (1938) Classical theory of radiating electrons. Proc. Roy. Soc. of London. A929:0148-0169.

- Many other approaches exist
  - semi-classical and QED
- Different approximations can yield different results
- most models agree that effects become important for $a_0 > 100$.

LAD (1904-1938)	$m\dot{u}^\alpha = eF^{\alpha\beta}u_\beta + m\tau_0\left(\delta^\alpha_\beta + u^\alpha u_\beta\right)\ddot{u}^\beta$
Landau-Lifshitz (1962)	$m\dot{u}^\alpha = eF^{\alpha\beta}u_\beta + e\tau_0\left[F^{\alpha\beta}_{,\gamma}u_\beta u^\gamma + e/m\left(\delta^\alpha_\beta + u^\alpha u_\beta\right)F^\beta_\gamma F^\gamma_\delta u^\delta\right]$
Mo-Papas (1971)	$m\dot{u}^\alpha = eF^{\alpha\beta}u_\beta + e\tau_0\left(F^{\alpha\beta}\dot{u}_\beta + F^{\beta\gamma}\dot{u}_\beta u_\gamma u^\alpha\right)$
Caldirola (1979)	$-m/\tau_0\left[u^\alpha(\tau-\tau_0) + u^\alpha(\tau)u_\beta(\tau)u^\beta(\tau-\tau_0)\right] = eF^{\alpha\beta}(\tau)u_\beta(\tau)$
Caldirola-Yaghjian (1992)	$m\dot{u}^\alpha = eF^{\alpha\beta}u_\beta + m/\tau_0\left[u^\alpha(\tau-\tau_0) + u^\alpha(\tau)u^\beta(\tau)u_\beta(\tau-\tau_0)\right]$

Jackson:

The difficulties presented by this problem touch one of the most fundamental aspects of physics, the nature of the elementary particle. Although partial solutions, workable within limited areas, can be given, the basic problem remains unsolved.

One might hope that the transition from classical to quantum-mechanical treatments would remove the difficulties. While there is still hope that this may eventually occur, the present quantum-mechanical discussions are beset with even more elaborate troubles than the classical ones.

It is one of the triumphs of comparatively recent years (~ 1948–1950) that the concepts of Lorentz covariance and gauge invariance were exploited sufficiently cleverly to circumvent these difficulties in quantum electrodynamics and so allow the calculation of very small radiative effects to extremely high precision, in full agreement with experiment. From a fundamental point of view, however, the difficulties remain.

* e.g. non-exhaustive list:

Abraham, M., Ann. Phys. (Leipzig) 315, 105–179 (1902);

Abraham, M., Ann. Phys. (Leipzig) 319, 236–287 (1904);

Dirac, P.A.M., Proc. R. Soc. Lond. Ser.A 167, 148 (1938);

Landau, L.D., Lifshitz, E.M.: Elsevier, Oxford (1975);

Caldirola, P., Rivista del nuovo cimento, (1979), 2, 13;

Jackson, E.A., J. Math. Phys. 25, 1584–1591 (1984);

Spohn, H., Europhys. Lett. 50, 287–292 (2000);

Di Piazza, A., Lett. Math. Phys (2008) 83:305-313;

Dorigo, M. et al., CP1084, Rarefied Gas Dynamics: 26th International Symposium, edited by T. Abe, (2009) AIoP 978-0-7354-0615-5/09/$25.00;

Sokolov, I., J. Exp. & Theor. Physics (2009), Vol. 109, pp. 207-212

...

Why Extreme Fields?

Nonlinear, many body QED

↓

Equivalence to QCD

↓

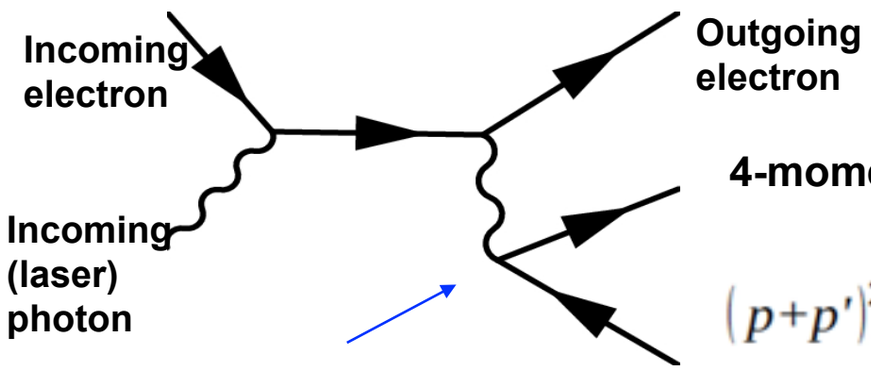
New particles in QED - massless scalars, low energy resonances, axions, …

↓

Quantum Field Theory & Gravity - Unruh effect, …

We are developing an effective Quantum Field Theory for Intense Laser Matter Interactions

Pair creation



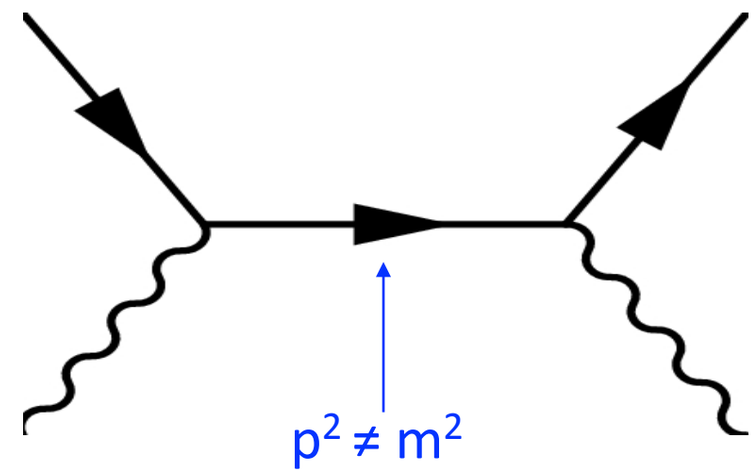
4-momentum of e+ e- pair:

$$(p+p')^2 \simeq 2\, p\cdot p' \simeq 8\, E^2$$

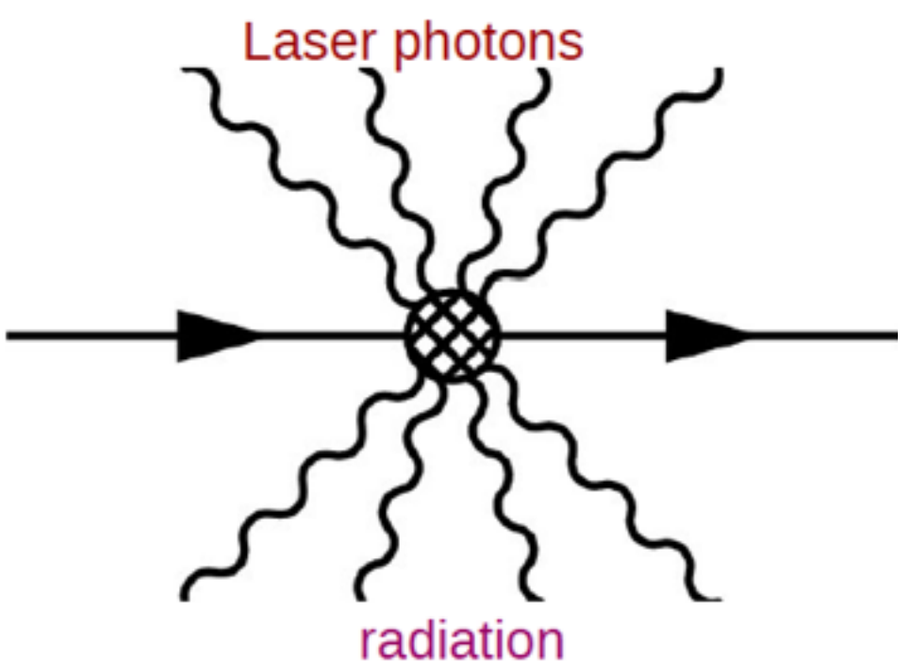
photon does not satisfy its Equation of Motion (momentum conservation)

$$\partial^2 A = 0 \Leftrightarrow k^2 = 0$$

High energy photon emission

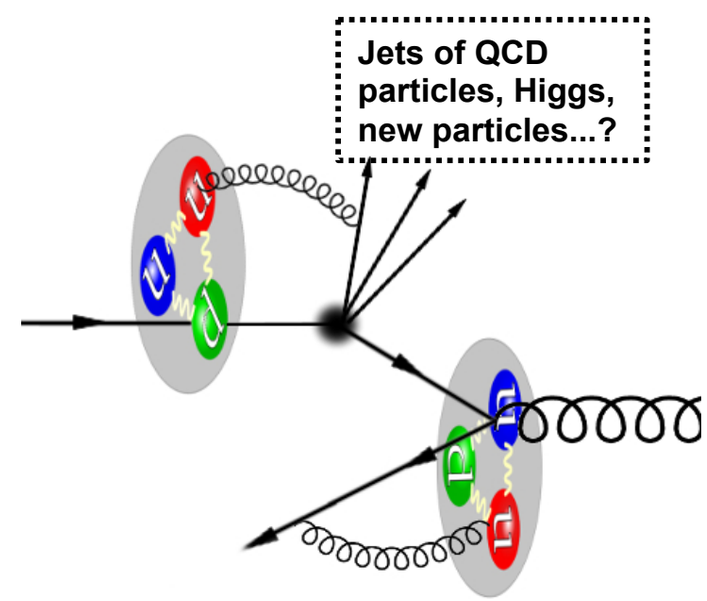


In general it looks more like this...



- many low energy, “non-quantum” photons
- large number of electrons → collective fields & behavior
- high energy quantum phenomena
- interacting with each other

Similar issues in QCD…



Our approach

to building an Analytic Quantum Field Theory for Ultra-intense Laser-Interactions

Start with full theory: $$L_{QED} = \overline{\psi}\left(i\partial_\mu \gamma^\mu - eA_\mu \gamma^\mu - m\right)\psi$$

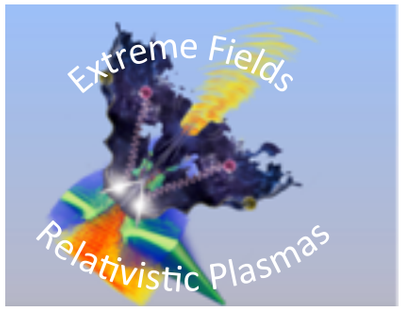
- **Separate photon fields A*μ* and electron fields ψ in Lagrangian into "classical" and "quantum" degrees of freedom**
- **Ask what observables we are interested in (leads to more refined definition of degrees of freedom)**

Example: a_0 ~ 50 laser pulse incident on underdense gas,
Looking for acceleration of electrons to high momentum p~0.1-1 GeV

Calculating electron and photon spectra involves (at least):

- Low frequency (ω ~ 1 eV) laser and "classical" radiation
- Low energy electrons ~eV - keV in initial (ionized) gas
- High momentum electrons in final (accelerated) state
- High frequency (k ~ 1 – 100 MeV) photons of associated radiation

Decouple the low energy dynamics using the widely-separated hierarchy of scales



$\hbar = c = 1$

$\frac{eA_{cl}}{m} = a_0 \gg 1$

QCD analogs

Integrate out "heavy" degrees of freedom as not fluctuating

Momentum

H → p_f Final (accelerated) electron momentum

Quantum corrections to H, f_q → eA_{cl}, p_T

For photon and pair emission, study quantum fluctuations in this momentum range

→ m_e

f_q, S → $a_0\omega$ Simulation resolution

Decouple dynamics at long wavelength – better handled by numerical simulation

→ ω_{laser}

(Like relativistic transparency: high energy particles respond slowly to low energy background)

We calculate e.g. the observable spectrum of positrons

- ✔ Decouple low energy components of classical field
- ✔ High energy processes calculated perturbatively in the small parameters and local on the scale of the inverse momentum

---

= ***factorization of the positron spectrum***

Electron distribution function – classical dynamics of electrons in the plasma (determined from simulations)

$$2p_0 \frac{dN_{\bar{e}}}{d^3p} = f_p \circ J \circ S$$

Final state "jet" function:
perturbative interactions of high energy electrons, positrons and photons,

"Soft" function:
low energy (E<m) and classical radiation

Quantum chromodynamics: similarity to strong field QED

Quark interactions inside the proton described by Quantum ChromoDynamics

1. QCD is a gauge theory: matter-radiation coupling is same as QED

$$L_{QCD} = \overline{\psi}\left(i\gamma\cdot\partial - e\gamma\cdot\hat{A}_a\tau_a - m\right)\psi - \frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu}$$

Quark (matter) gluon (radiation) radiation kinetic term

$$L_{QED} = \overline{\psi}\left(i\gamma\cdot\partial - e\gamma\cdot\hat{A} - e\gamma\cdot A_{cl} - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

electron photon high intensity classical potential

2. QCD is nonperturbative at long wavelength

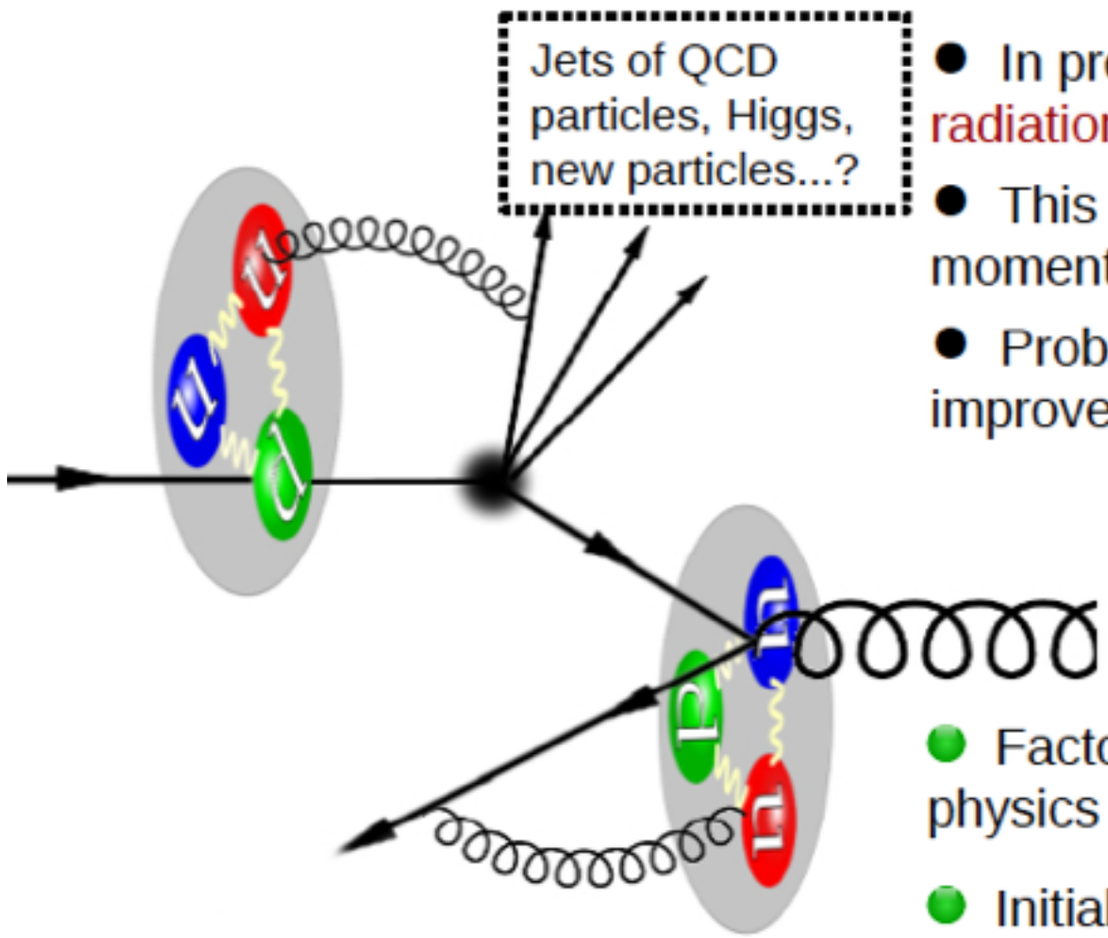
Analogy to strong field QED: High intensity classical field $\frac{eA_{cl}}{m} \gg 1$

means electron coupling to long wavelength radiation and classical potential must be treated nonperturbatively (dominates free-field *m*)

3. Search for new physics (in proton-proton collisions at LHC for example) is a search for high energy scattering/particle production in low-energy nonperturbative background

Understanding the proton and searching for new Physics requires understanding QCD

Discovery requires precision control of QCD radiation and initial state(=distribution of quarks inside proton)



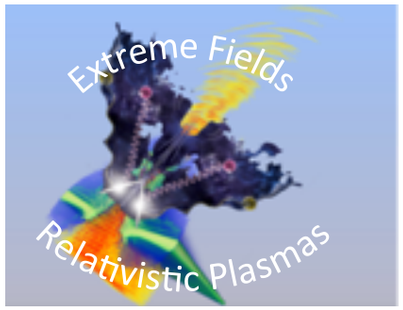
- In proton-proton collisions, low momentum QCD radiation couples to initial and final state.
- This nonperturbative effect decoupled from high momentum collision using QFTs derived from QCD*
- Probes distribution of quarks and systematically improves calculated cross-sections

$$\frac{d\sigma}{d^2 p_T} = f_q \circ f_{\bar{q}} \circ H \circ S$$

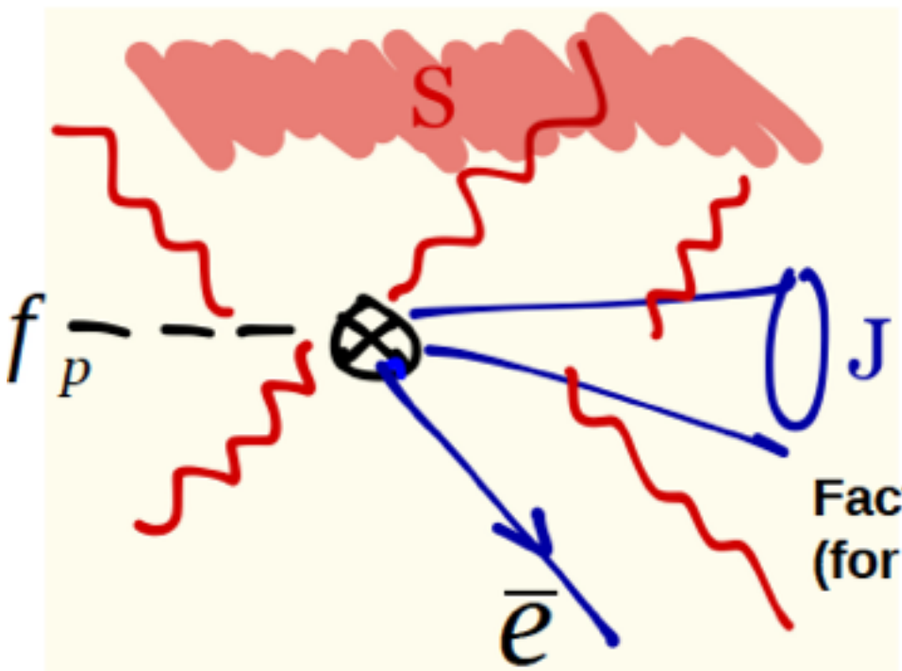
- Factorization of cross-section separates different physics at different length scales
- Initial state (quark distribution) may be studied on lattice or obtained from other experiments

* Example QFT: Soft-Collinear Effective Theory
[C. Bauer, S. Fleming, et al. Phys. Rev. D (2000, 2001, 2002)]

Some problems in QCD require similar formalism as strong field QED



Discovery of high energy (quantum) effects in strong field QED also requires precision control of QED radiation and long-wavelength dynamics



- In laser-plasma interaction, long-wavelength classical electromagnetic radiation affects particle dynamics before and after sought-for high-energy process (pair production or photon emission)
- This nonperturbative effect decoupled from high momentum collision using a QFT derived from QED*

Factorized positron spectrum (for electron acceleration)

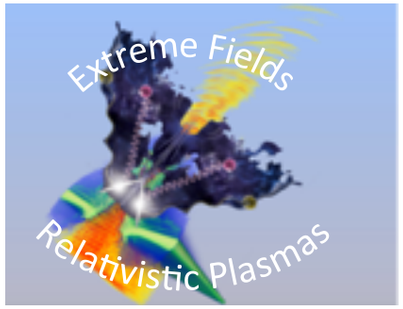
$$2p_0 \frac{dN_{\bar{e}}}{d^3p} = f_p \circ J \circ S$$

- Factorized long-wavelength classical radiation analog to low-energy QCD radiation: QED theory and laser experiments help understand physics content of ***S***

- Developing theory and experiment for strong-field QED supports QCD framework and validates theoretical description in (slightly) easier QED theory

* electron-strong laser effective theory under development
[L. Labun, O. Z. Labun, S. Fleming and B. M. Hegelich, paper in preparation]

Light scalar particles coupling to electromagnetism



Theory suggests several possible light bosonic particles interacting with photons

1. Axion, or axion-like particles (pseudoscalar)

Axion field

Interaction term: $$L = \frac{a}{f_a} F^{\mu\nu} \epsilon_{\mu\nu\kappa\lambda} F^{\kappa\lambda} = \frac{-a}{f_a} \vec{E} \cdot \vec{B}$$

Decay constant

- Axion arises from strong-CP problem, but similar particles theorized from other physics origins

2. Dilaton/Anomalon (scalar)

Interaction term: $$L = \frac{\sigma}{f_\sigma} F^{\mu\nu} F_{\mu\nu}$$

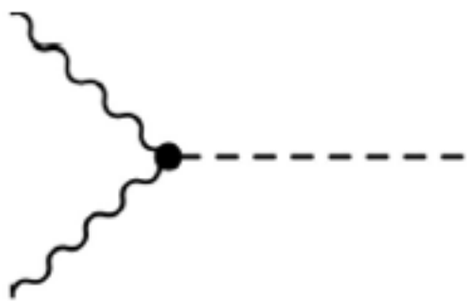
- Arises from anomalous breaking of conformal symmetry in classical electromagnetic fields, may be related also to breaking of chiral symmetry and presence of scalar condensate $\langle \overline{\psi}\psi \rangle$ in external fields

References: D. Blaschke and E. Mottola JHEP (2014),
L. Labun and J. Rafelski PRD (2010).

Laser plasma searches for light scalars

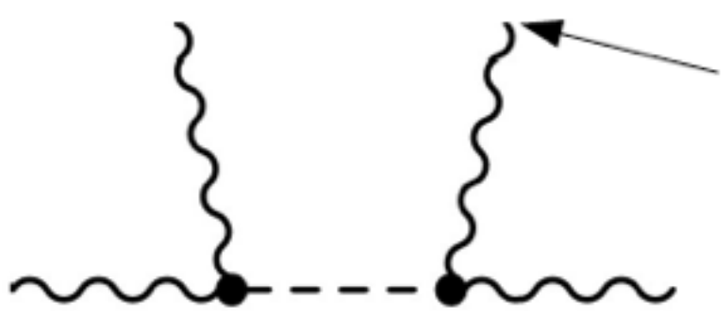
Question: do these particles exist? How can lasers be used to test their theory description?

i. Particle production



High intensity: many photons available to collide (also possible coherence enhancement and probably search for missing energy)

ii. Light-propagation effects



Could be enhanced by external leg being connected to strong background laser or plasma field

Similar to "light-shining through wall" experiments, here shining through plasma

Requires substantial theory work: improving theory description for these particles in strong field/plasma background and controlling standard QED plasma and radiation effects (see previous)

References: M. Ahlers, et al. PRD (2007).
J. Jaeckel and A. Ringwald Ann. Rev. Nucl. Part. Sci (2010).

Laser acceleration and the Equivalence Principle

Recall that General Relativity is constructed on principle that physics is the same in any local Lorentz frame; acceleration is curved coordinates

1. Lasers provide among the largest acceleration gradients available in the lab

- Accelerating particles using lasers may allow tests of the Equivalent Principle: is electromagnetic acceleration equivalent to gravitational acceleration?

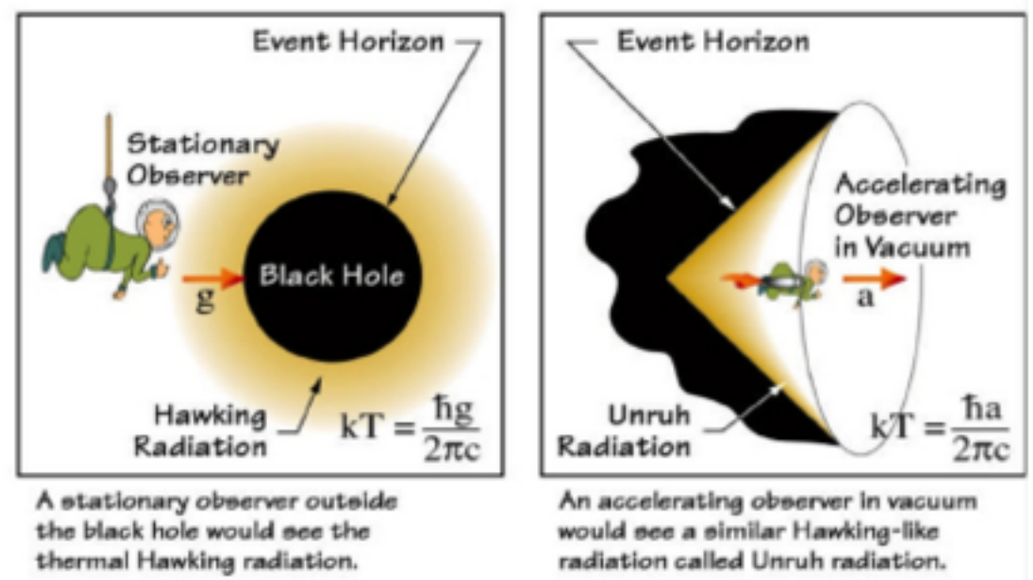


Image: P. Chen [1402.5823]

2. QED in strong fields is a controlled example of quantum field theory in a classical potential

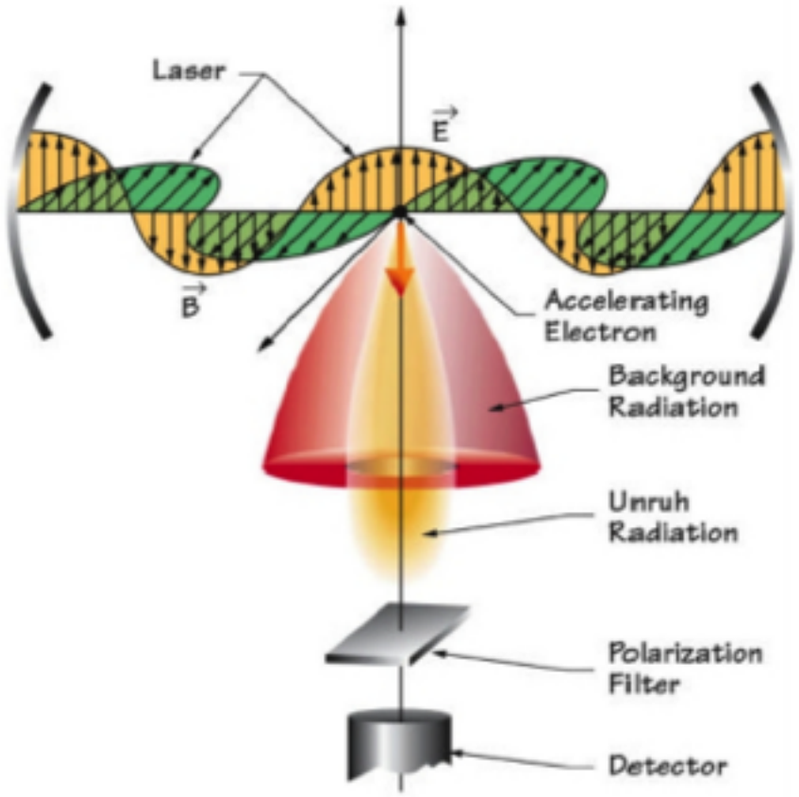
- Same formalism (different theories) describes pair production in strong E.M. fields as describe particle production by black holes and expanding universe

Laser experiments and gravity

1. Search for Unruh-effect impact on accelerated motion

- Experimental proposals to see radiation from accelerated observer (role played by probe electron) in laser experiments

References: P. Chen and T. Tajima PRL (1999)
R. Schutzhold et al. PRL (2008)
D. v. Ahluwalia et al. [1508.03091]



2. Discovery and measurement of pair production and quantum kinetic effects in strong fields validates theoretical framework for quantum field theory in classical potentials

References: P. Anderson and E. Mottola, PRD (2014a, 2014b)

Again requires substantial theory work: improving theory to make more clear and precise predictions of classical strong-acceleration effects and control standard QED plasma and radia effects (see previous)

Conclusions

- Possibly high impact, new physics to be studied with ultrahigh intensity laser generated extreme fields
  - method development and validation for QCD
  - low mass scalars in QED
  - Relation between Quantum Field Theories and General Relativity
- Dedicated theory effort required to figure out if this is really possible and how to exactly go about it.
- but …
- Requires detailed understanding of QED background
  - this can be investigated on Bella-i!
  - will likely require dedicated, long running experiment as standard in high energy physics: axion searches, light-shining-through wall, neutrino mass, Higgs, etc. all run for years to get good enough statistics